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Procedia Social and Behavioral Sciences 20 (2011) 372–380

Procedia
Social and Behavioral Sciences

14th EWGT & 26th MEC & 1st RH

Traffic equilibrium network design problem under uncertain constraints

Leonardo Caggiani[a,*], Michele Ottomanelli[a]

[a]*Dept. of Environmental Engineering and Sustainable Development - Technical University of Bari - Viale De Gasperi, Taranto, 74100, Italy*

Abstract

Network design models allow to define an optimal network configuration by means of objective functions subject to a series of constraints. The starting data and/or the constraints of the problem can be affected by uncertainty. These uncertain values are better managed through the use of fuzzy values/constraints. In this paper we present a fuzzy non-linear programming to solve the equilibrium Network Design problem for urban areas. This problem is formulated as a fixed point optimization subject to fuzzy constraints. The proposed method has been applied to a test network. The obtained results show that the proposed approach is very interesting.

Keywords: Network design; Congested networks; Uncertainty; Fuzzy programming; Signal settings optimization.

1. Introduction

Transportation planners have to manage different models in order to evaluate the network status or to suggest new network scenarios. Generally these models may be divided into two principal groups: demand/supply simulation models and supply design models. The simulation models represent the tool to predict users behaviour (supply and activity systems are given, such as facilities, services and prices). The supply design models are based on simulation models and allow to define supply and activity systems that best satisfy the prefixed objectives. In particular the models able to define the transportation network layout are named Network Design models (Cascetta, 2009). Starting from an existing transportation network, these models allow to define an optimal network configuration by means of objective functions subject to a series of constraints.

The variables of these optimization problems are continuous or discrete; they are divided into three categories: layout variables (i.e. link capacities, links directions), supply performance variables (i.e. signals parameters) and price variables (i.e. transport fares).

* Corresponding author. Tel.: +39 099 4773270; fax: +39 099 4773304.
E-mail address: l.caggiani@poliba.it.

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doi:10.1016/j.sbspro.2011.08.043

The objectives of the problem may be social objectives (minimization of users costs) or operators objective (maximization of receipts).

The constraints can be external, such as the maximum budget available or the maximum pollution admitted, technical (i.e. link flows-capacity fraction values) or consistency constraints among demand, flows and costs.

In literature different models, with one objective function or multi-objective (Meng and Yang, 2002; Cantarella and Vitetta, 2006; Paksoy et al. 2010), and algorithms have been presented to solve Network Design problems. These optimization problems usually have rigid ranges or minimum-maximum thresholds constraints, analytically defined with inequalities. Actually, both analysts available data and problem constraints can be affected by uncertainty; moreover within an interval, some values may better satisfy the purpose of the analysts (i.e. in a budget interval constraints the lower values are preferable to the others). These uncertain values are better managed with a soft computing approach in particular through the use of fuzzy values/constraints (Zimmermann, 1996).

Recently, great attention has been given to new paradigms developed in the theoretical framework of Fuzzy Set, in which Fuzzy Logic and Possibility Theory are the mathematical tools most used for solving transportation problems (Teodorovic and Vukadinovic, 1998) like a fuzzy optimization (Kikuchi and Kronprasert, 2007; Ottomanelli et al., 2009; Caggiani et al. 2011). Few authors have studied the opportunity to consider also this knowledge together with the transportation Network Design problem (Das et al., 1999; Mudchanatongsuk et al., 2008; Selim and Ozkarahan, 2008; Ghatee and Hashemi, 2009a and 2009b).

In this paper we present a fuzzy non-linear programming to solve the equilibrium Network Design problem for urban areas. This problem is formulated as a fixed point optimization subject to fuzzy constraints. The proposed method has been applied to a test network. The preliminary results show that the proposed approach is very interesting since it behaves as a multi-criteria optimization.

2. Statement of the problem

The general formulation of the supply design problem is (Cascetta, 2009):

$$\mathbf{x}^* = \arg \underset{\mathbf{x}}{opt}\, w(\mathbf{x}, \mathbf{f}^*) \tag{1}$$

subject to:

$$\mathbf{f}^* = \boldsymbol{\Delta}(\mathbf{x})\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f}^*, \mathbf{x}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{x})) \tag{1a}$$

$$\mathbf{x}, \mathbf{f}^* \in E \tag{1b}$$

$$\mathbf{x}, \mathbf{f}^* \in T \tag{1c}$$

where:

- $\mathbf{x}$ is the vector of the design variables;
- w is the objective function;
- $\mathbf{f}^*$ is the vector of equilibrium assignment traffic flows;
- $\boldsymbol{\Delta}$ is the link-path incidence matrix;
- $\mathbf{P}$ is the path choice probability matrix;
- $\mathbf{C}$ is the path cost vector;
- $\mathbf{d}$ is the demand vector;
- the (1a) represents the consistency constraint among demand, flows and supply parameters (set of possible configurations of network flows)
- the (1b) and the (1c) express the sets of supply parameters satisfying external (E) and technical (T) constraints.

In urban network design problems the decision variables vector may include traffic lights vector parameters (**g**) and topological vector parameters (**y**) such as links capacity, lanes width and/or numbers, street direction and so on.

3. Proposed optimization model

In this paper we introduce in problem (1) also uncertain, incomplete information about design variables, external and technical constraints assuming the congested network case. For example possible constraints can be:

$$B \approx a \tag{2a}$$

$$ca_{l1} >> ca_{l2} \tag{2b}$$

$$g_u/g_v \approx b \tag{2c}$$

where B is the available budget, ca_{l1} and ca_{l2} are the link l_1 and l_2 capacity, g_u and g_v are the effective green time of two phases traffic light regulation scheme, a and b are real numbers. These constraints represent the incomplete information and formalize quantitatively a linguistic/approximate expressions.

In our model we propose to specify these uncertain relations/expressions as a fuzzy set (Zadeh, 1965; Zimmermann, 1996). In fuzzy logic a crisp number belongs to a set (fuzzy set) with a certain degree of membership, named also satisfaction *h*. The degree of membership is defined by a "membership function". For example, the constraint (2a) can describe the statement "the budget B is approximately equal to a". If there is no additional specific information, a triangular membership function *h(B)* can be assumed to specify the fuzzy constraint and so the previous statement is analytically defined by the fuzzy set depicted in Fig. 1. The z^-, a and z^+ values could be based on availability budget forecasts.

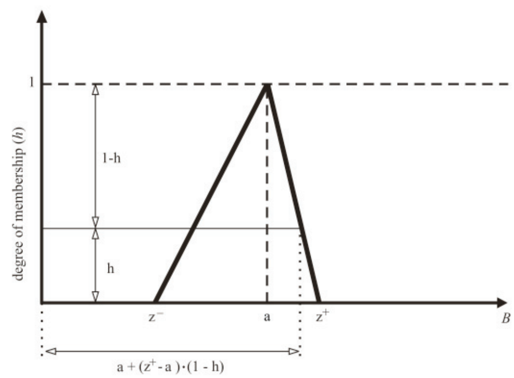

Figure 1. Fuzzy set $B \approx a$.

In fuzzy set theory, the closer to one the degree of membership is, the more the corresponding abscissa value belongs to the respective linguistic variable (fuzzy set).

If the membership functions are triangular then all the fuzzy constraints considered in problem (3) can be expressed as inequalities and depend on the satisfaction h.

In the previous example the inequalities representing the constraints are:

$$B \le a + (z^+ - a) \cdot (1 - h)$$
$$B \ge a - (a - z^-) \cdot (1 - h)$$

The closer to one the satisfaction is, the more the constraints are fulfilled. Therefore, in order to find the optimal solution to problem (1) subject to certain and fuzzy constraints, it is necessary to maximize the satisfaction h of the fuzzy constraints and, at the same time, to optimize the value of the objective function.

In fuzzy optimization, the problem (1) is equivalent to the following problem (3), where the objective function to be maximized is the satisfaction h, and the value of optimization function becomes a further constraint to the problem (see for ex. Teodorovic and Vukadinovic, 1998). This constraint depends on the type of optimization that is if the problem (1) is a minimization the constraint is the (3a) (membership function depicted in Fig. 2a) instead if it is a maximization the constraint is the (3b) (membership function depicted in Fig. 2b):

$$\max h \tag{3}$$

subject to:

$$\begin{cases} w(\mathbf{x}(h), \mathbf{f}^*) \le \overline{m} \cdot (1 - h) & (3a) \\ \text{or} \\ w(\mathbf{x}(h), \mathbf{f}^*) \ge \overline{m} \cdot h & (3b) \end{cases}$$

$$\mathbf{f}^* = \boldsymbol{\Delta}(\mathbf{x}(h))\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f}^*, \mathbf{x}(h)))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{x}(h))) \tag{3c}$$

$$x_e, \mathbf{f}^* \in E \quad \forall e = 1,2,\ldots,p \tag{3d}$$

$$x_t, \mathbf{f}^* \in T \quad \forall t = 1,2,\ldots,q \tag{3e}$$

$$x_i(h) \in F_i \quad \forall i = 1,2,\ldots,n \tag{3f}$$

where:

- $p+q$ is the number of certain design variables;
- n is the number of uncertain design variables;
- $F_i = \{(w, h(w)) \mid w \in W\}$ is the fuzzy set i;
- W is the set of all possible objective function values w.

In Fig. 2 the value of $\overline{m}$ represents the maximum value admitted for the objective function in problem (1).

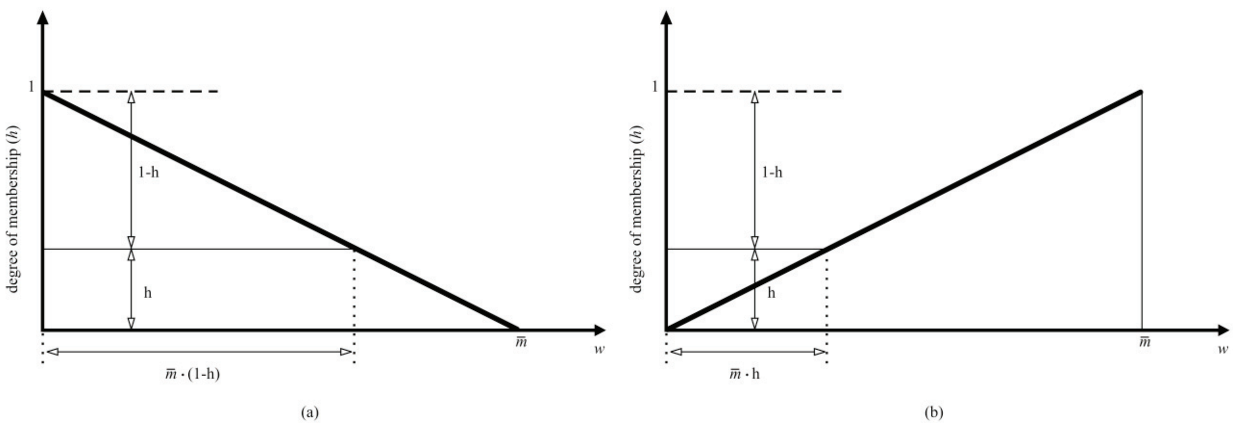

Figure 2. Fuzzy set representing the expression "satisfactory *w* optimization".

The assumed fuzzy constraints will definitely depend on the same value of the satisfaction *h*. The closer to one the value of *h* (maximization of satisfaction) is, the more the objective function of the (1) is optimized (through the (3a) or (3b)) in accordance with the consistency constraints (3c), with the certain relations (3d) and (3e) and with the fuzzy constraints (3f).

4. Algorithm for problem solution

In this section we present an algorithm for the solution of the problem (3). We propose to solve this optimization with a SQP algorithm (Sequential Quadratic Programming). At each iteration *s* of this algorithm an approximation of the Hessian of the Lagrangian function is used to generate a QP (Quadratic Programming) sub-problem. The solution of this sub-problem is adopted to form a search direction for a line search procedure. For a detailed description of this procedure refer to Schittkowski (1985) and Bonnans et al. (2006).

The fixed point formulation (3c) is solved, within each optimization step of QP sub-problem, to find the equilibrium flows. This problem is expressed assuming the congested network case with stochastic assignment model (Meng et al., 2004; Chen et al., 2009). The assignment matrix **H** (product between link-path incidence matrix **Δ** and path choice probability matrix **P**) in the constraint (3c), is considered as function of the path costs **C** and design variables **x**.

The solution algorithm of the assignment problem is based on the Method of Successive Averages – Flow Averaging (MSA – FA) with variable step length in order to improve the convergence speed (Cantarella, 1997; Cascetta et al., 2006). The algorithm is summarized in the following four steps:

Step 0 initialize: $k = 1$ and $k_i = k$;
choose the number of iterations after which the rise of the magnitude of step begins: N_i
initialize the flows with a feasible solution corresponding to free flow costs: $\mathbf{f}^{k=1} = \mathbf{f}^0$;

Step 1 update iteration counter: $k = k + 1$;

Step 2 compute link costs : $\mathbf{c}^k = \mathbf{c}[\mathbf{f}^{k-1}, \mathbf{x}(h^s)]$;

Step 3 compute the corresponding Stochastic Network Loading flows : $\mathbf{f}^k{}_{SNL} = \mathbf{\Delta P}(\mathbf{\Delta}^T\mathbf{c}^k)\mathbf{d}$;

Step 4 calculate the average of flows: $\mathbf{f}^{k+1} = [(k\text{-}1)\,\mathbf{f}^k + \mathbf{f}^k{}_{SNL}]/k$;

Step 5 if $|\,f_i{}^{k+1} - f_i{}^k{}_{SNL}| / f_i{}^{k+1} < \varepsilon_{MSA}$ then go to Step 8, else go to Step 6;

Step 6 if $k \leq N_i$ go to Step 1, else go to Step 7;

Step 7 change the iteration counter: $k_i = 2k$, $N_i = 2\,N_i$, $k = k_i$ and go to Step 2;

Step 8 stop the algorithm.

5. Numerical application and results

The following application aims to experimentally evaluate the performances of the proposed model. In this test we propose a network design optimization considering signal settings parameters as supply performance variables. The chosen approach to the problem is the global optimization of signal settings that consists in searching the optimal effective green time for all intersection signal settings (vector g*); these values are obtained through optimization of an objective function depending on signal settings and equilibrium flows as previously described in (1).

The proposed model has been applied to the network used in Yang et al. (2001) considered as urban network. The graph is made up of 9 nodes (3 origins and 3 destinations), and 14 links as depicted in Fig. 3. All the employed data are those proposed by Yang et al. (2001) except for links length (and free flow travel time) equal to 80 meters for all network links. The signalized intersections are the node 5 with three-phases regulation scheme and nodes 6 and 8 with a two-phases regulation scheme. For these three intersections the effective cycle time is fixed to C_t = 90 seconds and the starting effective green time is equally divided for each phase (starting values - Case 0).

The link cost values c_l employed for the numerical test are the sum of the free flow travel time and the waiting time due to the signalized intersections. The free flow travel time is calculated as the fraction between link length and link mean speed. The latter is computed with the experimental speed-flow relationship for Italian urban area proposed by Cartenì and Punzo (2007). The mean waiting time at signalized intersection is estimated using the Doherty (1977) delay function (4):

$$
\begin{aligned}
t^l_{wa} &= 0.5 \cdot C_t (1-\mu)^2 + \frac{1980}{\mu \cdot s} \cdot \frac{f_l}{\mu \cdot s - f_l} && \text{if} \quad f_l \le 0.95 \cdot \mu \cdot s \\
t^l_{wa} &= 0.5 \cdot C_t (1-\mu)^2 - \frac{198.55}{\mu \cdot s / 3600} + \frac{220 \cdot f_l}{(\mu \cdot s)^2 / 3600} && \text{if} \quad f_l > 0.95 \cdot \mu \cdot s
\end{aligned}
\qquad (4)
$$

where:

- t^l_{wa} is the waiting time at intersection on link l (s/veh);
- f_l is the traffic flow on link l (veh/h);
- s is the access saturation flow (veh/h);
- g is the effective green time;
- μ is the effective green ratio (g/C_t).

This function considers the under-saturated and over-saturated links with the linear extension of the delay curve for a saturation degree greater than 0.95.

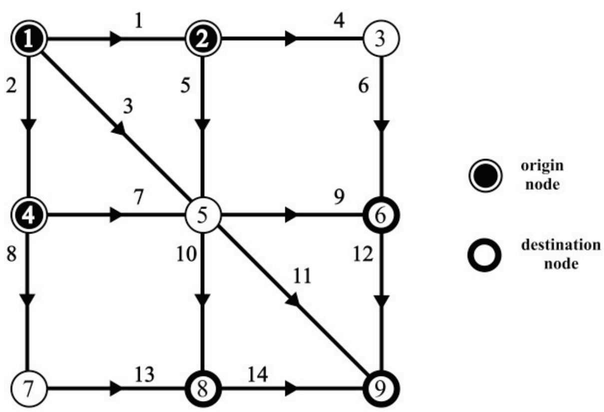

Figure 3. Test network.

The demand d (Table 1) has been assigned to the network using a Stochastic User Equilibrium traffic assignment model, with a Logit formulation for the path choice model (SUE-Logit Model).

Table 1 Demand vector

O-D	1-6	1-8	1-9	2-6	2-8	2-9	4-6	4-8	4-9
i	1	2	3	4	5	6	7	8	9
d	120	150	100	130	200	90	80	180	110

The value of the Logit path choice model parameter has been assumed equal to 1.5. The path choice set is composed of all the possible paths joining the considered origin and destination nodes of the network.

The aim of the test is to find the vector of effective green time for the considered signalized intersections that minimizes the equilibrium total users costs T^{uc} (sum of the product between equilibrium link flows f_l*and equilibrium link costs c_l*) under uncertain constraint.

In the experiment, we suppose that the analyst wants to reduce, compared to the equally divided effective green time, the total travel time of links 7 and 13. The linguistic assessment of the analyst in order to minimize these total travel time may be *"$T_7^{uc} = c_7^* \cdot f_7^*$ (and $T_{13}^{uc} = c_{13}^* \cdot f_{13}^*$) are smaller than starting values $T_7^{uc\_eq}$ (and $T_{13}^{uc\_eq}$)"*. If this problem is solved with a classical approach (Cascetta et al., 2006) the optimization problem (1) becomes:

$$\mathbf{g}^* = \arg\min_{\mathbf{g}} \sum_l f_l^* \cdot c_l^*(\mathbf{f}^*, \mathbf{g}) \tag{5}$$

subject to:

$$f_7^* \cdot c_7^* \le T_7^{uc\_eq} \tag{5a}$$

$$f_{13}^* \cdot c_{13}^* \le T_{13}^{uc\_eq} \tag{5b}$$

$$\mathbf{f}^* = \mathbf{\Delta}(\mathbf{g})\mathbf{P}(\mathbf{g}, \mathbf{C}(\mathbf{f}^*, \mathbf{g}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{g})) \tag{5c}$$

$$\sum g_l^{in} = C_t^{in} \qquad \forall \ in \in \{5,6,8\} \tag{5d}$$

$$g_l^{in} \le 85 \text{ [s]} \quad \text{and} \quad g_l^{in} \ge 5 \text{ [s]} \qquad \forall \ in \in \{5,6,8\} \tag{5e}$$

where:

- (5a) and (5b) describe with crisp (not fuzzy) thresholds the constraints set by analyst on link cost 7 and link cost 13;
- (5c) establishes consistency among demand, flows and supply parameters;
- (5d) ensures that the sum of the effective green time for each signalized intersection is equal to the prefixed effective cycle time;
- (5e) ensures maximum and minimum values for the effective green time;

The vague (uncertain) expression on total user costs can better formalized with fuzzy constraints through the proposed method. In this case, as previously discussed, the problem (5) can be formulated as follows:

$$\max h \tag{6}$$

subject to:

$$\sum_l f_l^* \cdot c_l^*(\mathbf{f}^*, \mathbf{g}(h)) \le \overline{m} \cdot (1-h) \tag{6a}$$

$$f_7^* \cdot c_7^* \le T_7^{uc\_eq} \cdot (1-h) \tag{6b}$$

$$f_{13}^* \cdot c_{13}^* \le T_{13}^{uc\_eq} \cdot (1-h) \tag{6c}$$

$$\mathbf{f}^* = \mathbf{\Delta}(\mathbf{g})\mathbf{P}(\mathbf{g}, \mathbf{C}(\mathbf{f}^*, \mathbf{g}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{g})) \tag{6d}$$

$$\sum g_l^{in} = C_t^{in} \qquad \forall \ in \in \{5,6,8\} \tag{6e}$$

$$g_l^{in} \le 85 \text{ [s]} \quad \text{and} \quad g_l^{in} \ge 5 \text{ [s]} \qquad \forall \ in \in \{5,6,8\} \tag{6f}$$

where:

(6a), (6b) and (6c) constraints are fuzzy constraints with the triangular shape of Figure 2(a).

The starting values of the proposed method and algorithm are:

- $\overline{m} = 600000$, $T_7^{uc\_eq} = 11000$, $T_{13}^{uc\_eq} = 3500$ (fixed from their starting values);
- algorithms starting values $h^0 = 0.5$ and $N_i = 10$;
- algorithm stop test parameters: $\varepsilon_{SQP} = 10^{-6}$, $\varepsilon_{MSA} = 10^{-3}$;

In order to compare our method with the classical signal settings optimization we have carried out numerical applications using the classical model (5) (Case 1) and numerical tests with our proposed method (6) (Case 2). For both methods two sub-cases have been proposed. The first sub-case (*a*) considers only the constraint on link 7; the second sub-case (*b*) has the constraints on links 7 and 13.

We have also carried out sensitivity analyses employing different membership function shapes such as trapezoidal membership functions. The results in these cases are very similar to those obtained with the triangular shapes and lead to the same conclusions.

Some results obtained in all cases for triangular membership functions are summarized in Table 2. The optimization does not influence heavily the values of equilibrium link flows, but affects the value of the costs on the network and intersection green times.

Table 2 Excerpt of the results

		h^*	g_3^5[s]	g_5^5[s]	g_7^5[s]	g_6^6[s]	g_9^6[s]	g_{10}^8[s]	g_{13}^8[s]	T_7^{uc}	T_{13}^{uc}	T^{uc}
Case 0		-	30	30	30	45	45	45	45	12480	3836	67275
Case 1	*a*	-	5	47.46	37.54	27.46	62.54	66.92	23.08	10917	6038	60950
	b	-	5	47.40	37.60	27.45	62.55	40.70	49.30	10857	3500	63376
Case 2	*a*	0.47	5	5	80	45.06	44.94	44.67	45.33	5816	3796	71074
	b	0.42	6.44	6.44	77.12	45.03	44.97	5	85	5890	2026	82295

The proposed method better satisfies the relative constraints compared to the method with rigid thresholds. Whereas in the classical method the algorithm stops when all the constraints are even slightly satisfied, the maximization of the satisfaction *h* leads to fulfill the fuzzy constraints as much consistently as possible with all the other constraints.

In Case 2 the values of the total link user costs are generally lower than the relative values of Case 1, although the network total user costs are respectively the highest. Nevertheless the classical optimization is not able to reduce link costs by more than about 8.6% – 13%. Conversely the proposed method allows to reduce link costs by about 47.2% – 52.8%. These first results show that the fuzzy optimization is suitable when a high travel time reduction on specific links is required such as emergency situations. Actually the proposed fuzzy optimization behaves as a multi-criteria optimization since it attends at the same time to a minimum network total user cost under the optimization of the other conflicting problem restraints.

6. Conclusions

In literature, network design problems with external, technical or consistency constraints have been largely discussed. These constraints are mainly presented with rigid minimum and/or maximum thresholds. Actually some parameters of the problem can be affected by uncertainty such as vague information on possible available budget, uncertain link flows-capacity fraction values, approximate prefixed objectives and so on.

In this paper we suggest to consider also flexible goal and constraints in a network design problem, jointly with crisp constraints (rigid thresholds). Uncertain data are explicitly represented by fuzzy numbers or fuzzy constraints. In order to include these uncertain/imprecise values, with different uncertainty levels, the network design problem is then specified as a fuzzy programming problem.

After the definition of the optimization model and its resolution algorithms we have applied the proposed approach to a test network in the case of global optimization of signal settings and compared the obtained results with the classical approach.

This first analysis has shown the ability of our method to take into account simultaneously different type of information (certain and uncertain) and conflicting constraints as a multi-objective optimization.

Ongoing research activities are dealing with the inclusion of uncertainty level about link capacity, as well as, about other available vague information (for example on stated preferences on users origin - destination costs). In addition, the proposed optimization method could be included in the framework of equity network design.

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