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Advanced numerical modeling and validation of dynamic thermal behavior in layered composites for active thermography test

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**Politecnico
di Bari**

Department of Mechanics, Mathematics and Management

Ph.D. Program

Aerospace Engineering and Sciences

SSD:IIND-03/A–Mechanical Design and Machine Construction

Final Dissertation

Advanced Numerical Modeling and Validation of Dynamic Thermal Behavior in Layered Composites for Active Thermography Test

by
Sidra Riaz

Supervisor:

Prof. Umberto Galietti

Prof. Maria Cinefra

Coordinator of Ph.D. Program

Prof. M. D. De Tullio

Academic Year (2021 - 2025)



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Abstract

Non-Destructive Evaluation (NDE) techniques, such as thermography, have become essential for detecting and characterizing defects in composite materials, especially in aerospace and other high-performance applications. While traditional numerical approaches, including those implemented in software like ANSYS and COMSOL, have provided valuable insights into thermal behavior, they face limitations in computational efficiency and accuracy when applied to anisotropic, layered composites.

This research introduces advanced numerical modeling methodologies based on the Carrera Unified Formulation (CUF) and the Sublaminar Generalized Unified Formulation (SGUF). For the first time, CUF and SGUF are implemented for transient thermal analysis using active thermography, specifically in Thermography. These formulations enable efficient and accurate simulations of dynamic thermal behavior, capturing interlaminar interactions and thermal gradients that are critical for defect detection. The study begins with a comprehensive review of traditional thermographic techniques and their numerical counterparts. Numerical approaches commonly implemented in finite element tools like Finite difference models (FDM), ANSYS and COMSOL are critically analyzed, revealing significant gaps in computational efficiency and their inability to fully capture interlaminar thermal interactions and anisotropic material behaviors. To address these challenges, CUF and SGUF-based models are developed, offering a more streamlined and accurate framework for transient thermal analysis. These models are further applied in various parametric studies, including lay-up sequence analysis, material anisotropy, and thermal gradient effects, to comprehensively evaluate their performance. Validation of the developed models is conducted through comparisons with experimental data and traditional numerical benchmarks, demonstrating their accuracy and robustness.

Key findings demonstrate that CUF and SGUF enhance computational efficiency and accuracy in predicting thermal responses, such as temperature distribution and heat flux, making them effective for thermal analysis of composite materials. Their efficiency enables the application to real defect scenarios, such as simulating air-filled gaps in T-joint structures for debonding analysis. Moreover, the versatility of these advanced formulations enables their application to complex geometries and real-world scenarios, providing critical insights into defect detection and thermal behavior in layered composites.

Keywords: Non-Destructive Testing (NDT), Lock-in Thermography (LIT), Carrera Unified Formulation (CUF), Sublaminar Generalized Unified Formulation (SGUF), Composite Materials, Thermal Analysis, Finite Element Method (FEM).

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Introduction

Composite materials are widely used in many fields, especially aerospace, due to their excellent strength-to-weight ratio. The increasing use of composite materials in various engineering applications has raised the importance of reliable and efficient Non-Destructive Testing (NDT) methods. However, composite components' safety, integrity, and durability are significantly reduced due to the complex internal structure and the possibility of hidden defects. In such cases, non-destructive testing (NDT) is required to detect defects and evaluate residual strength. Shear analysis, thermal imaging, ultrasonic, and X-ray are some of the NDT methods currently available for testing composite materials. Thermography has significant advantages over other well-known non-destructive testing methods. For example, both the excitation source and detector are contactless with the surface of the test piece. In addition, thermal imaging allows the scanning of large areas of the test sample within a few minutes. In this regard, stimulated or active thermography is based on the accumulation of heat on the surface of the test specimen by an external heat source, such as a halogen/flash lamp or a laser source. The heat waves generated inside the test specimen follow the Fourier heat conduction law. This phenomenon involves the propagation of heat waves into the material in the presence of defects such as delamination, cracks, artificial inclusions, debonding, and air gaps located under the surface of the test specimen. As a result, the defective area may appear warmer or colder than the surrounding area, and the thermophysical properties of the two materials directly affect the heat transfer between them. Depending on the thermal wave shape used for modulating the excitation of the heat source, the active thermography can be classified as Pulsed (PT), Step (ST), and Lock-in (LI).

Numerical modeling is widely used to predict the complex material behavior of anisotropic composites in structural dynamics and strength analysis. In this area, finite element (FE) modeling can also be used to predict the axial dependence of the anisotropic thermal diffusion parameter. Modeling of an accurate CFRP model requires significant computational effort for multilayer laminates, thus requiring the development of equivalent and simplified models for structures containing different fiber orientations in each layer and the corresponding anisotropic material properties. Such equivalent models require validation through experimental measurements, are time-consuming, and suffer from uncertainties due to modeling simplification. Recent trends in other NDE control methods use advanced optimization procedures to improve the accuracy of experimental control. These include established methods for updating finite element models in structural dynamics and advancements aimed at enhancing active thermog-

raphy. This technique can be utilized to update the equivalent finite element model to reflect the actual material properties of the inspected structure, enabling efficient calculation of lateral diffusivity losses and determination of diffusivity for various defect sizes and depths. Traditional numerical software used in thermography has significant limitations that limit its effectiveness in advanced composite material analysis. These tools frequently rely on simplified assumptions and low-fidelity models, which may not adequately capture the anisotropic thermal properties of composite laminates. Furthermore, when working with large-scale or multilayered structures, they can be computationally costly and time and resource-consuming. Their application for accurate defect detection is limited by their capacity to effectively adapt to complicated geometries and limitations in simulating multi-physics interactions.

To address these challenges, this study aims to develop an advanced numerical modeling approach and conduct comprehensive experimental and numerical investigations to enhance the capabilities of active thermography for composite inspection. This study aims to improve defect characterization and accuracy by optimizing inspection parameters through systematic validation with experimental data. The research will also provide a more robust framework for non-destructive testing in aerospace and other industries. Furthermore, this research will advance the fundamental understanding of heat transfer mechanisms in composite materials with complex geometries and various types of defects, potentially leading to improved design guidelines and maintenance strategies for composite structures.

Developing a computationally effective numerical modeling method for predicting the thermal behavior of composite materials with defects using the Sublaminar Generalized Unified Formulation (SGUF) and Carrera Unified Formulation (CUF) is one of the main motivating factors behind this research. Because these formulations accurately capture the complicated anisotropic and multi-layered nature of composite structures, they offer clear advantages over conventional commercial finite element analysis (FEA) tools, such as COMSOL Multiphysics.

CUF and SGUF-based models demonstrate how composites behave thermally, taking into account the effects of different layer orientations and material anisotropy. CUF and SGUF approaches provide a high level of accuracy while significantly reducing computational costs, unlike traditional FEA methodologies that typically require many degrees of freedom to model complex components. This efficiency is particularly useful for modeling delamination scenarios, such as air-filled gaps or other types of defects that are difficult to represent using traditional modeling methods. The low computational load allows the modeling of large or complex composite structures and allows for more realistic failure scenarios, such as delamination or localized voids, to be analyzed without excessive computing resources. This allows for more in-depth studies of the thermal response of composites using active thermography. This study develops these advanced numerical tools to improve the understanding and application of active thermography for non-destructive testing (NDT) of composite structures. By providing more accurate modeling of defects and thermal effects, these models have the potential to improve the reliability, efficiency, and effectiveness of NDT inspections, ultimately advancing

the inspection and maintenance of complex composite materials in engineering applications.

The primary objectives for the PhD work can be broadly outlined as follows:

Research Objectives

To develop Finite Element Method (FEM) models based on the Carrera Unified Formulation (CUF) and the Sublaminated Generalized Unified Formulation (SGUF) for non-destructive testing (NDT), focusing on thermography for dynamic analysis. The work includes implementing a transient solution and incorporating boundary conditions to advance the study of thermography.

Validate the FEM model through comparisons with traditional numerical methods and experimental results.

Verify the sensitivity of the proposed model to key thermographic test parameters, such as excitation frequency, defect size, and depth.

Apply the developed numerical model to realistic delamination scenarios and complex structures, such as debonding in T-joints, to accurately capture thermal behavior with computational efficiency, enabling applications in both industrial and academic settings.

This thesis is organized into four main chapters and a concluding chapter. A brief overview of each chapter is provided below:

Chapter 2 - State of the Art :Thermography and Traditional FEM methods:

This chapter begins with a state-of-the-art review of advanced numerical modeling techniques and their applications in composite materials. It introduces active thermography and nondestructive testing (NDT) techniques for composites, with a particular focus on numerical tools used for damage modeling. Traditional FEM-based modeling approaches in COMSOL and ANSYS are reviewed, highlighting their strengths and limitations in simulating thermal behavior and defect propagation in composite materials. In addition, the role of adhesion in composite structures is thoroughly examined, particularly in the context of adhesive joints, T-joints, and debonding scenarios. Adhesion significantly impacts the propagation of defects, debonding behavior, and overall structural integrity in composites.

Chapter 3 - Carrera Unified Formulation (CUF)

This chapter begins with a state-of-the-art review of advanced numerical modeling techniques, focusing on their implementation and applications in composite materials. It introduces advanced methods, including the Carrera Unified Formulation (CUF) and the Sublaminated Generalized Unified Formulation (SGUF), highlighting their potential for accurate damage prediction in composites. The discussion transitions into the background theory of these formu-

lations, detailing their theoretical foundations and computational advantages. While CUF and SGUF have demonstrated exceptional capabilities in structural analysis and damage prediction, their implementation in non-destructive testing (NDT) methods such as active thermography remains unexplored. This chapter delves into their potential applications, emphasizing how these formulations could expand the capabilities of defect detection and material condition monitoring. By bridging advanced numerical modeling with NDT techniques, this work aims to pave the way for innovative approaches to damage assessment in composite materials.

Chapter 4 - Methodology:

This chapter provides a detailed description of the experimental and numerical techniques employed. Along with important test material and defect criteria, the lock-in thermography setup is explained, including test specimens, infrared cameras, and the IRTA software for thermal image processing. The numerical modeling methodology in this study employs advanced techniques, including the Carrera Unified Formulation (CUF) and the Sublaminar Generalized Unified Formulation (SGUF), combined with heat transfer simulations in COMSOL. For the first time, a transit solution for dynamic thermal analysis has been implemented using CUF and SGUF methods. Boundary conditions have been incorporated to enhance the accuracy and efficiency of thermal behavior prediction. Furthermore, the proposed CUF-SGUF framework has been successfully applied to complex structures, such as T-joint configurations, making a significant contribution to the field. In terms of thermography, this methodology not only enhances defect detection but also helps reduce the number of degrees of freedom, improving the overall efficiency and accuracy of thermal analysis.

Chapter 5 Results and Discussion:

This chapter presents the outcomes of the research through four key sections:

Model Development and Validation:

This section focuses on implementing the advanced CUF and SGUF methodologies, validating them against traditional numerical tools like COMSOL and experimental results.

Sensitivity Analysis:

In this section, the robustness of the SGUF methodology is examined through sensitivity studies of subsurface defect detection under varying conditions.

Effect of Thermal Anisotropy and Lay-up Configuration:

This section explores the impact of material properties and lay-up sequences on defect detection using Lock-in Thermography test.

Application: Implementation of CUF Using MUL2 Code for Simulating Debonding in Complex T-Joint Structures

This section applies the developed methodology to simulate delamination in CFRP T-joints, and modeling of air-filled gaps to demonstrate its practical utility.

Conclusion and Outlooks:

This Chapter summarizes the key findings and insights from this study, emphasizing the effectiveness of the advanced numerical methodologies (CUF and SGUF) for defect detection and thermal analysis of composite materials. Additionally, this chapter outlines potential directions for future work.

References are listed at the end, with details of the transient solver and post-processing algorithms provided in the Appendix.

Chapter 2

Dynamic Thermal Behavior in Layered Composites for Active Thermography Test: A Literature Review

2.1 State of the Art and Numerical Approaches for Thermography

Nondestructive testing (NDT) has been widely studied as a critical methodology for assessing the integrity and reliability of materials across various industries. NDT is a method of detecting surface and internal defects or metallurgical conditions without compromising the performance of the material. NDT consists of multiple testing approaches that can be applied across different materials and applications. These include eddy current testing, radiography, magnetic particle inspection, and ultrasonics, each with its advantages and disadvantages. The larger field of Non-Destructive Evaluation (NDE) focuses on identifying, characterizing, and quantifying damages in materials' surfaces and interiors while maintaining that their original properties and functionality remain unaffected. Among various NDT techniques, thermography has proven to be highly effective for defect detection. In particular, its application to composite materials is of great interest due to its complex anisotropic nature and widespread use in aerospace, automotive, and structural applications. The literature contains an extensive range of details on the importance of composite materials in aeronautical applications. Table 2.1 summarizes the key NDT methods, their applications, advantages, and limitations, providing a comprehensive overview of their role in material inspection.

Thermal analysis of metallic materials differs significantly from that of composite materials due to the higher thermal conductivity and isotropic nature of metals. Numerical simulations for metallic structures often employ Finite Element Methods (FEM), Finite Difference Methods (FDM), and Analytical Models to study heat transfer, thermal stresses, and defect detection. In particular, Lock-in Thermography (LIT), Pulsed Thermography, and Eddy Current Thermography are widely used non-destructive testing (NDT) techniques for evaluating metallic materials.

These methods rely on the heat diffusion process to identify subsurface defects such as cracks, voids, and corrosion. For metals, apparent in-plane and through-thickness thermal conductivity play a crucial role in determining heat propagation, affecting the detectability of defects. Studies, such as those by Mendioroz et al., [1] have analyzed the impact of thermal diffusivity and defect depth in metallic samples, providing a framework for optimizing infrared thermography applications. Compared to composites, where anisotropy influences heat flow, metallic materials exhibit more uniform heat distribution, making defect detection more dependent on heat source modulation frequency and defect geometry. Incorporating simulation-based approaches for thermal analysis in metals enables better prediction of defect characteristics, complementing experimental NDT methods for quality assessment and structural health monitoring.

Kesarwani et al. [2] and Irving et al. [3] emphasize the unique qualities of these materials, especially their exceptional strength-to-weight ratio, corrosion resistance, and long-term durability. According to Harris et al. [4], their remarkable specialized qualities particularly their specific strength and stiffness have made their extensive use in the automotive, aerospace, and aviation industries. But Soni et al. [5] highlight a serious drawback. Engineered materials known as composites are created by joining two or more basic materials that have entirely distinct chemical or physical characteristics. When compared to its constituent parts, the combined material has better qualities.

Table 2.1: Summary of NDT methods, their applications, advantages, and limitations.

Method	Inspection Scope	Common Applications	Key Benefits	Key Limitations
Acoustic Emission (AE)	Crack growth, delamination	Composite and fiber-based materials	Identify fatigue cracks, fractures, and delamination	Requires skilled operators; data analysis can be time-intensive.
Ultrasonic Testing (UT)	Surface and internal flaws	Forgings, welded joints, and structural components	High accuracy in defect detection; versatile for various shapes	Challenging for complex geometries; testing can be slow.
X-ray Imaging	Internal discontinuities	Castings, non-metallic parts, and composites	Effective for porosity, slag, and inclusions	Limited for perpendicular cracks; depth measurement is difficult; high cost for on-site use.
Eddy Current Testing	Surface-level and shallow defects	Conductive materials	Quick testing; minimal surface preparation needed	Sensitive to geometry changes; noise in signals can affect results.
Infrared Thermography (IRT)	Surface and inter-layer evaluation	Metals and composites	Non-invasive, covers large areas efficiently	Complex algorithms needed for depth estimation.
Magnetic Particle Inspection	Surface and subsurface flaws	Ferromagnetic materials	Low-cost, portable; detect near-surface defects	Only applicable to ferromagnetic materials.

According to Connolly et al. [6], CFRP structures that have hidden defects may break catastrophically under service conditions. Expanding upon this basis, Fang et al. [7] have shown that although CFRPs have excellent mechanical qualities, their complicated internal structure presents major difficulties for inspection techniques. According to additional research by Marani et al. [8], the layered and heterogeneous features of CFRPs makes it particularly difficult to identify different kinds of defects, such as fiber breakage, matrix cracks, and delaminations. These issues have been specifically examined by Fang et al. [9], who point out that conventional inspection techniques frequently have trouble with the anisotropic characteristics and numerous failure mechanisms of CFRP materials. The complex and heterogeneous nature of composite

materials makes them susceptible to internal defects such as delaminations, voids, and fiber breakage, which can significantly impact their structural integrity. To ensure their reliability and performance, advanced non-destructive testing (NDT) techniques, particularly thermography, are extensively utilized for defect detection and damage assessment. Thermography, initially introduced in the aerospace industry for detecting defects in thin aluminum aircraft skins, has evolved into a versatile nondestructive evaluation (NDE) technique (Mandic et al.,) [10]. Over time, its application has expanded and proven effective in identifying structural defects such as delaminations and subsurface damage caused by impacts in advanced composite materials (Maldague and Marinetti, 1996; Maldague, 2001; Vavilov et al., 2000) [11–13]. Early foundational work by Maldague and Marinetti [14] established a framework for integrating computational methods with thermography. Their study demonstrated the relationship between thermal signal patterns and underlying defects, offering numerical modeling techniques to detect delaminations. This innovative approach laid the foundation for quantitative thermal analysis, paving the way for thermography’s application in various industrial contexts.

Pitarresi [15, 16] has presented an experimental analysis, while [17–20] has offered a theoretical framework for describing this technique. The anisotropic nature of composite materials, particularly their thermal conductivity, has a significant impact on heat propagation through the structures. This anisotropy is a direct result of the fiber orientation and layup sequence of the composite laminate. Consequently, the thermal wave propagation and the resulting surface temperature distribution can vary significantly depending on the thermal anisotropy of the material and the specific layup configuration of the composite plate [15, 16]. Ply layup’s effect on CFRPs has not yet been thoroughly studied. Unidirectionally reinforced layers are widely layered to make CFRPs. Depending on the application, different ply layups, such as unidirectional (UD), cross-ply, or angle-ply laminates, are used. The difference in the Layup could also affect the inspection results. Furthermore, in previous studies, only one or two-way thermal conductivity or thermal conductivity was evaluated. Using the laser flash approach, Ellis and McDanel [21] investigated the thermal conductivity of copper matrix composites both parallel and perpendicular to the fiber direction.

Pawer et al., [22] implemented 3D normalization to overcome the problems arising due to non-uniform heating and lateral heat diffusion in the case of pulsed IR thermographic NDT. In this case, normalization is carried out by dividing an experimental IR image sequence by the synthetic sequence which is calculated by solving the corresponding 3D problem of heat conduction. As a result of 3D normalization, the artefacts appearing due to uneven heating and absorption can be subdued through data normalization (division). Furthermore, fully automatic defect detection becomes possible as defining a reference point for sound area is not required, as opposed to thermal contrast methods. The 3D normalization procedure can be implemented as either sequence subtraction or division as seen in the Figure 2.1.

The mathematical model of classical PIRT (Figure 2.2) is trivial. The mathematical model for a three-layer body containing several parallelepiped shaped defects involves solving a 3D Cartesian-coordinate transient heat conduction problem. The body under inspection is heated

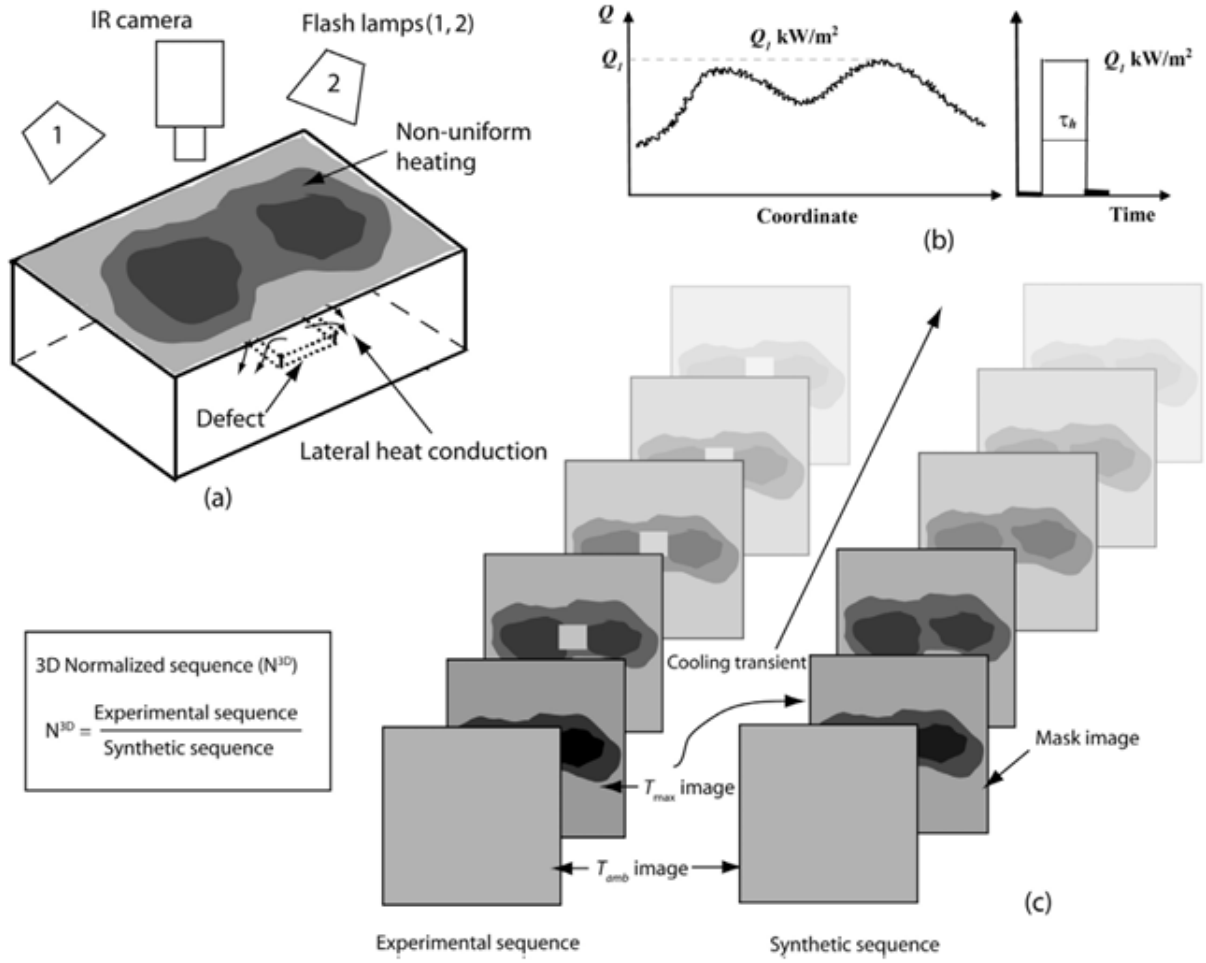


Figure 2.1: (a)Typical PIRT procedure, (b)heating profile and (c)3D normalization procedure.

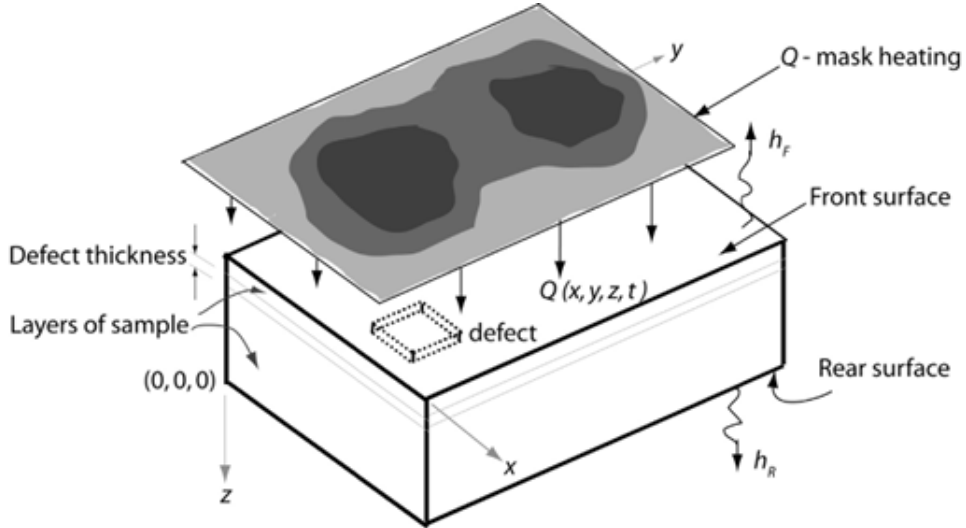


Figure 2.2: 3D thermal mathematical model.

on the front (F) surface, whereas both front and rear (R) surfaces exchange energy with the ambient according to Newton's law.

Lopez et al., [23], describe a methodology for the modeling and simulation of a pulsed thermography inspection of carbon fiber-reinforced composites (Figure 2.3). A thermal numerical

model based on the 3D transient heat conduction equation for heterogeneous media is proposed to model the thermal response of the composite medium when a thermal pulse is applied to its surface. The solution of the model is developed using the finite volume method, in which the approximated equations are obtained by performing energy balances for each elementary volume. The experimental validation of the thermal numerical model consisted of a comparison of the thermal decay curves for defective areas obtained through numerical simulation and with a pulsed thermography inspection of a laminated composite specimen with simulated defects (Teflon inserts). A parametric study was carried out to analyze the influence of the irradiation power density, non-uniform heating, and characteristics associated with the defects on quantitative variables such as the onset time and thermal contrast. Numerical results showed that the variable thermal contrast is highly sensitive to changes in the defect thickness, aspect ratio, and parameters related to the external stimulation. Furthermore, an increase in the detectability of defects, especially those with lower aspect ratios, can be achieved by increasing the irradiation power.

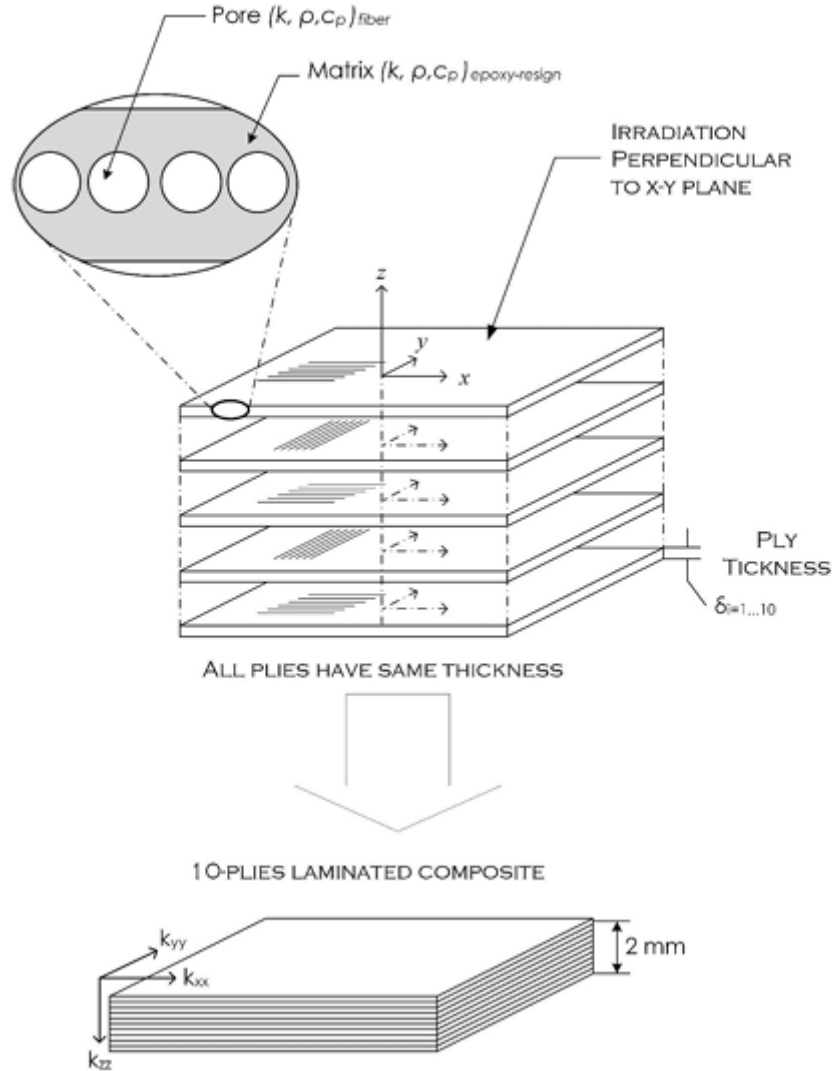


Figure 2.3: Schematic representation of the composite medium and its constituents.

Figure (2.4), which offers a thorough timeline of significant developments and techniques in

the discipline, provides a thorough overview of the development of numerical approaches in thermographic analysis.

One of the widely used forms of thermography is active infrared thermography, which employs external heat sources such as lamps or lasers to transfer heat through or across the object of interest (Almond et al.; Dell et al.; Dwivedi et al.; Ibarra-Castanedo et al.; Vavilov et al.,) [24–31]. An infrared imaging device is then used to capture the surface’s thermal gradients, with defects such as cracks, inclusions, voids, and delaminations producing observable thermal anomalies due to disrupted heat transfer rates (Mandic et al.,) [10]. For instance, delaminations between fiber-reinforced polymer (FRP) layers and concrete structures act as insulators, resulting in higher surface temperatures in delaminated regions compared to bonded ones. Maldague [11, 12] highlighted that the accuracy of defect detection in thermography is highly dependent on the consistency of the heat source. Uneven heating can introduce misleading thermal gradients, which may be misinterpreted as defects.

Advancements in thermography have further enhanced its capabilities. Vavilov et al. in 2000 [13] investigated three-dimensional heat flow modeling to improve defect characterization in complex materials. Their work highlighted the importance of integrating computational intelligence with thermographic evaluations, enabling more precise defect identification and classification. Such advancements emphasize the potential of thermography as a powerful tool for evaluating defects in various materials and structures.

Numerical modeling has emerged as a vital tool for complementing experimental thermography techniques, enabling the simulation and analysis of thermal behavior in complex systems [32–35]. The combination of numerical techniques with thermographic testing has greatly improved the ability to find and characterize flaws, from the earliest theoretical frameworks to contemporary software-driven simulations.

Studies based on:

- General developments in thermography
- COMSOL-based multiphysics modeling
- ANSYS-based thermal analysis
- Numerical methods and computational advances (Python)

- T Matarrese et al. (2023) - Comparison among four lock-in algorithms in transient regime on CFRP
- Chen et al. (2023) - Conducted industrial-scale modeling for composite laminates.
- Thompson et al. (2023) - Introduced digital twin technologies for thermal analysis.
- Martinez et al. (2023) - Comparison of numerical models with experiments.
- Gupta et al. (2023) - Hybrid techniques combining Lock-in thermography and ML.
- Hassan et al. (2023) - Ultrafast thermal wave propagation in composites.
- Alhammad, Muflih, et al. (2022) - Damage Detection Technique for Composites Based on Thermographic Image Processing.
- Liu et al. (2022) - Characterized defects using integrated machine learning approaches.
- Lee and Zhang (2022) - Investigated machine learning for automated defect.
- Chen et al. (2022) - Developed AI-driven algorithms for enhanced thermal imaging.
- Patel et al. (2022) - Defect detection accuracy using convolutional neural networks.
- He, Yunze et al. (2021) - Infrared machine vision and infrared thermography with deep learning.
- Zhang et al. (2021) - Analyzed layered composite structures for thermal stress.
- Kumar et al. (2021) - Coupled thermomechanical simulations for defect localization.
- Rodriguez et al. (2021) - Developed adaptive mesh refinement for thermal simulations.
- Singh et al. (2021) - Validated industrial-scale applications of Lock-in thermography.
- Bang et al. (2020) - Defect identification in composite materials via thermography.
- Wang et al. (2020) - Developed transient analysis techniques for CFRP materials.
- Park and Kim (2020) - Implemented GPU-accelerated FEM for real-time defect analysis.
- Xu et al. (2020) - Proposed hybrid thermal imaging methods for defect detection.
- Omar et al. (2019) - Developed multiphysics thermal wave models for defect detection.
- Sharma et al. (2019) - Investigated the influence of thermal diffusivity .
- Sirikham et al. (2018) - Estimation of damage thickness in fiber-reinforced composites.
- Bison et al.(2017) - Introduced thermal and mechanical model for defect detection.
- Davide Palumbo et al. (2015) - Quantitative damage evaluation of composite materials.
- U. Galietti et al.(2012) - Non destructive evaluation of composite materials with new thermal methods.
- He and Balageas (2012) - Application of Lock-in thermography for subsurface defect.
- Ibarra et al. (2010) -Advanced thermography techniques for defect characterization.
- Meola et al. (2008) - Integrated pulsed thermography for defect detection.
- Zalameda et al. (2007) - Thermographic signal reconstruction for aerospace applications.
- Krishnapillai et al. (2005) - Developed 2D finite element modeling for thermal analysis.
- Umberto Galietti(2004) Defect Detection in Composite Materials by Thermography and Neural Networks
- Maldague and Marinetti (1996) - First numerical model for delamination detection.

Figure 2.4: A Comprehensive Timeline of Numerical Approaches in Thermographic Analysis

Maldague and Marinetti's innovative study [11], which used numerical models to simulate thermal wave propagation and correlate thermal patterns with underlying defects, is where the use of numerical approaches in thermography first appeared. Their research showed that delamination and subsurface flaws could be accurately predicted by integrating computational methods with experimental data. The use of the Finite Element Method (FEM) for thermal analysis increased substantially in the late 1990s and early 2000s, building on this foundation. Advanced defect identification in thin-layered structures was made possible by Mandic et al. [10], who used FEM-based simulations to study heat transfer in aerospace materials. Three-dimensional heat flow simulations have been introduced by Vavilov and Burleigh [13] to these efforts, giving a more accurate depiction of thermal behavior in multilayered composites. By integrating machine learning techniques with numerical modeling, Galletti et al. [36] showed that mixing neural networks with FEM simulations improves defect classification. The significance of computational intelligence in thermographic analysis is shown by these developments.

According to history, the FEM was first used when the following two advancements were made available:

1. **Technological:** The developing of computational devices that can do mathematical tasks rapidly.
2. **Mathematical:** The creation of mathematical techniques for solving "approximated" or "weak" types of differential equations, including partial differential equations (PDEs) and ordinary differential equations (ODEs). The weight residual method (WRM) is arguably the most important of these techniques.

An important development in the thermal-mechanical analysis of materials is the development of finite element modeling (FEM). Initially concentrating on structural analysis, FEM has successfully extended into thermal mechanics. In contrast to one- and two-dimensional methods, Krishnapillai et al. [37] showed that three-dimensional finite element modeling of pulse thermography offers more thorough insights. Basic geometric models were created in one and two dimensions by Maillet et al. [38] in earlier work, but these were insufficient for complex component analysis. With notable contributions from Perez et al. [39] and Maldague et al. [14] emphasizing subsurface defect responses in thin composite materials, the study of flash thermography has been well documented in the literature. However, a significant drawback was pointed out by Krishnapillai et al. [37]: the difficulty of modeling defect information from experimental data when testing conditions differ from the initial configuration. This emphasizes the significance of creating thorough numerical models that accomplish two goals: they improve basic comprehension of the underlying processes and offer useful testing suggestions. Building on the advancements in numerical modeling, software such as ANSYS Finite difference method and COMSOL has been widely utilized to simulate and analyze thermal behavior, offering a solution for complex systems and enhancing the accuracy of thermographic defect detection.

ANSYS is a widely used computational tool for simulating transient thermal behavior in composite materials, particularly for thermographic applications aimed at defect detection. Its robust Finite Element Analysis (FEA) capabilities, which make use of complex meshing

strategies, solver algorithms, and parametric studies, enable the very accurate solution of heat transfer problems. A complete platform for thermal analysis and composite material defect identification is offered by ANSYS. An essential tool for thermographic applications, ANSYS enables researchers to precisely simulate and study the thermal response of composites by utilizing sophisticated meshing, solver capabilities, and parametric investigations.

By identifying how sensitive the thermal response is to changes in material properties, parametric studies can identify subsurface defects. The use of ANSYS for composite defect identification has been emphasized in many research studies. To improve the accuracy of defect detection, Fernandes et al. [40] used ANSYS simulations to compare changes in heat conductivity in composites. To investigate subsurface faults, Saleh et al., [41], used transient thermal analysis in ANSYS to simulate heat diffusion and compare it to experimental thermography. To improve defect characterization in composites and refine thermal parameters, Gupta et al. [42] showed how to integrate ANSYS with optimization techniques. Similar to this, Wang et al. stressed how crucial it is to describe accurate thermal behavior in fiber-reinforced composites employing anisotropic thermal characteristics in ANSYS simulations. In thermography, ANSYS has been widely used for thermal simulations. With an emphasis on defect depth estimates, researchers like Kumar et al. [43] showed how to use ANSYS to simulate heat wave propagation in composite laminates. The software's capacity to model heat distribution in anisotropic materials to characterize subsurface cracks was emphasized in another study by Thajeel et al. [44]. A systematic methodology to use finite element simulations for deep learning-based predictive modeling was presented by Qiang Fang et al. [7].

The software COMSOL Multiphysics is a perfect tool for simulating and optimizing defect identification procedures in composite laminates since it integrates several physics and parametric estimating aspects. Fernandes et al. [40] and Saeed et al. [45] demonstrated the software's effectiveness in modeling transient heat transfer inside multilayered composites, especially for defect detection applications. Their work demonstrated COMSOL's capacity to simulate complex thermal phenomena in composite structures. Cannas et al. [46] and Cannard et al. [47] emphasized the significance of coupled physics analysis in defect detection. They found that incorporating heat transfer and structural deformation improves the representation of delamination effects in composite materials. Saleh et al. [41] expanded the research area by studying how variations in thermal conductivity affect subsurface defect detection.

In recent research, Finite Element Method (FEM) modeling has demonstrated encouraging results in simulating thermographic responses of composite materials. Several researchers have used FEM to evaluate their numerical models, including Grys et al., Mulaveesala and Ghali, Peng et al., Streza et al., and Zhao et al. [48–52]. Commercial FEA software programs, particularly COMSOL, are growing popular in this discipline. Junyan et al. (2012), Aoukili et al. (2018), and Jinlong et al. (2015) used these platforms to model thermal behavior and estimate defect places in materials. However, it is important to note that, while these commercial software programs have powerful simulation capabilities, they frequently require large computational resources.

Traditional FEA modeling is computationally intensive, thus researchers are looking for more efficient ways that preserve accuracy while decreasing computational overhead. Several researchers, including Hernandez et al., Cheng et al., Chady et al., and Miskiewicz et al., have demonstrated this necessity. [53–56]. The combination of computer modeling and enhanced detection techniques resulted in a considerable advancement in the industry. Wei et al. [57] made a significant contribution by merging deep learning and infrared thermography for impact damage detection in curved CFRP specimens. The technique generated remarkable F1-scores of 92.74% and 87.39% for mid-wave and long-wave infrared pictures, respectively. Marani et al. [58] improved the field by inventing a new approach employing pulsed thermography and demonstrating outstanding accuracy through the deployment of decision forest classifiers.

Automated defect identification has made significant progress thanks to various computational methodologies. Saeed et al. [45] created a revolutionary CNN-based technique with transfer learning that achieved 88% accuracy in-depth estimation for CFRP materials. Pan et al. [59] successfully integrated IRT and Mask R-CNN to detect interior defects in FRP-reinforced concrete structures with an average accuracy of 96.28%. Li et al. [60] proved the effectiveness of CNN-based techniques in detecting pavement defects, attaining an impressive 97.36% global pixel accuracy. These computation approaches have improved significantly in the industrial domain. Jha et al. [61] offered a detailed analysis of the progression from traditional manual inspection methods to modern deep learning-based automated optical inspection. They addressed both the advantages and challenges associated with pixel-level segmentation approaches. While Tur et al. [62] concentrated on aerospace applications, their study was rather restricted to particular structural components, as opposed to subsequent studies that covered a wider range of composite models and defect kinds. Building on the previous discussion of numerical modeling, these techniques are increasingly applied to simulate real-life scenarios such as delamination and debonding in complex geometries, including critical components like turbine blades, where understanding the thermal behavior and structural integrity under operational conditions is essential for improving performance and safety.

Various intricate factors can cause blade failures during operation, and they are largely responsible for accidents and wind turbine damage [63]. The materials used to make blades are frequently woven fabrics with different patterns. Chou et al. [63] state that converting 2D materials into three-dimensional (3D) configurations is necessary for the creation of intricate interior forms including spars, shear webs, and other connections. This change, however, creates spaces in the structure where the load is transferred over regions that have little to no fiber reinforcement. These areas serve as natural places for the onset of delamination.

An adhesive layer is applied between surfaces during the adhesive bonding process, which creates a strong bind. The structural continuity, high strength-to-weight ratio, design flexibility, and ease of manufacture of adhesively bonded joints make them a desirable alternative for mechanical joints. These advantages were highlighted by Banea et al. [64] and Baker et al. [65], emphasizing the increasing significance of adhesively bonded joints in recent structural applications.

According to Toftegaard et al. [66], Justo et al. [67], and Koh et al. [68], composite T-joints are widely utilized in the aerospace and marine industries. Recent research has focused on employing different manufacturing techniques to increase the strength of these joints. For example, the effects of various manufacturing techniques on joint performance were examined by Owens et al. [69], Shenoi et al. [70], and Theotokoglou et al. [71]. Similarly, Aized et al. [72] and Denkana et al. [73] investigated robotic and automated techniques to enhance joint quality and efficiency.

Manufacturers have adopted cost-efficient procedures as a result of the need to create massive components at increasing production rates. Belnoue et al. [74] and Kozaczuk et al. [75] addressed improvements in manufacturing technologies that satisfy the needs of lower fabrication costs and reduced structural weight, especially when contrasted with conventional welded and bolted joints. Lichtinger et al. [76] highlighted how these developments contribute to increased industrial output.

Both the adhesive component and the bonding procedure have an impact on the quality of adhesively bonded joints, which is measured by maximum strength and stiffness. To assess joint quality, non-destructive testing (NDT) methods have been used extensively. Several NDT techniques for evaluating debonding and damage in composite structures were reviewed by Adams et al. [77], Mao et al. [78], and Ma et al. [79]. Stimulated or active thermography in particular has drawn interest because of its rapid and effective full-field providing of data. Alexis et al. [80] and Johnson et al. [81] examined the benefits of active thermography in terms of quick and precise defect detection, while Palumbo et al. [82] and Lai et al. [83] illustrated how well it works to identify debonded areas.

Finite element (FE) models have significantly simplified the simulation of delamination in composite structures. Advanced FE models were created by Allegri et al. [84] and Wang et al. [85, 86] to model the delamination behavior of composite T-joints. Additional research on detecting delamination in these joints was conducted by Bianchi et al. [87], Chen et al. [88, 89], and Baldi et al. [90].

The failure mechanisms of composite T-joints have also been investigated experimentally. Delamination initiation and propagation were studied from several angles by Cartié et al. [91], Burns et al. [92], and Shenoi et al. [93]. Marcadon et al. [94], Read et al. [95], and Wang et al. [86] investigated the function of stress concentration at the corner of the joint, an area that is dependent to the initiation of delamination because of an excessive amount of stress. According to Mouritz et al., [96], the reinforcing configuration affects how cracks spread, with cracks either following the flange-skin bond line or both the central web and the bond line. They also pointed out that the manufacturing process used has a big impact on the qualities of composite T-joints and how well toughening techniques work.

2.2 Theoretical Background

2.2.1 Composite Materials and Challenges in Defect Detection

Composite materials, known for their superior strength-to-weight ratio and versatility, are widely used in industries such as aerospace, automotive, and construction. However, defect detection in these materials presents unique challenges due to their heterogeneous nature. Composites are generally classified into three categories: particle-reinforced, fiber-reinforced, and structural composites. Each type has distinct characteristics that affect its behavior under stress and loading conditions. Defects such as delamination, voids, and fiber breakage can significantly impact the performance of these materials, making accurate and reliable inspection techniques crucial for ensuring their integrity. Advanced non-destructive testing methods, including thermography, are commonly employed to detect such defects and evaluate the overall health of composite structures. Figure 2.5 illustrates the classification of composites into particle-reinforced, fiber-reinforced, and structural composites. The common defects identified in CFRPs are depicted in Figure 2.6.

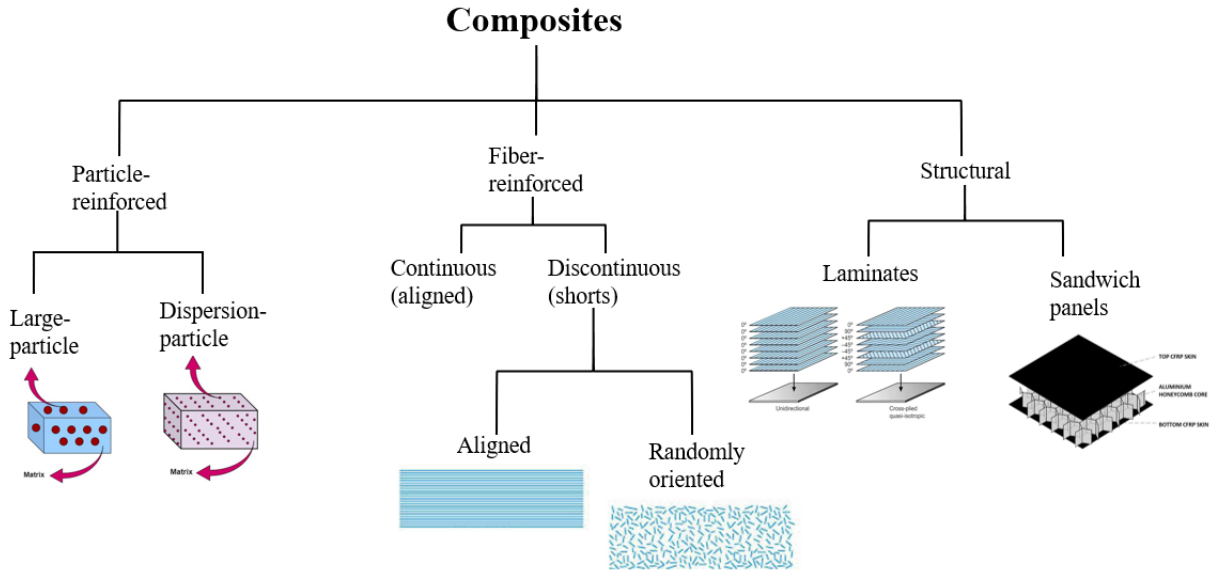


Figure 2.5: Classification of composites into particle-reinforced, fiber-reinforced, and structural composites

2.2.2 Lock-in Thermography Theory

Lock-in Thermography [31, 97, 98] has become an attractive technique for identifying subsurface defects in recent years by taking use of the heat diffusion properties of materials under thermal excitation [99–101]. LIT is particularly advantageous for inspecting composite materials, where conventional methods may struggle due to the heterogeneous and anisotropic nature of these materials [102, 103]. Despite its growing application, the sensitivity of LIT for defect detection remains influenced by multiple parameters, and understanding how these components

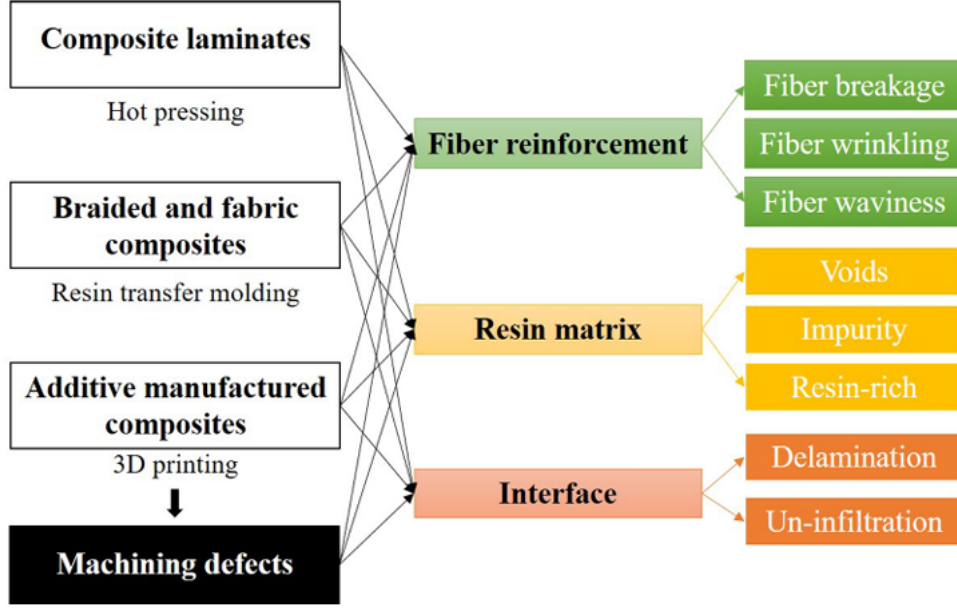


Figure 2.6: Common defect in composites

interact and affect detection capabilities is necessary to optimize LIT for particular inspection tasks [18, 29–31, 49, 104, 105]. LIT is performed by periodically exciting and capturing the resulting surface temperature response over time [30, 49–51, 104]. The presence of subsurface defects causes variations in heat flow, which are detected by the thermal waves as phase shifts in the thermal signal. The thermal response is significantly affected by the defect’s size and depth as well as the frequency of thermal excitation. In this method, thermal waves are created by periodically applying heat to the specimen surface, typically using an external heat source such as halogen lamps, as shown in the schematic diagram Figure 2.7.

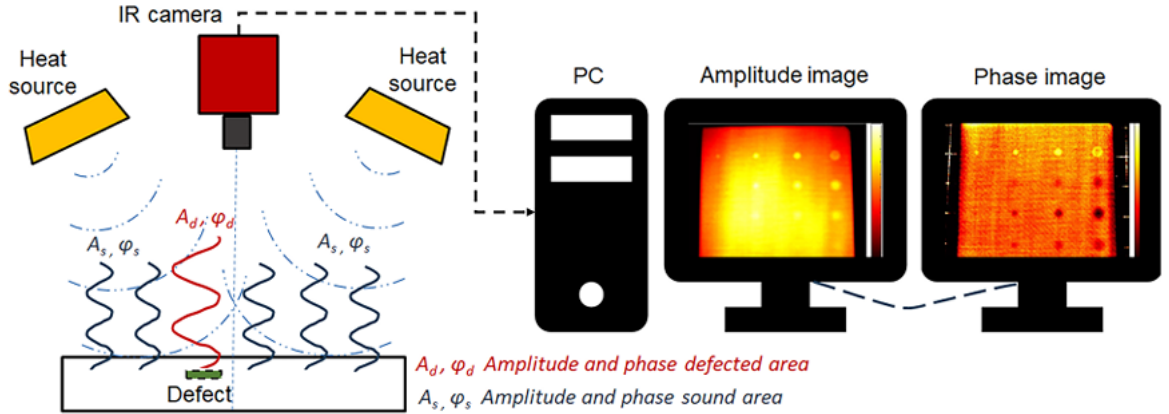


Figure 2.7: A schematic representation of the lock-in set-up

The significance of excitation frequency in LT has been emphasized by earlier research [50, 106–109], as it directly affects depth resolution for subsurface defects. A multi-frequency method can improve sensitivity over a range of defect profiles, whereas a single excitation frequency can limit the detection of defects to specific depths [18, 20, 110]. Phase contrast is a crucial parameter in detecting and measuring defects, and when the excitation frequency is

set properly, it can maximize the detection of specific defect depths. However, inappropriate excitation frequency settings might decrease sensitivity, particularly for smaller or deeper flaws. For this reason, frequency selection criteria based on defect and material characteristics must be developed. As a result, comprehending how these factors interact may allow inspectors to customize LIT settings to meet particular detection requirements and raise inspection accuracy. To study the temperature behavior on the component's surface and stimulate the material with thermal waves that cause heat flow, thermographic techniques all depend on an external heat source. This technique allows us to identify a defect by changing the heat distribution within the material. Consequently, there are typical heat distributions that change surface temperatures in the region. In this study, LT was used to identify the simulated defects in CFRP specimens. LT analysis uses a modulated heat source to excite the sample at an appropriate frequency to obtain information regarding the simulated defects. The literature has utilized this strategy to reduce test times. The phase of the thermal wave is directly related to the depth of the defect, z , through the thermal diffusion length μ . The relationship between the phase map and the thermal diffusion length μ (which represents the depth of penetration of thermal waves) is governed by the Equation 2.1:

$$\varphi(z) = \frac{z}{\mu} \quad (2.1)$$

The sample undergoes heat transfer, with the depth achieved (z) being defined by the thermal diffusion length (μ). The determination of μ relies on the thermophysical characteristics of the material and the modulation period T .

$$\mu = \sqrt{\frac{2k}{\omega \rho c_p}} = \sqrt{\frac{2\alpha}{\omega}} = \sqrt{\frac{\alpha T}{\pi}} \quad (2.2)$$

where α shows thermal diffusivity, ω indicates angular frequency, C_p represents specific heat at constant pressure, and k implies thermal conductivity. Although only numerical modeling results are used in this study, the principle of LIT as described provides the theoretical basis for simulating heat propagation and defect detection. By leveraging Finite Element Analysis (FEA) to replicate the LIT setup, this research examines how varying excitation frequency, defect size, and defect depth influence the phase and thermal response, thus enabling a quantitative sensitivity analysis for subsurface defect detection in composite materials. LIT is particularly suited for composite materials due to its ability to detect small defects, such as delaminations, voids, and cracks, with high precision. Its depth penetration capability depends on the thermal diffusivity of the material and the modulation frequency.

2.2.2.1 Experimental Test Parameters

In thermographic analysis, the experimental setup and test conditions are extremely important. It is essential to choose the correct equipment and accurately set all of the instrumentation to meet the investigation's major goals:

1. Identifying the defects
2. Characterizing defects in terms of the size, depth, and shape

The hardware is composed of two main components:

1. Heat sources
2. IR camera

The experimental setup can be enhanced by adding additional hardware, such as sensors for signal monitoring like a microwave sensor or luxometer, and hardware needed to manage the heat sources. Furthermore, we can employ additional heat source-related accessories such as IR filters and focus lenses to control the laser beam, among other things. The set-up configuration 2.8 is determined by the component's dimension and shape, as well as the ability to inspect both sides. Two primary configurations can be used:

1. **Reflection setup:** preventing reflections from heat sources.
2. **Transmission setup:** protecting the infrared camera from heat sources (usually masking the heat source for small samples).

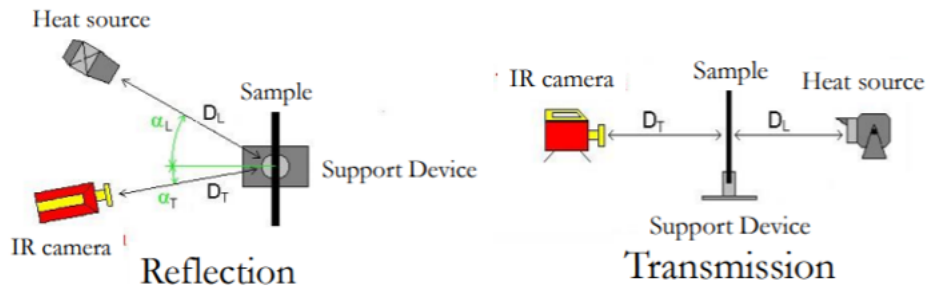


Figure 2.8: (a) Reflection and Transmission configurations

The IR camera should be set at a viewing angle of zero. High viewing angles can give a "deformed/distorted" image, which can cause inaccuracies in defect size and depth estimation.

2.2.2.2 IR Camera and heat sources distances

The influence of the heat source's distance is determined by the heat source used. The chosen heat source must be given special consideration. The energy density of halogen lamps ($1/m^2$) varies with distance from the sample. A laser source typically provides a constant energy density regardless of distance. A beam expander can also be used to adjust the spot size (increase or decrease the heated area). It is critical to understand the energy density that is given to the sample. Numerical models (FEM analysis) and master specimens can be used to estimate energy density. The geometric resolution, expressed in mm to pixels ratio (IR sensor + optic), is determined by the distance IR camera sample. This distance defines the smallest resolvable defect that can be recognized. For large components or structures, a high resolution results in limited FOVs and hence a high testing time.

2.2.2.3 Source power and IR detector

The heat source's power is a crucial factor; it must be sufficient to reach and identify the corresponding defects. To determine the correct power and heat source, 1D simplified analytical models or 3D numerical models can be utilized based on the required energy density [J/m^2]. Selecting the appropriate power value is critical to avoid damaging the samples/components.

Two kinds of IR detectors can be used:

1. photonic detectors
2. micro bolometric detectors

2.2.3 Bonding and Debonding in CFRP T-joint Structures

Bonded and debonded Carbon Fiber Reinforced Polymer (CFRP) T-joint structures are commonly used in various engineering applications, particularly in aerospace and automotive industries, due to their high strength-to-weight ratio and excellent fatigue resistance. However, the integrity of these structures can be compromised by issues such as adhesive debonding, which can lead to reduced performance and failure.

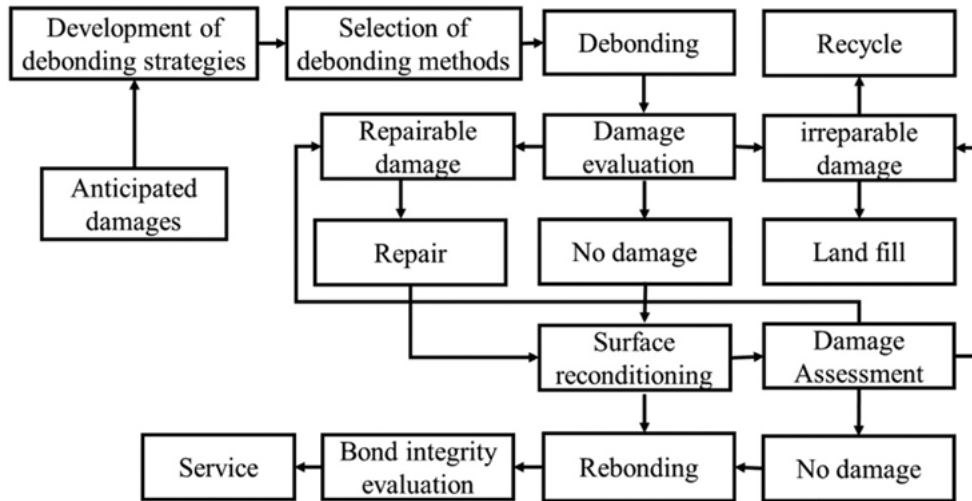


Figure 2.9: Typical repair flow chart of adhesive bonded assemblies

The repair and maintenance of adhesive-bonded assemblies are crucial to prolonging the life of CFRP components. Figure 2.9 shows A typical repair flow chart for adhesive-bonded assemblies that outlines the essential steps involved in identifying, assessing, and repairing damaged joints. This process ensures the continued reliability and safety of CFRP structures by addressing debonding and other adhesive failures.

2.2.3.1 Delamination and Failure in Composite Wind Turbine Blades

The failure processes of composite T-joints with different reinforcement schemes have not been thoroughly investigated, even though the mechanical properties of composite materials have been thoroughly explored. Mouritz [111] pointed out that there are still several scientific and

technical obstacles to overcome, especially when it comes to creating predictive finite element (FE) models that can precisely anticipate the strength and failure behavior of composite T-joints.

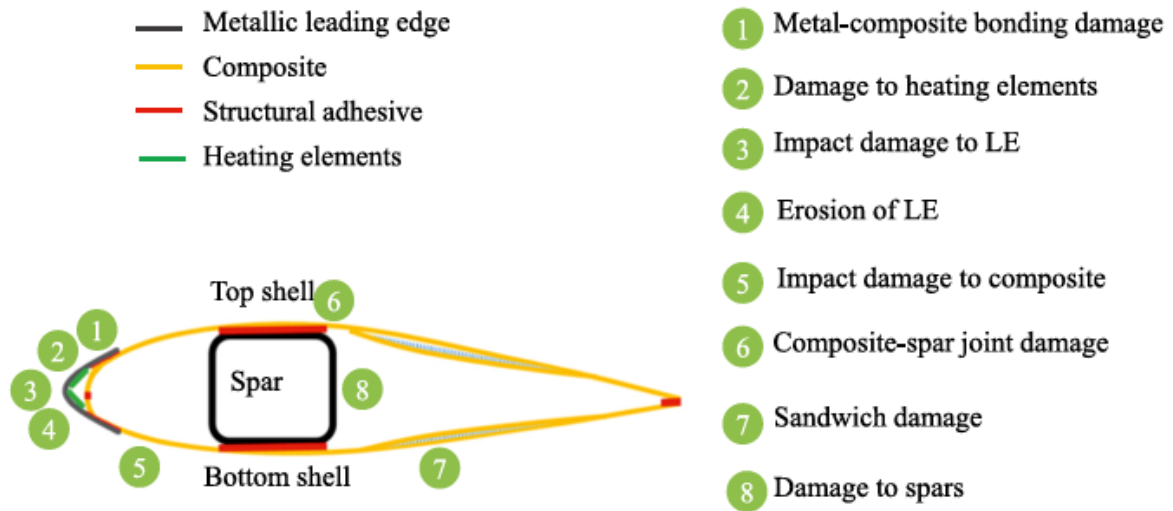


Figure 2.10: Damages in the composite wind turbine blades

These drawbacks highlight the need to learn more about how composite T-joints degrade under various loading scenarios. Furthermore, increasing wind turbines' overall durability and dependability requires the development of practical strategies to extend their fatigue life. Figure 2.10 shows some common damages that can occur in composite wind turbine blades that are bonded with various sub-components.

2.2.3.2 Failure Modes in Adhesively Bonded Composite Joints

The typical failure modes seen in adhesively bonded composite joints are depicted in Figure 2.11. Understanding these failure mechanisms is essential to comprehending the structural integrity and mechanical performance of bonded joints under varied stress scenarios. Among the primary failure modes are:

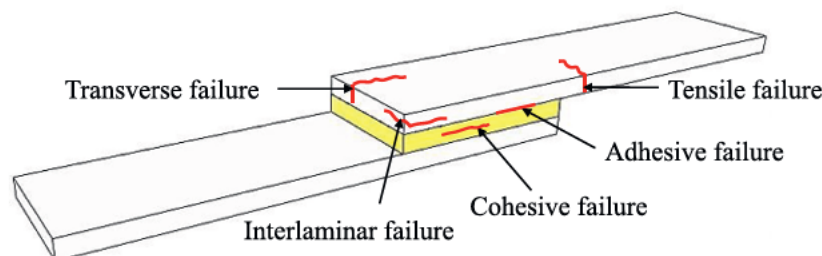


Figure 2.11: Failure modes of a typical adhesively bonded composite joint.

1. Tensile Failure:

When the bonded materials, or composite adherends, are exposed to high tensile stress, the

composite material itself breaks, a phenomenon known as tensile failure. Both the amount of applied load and the adherends' material characteristics affect this kind of failure [64,112].

2. Transverse Failure:

The adherends' cracks or fractures propagating perpendicular to the load direction are indicative of transverse failure. Composite materials with low transverse strength frequently exhibit this mode, which is frequently brought on by internal flaws or fiber orientation [67,113].

.

3. Adhesive Failure:

The interface between the adhesive layer and the adherends is where adhesive failure happens, which suggests inadequate surface preparation or poor adherence. This mechanism of failure implies that the cohesive strength of the adhesive or adherends is stronger than the bond strength at the interface [65,114].

4. Cohesive Failure:

Cohesive failure happens within the adhesive layer itself when the stress exceeds the adhesive's internal strength. This mode shows that the adherends were well-bonded to the adhesive, and failure was caused by the adhesive's inherent characteristics [70,115].

5. Interlaminar Failure:

Interlaminar failure occurs within the composite adherends, typically at the interface between different laminate layers. This failure mode is often associated with delamination caused by shear or peeling stresses [77,79].

Each of these failure scenarios offers important insights into the bonding process's effectiveness and the bonded joint's performance. The predominant failure mode is greatly influenced by variables such as joint shape, loading circumstances, surface treatment, and adhesive choice. Engineers can improve adhesively bonded composite joint design and repair techniques by better understanding these modes.

2.2.4 Heat Transfer Analysis

Heat is a form of energy that can be transferred from one point to another in different ways due to temperature differences. The science that determines the time taken to transfer the energy is "heat transfer" [116]. Thermodynamics deals with the amount of heat transfer that takes place when a system undergoes a process to change from one equilibrium state to another [116]. We are generally interested in how much time is needed to undergo this change. There are three mechanisms by which heat can be transferred namely conduction, convection, and radiation. The basic requirement for heat transfer to occur by any of these modes is the temperature difference. Heat is transferred from a medium at a higher temperature to a medium at a lower temperature. This flow of energy stops when the two mediums are at the same temperature.

1. Conduction

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones due to interactions between the particles [116]. Conduction can take place in solids, liquids, and gases. In solids, conduction occurs due to free electrons

and vibrations of the molecules. When the molecules vibrate they transfer their energy to the neighboring molecule and this process continues. This phenomenon occurs in liquids and gases due to the collision and diffusion of the molecules in random motion [116]. The rate at which conduction will occur depends on the material of the medium, its geometry and thickness, and the temperature difference across the medium. Keeping all this in mind, the heat conduction equation can be written as;

$$\dot{Q} = -kA \frac{\Delta T}{\Delta x} \quad (2.3)$$

In Equation 2.3, $\dot{Q}$ is the heat conduction rate (W), k is the thermal conductivity of the material (W/(m·K)), A is the cross-sectional area (m²), ΔT is the temperature difference (K), Δx is the thickness (m). The negative sign indicates that heat is conducted in the direction of decreasing temperature. In the limiting case as $\Delta x \rightarrow 0$, Equation 2.3 reduces to the following differential form, known as Fourier's law of heat conduction:

$$\dot{Q} = -kA \frac{dT}{dx} \quad (2.4)$$

2. Convection

This mode of heat transfer is between a solid surface and a moving fluid. The faster the movement of fluid, the greater the rate of convective heat transfer. Convection can be forced or free. Forced convection occurs when the fluid is forced to flow over the surface using a pump, fan, or wind. Free convection, also called natural convection, occurs if fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid [116]. In such cases, warmer light air near the surface rises and cooler heavy air falls to replace it. The convective heat transfer rate is expressed by Newton's law of cooling as follows.

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \quad (2.5)$$

In Equation 2.5, $\dot{Q}_{conv}$ is the heat transfer rate due to convection (W), h is the heat transfer coefficient (W/(m²·K)), A_s is the surface area (m²), T_s is the surface temperature (K), T_∞ is the temperature of the fluid far from the surface (K).

3. Thermal radiation

Thermal radiation is electromagnetic radiation generated by the thermal motion of charged particles in matter. This mechanism of heat transfer does not require a medium for its transport. Radiation heat emitted is expressed by Stefan-Boltzmann law for an absolute surface temperature T_s in K. The net rate of heat transfer between a body and its surroundings is given by Equation 2.6, in which ϵ is emissivity and A_s is the surface area.

$$\dot{Q}_{rad} = \epsilon A_s \sigma (T_s^4 - T_{surr}^4) \quad (2.6)$$

In Equation 2.6, σ is the Stefan-Boltzmann constant whose value is $5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^4)$.

$\dot{Q}_{\text{rad}}$ is the radiation rate in watts (W), and T_{surr} is the surrounding temperature in kelvins (K). The emissivity of a blackbody is equal to 1. Emissivity is a radiation property that measures how closely a body approximates a blackbody. Its value lies in the range $0 \leq \epsilon \leq 1$. Another property related to radiation is the absorptivity α , which is the fraction of the incident energy absorbed by the surface. Its value lies within the range $0 \leq \alpha \leq 1$, and its value is also 1 for a blackbody. Thus, a blackbody is a perfect emitter as well as a perfect absorber [116]. All three modes of heat transfer can occur simultaneously; however, there can be cases where only two modes take place. For non-pressurized spacecraft, conduction and thermal radiation are the only means of heat transfer. Convection is not possible as it requires a medium to occur, and space is a vacuum.

2.2.4.1 Governing Equation

Heat transfer is the main physical process that thermography analyzes to identify and evaluate defects. Transient heat conduction is a key factor in how heat moves through composite materials and how internal flaws affect this process. The heat conduction equation, which derives from the conservation of energy principle, governs the transfer of heat in a solid body:

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q + Q_{\text{ted}} \quad (2.7)$$

where the heat flux vector $\mathbf{q}$ is defined as:

$$\mathbf{q} = -k \nabla T \quad (2.8)$$

Here:

- ρ is the material density (kg/m³),
- c_p is the specific heat capacity (J/(kg · K)),
- T is the temperature (K),
- $\mathbf{u}$ is the velocity field of the material (m/s),
- k is the thermal conductivity (W/(m · K)),
- Q is the external heat source term (W/m³),
- Q_{ted} is the thermoelectric damping heat source (W/m³).

The term $\rho c_p \frac{\partial T}{\partial t}$ represents the rate at which temperature changes within the material over time, where ρ is the material density, c_p is the specific heat capacity, and T is the temperature. The term $\rho c_p \mathbf{u} \cdot \nabla T$ accounts for heat transfer due to the material's motion, with $\mathbf{u}$ denoting the velocity vector. The heat conduction within the material is described by $\nabla \cdot \mathbf{q}$, where $\mathbf{q} = -k \nabla T$ is the heat flux governed by Fourier's law, with k being the thermal conductivity and ∇T the temperature gradient. Finally, the terms Q and Q_{ted} represent external and internal heat generation, respectively, encompassing energy sources such as external heating and thermomechanical effects.

For anisotropic materials, such as composites, thermal conductivity k becomes a tensor:

$$\mathbf{k} = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}$$

The heterogeneous structure of composite materials, which results in significant changes in heat conductivity in various directions, makes this formulation particularly important.

2.2.4.2 Boundary and Initial Conditions

To solve the heat conduction equation, appropriate boundary, and initial conditions are defined:

1. Temperature Boundary Condition:

$$T(x, y, z, t) = T_0 \quad \text{on } \Gamma_T \quad (2.9)$$

where T_0 is the prescribed temperature at the boundary Γ_T .

2. Heat Flux

$$-\mathbf{k}\nabla T \cdot \mathbf{n} = q \quad \text{on } \Gamma_q \quad (2.10)$$

where q is the heat flux across the boundary Γ_q , and $\mathbf{n}$ is the outward normal to the boundary.

3. Convective Boundary Condition:

$$-\mathbf{k}\nabla T \cdot \mathbf{n} = h(T - T_\infty) \quad \text{on } \Gamma_h \quad (2.11)$$

where h is the convective heat transfer coefficient, and T_∞ is the ambient temperature.

Initial conditions define the temperature distribution at the start of the simulation:

$$T(x, y, z, t = 0) = T_{\text{init}}$$

2.2.5 Numerical Approaches in Thermography Analysis

2.2.5.1 Approximation of the Domain

Due to technological advancements, weak-form problems must be solved automatically through the use of automated techniques. Therefore, it is possible to "discretize" the problem into many finite areas, or elements, where mathematical methods are effective. A computer can then be used to "assemble" and ultimately "solve" the issue. The problem in the theory of structures pertains to a 3D, 2D, or 1D domain and can be quite complicated in terms of geometry, mechanical boundary conditions (loadings), and geometrical boundary constraints (conditions on generalized displacements in a given collection of points). Figures 2.12, which display common 3D, 2D, and 1D domains, provide examples of geometrical domains.

The FEM's basic concept is to "discretize" complicated domains into simpler ones.

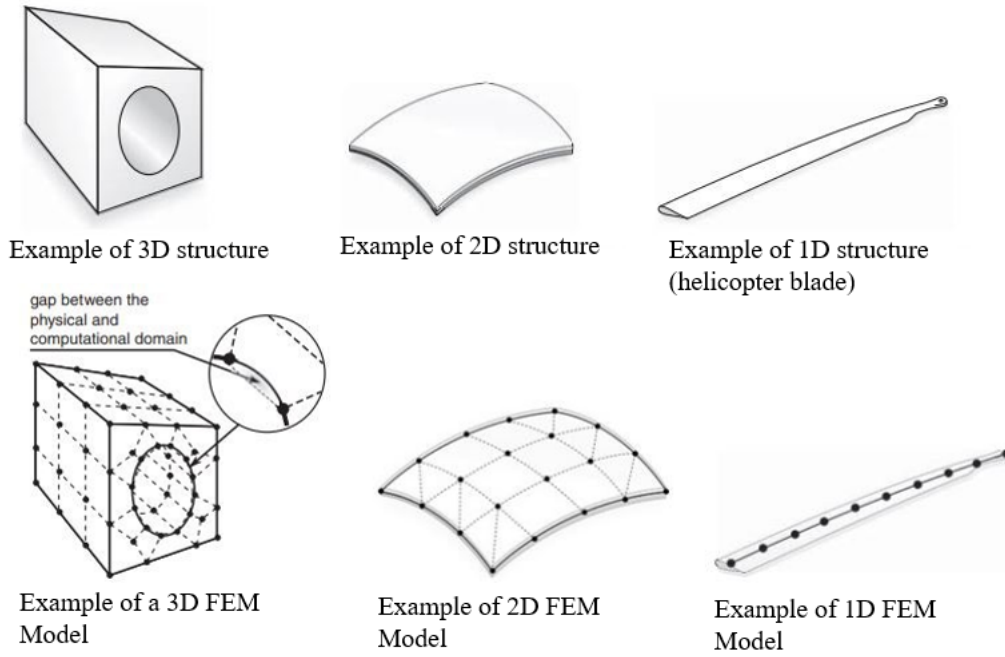


Figure 2.12: Geometrical domain examples typical 3D, 2D and 1D domains

2.2.6 Finite Difference Models

Finite Difference Models (FDM) are widely used numerical techniques for solving partial differential equations, particularly in heat transfer and diffusion problems. These models discretize the domain into a grid and approximate derivatives using finite differences, making them highly suitable for problems where analytical solutions are difficult to obtain. In the context of thermography and defect detection, FDM is used to model heat propagation through materials, providing insights into temperature distributions and identifying potential flaws or anomalies. The simplicity and flexibility of FDM make it a powerful tool for simulating complex geometries and boundary conditions, allowing for the analysis of transient heat behavior in both one-dimensional and two-dimensional systems, as shown in Figures 2.13 and 2.14 in the study by Maillet et al. These models enable accurate predictions of thermal profiles, essential for evaluating the effectiveness of non-destructive testing methods such as Lock-in Thermography.

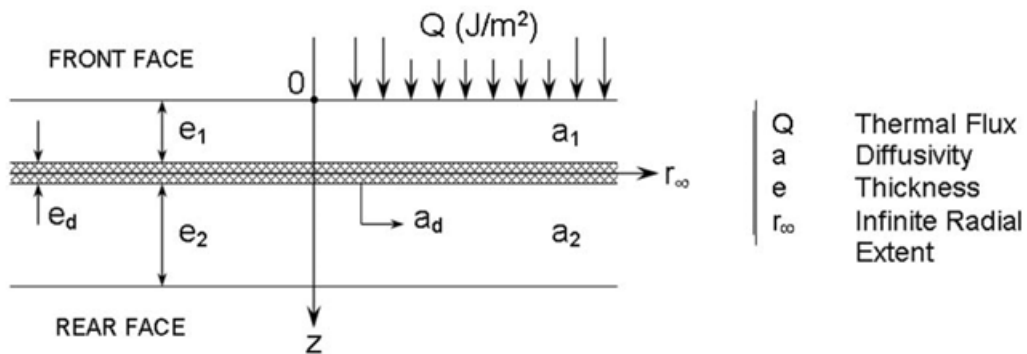


Figure 2.13: Geometry of a two-dimensional numerical model

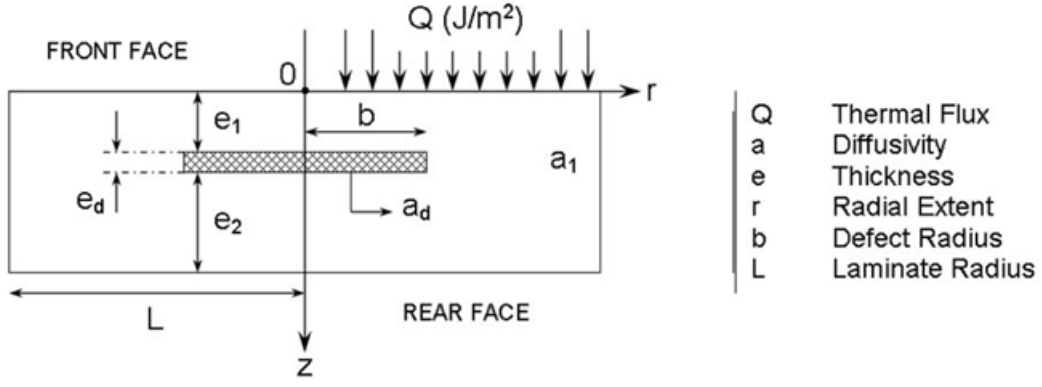


Figure 2.14: Geometry of a two-dimensional numerical model

Figures 2.15 and 2.16 provide a detailed illustration of the study framework and show the numerical models used to simulate delamination. The discretized form of the heat equation for transient analysis in a three-dimensional domain is as follows:

$$T_{i,j,k}^{n+1} = T_{i,j,k}^n + \Delta t \left[\frac{k_x}{\Delta x^2} (T_{i+1,j,k}^n - 2T_{i,j,k}^n + T_{i-1,j,k}^n) + \frac{k_y}{\Delta y^2} (T_{i,j+1,k}^n - 2T_{i,j,k}^n + T_{i,j-1,k}^n) + \frac{k_z}{\Delta z^2} (T_{i,j,k+1}^n - 2T_{i,j,k}^n + T_{i,j,k-1}^n) + Q \right], \quad (2.12)$$

where:

- $T_{i,j,k}^n$: Temperature at point (i, j, k) at time step n ,
- Δt : Time step (s),
- $\Delta x, \Delta y, \Delta z$: Spatial discretization in x, y, z directions (m).

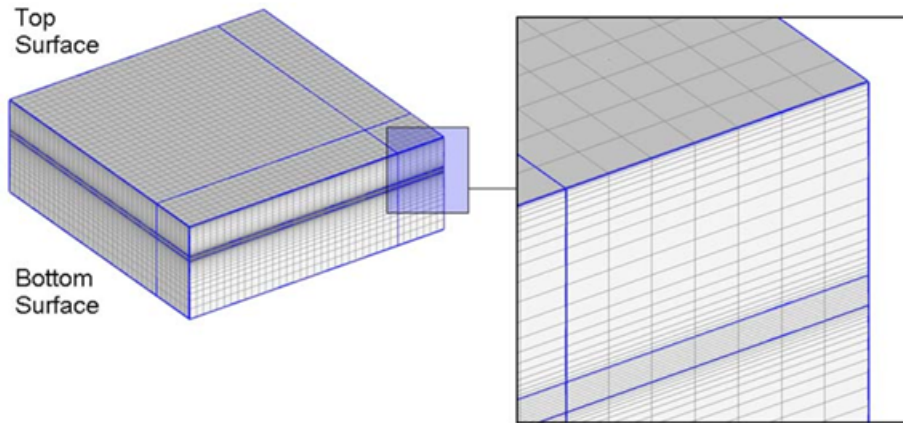


Figure 2.15: FE delamination model: bias element spacing configuration.

Fundamental methods for simulating delamination in composite materials using finite elements have been developed by researchers. Expanding on the approach created by Krishnapillai [37], considerable adjustments were required to account for finite-width and through-

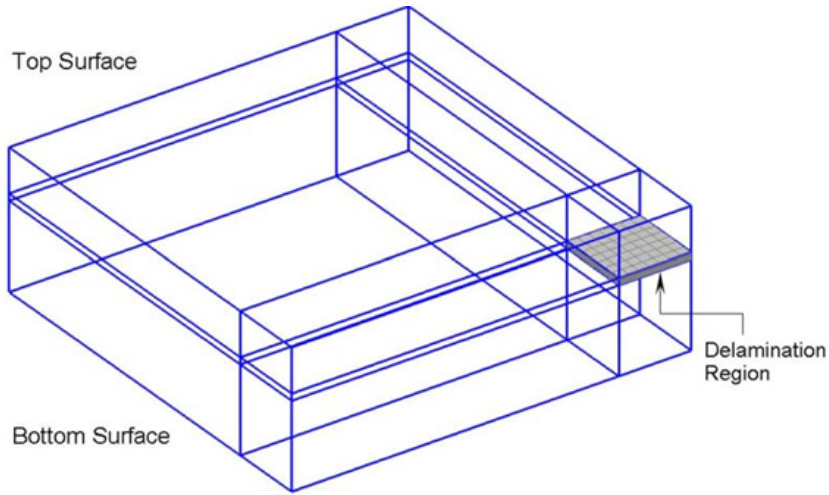


Figure 2.16: FE delamination model: defect slice embedded within a composite.

thickness defects in the model. The literature has an extensive amount of information on the crucial elements of mesh creation and optimization. In transient thermal analysis, the mesh configuration in particular needs to be carefully considered to achieve a compromise between accuracy and computing efficiency. A key factor in model accuracy is the precise use of mesh spacing. Investigations have indicated that the interface areas between the delamination and composite zones require special consideration, demanding a progressive change in mesh size Figure 2.15. A dual-mesh technique is emphasized in the literature: a biased mesh Figure 2.16 in the composite areas and a fine mesh in the delamination zone. Inaccurate thermal contours that may result from built-up errors in coarse mesh spacing have been prevented because of in large part this technology. In their fundamental work, [37] also developed significant assumptions about thermal properties. Their study showed that, when comparing models with and without these thermal effects, convection and radiation impacts did not affect surface contours.

Although they offer a simple but efficient approach to resolving thermal issues, finite difference models (FDM) have several drawbacks. Because they rely on structured grids, they have trouble with complicated geometries and irregular boundaries, which makes them inappropriate for simulating composite heterogeneity. The accuracy of defect detection is decreased by FDM's sensitivity to numerical instability and truncation errors, especially in transient situations or steep material gradients. FDM also lacks scalability for huge 3D challenges and flexibility for advanced physics integration. These drawbacks emphasize the necessity of more advanced formulations that provide more precision and versatility for composite thermal analysis, such as finite element techniques (FEM) or isogeometric analysis (IGA).

2.2.7 Thermal Transient Analysis using ANSYS

Thermal transient analysis plays a crucial role in understanding the heat transfer dynamics within materials and structures, especially when subjected to varying thermal conditions. In

engineering applications, such as defect detection and material characterization, accurately simulating thermal behavior is essential for predicting temperature distributions and identifying potential flaws. ANSYS, a powerful finite element analysis tool, is commonly employed for performing such thermal simulations. The thermal transient analysis using ANSYS involves generating temperature profiles under transient thermal conditions, which are essential for evaluating the thermal response of materials and structures over time. This data is crucial for both material design and defect detection, as it helps in understanding the effects of heat propagation and identifying areas of interest for further inspection. As depicted in Figure 2.17, the process begins with the generation of data through finite element simulations, followed by preprocessing and data preparation to ensure high-quality, consistent results, which are then used to train deep learning models for enhanced analysis and decision-making.

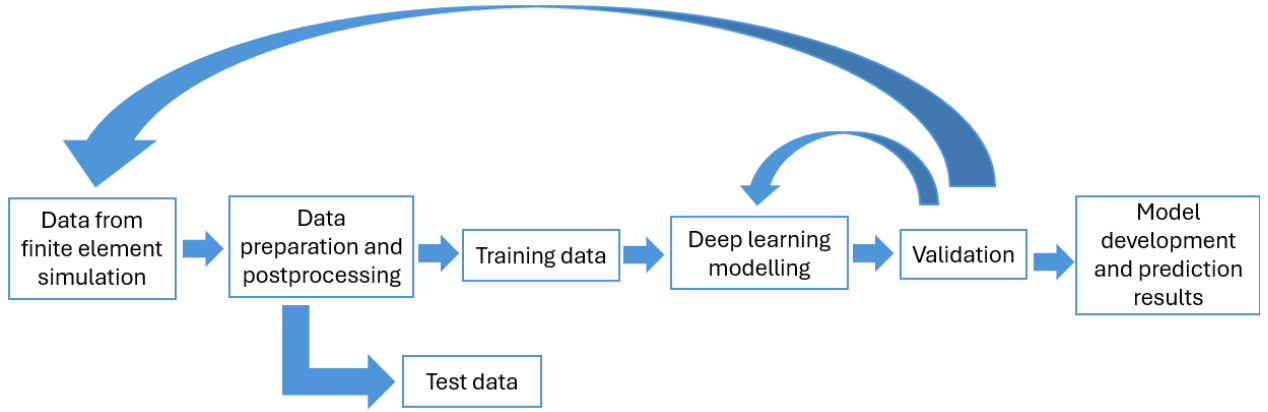


Figure 2.17: A suggested procedure for training a deep learning model using data produced by finite element modeling

Finding trends and creating reliable models with prediction capabilities are the main goals of the deep learning modeling stage. The accuracy and performance of these models are then evaluated by validating them on different validation datasets. Ultimately, the simulation-based predictive modeling cycle is completed when the validated models are used to deliver prediction outcomes for real-world applications.

The process of discretizing the domain into finite elements and solving the heat conduction equation in a weak form is known as the finite element formulation for heat transfer problems. The governing equation for a typical element is changed to:

$$\int_{\Omega^e} \rho c \frac{\partial T^e}{\partial t} N^T N d\Omega^e + \int_{\Omega^e} \nabla N^T \mathbf{k} \nabla T^e d\Omega^e = \int_{\Omega^e} N^T Q d\Omega^e \quad (2.13)$$

where:

- N is the shape function matrix,
- Ω^e is the domain of element e ,
- T^e is the temperature vector for the element.

After assembly for all elements, the system of equations can be written in matrix form:

$$\mathbf{C} \frac{d\mathbf{T}}{dt} + \mathbf{K}\mathbf{T} = \mathbf{Q} \quad (2.14)$$

where:

- $\mathbf{C}$ is the heat capacity matrix,
- $\mathbf{K}$ is the conductivity matrix,
- $\mathbf{Q}$ is the heat source vector,
- $\mathbf{T}$ is the nodal temperature vector.

2.2.8 COMSOL Multiphysics

Heat transfer in thermographic applications is one of the numerous physical processes that can be simulated with the help of the flexible finite element analysis (FEA) program COMSOL Multiphysics [117]. COMSOL's capabilities are especially useful for simulating heat transfer processes and assessing thermal reactions in the context of defect detection in composite materials. This allows for the identification of subsurface defects including cracks, voids, and delaminations.

Numerical modeling with software like COMSOL requires significant computational resources, such as advanced hardware and long processing time, particularly when dealing with complicated geometries and materials. This problem becomes considerably more significant when optimization or sensitivity analysis is necessary, as these processes require several iterative simulations to adjust model parameters and examine the influence of various factors. The high computing cost can be a limitation, particularly for large-scale or high-resolution simulations.

This enhanced formulation enables COMSOL to simulate complicated thermal phenomena, such as the effects of material motion (if present) and internal heat generation. Thus, the equation simplifies to:

$$\rho c_p \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = Q + Q_{\text{ted}} \quad (2.15)$$

2.2.8.1 Parametric Estimation and Optimization

Parametric estimation and optimization are critical components of numerical simulations, particularly when assessing heat transfer in composite materials to identify thermographic defects. In COMSOL Multiphysics, parametric estimates can be used to determine unknown material properties, boundary conditions, or model parameters using experimental or simulated data. Optimization approaches, on the other hand, allow for the improvement of model predictions and system performance by altering parameters to reduce errors or enhance efficiency.

Parametric estimation and optimization in COMSOL are critical for accurately modeling and analyzing thermographic data for defect detection. Both approaches use an objective

function based on the least-squares method to minimize the gap between simulated and experimental data.

$$f(\mathbf{p}) = \sum_{i=1}^N (T_{\text{sim}}(x_i, t_i, \mathbf{p}) - T_{\text{exp}}(x_i, t_i))^2 \quad (2.16)$$

Here:

- $T_{\text{sim}}(x_i, t_i, \mathbf{p})$ is the simulated temperature at position x_i and time t_i based on the parameter vector $\mathbf{p}$,
- $T_{\text{exp}}(x_i, t_i)$ is the experimentally measured temperature at the same position and time,
- N is the total number of measurement points.

In parametric estimation, the parameter vector $\mathbf{p}$ contains material characteristics like thermal conductivity (k) and specific heat capacity (c). The properties are iteratively improved to reduce $f(\mathbf{p})$, which aligns the simulated temperature field with the experimental values.

This approach is crucial in thermography-based defect detection since it allows for accurate material property estimation and dependable thermal analysis. COMSOL provides excellent solutions for this inverse problem using optimization algorithms such as gradient descent and genetic algorithms.

Optimization involves applying the same objective function to a broader set of design factors, such as material qualities, geometric parameters, boundary conditions, and heat flux. The optimization problem is stated as follows:

$$\text{minimize } f(\mathbf{p}) \quad (2.17)$$

To ensure practical solutions, physical limitations are often included, such as the maximum permitted temperature or thermal flux. Optimizing the parameter vector $\mathbf{p}$ improves model accuracy and prediction performance, especially for detecting defects in composite materials.

2.2.8.2 Optimization Methods in COMSOL

An optimization method's choice is affected by the problem's complexity, the required degree of precision, and the available computing power. An appropriate method addresses the particular difficulties presented by the problem, such as non-linearity, high-dimensional parameter spaces, or multi-physics interactions, while striking a balance between accuracy and efficiency. Advanced optimization approaches may be necessary for complex simulations to ensure stable results within the limits of available computer capacity. COMSOL provides various optimization techniques, including:

2.2.8.3 Sensitivity Analysis

Parameter sensitivity analysis is critical for determining which parameters have the most influence on simulation outcomes, allowing for efficient optimization. Sensitivity analysis in heat transfer identifies the parameters that have the greatest impact on heat distribution, such

Table 2.2: Optimization Methods Overview

Method	Description
Gradient-Based Methods	Utilize the gradient of the objective function to design variables to iteratively search for optimal values. Common techniques include the steepest descent and Newton’s method.
Genetic Algorithms	Used for non-linear, non-convex problems, these methods employ random search strategies and are particularly effective when the parameter space is large and poorly understood.
Particle Swarm Optimization	Simulates the behavior of a swarm of particles to explore the parameter space and find optimal solutions.

as thermal conductivity k . The sensitivity of the objective function $J(\mathbf{p})$ with respect to a given parameter p_j is expressed as:

$$S_j = \frac{\partial J(\mathbf{p})}{\partial p_j} \quad (2.18)$$

This derivative provides insight into how changes in a given parameter affect the optimization outcome, enabling the choice of the most influential factors to optimize.

To summarize, COMSOL Multiphysics offers advanced capabilities for parametric estimation and optimization in heat transfer simulations, which are critical for improving defect detection in composite materials. Combining inverse problem-solving for parameter estimation with optimization approaches such as genetic algorithms and particle swarm optimization provides a diverse and effective approach to enhancing the accuracy and resolution of thermographic defect detection methods.

2.2.8.4 Limitations of Using COMSOL Simulations

While COMSOL Multiphysics is a versatile and extensively used tool for simulating thermal phenomena in composite materials, it has some limitations that may restrict its usefulness, notably for defect detection and thermal analysis in complex systems. The main disadvantages are summarized in Table 2.3.

These limitations underscore the importance of exploring advanced numerical methods and hybrid frameworks to overcome the challenges associated with COMSOL simulations. Techniques such as isogeometric analysis (IGA), extended finite element methods (XFEM), and hybrid approaches integrating COMSOL with machine learning algorithms offer promising avenues for improving scalability, accuracy, and automation in thermal defect detection for composite materials. These advancements could address the computational and modeling challenges, paving the way for more robust and efficient simulations.

Table 2.3: Drawbacks of Using COMSOL for Thermal Simulations in Composite Materials

Limitation	Description
High Computational Cost	COMSOL simulations often require significant computational resources, particularly for high-fidelity models or large-scale problems involving transient thermal analysis in multi-layered composites. Parametric studies or optimization tasks exacerbate this issue by further increasing the computational demand.
Steep Learning Curve and User Dependency	Despite its user-friendly interface, obtaining reliable and accurate results requires extensive expertise in physics modeling, numerical techniques, and solver configuration. Improper settings for meshing, boundary conditions, or solver parameters can lead to convergence issues or inaccurate outputs [118].
Limited Solver Flexibility	The built-in solvers in COMSOL may lack the flexibility needed for advanced problems, such as highly anisotropic materials, non-linear thermal properties, or coupling with external machine learning frameworks for automated defect detection [60].
Material Modeling Limitations	While COMSOL offers an extensive material library, it struggles to fully capture the anisotropic and heterogeneous nature of advanced composite materials. This limitation is particularly evident when modeling spatially varying thermal properties or multi-scale phenomena [47].

2.3 Conclusion

Chapter 2 examined current approaches in transient thermal modeling and validation in composite materials utilizing active thermography. This chapter began by outlining thermography as a non-destructive evaluation (NDE) technology, highlighting its importance in detecting defects such as delaminations, cracks, and voids in multilayer composites. Among thermographic approaches, Lock-in Thermography (LIT) stands out for its ability to perform quantitative defect analysis with high precision, making it a popular technique for complicated composite constructions. The contributions of conventional numerical tools, such as the Finite difference model, ANSYS, and COMSOL Multiphysics, to the modeling of heat transfer and the identification of defects in composites, were examined. These approaches have provided useful insights into defect characterization; however, their computational limits, particularly in dealing with the anisotropic and heterogeneous nature of composite materials, have been identified. Significant computational resources are needed for high-fidelity models, and conventional finite element techniques frequently fall short of adequately capturing heat gradients and interlaminar stresses, particularly in multilayered structures. Additionally, simulating complex phenomena such as real delamination or debonding in composite structures presents substantial challenges, as these models struggle to account for intricate material behaviors and damage propagation in these advanced composites. Due to these limitations, performing fast computations and

creating virtual experimentation becomes essential. Advanced numerical modeling techniques, such as the Carrera Unified Formulation (CUF), are necessary to overcome these challenges, providing a more efficient and accurate approach for simulating the thermal behavior of complex composite structures and enabling effective virtual defect analysis.

Chapter 3

Carrera Unified Formulation(CUF) and Sublaminated Generalized Unified Formulation (SGUF): A Literature Review

3.1 State of the Art in Advanced Numerical Modeling: Implementation and Applications

The Carrera Unified Formulation (CUF) has been widely used and refined to analyze composite and multilayered structures. This chapter introduces a novel way of deriving finite element (FE) matrices. This method is based on the Carrera Unified Formulation (CUF), which allows FE matrices and vectors to be expressed in terms of fundamental nuclei. The CUF is illustrated by extending the index notation widely used in FE procedures (indexes i and j) to structural theory (indexes τ and s). As a result, the fundamental nucleus (FN) is expressed in terms of four indexes (τ , s , i , and j).

The FN is a 3×3 array (3×1 in the case of a vector), and its shape remains consistent for one-dimensional (1D), two-dimensional (2D), and three-dimensional (3D) problems as predicted in the Figure 3.1.

Carrera and Petrolo [119] and Carrera et al. [120–122] studied refined beam elements for static and dynamic analyses of laminated composite structures. Ibrahim et al. [123] researched the buckling behavior of composite thin-walled beams, whereas Varello et al. [124] examined the static aeroelastic response of composite wings. Carrera et al. [125] made significant advances in beam element systems, particularly for structural analysis of single and multiple fiber/matrix cells. Carrera (2013) revised the "component-wise" (CW) technique for anisotropic laminated composites. Carrera et al. [124, 126, 127] applied refined and CW beam models to analyze reinforced and wing structures. Carrera and Petrolo investigated the efficiency of higher-order terms in modified beam theories [121]. Carrera [128] discusses theories and finite elements (FEs) for multilayered plates and shells using a unified formulation, including numerical assessments and benchmarks. Carrera et al. [129] and Botshekanan Dehkordi et al. [130] assessed CUF plate

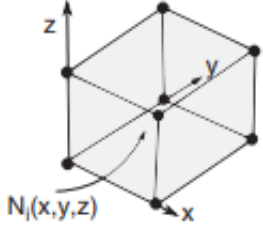
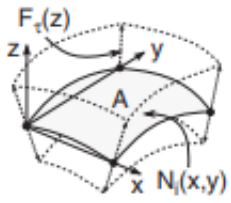
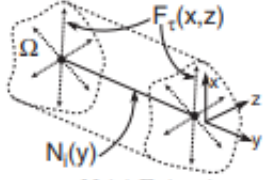
Equilibrium equations in Strong Form $\rightarrow \delta L_i = \int_V \delta u k u dV + \int_S \dots dS$	
$\underbrace{\begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}}_k \underbrace{\begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}}_u = \underbrace{\begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix}}_p$	$\begin{aligned} k_{xx} &= -(\lambda + 2G) \partial_{xx} - G \partial_{zz} - G \partial_{yy}; \\ k_{xy} &= -\lambda \partial_{xy} - G \partial_{yx}; \\ k_{xz} &= \dots \end{aligned}$
$\lambda = (Ev)/[(1+\nu)(1-2\nu)]; \quad G = E/[2(1+\nu)]$	
$u = u(x, y, z)$ $\delta u = \delta u(x, y, z)$	The diagonal (e.g. k_{xx}) and the non-diagonal (e.g. k_{xy}) terms can be obtained through proper index permutations.
 $u = N_i(x, y, z)u_i$ $\delta u = N_j(x, y, z)\delta u_j$	3D FEM Formulation $\rightarrow \delta L_i = \delta u_j k^{ij} u_i$ $k_{xx}^{ij} = (\lambda + 2G) \int_V N_{j,x} N_{i,x} dV + G \int_V N_{j,z} N_{i,z} dV + G \int_V N_{j,y} N_{i,y} dV;$ $k_{xy}^{ij} = \lambda \int_V N_{j,y} N_{i,x} dV + G \int_V N_{j,x} N_{i,y} dV$
 $u = N_i(x, y)F_\tau(z)u_{\tau i}$ $\delta u = N_j(x, y)F_s(z)\delta u_{sj}$	2D FEM Formulation $\rightarrow \delta L_i = \delta u_{sj} k^{rsij} u_{\tau i}$ $k_{xx}^{rsij} = (\lambda + 2G) \int_\Omega N_{i,x} N_{j,x} d\Omega \int_h F_\tau F_s dz$ $+ G \int_\Omega N_i N_j d\Omega \int_h F_{\tau,z} F_{s,z} dz + G \int_V N_{i,y} N_{j,y} d\Omega \int_h F_\tau F_s dz;$ $k_{xy}^{rsij} = \lambda \int_\Omega N_{i,y} N_{j,y} d\Omega \int_h F_\tau F_s dz + G \int_\Omega N_{i,x} N_{j,y} d\Omega \int_h F_\tau F_s dz$
 $u = N_i(y)F_\tau(x, z)u_{\tau i}$ $\delta u = N_j(y)F_s(x, z)\delta u_{sj}$	1D FEM Formulation $\rightarrow \delta L_i = \delta u_{sj} k^{rsij} u_{\tau i}$ $k_{xx}^{rsij} = (\lambda + 2G) \int_l N_i N_j dy \int_A F_{\tau,x} F_{s,x} dA$ $+ G \int_l N_i N_j dy \int_A F_{\tau,z} F_{s,z} dA + G \int_l N_{i,y} N_{j,y} dy \int_A F_\tau F_s dA;$ $k_{xy}^{rsij} = \lambda \int_l N_{i,y} N_{j,y} dy \int_A F_\tau F_{s,x} dA + G \int_l N_i N_{j,y} dy \int_A F_{\tau,x} F_s dA$

Figure 3.1: A schematic description of the CUF and the related fundamental nucleus of the stiffness matrix for 3D, 2D and 1D models

elements for composite and sandwich structures, respectively. Ferreira et al. [118] analyzed viscoelastic sandwich laminates using CUF plate elements, while Carrera et al. [131] employed them to identify laminated anisotropic plate failures. Carrera et al. [132] conducted comparative studies of multilayered plate elements with various 2D assumptions, and Carrera et al. [122] discussed the accuracy further. Carrera and Miglioretti [119] suggest using the best-refined plate elements for a certain challenge. Carrera and Nali [133] developed advanced multilayered plate elements for multifield issues, including electro-mechanical analyses in piezo materials.

The scope of research highlights CUF's versatility and robustness in dealing with the complexities of composite and multilayered structural analysis. It is still expanding, driven by its versatility and precision in capturing complex material phenomena. Carrera et al. [128] proposed the Generalized Unified Formulation (GUF), which is a systematic and efficient

framework for studying structures across many dimensions (1D, 2D, and 3D) and complexities. GUF simplifies the derivation of finite element matrices by illustrating them as invariant basic nuclei across structural mechanics theories. The Sublaminated Generalized Unified Formulation (SGUF) extends this framework by integrating sublaminated theories and treating each composite layer as an independent domain [134]. This approach is very useful in thermal transient analysis, as it addresses essential interlaminar thermal gradients and interactions.

The study of heat transfer on composite materials has long been a research priority. Carrera et al. [135] developed SGUF to overcome the shortcomings of standard lamination models, which frequently fail to account for interlayer effects. Earlier investigations, such as those by Whitney and Sun [136], gave insights into thermal stresses in laminated plates but failed to adequately address interlayer interactions. Classical Equivalent Single-layer (ESL) models, including Classical Lamination Theory (CLT) and First-order Shear Deformation Theory (FSDT), have been widely researched [137]. While these models rely on plane stress assumptions, FSDT incorporates shear correction factors (SCF) to account for transverse shear deformability, which is often problem-dependent (Birman 2002, Choice). Higher-order Shear Deformation Theories (HSDT), such as Ambartsumyan's and Reddy's third-order theories [138, 139], incorporate quadratic transverse shear stresses and cubic in-plane displacements, eliminating the requirement for SCFs. These theories provide increased accuracy in capturing transverse effects while maintaining the plane stress state [140, 141].

Recent improvements, such as Zig-Zag theories, have further advanced ESL models by integrating piecewise linear displacement fields to account for discontinuities at layer interfaces (demasi 2012, partly). The Zig-Zag approach, incorporating Murakami's Zig-Zag function [142, 143], enhances the model's ability to handle the mismatch in mechanical properties between neighboring plies, as observed in sandwich structures [144, 145]. Carrera and Brischetto [146] found that the accuracy of ESL models, even those with Zig-Zag effects, decreases with increasing panel thickness and material property mismatches.

Layer-wise (LW) techniques are frequently used with sandwich structures. Librescu and Hause [147] used FSDT, and Pai and Palazotto [148] improved the refined model proposed by Lo et al. [141]. Researchers like D'Ottavio and Polit [149] have used advanced CUF-based models to conduct accurate analyses. Simplified methods, such as pure membrane or CLT for thin faces [150, 151], and complicated 3D elastic continuum models for the core [152, 153], have also been investigated. These different models demonstrate the Sublaminated-GUF's adaptability and utility in capturing complicated processes in composite materials.

The advances in SGUF and similar models highlight their importance in solving the issues of thermal and mechanical analysis in composite structures, especially when standard models fail.

The continuing movement of composite structures from secondary to principal load-bearing roles needs the creation of advanced models capable of accurately capturing both global and localized reactions. Several comprehensive reviews (Kapania et al., 1989; Noor et al., 1990; Reddy, 1994; Carrera, 2002; Sayyad et al., 2015) reflect the growing interest in theoretical and

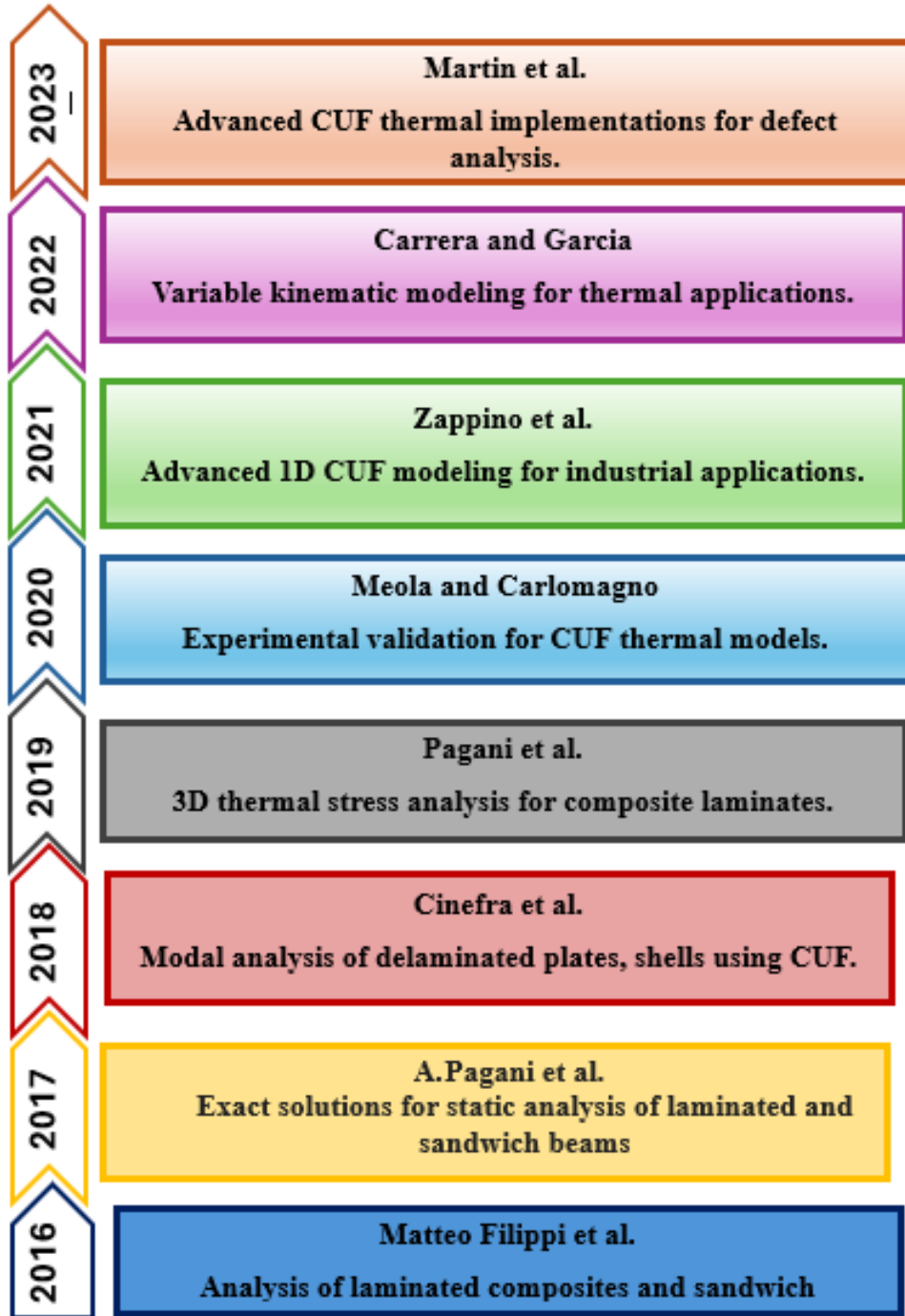


Figure 3.2: A Comprehensive Timeline of Numerical Approaches Using CUF and SGUF Formulations

computational models for multilayered panels [154–158]. Most of these models are based on an axiomatic approach, which makes certain assumptions about the behavior of the displacement field, stress field, or a combination of both along the panel’s thickness (Carrera et al., [157]). This contribution also uses an axiomatic framework. A convenient classification of axiomatic models for composite multilayered panels distinguishes between Equivalent Single Layer (ESL) and Layer-Wise (LW) models (Reddy et al., [159]). In the ESL approach, the heterogeneous layup is homogenized, representing it as an anisotropic material with membrane-bending and

extension-shear coupling. This results in computational models where the number of unknowns remains independent of the number of plies in the panel. In contrast, LW models apply constraining assumptions to each layer of the composite layup, leading to a total number of unknowns that scale with the number of layers.

Zig-Zag theories provide an intermediary between these two approaches, partially capturing the underlying variability of composite structures while keeping numerous unknowns independent of the number of plies [128]. A straightforward implementation of the LW technique presupposes consistent behavior across all physical plies in a multilayered structure. This method, however, may result in an impractical proliferation of unknowns for composites with a large number of plies.

To overcome this issue, the overall number of unknowns in LW models can be optimized in two approaches. First, physical plies can be divided into sublaminates, reducing the number of layers. Second, the models can make simple assumptions to reflect the geometric and material features of each sublaminate. This is especially effective in sandwich panels, where different assumptions are made for the thin, stiff faces and the thick, soft core (Allen et al., and Noor et al., [155, 160]). Reddy’s LW model exhibits this optimization by separating the layup into an arbitrary number of sublaminates, each with independently determined through-thickness displacement characteristics (Reddy et al., 1987, 2003).

Carrera [135, 157] introduced a highly versatile modeling framework for composite structures, the Unified Formulation (CUF). This approach uses a compact index notation to express the governing equations of numerous models within a single computing framework. The CUF offers multiple advantages: it facilitates direct assessment of different models using the same computational code to establish their range of applicability (De Masi et al., D’Amore et al., and Carrera et al., [161–163]); it allows identification of the optimal model for a specific problem based on the trade-off between accuracy and computational cost (Carrera et al., and Vidal et al., [164, 165]); and it supports a global-local sub-modeling strategy by coupling less accurate, computationally efficient models with more precise, resource-intensive ones (Biscani et al., and Wenzel et al., [166, 167]). The original CUF incorporates affecting hypotheses for the displacement field into weak forms expressed by either the Principle of Virtual Displacements (PVD) or Reissner’s Mixed Variational Theorem (RMVT) (Reissner [168]). The RMVT is especially useful since it permits independent approximations for transverse stress fields, assuring interlayer equilibrium from the start. Within CUF, the displacement field can be characterized using either the Layer-Wise (LW) or Equivalent single-layer (ESL) approaches, which can be supplemented by a Zig-Zag function to improve the ESL representation. However, transverse stresses are always represented using the LW technique.

Another characteristic of CUF is the ability to arbitrarily choose the polynomial order for through-thickness panel (ESL) or layer (LW) approximations, even though the same order is applied uniformly to all unknown functions. The Generalized Unified Formulation (GUF), proposed by Demasi [169, 170], addresses this problem by allowing multiple approximation orders for different field variables. Demasi [171] extended his PVD-based GUF by allowing

individual ESL or LW descriptions for each displacement component. All plies of the composite, however, employ the same hypotheses (such as ESL or LW descriptions and approximation orders) in both GUF and CUF.

With recent developments specifically designed for sandwich plates, the RMVT-based CUF has been expanded to include an ESL description of the displacement field for the face sheets and an LW description for the sandwich core (Dehkordi et al., [130]). Messina et al. [172] used Adaptive-Global Piecewise-Smooth Functions (A-GPSF) to improve the modeling of complicated composite structures. The RMVT-based CUF was recently modified for sandwich plates to use a Layer-Wise description for the sandwich level and an ESL description of the displacement field at the composite face sheets level. Messina et al. recently developed similar ideas using Adaptive-Global Piecewise-Smooth Functions (A-GPSF). Through the use of Reddy's sublaminate notion [173], the current work methodically expands upon the Unified Formulations. This technique formulates different hypotheses at the sublaminate level using the most flexible Generalized Unified Formulation (GUF). Therefore, each field variable can be independently described using either an Equivalent Single Layer (ESL) or Layer-Wise (LW) approach, with a freely selectable polynomial approximation order within each sublaminate, regardless of whether it is derived from the Principle of Virtual Displacements (PVD) or Reissner's Mixed Variational Theorem (RMVT). This approach defines a multilayered plate as a combination of N_p physical plies and N_l numerical layers. The model uses global z , layer-specific z_k , and ply-specific z_p coordinates for structural description and non-dimensional layer- and ply-specific coordinates ζ_k and ζ_p . Comparing this work to recent developments in GUF and CUF, as described by Demasi [171] and Dehkordi et al. [130], respectively, highlights its novelty and advancements.

On one hand, the PVD-based technique given in [171] is expanded by including sublaminate to improve modeling flexibility, while also addressing RMVT-based models. It is important to note that the suggested formulation takes all established GUF models as special instances when the entire composite section is considered as a single sublaminate. In contrast, the methodology introduced in [130] is improved by using GUF instead of CUF for expressing assumptions and allowing the use of PVD-based models. The proposed S-GUF approach improves computational efficiency by extending [171] to allow different approximations for each sublaminate and advancing [130] by allowing distinct approximations for various field variables within each sublaminate.

3.2 Theoretical Background

3.2.1 Carrera Unified Formulation (CUF)

The Carrera Unified Formulation (CUF) significantly improves computational mechanics, specifically in the modeling and analysis of multilayered composite structures. E. Carrera introduced CUF in the late 1990s, providing an adaptable and methodical framework for

constructing enhanced models for structural analysis. Its foundation is based on expanding the displacement field using polynomial functions, allowing for a smooth transition between classical and higher-order theories. The Euler-Bernoulli beam theory and the Kirchhoff-Love plate theory are two examples of classical theories that frequently fail to capture the complex behaviors of advanced composite materials. This is why the CUF was developed. Traditional theories rely on simplified displacement fields, which can result in mistakes when examining multilayered or complicated structures with high heterogeneity.

Carrera's early work (1996-2000) proposed the idea of employing hierarchical kinematic expansions to characterize the displacement field within structures. This method enabled analysts to include higher-order terms in the displacement field, increasing prediction accuracy while reducing computational cost. Early CUF versions were applied to simple beam and plate models, demonstrating their advantages over traditional models in terms of stress and strain distributions. CUF's capacity to accommodate varied kinematic models was a significant achievement. Carrera [135] included the Principle of Virtual Displacements (PVD) into the CUF framework, allowing for systematic derivation of governing equations for diverse structure configurations. This development enabled CUF to be used for:

1. **Linear Elastic Problems:** Making reliable predictions about stress and deformation in isotropic and orthotropic materials.
2. **Nonlinear Problems:** Extending the concept to include geometric and material nonlinearities.
3. **Thermomechanical Analysis:** Thermal effects are being incorporated into multilayered composite analysis.

Because of CUF's hierarchical nature, analysts might adjust the level of approximation by changing the order of the polynomial expansion. Because of its versatility, CUF is an effective tool for establishing a balance between computing efficiency and accuracy.

3.2.1.1 CUF Assembly Technique

The Carrera Unified Formulation (CUF) simplifies matrix assembly, making it easy to incorporate into computer algorithms. The matrix assembly process uses four assembled loops over the following indices:

- i (node index)
- j (node index)
- τ (expansion index)
- s (expansion index)

Each combination of these indices generates a Fundamental Nucleus (FN). This procedure can be mathematically described as follows:

$$\mathbf{K}_{ij}^{\tau s} = \text{FN}(i, j, \tau, s) \quad (3.1)$$

As illustrated in Figure 3.3, this methodology enables the hierarchical construction of:

1. The nodal matrix
2. The elemental matrix
3. The global stiffness matrix

This systematic approach leverages the fundamental nucleus to build complete structural matrices efficiently. The process naturally progresses from the most basic computational unit (the fundamental nucleus) to the full global system matrix.

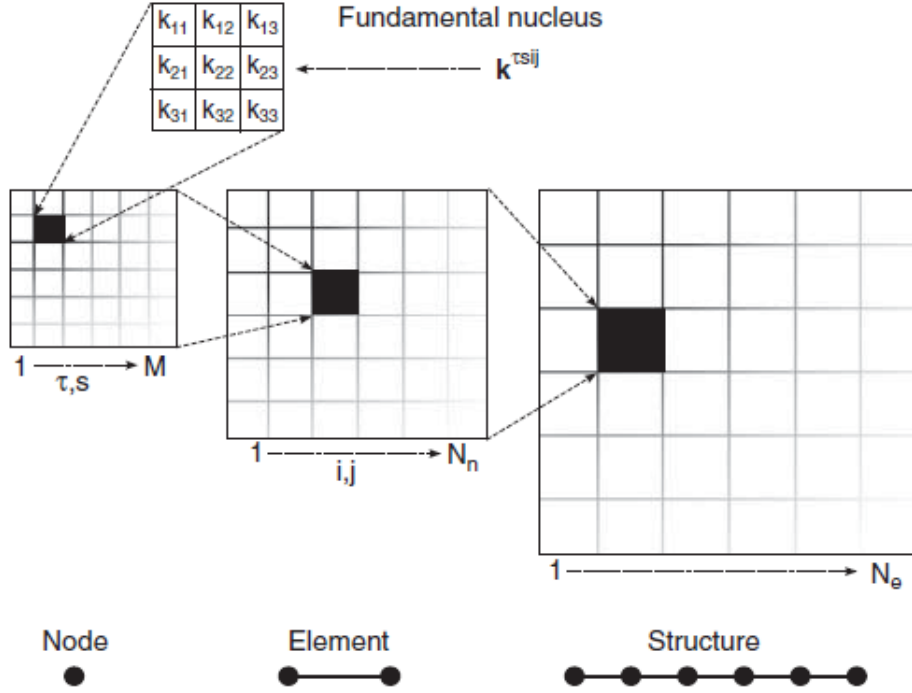


Figure 3.3: Assembly procedure: The FN is the core, the loops on τ and s build the matrix for a given pair of i and j , the loops on i and j give the matrix of the elements, and the loop on the elements gives the global stiffness matrix

3.2.1.2 Volume Integration for Stiffness and Mass Matrices

Carrera [135] developed a unified formulation approach for calculating FNs (stiffness and mass matrices), which involves calculating integrals. The nuclear elements have a common form that can be expressed as

$$FN = C \int_V f(x, y, z) dV \quad (3.2)$$

where C is a constant that can be either a mass parameter or a material stiffness constant, and $f(x, y, z)$ is a generic function that combines the derivatives of the form functions. This integral can only be calculated exactly for elements with simple geometry. For arbitrary-shaped elements, the integral must be evaluated numerically. The approach provided here follows the methodology established in [135] and uses the Gauss quadrature formulation.

3.2.1.3 Expansion of CUF for Multilayered Composite Structures

As CUF developed, its applications extended to include increasingly complicated challenges like multilayered structures and composite materials. The formulation's flexibility made it ideal for studying layered composites, where layers have different mechanical properties and interactions between layers can result in complex stress distributions. The application of CUF to multilayered composite structures was an important milestone in its evolution. Composite materials, which are widely employed in the aerospace, automotive, and marine industries, provide particular problems due to their anisotropic and heterogeneous properties.

To accurately analyze these materials, determine:

1. Interlaminar stresses are important for forecasting delamination.
2. The impact of ply orientations and stacking sequences.
3. Thermomechanical coupling effects.

The ability of CUF to simulate separate layers with different material properties and correctly capture interlaminar stresses made it a good choice for composite analysis. Notably, Carrera and his colleagues used CUF to:

1. **Thermal Stress Analysis:** Providing information about how temperature gradients influence stress distributions in multilayered composites.
2. **Delamination Studies:** Analyzing the beginning and progression of delamination under mechanical and thermal loads.
3. **Dynamic Analysis:** Vibrational properties of laminated plates and shells are being investigated, including damping and resonance behavior.

As previously noted, the assumptions for displacements and stresses can be made at either the layer or multilayer levels. The first situation yields an LW(layer wise) description, while the latter yields an ESL(equivalent single layer) description. If a LW description is used, u_i and σ_{ij} are layer variables. These are distinct in each layer. According to an ESL explanation, u_i and σ_{ij} represent plate/shell variables. These are the same for the entire multilayer.

3.2.1.4 ESLMs

Higher-order ESLMs generally take the following form:

$$\begin{aligned}
 u_x(x, y, z) &= u_{x0}(x, y) + zu_{x1}(x, y) + z^2u_{x2}(x, y) + \dots + z^Nu_{xN}(x, y) \\
 u_y(x, y, z) &= u_{y0}(x, y) + zu_{y1}(x, y) + z^2u_{y2}(x, y) + \dots + z^Nu_{yN}(x, y) \\
 u_z(x, y, z) &= u_{z0}(x, y) + zu_{z1}(x, y) + z^2u_{z2}(x, y) + \dots + z^Nu_{zN}(x, y)
 \end{aligned} \tag{3.3}$$

These can be written in the following compact manner, as per the unified formulation:

$$u = F_0u_0 + F_1u_1 + \dots + F_Nu_N = F_\tau u_\tau, \quad \tau = 0, 1, 2, \dots, N \tag{3.4}$$

where N is the order of the expansion and

$$F_0 = 1, \quad F_1 = z, \quad F_2 = z^2, \dots, F_N = z^N \quad (3.5)$$

3.2.1.5 Theories for Multilayered Structures

Theories for multilayered structures can be composed by making proper decisions about the following points:

- **Choice of the unknown variables:**
 - Displacement formulation
 - Mixed formulation
- **Choice of the variable description:**
 - Equivalent single-layer (ESL) models
 - Layer-wise (LW) model

3.2.1.6 Layer-Wise Theory

The previously described theories take into account several unknown factors that are not affected by the number of constitutive layers N . These are all referred to as equivalent single-layer models or ESLMs. Although these kinematic theories may describe transverse shear and normal strains, as well as transverse warping of the cross-section, their approach is kinematically homogeneous in the sense that it is insensitive to individual layers, unless ZZ models are utilized. If a detailed response from the individual layers is required, and if there are significant variations in the displacement gradients between layers, as in the case of a local phenomenon description, this approach necessitates the use of special HOTs in each of the constitutive layers, as well as an increase in the number of unknowns in the solution process, which increases the complexity of the analysis. Layer-wise models, also known as LWMs, are produced in these cases. Articles by Han and Wang [174], Cheung and Wu [175], Srinivas [176], Cho et al. [177], and Robbins and Reddy [178] provide examples of these types of theories. To enforce the compatibility conditions at each interface, these displacement models require the inclusion of constraint conditions. Nosier et al. [179] and Reddy [180] provided generalizations of these theories. They expressed the displacement variables in the thickness direction in terms of Lagrange polynomials.

$$u_i^k(x, y, z) = L_1(z_k)u_i^k \Big|_{k/2} + L_2(z_k)u_i^k \Big|_{-k/2} + L_3(z_k)u_i^k + \dots + L_N(z_k)u_i^{k_N} \quad (3.6)$$

Let $k = 1, \dots, N$. Interface values are employed as unknown variables (the first two terms of the expansion $u_i^k|_{k/2}$ and $u_i^k|_{-k/2}$), allowing for easy connection of the compatibility criteria at each interface. L_1 and L_2 correspond to linear Lagrangian polynomials, whereas $L_3, \dots, L_N$ should represent an independent base of polynomials beginning with the parabolic one (L_3). Figure 3.4 depicts the notation for multilayered structures.

The Carrera Unified Formulation's layerwise (LW) theory treats each composite structure layer as an independent subdomain with its displacement field. This technique is highly

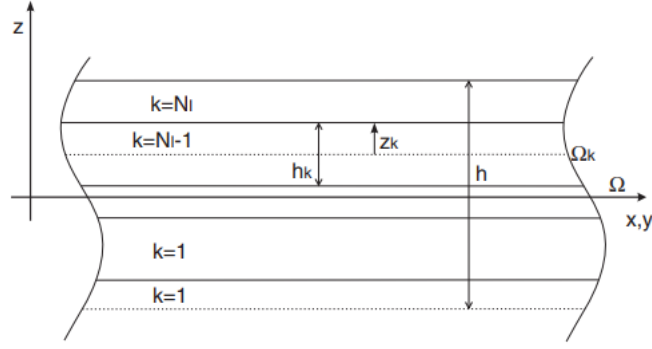


Figure 3.4: Notation for multilayered structures

accurate, especially for laminated composites, because it captures complex interactions between layers and adjusts for discontinuities at layer interfaces. Laminated structures are a defining feature of composite materials, containing multilayered constructions formed of flat or curved panels, where multiple layers or laminae are perfectly bonded together (see Figure 3.4). The matrices of each layer are assembled at the multilayer level by imposing the continuity of the displacements at the interface between the layers (for more details, refer to [157]). Schematic of an assembled block matrix according to the layer-wise approach is shown in Figure 3.5.

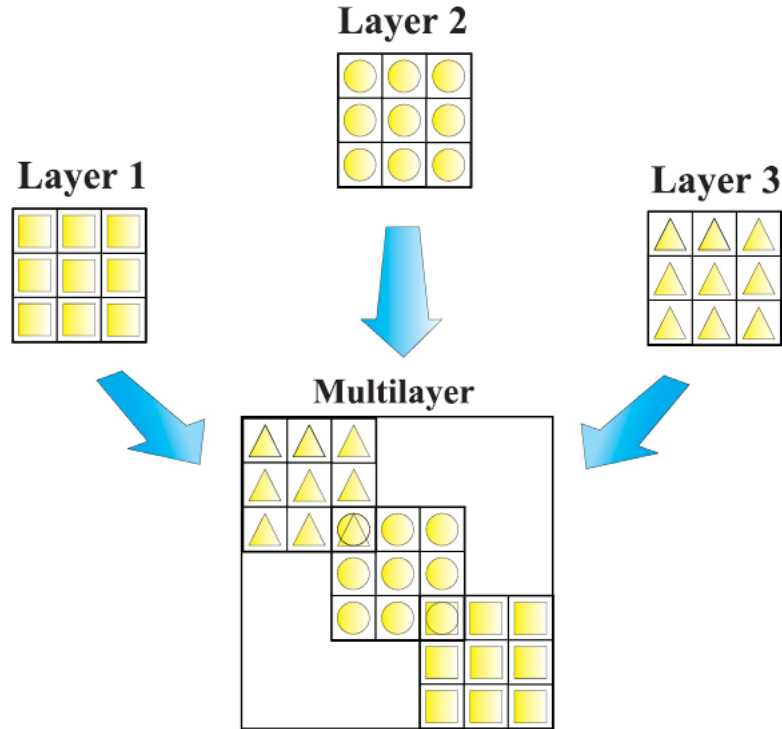


Figure 3.5: Assembling procedure at a multilayer level according to layer-wise approach.

Each lamina is made up of fibers that are embedded within a matrix. These fibers are generated by particular technological methods, offering high mechanical qualities in the longitudinal direction (denoted as L) in Figure 3.6. The matrix keeps the fibers together and is frequently constructed of epoxy materials, whilst the fibers are typically formed of carbon, boron, or glass. Fibers and the matrix can be combined in various ways, including unidirectional

laminae or laminates with varied orientations. These configurations are frequently utilized in aerospace, automotive, and marine industries. The arrangement of laminae, or the lay-up sequence, enables material tailoring, optimizing the composite for specific design requirements. This versatility is a significant advantage in engineering applications, allowing composites to meet a wide range of structural requirements effectively.

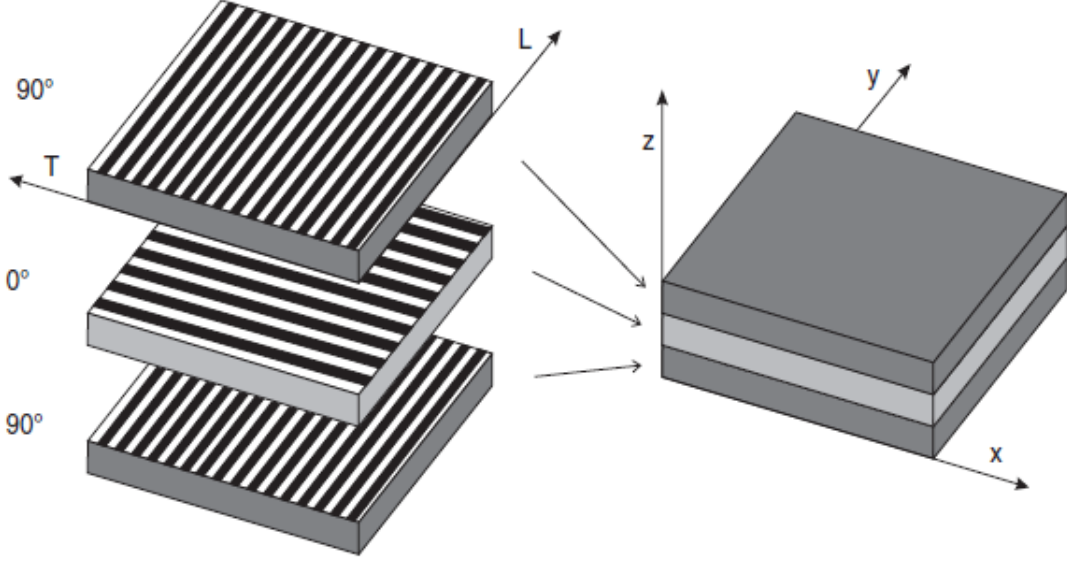


Figure 3.6: Layered plate made of unidirectional laminae

3.2.1.7 C_z^0 Requirements

Layered structures are referred to as 'transversely anisotropic' because they have varied mechanical and physical properties throughout their thickness. Transverse discontinuous mechanical characteristics generate a displacement field, u_i , in the thickness direction, resulting in a rapid change in slope at each layer interface. Figure 3.7b illustrates the zig-zag (ZZ) displacement field in the thickness direction. In-plane stresses $\sigma_p = (\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ can typically be discontinuous at each layer interface. Transverse stresses $\sigma_n = (\sigma_{xz}, \sigma_{yz}, \sigma_{zz})$ must be continuous at each layer interface for equilibrium considerations, such as the Cauchy theorem. This is commonly referred to in the literature as interlaminar continuity (IC) of transverse strains. Figure 3.7 depicts the qualitative distribution of probable displacements and transverse stresses in a multilayered structure, based on the variable description. Displacement and transverse stresses are C_z^0 continuous functions in the thickness direction, indicating compatibility and equilibrium. It is also clear that the displacement u has a discontinuous first derivative at each interface, where mechanical properties change suddenly (1995-1996, 1997). C_z^0 and IC are the C_z^0 requirements. Ensuring that the C_z^0 conditions are met is critical for developing multilayered structure theories.

The Carrera Unified Formulation (CUF) has made major contributions to the modeling of multilayered composite structures by offering a generalized framework for structural analysis using variable kinematic theory. Building on CUF's flexibility and efficiency, the Sublaminar

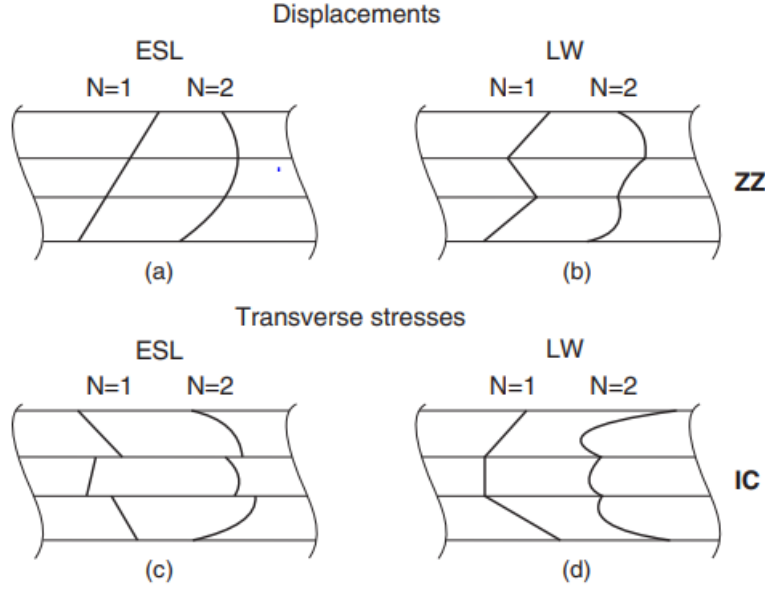


Figure 3.7: C_z^0 requirements

Generalized Unified Formulation (SGUF) expands its possibilities by proposing a hierarchical approach to account for sublaminates behavior inside a composite. SGUF treats each sublaminates as a separate layer group, allowing for more precise simulation of intra-layer phenomena such as localized stresses, delaminations, and thermal gradients. This improvement makes SGUF a strong tool for advanced composite material analysis by enabling more precision in the analysis of complex multilayered composites while maintaining the computational efficiency of CUF.

3.2.2 Sublaminates Generalized Unified Formulation (SGUF)

Composite materials have received a lot of attention due to their better mechanical and thermal qualities, particularly in applications that require high strength-to-weight ratios, such as aerospace, automotive, and marine structures. Thermal transient analysis in such materials is required to assure their reliability and performance under varying thermal loads. Among the different analytical and numerical methodologies employed, the Sublaminates Generalized Unified Formulation (GUF) has emerged as an effective framework. This section discusses the development and application of GUF in thermal transient analysis for composite materials, emphasizing its benefits and limitations. To perform thermal analysis on composite materials, heat conduction, convection, and radiation phenomena in anisotropic and heterogeneous layers must be studied. Traditional techniques, such as Classical Laminates Theory (CLT), frequently fail to capture the specific thermal behavior of layer interfaces, resulting in erroneous predictions. The SGUF framework is built around the following essential principles:

$$\mathbf{u}(x, y, z) = F_\tau(z)\mathbf{u}_\tau(x, y), \quad \tau = 1, 2, \dots, N \quad (3.7)$$

where the displacement vector is represented by $\mathbf{u}$, the generalized displacement vectors are

$\mathbf{u}_\tau$, and the expansion functions are F_τ . The thermal field can be expressed as follows:

$$T(x, y, z, t) = \Phi_s(z)T_s(x, y, t), \quad s = 1, 2, \dots, M \quad (3.8)$$

3.2.2.1 Application to Thermal Analysis

Different approaches have been used to incorporate thermal effects into the SGUF architecture. Specifically, studies have focused on:

3.2.2.2 Steady-State Thermal Analysis

SGUF has been widely used to study the steady-state thermal response of composite structures. The governing equations for steady-state conditions are written as follows:

$$\nabla \cdot (k_{ij} \nabla T) + Q = 0 \quad (3.9)$$

The thermal conductivity tensor is denoted by k_{ij} , whereas the internal heat generation is given by Q .

3.2.2.3 Transient Thermal Analysis

Because of time-dependent effects, the transient thermal behavior adds even more complexity. The governing equation becomes:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k_{ij} \nabla T) + Q \quad (3.10)$$

The ability of SGUF to handle many scales and physical phenomena has been used to analyze composite materials with embedded temperature sensors. The governing equations of such systems frequently have the form:

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{uT} \\ \mathbf{K}_{Tu} & \mathbf{K}_{TT} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{T} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_u \\ \mathbf{F}_T \end{bmatrix} \quad (3.11)$$

The numerical implementation of SGUF for thermal analysis usually includes:

$$\mathbf{K}\mathbf{T} + \mathbf{C}\dot{\mathbf{T}} = \mathbf{F} \quad (3.12)$$

where $\mathbf{F}$ denotes thermal loads, $\mathbf{K}$ is the conductivity matrix, and $\mathbf{C}$ is the capacity matrix.

3.2.2.4 Variable Kinematics Models

The flexibility of SGUF in implementing variable kinematics models is one of its key advantages. This method provides for:

- Selective refinement of the model in critical regions
- Optimization of computational resources

- Enhanced accuracy in thermal stress prediction

Cinefra et al. [181] demonstrated that variable kinematics models can be as accurate as full 3D solutions while maintaining computational efficiency.

3.2.2.5 Interface Continuity

A key component of the development of SGUF has been handling interface continuity in thermal difficulties. The following is an expression for the interface conditions:

$$T^{(k)}(z_k) = T^{(k+1)}(z_k), \quad k = 1, 2, \dots, N - 1 \quad (3.13)$$

3.2.2.6 Formulation for Physical and Numerical Layers

Figure 3.8 provides a schematic representation of adaptive refining. It is important to note that the suggested formulation is a major extension of existing variable kinematic modeling methodologies. Figure 3.9 depicts the link between physical and numerical layers. The computational efficiency of the model is maximized by selecting supplementing the approximation within sublaminates that require a more refined description. This selective refinement fully utilizes the Unified Formulation's kinematic modeling flexibility.

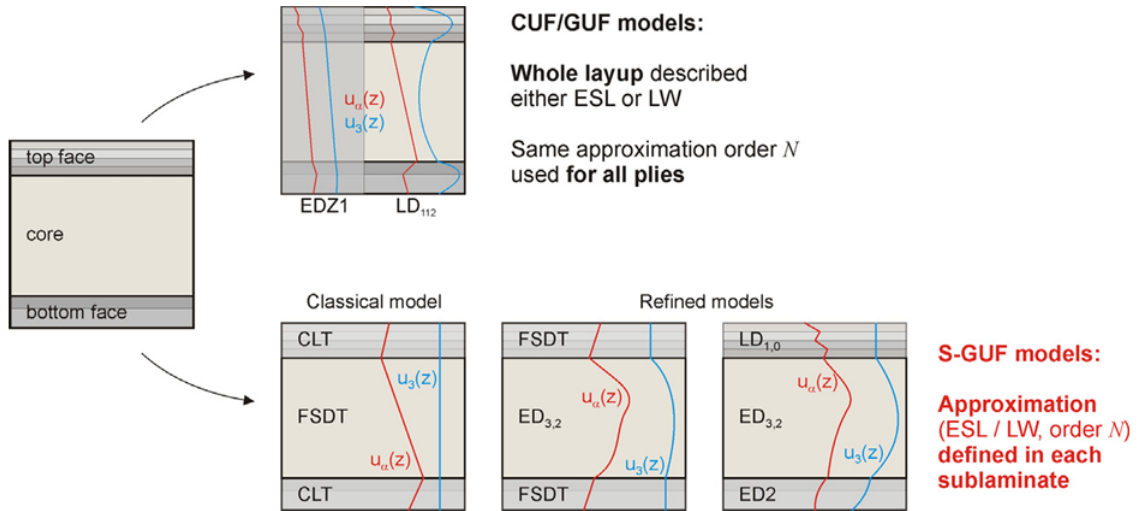


Figure 3.8: The proposed Sublaminate-GUF (S-GUF) broadens the flexibility of CUF/GUF by allowing to formulation of different GUF-type models for separate sublaminates.

3.2.2.7 Comparison of Sublaminate GUF with Other Methods

There are several widely used approaches for thermal analysis of composite materials, each having unique benefits and drawbacks. Table 3.1 presents a comparative analysis that demonstrates the unique features of several approaches utilized in the thermal analysis of composite materials. Every approach has a unique mix of benefits and drawbacks, making them appropriate for various uses based on the particular needs of the analysis.

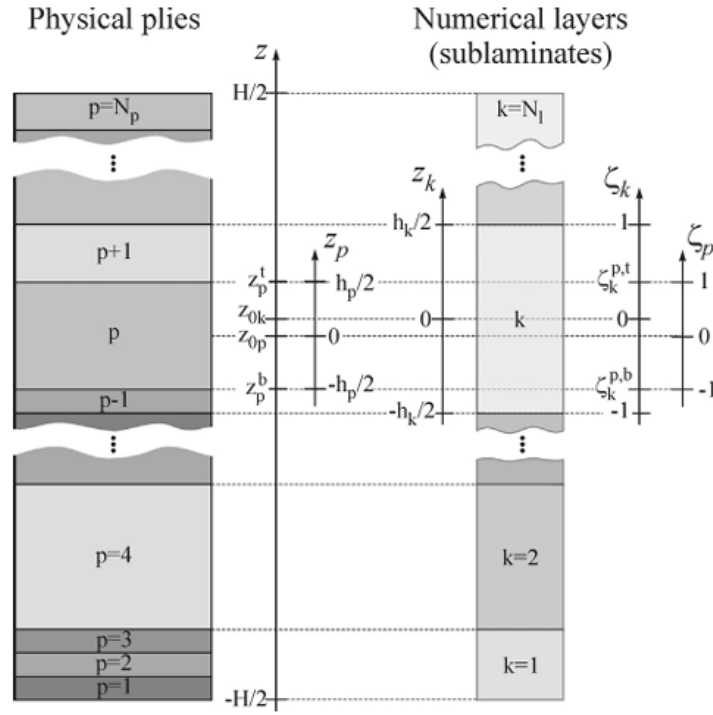


Figure 3.9: A Multilayered plate as an assembly of N_p physical plies (left) and N_l numerical layers (right).

3.3 Discussion and Open Points

While the application of the Carrera Unified Formulation (CUF) and Sublaminates Generalized Unified Formulation (SGUF) has led to significant advancements in the modeling of composite laminates for thermographic defect detection, several key gaps remain. These gaps highlight areas that require further research and development, particularly in transient thermal solutions, boundary conditions, and integration with experimental thermography.

1. **Transient Thermal Solutions:** CUF and SGUF have not been fully explored in the context of transient thermal analysis for thermography. While steady-state thermal models provide useful insights, thermographic defect detection is inherently time-dependent. The ability to accurately model transient heat propagation is critical for identifying defects such as delaminations, voids, and fiber-matrix separations. Future research should focus on refining CUF and SGUF implementations to include time-dependent thermal effects with realistic material responses.
2. **Boundary Condition Implementation:** The accurate definition of boundary conditions in CUF and SGUF remains a challenge, particularly in thermal analysis. Real-world thermographic inspections involve complex boundary conditions such as localized heat sources, convective heat transfer, and variable environmental influences. Unlike traditional FEM models, which often rely on predefined boundary conditions, CUF and SGUF require further development to incorporate realistic, application-specific boundary conditions that align with experimental thermographic setups.
3. **Complex Delamination Geometries:** While CUF and SGUF have demonstrated accu-

Table 3.1: Comparative Analysis of Different Methodologies for Thermal Analysis in Composites

Method	Advantages	Limitations
Classical Laminate Theory (CLT)	Simple implementation Computationally efficient Suitable for preliminary analysis	Cannot capture layerwise interactions Poor prediction of inter-laminar thermal gradients Limited accuracy for thick laminates
First-order shear Deformation Theory (FSDT)	Includes shear deformation effects. Good for moderately thick laminates. Better accuracy than CLT.	Limited accuracy for thick laminates. Poor prediction of high thermal gradients. Insufficient for complex layered structures.
Finite Element Method (FEM)	High accuracy. Handles complex geometries. Excellent for anisotropic materials. Versatile application range.	Computationally intensive. Resource-heavy for large systems. Time-consuming for multilayered analysis.
Sublaminar GUF	Accurate layerwise modeling. Excellent capture of inter-laminar effects Scalable to complex systems. Better handling of thermal gradients.	Higher computational cost than CLT/FSDT. More complex implementation. Requires more detailed input parameters.

racy in modeling composite laminates, their application to complex delamination geometries remains underdeveloped. Defects in real-world applications are often irregular and span multiple layers with varying orientations. The interaction between transient heat flow and such defects is not fully captured using current CUF and SGUF implementations. Future work should explore advanced defect modeling techniques to improve the accuracy of thermographic predictions in composites with complex damage scenarios.

4. **Integration with Experimental Thermography:** One of the major gaps in current CUF and SGUF-based simulations is their validation with experimental thermography. Numerical models must be benchmarked against experimental data to ensure their accuracy in predicting thermal signatures of defects. This requires a systematic approach to compare simulated and real thermographic images, considering factors such as equipment calibration, noise reduction, and environmental effects. The development of standardized validation procedures for CUF and SGUF in thermographic applications is necessary for their widespread adoption.
5. **Thermal and Mechanical Coupling:** CUF and SGUF have primarily been used for thermal or mechanical analysis separately, but real-world composite structures experience coupled thermo-mechanical effects. In thermographic inspections, localized heating can induce thermal stresses that influence defect growth and material degradation. Current formulations do not fully account for these interactions, limiting their predictive capability in realistic operational conditions. Future research should focus on developing coupled thermal-mechanical models using CUF and SGUF to better represent these effects.

6. **Material Property Variability and Aging Effects:** The current implementation of CUF and SGUF does not fully account for temperature-dependent material properties, environmental aging, or degradation over time. In real-world thermographic inspections, material properties such as thermal conductivity and diffusivity change due to factors like prolonged exposure to heat, moisture, or mechanical loading. Incorporating these time-dependent material variations into CUF and SGUF simulations would improve the reliability of defect detection in aged composite structures.
7. **Digital Twinning for Thermographic Analysis:** The integration of CUF and SGUF into digital twin frameworks remains an open research area. Digital twinning involves creating real-time, physics-based models that continuously update based on experimental data. This approach is highly valuable for thermographic inspections, allowing predictive maintenance and real-time defect monitoring in aerospace, automotive, and structural health monitoring applications. Future work should focus on embedding CUF and SGUF within digital twin architectures to enhance the accuracy and adaptability of thermographic defect detection systems.
8. **Computational Efficiency for Large-Scale Structures:** While CUF and SGUF offer advantages in reducing computational cost compared to conventional FEM, their application to large-scale or highly detailed composite structures can still be computationally expensive. Optimizing these methods for high-performance computing (HPC) environments and GPU-based parallel processing is necessary to make CUF and SGUF more viable for industrial-scale thermographic applications.

In conclusion, while CUF and SGUF present a significant advancement in composite material modeling for thermographic defect detection, several gaps must be addressed for their broader application. The development of transient thermal solutions, improved boundary condition modeling, coupled thermal-mechanical analysis, digital twinning, and computational optimization are key areas for future research. Addressing these challenges will enhance the predictive accuracy, computational efficiency, and practical implementation of CUF and SGUF in real-world thermographic applications.

3.4 Conclusion

This study introduces the Carrera Unified Formulation (CUF) and Sublaminated Generalized Unified Formulation (SGUF) as advanced numerical techniques for thermographic defect detection in multilayer composite laminates. CUF provides a flexible framework for deriving finite element matrices based on invariant basic nuclei, enabling smooth transitions between classical and higher-order theories. SGUF extends this approach by incorporating layer-wise modeling, offering a more accurate representation of interlaminar interactions and anisotropic thermal conductivities.

A key novelty of this work is the first-ever application of CUF and SGUF to transient thermal analysis for defect detection using Thermography. Unlike traditional FEM-based

approaches in software like ANSYS and COMSOL, which rely on predefined material models and require extensive computational resources, CUF and SGUF offer a more efficient and adaptable alternative. These methods reduce computational costs while preserving accuracy, making them highly suitable for parametric studies involving variable lay-up sequences, anisotropic thermal behavior, and realistic defect geometries.

By integrating CUF and SGUF into thermographic analysis, this study overcomes existing computational limitations and enhances the understanding of dynamic thermal behavior in composites. The developed models not only improve defect detection accuracy but also pave the way for future advancements, such as coupled thermal-mechanical analysis and digital twinning for real-time structural health monitoring.

In the following chapters, CUF and SGUF are implemented and validated against conventional numerical methods and experimental data. The insights gained from this research mark a significant step toward improving the accuracy, efficiency, and practicality of thermographic defect detection in aerospace, automotive, and structural applications.

Chapter 4

Methodology

4.1 Experimental Campaign

4.2 Simulated Delamination in CFRP Laminate

4.2.1 Material and geometry

The sample considered for the following investigations is made of CFRP composite material and has been laminated using fabric prepreg with fiber T300. It is a symmetric cross-ply laminate consisting of 25 plies for a total thickness of 5 mm. The geometry of the defect is circular. The defect is produced with 1 layer of release film MR-1 RED (0.01mm thickness) and 2 layers of flash breaker (0.03mm thickness). A summary of these layer thicknesses is presented in Table 4.1. A lateral view of the plate with the defect, along with the defect's diameter and depth corresponding to the ply is presented in Figure 4.1.

Table 4.1: Layer thicknesses of the multilayered structure.

Layer	Thickness (mm)
Halar layer	0.03
Air layer	0.01
Remaining layers (undamaged)	~0.2

In Figure 4.2, the lock-in thermographic setup is shown. The plate's surface was heated using a square-modulated wave. A total of six 4000W halogen lamps were used to stimulate the specimen. The FLIR System's A655sc IR camera, which uses a microbolometer detector, was employed for thermographic experiments. The final geometric resolution was 0.65 mm/pixel. The specimen was configured as a cantilever beam to avoid any potential heat conduction effects from supports coming into direct contact with the opposite side. The MultiDES system coordinated the data acquisition (DES Diagnostic Engineering Solutions S.r.l.) and its proprietary software IRTA with the control of the halogen lamps.

A suitable experimental plan was studied, as shown in Table 4.2. For each excitation

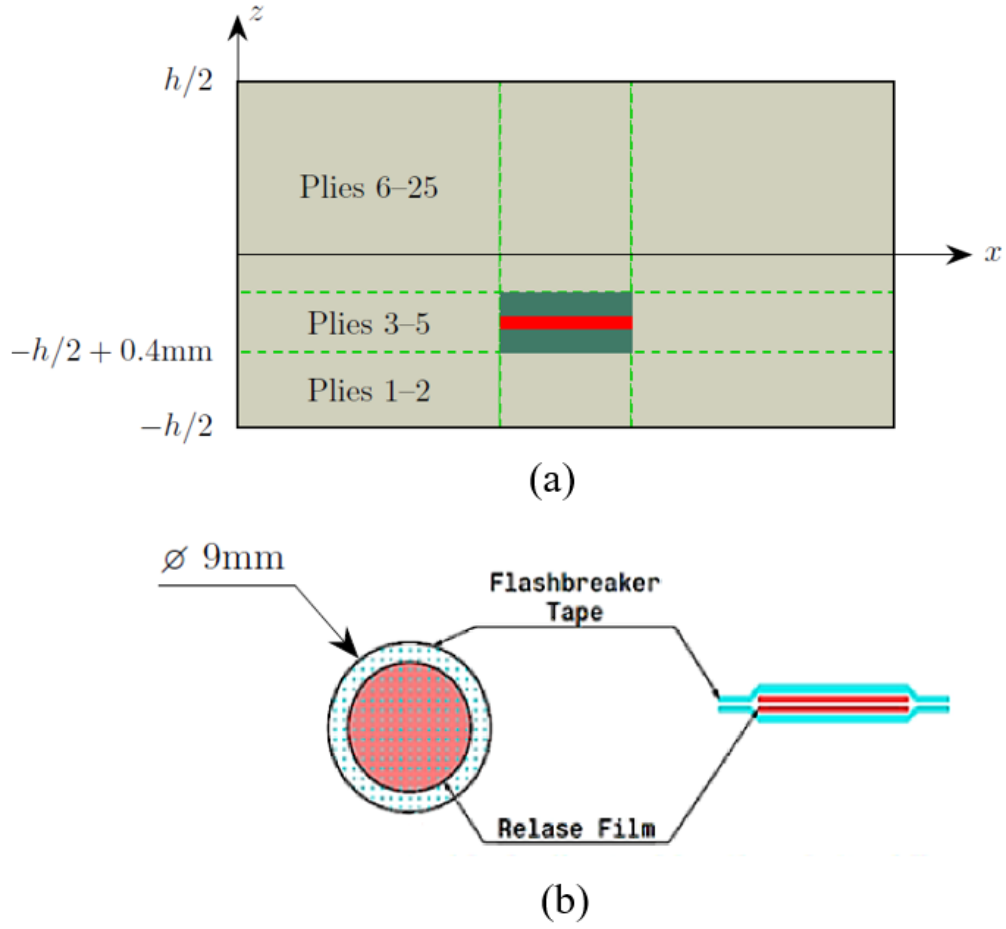


Figure 4.1: (a) Multilayered plate with defect (b) Experimental Scheme of the defect used to simulate delamination



Figure 4.2: Lock-in thermography set-up

frequency, the test piece was periodically heated with three excitation cycles to perform the experimental tests in the transient regime. Additionally, to evaluate the reliability and repeatability of the obtained results, each test was repeated three times. It should be noted that each test was started after the test piece's surface temperature reached the laboratory

environment temperature of approximately 24°C.

Table 4.2: Input parameters for the acquisition of thermal sequences

Waveform	Excitation frequency (Hz)	Acquisition duration (s)	Frame Rate	No. of cycles
Square	0.033	90	5 Hz	3
	0.022	135	5 Hz	3
	0.0125	240	5 Hz	3

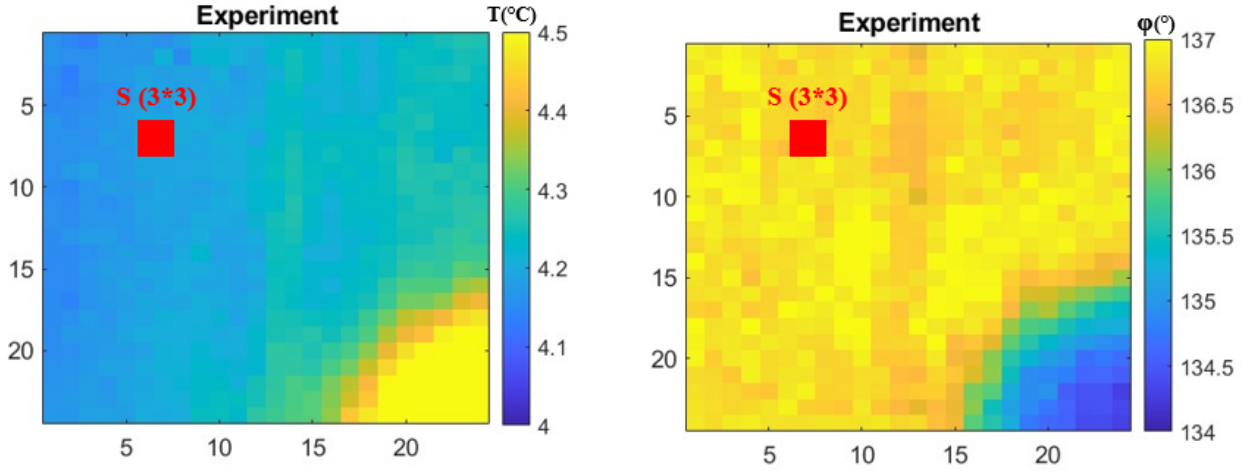


Figure 4.3: Amplitude and phase map of excitation frequency 0.0125Hz

4.2.2 Post processing Algorithm: Signal reconstruction (SR)

Subsequently, the thermal data were processed by IRTA software to recover amplitude and phase images from lock-in tests. In this work, we propose the use of a modulated square wave as a heat source to reduce the testing time. In fact, in this case, as demonstrated in other works [15], [16] the thermal response of the material contains information about high-order frequencies proportional to the main frequency. By decomposing the thermal response in the time domain as the sum of a singular sinusoidal wave, we can write

$$T_m(t) = a + bt + \sum_{n=1}^{\infty} \Delta T_n \sin(n\omega t + \varphi_n) \quad \text{with } n = 1, 3, 5, \dots \quad (4.1)$$

where a and b are two constants used to model the average temperature growth of the material, ω is the modulated frequency of the main harmonic (first Fourier component), and ΔT_n and φ_n represent the amplitude and the phase of n -th Fourier component. In this work, a 5th-order expansion is considered for Equation 4.1:

$$T_m(t) = a + bt + \Delta T_1 \sin(\omega t + \varphi_1) + \Delta T_3 \sin(3\omega t + \varphi_3) + \Delta T_5 \sin(5\omega t + \varphi_5) \quad (4.2)$$

The 8 unknown coefficients $(a, b, \Delta T_1, \varphi_1, \Delta T_3, \varphi_3, \Delta T_5, \varphi_5)$ are determined for each pixel through a least-squares fit of the thermal signal, which eventually yields the amplitude and phase information of high-order excitation frequencies for the main one by a single lock-in test.

The phase and amplitude map from the experiment is given in Figure 4.3. The thermal and phase-contrast maps that will be used for both, the experiment and the numerical results, are defined as

$$\Delta T = (T - \text{mean}(S)) ; \Delta \varphi = (\varphi - \text{mean}(S)) \quad (4.3)$$

where “mean(S)” is the average value evaluated in the 3x3 pixels area of the sound region illustrated in Figure 4.3.

4.3 Debonding in CFRP T-joints

Debonding in CFRP T-joints can be efficiently detected utilizing thermography testing, a non-destructive technology that shows thermal variations induced by changes in heat flow. Active thermography technologies, such as lock-in or pulse thermography, are especially useful because they exhibit differences in thermal conductivity caused by debonded regions. Because they are air-filled, these regions prevent heat transfer and produce observable contrasts in thermal images. This method makes it possible to precisely identify and characterize de-bonding defects offering essential details for CFRP T joint quality assurance and maintenance in automotive, aerospace, and other structural applications.

4.3.1 Material and geometry

The material investigated in this study consists of T-joint composite specimens commonly used in aerospace and structural applications due to their high strength-to-weight ratio and durability. These specimens are fabricated using carbon fiber-reinforced polymer (CFRP) laminates, a material known for its anisotropic thermal properties.

Figure 4.4a provides a detailed schematic of the T-joint specimen, illustrating its geometric dimensions, which are critical for ensuring consistency in experimental and numerical analyses. A key aspect of this study is the detection and characterization of de-bonding defects within the T-joint structure. De-bonding refers to the delamination or separation at the adhesive interface between the composite components, which can significantly impact the structural integrity and thermal response of the joint. Figure 4.4b presents both a schematic representation and actual sample images, marking Side A and Side B, which are used for the de-bonding analysis. These images help identify the regions where adhesive failure may occur, affecting heat diffusion and phase contrast patterns during Lock-in Thermography (LIT) testing.

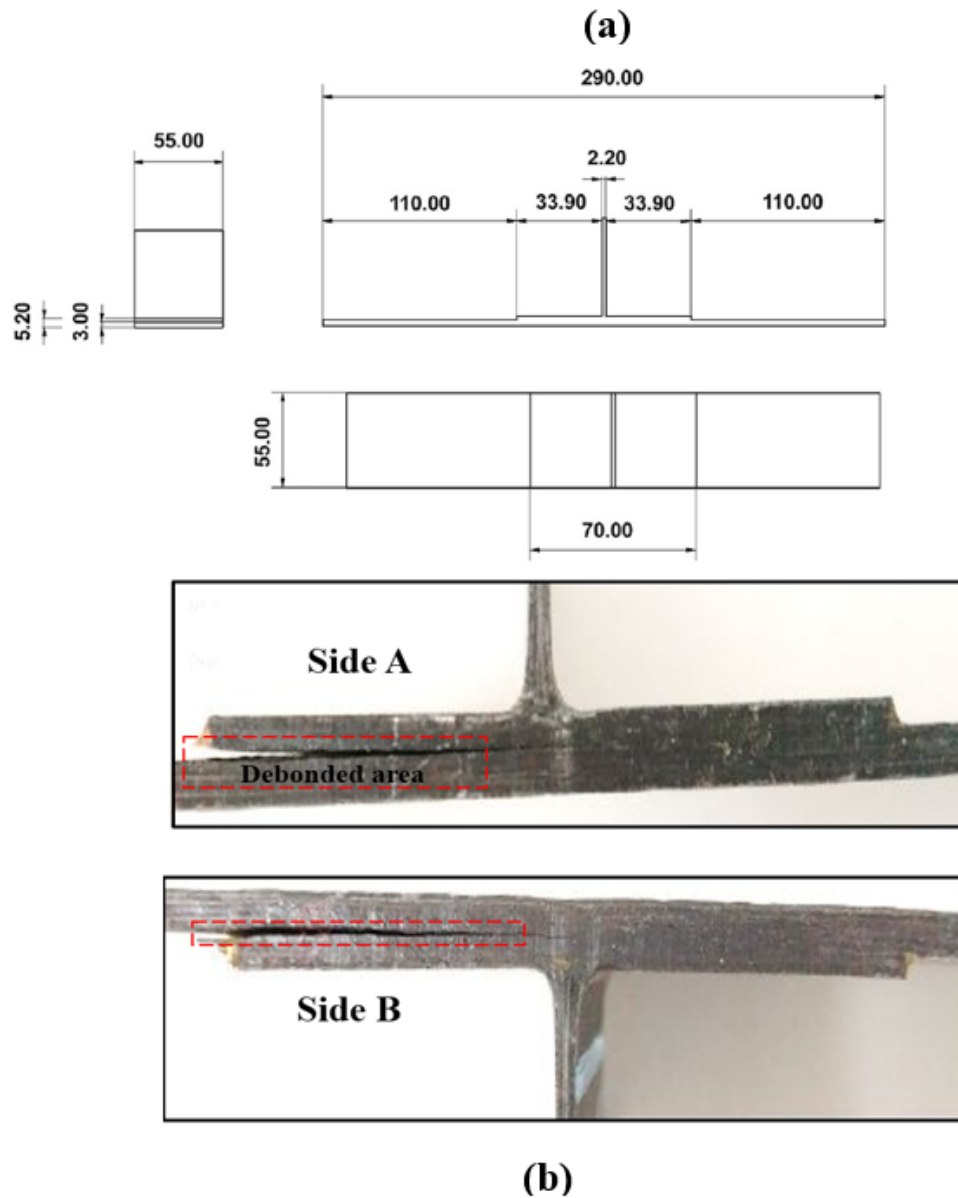


Figure 4.4: T-joint structure:(a) Dimensions of the specimen (c) Actual test specimen indicating Side A and B

The main components included:

Skin and Stringer:

Modeled as orthotropic CFRP layers with defined thermal conductivities in the longitudinal (fiber) and transverse (matrix) directions.

Web:

Representing the vertical connection between the springer and skin, with similar material properties to the springer.

De-bonded Interface:

The air-filled gap between the skin and flange was explicitly simulated to match the experimental conditions. The thickness of this gap was varied iteratively during simulations to align with observed phase contrast results.

The thermophysical properties of the CFRP material used in this study, obtained from the literature, are summarized in Table 4.3. This table provides essential parameters such as thermal conductivity (k), specific heat capacity, and density, which are fundamental for modeling the Lock-in Thermography test. It is important to highlight that thermal conductivity in CFRP is direction-dependent. The value of k presented in Table 4.3 corresponds to the orthogonal heat conduction relative to the fiber direction, as CFRP materials exhibit significantly different heat conduction properties along and across the fibers. Understanding this anisotropic behavior is crucial for accurately interpreting phase contrast data and assessing de-bonding defects in the experimental and numerical analyses [182].

Table 4.3: Thermo-physical constants of materials

Materials	k (W/m.k)	ρ (kg/m ³)	c_p (J/kg.k)
CFRP	0.800	1600	1200
Air	0.024	1.200	1006

4.3.2 Lock in Test

Lock-in Thermography (LT) is defined as using an external heat source to generate periodic thermal waves. Temperature variations across the specimen surface can be used to determine the amplitude and phase of the material's thermal response. In a one-dimensional model, the magnitude R of the amplitude signal, normalized to a thermally thick and opaque reference specimen, and phase ϕ can be analytically determined as follows [183, 184]:

$$R = \sqrt{\frac{(1 - R_1)e^{-2h_n}}{1 - R_1R_2e^{-2h_n}}} \cdot \frac{F \sin^2(h_n)}{1 + F \sin^2(h_n)} \quad (4.4)$$

$$\phi = \arctan \left\{ \frac{-R_2(1 + R_1)e^{-2h_n} \sin(2h_n)}{1 - R_1(R_2e^{-2h_n}) + R_2(1 - R_1) \cos(2h_n)} \right\} \quad (4.5)$$

where

$$F = \frac{4R_1R_2e^{-2h_n}}{(1 - R_1R_2e^{-2h_n})^2}, h_n = \frac{h}{\mu}, \mu = \sqrt{\frac{2\alpha_d}{\omega}} = \sqrt{\frac{\alpha_d}{\pi f}} = \sqrt{\frac{\alpha_d}{\pi}} T, \alpha_d = \frac{k}{\rho c_p} \quad (4.6)$$

The LT test consists of applying a modulated heat source to the specimen's surface at a specified frequency (or period). Equation (4.6) uses the thermal diffusivity length μ to calculate the excitation frequency to the examined depth. As a result, a series of tests at various excitation frequencies are required to study the entire specimen. Numerical simulations that estimate the amplitude and phase can be used to minimize the number of tests. These simulations can provide valuable information regarding the range of excitation frequencies (modulated periods) with a large phase signal contrast.

To simulate the phase data of a thermally thin specimen and a reference specimen, which is supposed to be thermally thick, equation (4.6) was utilized to generate synthetic experimental

data [184]. When the front and rear surfaces of a thermally thin specimen (CFRP) with thickness h are exposed to gas (air), the behavior of the specimen is described by the one-dimensional model. The reflection coefficients in this instance coincide, as $R1 = R2$. To determine the phase values, two ranges of thickness and period were fixed. Given the reflection setup arrangement, the bonded area is up to 3 mm deep from the heated skin surface, with the de-bonded area being between the skin and the stringers. At a thickness of 3 mm, phase contrast becomes substantial at a modulation period of 50 s. The phase contrast rises with the modulation period, as expected. Palumbo [20] demonstrated that using longer periods can increase in-plane diffusion of thermal waves, resulting in phase contrast variations. Two modulation periods—the 90s and the 60s—were selected for the lock-in testing in consideration of these factors.

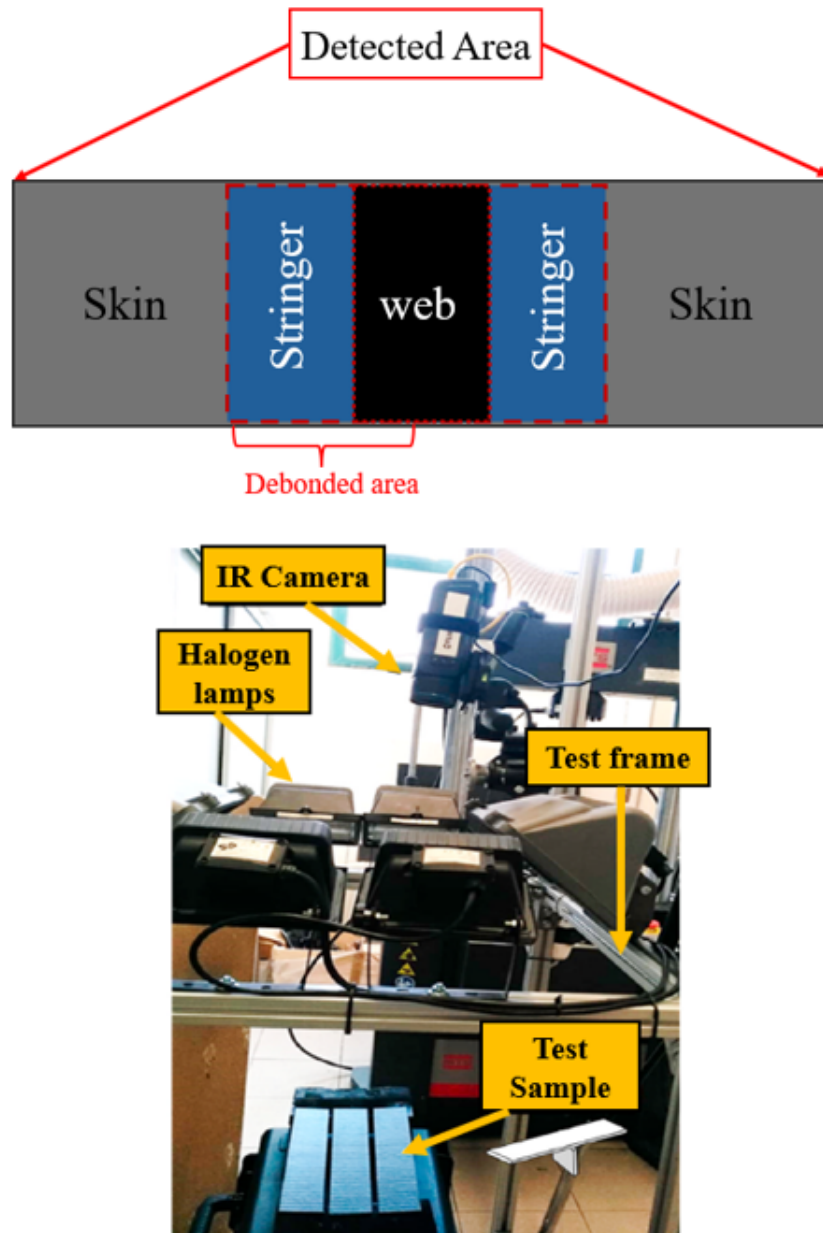


Figure 4.5: Experimental set-up used for lock-in tests

The setup in Figure 4.5 consists of four halogen lamps with a power of 4 kW controlled by a MultiDES system, modulating the heat source using a sequence of square waves. Thermal sequences were captured using a FLIR sc655sc IR camera with a 640×480 -pixel micro-bolometer detector and a thermal sensitivity (NETD- Noise Equivalent Temperature Difference) of less than 30 mK. The distances of the lamps and the infrared camera from the specimen were 730 mm and 1520 mm, respectively. The geometrical resolution reached was 0.6 mm per pixel. Table 4.4 summarizes the experimental LT tests.

Table 4.4: Experimental Parameters used for lock-in thermography (LT) tests

Test number	Modulation Period (s)	Modulation Frequency (Hz)	Number of (cycles)
1	60	0.017	3

4.3.2.1 Data processing

The thermal data from the lock-in thermography tests were processed using IRTA® software to generate phase images. The phase map was obtained by solving the thermal reconstruction algorithm embedded within the software, as described in Equations 4.1 and 4.2.

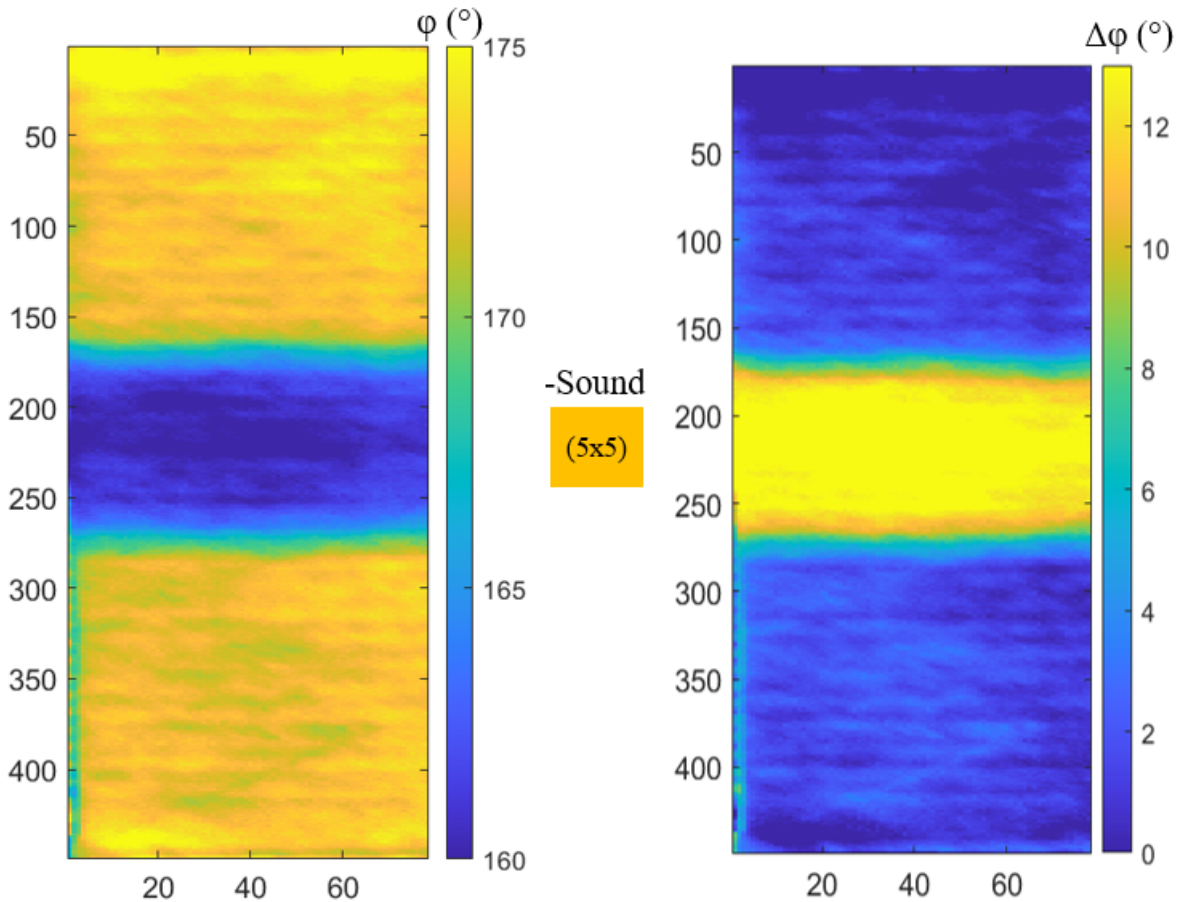


Figure 4.6: Phase map from experimental data

In the context of debonding analysis in composite T-joint structures, phase contrast plays a critical role due to its high sensitivity to subsurface defects. Unlike raw thermal images, which surface emissivity variations and non-uniform heating can influence, phase-based imaging effectively enhances defect visibility by isolating thermal anomalies from background noise. This makes it particularly effective for detecting and characterizing debonding regions, as phase contrast highlights discontinuities in thermal diffusion caused by weak or detached interfaces. Consequently, phase analysis provides a more reliable and precise evaluation of debonding in composite structures compared to conventional thermographic approaches.

The phase map from the experiment is given in Figure (4.6). The phase-contrast maps that will be used for both, the experiment and the numerical results, are defined as

$$\Delta\varphi = (\varphi - \text{mean}(S)) \quad (4.7)$$

where “ $\text{mean}(S)$ ” is the average value evaluated in the 5x5 pixels area of the sound region illustrated in Figure (4.6).

4.4 Numerical Simulation

Numerical simulation is critical for understanding the thermal behavior of composite materials and finding defects with thermographic techniques. Advanced modeling methods can correctly reflect the complex interactions of heat transmission, material properties, and defect characteristics. The simulation includes a complex multilayered defect structure, accurate material characteristics, and boundary conditions modeled after experimental circumstances. Furthermore, the model is designed to maximize computing efficiency while maintaining accuracy. This allows for the investigation of large-scale issues using lower computational resources, making it appropriate for parametric studies and optimization tasks.

4.5 COMSOL Multiphysics

4.5.1 Geometry and Model Setup

The geometry utilized in this study represents a composite plate with overall dimensions of $30\text{ mm} \times 30\text{ mm}$ and a thickness of 5 mm . A circular defect is incorporated at the center of the plate to simulate a delamination as shown in Figure 4.7a. The defect is modeled with a multilayer structure to replicate realistic material behavior, consisting of the following layers: An outer layer of halar material with a thickness of 0.03 mm , An air gap with a thickness of 0.01 mm and another layer of halar material with a thickness of 0.03 mm .

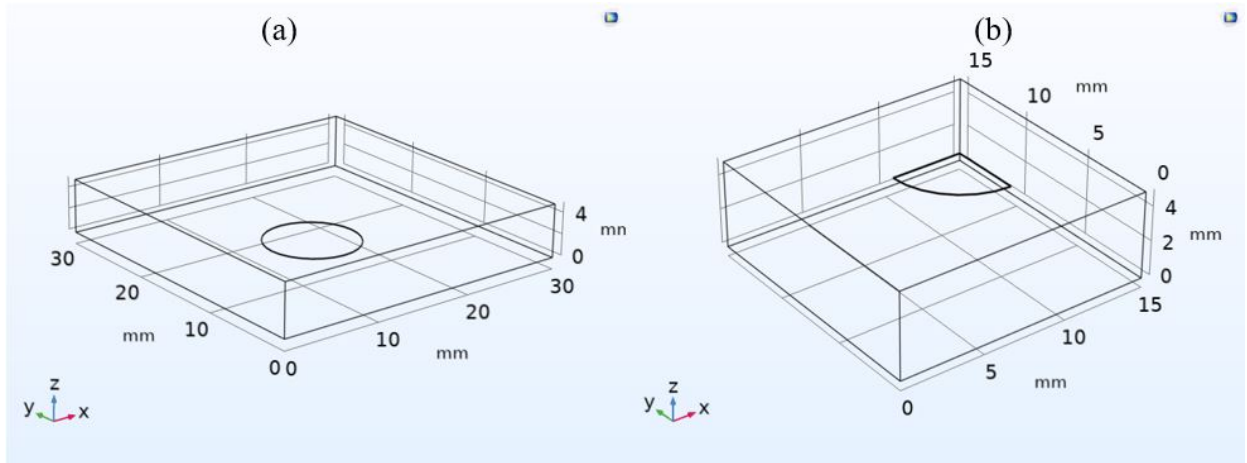


Figure 4.7: (a) Geometry of the composite plate showing the centrally placed circular defect
(b) Quarter-symmetric geometry

This layered configuration is designed to mimic the interface properties between the composite material and the air void, providing a representative thermal boundary condition. The model employs quarter symmetry to enhance computational efficiency while maintaining accuracy. The computational domain is reduced to a quarter of the plate by exploiting symmetry in the geometry and boundary conditions. The effective dimensions of the reduced domain are $15\text{ mm} \times 15\text{ mm}$, significantly decreasing the computational resources required for

simulation while preserving the essential physical characteristics of the problem. The reduced geometry, including the location and structure of the defect, is shown in Figure 4.7b. This figure illustrates the quarter-symmetric composite plate and highlights the circular defect centrally placed in the model domain. The symmetric boundary conditions ensure that the thermal behavior of the reduced domain accurately represents that of the full plate. This modeling approach allows for detailed analysis of the heat transfer behavior and defect detection while optimizing computational efficiency. The layered defect structure and quarter symmetry enable the study of localized thermal anomalies under realistic conditions.

The flowchart illustrating the model validation steps in the study is shown in Figure 4.8.

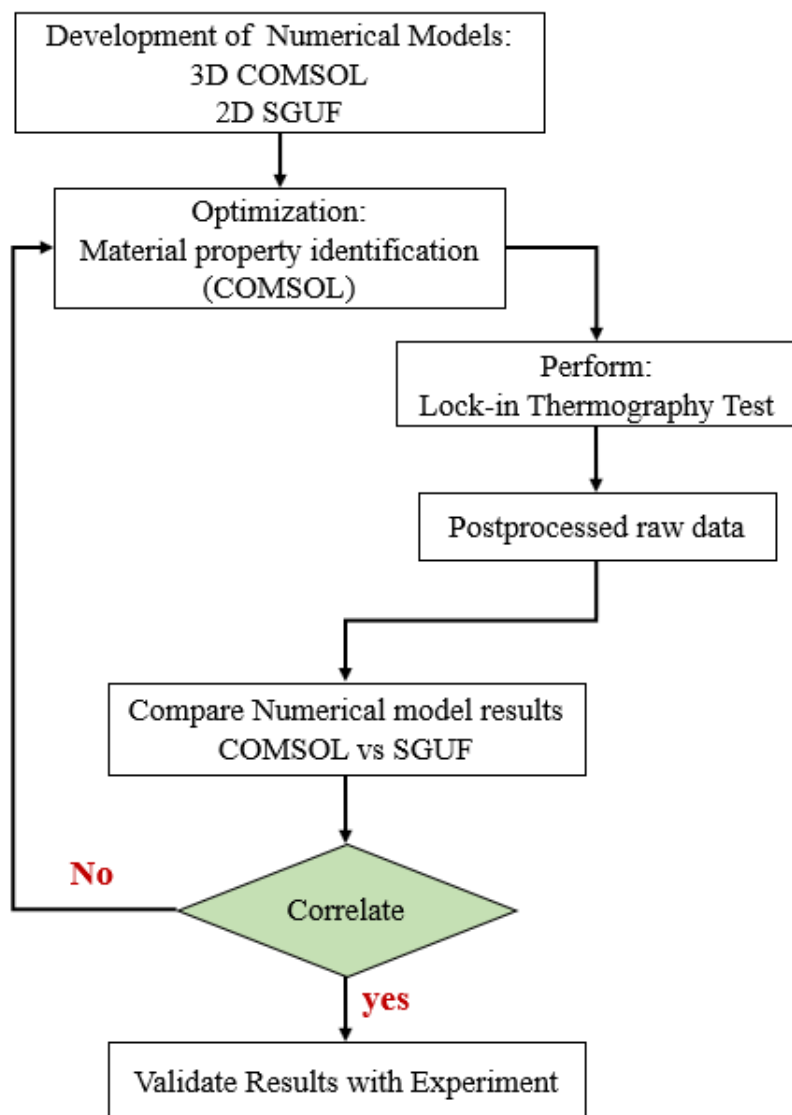


Figure 4.8: Flowchart of Methodology and Scheme for Material Property Determination used in Numerical Models

4.5.2 Material Properties

The numerical model integrates three distinct materials: Carbon Fiber Reinforced Polymer (CFRP T300), air, and halar, each characterized by unique thermal and physical properties sourced from manufacturer datasheets. The properties were calibrated using the COMSOL Multiphysics optimization interface, incorporating experimental data from both defective and non-defective regions. This process optimized the material parameters to ensure the numerical model accurately simulates heat transfer behavior and defect detection, achieving close alignment with experimental results. The optimization process employed the SNOPT solver, a Sequential Quadratic Programming (SQP)-based algorithm, which is particularly effective for solving large-scale nonlinear optimization problems with constraints. This solver is well-suited for applications requiring precise parameter fitting, as it efficiently minimizes the discrepancy between experimental and numerical results. The thermal conductivity (k_z) determines the material's ability to conduct heat in the through-thickness direction, while the specific heat capacity (C_p) reflects its capacity to store thermal energy. The density (ρ) influences the material's thermal inertia, playing a critical role in the dynamic response of the system. The adopted optimization process, designed to fine-tune these parameters for accurate thermal modeling, is illustrated in Figure 4.9. This process ensures that the simulated thermal behavior aligns closely with experimental observations, enabling reliable predictions of defect characteristics. These parameters are crucial for solving the heat transfer Equation 2.7 under transient conditions using the finite element method (FEM). The heat transfer analysis, based on the lock-in thermography concept, forms the foundation for defect detection. Table 4.5 summarizes the material properties identified through this optimization process. Detailed theoretical information about the SNOPT solver and its advantages is provided in the Appendix B.

Table 4.5: Material properties used in the numerical model

	Density (kg/m^3)	Thermal Conductivity ($W/(m*K)$)	Specific heat ($J/(kg*K)$)
CFRP (T300)	1760	0.57	820
Halar	1680	0.13	946
Air	1.2	0.013	1005

4.5.3 Heat Source

To simulate the heat source, a rectangular pulse function was employed, as shown in Figure 4.10a. The rectangular pulse was defined using the built-in *Rectangle* function in the software. To implement periodic heating and cooling, an analytical function was defined, as illustrated in Figure 4.10b. The expression used for the analytical function is:

$$\text{rect1}(\text{mod}(\frac{t}{1s}, t_c))$$

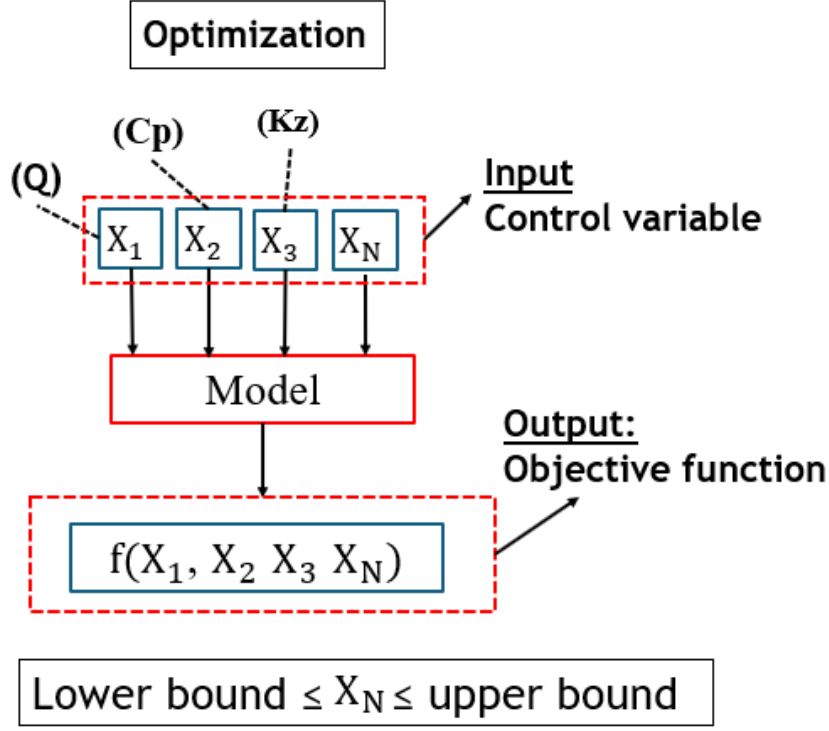


Figure 4.9: Optimization process to get material properties

where t_c denotes the duration of one complete cycle of heating and cooling. The analytical function was used to repeat the rectangular pulse periodically over time. The heat flux (Q) Watt/m² is applied at the bottom surface of the plate. The heat flux is defined as:

$$Q_{\text{applied}} = Q \cdot \text{an1} \left(\frac{t}{1[s]} \right) \quad (4.8)$$

where an1 represents the analytical function. The defect was placed at a depth of 0.4 mm within a plate of thickness 5 mm as shown in Figure 4.10c. The heat flux is applied to the bottom surface of the plate at $z = 0$. This configuration ensures a realistic simulation of heat propagation through the composite laminate and interaction with the defect region.

4.5.4 Meshing Strategy

A user-controlled meshing strategy was employed for the simulation to ensure accurate resolution of the thermal gradients, particularly around the defect region. The meshing approach was carefully designed to balance computational efficiency and accuracy. For the defect region, a highly refined mesh was used to capture the temperature gradient accurately. More than two elements were applied through the thickness of the defect to resolve the thermal distribution. Tetrahedral elements were selected to represent the complex geometry of the defect. Table 4.6 illustrates the mesh used for the defected region and the rest of the plate. Throughout the entire simulation, tetrahedral elements were utilized for consistent results.

COMSOL Multiphysics automatically refines the mesh based on user-defined parameters, ensuring that the resolution adapts to the complexity of the geometry and the physical gradients.

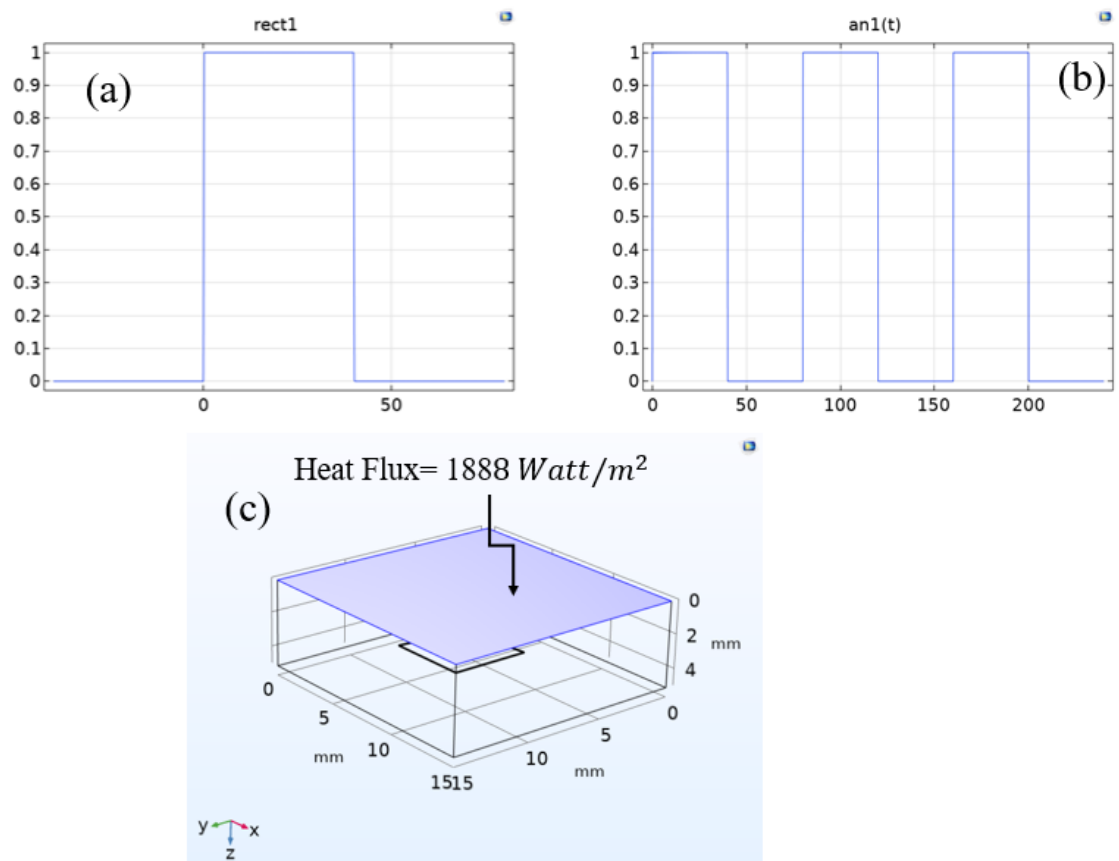


Figure 4.10: (a) Rectangular pulse (b) analytical function used for heat flux for period 80s (c) imposed heat at bottom surface

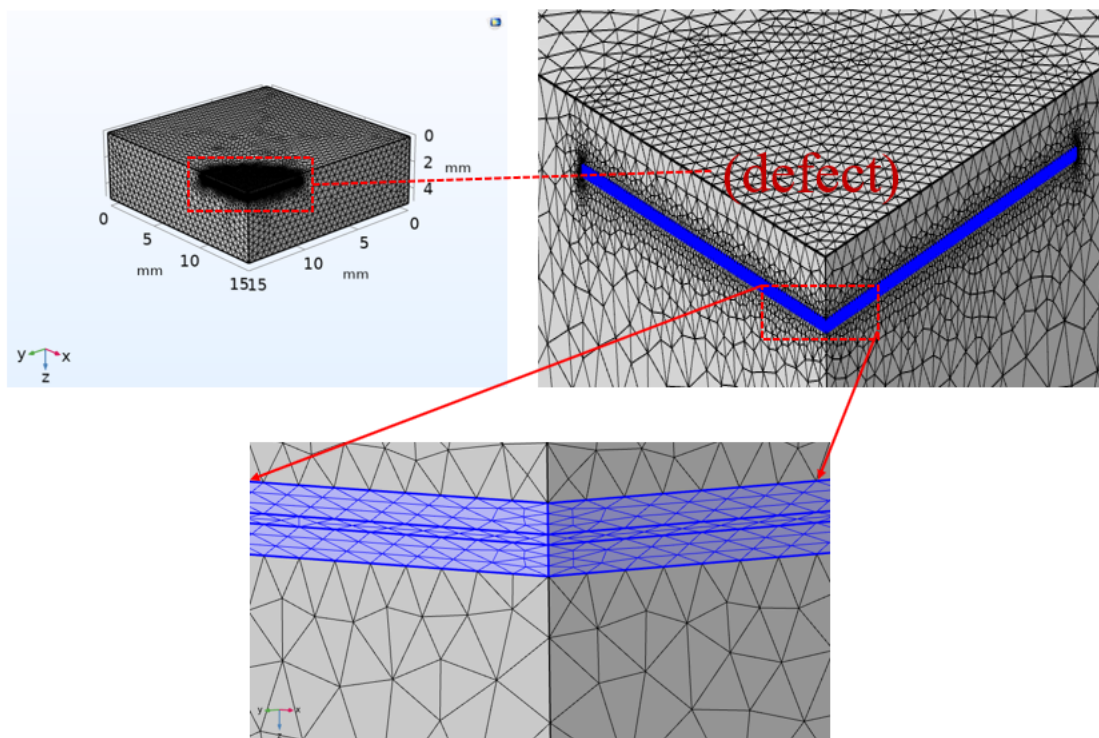


Figure 4.11: Mesh for the whole plate and the Defect Region

The meshing process involves generating a triangular surface mesh, which is then extended into a volumetric tetrahedral mesh. This approach ensures that areas with steep gradients, such as the defect, are resolved with higher precision.

Table 4.6: Mesh Parameters for Defect Region and Remaining Plate

Parameter	Defect Region	Remaining Plate
Element Type	Tetrahedral	Tetrahedral
Maximum Element Size	0.3 mm	0.525 mm
Minimum Element Size	0.003 mm	0.0225 mm
Maximum Element Growth Rate	1.3	1.35
Curvature Factor	0.2	0.3
Resolution of Narrow Regions	1.0	0.85

4.5.5 Time Dependent Solver: Backward Differentiation Formula (BDF)

The time-dependent solver in COMSOL Multiphysics utilizes numerical methods to solve partial differential equations (PDEs) that evolve with time, such as the transient heat transfer equation. For solving such time-dependent problems, COMSOL primarily employs the Backward Differentiation Formula (BDF) method, which is particularly effective for stiff differential equations commonly encountered in heat transfer analyses. This method ensures stability and accuracy, even for highly transient and nonlinear problems, making it ideal for the lock-in thermography simulations performed in this study. Detailed information about the BDF solver and its theoretical background is provided in Appendix A.

The BDF method is an implicit multi-step method that approximates the time derivative using previous time steps. For a differential equation of the form:

$$\frac{dy}{dt} = f(t, y) \quad (4.9)$$

The k-th order BDF formula can be expressed as:

$$\sum_{j=0}^k \alpha_j y_{n+j} = h\beta f(t_{n+k}, y_{n+k}) \quad (4.10)$$

where h is the time step, α_j and β are the method coefficients, and y_n represents the solution at time step n . The derivation of the BDF formula is given in appendices.

In the present analysis, several key parameters were configured to optimize the solver for transient heat transfer simulations. The maximum BDF order was set to 2, striking a balance between accuracy and computational efficiency, while the minimum BDF order was set to 1, allowing the solver to adapt dynamically between first and second-order methods. An initial time step of 10^{-6} seconds was chosen to ensure accurate solution initialization, and an event tolerance of 10^{-6} was specified to maintain precise event detection during the simulation.

The BDF method offers significant advantages for transient heat transfer analysis, making it well-suited for the lock-in thermography simulations in this study. Its inherent stability is particularly valuable for solving stiff problems involving multiple time scales. The method also features adaptive time-stepping, allowing the solver to adjust step sizes based on solution behavior, optimizing computational efficiency. Furthermore, the built-in error estimation ensures that the solution accuracy remains within specified tolerances throughout the simulation.

To ensure a stable and efficient solution process, the solver employs backward Euler initialization, which provides a consistent and robust startup for the simulations. The singular mass matrix option was configured as "Maybe" to handle potential singularities in the system matrix. Additionally, error estimation was set to exclude algebraic variables, further improving computational efficiency by focusing only on relevant degrees of freedom.

4.6 Sublamine Generalized Unified Formulation (SGUF)

The Sublamine Generalized Unified Formulation (SGUF) is extended to the framework of Finite Element Method (FEM) for both displacement-based and mixed (RMVT) formulation. The variable kinematics approach allows to choose of different plate models according to the desired level of accuracy. Furthermore, the mixed ESL/LW approach of SGUF makes the model particularly convenient for sandwich structure analysis.

4.6.1 Geometric description of the composite plate

A square composite plate of size 30x30 mm is considered and described in a Cartesian reference frame, indicated as (x, y, z) . The composite laminate is a cross-ply stack of 25 CFRP plies with alternating 0° and 90° orientations to the x -axis. The symmetric in-plane geometry allows us to reduce the computational model to a $15 \times 15 \text{ mm}^2$ area by considering only a quarter of the laminate. A defect of a radius of 4.5 mm (see Figure 4.1) is located at the plate's center and it occupies the region from 10.5 mm to 15 mm along both the x and y axes.

The plate thickness along the z direction is denoted by h . The defect configuration shown in Figure 4.1 describes the defect in the experimental setup. The same concept was applied to the SGUF models to simulate the delamination, as shown in Figure 4.12. The 2D mesh has been refined towards the central region to better resolve the expected thermal gradients induced by the defect, see Figure 4.13, and 12 elements were used to mesh the defective area. The plate is heated at its bottom surface through a prescribed heat flux representing the power input from the halogen lamps.

4.6.2 2D Mesh configuration

The 2D shape functions used in the present model define an 8-node isoparametric Serendipity interpolation for arbitrarily shaped quadrilateral element (see Figure 4.13b). Figure 4.13c illustrates the temperature distribution across the laminate thickness. The integral over the

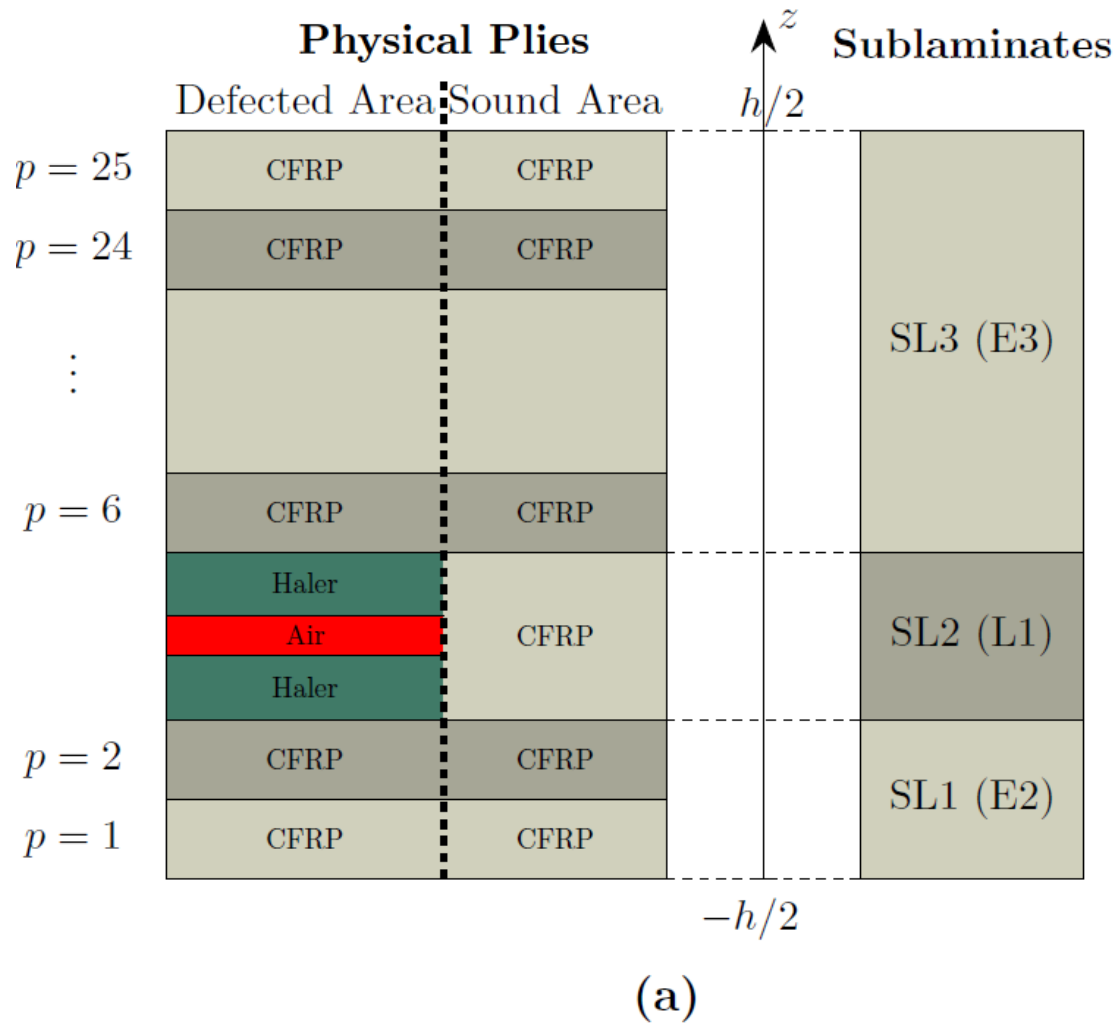


Figure 4.12: Multilayered plate as an assembly of physical plies and numerical layers

in-plane plate domain is computed by exact Gauss-Legendre quadrature. After a convergence analysis, a mesh with 24 (Q8) elements for the quarter plate has been used for the present study, i.e. a total of 1728 DOF.

4.6.3 Approximation for Temperature Profile

Sublaminates Generalized Unified Formulation is used to create a two-dimensional (2D) Finite Element (FE) model for the simulation of transient heat transfer analysis to accurately evaluate defects in composite materials. Numerical layers are used in the traditional CUF or GUF framework to divide physical plies [162, 185]. The present approach expands on the concept of a “numerical layer” by enabling the combination of multiple physical plies into a single numerical layer, resulting in the definition of sublaminates, which thus allows to reduce the number of degrees of freedom. Figure (4.12) gives exemplarily a model in which the defected area is modeled with a layer-wise (LW) approach (L1) to grasp the temperature variations between these strongly heterogeneous materials, while an Equivalent Single-Layer (ESL) description with quadratic and cubic expansion (E2, E3) is adopted for the remaining plies of the stack. The model across the thickness is complemented by the approximations used for the FEM and

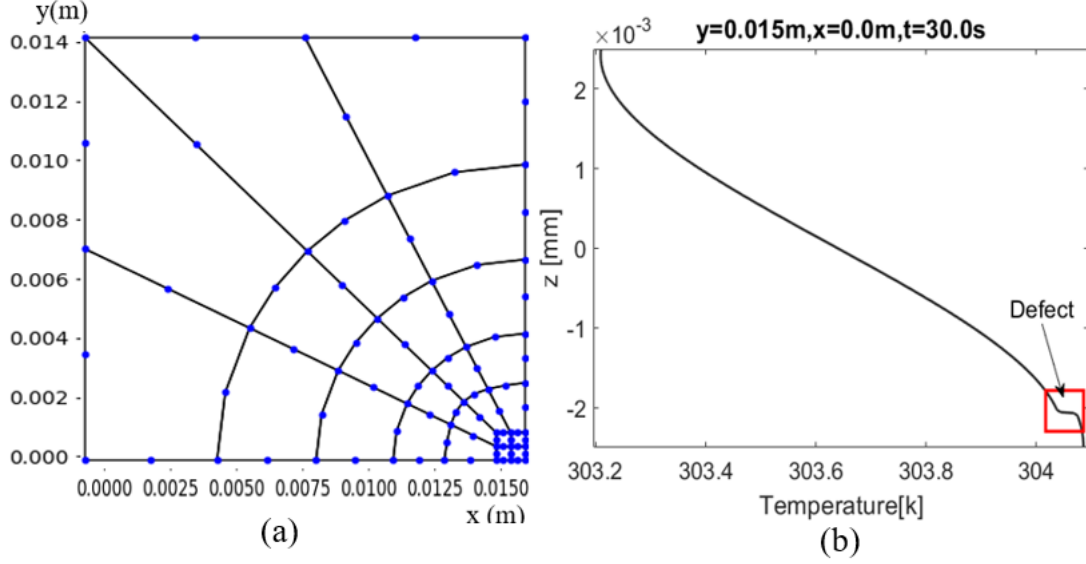


Figure 4.13: (a) 2D Quad Mesh (b) Temperature through thickness at coordinate (0.015, 0) and at time duration 30s

is expressed Eq. 4.13 in the compact notation of Unified Formulation:

$$\theta^{p,k}(x, y, z; t) = F_{\mu_{\theta}^k}(z) \hat{\theta}_{\mu_{\theta}^k}^{p,k}(x, y; t) = F_{\mu_{\theta}^k}(z) N_i(x, y) \theta_{\mu_{\theta}^k i}^{p,k}(t) \quad (4.11)$$

In this context:

- “ p ” is the index for the physical plies, which are the actual layers of the composite material.
- “ k ” is the index for the numerical layers (sublaminates)
- $F_{\mu_{\theta}^k}(z)$ are the thickness functions constructed from Legendre polynomials involving unknown functions at the top and bottom of the layer and the higher-order terms of the expansion:

$$\mu_{\theta}^k = t, l, b \text{ with } l = 2, \dots, N_{\theta}^k$$

- $\theta_{\mu_{\theta}^k}^{p,k}(t)$ is the time-dependent nodal temperature vector
- “ i ” is the index for the FE nodes and $N_i(x, y)$ the 2D shape functions of the FE interpolation

The specific SGUF model illustrated in Figure 4.12 has a total of 9 degrees of freedom (DOF) per node. One has 4 DOF for the bottommost sublaminate comprising 20 plies (i.e., the temperature at the bottom of ply 1 and the top of ply 20, plus 2 internal temperatures defining the amplitudes of the quadratic and cubic Legendre polynomials), 4 DOF for the 3 plies representing the defect (for each ply, top and bottom temperature), and 3 DOF for the topmost sublaminate comprising the plies 24 and 25 (i.e., top of ply 25, the bottom of ply 24 and the internal temperature of the quadratic Legendre polynomial); enforcing the continuity of the temperature at the 2 interfaces between sublaminates reduces eventually the total number of DOF from $4+4+3=11$ to $11-2 = 9$.

4.6.4 Governing Equation solution in SGUF

Let us consider a plate occupying the volume $V = \Omega \times_{0 \leq z \leq h}$, where h is the total thickness and Ω is the two-dimensional region in the (x, y) –plane. Furthermore, let us decompose the total boundary region $\partial\Omega$ into two regions, $\partial\Omega_\theta$ and $\partial\Omega_q$, where temperature and heat flux are prescribed, respectively, such that $\partial\Omega_\theta \cup \partial\Omega_q = \partial\Omega$ and $\partial\Omega_\theta \cap \partial\Omega_q = \emptyset$. The following partial differential equation (PDE) describes the heat transfer mechanism in an orthotropic medium:

$$\kappa_{11} \frac{\partial^2 \theta(t)}{\partial x^2} + \kappa_{22} \frac{\partial^2 \theta(t)}{\partial y^2} + \kappa_{33} \frac{\partial^2 \theta(t)}{\partial z^2} + s(t) + h(\theta) = \rho c_p \left(\frac{\partial \theta}{\partial t} \right) \quad (4.12a)$$

$$\theta = \bar{\theta}(t) \quad \text{on } \Omega_\theta \quad (4.12b)$$

$$\mathbf{n} \cdot \mathbf{q} = \bar{q}_n(t) \quad \text{on } \Omega_q \quad (4.12c)$$

where ρ is the specific mass, c_p is the material heat capacity at constant pressure, s is the volume-specific internal heat generation, $\mathbf{n}$ is the unit vector normal to $\partial\Omega$ and directed outwards, $\mathbf{q}$ is the heat flux vector and h is the heat convection coefficient. The coefficients κ_{11} , κ_{22} and κ_{33} are the thermal conductivities along the coordinate axes, which are considered constants throughout the volume V . Equation 4.12b and Equation 4.12c are the boundary conditions in terms of prescribed temperature and heat flux, respectively, with the bar/overline symbol $\bar{\square}$ denoting prescribed quantities. The FEM is based on the weak form of the PDE as described in [186, 187], [188]:

$$\int_V \delta\theta \rho c_p \dot{\theta} dV + \int_V \delta\boldsymbol{\vartheta}^T \boldsymbol{\kappa} \boldsymbol{\vartheta} dV = - \int_{\Omega_q} \delta\theta \bar{q}_n d\Omega + \int_V \delta\theta Q dV \quad (4.13)$$

where $\boldsymbol{\kappa}$ denotes the thermal conductivity matrix and $\boldsymbol{\vartheta}$ represents the temperature gradient vector,

$$\boldsymbol{\kappa} = \text{diag}(\kappa_{11}, \kappa_{22}, \kappa_{33}) \quad \boldsymbol{\vartheta} = \nabla\theta \quad (4.14)$$

In the following analysis, we consider that no heat is generated in the plate, which shall be thermally stimulated by a spatially uniform heat flux applied at the bottom surface of the plate, thus acting along the z – direction. This assumption leads to a simplified form of 4.13:

$$\int_V \delta\theta \rho c_p \dot{\theta} dV + \int_V \delta\boldsymbol{\vartheta}^T \boldsymbol{\kappa} \boldsymbol{\vartheta} dV = -\bar{q}_z \int_{\Omega_q} \delta\theta d\Omega \quad (4.15)$$

where $\bar{q}_z$ is the prescribed heat flux as given in Figure 4.13. The expansion in equation 4.11 is introduced into the weak form and the integrals across the thickness are carried out. The procedure yields the so-called fundamental nuclei, which are elementary 1x1 arrays used to compute the conductivity (“stiffness”) and heat capacity (“mass”) matrix of the whole structure upon cycling over the indices k and assembling the layers’ contributions at the common interfaces. Explicit expressions of the FE matrices are given in Appendix C.

4.6.5 Solution procedure

Eq.4.15 can be recast in a more compact form upon introducing FE matrices defined in C:

$$M\dot{\Theta} = -K\Theta + Q \Rightarrow \dot{\Theta} = M^{-1}(-K\Theta + Q) \quad (4.16)$$

where Θ is the vector of unknowns nodal temperatures and Q the vector of nodal heat flows. The transient heat flux problem requires specifying appropriate initial conditions by defining the initial temperature field $\theta(x, y, z; t = t_0)$. Since heat transfer problems are not inherently stiff, the time integration is carried out using the Adams-Bashforth-Moulton method and Runge-Kutta 4th Order Method as detailed in Algorithm 1 and 2. The Adams-Bashforth-Moulton method belongs to the family of linear multistep methods designed for solving ordinary differential equations. It is based on the idea of combining numerical integration techniques with predictor-corrector schemes. In particular, the value of Θ_{i+1} at t_{i+1} is estimated by using the explicit Adams-Bashforth method. The Adams-Moulton method is then used to correct the predicted value.

Algorithm 1 Adams-Bashforth-Moulton method.

Require: Initial vector Θ_0 , initial time t_0 , step size h , order k , number of steps N , function $f(t, \Theta)$

Ensure: Approximate solutions Θ_i at $t_i = t_0 + ih$, for $i = 0, 1, \dots, N$

- 1: Initialize $t_i \leftarrow t_0$, $\Theta \leftarrow \Theta_0$
 - 2: **for** $i = 0$ to $k - 2$ **do** ▷ Bootstrap initial values using RK4
 - 3: Compute $t_{i+1}, \Theta_{i+1} \leftarrow \text{RK4_step}(f, \Theta_i, t_i, h)$
 - 4: Store t_{i+1}, Θ_{i+1}
 - 5: **end for**
 - 6: **for** $i = k - 1$ to $N - 1$ **do** ▷ Predictor-Corrector loop
 - 7: Compute $t_{i+1} \leftarrow t_i + h$
 - 8: Compute $\Theta_{\text{pred}} \leftarrow \Theta_i + h \sum_{j=0}^{k-1} \beta_j f(t_{i-j}, \Theta_{i-j})$ ▷ Predictor step (Adams-Bashforth)
 - 9: Compute $\Theta_{\text{corr}} \leftarrow \Theta_i + h \alpha_0 f(t_{i+1}, \Theta_{\text{pred}}) + h \sum_{j=1}^{k-1} \alpha_j f(t_{i+1-j}, \Theta_{i+1-j})$ ▷ Corrector step
(Adams-Moulton)
 - 10: Update $\Theta_{i+1} \leftarrow \Theta_{\text{corr}}$
 - 11: Store t_{i+1}, Θ_{i+1}
 - 12: **end for**
-

Note: Coefficients α_j and β_j depend on the chosen order of the method.

Algorithm 2 Step of the Runge-Kutta 4th Order Method (RK4).

Require: function $f(t, \Theta)$, Θ_i , t_i , step size h

Ensure: Approximate solution Θ_{i+1} at $t_{i+1} = t_i + h$

- 1: Compute $k_1 \leftarrow h f(t_i, \Theta_i)$
 - 2: Compute $k_2 \leftarrow h f(t_i + \frac{h}{2}, \Theta_i + \frac{k_1}{2})$
 - 3: Compute $k_3 \leftarrow h f(t_i + \frac{h}{2}, \Theta_i + \frac{k_2}{2})$
 - 4: Compute $k_4 \leftarrow h f(t_i + h, \Theta_i + k_3)$
 - 5: Update $t_{i+1} = t_i + h$, $\Theta_{i+1} \leftarrow \Theta_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
 - 6: Output t_{i+1}, Θ_{i+1}
-

The Adams-Bashfort method, being an explicit method, is not unconditionally stable and requires a sufficiently small time step to avoid stability issues. A time step $h = 0.5$ s has been

proven to ensure the stability of the method and it is used for all the analyses presented in the subsequent section. The number of steps N in Algorithm 1 depends on the excitation frequency and number of cycles and can be calculated as follows:

$$N = \frac{n_c}{f_{ex}h} \quad (4.17)$$

where f_{ex} is the excitation frequency, n_c the number of cycles and h is the time step.

The resulting temperature profile over time at the bottom surface of the plate is then post-processed to obtain a pixels-wise representation of the temperature field in the x - and y -directions. The dimension of the pixel is set to 0.625 mm, resulting in a 24×24 discretization of the quarter plate. The raw data is then processed using IRTA software which solves the Eq. 4.2.

4.7 Carrera Unified Formulation (CUF)

To complement the experimental investigations, numerical simulations were conducted using the Carrera Unified Formulation (CUF). CUF provides a robust, versatile, and computationally efficient framework for the modeling and analysis of composite structures, including the complex geometry of CFRP T-joints. This advanced formulation enables the accurate representation of the material's anisotropic thermal properties and the intricate interfacial interactions between bonded and de-bonded regions.

4.7.1 MUL2 CODE

The MUL2 (Multilayer and Multifield Problems) CODE is implemented through the Carrera Unified Formulation (CUF) for 2D structures based on Hierarchical Legendre Expansion (HLE) polynomials. An efficient method for the Structural Health Monitoring (SHM) of bonded and debonded composite joints is experimentally verified and simulated by commercial FE software by the CUF approach. The transient heat transfer mechanism through the investigated specimens is numerically simulated with a simple Finite Elements (FE) model and it is solved via MUL2 CODE. Experimental campaign data are compared with CUF results demonstrating the consistency of the MUL2 formulation that is computationally simpler, but very efficient for the joint analysis. The presented and discussed CUF application can quantify with high accuracy the debonding extension in the damaged structure.

Multilayered structures are transversely anisotropic, and their mechanical properties are discontinuous along the thickness. The Layer-Wise (LW) approach [176–178] has been proposed to deal with these mechanical behaviors. In the LW, each layer is seen as an independent plate, and compatibility of displacement components is imposed at the interfaces. Compatibility and equilibrium conditions are then used at the interfaces to reduce the number of unknown variables. The present work proposes the use of layer-wise theories in the framework of Carrera Unified Formulation (CUF) [135] to overcome most of the issues related to classical theories

for the simulation of complex structures. In particular, the present research aims to exploit the accuracy and the computational efficiency of layer-wise CUF models to simulate transit thermal behavior in two debonded composite T-joints, to detect a debonding region.

Methodology adopted in this work is as follows:

1. An advanced numerical model is developed in the framework of CUF that uses 2D elements based on a layer-wise approach.
2. To validate the accuracy of the numerical model by comparing simulated phase contrast results with experimental results from Lock-in Thermography (LT) tests.
3. To simulate the influence of air gap thickness in debonded regions on the phase contrast, enabling precise determination of debond size and geometry.
4. To optimize defect detection techniques by establishing a correlation between air gap properties (thickness, thermal conductivity) and phase contrast values.

4.7.2 CUF Implementation and Fundamental matrices

The structural model under investigation is a CFRP T-joint, consisting of a web, stringer, and skin. This critical composite joint is commonly used in aerospace and automotive applications. The primary focus of the study is to analyze de-bonding at the interface between the skin and the stringer. The de-bonded regions, modeled as air-filled gaps, are crucial for evaluating the phase contrast in Lock-in Thermography (LT) tests.

The Carrera Unified Formulation (CUF) is applied to simulate the dynamic behavior of a composite T-joint structure. By reducing the 3D problem to a 2D representation, the approach enables the development of computationally efficient FEM models with significantly lower degrees of freedom compared to traditional FEM methods. A through-thickness approximation is employed to address the governing equation effectively. The functions $F_r(z)$ depend only on the thickness coordinates z , so they are called thickness functions. The ESL (Equivalent Single Layer) approach, typically adopted in classical models, uses the Taylor series as thickness functions, with z that varies from $-h/2$ to $h/2$ (h is the whole plate thickness). The LW (Layer Wise) models suppose the variables are independent in each layer and Lagrange polynomials are assumed as thickness functions. So z is defined as a local variable for the thickness of the k^{th} layer.

The structural mass matrix $\mathbf{M}$, the structural stiffness matrix $\mathbf{K}$ and the load vector $\mathbf{F}$ are obtained in terms of fundamental nuclei, which are independent of expansion order in the thickness direction. Once the fundamental nuclei are expanded on the indexes τ , s , i , and j , we obtain the final governing equations of the element at layer level that include the FE approximation and the CUF:

$$[\mathbf{M}^k] \cdot \{U\} + [\mathbf{K}^k] \cdot \{U\} = \{F\} \quad (4.18)$$

4.7.2.1 Model Description in Mul2

The T-joint was modeled numerically to replicate the actual geometry of the test specimen used in the experimental setup.

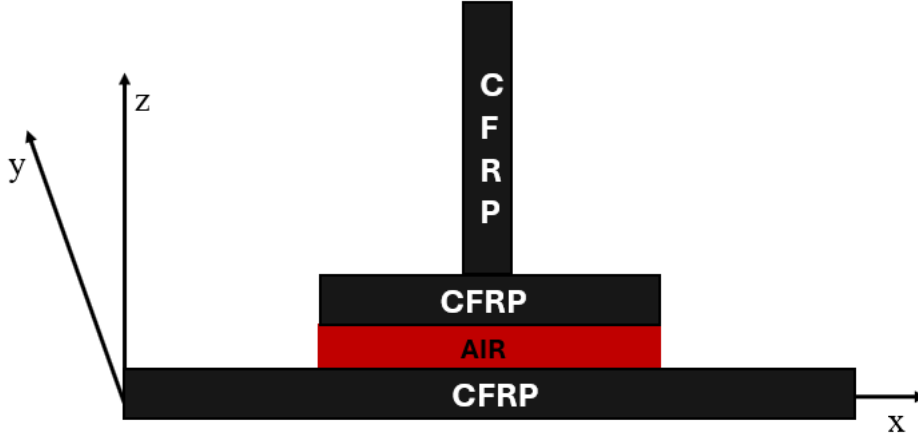


Figure 4.14: Schematic representation of CFRP stiffened panel with air gap

Figure 4.14 illustrates the schematic cross-section of the panel, comprising three distinct CFRP regions separated by an air gap, oriented along the (x, y, z) coordinate system. The primary horizontal CFRP base layer extends along the x -axis, supporting a vertical stiffener element. An air gap is introduced between the base and the intermediate horizontal CFRP layer to simulate potential delamination or manufacturing defects.

4.7.2.2 FEM Geometry and Mesh

Figure 4.15(b) presents the discretized model developed using the MUL2 code, implementing the Carrera Unified Formulation (CUF). The mesh consists of nine-node quadrilateral elements ($Q9$), providing higher-order interpolation capabilities for enhanced accuracy in structural analysis. This advanced meshing approach enables precise representation of the complex geometry while maintaining computational efficiency. The model incorporates:

- Nine-node quadrilateral elements for superior geometric approximation
- Refined mesh density in critical regions
- Full three-dimensional representation of the stiffened panel
- Accurate modeling of material interfaces and geometric discontinuities

4.7.2.3 Boundary and Loading Conditions

The Lock-in Thermography (LT) simulation was conducted under transient heat transfer conditions to replicate experimental defect detection scenarios. A sinusoidal heat flux was applied to the bottom surface of the composite laminate at $z = -h/2$, where h denotes the total laminate thickness. This periodic thermal excitation ensured consistent heat modulation across the specimen, mimicking real-world testing conditions. The applied heat flux played a crucial role in generating phase contrast maps, which were analyzed to detect and quantify

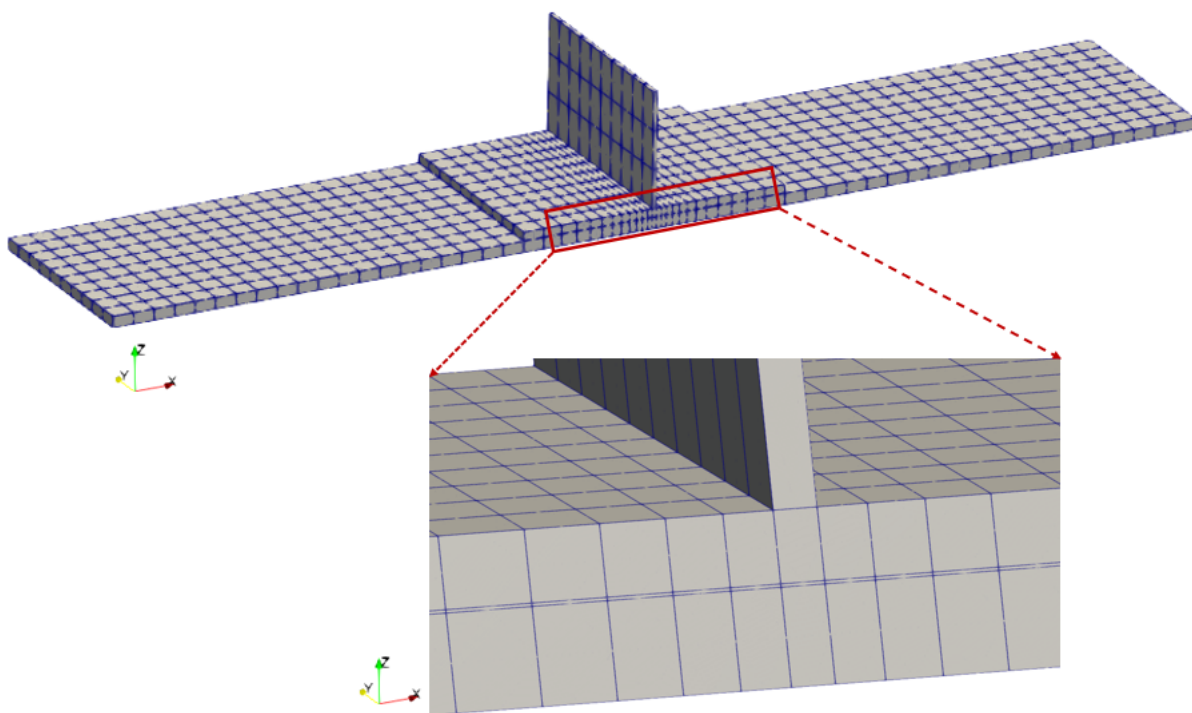


Figure 4.15: Geometric modeling and mesh of CFRP stiffened panel

defects within the laminate structure. These boundary conditions are implemented in Figure 4.16, with a detailed description provided below.

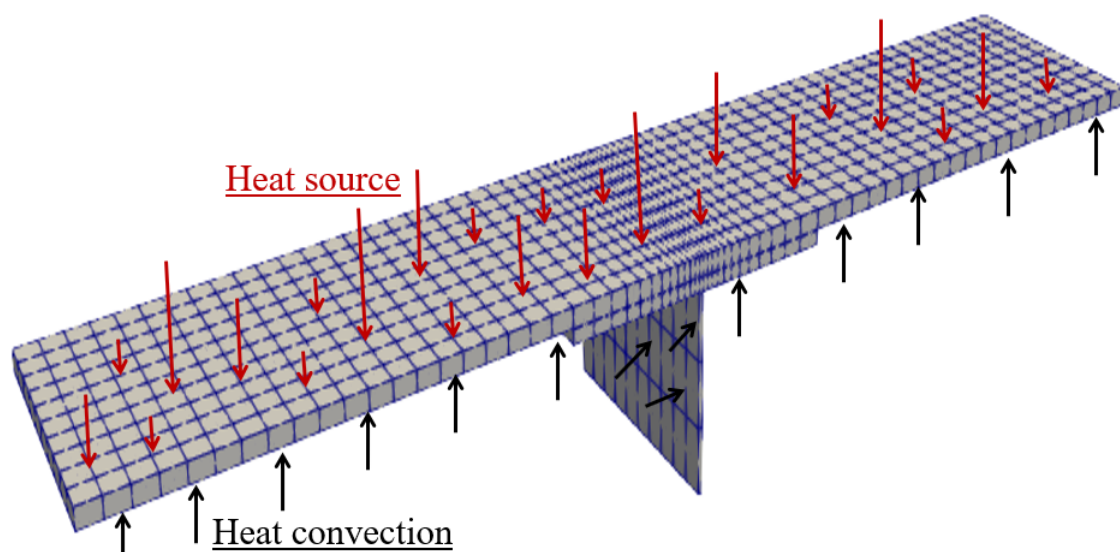


Figure 4.16: Implementation of boundary conditions

Heat Flux Modulation:

A sinusoidal heat source was applied to the surface of the T-joint at a constant frequency, matching the experimental setup (e.g., 60 s modulation period).

Boundary Conditions:

The bottom surface was thermally insulated, while the other surfaces were exposed to convective cooling, with a heat transfer coefficient of $9 \text{ W/m}^2\cdot\text{K}$.

Air Gap Simulation:

The de-bonded regions were modeled with a uniform air layer, and the thickness of the layer was varied to capture the thermal discontinuity observed in the experimental phase-contrast maps.

The aim is to validate the CUF model by comparison with experimental results by demonstrating that it permits to investigate of the composite T-joint in an accurate and fast way. Additionally, the study shows the CUF model's precision and computational effectiveness in detecting delaminations and debonding in various multilayered composite structures.

Chapter 5

Results and Discussion

5.1 Model Development and Validation

In this study, we used both experimental and finite element analysis (FEA) to explore the thermal responses of materials subjected to cyclic excitation throughout three cycles at frequencies ranging from 0.033 Hz to 0.0125 Hz. During these studies, infrared (IR) thermal imaging was used to take subsequent thermal images of the specimen's surface, which were captured at precise x and y positions. These thermal images were crucial for visualizing and analyzing temperature distributions throughout time.

5.1.1 Convergence analysis

Convergence analysis is a critical step in developing and validating numerical models. It is assessed in three key areas: model formulation, time step, and mesh refinement. Model convergence evaluates different mathematical representations to determine which best captures the physical behavior. Time-step convergence analyzes the impact of temporal discretization on solution accuracy and stability. Mesh convergence examines how spatial FE discretization affects the results, balancing solution accuracy with computational efficiency. As the model, time step, and mesh are refined, the numerical solution should approach the true solution of the governing equations. However, excessive refinement can dramatically increase computational costs. Therefore, the goal of convergence analysis is to identify the optimal configuration that provides sufficiently accurate results with reasonable computational resources. To conduct these analyses, the thermal contrast ΔT as defined in Eq.4.3 was calculated by averaging the values over a small area (3x3 pixels) within the defective region, specifically between the 20th and 22nd pixels in the x-y direction. The pixel dimensions are expressed in millimeters, corresponding to the scale of the image. An excitation frequency of 0.033 Hz was used for the convergence studies, with amplitude measurements taken at the end of each heating cycle. Convergence studies for model formulation, time step, and mesh refinement will be presented in detail, demonstrating how these specific parameters were chosen to balance solution accuracy with computational efficiency. Following the convergence analysis, we will proceed with COMSOL

comparisons and experimental validations using these optimized parameters.

Table 5.1: Convergence Analyses for Time Step

Category	Parameter	FEM mean(ΔT)°C	Solution Time (s)	Error (%)
Reference Value: mean(ΔT) = 0.280 °C				
Mesh Convergence	24 Elements	0.264	39	5.71
	56 Elements	0.262	910	6.42
	64 Elements	0.264	1,025	5.71
	72 Elements	0.265	1,407	5.35
Model Convergence	L1/L1/L1	0.255	1,111	8.92
	E2/E3/E3	0.147	34	47.50
	E2/L1/E3	0.256	32	8.57
	L1/L1/E3	0.253	30	9.64
	L2/L2/L2	0.259	26,817	7.5
	E2/E1/E3	0.143	16	49.02
Reference Value: mean(ΔT) = 0.301 °C				
Time Step Convergence	0.15 s	0.300	260	0.19
	0.5 s	0.302	32	0.58
	1.5 s	0.293	9	2.46

5.1.1.1 Mesh convergence

The results from the mesh convergence analysis, presented in Table 5.1), show that as the number of elements increases, the mean temperature change (ΔT) remains relatively stable, with only minor fluctuations. This indicates that the numerical solution has converged and the selected mesh density is adequate for the analysis. Specifically, for 24 elements, the mean temperature change is 0.264°C, and the error is 5.71%. Increasing the mesh to 56 elements results in a slightly lower ΔT of 0.262°C, with an increased solution time of 910 seconds, but the error also increases to 6.42%. For 64 elements, the mean ΔT again stabilizes at 0.264°C, with an error of 5.71%, while the solution time increases to 1,025 seconds. Finally, with 72 elements, the temperature change is 0.265°C, with the smallest error of 5.35%, although the solution time increases significantly to 1,407 seconds.

Based on this analysis, the choice of 24 elements was finalized, as it provides an acceptable balance between accuracy (error of 5.71%) and computational efficiency, minimizing solution time without compromising significant accuracy.

5.1.1.2 Model convergence

Following 5.1 the model convergence analysis examines the impact of different model configurations on the simulation results. The L2/L2/L2 configuration yields a mean ΔT of 0.259°C (7.5% error), with a solution time of 26,817 seconds. The configuration L1/L1/L1 results in a mean ΔT of 0.255°C (8.92% error) and a solution time of 1,111 seconds. A reduction of the computational time is achieved upon adopting an ESL description for the sublaminae.

When using the E2/E3/E3 and E2/E1/E3 configurations, the mean ΔT significantly reduces to 0.147°C and 0.143°C, respectively, i.e., the error increases drastically to 47.5% and 49.02%. Even if the solution time is much shorter, with 34 seconds and 16 seconds, respectively, the errors for these models remain in acceptable. The E2/L1/E3 configuration retains the LW description only for the defect region. It yields a mean ΔT of 0.256°C (8.57% error) with a much shorter solution time of 32 seconds. The L1/L1/E3 configuration requires a similar computational time (30 seconds) but provides a less accurate result ($\Delta T = 0.253^\circ\text{C}$, 9.64% error).

Based on these results, it appears that the most reasonable choice is to use an LW description for the plies representing the defect and an ESL description for the CFRP sublaminates, for which at least quadratic approximation should be used. So, the E2/L1/E3 configuration was selected, as it provides a good balance between accuracy (error of 8.57%) and computational efficiency, with a reasonable solution time and a minimal reduction in accuracy compared to the other configurations.

5.1.1.3 Time Step Convergence

The time step convergence analysis evaluates the impact of varying time steps on simulation accuracy and computational efficiency. A reference value of $\Delta T = 0.300^\circ\text{C}$ was used, which has been obtained with a simulation with a small time step of 0.2 s. A smaller time step of 0.15 s produces a mean ΔT of 0.299°C, with a negligible error of 0.19%. Although this step size offers high accuracy, the computational time is relatively high at 260 seconds. Increasing the time step to 0.5 s slightly increases the error to 0.58%, resulting in a mean ΔT of 0.302°C. However, this step size significantly reduces the computation time to 32 seconds, making it a practical choice for simulations over short periods (30 seconds in this study). A larger time step of 1.5 s results in a mean ΔT of 0.293°C and an error of 2.46%. While computational time is further reduced to 9 seconds, the increased error indicates a trade-off between efficiency and accuracy. Based on this analysis, a time step of 0.5 s was finalized for subsequent studies. Although it has a slightly higher error compared to the smaller time step of 0.15 s, it offers acceptable accuracy while being computationally efficient. It is noted that for longer simulation periods, smaller time steps may lead to prohibitive computational costs, reinforcing the practicality of selecting an intermediate time step like 0.5 s for this study. The final configuration for the numerical model is shown in Table 5.2.

Table 5.2: Parameters after convergence analysis using a period of 30s

Parameter	Value
Model	E2/L1/E3
Time step	0.5 s
Mesh	24 elements

5.1.2 COMSOL Multiphysics - FE Model

A first validation study is conducted by comparing the numerical models of SGUF and the commercial package COMSOL Multiphysics, which is a reference FEM-based tool for multifield computations. However, the use of the standard 3D FEs implies a non-trivial modeling and meshing step. The adopted computational model for the transient heat transfer on the quarter plate is depicted in Figure 4.11.

5.1.2.1 Mesh refinement analysis for COMSOL

A mesh refinement study was performed using the COMSOL parametric sweep feature to analyze the temperature gradient in the defected region accurately. Tetrahedral elements were utilized ensuring high mesh density in the defected area with at least two elements across the thickness of the plies. The mesh parameters, including the maximum element size, minimum element size, element growth rate, curvature factor, and narrow region resolution, were systematically varied, as shown in Table 5.3. The results were analyzed by monitoring the temperature at a surface point located at $(x, y, z) = (13, 13, 0)$, which lies within the defected region. These coordinates were specifically chosen to focus on the area of interest where the defect is located. The convergence behavior was evaluated, and the temperature distribution for different mesh configurations is plotted in Figure 5.1. Based on the results, the solution converged satisfactorily for the second mesh configuration (green line), with a maximum element size of 1.2 mm and a minimum element size of 0.15 mm. This mesh was selected as the optimal choice due to its balance between computational accuracy and efficiency, minimizing computational time while maintaining solution accuracy. The finalized mesh consisted of 273,180 domain elements, 50,888 boundary elements, and 1,117 edge elements, with a total of 680,686 degrees of freedom (DOF).

Table 5.3: Mesh convergence analysis for the defected region using a period of the 30s

Parametrs	Symbols	Range
Maximum element size	max_e	2.85, 1.2, 0.525
Minimum element size	min_e	0.6, 0.15, 0.0225
Element growth rate	GR	1.7, 1.45, 1.35
Curvature factor	Cf	0.8, 0.5, 0.3
Resolution of narrow region	Rnr	0.3, 0.6, 0.85

The same time step (0.5 s) was eventually chosen for the SGUF model and COMSOL computation. For the COMSOL simulations, the implicit Backward Differentiation Formula (BDF) was employed as the time integration scheme, chosen for its effectiveness in handling differential equations and stability in time-dependent simulations.

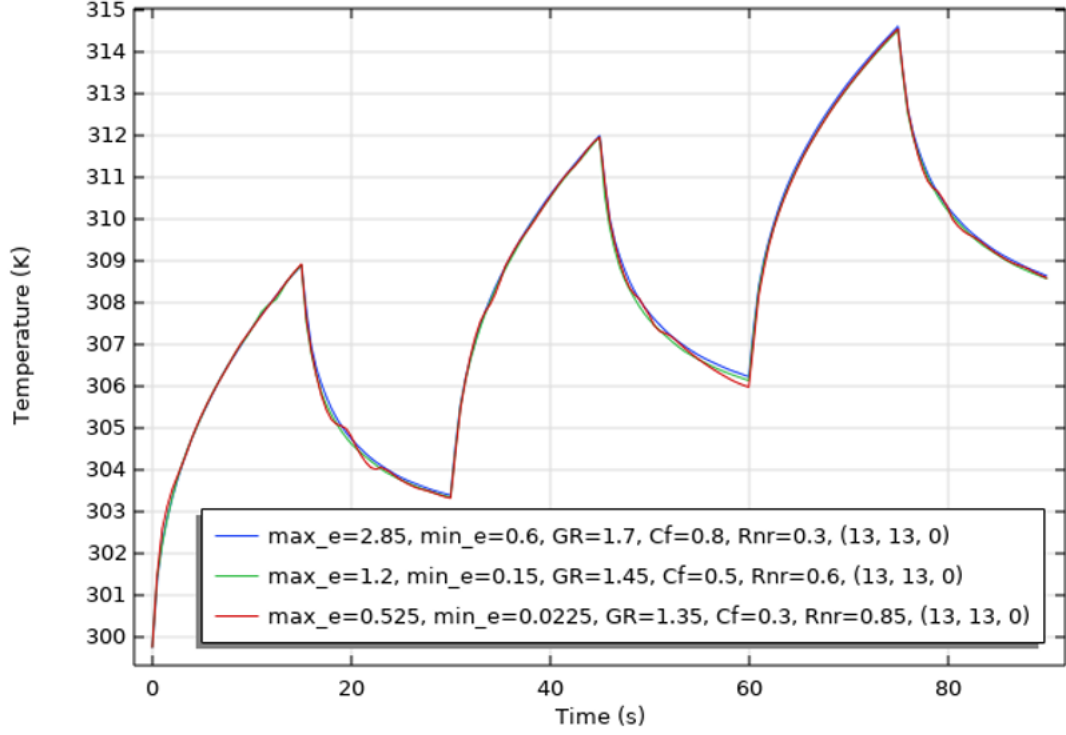


Figure 5.1: Mesh refinement analysis using parametric sweep

5.1.3 Model Validation: SGUF vs. COMSOL and Experiment

5.1.3.1 Comparison between SGUF and COMSOL

This initial comparison served as a validation step for numerical approaches. To analyze and compare the results, two specific locations were selected within the sound area (point S) and the defective area (point D). The coordinates of these points are given in Figures 5.2a, 5.2a for the 3D COMSOL model and the 2D SGUF model. The thermal history curves for both locations are plotted in Figure 5.2c. The comparison of temperature profiles for both defective and sound regions demonstrates a close agreement between the 3D COMSOL and the 2D SGUF simulations. The much more complex mesh required by COMSOL (Figure 5.2a) is thus seen to be effectively replaced by the high-order SGUF model which comes along a simple 2D FE mesh (Figure 5.2b). Following this numerical validation, we will subsequently compare the results from both models with experimental data. This comprehensive approach allowed us to assess the accuracy and reliability of both numerical methods against real-world observations.

5.1.3.2 Qualitative analysis

The FEM models were validated against experimental results at different excitation frequencies. Gaussian random noise was carefully added to the temperature data throughout our numerical simulations to replicate real-world measurements and errors.

The noise level was calculated empirically using data from early experimental observations, in which we identified the mean, variance, and standard deviation of a noisy section based on the first few seconds of temperature data. Based on this analysis, we calculated that the

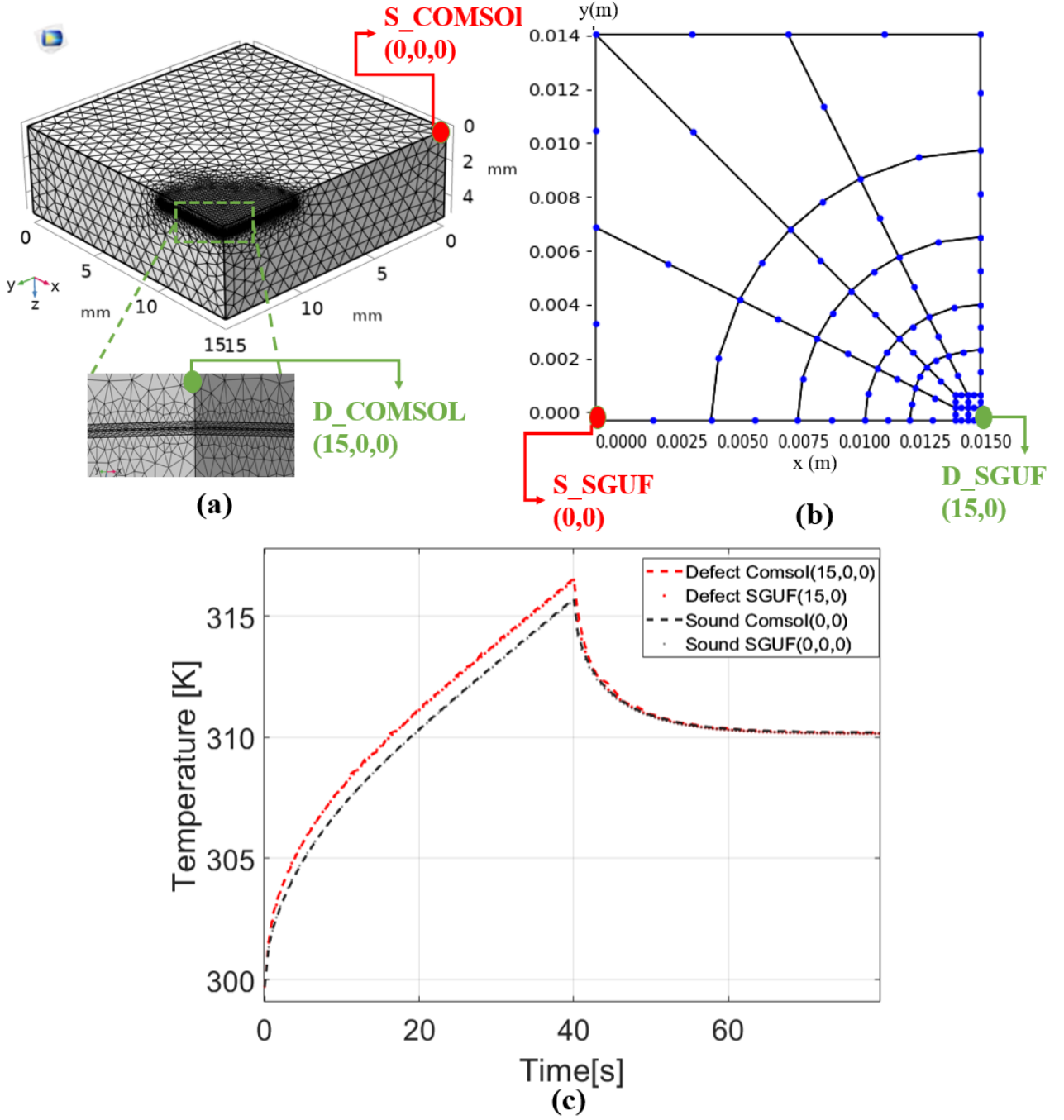


Figure 5.2: (a) 3D Mesh for the plate using COMSOL Model (b) 2D Mesh for the plate using SGUF Model (c) Thermal response of the material at 45s, obtained from both models at points located in the sound (S) and defective (D) regions

standard deviation of noise in our experimental data was 0.04°C . To incorporate equivalent noise into our FEM simulation findings, we used a noise level of 4% of the temperature range in our simulations. This noise was generated using a normal distribution with a mean of 0 and the calculated standard deviation and then added to each data point in the temperature dataset. This deliberate use of noise allowed us to comprehensively examine the performance, precision, and dependability of both the SGUF and COMSOL techniques in situations that are quite similar to real-world scenarios. This approach emphasized the importance of designing

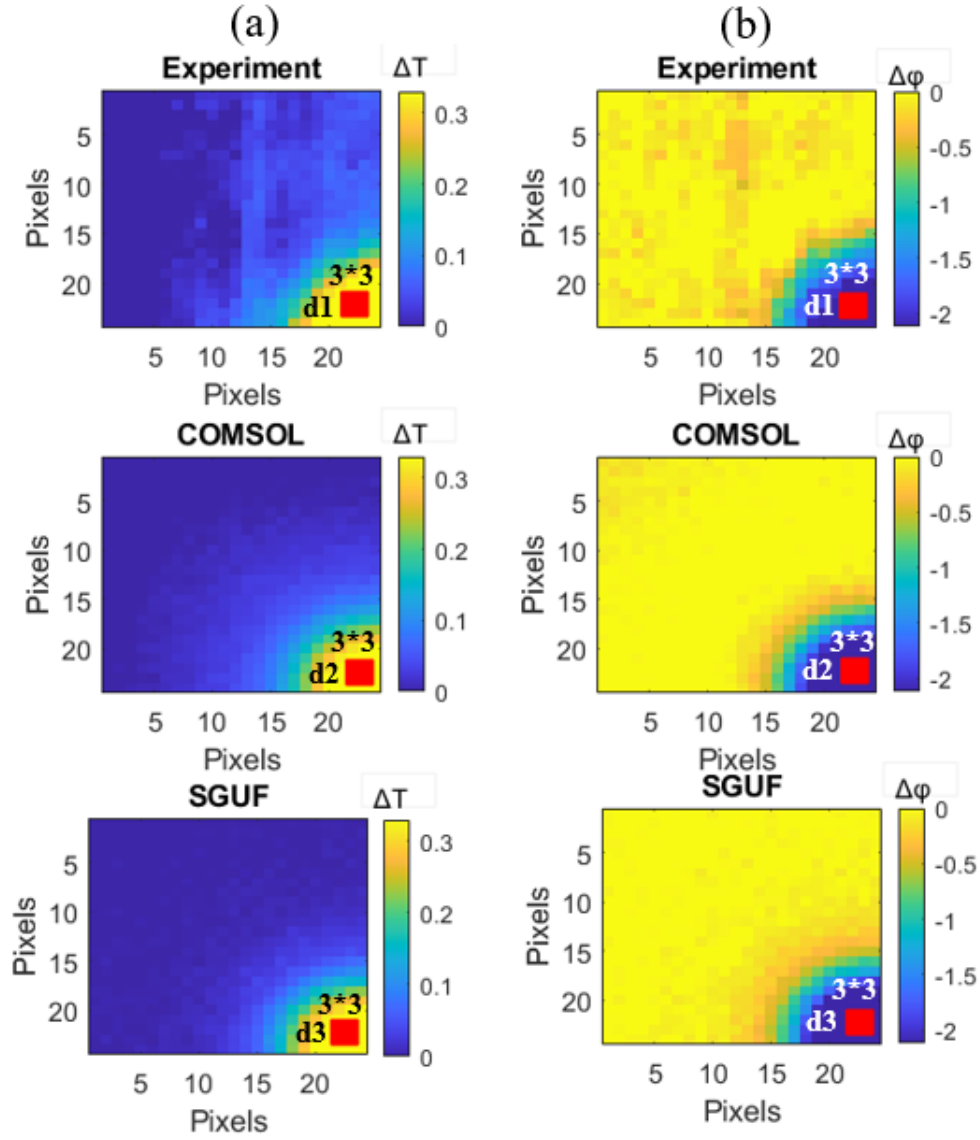


Figure 5.3: Maps for an excitation frequency of 0.0125 Hz: (a) Thermal Contrast ΔT ($^{\circ}\text{C}$), (b) Phase Contrast $\Delta\varphi$ ($^{\circ}$)

noise-resistant algorithms to provide consistent results in real-world applications.

The thermal contrast can be used to select the most adequate excitation frequency since a longer period produces more thermal and phase contrast, which is crucial for defect identification. The lowest frequency of 0.0125 Hz has been shown to give the greatest thermal and phase contrast, making defects easier to identify due to a more clear distinction between defective and surrounding areas. To compare the experimental results with the numerical simulations, We compared the temperature and phase contrast values across the experimental, COMSOL, and SGUF images for all frequencies. Figures 5.3, 5.4, 5.5 show significant correlations between experimental and numerical results for both amplitude and phase, indicating a good match. This first successful qualitative comparison is further substantiated by a quantitative analysis.

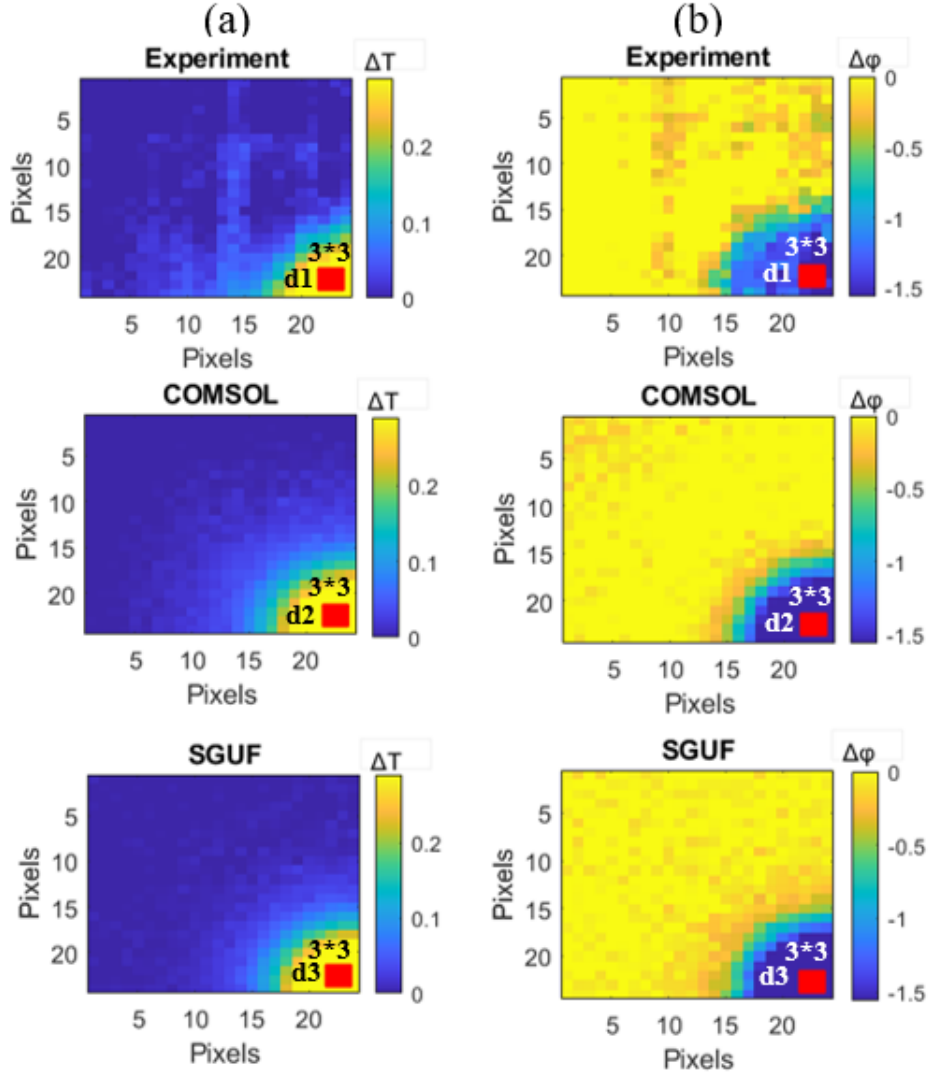


Figure 5.4: Maps for an excitation frequency of 0.022 Hz: (a) Thermal Contrast ΔT ($^{\circ}\text{C}$), (b) Phase Contrast $\Delta\varphi$ ($^{\circ}$)

5.1.3.3 Quantitative Analysis

The investigated algorithms provided two maps: Thermal and phase contrast, respectively. The thermal contrast map indicates the size of the defect, while the phase contrast map is effective because its signal is less impacted by uneven heating and can identify defects located deeper within the material. Using the maps from experimental data, COMSOL simulations, and SGUF models, red rectangular boxes were placed over the defective areas and labeled d1, d2, and d3, respectively (see Figure 5.3, 5.4, 5.5). Small regions of size 3 x 3 pixels were selected within the defected area (pixels 21 to 23) to quantitatively assess the accuracy of thermal and phase contrast measurements. The thermal contrast (ΔT) increases as the excitation frequency decreases, as shown in Table 5.4. Both the COMSOL and SGUF models exhibit good agreement with experimental data at 0.033 Hz, with COMSOL and SGUF uncertainties of about 3%. This suggests that both models are capable of accurately representing the higher-frequency thermal response. The COMSOL and SGUF models show comparable accuracy as the frequency

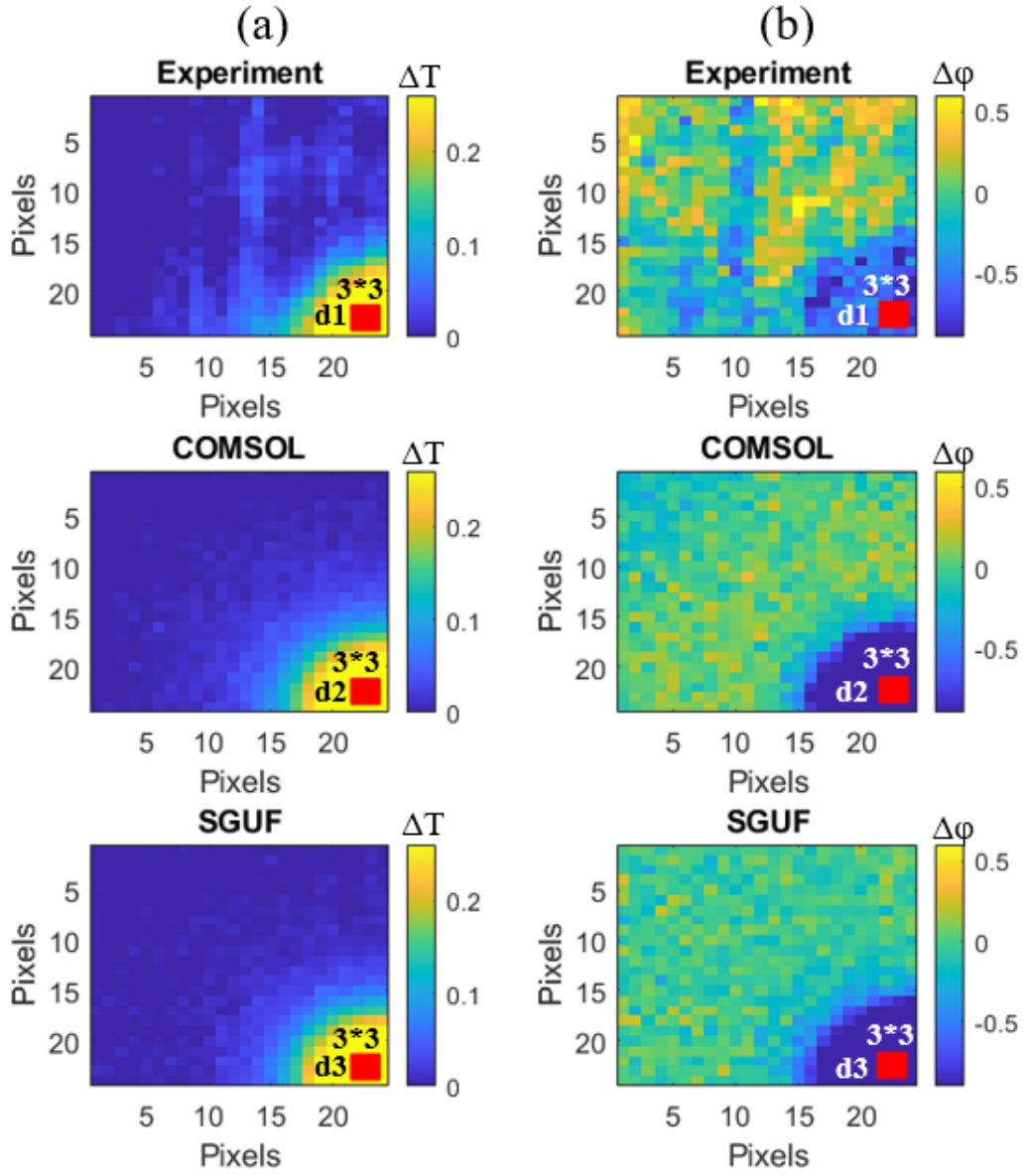


Figure 5.5: Maps for an excitation frequency of 0.033 Hz: (a) Thermal Contrast ΔT ($^{\circ}\text{C}$), (b) Phase Contrast $\Delta\varphi$ ($^{\circ}$)

decreases to 0.022 Hz, with 2% errors, respectively. At the lowest frequency of 0.0125 Hz, both models continue to show good agreement, with COMSOL exhibiting slightly lower uncertainty (1%) compared to SGUF (2%).

The phase contrast ($\Delta\varphi$) results are summarized in Table 5.5. Phase contrast, a crucial parameter in lock-in thermography, represents the temporal delay of the thermal wave relative to the modulation signal. Phase contrast for experiment, COMSOL, and SGUF are approximately the same at 0.033 Hz, with SGUF showing a little lower uncertainty of 1% and COMSOL displaying an even smaller uncertainty of 3%. The COMSOL and SGUF uncertainties are 2% and 1%, respectively, indicating that both numerical models are very precise at 0.022 Hz. At the lowest frequency of 0.0125 Hz, phase contrast rises and both models remain accurate, with SGUF slightly higher at 2% and COMSOL displaying 1% uncertainty. The overall lower error margins in the COMSOL simulations can be attributed to the use of a detailed 3D

Table 5.4: Summary of ΔT mean values with uncertainty ranges

Frequency (Hz)	Experiment mean ΔT (K)	COMSOL mean ΔT (K)	SGUF mean ΔT (K)
0.033	0.26	$0.25 \pm 3\%$	$0.24 \pm 3\%$
0.022	0.29	$0.28 \pm 2\%$	$0.28 \pm 2\%$
0.0125	0.33	$0.32 \pm 1\%$	$0.31 \pm 2\%$

Table 5.5: Summary of $\Delta\varphi$ mean values with uncertainty ranges

Frequency (Hz)	Experiment mean $\Delta\varphi$ ($^\circ$)	COMSOL mean $\Delta\varphi$ ($^\circ$)	SGUF mean $\Delta\varphi$ ($^\circ$)
0.033	1.42	$1.38 \pm 3\%$	$1.40 \pm 1\%$
0.022	1.56	$1.54 \pm 2\%$	$1.55 \pm 1\%$
0.0125	2.12	$2.10 \pm 1\%$	$2.08 \pm 2\%$

model that more accurately represents the physical geometry and boundary conditions of the experimental setup, leading to improved prediction accuracy, especially at higher modulation frequencies where spatial temperature variations are more pronounced. The SGUF model, while relying on a 2D computational mesh with simplified assumptions, still performs reliably across all frequencies, with overall uncertainties within 3%. Both models demonstrate strong validity, with error margins below 3% for all frequencies, underscoring their reliability in predicting both thermal and phase contrasts in lock-in thermography experiments. However, both the COMSOL and SGUF models yield errors of less than 5%, a threshold value considered indicative of good agreement between the simulations and the experimental data, for all tested frequencies. This underscores their validity and reliability in predicting thermal contrasts in lock-in thermography experiments.

The sophisticated 3D COMSOL model with 680686 degrees of freedom requires a dedicated geometry model and utilizes the automatic mesh generator. The complex details of the physical system under investigation can thus be resolved up to the requested level of refinement. In contrast, the SGUF model counts only 1728 DOF; it follows a much simpler 2D computational approach, where the complexity of the response across the thickness is resolved by the adopted model separately from the in-plane meshing. The implementation of SGUF FEM software has obvious limitations concerning physics and geometry that can be represented, in comparison to the general-purpose commercial package, COMSOL. However, the SGUF model's simple set-up makes it a suitable alternative for scenarios that require quick, iterative simulations or have limited processing resources. Despite its simplicity, the SGUF model has proven to maintain an adequate level of accuracy, making it a practical choice for early analysis and optimization study. The significant reduction in degrees of freedom from the COMSOL to the SGUF model shows the compromise between computing efficiency and precision. The balance between precision and efficiency is a principal factor when choosing the best modeling approach, and the concurrent analysis of Figures 5.3, 5.4, 5.5 shows the practical benefits of the

SGUF model.

5.1.4 Conclusion

This study demonstrated the effectiveness of employing lock-in thermography combined with numerical simulations to investigate temperature and phase contrast in non-destructive testing. The experimental results served as a robust foundation for validating the numerical simulations across different modulation frequencies. Key findings include:

1. The COMSOL model's thorough 3D simulation has lower error margins than the SGUF model, indicating higher accuracy in predicting thermal response.
2. While the COMSOL model has higher accuracy, the SGUF model is more computationally efficient and provides reasonable agreement with experimental data within an acceptable error level of 6 %.
3. Both models were found to be reliable for analyzing phase maps in lock-in thermography since they agreed well with experimental results.

The primary goal of this work was to create a 2D model with much fewer degrees of freedom (DOF) to reduce computational costs while maintaining acceptable precision. The SGUF model accomplished this objective, confirming its importance as a tool for quick and efficient thermal analysis.

5.2 Sensitivity Analysis

Finite Element Modelling (FEM) is a powerful method for investigating heat transfer mechanisms in composite materials and has shown potential in predicting thermal wave propagation under LIT conditions. By creating controlled numerical models, researchers can systematically analyze the effects of defect size, depth, and excitation frequency on the resulting thermal and phase contrast. These variables can be precisely controlled via simulations, which may provide insights that are difficult to acquire through experimentation. For instance, FEM makes it possible to evaluate the sensitivity of LIT in a variety of controlled conditions by altering defect characteristics in a simulated environment, reducing the impact of experimental limitations and external noise. Furthermore, numerical modeling makes an iterative approach feasible, allowing for the repeated testing of various scenarios to create solid and quantitative correlations between the parameters affecting the sensitivity of defect detection. These simulations are especially useful in designing LIT systems and protocols customized to the specific defect profiles within composite materials. The reliability of defect identification in industrial applications can be increased by optimizing LIT inspections by focusing on the most important characteristics.

5.2.1 Parametric study

Using the Lock-in thermography test, an extensive parametric analysis was carried out to verify the sensitivity of the model to the imposed excitation frequency and defect depth and

size. With the material properties Table 4.5, geometry, and boundary conditions set within this initial configuration worked as the base case. The parametric investigation was carried out by systematically changing the defect size, depth, and excitation frequency from this baseline to assess their impact on the sensitivity of defect detection.

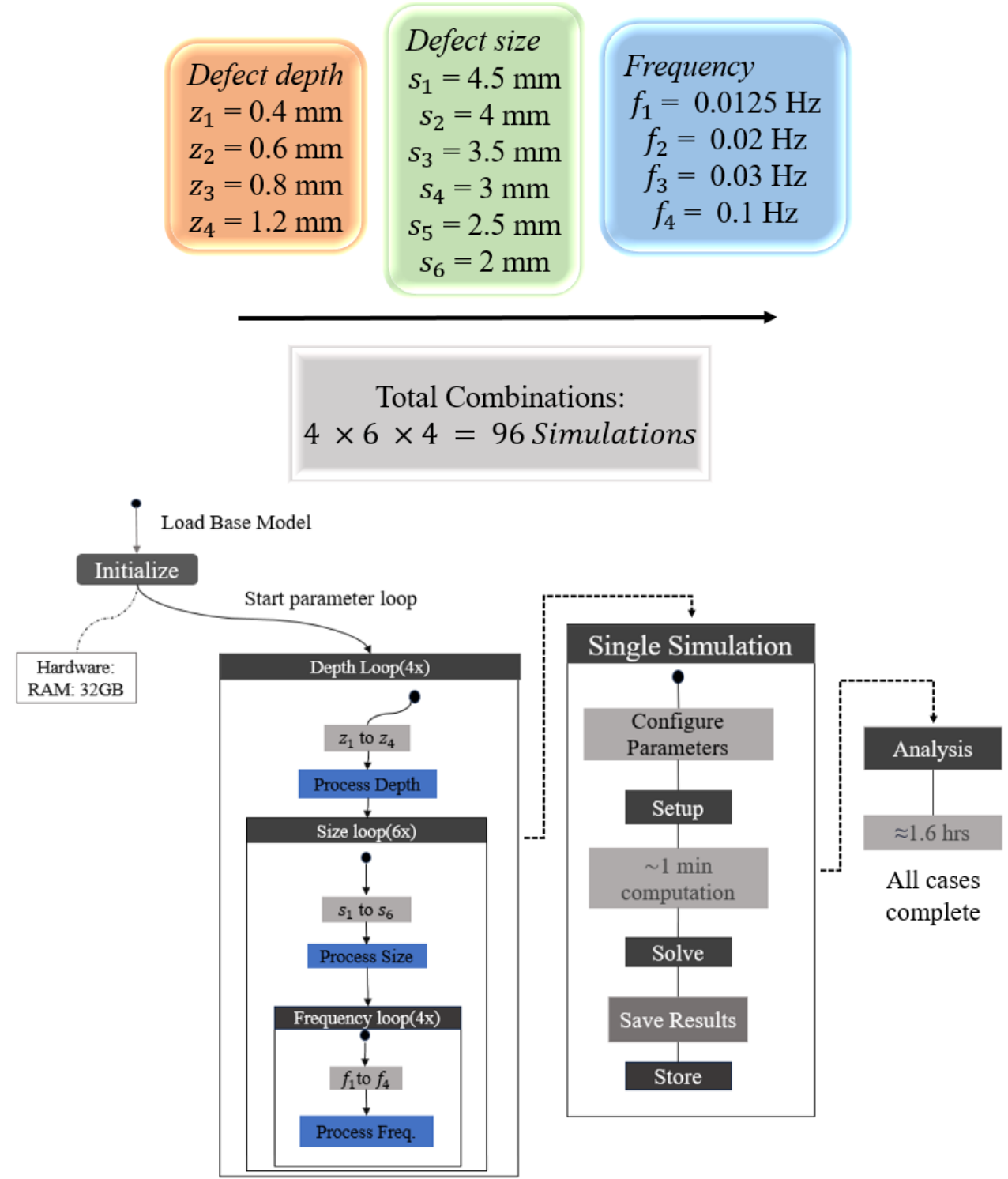


Figure 5.6: Parametric study configuration and computational workflow

Figure 5.6 illustrates the computational method and settings for the parametric studies. Four levels of defect depth variation (0.4 mm to 1.2 mm), six steps of defect size variation (2.0

mm to 4.5 mm), and four excitation frequency values (0.0125 Hz to 0.1 Hz) were examined. The time integration was carried out using the Adams-Bashforth-Moulton method and Runge-Kutta 4th Order Method (RK4) as explained in algorithm 1,2 and equation 4.17. With a total investigation period of almost 1.6 hours, this full factorial design produced 96 distinct simulation cases each of which was calculated using a time step of 0.5 seconds and used roughly 1 minute of processing time.

5.2.2 Post-processing

The Diagnostic Engineering Solutions (DES) and its proprietary software, IRTA [189], employ the Signal Reconstruction (SR) algorithm for the post-processing of raw data 4.1,4.2. This algorithm extracts important parameters, such as the thermal wave's amplitude (A) and phase (φ). This study's geometric resolution of 0.68 mm/pixel provides sufficient data to detect subsurface features. The algorithm processes temperature data over time to separate the phase and amplitude images. Phase signals are less impacted by surface characteristics like surface emissivity or reflectivity, which makes it simpler to identify subsurface defects. This makes phase data especially useful for defect detection. Furthermore, phase information penetrates the material more deeply than amplitude, which is more sensitive to surface characteristics. Significant contrast in phase images is made possible by defects, which appear as regions with distinct phase values from the surrounding homogenous material. Thermal and phase-contrast maps are computed using equation 5.1, respectively where i and j represent the i -th and j -th pixels. The reference value is derived from the mean value over small 3×3 pixel regions in the sound region, designated as S . Following that, we define the temperature and phase contrast as follows:

$$\Delta T_{ij} = T_{ij} - \text{mean}(S), \quad \Delta \varphi_{ij} = \varphi_{ij} - \text{mean}(S) \quad (5.1)$$

5.2.3 Results and Discussion

To evaluate the depth-dependent properties of subsurface defects, phase contrast analysis was used. Defect characterization capabilities were significantly improved by the investigation, which included systematic changes in defect sizes, depths, and excitation frequencies. At different frequencies (f) and depths (z), the phase contrast maps show unique patterns. This result is in line with thermal wave theory, which suggests that phase delay increases with depth.

The phase contrast at a depth of $z = 0.4$ mm and varying frequencies exhibits distinct frequency-dependent behavior across the range of 0.0125–0.05 Hz, as illustrated in Figure 5.7a. At the lowest frequency (0.0125 Hz), the phase contrast maps show a more uniform distribution and a clear phase transition zone, reflecting better heat penetration and thermal wave propagation. As the frequency increases to 0.05 Hz, the phase contrast patterns near the defect region become more concentrated, signifying enhanced detection sensitivity for

near-surface defects.

To ensure consistency across qualitative results (Figure 5.7, Appendix D.1 and D.2), the phase and thermal contrast data have been normalized using a simple linear normalization approach, given by:

$$\text{Normalized Matrix} = \frac{M - \min(M(:))}{\max(M(:)) - \min(M(:))} \quad (5.2)$$

where M represents the phase and thermal contrast matrix. This normalization scales the matrix values to the range $[0, 1]$, enabling clearer visualization without incorporating standard deviation-based normalization. The complete set of phase contrast results for various

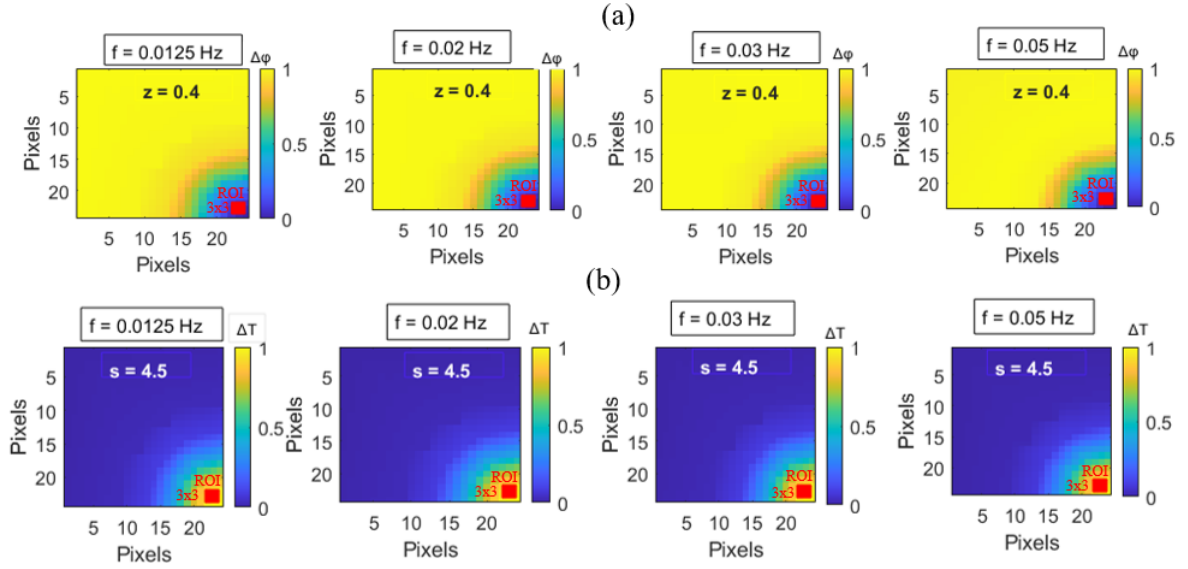


Figure 5.7: (a) Normalized Phase contrast for depth=0.4 at different frequencies (b) Normalized Thermal contrast for size=4.5 at different frequencies

depths and frequencies is presented in Appendix D.1, showing patterns consistent across depth configurations. Notably, the phenomenon of “Blind Frequency” is observed, where the phase contrast becomes null. For a fixed depth and varying frequencies, a change in the sign of the phase contrast can be seen, indicating that it passes through zero. This behavior is evident in Appendix D.1, highlighting the importance of selecting appropriate frequencies to avoid blind spots in defect characterization.

$$\Delta T = \text{mean}(\Delta T_{ij}), \quad \Delta \varphi = \text{mean}(\Delta \varphi_{ij}) \quad (5.3)$$

The statistical analysis results from the ANOVA Test for the phase map are shown in Table 5.6. The analysis focused on an ROI (3×3 pixel) within the defective region. The selected values and their distribution can be observed in Figure 5.7, while the methodology for calculating these values is detailed in Equation 5.3. With a very high F-value of 4600.15, the depth of the defect is found to be the most significant component. This finding highlights the critical importance of accurately accounting for the defect’s subsurface position when analyzing phase contrast data. The thermal wave propagation is highly sensitive to the depth

of the feature, and this depth-dependent behavior must be a primary consideration in defect characterization strategies. As expected, the F-value of 14510.63 indicates that frequency also shows a significant main impact. The size component has a slightly lesser impact than depth and frequency, but still being highly significant ($F = 228.78$). This indicates that although defect size has a significant influence on altering the phase contrast, the depth and frequency parameters have a greater influence. The complex connection between defect size, depth, and the selected excitation frequency is further demonstrated by the considerable size: depth ($F = 129.35$) and size: frequency ($F = 21.1$) interactions. According to the depth: frequency interaction ($F = 22.11$), the frequency response is dependent on the depth of the defect. This finding aligns with the expected physics of thermal wave propagation, where the depth-dependent attenuation and phase lag of the thermal waves is influenced by the excitation frequency.

Table 5.6: Analysis of Variance (ANOVA Test) for Phase Map

Source	Sum Sq.	d.f.	Mean Sq.	F	p-value
Size	4.34	5	0.88	228.78	8.36e-31
Depth	52.94	3	17.65	4600.15	5.67e-56
Frequency	166.99	3	55.67	14510.63	3.54e-67
Size:Depth	7.44	15	0.49	129.35	7.58e-32
Size:Frequency	1.21	15	0.08	21.1	1.68e-15
Depth:Frequency	0.76	9	0.085	22.11	9.91e-14
Error	0.17	45	0.004		
Total	233.92	95			

The thermal contrast images are displayed with different frequencies and a constant depth and size ($s=4.5\text{mm}$) in Figure 5.7b. Defect detectability significantly increases across the frequency range, as shown by the thermal contrast maps. Higher frequencies (0.05 Hz) result in more precise and strong contrast patterns, whereas lower frequencies (0.0125 Hz) offer a more diffuse but deeper thermal contrast pattern. The thermal diffusion length principle, which suggests that lower frequencies provide deeper penetration but lesser spatial resolution, is by this behavior. Appendix D.2 provides a full thermal contrast analysis for a range of defect sizes and frequencies, showing consistent behavior across size configurations.

Table 5.7: Analysis of Variance (ANOVA) for Thermal Map

Source	Sum Sq.	d.f.	Mean Sq.	F	P-value
Size	0.31	5	0.06	22837.01	2.17e-57
Depth	0.56	3	0.19	68922.6	2.15e-82
Frequency	63.62	3	7772197.73	270.28	1.44e-128
Size:Depth	0.09	15	2063.98	114.13	1.21e-58
Size:Frequency	0.0024	15	59.63	17.63	1.32e-24
Depth:Frequency	0.0014	9	57.23	26.38	1.12e-21
Error	0.0001	45	0.0		
Total	64.59	95			

The Analysis of Variance (ANOVA) results in Table 5.7 provide a statistical analysis of the significance of the thermal maps. The influence of defect size on the thermal signal response is significant, as evidenced by an F-value of 22837.01 and a highly significant p-value. Larger defects generate more pronounced thermal wave disruptions, which are easier to detect on the thermal maps. Another important factor is defect depth, which has an F-value of 668922.6. This suggests that the attenuation of thermal waves through the material significantly decreases the detectability of deeper defects. This attenuation effect is well captured by the numerical model's through-thickness approximation. The thermal response is strongly influenced by the excitation frequency, which has the largest mean square and an F-value. This confirms that it is the most significant element in influencing the behavior of thermal waves. To maximize the identification of defects at different depths, frequency is crucial since it impacts the penetration depth and resolution of thermal signals. Higher frequencies enhance resolution for defects close to the surface, whereas lower frequencies allow for deeper wave penetration.

With an F-value of 2,063.98, the relationship between depth and size is complicated. Larger defects improve detectability, but as depth increases, wave attenuation reduces their effect. This emphasizes that to optimize detection sensitivity, both factors must be taken into account at the same time in the analysis, particularly when dealing with larger but deeper defects. The significant mean square value of 59.63 for the size-frequency interaction indicates that defect size affects the most appropriate frequency selection. Higher frequencies improve resolution for smaller defects, whereas lower frequencies could be sufficient for bigger defects. However, as independent factors, size and frequency have stronger effects than this interaction, which is less noticeable. With an F-value of 26.38, the depth-frequency interaction illustrates how detection sensitivity depends on frequency modulation about defect depth. Higher frequencies improve thermal contrast for small defects, while lower frequencies effectively treat deeper defects. This illustrates the importance of adaptive frequency techniques for a range of fault depths in industries.

Lock-in Thermography testing procedures will be significantly affected by these findings, especially for applications that require precise depth analysis. The phase contrast analysis provides a robust method for depth estimation, with the caveat that inspection parameters must be carefully selected based on the expected defect characteristics. Since both amplitude and phase information can be used to achieve optimum detection and sizing capabilities, the results strongly represent the implementation of multi-frequency techniques for extensive defect characterization and to avoid blind frequencies.

5.2.4 Conclusion

This study proposed a numerical model based on Sublaminar Generalized Unified Formulation (SGUF) to investigate the lock-in technique in composite laminates. In particular, the lock-in technique's sensitivity to the excitation frequency and defect size and depth has been proven. The results show that the proposed approach can be used to quickly optimize the test parameters as a function of the defect size and depth. Three key factors were examined: depth (0.4-1.2

mm), excitation frequency (0.0125-0.05 Hz), and defect size (2.0-4.5 mm). With significant statistical support ($F = 4600.15$, $p < 5.67e-56$), key findings showed that larger defects (>3.5 mm) produced higher thermal contrast values, whereas phase contrast analysis offered efficient depth characterization. The best frequency ranges were found to be 0.03-0.05 Hz for deeper defects (>0.8 mm) and 0.02-0.033 Hz for thermal contrast. Significant parameter interactions were found in the study, highlighting the significance of multi-parameter optimization in evaluation methods.

5.3 Effect of Thermal Anisotropy and Lay-up Configuration

In this study, we focused on the impact of CFRP ply layup and anisotropic thermal conductivity on active thermographic inspections. Several studies have been carried out to assess thermal diffusivity or thermal conductivity using thermography. A fitted model for transient heat conduction was used by Miettinen et al. [102, 190] and Rémy et al. [103] for measuring the one-dimensional in-plane thermal diffusivity of a thin plate-like specimen using thermography. To overcome this gap, we use a 3D numerical model based on the Generalized Unified Formulation (GUF). The GUF framework provides a versatile and computationally efficient approach to modeling the thermal behavior of composite laminates with an extensive range of layup sequences and material parameters. We used extensive finite element modeling to study the effects of thermal anisotropic conductivity and layup patterns on defect detection in CFRP composites. The IRTA® [189] software was used to analyze the thermal data to generate amplitude and phase images from the lock-in test.

This study presents the phase map results. The phase is the time delay between the thermal input and the thermal response of the material. Usually, only the phase maps are used for characterizing defects in terms of shape, size, and depth. The Input parameters for the acquisition of thermal sequences are shown in Table 5.8.

Table 5.8: Input parameters for the acquisition of thermal sequences

Waveform	Number of Cycles	Frames per Cycle	Total Frames (Hz)
Square	3	160	481

5.3.1 Thermal conductivity analysis

Thermal conductivities in-plane and through thicknesses have significant effects on heat diffusion and the detection of defects in composite materials. Assuming equal in-plane thermal conductivities ($k_{xx} = k_{yy}$), we may simplify the analysis while understanding how differences in these parameters affect the thermal response. A statistical analysis using these factors allows

for more precise results in thermographic inspections. Table 5.9 summarizes the ranges and sensitivity grids used in this investigation.

Table 5.9: Sensitivity Analysis of Thermal Conductivities

Parameter	Range (W/m·K)	Sensitivity Grid (W/m·K)
$k_{xx} = k_{yy}$	10 – 20	10, 15, 20
k_{zz}	0 – 1	0.3, 0.7, 0.9

5.3.2 Lay-up configurations

In Figure 5.8, various layup configurations $[0^\circ/0^\circ]$, $[0^\circ/90^\circ]$, $[0^\circ/\pm 45^\circ/90^\circ]$, and $[\pm 45^\circ]$ were investigated for their impact on thermal anisotropy and defect identification. The model considers the directional anisotropic thermal conductivities, with $k_{xx} = 15 \text{ W/m}^2$, $k_{yy} = 5 \text{ W/m}^2$, and $k_{zz} = 0.5 \text{ W/m}^2$, for each layer. This setup allows us to predict heat propagation in three dimensions. This method enables a detailed investigation of the effect of fiber orientation on temperature gradients, particularly near embedded defects, using a fully layer-wise model. By comparing the simulated thermal responses of various layups, we can determine which configurations improve or hinder defect visibility.

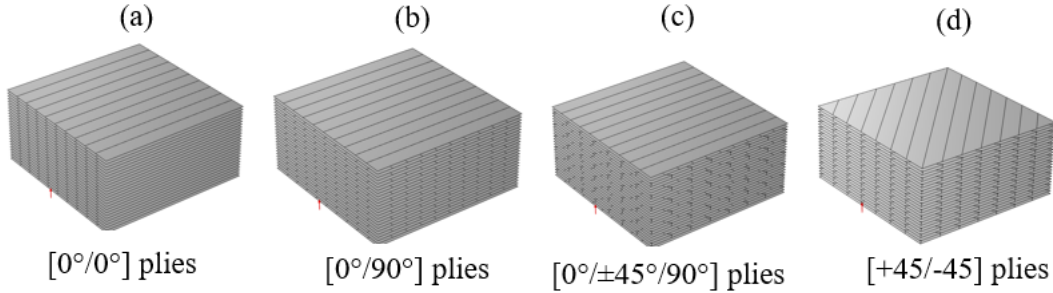


Figure 5.8: Lay up configurations

5.3.3 Discussion

The relationship between the in-plane and through-thickness thermal conductivities is shown in Figure 5.9. As the thermal conductivity increased, the phase contrast decreased. This observation indicates that an enhanced in-plane conductivity leads to more uniform thermal diffusion across the laminate structure. The diagrams indicate that an increase in the lateral thermal conductivity reduces the defect boundaries. In the x-direction, heat dissipated more quickly.

The ANOVA results, which show that the p-value is 0.1783, and the F-value for in-plane thermal conductivities ($k_{xx} = k_{yy}$) is 1.7257, provide quantitative validation for this. This shows that at the 95% confidence level, there is some variance between groups, but this is not enough to be considered statistically significant. To perform the statistical analysis, a small

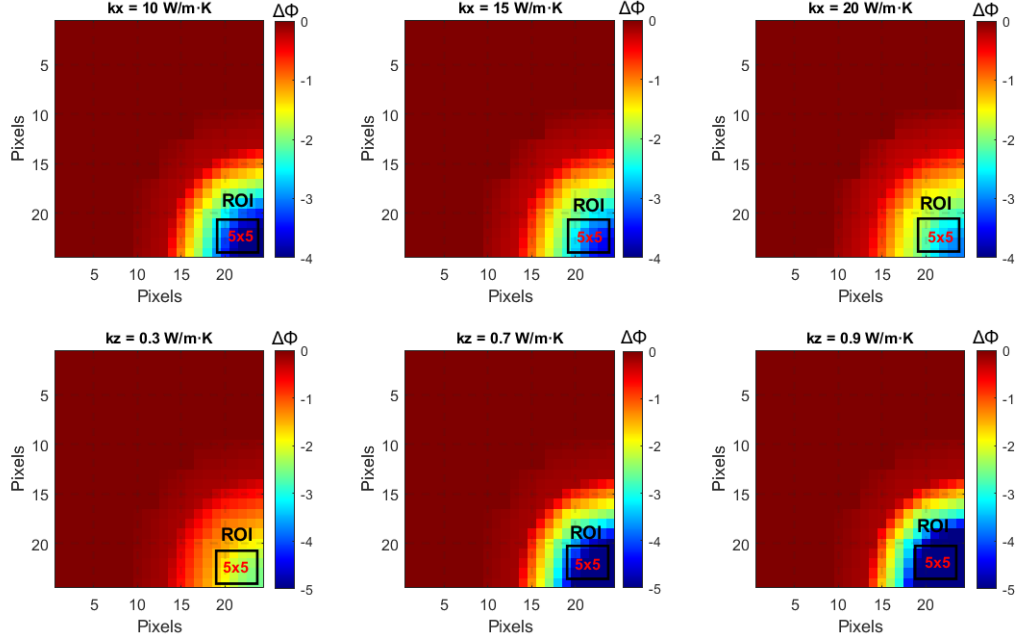


Figure 5.9: Phase contrast maps for in-plane and through-thickness thermal conductivities

region of 5×5 pixels was considered in each case, and the mean values were calculated for that region as shown in Figure 5.9. These mean values were then used to conduct the ANOVA test. The results indicate that, in comparison with k_{zz} , the in-plane thermal conductivity had a minimal effect on the phase contrast sensitivity. The through-thickness direction was crucial for defect detection, as heat must penetrate deeper into the material and return to its surface. This makes subsurface defects more visible.

Table 5.10: ANOVA Results for Thermal Conductivities in-plane and through thickness

Source	SS	df	MS	F	p-value
$k_{xx} = k_{yy}$					
Groups	1.9818	2	0.9909	1.7257	0.1783
Error	990.4854	1725	0.5742		
Total	992.4672	1727			
k_{zz}					
Groups	47.1281	2	23.5641	15.2330	2.7678e-07
Error	2.6684×10^3	1725	1.5469		
Total	2.7155×10^3	1727			

The temperature diffusion through the thickness was slower with decreasing levels of k_{zz} . As a result, significant temperature differences may develop around defects. Defects were visible at lower k_{zz} levels because of the more pronounced phase contrast. The sensitivity of k_{zz} to the through-thickness thermal conductivity was demonstrated by ANOVA results in Table 5.10. Both are extremely small and statistically significant with an F-value of 15.2330 and a corresponding p-value of 2.7678×10^{-7} . This result indicates that k_{zz} has a significant influence on the defect detection sensitivity.

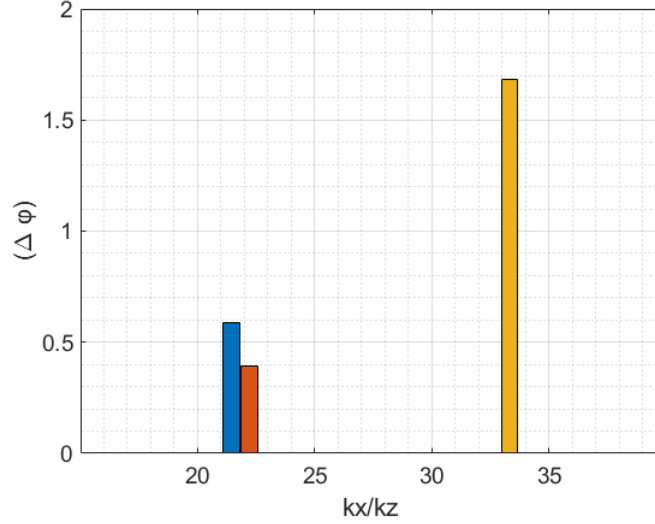


Figure 5.10: Thermal conductivities ratio as a function of phase contrast values

The correlation between mean phase contrast and thermal conductivity ratio analysis provides significant details about the capability of lock-in thermography to detect defects in composite laminates. Figure 5.10 indicates that variations in thermal conductivity significantly influence the visibility of defects, with the ratio (k_{xx}/k_{zz}) serving as a crucial parameter in this context. The statistically significant ANOVA results and clear contrast in the heat maps indicate that the through-thickness thermal conductivity k_{zz} is the most sensitive and important factor in defect detection. To optimize composite laminates for non-destructive testing applications, it is important to consider the significance of k_{zz} . Improving through-thickness conductivity can help identify subsurface defects more easily, especially in materials where heat dissipation through the thickness is difficult.

The presented heat and binary maps in Figure 5.11 offer valuable insights into the thermal behavior and defect detection capabilities across different composite layups. Due to the fibers' unidirectional alignment, heat can propagate across the plane more effectively due to its unidirectional nature $[0^\circ/0^\circ]$, which also makes the temperature gradient surrounding the defect more apparent. The stacking sequence $[0^\circ/\pm 45^\circ/90^\circ]$ displayed a comparable dispersion of heat. Nonetheless, compared with the $[0^\circ/0^\circ]$ layup, the phase difference values were comparatively greater.

Table 5.11: ANOVA result for layup sequence

Source	SS	df	MS	F	p-value
Groups	0.253	3	0.08435	0.2	0.8967
Error	971.745	2300	0.4225		
Total	971.998	2303			

The multi-directional structure of this layup allows a more even heat transfer. This reduced the contrast surrounding the defect. The phase contrast was lower in the $[0^\circ/90^\circ]$ layup than in the $[0^\circ/0^\circ]$ and $[0^\circ/\pm 45^\circ/90^\circ]$ layups. Because of the orthogonal fibers in this structure, the

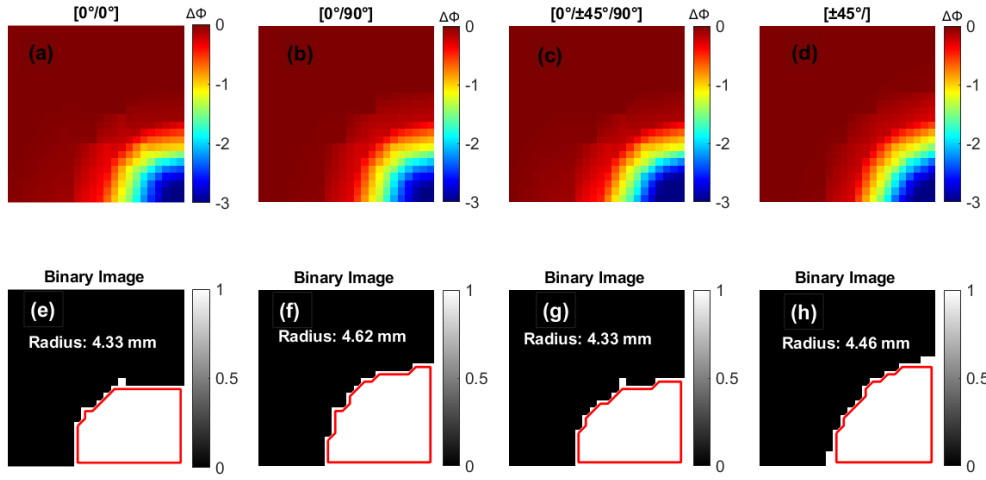


Figure 5.11: Original phase contrast maps (a),(b),(c),(d) and corresponding binary images (e),(f),(g),(h)

Table 5.12: Error Analysis of Detected Defect Radius for Different Layups

Layup	Original Radius (mm)	Detected Radius (mm)	% Error
[0°/0°]	4.5	4.333	3.71
[0°/90°]	4.5	4.6245	2.77
[0°/±45°/90°]	4.5	4.333	3.71
[±45°/±45°]	4.5	4.4603	0.88

heat distribution is restricted in some directions, which reduces the contrast around the defect. The defect is still visible, but it responds less to the phase contrast than layups with more aligned fiber directions. The $[\pm 45^\circ/]$ layer showed the lowest phase contrast values in the final heat map. Heat diffuses across the laminate in multiple directions when the fibers are aligned diagonally, which makes the defect least noticeable in this arrangement. This implies that multi-directional fiber layouts are useful for defect detection, even though their sensitivities are lower.

A statistical comparison of the mean phase contrast values for each of the four layup sequences is presented in Table 5.11. The error variance was 971.745, suggesting that the variation within the groups was significantly greater than that between them. The variance between the groups (lay-up sequences) was 0.253. The effect of layup sequencing on defect identification in composite laminates is affected by these findings, leading to numerous significant conclusions. Higher phase-contrast values indicate that layup sequences with aligned fiber orientations, such as $[0^\circ/0^\circ]$ and $[0^\circ/\pm 45^\circ/90^\circ]$, offer greater sensitivity for defect detection. More precise temperature gradients surrounding the defects are produced by these arrangements because they enhance more efficient thermal diffusion. Layup sequences such as $[0^\circ/90^\circ]$ and $[\pm 45^\circ/]$, which have fiber orientations that are orthogonal or diagonal, respectively, reduce the sensitivity to defects because the heat energy is distributed more uniformly across the laminate. This complicates the process of identifying subsurface defects. The ANOVA test demonstrated no statistically significant differences between the layup sequences despite variations in the

mean phase contrast. This shows that although the layup sequence has some influence on defect identification, it does not have a sufficiently significant impact to be regarded as statistically significant. This may suggest that the more important variables for defect detection using Lock-in Thermography than layup are the thermal properties or depth of the defect. According to phase contrast studies for various layup sequences, fiber orientation influences the thermal response of composite laminates with aligned fiber configurations ($[0^\circ/0^\circ]$ and $[0^\circ/\pm 45^\circ/90^\circ]$), demonstrating better sensitivity to defects.

The binary images in Figure 5.11 provide a clear visualization of the detected defect boundaries after image processing. These maps are crucial for quantifying defect dimensions. All layups show a generally similar defect shape, validating the consistency of the detection method. Slight variations in the defect outline are evident, particularly in the curvature and regularity of the boundary. The $[0^\circ/90^\circ]$ layup (f) appears to yield a slightly larger detected area compared to the others, which aligns with its higher detected radius of 4.62 mm as shown in Table 5.12. The differences in defect representation across these binary maps highlight the impact of layup configuration on defect boundary definition in thermal imaging. The binary images effectively isolate the defect region visible in the heat maps, demonstrating the efficacy of the image processing technique. The slight variations in defect size and shape between layups in the binary images correlate with the subtle differences in thermal gradients observed in the heat maps. The $[\pm 45^\circ/]$ configuration (5.12d, h) shows the closest match between the original and detected radius, suggesting that this layup may offer optimal conditions for accurate defect sizing using lock-in thermography.

5.3.4 Conclusion

Our study indicates that through-thickness and in-plane thermal conductivity significantly impact defect detection using lock-in thermography. This suggests that through-thickness thermal conductivity variations are more sensitive to defect detection in composite laminates than the in-plane thermal conductivity variations. Greater heat diffusion through the thickness produces a more noticeable phase contrast, improving defect visibility. Lay-up sequences with aligned fibers, such as $[0^\circ/0^\circ]$ and $[0^\circ/\pm 45^\circ/90^\circ]$, showed better defect detection capabilities because of more effective heat diffusion. In contrast, lay-ups with diagonal or orthogonal fiber orientations produced a less pronounced phase contrast. Utilizing lamination angles, we performed image processing to compare actual and detected defect sizes, demonstrating the potential for enhanced accuracy in defect characterization. Combined with these aspects, we may improve our comprehension of defect dimensions and characteristics by utilizing advanced image processing algorithms, which will result in evaluations of composite materials that are more reliable.

5.4 Application: MUL2 Code for Simulating Debonding in Complex T-Joint Structures

5.4.1 Initial validation of Model

The thermal imaging results presented in Figure 5.12 provide valuable insights into the structural integrity of the sample. The image highlights two key regions of interest: Spot1 (sound area) and Spot2 (debonded area), with their respective spatial coordinates explicitly indicated. Spot1, located at $(x = 45, y = 28)$, represents the intact region where heat conduction follows a uniform pattern, whereas Spot2, positioned at $(x = 145, y = 28)$, corresponds to the debonded area, where disrupted heat flow leads to distinct thermal anomalies.

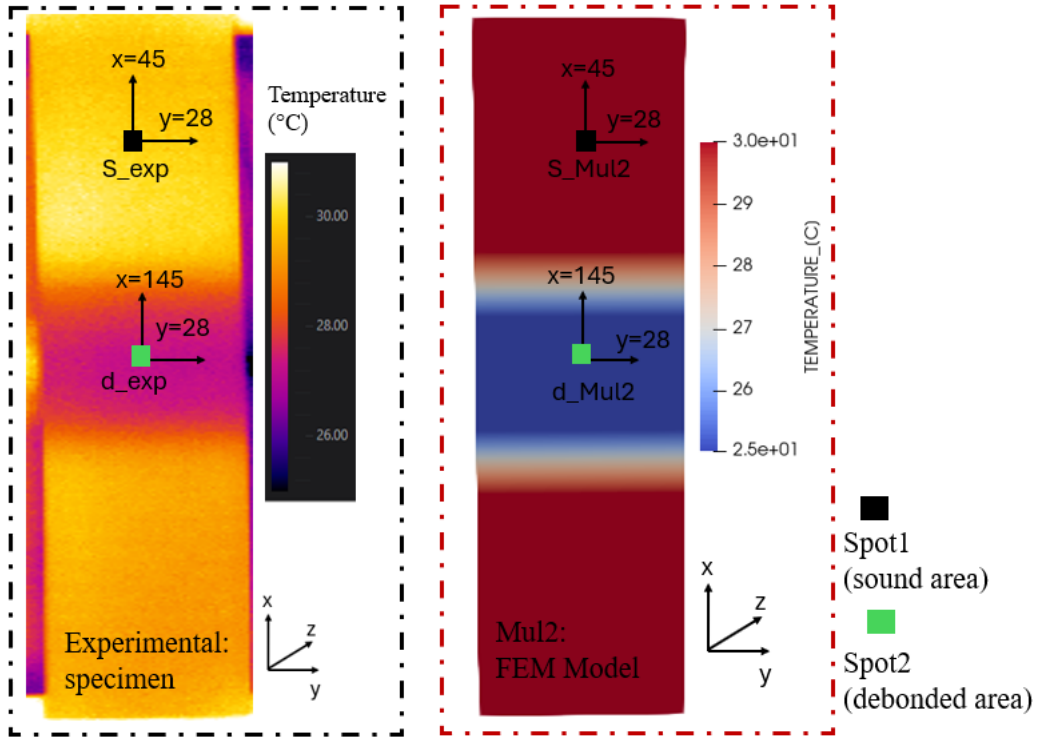


Figure 5.12: Thermal images from the lock-in thermography experiment.

To further analyze the experimental findings, Figure 5.14 compares the time-domain temperature variations at the two spots, contrasting the experimental (S_{Exp} and d_{Exp}) and simulated (S_{Mul2} and d_{Mul2}) data. In the sound area (Spot1), the experimental and simulated temperature curves show good agreement regarding the overall trend and magnitude of the temperature oscillations. A comprehensive analysis of transient thermal simulation is provided in Appendix G, offering detailed insights into the thermal behavior and numerical implementation. This suggests that the simulation (*Mul2* model) accurately captures the thermal behavior of the intact region of the sample. The explicit identification of spatial locations for defect analysis enhances the accuracy of thermal evaluations, providing a quantitative basis for refining numerical simulations. The results suggest that additional factors, such as localized

heat dissipation mechanisms and contact resistance effects, may need to be incorporated to improve the predictive capabilities of the *Mul2* model for debonded regions.

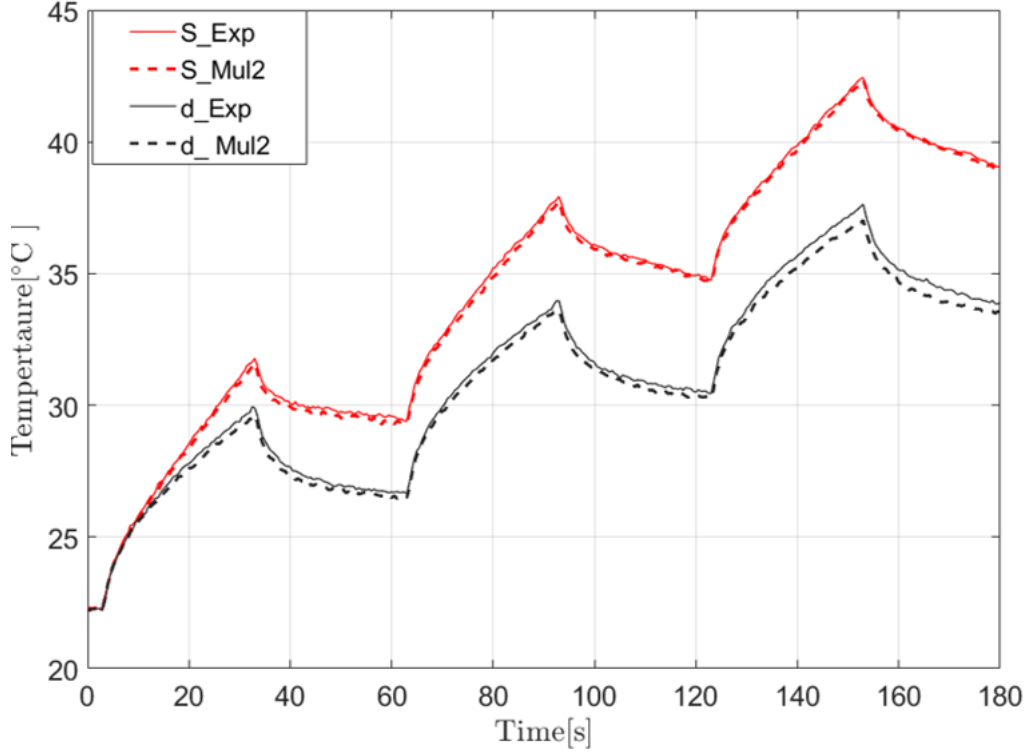


Figure 5.13: Comparison of experimental and simulated temperature variations.

The experimental and simulated results show a modulation period of 60 seconds, consistent with the modulation frequency used in the lock-in thermography setup. This temporal correlation between the experimental and simulated data provides confidence in the overall experimental and modeling approach. This analysis initially validates the *Mul2* model.

5.4.2 Phase contrast: Experiment vs FEM

The phase contrast results obtained from the Lock-in Thermography (LIT) test and Finite Element Model (FEM) simulations exhibit a strong correlation, demonstrating the effectiveness of the numerical model in predicting thermal phase variations associated with debonding defects. A focused analysis was conducted on a 3×3 pixel region within the debonding area, where the mean phase contrast values ($\Delta\phi$) were computed for both the experimental and FEM data.

The experimental results indicate a mean $\Delta\phi$ of 12.94°C , while the FEM simulation produced a mean $\Delta\phi$ of $12.57^\circ\text{C} \pm 1\%$, showing a close agreement with a minor deviation. This slight difference can be attributed to factors such as measurement noise in experimental data, material property variations, and potential discrepancies in the air thickness between the experimental setup and the numerical model. Despite these uncertainties, the FEM results closely follow the experimental trend, reinforcing the reliability of the numerical approach in defect characterization.

These findings validate the potential of FEM-based thermal analysis in predicting phase contrast variations in composite laminates. The strong agreement between experiment and simulation underscores the applicability of FEM as a robust tool for quantitative defect detection in structural health monitoring applications, particularly in composite materials subjected to Lock-in Thermography testing.

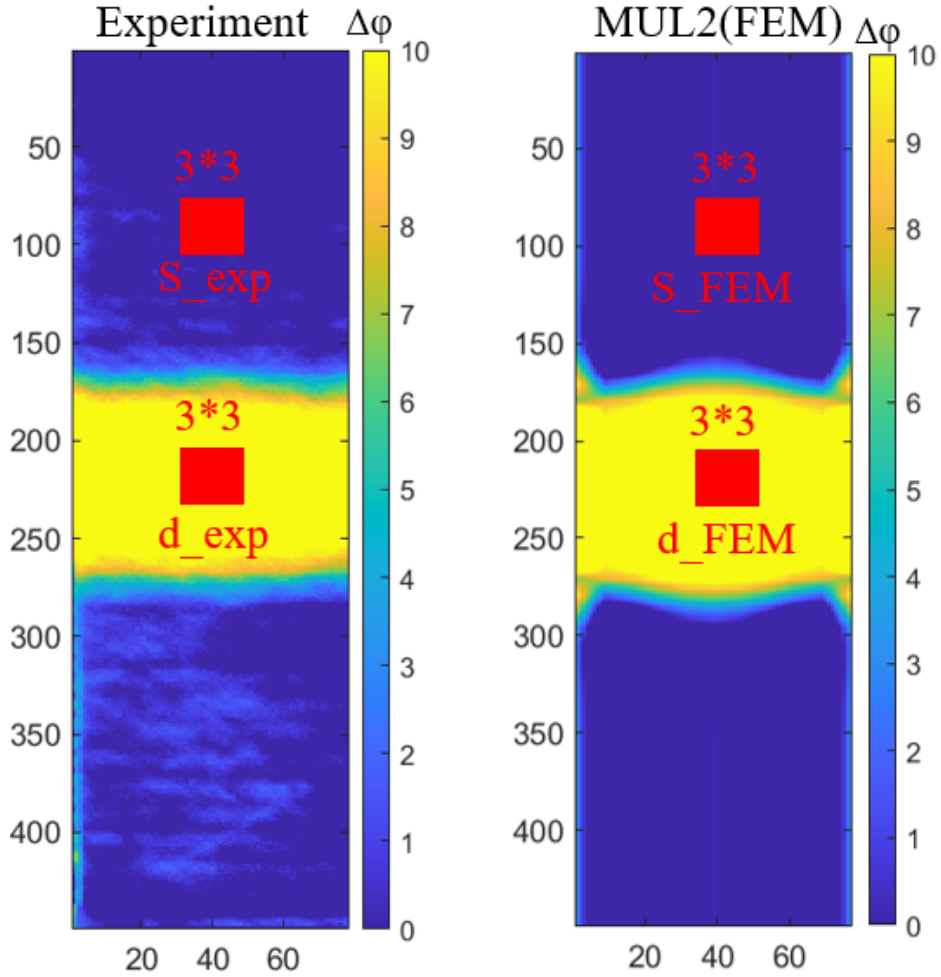


Figure 5.14: Phase contrast results: Comparison between experimental and FEM

Table 5.13: Summary of $\Delta\varphi$ mean values with uncertainty ranges

Frequency (Hz)	Experiment mean $\Delta\varphi$ ($^{\circ}$)	MUL2(FEM) mean $\Delta\varphi$ ($^{\circ}$)
0.017	12.94	$12.57 \pm 1\%$

5.4.3 Conclusion

This study implemented the *Mul2* code for simulating debonding in complex T-joint structures, integrating heat transfer analysis and boundary conditions based on the Carrera Unified Formulation (CUF) in the context of thermography. This approach has not been previously

implemented for thermal defect detection, making it a novel contribution to the field. The results demonstrate that the *Mul2* model effectively captures the thermal response with an agreement between experimental and simulated temperature profiles.

The findings highlight the potential of the *Mul2* model for defect detection and characterization in composite structures. However, further refinements are necessary to enhance its predictive accuracy for debonded regions. Future work will focus on performing a sensitivity analysis on the thickness of the air layer in the debonded part to better understand its influence on thermal behavior. Additionally, a convergence study will be conducted to assess the stability and accuracy of the *Mul2* model under varying computational parameters. These improvements will contribute to optimizing the simulation approach and increasing its reliability for defect evaluation in complex composite structures.

Conclusion

The advanced numerical modeling and validation of multilayer composites employing active thermography for non-destructive testing (NDT) are thoroughly examined in this thesis. The work demonstrates how advanced modeling tools can transform defect identification and material characterization through in-depth numerical simulations and comparisons with conventional experimental methods. Below, the important contributions and insights of this work are summarized:

Key Contributions:

1. Development of Advanced Numerical Models:

We used advanced numerical methods based on the Sublaminar Generalized Unified Formulation (SGUF) and Carrera Unified Formulation (CUF) to describe the thermal behavior of composite laminates accurately. These models accurately simulated the propagation of thermal waves in anisotropic materials while using lower computational costs.

2. Advantages Over Traditional Numerical Methods:

Unlike conventional software such as COMSOL Multiphysics, which uses highly computational 3D element-based meshes, the proposed models use 2D meshes for thermal simulations. This allows for extensive layerwise investigations while significantly reducing the overall processing time and resources. Furthermore, enhanced CUF capabilities reduce the need to re-mesh the model when defect positions change, allowing for the smooth and efficient simulation of a wide range of defect scenarios.

3. Efficient Layerwise Analysis:

The layerwise modeling approach, combined with advanced computational tools, enabled the exact characterization of the thermal behavior of each layer in a composite structure. This capability is critical for detecting delaminations and defects related to specific layers, which previous approaches frequently struggle to handle without expending excessive computing effort.

4. Support for Virtual Testing:

The advanced numerical models' low computational cost enabled comprehensive virtual testing, avoiding the need for several experimental trials. Simulations were used to recreate real-world conditions, such as air-filled defects modeled as delaminations. These virtual tests are important in evaluating and optimizing NDT procedures before actual installation.

5. Application to Real-World Problems:

The developed models were applied to complex defect scenarios, such as air-filled delaminations and defects in different places, without needing any changes to the computational mesh.

This capability demonstrates its durability and suitability for industrial applications. In-depth sensitivity analysis and optimization studies are made possible by the high-throughput simulations made possible by a reduced computational load.

Significance of Findings

This study shows how advanced numerical modeling can improve active thermography for nondestructive testing. Key advantages include:

1. **Improved Computational Efficiency:**

In comparison to standard 3D element-based approaches, using 2D meshes and efficient formulations significantly reduced computational costs.

2. **Flexibility in Defect Analysis:**

Simulating defects in various positions without remeshing allows for rapid prototyping and optimization of inspection approaches.

3. **Enhanced Defect Characterization:**

The models accurately represented defect size, depth, and shape, which addressed the limitations of traditional approaches.

4. **Practical Relevance:**

Applications to scenarios such as air-filled delaminations demonstrate the models' applicability in solving real-world problems.

Future Work

Based on the results of this investigation, several directions for further research are suggested:

1. **Integration of Machine Learning:**

Using machine learning techniques can greatly enhance defect identification and classification. Training models using phase contrast data allows for the development of prediction algorithms that can identify problems with high accuracy and reliability. Defect characterization can be advanced by using these models to find hidden patterns and correlations in the data.

2. **Digital Twin Development:**

The CUF-based numerical framework, together with digital twin technologies, may generate real-time, dynamic duplicates of composite structures. This integration allows for predictive maintenance, continuous monitoring, and optimization of composite laminates in service situations, resulting in increased performance and safety.

3. **Multi-Physics Simulationss:**

Expanding the models to include coupled thermal, mechanical, and electrical impacts for more diverse applications.

To summarize, this thesis advances the state of the art in numerical modeling and experimental validation of active thermography for composite NDT. The advanced CUF-based numerical modeling framework not only provides a computationally efficient solution for identifying defects in composite laminates, but it also opens up possibilities for incorporating modern technologies like machine learning and digital twins. These developments will allow the industry to shift from traditional experimental testing to virtual and predictive methods, resulting in cost-effective and dependable solutions for real-world applications.

Appendix

Appendix A

Derivation of k-th Order BDF Formula

The Backward Differentiation Formula (BDF) is derived from the principle of approximating the derivative using backward differences. The derivation follows these key steps:

Step 1: Taylor Series Expansion We begin with the Taylor series expansion of $y(t)$ around time t_n :

$$y(t) = y(t_n) + \left. \frac{dy}{dt} \right|_{t_n} (t - t_n) + \frac{1}{2!} \left. \frac{d^2y}{dt^2} \right|_{t_n} (t - t_n)^2 + \cdots + \frac{1}{k!} \left. \frac{d^k y}{dt^k} \right|_{t_n} (t - t_n)^k + O(h^{k+1}) \quad (\text{A.1})$$

Step 2: Backward Difference Operator Define the backward difference operator ∇ as:

$$\nabla y_n = y_n - y_{n-1} \quad (\text{A.2})$$

Higher-order differences are defined recursively:

$$\nabla^2 y_n = \nabla(y_n - y_{n-1}) = y_n - 2y_{n-1} + y_{n-2} \quad (\text{A.3})$$

$$\nabla^3 y_n = y_n - 3y_{n-1} + 3y_{n-2} - y_{n-3} \quad (\text{A.4})$$

Step 3: Newton Backward Difference Formula Express y_{n+1} in terms of backward differences:

$$y_{n+1} = y_n + \nabla y_n + \frac{\nabla^2 y_n}{2!} + \frac{\nabla^3 y_n}{3!} + \cdots + \frac{\nabla^k y_n}{k!} \quad (\text{A.5})$$

Step 4: Derivative Approximation The first derivative can be approximated using the backward differences:

$$\left. \frac{dy}{dt} \right|_{t_{n+1}} \approx \frac{1}{h} \sum_{j=1}^k \alpha_j \nabla^j y_{n+1} \quad (\text{A.6})$$

where h is the step size and α_j are coefficients to be determined.

Step 5: BDF Formula Derivation For a k-th order BDF method, we express the

derivative approximation in terms of solution values:

$$\left. \frac{dy}{dt} \right|_{t_{n+1}} \approx \frac{1}{h} \sum_{j=0}^k \beta_j y_{n+1-j} \quad (\text{A.7})$$

The coefficients β_j are determined by matching terms in the Taylor series expansion.

Step 6: Explicit Forms For specific orders, the formulas are:

First-order BDF (BDF1):

$$\left. \frac{dy}{dt} \right|_{t_{n+1}} \approx \frac{y_{n+1} - y_n}{h} \quad (\text{A.8})$$

Second-order BDF (BDF2):

$$\left. \frac{dy}{dt} \right|_{t_{n+1}} \approx \frac{3y_{n+1} - 4y_n + y_{n-1}}{2h} \quad (\text{A.9})$$

Third-order BDF (BDF3):

$$\left. \frac{dy}{dt} \right|_{t_{n+1}} \approx \frac{11y_{n+1} - 18y_n + 9y_{n-1} - 2y_{n-2}}{6h} \quad (\text{A.10})$$

Step 7: General k-th Order Formula The general k-th order BDF formula can be written as:

$$\sum_{j=0}^k \alpha_j y_{n+1-j} = h\beta f(t_{n+1}, y_{n+1}) \quad (\text{A.11})$$

where the coefficients α_j and β are determined by:

$$\sum_{j=0}^k \frac{\alpha_j}{j!} (-j)^m = \begin{cases} 0 & \text{for } m = 0, 1, \dots, k-1 \\ k!\beta & \text{for } m = k \end{cases} \quad (\text{A.12})$$

Error Analysis The local truncation error for a k-th order BDF method is:

$$\text{LTE} = O(h^{k+1}) \quad (\text{A.13})$$

The stability region for BDF methods increases with order k, but methods higher than order 6 are unstable. For practical applications in heat transfer problems, orders 1 to 5 are most commonly used, with order 2 being particularly popular due to its balance of accuracy and stability.

Appendix B

SNOPT Solver Theoretical Foundation

Optimization Problem Formulation:

The general nonlinear optimization problem addressed by SNOPT can be mathematically expressed as:

$$\min_x f(x) \tag{B.1}$$

Subject to:

$$c_L \leq c(x) \leq c_U \tag{B.2}$$

$$x_L \leq x \leq x_U \tag{B.3}$$

Where:

- x is the vector of design variables
- $f(x)$ is the objective function
- $c(x)$ represents the constraint functions
- c_L and c_U are lower and upper bounds of constraints
- x_L and x_U are lower and upper bounds of design variables

Sequential Quadratic Programming (SQP) Methodology:

SNOPT employs the Sequential Quadratic Programming (SQP) method, which can be mathematically described through the following iterative framework:

$$x_{k+1} = x_k + \alpha_k p_k \tag{B.4}$$

Where:

- x_{k+1} is the next iteration point
- x_k is the current iteration point
- α_k is the step length
- p_k is the search direction

The search direction p_k is determined by solving a quadratic subproblem:

$$\min_p \nabla f(x_k)^T p + \frac{1}{2} p^T B_k p \quad (\text{B.5})$$

Subject to:

$$\nabla c_i(x_k)^T p + c_i(x_k) = 0, \quad i \in \text{equality constraints} \quad (\text{B.6})$$

$$\nabla c_j(x_k)^T p \leq 0, \quad j \in \text{inequality constraints} \quad (\text{B.7})$$

Key Algorithmic Components:

1. Lagrangian Function

$$L(x, \lambda) = f(x) + \sum_i \lambda_i c_i(x) \quad (\text{B.8})$$

2. Karush-Kuhn-Tucker (KKT) Conditions

$$\nabla_x L(x, \lambda) = \nabla f(x) + \sum_i \lambda_i \nabla c_i(x) = 0 \quad (\text{B.9})$$

$$c_i(x) = 0 \quad (\text{B.10})$$

$$\lambda_i \geq 0 \quad (\text{B.11})$$

Numerical Implementation Specifics

1. Sparse Matrix Representation:

SNOPT utilizes a specialized approach for handling sparse matrices. The Jacobian matrix $J(x)$ is stored in compressed sparse row (CSR) or compressed sparse column (CSC) format to minimize memory usage and computational complexity.

2. Quasi-Newton Update:

The Hessian matrix is approximated using a quasi-Newton method (typically BFGS update):

$$B_{k+1} = B_k + \frac{y_k y_k^T}{y_k^T s_k} - \frac{B_k s_k s_k^T B_k^T}{s_k^T B_k s_k} \quad (\text{B.12})$$

Where:

- $y_k = \nabla f(x_{k+1}) - \nabla f(x_k)$
- $s_k = x_{k+1} - x_k$

Convergence Criteria

The optimization process terminates when:

1. Gradient Norm Condition:

$$\|\nabla f(x_k)\| \leq \epsilon_1 \quad (\text{B.13})$$

2. Constraint Violation Condition:

$$\|c(x_k)\| \leq \epsilon_2 \quad (\text{B.14})$$

Where ϵ_1 and ϵ_2 are user-defined tolerance levels.

Advantages in Material Property Estimation

For thermal property optimization, SNOPT offers distinct advantages:

1. Handles complex, nonlinear objective functions
2. Manages multiple constraints simultaneously
3. Provides efficient sparse matrix computations
4. Ensures global convergence through sophisticated line search mechanisms

Objective Function Formulation:

$$\text{Objective Function} = \min \sum_{i=1}^N \left| \frac{\text{Experimental Data}_i - \text{Simulated Data}_i}{\text{Experimental Data}_i} \right| \quad (\text{B.15})$$

This formulation allows precise estimation of thermal properties (conductivity, specific heat, density) by minimizing the relative error between experimental and simulated thermal responses.

Appendix C

Fundamental nuclei

By introducing the expansion from Eq. 4.11 into both the first and second terms of Eq. 4.15, and subsequently decomposing the volume integral into separate integrals over the plate's thickness and the plane, we derive:

$$\begin{aligned}
\delta \Theta^T \mathbf{K} \Theta &= \int_{\Omega} \left[\sum_{k=1}^{N_l} \sum_{p=1}^{N_p^k} \int_{h_{p,k}} \delta \vartheta^{p,kT} \boldsymbol{\kappa}^{p,k} \vartheta^{p,k} dz \right] dx dy \\
&= \int_{\Omega} \left[\sum_{k=1}^{N_l} \sum_{p=1}^{N_p^k} \int_{h_{p,k}} \left(\delta \vartheta_x^{p,k} \kappa_{11}^{p,k} \vartheta_x^{p,k} + \delta \vartheta_y^{p,k} \kappa_{22}^{p,k} \vartheta_y^{p,k} + \delta \vartheta_z^{p,k} \kappa_{33}^{p,k} \vartheta_z^{p,k} \right) dz \right] dx dy \\
&= \delta \theta_{\mu_{\theta}^k i}^{p,k} \left[\int_{\Omega} N_{i,x} \left(\sum_{k=1}^{N_l} \sum_{p=1}^{N_p^k} \kappa_{11}^{p,k} \int_{h_{p,k}} F_{\mu_{\theta}^k} F_{\tau_{\theta}^k} dz \right) N_{j,x} dx dy \right] \theta_{\tau_{\theta}^k j}^{p,k} \\
&\quad + \delta \theta_{\mu_{\theta}^k i}^{p,k} \left[\int_{\Omega} N_{i,y} \left(\sum_{k=1}^{N_l} \sum_{p=1}^{N_p^k} \kappa_{22}^{p,k} \int_{h_{p,k}} F_{\mu_{\theta}^k} F_{\tau_{\theta}^k} dz \right) N_{j,y} dx dy \right] \theta_{\tau_{\theta}^k j}^{p,k} \\
&\quad + \delta \theta_{\mu_{\theta}^k i}^{p,k} \left[\int_{\Omega} N_i \left(\sum_{k=1}^{N_l} \sum_{p=1}^{N_p^k} \kappa_{33}^{p,k} \int_{h_{p,k}} F_{\mu_{\theta}^k, z} F_{\tau_{\theta}^k, z} dz \right) N_j dx dy \right] \theta_{\tau_{\theta}^k j}^{p,k}
\end{aligned} \tag{C.1}$$

$$\begin{aligned}
\delta \Theta^T \mathbf{M} \dot{\Theta} &= \int_{\Omega} \left[\sum_{k=1}^{N_l} \sum_{p=1}^{N_p^k} \int_{h_{p,k}} \delta \boldsymbol{\theta}^{p,kT} \rho^{p,k} c_p^{p,k} \dot{\boldsymbol{\theta}}^{p,k} dz \right] dx dy \\
&= \delta \theta_{\mu_{\theta}^k i}^{p,k} \left[\int_{\Omega} N_i \left(\sum_{k=1}^{N_l} \sum_{p=1}^{N_p^k} \rho^{p,k} c_p^{p,k} \int_{h_{p,k}} F_{\mu_{\theta}^k} F_{\tau_{\theta}^k} dz \right) N_j dx dy \right] \theta_{\tau_{\theta}^k j}^{p,k}
\end{aligned} \tag{C.2}$$

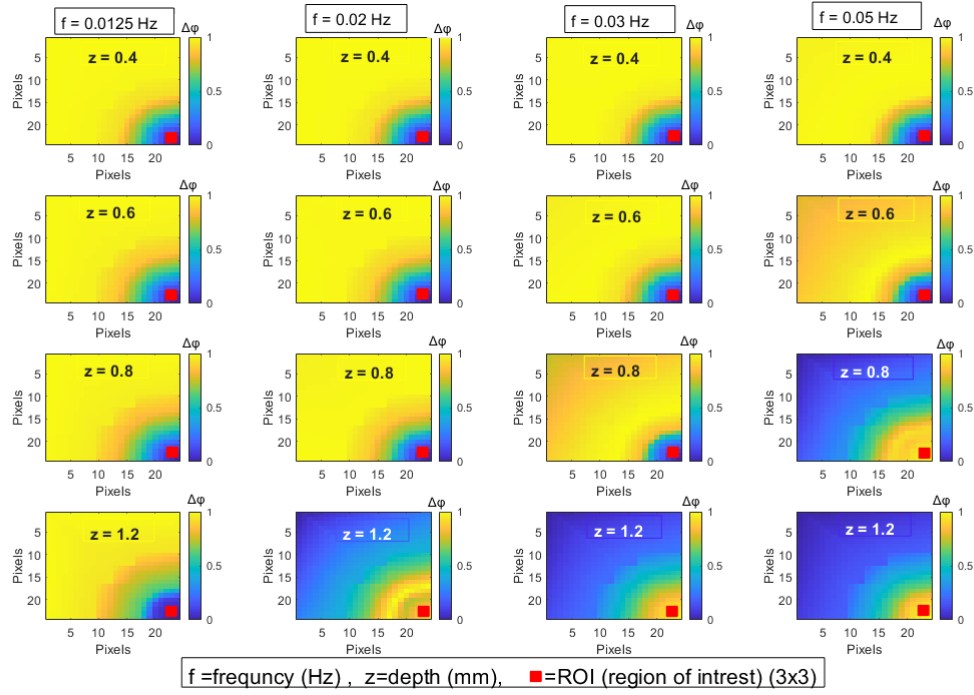
Upon introducing thickness and in-plane integrals according to the notation given in [191], which is omitted here for the sake of brevity, the explicit expressions for the thermal “stiffness” and “mass” matrices are given below where the symbol $\otimes$ denotes the Kronecker product.

$$\mathbf{K} = Z_{\theta\theta}^{\kappa_{11}} \otimes I^{1010} + Z_{\theta\theta}^{\kappa_{22}} \otimes I^{0101} + Z_{\theta,z\theta,z}^{\kappa_{33}} \otimes I^{0000}; \quad \mathbf{M} = Z_{\theta\theta}^{\rho c} \otimes I^{0000} \tag{C.3}$$

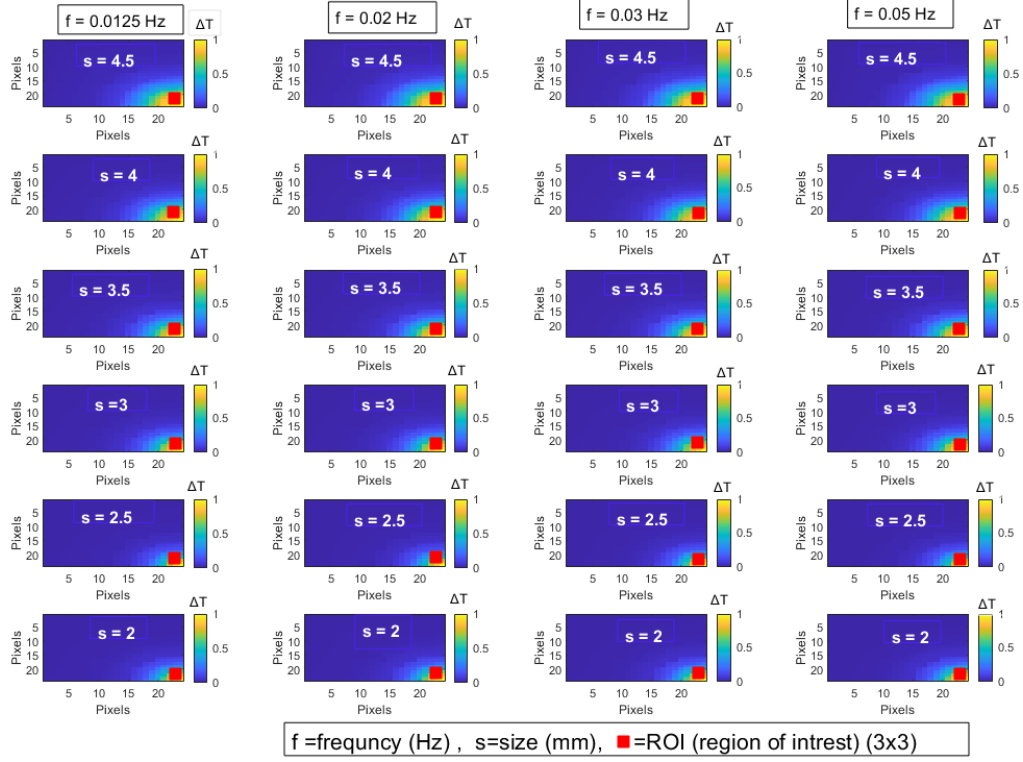
Appendix D

Phase and Thermal Contrast

D.1 Normalized Phase Contrast as a Function of Varying Depth and Frequencies (Constant Defect Size)



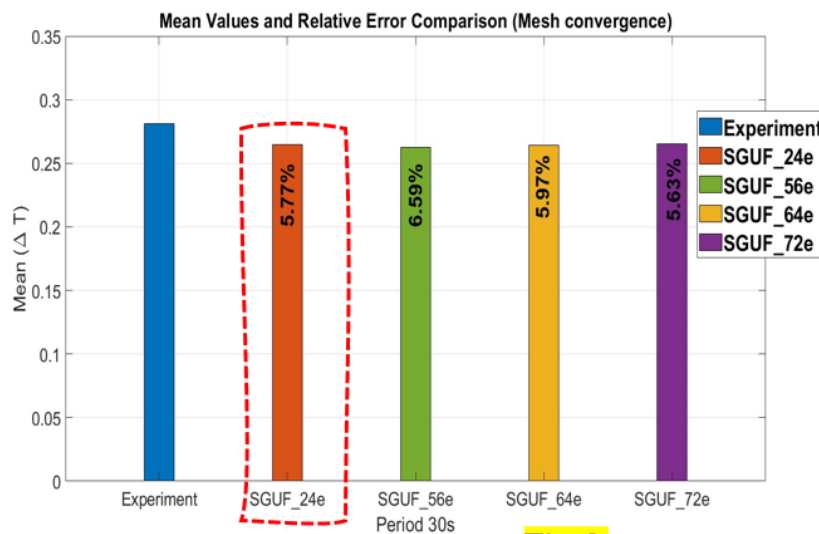
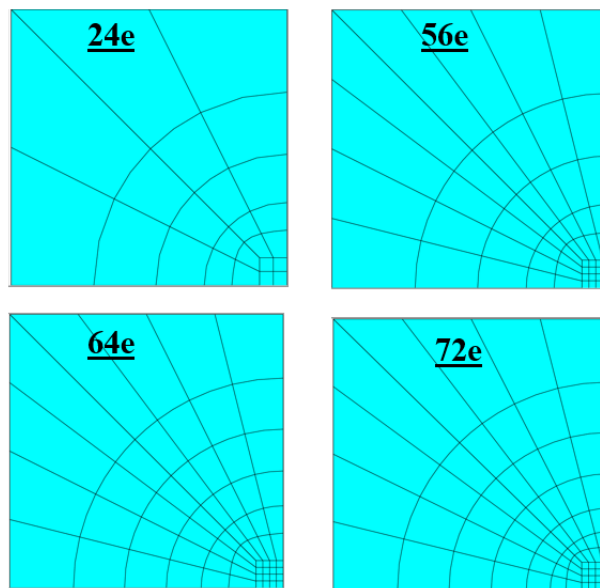
D.2 Normalized Thermal Contrast as a Function of Varying Size and Frequencies (Constant Defect Depth)



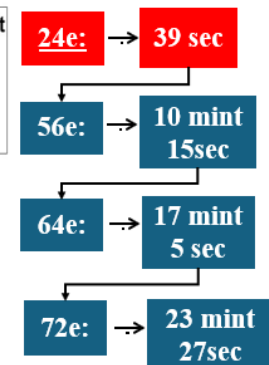
Appendix E

Convergence Analysis

E.1 Mesh Convergence

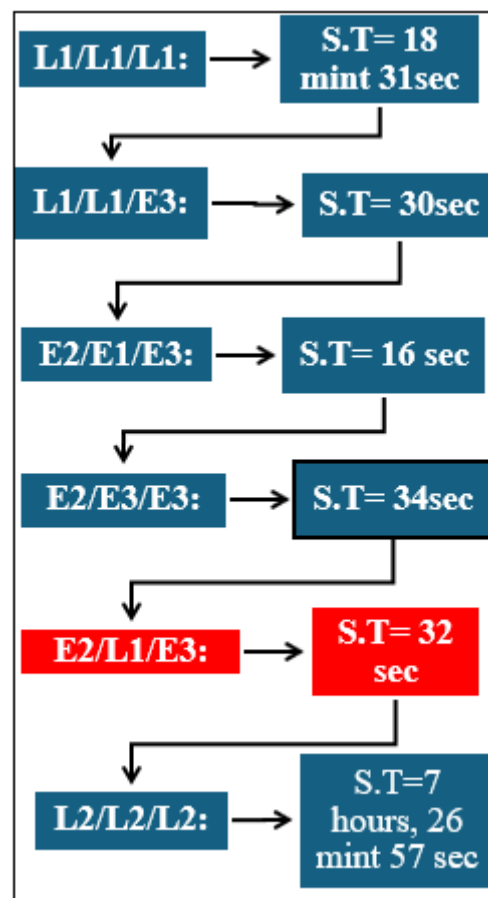
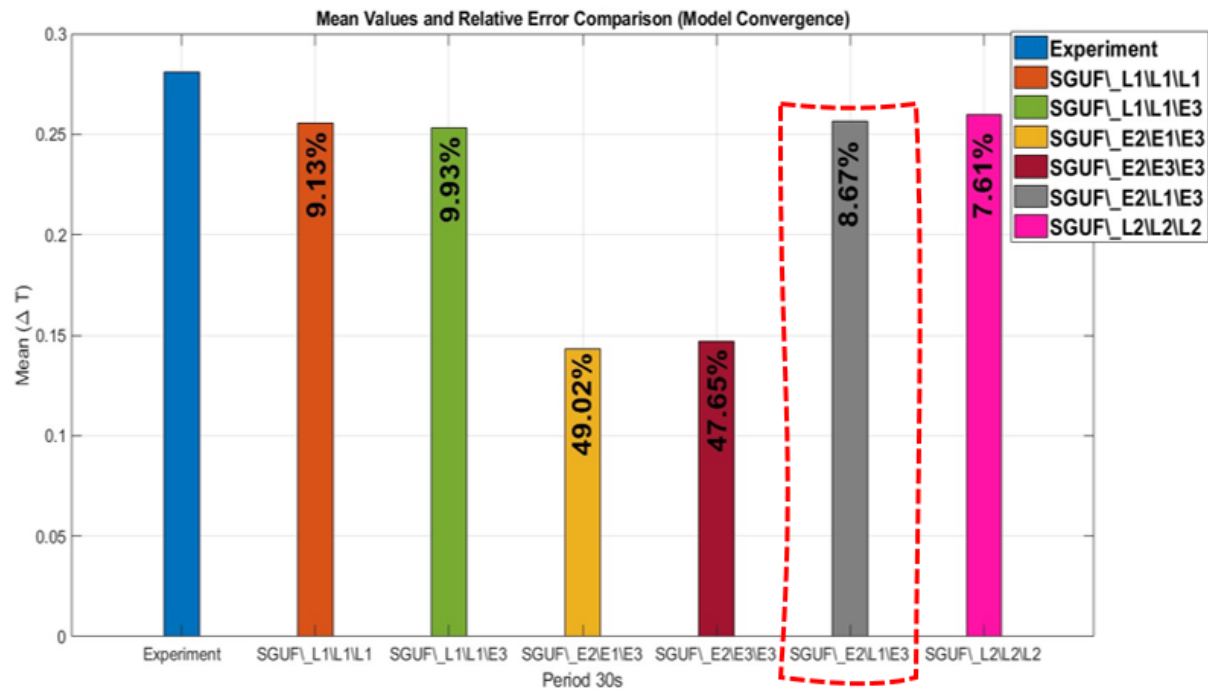


Computational cost:



Final:
Mesh 24 element

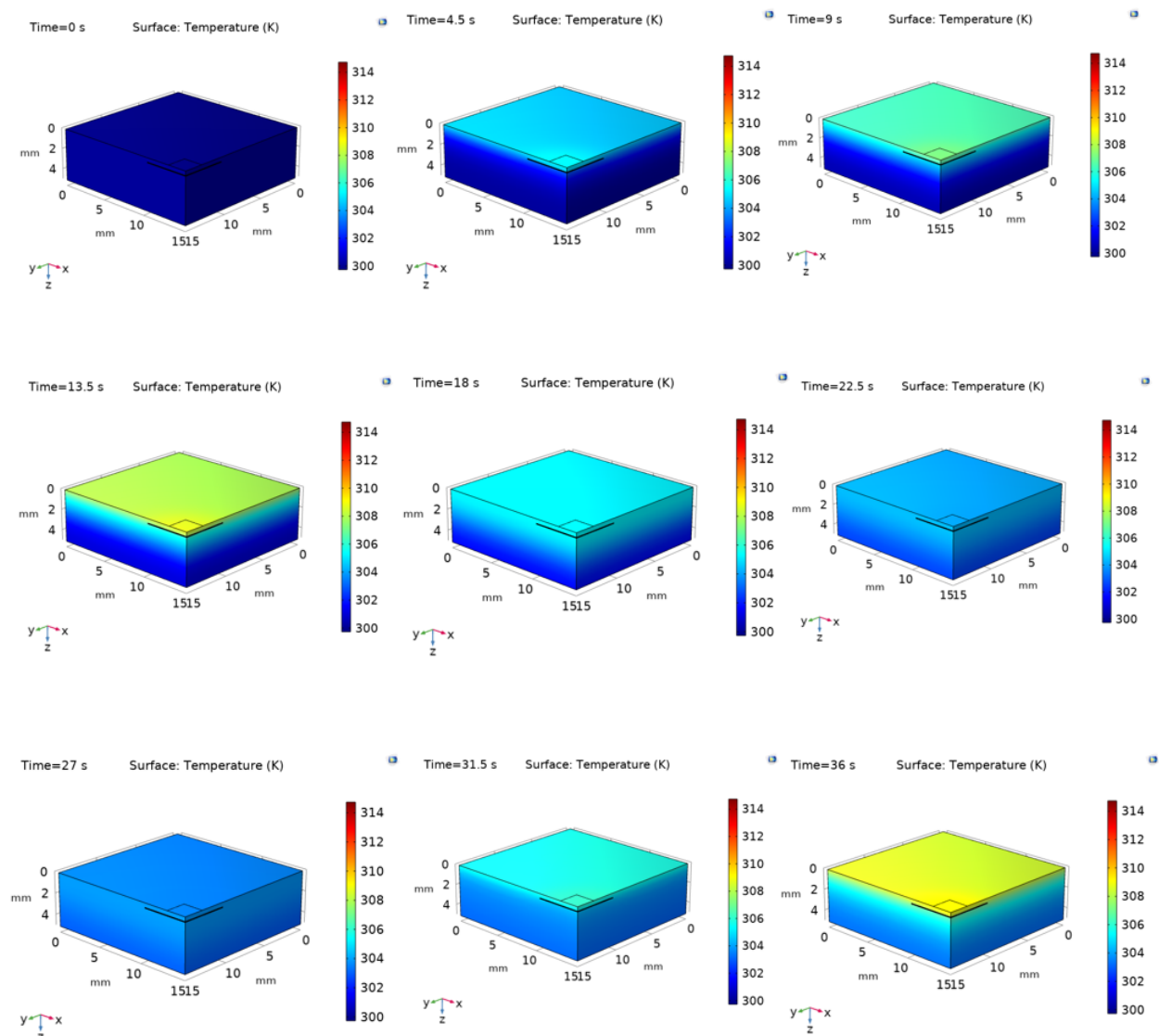
E.2 Model Convergence



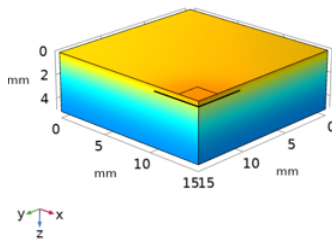
Appendix F

Thermal Maps for Different Time Step

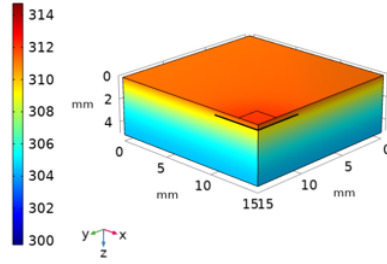
F.1 COMSOL Multiphysics Maps



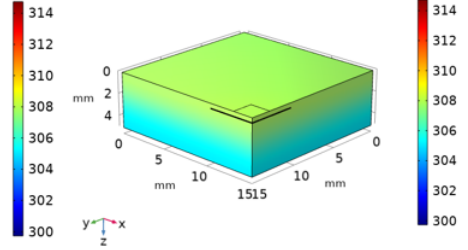
Time=40.5 s Surface: Temperature (K)



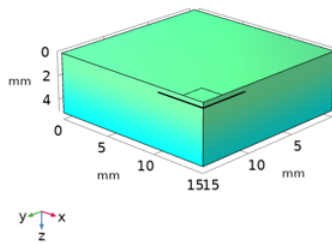
Time=45 s Surface: Temperature (K)



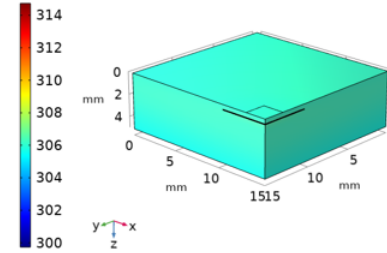
Time=49.5 s Surface: Temperature (K)



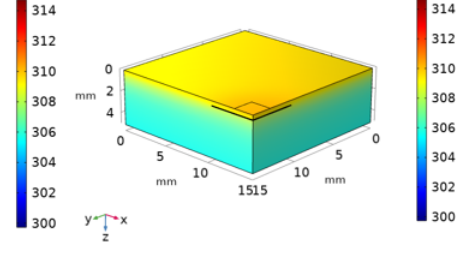
Time=54 s Surface: Temperature (K)



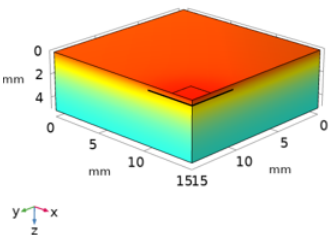
Time=58.5 s Surface: Temperature (K)



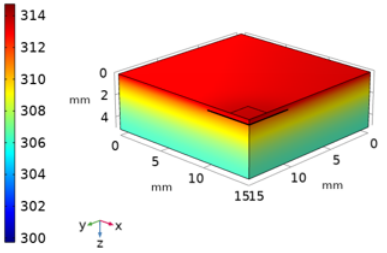
Time=63 s Surface: Temperature (K)



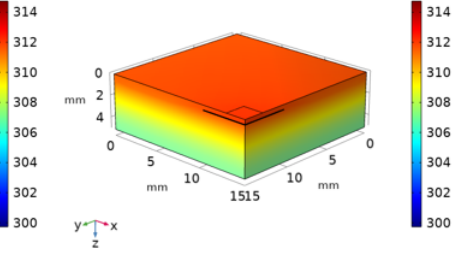
Time=67.5 s Surface: Temperature (K)



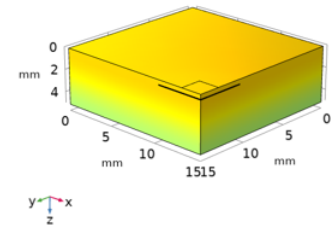
Time=72 s Surface: Temperature (K)



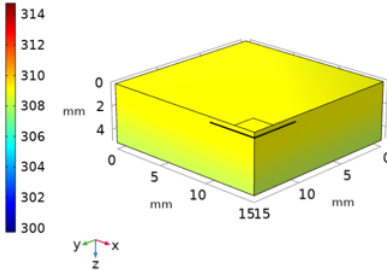
Time=76.5 s Surface: Temperature (K)



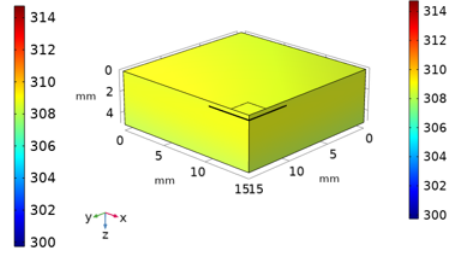
Time=81 s Surface: Temperature (K)



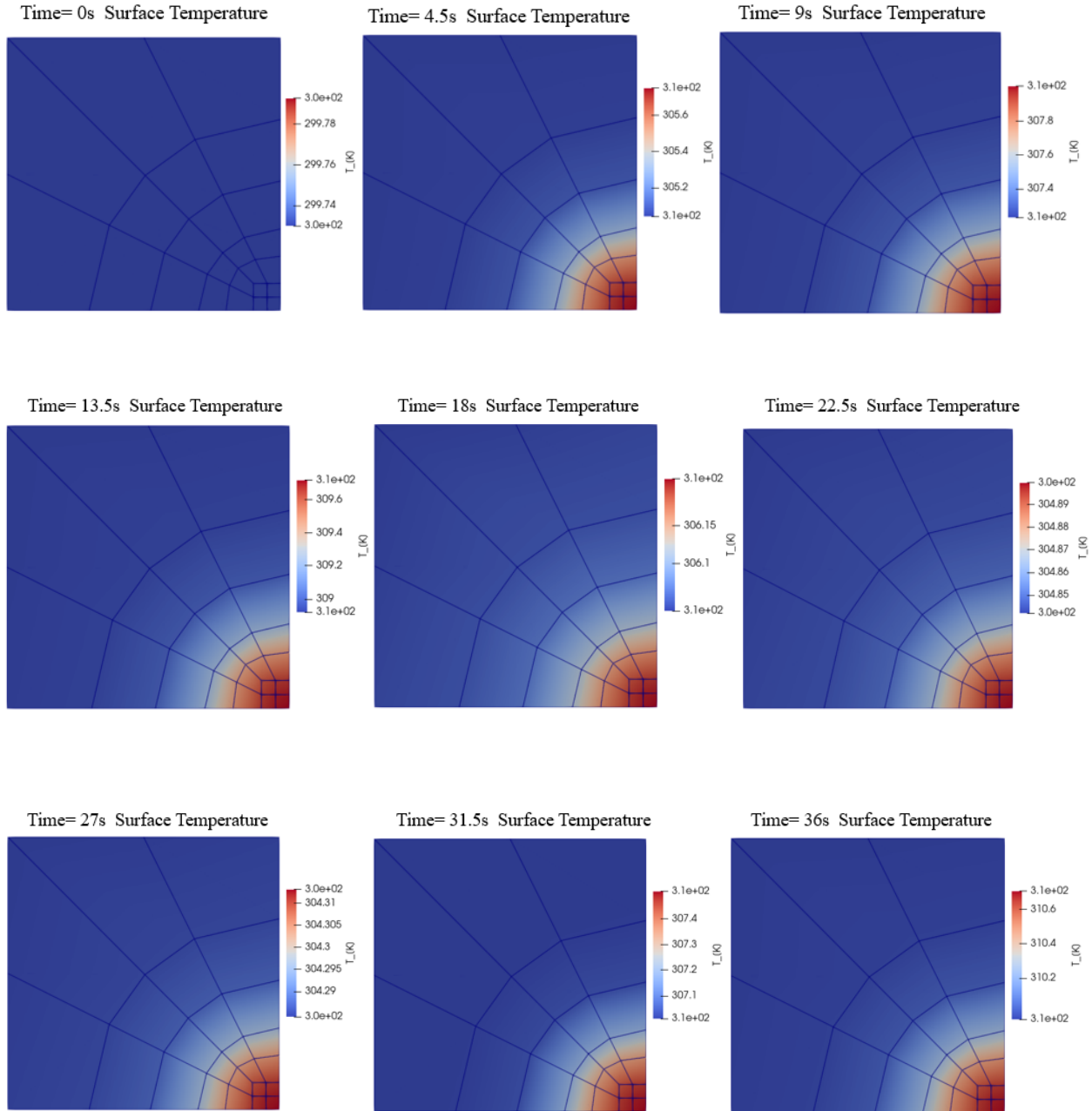
Time=85.5 s Surface: Temperature (K)



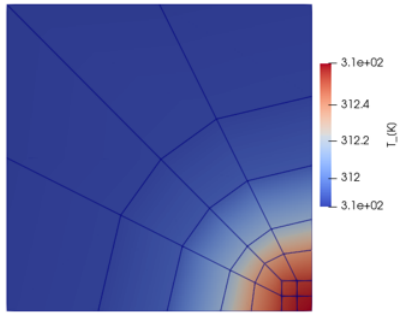
Time=90 s Surface: Temperature (K)



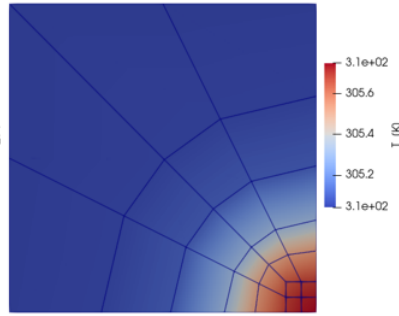
F.2 Sublimate Generalized Unified Formulation (SGUF) Maps



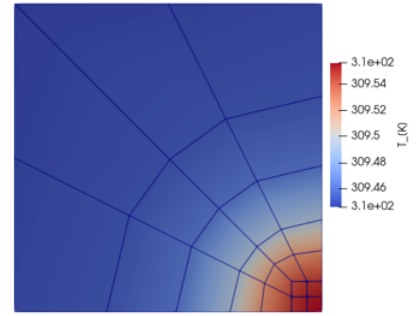
Time= 40.5s Surface Temperature



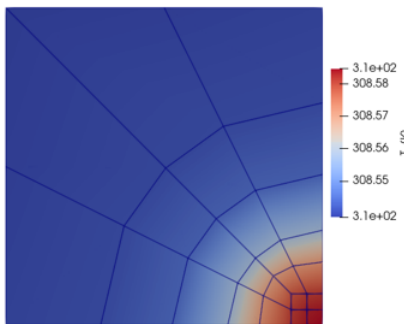
Time= 45s Surface Temperature



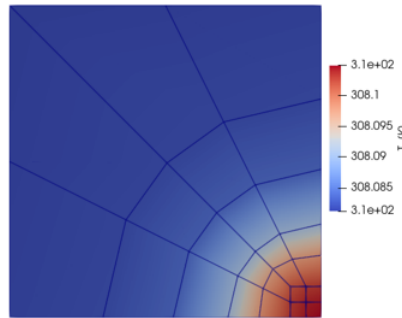
Time= 49.5s Surface Temperature



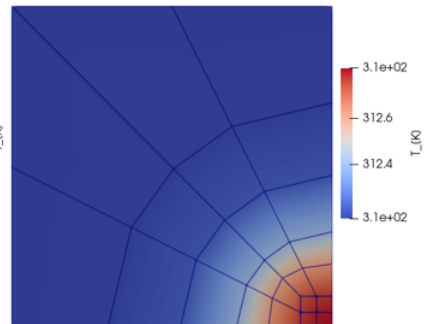
Time= 54s Surface Temperature



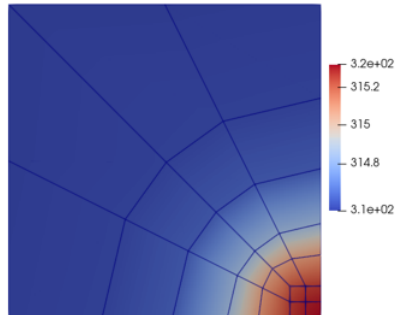
Time= 58.5s Surface Temperature



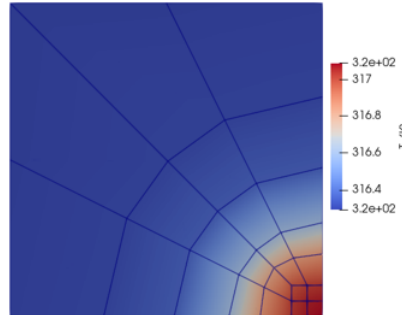
Time= 63s Surface Temperature



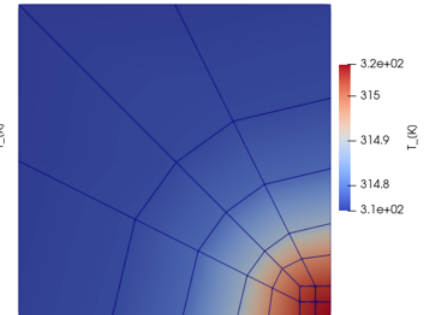
Time= 67.5s Surface Temperature



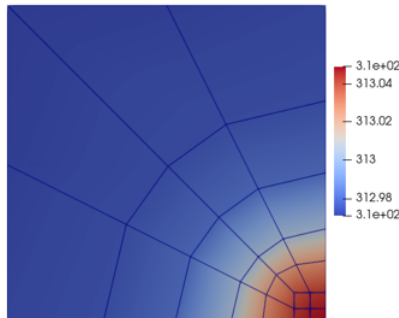
Time= 72s Surface Temperature



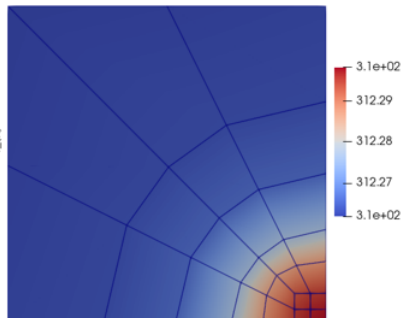
Time= 76.5s Surface Temperature



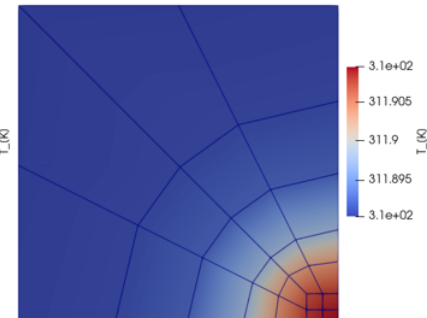
Time= 81s Surface Temperature



Time= 85.5s Surface Temperature

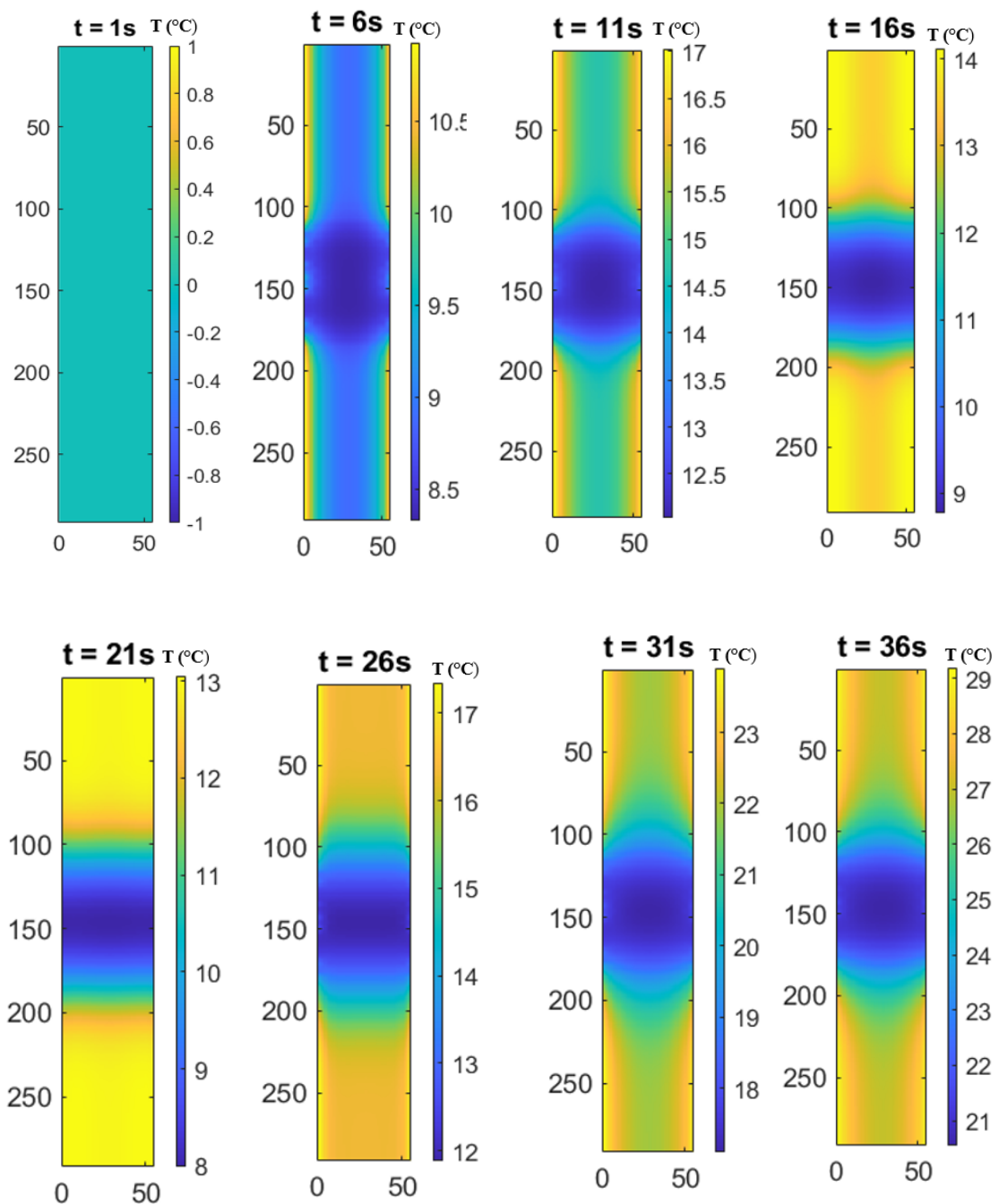


Time= 90s Surface Temperature

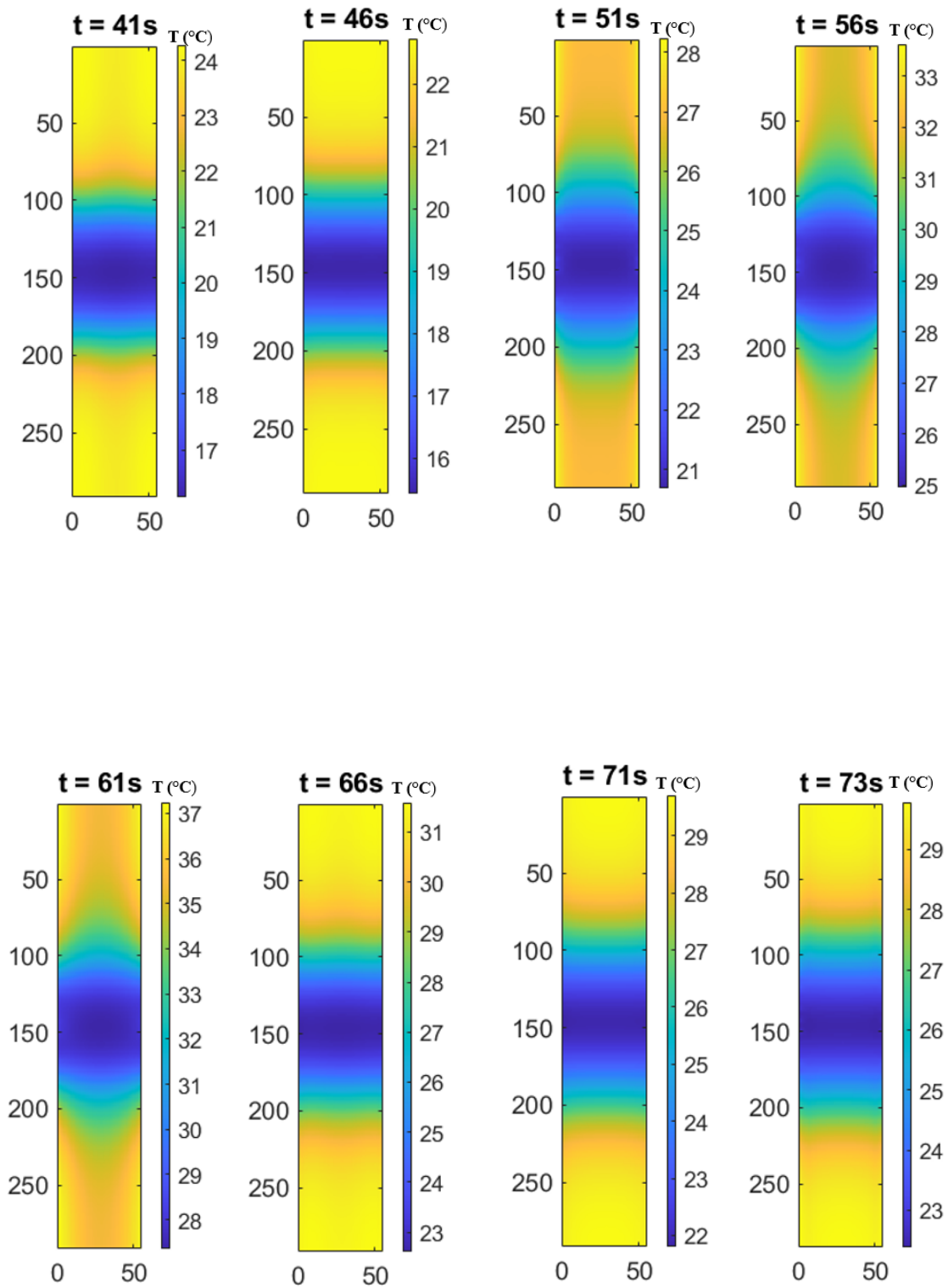


Appendix G

Thermal maps for T-joint structure simulation using MUL2 Code



Thermal maps for T-joint structure simulation using MUL2 Code



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