Optimized AHP to Overcome Limits in Weight Calculation: Building Performance Application

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Valentino Sangiorgio[1]; Giuseppina Uva[2]; and Fabio Fatiguso[3]

Abstract: Multicriteria decision making methods, in particular the analytic hierarchy process (AHP), are widely used in many sectors, and increasingly in the field of construction. The AHP is easily adaptable to many decision problems, but it presents great difficulties when the number of criteria or alternatives is large. This issue affects the consistency of the judgment matrix, which is related to the acceptability of the results. To overcome such a drawback of basic importance in the construction field, this paper proposes a novel optimized-AHP (O-AHP) method that redefines the judgment assignments and the generation of the judgment matrix by using a mathematical programming formulation. The validation benchmark tests show that the O-AHP exhibits the same effectiveness of the standard AHP, and it can be applied successfully when the number of alternatives is more than nine. Furthermore, the presented technique is applied to the building performance assessment in the field of antiseismic behavior. The results show that the O-AHP is an approach to simplify the decision-maker operations by defining a numerical value of the building pathology criticality index, which is useful for large-scale applications. **DOI: 10.1061/(ASCE)CO.1943-7862.0001418.**

Author keywords: Analytic hierarchy process; Optimization; Building performance; Quantitative methods.

Introduction

The analytic hierarchy process (AHP) provides the objective mathematics to process the inescapably subjective and personal preferences of an individual, the decision maker, or a group in making a decision (Saaty and Vargas 2001). The AHP is based on the decomposition of the problem into independent criteria: Such an operation allows the transformation of a multidimensional scaling problem into a one-dimensional scaling problem. Therefore, every criterion is analyzed individually to identify the related priority vectors (i.e., the weights assigned to each alternative or criterion) (Saaty and Vargas 2001). The AHP uses the principal eigenvalue method for deriving ratio scale priority vectors from positive reciprocal matrices. In particular, such matrices, called *comparison matrices* or *judgment matrices*, are established through pairs of comparisons (Barzilai et al. 1987; Saaty and Hu 1998).

Multicriteria decision making, and in particular the AHP application, to assess the criterion weightings became popular in different areas of construction (Schöttle and Arroyo 2016). In the project stage, Fong and Choi (2000) and Hsieh et al. (2004) used AHP for the contractor-selection decision, and in a similar way, Plebankiewiczm and Kubek (2015) and Kahraman et al. (2003) identified the best material supplier choice. Moreover, Wong and Li (2008) analyzed the selection of the intelligent building systems by identifying key selection criteria using AHP methodology. In addition, different contributions provide adequate solutions by AHP for the systematic evaluation of many factors, which include efficiency, user comfort, safety, reliability, functionality, and maintainability, to characterize the design work and the weighting of soft benefits in comparison with costs (Kwon et al. 2014; Shapira and Goldenberg 2005) and environmental impact (Reza et al. 2011; Chang et al. 2007).

Furthermore, AHP is used to support maintenance and intervention by comparing many measures, such as multiattribute, multivariate qualitative and quantitative data (Das et al. 2010). Such performance assessments are used to evaluate different aspects of construction, such as safety evaluation and management in construction sites (Teo and Ling 2006; Dağdeviren and Yüksel 2008; Li et al. 2013), green building rating (Ali and Al Nsairat 2009; Chang et al. 2007), energetic rehabilitation (Gigliarelli et al. 2011; Wang et al. 2012), construction management (Das et al. 2009; Wu et al. 2007; Kettner and Diaz 2000), large-scale structural vulnerability analysis (Panahi et al. 2014; Aghataher et al. 2008), and criticalities identification at the seismic and volcanic vulnerability assessment (Faggiano et al. 2011; Formisano and Mazzolani 2015).

In the field of construction, it is often necessary to compare a large number of criteria, and the trend is to increase the involved criteria to perform complex multi attribute analysis. Some authors (Arroyo et al. 2014) have deeply examined the limits and incongruences of the AHP methodology for specific decisions in construction design, such as the trade-off among factors, a problem that is not addressed in this paper. In addition, other applicability limits are derived from the consistency requirement of the judgment matrices, which are necessary for the reliability of the assigned judgments and the relative obtained weights (Saaty 2003, 1990; Saaty and Vargas 1984). Moreover, such a consistency depends on the pairwise comparisons performed by the decision maker, and as some authors discovered (Li et al. 2013; Ohnishi et al. 2007), the results of AHP often lose their reliability because the judgment matrix does not always satisfy the consistency requirement. Mathematical and psychology studies determined that the human mind is limited to 7 ± 2 alternatives in simultaneous comparison

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(Saaty 1977; Miller 1994), and the reliability of the results decreases as long as the number of criteria of comparison becomes large (more than nine criteria). To this aim, it is particularly important to study methodologies that are able to improve the consistency of judgments (Li et al. 2013) by transforming an inconsistent matrix into a consistent one (Saaty 2003; Saaty and Vargas 2001). Hence, the academic community is interested in improving the AHP applicability to the building performance evaluation by exploiting the AHP ability in simplifying complex and unstructured multicriteria decision analysis. Indeed, in building applications, the number of criteria often overcomes the maximum number that allows one to obtain coherent results.

To make AHP more suitable for construction-field applications, the main objective of this paper is to investigate a methodology that is able to overcome the limits of the standard AHP concerning large system consistency requirements. To this aim, the AHP weights evaluation is revised by proposing a new procedure, called optimized AHP (O-AHP), to determine the judgment matrix. In particular, the limits of the human mind for simultaneous comparison are faced with defining a set of inequalities that substitute the judgment scale proposed by Saaty. In such a way, the decision maker can use just a range of judgments and focus on the main judgments instead of carrying out all of the pairwise comparisons. This approach can be useful in complex problems, when the decision maker does not have a full system knowledge. In addition, a mathematical programming (MP) problem is formalized to deal with the inconsistency of the judgment matrix by minimizing the inconsistency performance index, subject to the inequalities imposed on the judgments. To validate the proposed methodology, a benchmark test proposed by Whitaker (2007) is used: The validation shows that good results are obtained in the case of few alternatives of comparison, and in the case of more than nine alternatives.

Finally, the proposed methodology is developed in the field of vulnerability assessment (Uva et al. 2016; Porco et al. 2015), to establish buildings' information systems (Lombillo et al. 2016) and decision support systems (Sangiorgio et al. 2016a, b). Hence, this paper considers an application to the building performance assessment in the area of antiseismic behavior, and shows how the presented approach can be applied usefully in the construction context.

A	1	2	…	n
1	1	$a_{1,2}$	…	$a_{1,n}$
2	$1/a_{1,2}$	1	…	$a_{2,n}$
…	…	…	1	…
n	$1/a_{n,1}$	$1/a_{n,2}$	…	1

Fig. 1. Generic judgment matrix A

Standard AHP Methodology

The proposed O-AHP methodology follows the well-known standard AHP four steps (Saaty 1980) that are specified in the following section: (1) problem structuring, (2) weights evaluation, (3) summary of priorities, and (4) sensitivity analysis. The weights evaluation is described in detail, as it concerns the mathematical formulations that are used in the proposed method. Furthermore, the consistency requirement issues of the judgment matrix are discussed.

Four Steps of Standard AHP

Beginning with a decision problem, the first step consists of structuring the problem according to a hierarchical scheme, to provide a detailed, simple, systematic, and structured decomposition of the general problem into its basic components. To this aim, the goal of the AHP is identified and the related criteria, subcriteria, and alternatives to reach the goal are determined.

The second step of weight evaluation is the core of the method, and provides the weights that are necessary for generating the ranking. More precisely, it is possible to individually analyze each aspect of the decision problem. Considering n ordered criteria of comparison (i.e., criteria, subcriteria, or alternatives in relation to the criteria or subcriteria), a $n \times n$ *judgment matrix* A is defined, in which each upper diagonal element $a_{ij} > 0$ is generated by comparing the ith element with the jth one through the fundamental scale of absolute numbers (Fig. 1). This semantic scale consists of verbal scales that are associated with numerical values (1, 3, 5, 7, 9) and compromises (2, 4, 6, 8) between such values (Table 1) (Saaty 2008).

The inferior triangular part of matrix A is completed with the reciprocal values of the upper triangular part, by obtaining reciprocal matrix elements, as follows: $a_{ji} = 1/a_{ij}$ or $a_{ji} \cdot a_{ij} = 1$. Moreover, if $a_{ij} \cdot a_{jk} = a_{ik}$, then matrix A is said to be *perfect consistent* and its principal eigenvalue is $\lambda_{\max} = n$.

In the standard AHP, the weights are obtained by solving the following eigenvector problem:

$$\mathrm{A}\mathrm{w} = \lambda_{\max}\mathrm{w} \tag{1}$$

where w = priority eigenvector associated with the principal eigenvalue $\lambda_{\max}$. If slight inconsistencies are introduced, then it holds $\lambda_{\max} \neq n$ (Saaty 2003).

In the AHP methodology, a judgment matrix A is obtained for each set of criteria, subcriteria, and alternatives that are considered in each criterion and subcriterion. Operatively, approximate formulation methods are used to calculate the weights from the judgments matrix.

Table 1. Fundamental Scale of Absolute Numbers of Saaty

Number value a_{ij}	Verbal scale	Explanation
$a_{ij} = 1$	Equal importance	Two activities contribute equally
$a_{ij} = 3$	Moderate importance of one over another	Experience and judgment slightly favor one activity over another
$a_{ij} = 5$	Strong importance	Experience and judgment strongly favor one activity over another
$a_{ij} = 7$	Very strong importance	An activity is favored very strongly over another
$a_{ij} = 9$	Extreme importance	An activity is favored by at least an order of magnitude
1.5–4–6–8	Intermediate value	A compromise between two judgments
$1/9, 1/8, \ldots, 1/2$	The reciprocal number expresses an opposite judgment	Experience and judgment "unfavor" one activity over another

First, the elements of matrix A are normalized as follows:

$$x_{ij} = \frac{a_{ij}}{\sum_i a_{ij}} \tag{2}$$

Second, the weight w_i are calculated as the average of the elements of the rows of the normalized matrix as follows:

$$w_i = \sum_{j=1}^{n} x_{ij}/n \tag{3}$$

At this point, it is necessary to evaluate the reliability of the obtained weights by measuring the inconsistency of matrix A: The inconsistency increases if the judgments are badly posed. In the approximate method, the principal eigenvalue is approximately evaluated as follows (Ishizaka and Lusti 2006):

$$\lambda_{\max} = \frac{1}{n}\sum_{i=1}^{n}\frac{a_{ij}\times w_j}{w_i} \tag{4}$$

Now, the consistency index (CI) is defined according to Saaty (1980), and it increases proportionally with the inconsistency of the matrix as follows:

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{5}$$

To provide a measure of the inconsistence that is independent of the matrix order, Saaty (1980) proposed the consistency ratio (CR). This is obtained by considering the ratio between CI and its expected value [random consistency index (RI)] determined over a large number of positive reciprocal matrices of order n, whose entries are randomly chosen in the set of values $n \in \{1, 2, \ldots, 10\}$, as follows:

$$CR = CI/RI(n) \tag{6}$$

In this paper, to face a large number of alternatives, it is necessary to consider values of RI(n) for $1 \leq n \leq 15$. Hence, among the different values of RI proposed in the related literature (Alonso and Lamata 2006), the values of Noble and Sanchez (1993) are used (Table 2).

On the basis of several empirical studies, Saaty (1980) concluded that the value of $CR < 0.10$ is acceptable. Such a test about CR is crucial for establishing the reliability of assigned judgments, and it is the parameter that mathematically determines the incoherence of the decision-maker judgments.

Table 2. Random Consistency Index (Data from Noble and Sanchez 1993)

n	RI
1	0
2	0
3	0.49
4	0.82
5	1.03
6	1.16
7	1.25
8	1.31
9	1.36
10	1.39
11	1.42
12	1.44
13	1.46
14	1.48
15	1.49

In the next step, the summary of priority is performed to determine the rankings and the global weights for each alternative. To this aim, the weights of each criterion and of each subcriterion are combined with the weights of the alternatives (weights aggregation).

Finally, the procedure is verified by conducting a sensitivity analysis of the results, to evaluate the stability of the solution with respect to possible excursions of the values associated with the judgments. The study of the methods to modify the input data, to observe the effect on the results, is an important research topic of the related literature (Butler et al. 1997; Triantaphyllou and Sánchez 1997).

Consistency Requirement Issues

As already mentioned, the consistency of matrix A is required for the reliability of the assigned judgments and the relative obtained weights. It is proved that the reliability of the method is measured by the values of CR that for $n > 4$ it is $CR < 0.10$ (Saaty 2003, 1990; Saaty and Vargas 1984). Consistency ratio numerically represents the consistency of matrix A. Moreover, such consistency depends on the pairwise comparisons performed by the decision maker. As some authors remark (Li et al. 2013; Ohnishi et al. 2007), the results of AHP often lose their reliability because the matrix does not always satisfy the consistency requirement. The decision maker could improve the consistency by applying a time-consuming trial and error method and losing the focus on judgments.

Optimization Procedure for Weight Evaluation

To overcome the consistency issue of AHP for problems with a large number of criteria or alternatives, an optimized AHP methodology for generating the judgment matrix A is proposed. In particular, the presented method focuses on the second step of AHP (i.e., the optimized weights evaluation). The methodology can be described by considering three main phases: (1) rough ranking evaluation of the alternatives; (2) determination of a set of judgment ranges; and (3) MP formulation of the O-AHP.

Rough Ranking Evaluation of the Alternatives

In some cases, it is useful to preliminarily assume an approximate ranking of the criteria of comparison. To this aim, such criteria can be positioned starting from the first rows and columns of the judgment matrix A in descending order of importance: This optional operation can simplify the subsequent steps.

Estimation of the Judgment Ranges

To consider a large number of alternatives, the decision maker can express the judgments not by single values but through the determination of ranges of values, named in this paper as *judgment ranges*.

In Phase 2, the judgment-range setting is formalized through inequalities, by proposing two new semantic ranges of the O-AHP, inspired by the fundamental scale of absolute numbers of Saaty: the O-AHP semantic ranges of the lower bounds (Table 3) and the O-AHP semantic ranges of the upper bounds (Table 4). Therefore, unlike the Saaty method, in which the decision maker performs the judgment by a crisp number, in the O-AHP method the decision

Table 3. Optimized-AHP Semantic Ranges of the Lower Bounds

Lower bound constraint	Verbal constraint	Explanation
$a_{ij} \geq 1$	Equal or more importance of i over j	A criterion is at least equally favored
$a_{ij} \geq 2$	More importance, even slightly, of i over j	A criterion is at least slightly favored
$a_{ij} \geq 3$	At least moderate importance of i over j	A criterion is at least favored
$a_{ij} \geq 5$	At least strong importance of i over j	A criterion is at least strongly favored
$a_{ij} \geq 7$	At least very strong importance of i over j	A criterion is at least very strongly dominant
$a_{ij} \geq 9$	Maximum importance of i over j	A criterion is at least favored by at least an order of magnitude
1.5–4–6–8	Intermediate value	A compromise between two judgments
$a_{ij} \leq (1/9, 1/8, \ldots, 1)$	The reciprocal number to the "≤" sign expresses an opposite judgment (becomes upper bound)	Experience and judgment certainly "unfavor" one criterion over another

Table 4. Optimized-AHP Semantic Ranges of the Upper Bounds

Upper-bound constraint	Verbal constraint	Explanation
$a_{ij} \leq 2$	The importance of i over j does not exceed the "minimum importance"	The importance of i over j is less than slightly favored
$a_{ij} \leq 3$	The importance of i over j does not exceed the "moderate importance"	The importance of i over j is less than favored
$a_{ij} \leq 5$	The importance of i over j does not exceed the "strong importance"	The importance of i over j is less than strongly favored
$a_{ij} \leq 7$	The importance of i over j does not exceed the "very strong importance"	The importance of i over j is less than very strongly dominant
$a_{ij} < 9$	The importance of i over j does not exceed the maximum importance	The importance of i over j is less favored by an order of magnitude
1.5–4–6–8	Intermediate value	Compromise between two judgments
$aij \geq (1/9, 1/8, \ldots, 1/2)$	The reciprocal number to the "≥" sign expresses an opposite judgment (becomes lower bound)	Experience and judgment certainly "unfavor" one criterion over another

maker expresses a judgment range (i.e., the range in which the judgment is assumed to belong).

Mathematical Formulation

An MP problem is formulated to determine the elements of the judgment matrix A on the basis of the inequalities assigned at Phase 2. To this aim, the sets of pairs of subcriteria that are subject to the judgment range inequalities are defined as follows:

$$C^L = \{(i,j)|a_{ij} > K_{ij}^L \text{ with } i < j\}$$

set of pairs of sub-criteria involved in lower bound inequalities

$$C^U = \{(i,j)|a_{ij} \leq K_{ij}^U \, with \, i < j\}$$

set of pairs of sub-criteria involved in upper bound inequalities

where K_{ij}^L and K_{ij}^U = values assigned by the decision maker to the lower and upper-bound inequalities, respectively, as detailed in Tables 3 and 4.

Hence, it is possible to specify the entries of **A** by the following set $\Gamma(A)$ of mathematical constraints:

$$\Gamma(A):$$

$$a_{ij} = 1 \quad \text{for } i,j = 1, \ldots, n \quad \text{with i} = j \tag{7a}$$

$$1/9 < a_{ij} < 9 \quad \text{for } i,j = 1, \ldots, n \quad \text{with } i < j \tag{7b}$$

$$a_{ij} \leq K_{ij}^U \quad \text{for } (i,j) \in C^U \tag{7c}$$

$$a_{ij} > K_{ij}^L \quad \text{for } (i,j) \in C^L \tag{7d}$$

$$a_{ij} = \frac{1}{a_{ji}} \quad \text{for } i,j = 1, \ldots, n \quad \text{with } i > j \tag{7e}$$

In particular, constraints (7a)–(7e) are derived from the matrix A definition and constraints (7c) and (7d) are determined by the decision maker.

Next, to calculate a judgment matrix that exhibits a low CI, the following MP problem is formulated:

$$\min CI(A) \tag{8a}$$

$$\text{subject to } \Gamma(A) \tag{8b}$$

The value of CI is calculated according to Eq. (5) and the approximated formula (4).

The MP problem solution can be found by a standard optimization solver. In this paper, the MP problem is solved in the *MATLAB* environment on a personal computer equipped by a 2.6-GHz Intel Core i5 with 8 GB of memory, in the single-thread mode by using the optimization toolbox of *MATLAB*.

The optimized judgment matrix A^{opt} of generic element a_{ij}^{Opt} is provided by solving the MP problem Eqs. (8a) and (8b), and consequently the optimized weights w^{opt} are calculated.

The methodological simplifications of the proposed O-AHP are the following:

1. The judgments necessary to obtain the entries of **A** are not determined by single values of comparison, but they are expressed by a set of ranges; and
2. The consistency of matrix A is not obtained through a trial and error method as in the standard AHP, but it is optimized by the solution of the MP problem, improving the coherence of the judgments.

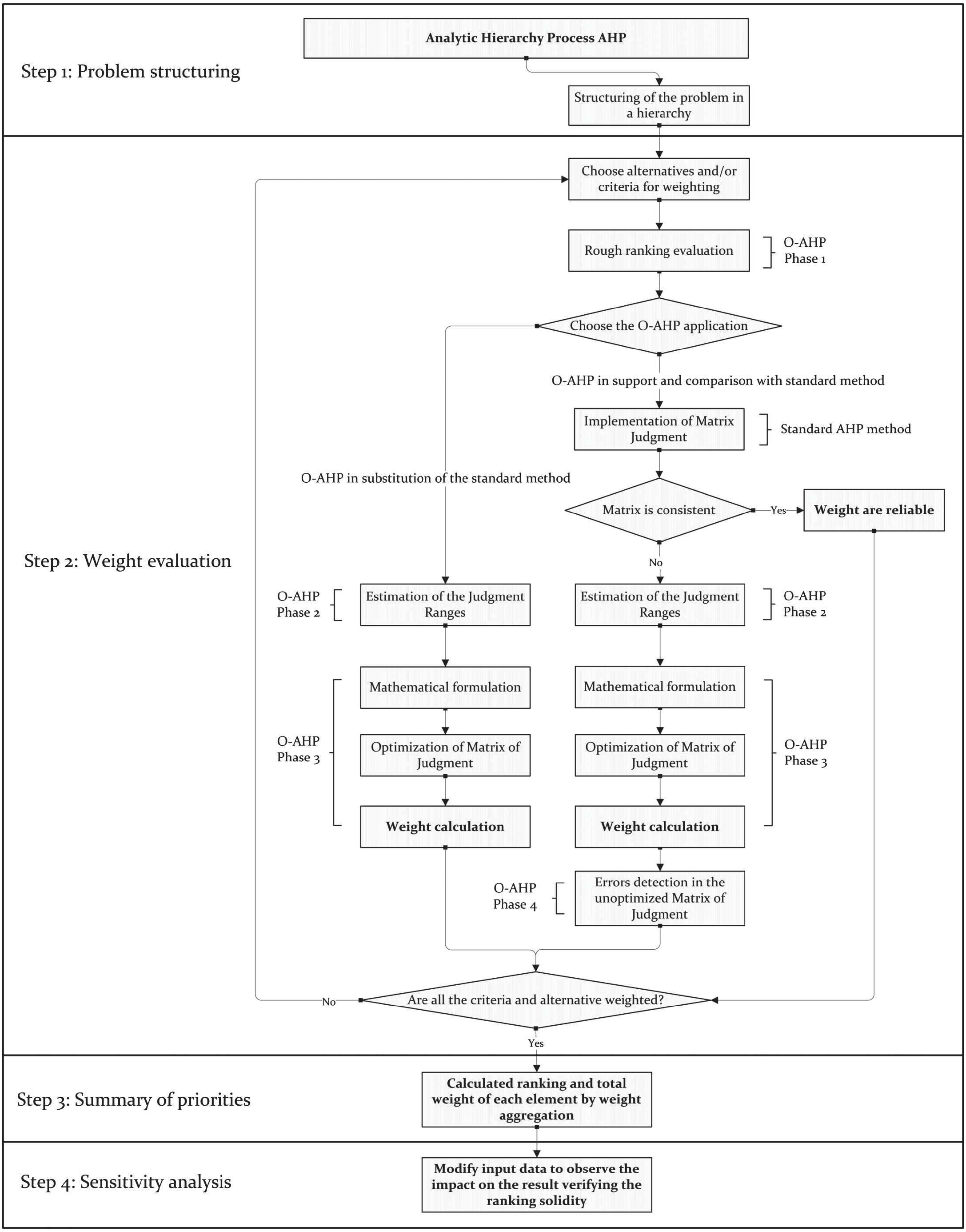

Fig. 2. Optimized-AHP new methodology, schematized for the two different applications

Optimized-AHP Methodology

In this section, the authors describe how the O-AHP methodology can be used for two purposes: (1) improving the classical method to obtain more consistent matrices of comparison; and (2) solving large system problems that are very difficult to be managed by the classical method. In the flowchart of Fig. 2, the new methodology is schematized for the two different applications. In particular, four steps are described: (1) problem structuring; (2) weights calculation, by O-AHP method; (3) summary of priority; and (4) sensitivity analysis.

Step 1 consists of structuring the problem in a hierarchy of criteria, subcriteria, and alternatives to provide a detailed, simple, systematic, and structured decomposition of the general problem into its basic components.

Step 2 regards the weight calculation of all criteria and alternatives through the proposed innovative process. The operations involving the new procedure in Step 2 are divided into four phases.

The first O-AHP phase is the row-ranking evaluation.

In the second phase, the O-AHP can be applied to two different situations: O-AHP in substitution of the standard method, or the O-AHP in support and comparison with standard AHP.

If the choice is the "O-AHP in substitution of the standard method," the decision maker proceeds with the O-AHP Phase 2, in which the judgment ranges are determined.

In the O-AHP Phase 3, the MP problem is formulated and solved: matrix A^{opt} and the weights w^{opt} are provided by the MP problem solution.

If the choice is the "O-AHP in support of standard method," then the decision maker should perform the standard process for obtaining matrix A. Hence, if matrix A satisfies the consistency ratio (i.e., CR < 0.10), then Step 2 goes to an end. If the matrix is not consistent, then Phases 2 and 3 of the O-AHP are applied and the MP problem is formulated and solved.

However, in this case, an additional Phase 4 has to be performed by the decision maker that executes a comparison between the matrices A and A^{opt} to detect errors or inaccuracies. To easily detect the incoherent judgments of matrix A, the additional difference matrix E is computed as follows:

$$e_{ij} = \begin{cases} |a_{ij} - a_{ij}^{Opt}| \text{ if } a_{ij} > 1 \quad \text{or} \quad a_{ij}^{Opt} > 1 \\ |1/a_{ij} - 1/a_{ij}^{Opt}| \text{ if } a_{ij} > 1 \quad \text{and} \quad a_{ij}^{Opt} > 1 \end{cases} \tag{9}$$

The definition of the additional matrix E follows from the necessity of determining the pairwise comparisons of A that have primarily determined the inconsistency. To this aim, if $a_{ij} > 1$ or $a_{ij}^{Opt} > 1$, then the element of E is given by the absolute value of the difference between a_{ij} and a_{ij}^{Opt}. On the contrary, if the jth criterion is more important than the ith one in the two matrices, then $a_{ij} < 1$ and $a_{ij}^{Opt} < 1$. In this case, it holds $|a_{ij} – a_{ij}^{Opt}| < 1$, even if the

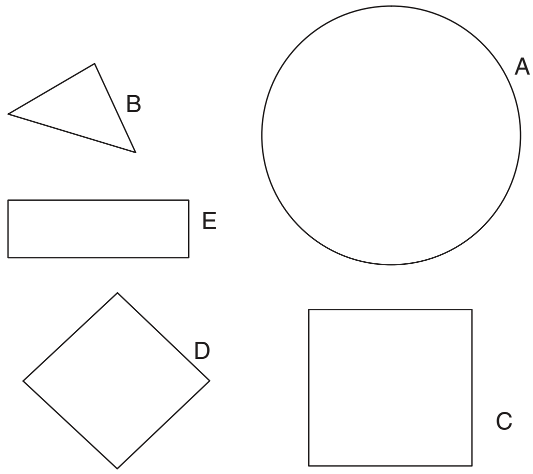

Fig. 3. Comparison of five areas

Table 5. Judgment Ranges for the Validation Test

ith element	K_{ij}^{L}	a_{ij}	K_{ij}^{U}	jth element
A	9.0 ≤	$a_{1,2}$	≤9.0	B
A	2.5 ≤	$a_{1,3}$	≤3.0	C
A	2.0 ≤	$a_{1,4}$	≤6.0	D
A	4.0 ≤	$a_{1,5}$	≤8.0	E
B	1/6 ≤	$a_{2,3}$	≤1/5	C
B	1/4 ≤	$a_{2,4}$	≤1/3	D
B	1/2 ≤	$a_{2,5}$	≤1/1.5	E
C	1.6 ≤	$a_{3,4}$	≤1.8	D
C	1.0 ≤	$a_{3,5}$	≤3.0	E
D	1.0 ≤	$a_{4,5}$	≤2.0	E

Table 6. Validation Benchmark Test Matrix A_{test}^{Opt} and Result

A_{test}^{Opt}	A	B	C	D	E	w_{test}^{Opt}	rv	$Pe(w_{test}^{Opt})$	w_{test}	$Pe(w_{test})$
A	1.0	9.0	2.5	3.4	6.0	0.498	0.47	4.8%	0.488	4.7%
B	0.1	1.0	0.2	0.3	0.6	0.049	0.05		0.049	
C	0.4	5.0	1.0	1.6	2.7	0.225	0.24		0.233	
D	0.3	3.0	0.6	1.0	1.7	0.144	0.14		0.148	
E	0.2	1.7	0.4	0.6	1.0	0.083	0.09		0.082	

Note: CI = 0.0019; CR = 0.0018; RI = 1.03.

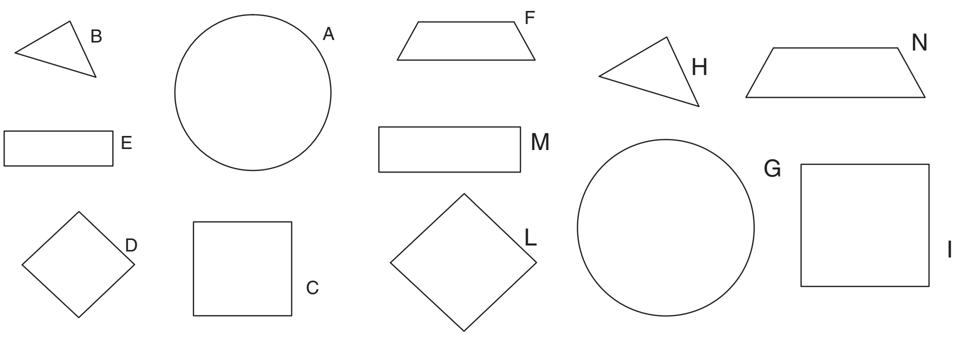

Fig. 4. Comparison of 12 areas

relative difference between the two judgments is large. Hence, the corresponding element is defined as $e_{ij} = |1/a_{ij} - 1/a_{ij}^{Opt}|$.

Step 2 is repeated until all of the criteria, subcriteria, and alternatives are weighted.

Finally, the decision maker performs the last two steps according to the classical method: In Step 3 the final rankings are calculated by the sum of the priorities, and in Step 4 the sensitivity analysis is performed to verify the robustness of the application.

Validation Benchmark Test

To validate Step 2 of the O-AHP, a benchmark validation test is used, based on the methodology proposed by Saaty (2005), which is applied by Whitaker (2007) to show that the results predicted by the theory give correct answers by matching known results. The considered validation test is chosen for its relative simplicity and importance among the many that are proposed in the related literature. More precisely, the five geometric shapes shown in Fig. 3 are considered and a set of individuals is asked to estimate the relative areas. In Whitaker (2007), the actual relative values of these areas are approximately evaluated as $A = 0.47$, $B = 0.05$, $C = 0.24$, $D = 0.14$, and $E = 0.09$, where such values are normalized with respect to the overall area of the five geometric shapes.

Moreover, the text is replicated by using the O-AHP. The mathematical constraints of Eq. (8*b*) are based on the judgment ranges that are evaluated by one of the decision makers, and are provided in Table 5. The optimized judgment matrix A_{test}^{Opt} is obtained by the MP Eqs. (8*a*) and (8*b*) solution, and the values of w_{test}^{Opt} are determined according to Eq. (3).

Next, to compare the solutions obtained by the standard AHP, the O-AHP and the real values, the following performance index Pe is defined as

$$Pe(w) = \sum_i \frac{|w_i - rv_i|}{rv_i} \frac{1}{n} 100\%$$

where w_i = ith component of the weight vector obtained by the standard AHP (denoted by w_{test}) or the O-AHP (denoted by w_{test}^{Opt}); rv_i = elements of vector rv associated with the real values of the geometric areas; and n = dimension of the judgment matrix.

Table 6 lists, on the left side, the judgment matrix obtained by the MP problem solution, the values of w_{test}^{Opt}, and the real values of the geometric shape areas. In addition, the right side lists the values obtained by the standard AHP provided by Whitaker (2007). Moreover, Table 6 reports the obtained indices $Pe(w_{test}^{Opt})$ and $Pe(w_{test})$: The outcomes show that the two methodologies give the same results, both very close to the real values. The consistency of the optimized A_{test}^{Opt} is satisfied as CR $= 0.0013 < 0.1$.

Moreover, the results demonstrate that all of the individuals provide a $Pe(w_{test}^{Opt})$ less than 5%. Hence, it is possible to conclude that the results obtained by the O-AHP can give accurate results as the standard AHP.

Because the most significant advantage of the proposed methodology is providing reliable results also with a large number of comparisons, an additional validation test is performed considering 12 geometric shapes. Fig. 4 shows the different shapes that are chosen to repeat the test with the same set of individuals that in this case determine 66 judgment ranges. To simplify the judgment assignment, in this complex case it is useful to preliminarily perform a rouge ranking of the shapes as listed in Table 7. The judgment ranges used for the formalization of the optimization are reported in Table 7. Moreover, the left part of Table 8 lists

Table 7. Judgment Ranges for the Validation Test 2

ith element	K_{ij}^L	a_{ij}	K_{ij}^U	jth element
G	1.2 ≤	$a_{1,2}$	≤1.5	A
G	1.0 ≤	$a_{1,3}$	≤2.0	I
G	2.0 ≤	$a_{1,4}$	≤3.0	L
G	2.0 ≤	$a_{1,5}$	≤3.0	C
G	3.0 ≤	$a_{1,6}$	≤4.0	N
G	3.0 ≤	$a_{1,7}$	≤4.0	M
G	4.0 ≤	$a_{1,8}$	≤4.0	D
G	5.0 ≤	$a_{1,9}$	≤7.0	F
G	6.0 ≤	$a_{1,10}$	≤8.0	E
G	7.0 ≤	$a_{1,11}$	≤8.0	H
G	9.0 ≤	$a_{1,12}$	≤9.0	B
A	1.0 ≤	$a_{2,3}$	≤1.0	I
A	1.5 ≤	$a_{2,4}$	≤3.0	L
A	2.0 ≤	$a_{2,5}$	≤2.0	C
A	2.0 ≤	$a_{2,6}$	≤3.0	N
A	3.0 ≤	$a_{2,7}$	≤4.0	M
A	3.0 ≤	$a_{2,8}$	≤5.0	D
A	2.0 ≤	$a_{2,9}$	≤4.0	F
A	5.0 ≤	$a_{2,10}$	≤6.0	E
A	6.0 ≤	$a_{2,11}$	≤7.0	H
A	8.0 ≤	$a_{2,12}$	≤9.0	B
I	1.5 ≤	$a_{3,4}$	≤2.0	L
I	2.0 ≤	$a_{3,5}$	≤2.0	C
I	2.0 ≤	$a_{3,6}$	≤3.0	N
I	2.0 ≤	$a_{3,7}$	≤4.0	M
I	3.0 ≤	$a_{3,8}$	≤4.0	D
I	4.0 ≤	$a_{3,9}$	≤5.0	F
I	4.0 ≤	$a_{3,10}$	≤6.0	E
I	6.0 ≤	$a_{3,11}$	≤8.0	H
I	7.0 ≤	$a_{3,12}$	≤8.0	B
L	1.0 ≤	$a_{4,5}$	≤1.5	C
L	1.0 ≤	$a_{4,6}$	≤1.5	N
L	1.5 ≤	$a_{4,7}$	≤1.5	M
L	1.5 ≤	$a_{4,8}$	≤2.0	D
L	2.0 ≤	$a_{4,9}$	≤4.0	F
L	2.0 ≤	$a_{4,10}$	≤3.0	E
L	3.0 ≤	$a_{4,11}$	≤5.0	H
L	4.0 ≤	$a_{4,12}$	≤6.0	B
C	1.0 ≤	$a_{5,6}$	≤1.5	N
C	1.0 ≤	$a_{5,7}$	≤1.5	M
C	1.5 ≤	$a_{5,8}$	≤2.0	D
C	2.0 ≤	$a_{5,9}$	≤3.0	F
C	2.0 ≤	$a_{5,10}$	≤3.0	E
C	3.0 ≤	$a_{5,11}$	≤4.0	H
N	4.0 ≤	$a_{5,12}$	≤6.0	B
N	1.2 ≤	$a_{6,7}$	≤1.5	M
N	1.0 ≤	$a_{6,8}$	≤1.5	D
N	1.5 ≤	$a_{6,9}$	≤2.0	F
N	2.0 ≤	$a_{6,10}$	≤2.0	E
N	2.5 ≤	$a_{6,11}$	≤3.0	H
N	3.0 ≤	$a_{6,12}$	≤4.0	B
M	1.0 ≤	$a_{7,8}$	≤1.5	D
M	1.0 ≤	$a_{7,9}$	≤2.0	F
M	1.5 ≤	$a_{7,10}$	≤2.0	E
M	1.5 ≤	$a_{7,11}$	≤3.0	H
M	1.0 ≤	$a_{7,12}$	≤3.0	B
D	1.2 ≤	$a_{8,9}$	≤1.5	F
D	1.5 ≤	$a_{8,10}$	≤2.0	E
D	2.0 ≤	$a_{8,11}$	≤2.0	H
D	2.0 ≤	$a_{8,12}$	≤3.0	B
F	1.0 ≤	$a_{9,10}$	≤1.5	E
F	1.5 ≤	$a_{9,11}$	≤2.0	H
F	1.5 ≤	$a_{9,12}$	≤2.0	B
E	1.0 ≤	$a_{10,11}$	≤1.5	H
E	1.5 ≤	$a_{10,12}$	≤2.0	B
H	1.0 ≤	$a_{11,12}$	≤1.5	B

Table 8. Validation Benchmark Test 2 Matrix $A^{Opt}_{test,2}$ and Result

$A^{Opt}_{test,2}$	G	A	I	L	C	N	M	D	F	E	H	B	$w^{Opt}_{test,2}$	rv	$Pe(w^{Opt}_{test,2})$
G	1.0	1.3	1.2	2.3	2.5	3.2	3.6	4.0	5.7	6.8	7.7	9.0	0.206	0.208	2.40%
A	0.8	1.0	1.0	1.9	2.0	2.5	3.1	3.3	3.9	5.4	6.6	8.5	0.167	0.167	—
I	0.8	1.0	1.0	1.8	2.0	2.5	2.9	3.4	4.4	5.1	6.9	7.6	0.165	0.160	—
L	0.4	0.5	0.5	1.0	1.1	1.3	1.5	1.8	2.3	2.8	3.6	4.6	0.089	0.089	—
C	0.4	0.5	0.5	0.9	1.0	1.3	1.4	1.7	2.3	2.6	3.4	4.3	0.084	0.082	—
N	0.3	0.4	0.4	0.8	0.8	1.0	1.2	1.3	1.8	2.0	2.7	3.4	0.067	0.066	—
M	0.3	0.3	0.3	0.7	0.7	0.8	1.0	1.1	1.5	1.8	2.3	2.7	0.056	0.056	—
D	0.3	0.3	0.3	0.6	0.6	0.8	0.9	1.0	1.3	1.6	2.0	2.5	0.050	0.053	—
F	0.2	0.3	0.2	0.4	0.4	0.6	0.7	0.7	1.0	1.2	1.6	1.8	0.038	0.039	—
E	0.1	0.2	0.2	0.4	0.4	0.5	0.6	0.6	0.8	1.0	1.3	1.6	0.032	0.033	—
H	0.1	0.2	0.1	0.3	0.3	0.4	0.4	0.5	0.6	0.8	1.0	1.2	0.025	0.026	—
B	0.1	0.1	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.8	1.0	0.020	0.021	—

Note: CI = 0.0006; CR = 0.0004; RI = 1.44.

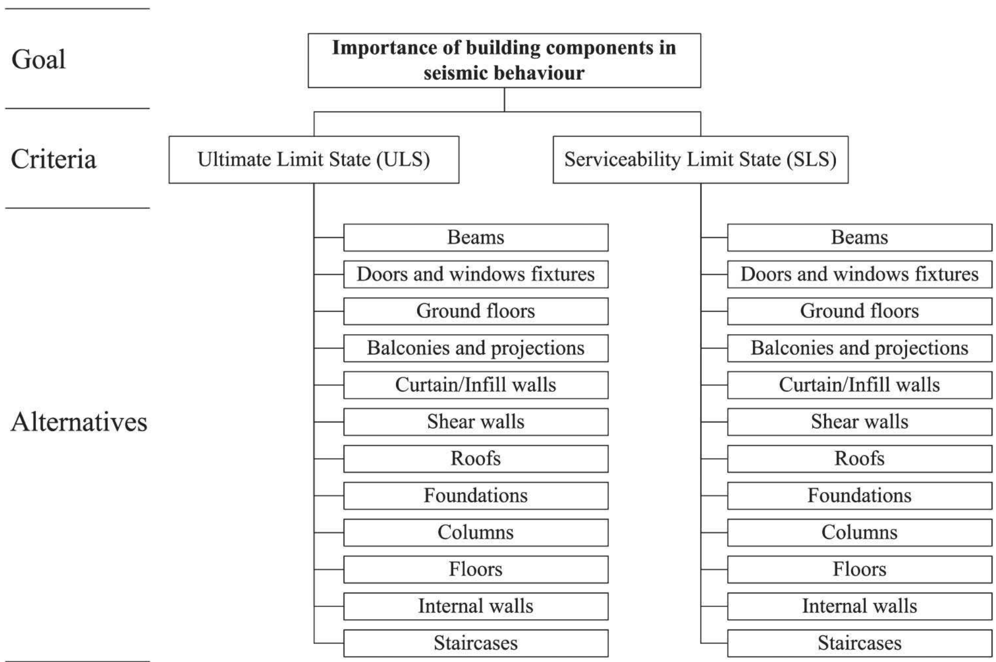

Fig. 5. Diagram of the decision problem of the considered application structured in a hierarchy

Table 9. Nonoptimized Matrix of Judgments A_1 and Result

A_1	F.l.	S.w.	C.	B.	F.	R.	S.	C.w.	G.f.	I.w.	B.p.	D.f.	w_1	w_{1n}
Foundation level	1.0	2.0	2.0	3.0	4.0	5.0	7.0	8.0	8.0	8.0	8.0	9.0	0.221	10.0
Shear walls	0.5	1.0	2.0	4.0	5.0	7.0	7.0	8.0	8.0	8.0	8.0	8.0	0.211	9.5
Columns	0.5	0.5	1.0	2.0	3.0	4.0	5.0	6.0	6.0	7.0	8.0	9.0	0.141	6.4
Beams	0.3	0.3	0.5	1.0	2.0	2.0	3.0	5.0	6.0	7.0	8.0	9.0	0.100	4.5
Floors	0.3	0.2	0.3	0.5	1.0	2.0	2.0	3.0	3.0	7.0	8.0	9.0	0.075	3.4
Roofs	0.2	0.1	0.3	0.5	0.5	1.0	5.0	5.0	6.0	8.0	8.0	9.0	0.085	3.8
Staircases	0.1	0.1	0.2	0.3	0.5	0.2	1.0	3.0	3.0	6.0	8.0	9.0	0.055	2.5
Curtain/infill walls	0.1	0.1	0.2	0.2	0.3	0.2	0.3	1.0	3.0	3.0	8.0	8.0	0.041	1.9
Ground floors	0.1	0.1	0.2	0.2	0.3	0.2	0.3	0.3	1.0	2.0	4.0	5.0	0.026	1.2
Internal walls	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.5	1.0	3.0	4.0	0.020	0.9
Balconies and projections	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.3	1.0	3.0	0.014	0.6
Doors and windows fixtures	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.3	1.0	0.010	0.5

Note: B. = beams; B.p. = balconies and projections; C. = columns; $CI_1 = 0.18$; $CR_1 = 0.123$; C.w. = curtain/infill walls; D.f. = doors and windows fixtures; F. = floors; F.l. = foundation level; G.f. = ground floors; I.w. = internal walls; R. = roofs; RI = 1.44; S. = staircases; S.w. = shear walls.

the judgment matrix $A^{Opt}_{test,2}$ and the optimized values of $w^{Opt}_{test,2}$ obtained by solving Eqs. (8*a*) and (8*b*). In addition, the right side lists the real values of the geometric shapes areas and the average percentage error of the results, in comparison with the real value of the geometric shape. The results point out that the O-AHP weights are very close to the real values with an average percentage error $Pe(w^{Opt}_{test,2}) = 2.4\%$ and the consistency test is satisfied: $CR = 0.0004 < 0.1$. All of the consulted decision makers obtain a result subject to an average percentage error less than 5%.

In conclusion, the considered test involving a large number of judgments confirms that the O-AHP exhibits the same effectiveness than in the first benchmark test.

Furthermore, the methodology of O-AHP allows omitting some judgment ranges when the decision maker is not able to assign all of the ranges. Then, the decision makers are asked to progressively remove the uncertain judgment ranges. Hence, 30 constraints are removed: $a_{1,7}$, $a_{1,8}$, $a_{1,9}$, $a_{1,10}$, $a_{1,11}$, $a_{2,6}$, $a_{2,7}$, $a_{2,8}$, $a_{2,9}$, $a_{2,10}$, $a_{3,9}$, $a_{3,10}$, $a_{3,11}$, $a_{4,7}$, $a_{4,8}$, $a_{4,9}$, $a_{4,11}$, $a_{4,12}$, $a_{5,9}$, $a_{5,10}$, $a_{5,11}$, $a_{5,12}$, $a_{6,10}$, $a_{6,11}$, $a_{6,12}$, $a_{7,10}$, $a_{7,11}$, $a_{7,12}$, $a_{8,10}$, and $a_{8,12}$. Also in this case, the resulting weights are very close to the real values, and the average percentage error $Pe(w_{test,2}^{Opt})$ becomes the worst cases of 7%.

Application: Building Performance Assessment

The O-AHP is applied to the crucial problem of the anti-seismic performance assessment in the building construction field (Sangiorgio et al. 2016a, b). In this field, it is not only important to rank alternatives and criteria, but also to evaluate the related weights useful in a seismic vulnerability assessment at the regional scale (Uva et al. 2016).

In the following, the steps proposed in Fig. 2 are applied to the considered problem.

Step 1. The goal is to determine the ranking of the most sensitive building components regarding the seismic behavior of the whole building. To his aim, two main criteria are identified in the O-AHP, and the problem is structured according to the European Building Code (CEN 2005): the ultimate limit state (ULS) and the serviceability limit state (SLS). Therefore, Fig. 5 shows the hierarchical scheme of the decision problem.

Step 2. Starting from the ULS criterion, O-AHP Phase 1 is performed by proposing a possible approximate ranking of the most important building criteria, in partial agreement with European Committee for Standardization (CEN 2005). In the rows and columns of matrix A, the elements of comparison are located in descending order of importance as listed in Table 9.

In the considered criterion, the O-AHP in support and comparison with standard AHP is applied to show the differences of the new method with respect to the standard one.

Following the standard AHP, 66 judgments are necessary and assigned by the decision maker on the basis of the pairwise comparisons to obtain the judgment matrix and the related weights. Table 9 lists the judgment matrix A_1, the relative weights w obtained by solving Eq. (3), and the values that quantify the inconsistency CI_1 and CR_1 calculated according to Eq. (5) and (6). The used value of RI corresponds to $n = 12$ (Table 2). The inconsistency of this matrix exceeds the limit of the Saaty theory: $\mathrm{CR}_1 = 0.123 > 0.1$. Hence, it is necessary to apply Phase 2 of the O-AHP.

In the O-AHP Phase 2, the decision maker uses the semantic ranges of O-AHP (Tables 3 and 4) to define the judgment ranges and determine the upper bounds K_{ij}^U and the lower bounds K_{ij}^L. The 66 determined judgment ranges are reported in Table 10.

In the subsequent O-AHP Phase 3, the problem is formalized by the mathematical constraints expressed in Eqs. (7a)–(7e).

Table 11 provides the solution to the optimization Eqs. (8a) and (8b), and provides the optimized judgment matrix A_1^{Opt}, the optimized values of w_1^{Opt} obtained by Eq. (3), the values of w_1^{Opt} normalized to 10 (denoted by w_{1n}^{Opt}), and the indices CI_1^{Opt} and CR_1^{Opt}.

Table 10. Judgment Ranges Specifying K_{ij}^U and K_{ij}^L for the Alternatives in Relation to ULS Criterion

ith element	K_{ij}^L	a_{ij}	K_{ij}^U	jth element
Foundation	2.0 ≤	$a_{1,2}$	≤3.0	Shear walls
Foundation	2.0 ≤	$a_{1,3}$	≤3.0	Columns
Foundation	3.0 ≤	$a_{1,4}$	≤4.0	Beams
Foundation	4.0 ≤	$a_{1,5}$	≤5.0	Floors
Foundation	5.0 ≤	$a_{1,6}$	≤6.0	Roofs
Foundation	7.0 ≤	$a_{1,7}$	≤8.0	Staircase and catwalks
Foundation	7.0 ≤	$a_{1,8}$	≤8.0	Curtain/Infill walls
Foundation	8.0 ≤	$a_{1,9}$	≤9.0	Ground floors
Foundation	8.0 ≤	$a_{1,10}$	≤9.0	Internal walls
Foundation	8.0 ≤	$a_{1,11}$	≤9.0	Balcony and projections
Foundation	9.0 ≤	$a_{1,12}$	≤9.0	Doors and windows f.
Shear walls	1.5 ≤	$a_{2,3}$	≤2.0	Columns
Shear walls	4.0 ≤	$a_{2,3}$	≤5.0	Beams
Shear walls	5.0 ≤	$a_{2,4}$	≤6.0	Floors
Shear walls	5.0 ≤	$a_{2,5}$	≤7.0	Roofs
Shear walls	6.0 ≤	$a_{2,6}$	≤8.0	Staircase and catwalks
Shear walls	7.0 ≤	$a_{2,7}$	≤8.0	Curtain/Infill walls
Shear walls	7.0 ≤	$a_{2,8}$	≤8.0	Ground floors
Shear walls	8.0 ≤	$a_{2,9}$	≤9.0	Internal walls
Shear walls	8.0 ≤	$a_{2,10}$	≤9.0	Balcony and projections
Shear walls	9.0 ≤	$a_{2,11}$	≤9.0	Doors and windows f.
Columns	2.0 ≤	$a_{3,4}$	≤4.0	Beams
Columns	3.0 ≤	$a_{3,5}$	≤5.0	Floors
Columns	4.0 ≤	$a_{3,6}$	≤6.0	Roofs
Columns	5.0 ≤	$a_{3,7}$	≤6.0	Staircase and catwalks
Columns	6.0 ≤	$a_{3,8}$	≤8.0	Curtain/Infill walls
Columns	6.0 ≤	$a_{3,9}$	≤8.0	Ground floors
Columns	7.0 ≤	$a_{3,10}$	≤9.0	Internal walls
Columns	8.0 ≤	$a_{3,11}$	≤9.0	Balcony and projections
Columns	9.0 ≤	$a_{3,12}$	≤9.0	Doors and windows f.
Beams	2.0 ≤	$a_{4,5}$	≤2, 5	Floors
Beams	2.0 ≤	$a_{4,6}$	≤3.0	Roofs
Beams	3.0 ≤	$a_{4,7}$	≤5.0	Staircase and catwalks
Beams	5.0 ≤	$a_{4,8}$	≤7.0	Curtain/Infill walls
Beams	6.0 ≤	$a_{4,9}$	≤8.0	Ground floors
Beams	7.0 ≤	$a_{4,10}$	≤9.0	Internal walls
Beams	8.0 ≤	$a_{4,11}$	≤9.0	Balcony and projections
Beams	9.0 ≤	$a_{4,12}$	≤9.0	Doors and windows f.
Floors	1.0 ≤	$a_{5,6}$	≤3.0	Roofs
Floors	2.0 ≤	$a_{5,7}$	≤3.0	Staircase and catwalks
Floors	3.0 ≤	$a_{5,8}$	≤4.0	Curtain/Infill walls
Floors	3.0 ≤	$a_{5,9}$	≤4.0	Ground floors
Floors	7.0 ≤	$a_{5,10}$	≤8.0	Internal walls
Floors	8.0 ≤	$a_{5,11}$	≤9.0	Balcony and projections
Floors	9.0 ≤	$a_{5,12}$	≤9.0	Doors and windows f.
Roofs	2.0 ≤	$a_{6,7}$	≤5.0	Staircase and catwalks
Roofs	2.0 ≤	$a_{6,8}$	≤5.0	Curtain/Infill walls
Roofs	2.0 ≤	$a_{6,9}$	≤6.0	Ground floors
Roofs	7.0 ≤	$a_{6,10}$	≤8.0	Internal walls
Roofs	7.0 ≤	$a_{6,11}$	≤8.0	Balcony and projections
Roofs	9.0 ≤	$a_{6,12}$	≤9.0	Doors and windows f.
Staircases	1.0 ≤	$a_{7,8}$	≤3.0	Curtain/Infill walls
Staircases	1.0 ≤	$a_{7,9}$	≤3.0	Ground floors
Staircases	6.0 ≤	$a_{7,10}$	≤8.0	Internal walls
Staircases	6.0 ≤	$a_{7,11}$	≤8.0	Balcony and projections
Staircases	7.0 ≤	$a_{7,12}$	≤9.0	Doors and windows f.
Curtain/Infill walls	2.0 ≤	$a_{8,9}$	≤3.0	Ground floors
Curtain/Infill walls	2.0 ≤	$a_{8,10}$	≤3.0	Internal walls
Curtain/Infill walls	5.0 ≤	$a_{8,11}$	≤8.0	Balcony and projections
Curtain/Infill walls	6.0 ≤	$a_{8,12}$	≤8.0	Doors and windows f.
Ground floors	2.0 ≤	$a_{9,10}$	≤3.0	Internal walls
Ground floors	2.0 ≤	$a_{9,11}$	≤4.0	Balcony and projections
Ground floors	2.0 ≤	$a_{9,12}$	≤5.0	Doors and windows f.
Internal walls	2.0 ≤	$a_{10,11}$	≤4.0	Balcony and projections
Internal walls	2.0 ≤	$a_{10,12}$	≤4.0	Doors and windows f.
Balcony and projections	1.0 ≤	$a_{11,12}$	≤3.0	Doors and windows f.

Table 11. Optimized Judgment Matrix, Weights, and CR Obtained by O-AHP in ULS Criterion

A_1^{Opt}	F.l.	S.w.	C.	B.	F.	R.	S.	C.w.	G.f.	I.w.	B.p.	D.f.	w_1^{Opt}	w_{1n}^{Opt}	w_1	w_{1n}
Foundation level	1.0	2.0	2.0	3.1	4.2	5.2	7.3	7.4	8.6	8.8	8.9	9.0	0.239	10.0	0.221	10.0
Shear walls	0.5	1.0	1.6	4.1	5.1	5.1	6.4	7.4	7.6	8.8	8.9	9.0	0.214	9.0	0.211	9.5
Columns	0.5	0.6	1.0	2.1	3.1	4.1	5.2	6.5	7.0	8.4	8.8	9.0	0.161	6.7	0.141	6.4
Beams	0.3	0.2	0.5	1.0	2.1	2.1	3.1	5.1	6.4	7.9	8.6	9.0	0.107	4.5	0.100	4.5
Floors	0.2	0.2	0.3	0.5	1.0	1.2	2.1	3.1	3.4	7.2	8.3	9.0	0.073	3.1	0.075	3.4
Roofs	0.2	0.2	0.2	0.5	0.8	1.0	2.1	2.2	3.0	7.2	7.3	9.0	0.065	2.7	0.085	3.8
Staircases	0.1	0.2	0.2	0.3	0.5	0.5	1.0	1.3	1.9	6.1	6.2	7.2	0.045	1.9	0.055	2.5
Curtain/infill walls	0.1	0.1	0.2	0.2	0.3	0.5	0.7	1.0	2.1	2.4	5.1	6.1	0.034	1.4	0.041	1.9
Ground floors	0.1	0.1	0.1	0.2	0.3	0.3	0.5	0.5	1.0	2.1	2.2	2.3	0.022	0.9	0.026	1.2
Internal walls	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.4	0.5	1.0	2.0	2.1	0.015	0.6	0.020	0.9
Balconies and projections	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.5	0.5	1.0	1.2	0.012	0.5	0.014	0.6
Doors and windows fixtures	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.4	0.5	0.8	1.0	0.011	0.5	0.010	0.5

Note: B. = beams; B.p. = balconies and projections; C. = columns; $CI_1^{Opt} = 0.084$; $CR_1^{Opt} = 0.058$; C.w. = curtain/infill walls; D.f. = doors and windows fixtures; F. = floors; F.l. = foundation level; G.f. = ground floors; I.w. = internal walls; R. = roofs; RI = 1.44; S. = staircases; S.w. = shear walls.

Matrix *E* of the deviation from consistency for ULS criterion	Foundation level	Shear walls	Columns	Beams	Floors	Roofs	Staircases	Curtain/Infill walls	Ground floors	Internal walls	Balconies and projections	Doors and windows fixtures
Foundation level	0	0	0	0	0	0	0	1	1	1	1	0
Shear walls	0	0	0	0	0	2	1	1	0	1	1	1
Columns	0	0	0	0	0	0	0	0	1	1	1	0
Beams	0	0	0	0	0	0	0	0	0	1	1	0
Floors	0	0	0	0	0	1	0	0	0	0	0	0
Roofs	0	2	0	0	1	0	3	3	3	1	1	0
Staircases	0	1	0	0	0	3	0	2	1	0	2	2
Curtain/Infill walls	1	1	0	0	0	3	2	0	1	1	3	2
Ground floors	1	0	1	0	0	3	1	1	0	0	2	3
Internal walls	1	1	1	1	0	1	0	1	0	0	1	2
Balconies and projections	1	1	1	1	0	1	2	3	2	1	0	2
Doors and windows fixtures	0	1	0	0	0	0	2	2	3	2	2	0

Fig. 6. Matrix of the deviation from consistency, in gray and the highest values in dark gray

The optimized matrix A_1^{Opt} respects the consistency limits, as $CR_1^{Opt} = 0.058 < 0.1$. Hence, it is evident that the O-AHP method leads to a more consistent matrix with logically corrected weights than the standard AHP. For instance, one of the major contradictions in the nonoptimized and inconsistent judgment matrix is the comparison between floors and roofs (Table 9). In the pair comparison, floors are considered to be more important than the roofs; on the contrary, in the result, roofs obtain a greater weight than floors. Such a situation is an evident drawback that can be found in the inconsistent matrices. However, the O-AHP methodology solves such a problem by introducing the judgment ranges and the optimization.

The last O-AHP Phase 4 reveals the comparison between matrices A_1 and A_1^{Opt}, to detect errors or inaccuracies that affect matrix A_1. The defined additional matrix E_1 is shown in Fig. 6, where the values of e_{ij} are rounded to integer values. More precisely, high values of e_{ij} (in gray and dark gray) correspond to inconsistent judgment between the elements i and j of the nonoptimized judgment matrix.

Next, Step 2 is repeated for the SLS criterion. The O-AHP Phase 1 of rouge ranking evaluation provides the same results as the first ULS criterion. Moreover, in this second criterion, the O-AHP in substitution of the standard method is applied. Therefore, the decision maker proceeds with Phases 2 and 3 of the O-AHP by determining the judgment ranges schematized in Table 12 and the optimized judgment matrix A_2^{Opt} given in Table 13.

The building component weights as related to the SLS criterion have few differences among them compared with the ULS criterion weights. Such a result is in accordance with CEN (2005): In the SLS, all nonstructural elements assume more importance in the seismic behavior, as they have to ensure functionality and must not cause occupant discomfort under routine conditions.

The final weights to be assigned concern the relationship between the two criteria: As they are assumed to have the same importance, it holds 0.5 for ULS and 0.5 for SLS.

Summary of Priorities and Weight Exploitation in Construction

The O-AHP methodology ends by performing the summary of priority as the standard AHP shown in Fig. 7, which shows the global weight of each alternative. Such a global weight is obtained by multiplying each criteria weight to the alternative weight and by summing the results for each alternative (Saaty 1987).

Table 12. Judgment Ranges Specifying K_{ij}^U and K_{ij}^L for the Alternative in Relation to SLS Criterion

*i*th element	K_{ij}^L	a_{ij}	K_{ij}^U	*j*th element
Foundation	1.0 ≤	$a_{1,2}$	≤1.0	Shear walls
Foundation	1.0 ≤	$a_{1,3}$	≤1.0	Columns
Foundation	1.0 ≤	$a_{1,4}$	≤1.5	Beams
Foundation	1.0 ≤	$a_{1,5}$	≤1.5	Floors
Foundation	1.0 ≤	$a_{1,6}$	≤2.0	Roofs
Foundation	1.5 ≤	$a_{1,7}$	≤2.0	Staircase and catwalks
Foundation	1.5 ≤	$a_{1,8}$	≤2.0	Curtain/Infill walls
Foundation	1.5 ≤	$a_{1,9}$	≤4.0	Ground floors
Foundation	2.0 ≤	$a_{1,10}$	≤4.0	Internal walls
Foundation	2.0 ≤	$a_{1,11}$	≤4.0	Balcony and projections
Foundation	5.0 ≤	$a_{1,12}$	≤8.0	Doors and windows f.
Shear walls	1.0 ≤	$a_{2,3}$	≤1.0	Columns
Shear walls	1.0 ≤	$a_{2,4}$	≤1.0	Beams
Shear walls	1.0 ≤	$a_{2,5}$	≤1.0	Floors
Shear walls	1.0 ≤	$a_{2,6}$	≤1.5	Roofs
Shear walls	1.0 ≤	$a_{2,7}$	≤1.5	Staircase and catwalks
Shear walls	1.5 ≤	$a_{2,8}$	≤2.0	Curtain/Infill walls
Shear walls	1.5 ≤	$a_{2,9}$	≤3.0	Ground floors
Shear walls	2.0 ≤	$a_{2,10}$	≤3.0	Internal walls
Shear walls	2.0 ≤	$a_{2,11}$	≤3.0	Balcony and projections
Shear walls	5.0 ≤	$a_{2,12}$	≤8.0	Doors and windows f.
Columns	1.0 ≤	$a_{3,4}$	≤1.0	Beams
Columns	1.0 ≤	$a_{3,5}$	≤1.5	Floors
Columns	1.0 ≤	$a_{3,6}$	≤1.5	Roofs
Columns	1.0 ≤	$a_{3,7}$	≤1.5	Staircase and catwalks
Columns	1.5 ≤	$a_{3,8}$	≤2.0	Curtain/Infill walls
Columns	1.5 ≤	$a_{3,9}$	≤3.0	Ground floors
Columns	2.0 ≤	$a_{3,10}$	≤3.0	Internal walls
Columns	2.0 ≤	$a_{3,11}$	≤3.0	Balcony and projections
Columns	4.0 ≤	$a_{3,12}$	≤8.0	Doors and windows f.
Beams	1.0 ≤	$a_{4,5}$	≤1.5	Floors
Beams	1.0 ≤	$a_{4,6}$	≤1.5	Roofs
Beams	1.0 ≤	$a_{4,7}$	≤1.5	Staircase and catwalks
Beams	1.0 ≤	$a_{4,8}$	≤1.5	Curtain/Infill walls
Beams	1.0 ≤	$a_{4,9}$	≤3.0	Ground floors
Beams	2.0 ≤	$a_{4,10}$	≤3.0	Internal walls
Beams	2.0 ≤	$a_{4,11}$	≤3.0	Balcony and projections
Beams	4.0 ≤	$a_{4,12}$	≤8.0	Doors and windows f.
Floors	1.0 ≤	$a_{5,6}$	≤1.0	Roofs
Floors	1.0 ≤	$a_{5,7}$	≤1.5	Staircase and catwalks
Floors	1.0 ≤	$a_{5,8}$	≤1.5	Curtain/Infill walls
Floors	2.0 ≤	$a_{5,9}$	≤3.0	Ground floors
Floors	2.0 ≤	$a_{5,10}$	≤3.0	Internal walls
Floors	2.0 ≤	$a_{5,11}$	≤4.0	Balcony and projections
Floors	4.0 ≤	$a_{5,12}$	≤8.0	Doors and windows f.
Roofs	1.0 ≤	$a_{6,7}$	≤1.5	Staircase and catwalks
Roofs	1.0 ≤	$a_{6,8}$	≤1.5	Curtain/Infill walls
Roofs	2.0 ≤	$a_{6,9}$	≤3.0	Ground floors
Roofs	2.0 ≤	$a_{6,10}$	≤3.0	Internal walls
Roofs	2.0 ≤	$a_{6,11}$	≤4.0	Balcony and projections
Roofs	4.0 ≤	$a_{6,12}$	≤8.0	Doors and windows f.
Staircases	1.0 ≤	$a_{7,8}$	≤2.0	Curtain/Infill walls
Staircases	1.0 ≤	$a_{7,9}$	≤2.0	Ground floors
Staircases	1.0 ≤	$a_{7,10}$	≤2.0	Internal walls
Staircases	1.0 ≤	$a_{7,11}$	≤2.0	Balcony and projections
Staircases	3.0 ≤	$a_{7,12}$	≤8.0	Doors and windows f.
Curtain/Infill walls	1.0 ≤	$a_{8,9}$	≤1.5	Ground floors
Curtain/Infill walls	1.0 ≤	$a_{8,10}$	≤1.5	Internal walls
Curtain/Infill walls	1.0 ≤	$a_{8,11}$	≤1.5	Balcony and projections
Curtain/Infill walls	3.0 ≤	$a_{8,12}$	≤8.0	Doors and windows f.
Ground floors	1.0 ≤	$a_{9,10}$	≤1.5	Internal walls
Ground floors	1.0 ≤	$a_{9,11}$	≤1.5	Balcony and projections
Ground floors	3.0 ≤	$a_{9,12}$	≤7.0	Doors and windows f.
Internal walls	1.0 ≤	$a_{10,11}$	≤1.5	Balcony and projections
Internal walls	3.0 ≤	$a_{10,12}$	≤7.0	Doors and windows f.
Balcony and projections	3.0 ≤	$a_{11,12}$	≤7.0	Doors and windows f.

In addition, the last column of Fig. 7 shows the global weights that are normalized to 10, to evaluate and quantify any building pathology or damage for a large-scale vulnerability assessment.

For instance, in considering the problem of evaluating the building pathologies on a large-scale application by using visual survey photos, it is necessary to determine the numerical values that quantify each criticality in relation to building seismic behavior by using a pathology criticality index. To this aim, the O-AHP is applied to the following goals, to determine the ranking and weights (starting from Step 1): pathologies extension, room crowding, and severity of pathology. The global weights can be computed for each goal alternative. Fig. 8 shows the school building “Cirielli” of Bari, Italy (Sangiorgio 2015). The pathology quantification of a visual survey presents the following alternatives: the beam for the building component, 100% for the pathology extension, the hallway for the room crowding, and the heavy expulsion of concrete cover for the severity of pathology. The last column of Fig. 7 reports the global weights computed by the O-AHP and their associated alternatives. For example, the value 6.1 corresponds to the normalized weight of the beams.

The pathology CI indicates that the considered criticality can be obtained by the average of the alternative global weights (assuming that each goal has the same importance). In the diagnostic sheet of Fig. 8, such a pathology CI is equal to 6.9 and represents the damage quantification of the visual survey in a scale from 0 to 10. Hence, it is possible to evaluate the damage or degradation of different building components by replicating the described calculation. Finally, the result of the presented methodology on a large scale can be applied with the post–seismic event survey data (Grimaz 2009), to perform a quick analysis of the severity of damages.

Conclusions

This paper presents a new strategy, the optimized AHP, which allows the application of the AHP methodology even if a large number of criteria and comparisons are necessary and the consistency is not easy to obtain. The work focuses on the drawback of the standard AHP, determined by the limited capacity of the human mind to assign coherent judgments when the number of simultaneous comparisons is large. Moreover, from a mathematical point of view, the reliability and coherence of the resulting weights evaluated by the judgment matrix are critical issues when the number of alternatives increases and the standard AHP is applied.

The proposed methodology overcomes the standard AHP drawbacks by revising the AHP weights evaluation procedure. First, the exact judgment assignments are replaced by judgment ranges. Second, the entries of the judgment matrix are provided by an MP formulation that minimizes the inconsistency of the matrix subject to the constraints imposed on the weights.

The main benefits of the resulting approach are the simplification of the decision-maker operations in the case of a large number of alternatives, and the determination of the judgment matrix that is obtained by optimizing the congruence and respecting the constraints imposed by the decision maker. In contrast, the O-AHP is able to generate a judgments matrix that exhibits the minimum value of inconsistency, regardless of whether the decision maker omits some judgment ranges. Obviously, this situation should be avoided and the decision maker has to define all of the possible judgment ranges.

To validate the presented methodology, a benchmark test proposed in the related literature is used: The results show the

Table 13. Optimized Judgment Matrix, Weights, and CR Obtained by O-AHP in SLS Criterion

A_2^{Opt}	F.l.	S.w.	C.	B.	F.	R.	S.	C.w.	G.f.	I.w.	B.p.	D.f.	w_1	w_{1n}
Foundation level	1.0	1.0	1.0	1.1	1.2	1.2	1.6	1.8	2.3	2.6	2.7	7.3	0.123	10.0
Shear walls	1.0	1.0	1.0	1.0	1.0	1.1	1.4	1.7	2.2	2.4	2.5	7.0	0.115	9.4
Columns	1.0	1.0	1.0	1.0	1.1	1.1	1.4	1.7	2.2	2.4	2.5	7.0	0.115	9.4
Beams	0.9	1.0	1.0	1.0	1.1	1.1	1.3	1.4	2.1	2.3	2.4	6.8	0.112	9.1
Floors	0.8	1.0	0.9	0.9	1.0	1.0	1.3	1.4	2.1	2.2	2.4	6.6	0.107	8.7
Roofs	0.9	0.9	0.9	0.9	1.0	1.0	1.2	1.4	2.1	2.2	2.3	6.5	0.105	8.5
Staircases	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.2	1.6	1.7	1.8	5.3	0.084	6.8
Curtain/infill walls	0.6	0.6	0.6	0.7	0.7	0.7	0.9	1.0	1.3	1.4	1.4	4.6	0.072	5.8
Ground floors	0.4	0.5	0.5	0.5	0.5	0.5	0.6	0.8	1.0	1.1	1.2	3.4	0.053	4.3
Internal walls	0.4	0.4	0.4	0.4	0.4	0.5	0.6	0.7	0.9	1.0	1.1	3.1	0.049	4.0
Balconies and projections	0.4	0.4	0.4	0.4	0.4	0.4	0.6	0.7	0.9	0.9	1.0	3.1	0.047	3.8
Doors and windows fixtures	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.3	0.3	0.3	1.0	0.016	1.3

Note: B. = beams; B.p. = balconies and projections; C. = columns; $CI_2^{Opt} = 0.0005$; $CR_2^{Opt} = 0.00034$; C.w. = curtain/infill walls; D.f. = doors and windows fixtures; F.= floors; F.l. = foundation level; G.f. = ground floors; I.w. = internal walls; R. = roofs; RI = 1.44; S. = staircases; S.w. = shear walls.

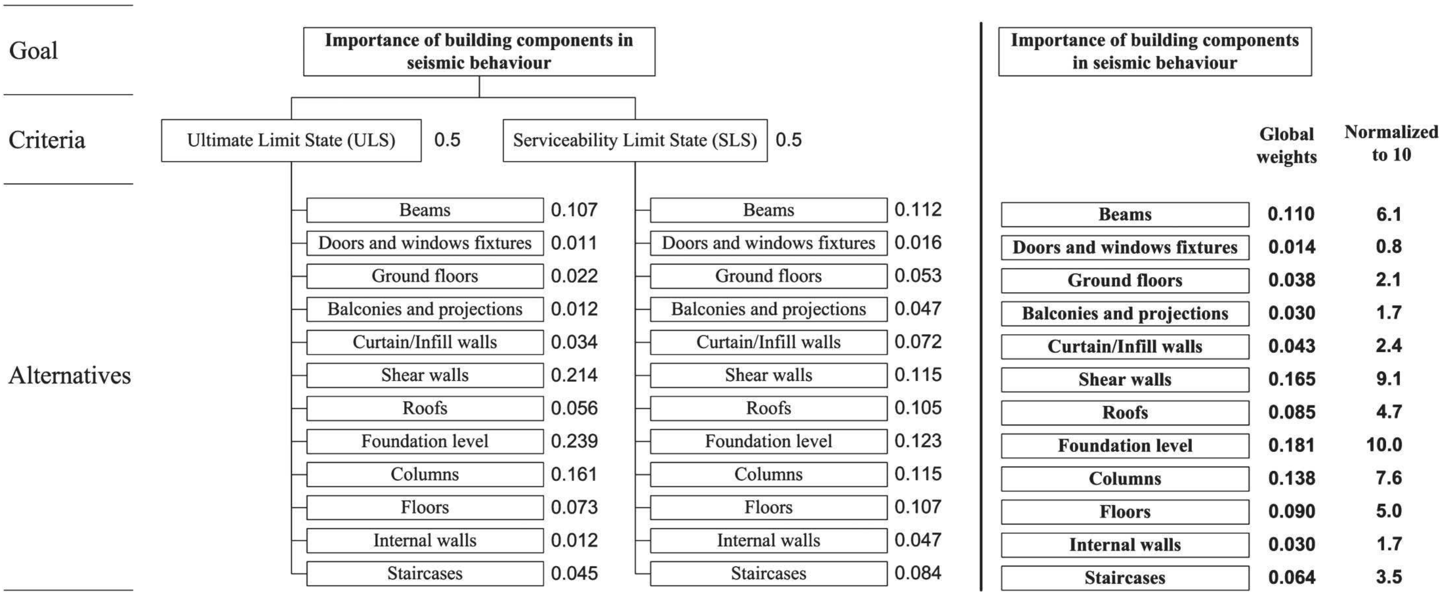

Fig. 7. Summary of priority of the O-AHP application

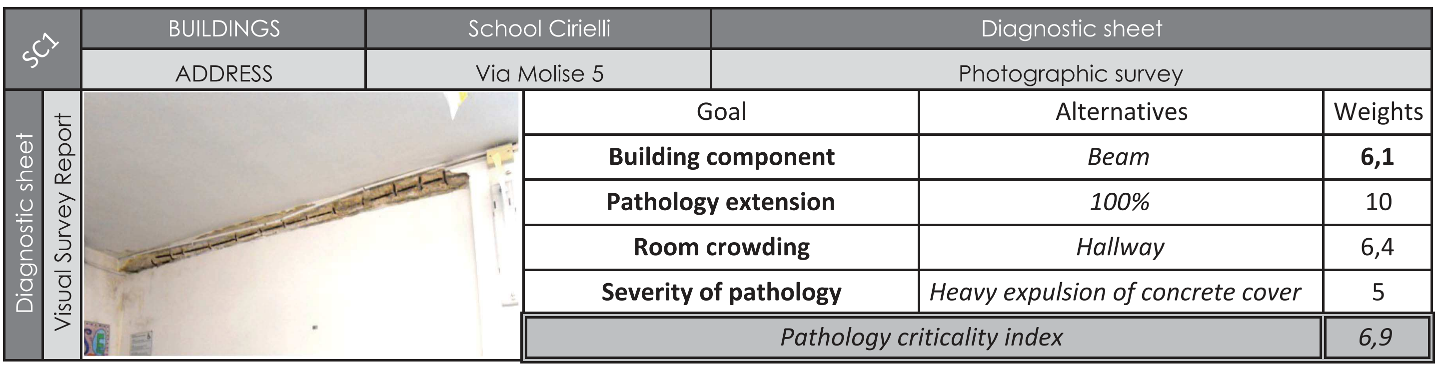

Fig. 8. Pathology quantification of a beam surveyed in school building "Cirielli"

effectiveness of the O-AHP also in the case of a large number of criteria. Moreover, the crucial problem of the identification and classification of the most important building components related to antiseismic performance is presented, and a possible application of the resulting weights in the field of building pathology evaluation is shown. In this context, a large number of alternatives is considered, and the standard AHP provides an inconsistent matrix. In contrast, the application of the O-AHP allows one to obtain consistent results and detect the imprecisions achieved by using the classic judgment assignment.

Therefore, the O-AHP method opens up new possibilities to AHP application in the field of construction, in which a large number of alternatives and criteria have to be evaluated.

Limitations of the Method and Future Research

The proposed O-AHP method does not overcome all of the limitations of the standard AHP, such as (1) the linear trade-offs among factors, (2) the zero as a natural scale of factors, and (3) the accomplishment of subjective judgments earlier in the decision-making process (Arroyo et al. 2014). In addition, the O-AHP method is actually not able to specify the minimum number of judgment ranges to perform effective results.

Future research will focus on the following main issues. From the knowledge point of view, in a first work, a detailed study will be performed that focuses on a complete sensitivity analysis of the O-AHP methodology and an exhaustive investigation of the O-AHP effectiveness in relation to the amplitude and the necessary number of judgment ranges.

In addition, in a further work, the application of O-AHP will be exhaustively implemented to all of the building safety performance goals related to the seismic and structural behavior. To this aim, an iterative evaluation will be performed by a team of expert decision makers in the field of building safety. The result will then be applied to several case studies, to converge toward comprehensive, shared, and validated outcomes.

Data Availability Statement

Data generated and analyzed during the study are available from the corresponding author by request. Information about the *Journal*'s data sharing policy can be found here: http://ascelibrary.org/doi/10.1061/%28ASCE%29CO.1943-7862.0001263.

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