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Modeling equity in the transportation Network Design Problem: new paradigms and challenges

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Doctor of Philosophy in Environmental and
Building Risk and Development

Coordinator: Prof. Michele Mossa

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2017

Modeling equity in the transportation Network Design Problem: New paradigms and challenges

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La modellazione dell'equità nel problema di progettazione delle reti di trasporto: Nuovi paradigmi e nuove sfide

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EXTENDED ABSTRACT

Transport related equity refers to the distribution of impacts (benefits and/or costs) throughout all sectors of society, and whether that distribution is considered fair and appropriate. Horizontal equity concerns distribution among individuals or groups considered equal in ability and necessities, whereas vertical equity should be considered in situations with different levels of needs (income, social class or mobility ability).

This dissertation deals with the network design problem, looking at how to make it equitable from a spatial and social point of view. It wants to propose a methodology to design transportation systems meeting the needs of communities, supporting fair accessibility, incorporating the equity concepts since the earliest stage of planning.

Sequentially, three main frameworks have been taken into account. The first part considers the presence of private vehicles on the network only; the second one, according to a multimodal vision, considers the coexistence of public transport alongside the private one. These two first formulations cope with the uncertainties related to the assessment of the problem, proposing and comparing two different specifications (rigid/crisp and flexible/fuzzy) of the same model, and enclosing a sensitivity analysis and a numerical application to highlight the usefulness and investigate the performances and robustness of the proposed method.

The third and last approach shows a shift towards the public transportation planning process, dealing with equity considerations applied to the transit network design problem, aiming at achieving a spatially and socially balanced service. Accordingly, a novel comprehensive equity indicator is also specified. Through a further sensitivity analysis, it has been investigated how the costs of the system vary according to the pursued level of equity, finding that higher overall costs must be born if more equitable service provision has to be achieved. A conclusive application to a real case study is presented, and future researches and suggestions for further studies are proposed.

keywords

Horizontal and vertical equity; equity indicator specification; network design problem; multi-modality; equitable transit planning.

SOMMARIO

L'equità relativa al settore dei trasporti riguarda la distribuzione di impatti (benefici e/o costi) a tutti i settori della società, ed indaga se tale distribuzione possa essere reputata giusta ed appropriata. L'equità orizzontale si riferisce alla distribuzione tra individui o gruppi considerati uguali per abilità e necessità, mentre l'equità verticale dovrebbe essere considerata in situazioni in cui sono individuabili livelli di bisogni differenti (quali fasce di reddito, classi sociali o diverse esigenze di mobilità).

Questa tesi affronta il problema della progettazione delle reti di trasporto (network design), cercando di capire come renderlo equo da un punto di vista sia spaziale che sociale. Vuole suggerire una metodologia per progettare i sistemi di trasporto venendo incontro ai bisogni delle comunità, supportando un adeguato livello di accessibilità, ed incorporando i concetti di equità sin dalle prime fasi della pianificazione secondo un approccio quantitativo.

In sequenza, sono stati affrontati tre differenti contesti principali. Il primo, considera il caso della rete stradale, con la presenza sulla rete di soli veicoli privati; un secondo, seguendo una visione multimodale, affronta il problema della progettazione tenendo conto della coesistenza sulla rete del trasporto pubblico e di quello privato. Queste due prime formulazioni del problema di network design fanno fronte alle incertezze intrinseche nella valutazione di un problema di questo tipo, e propongono, confrontan-

dole, due specificazioni diverse (a soglie rigide o flessibili/fuzzy) dello stesso modello, e presentano un'analisi di sensitività ed un'applicazione numerica per evidenziare l'utilità, le prestazioni e la robustezza del metodo proposto.

Il terzo ed ultimo modello si sposta in modo più deciso verso il processo di pianificazione del trasporto pubblico a livello tattico-operativo, dato che si occupa di applicare considerazioni di equità al problema di transit network design, puntando ad ottenere un servizio spazialmente e socialmente bilanciato. Allo scopo viene specificato un nuovo indicatore globale di equità. Attraverso una ulteriore analisi di sensitività, viene investigato come i costi del sistema variano a seconda del livello di equità raggiunto, riscontrando che si devono sostenere costi generalmente più alti nel momento in cui si vuole garantire un servizio di trasporto più equo. Un'applicazione conclusiva ad un caso di studio reale viene infine proposta, seguita da spunti per ricerche future e suggerimenti per ulteriori studi.

parole chiave

Equità orizzontale e verticale; Indicatori di equità; problema di progettazione delle reti di trasporto; multi-modalità; pianificazione equa del trasporto pubblico.

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1.0 INTRODUCTION

The role of transportation in human life is significant: advances in this field shape the life of individuals and the organization of societies, crucially affecting the development of civilizations. The increase in world population and the diffusion of western development models are generating a rise in demand for transportation. National and local governments must take decisions concerning transport networks in a careful way, above all in urban contexts, to grant mobility and, at the same time, reduce and mitigate mobility-related problems such as congestion, air pollution, noise, accidents, and segregation of vulnerable categories.

This dissertation focuses on the concept of equity, that needs to be applied to transportation networks, both public and private. The aim is to incorporate this idea of fairness since the planning stages, trying to achieve a system that could better cope with users' needs, both from a spatial and social point of view. In the next subsections, the problem will be more extensively introduced, and the thesis main objectives clarified.

1.1 PROBLEM STATEMENT

“Transportation equity is a civil and human rights priority. Access to affordable and reliable transportation widens opportunity and is essential to addressing poverty, unemployment, and other equal opportunity goals such as access to good schools and health care services.[...] Providing equal access to transportation means

providing all individuals with an equal opportunity to succeed” (The Leadership Conference on Civil and Human Rights 2015).

Although the relevance of transportation equity has been clearly highlighted, it happens that most of the traditional approaches that it is possible to find in the pertinent literature reviews often neglect equity goals. This is true for network design problems related to both private and public sector. Considering the societal function of transportation systems, however, it is of paramount importance that the planning stage of networks, even at operational level, is addressed to the research of solutions in which the outcomes of the design (costs and/or benefit) are distributed as much as possible among the potential users or classes of users.

That is the reason that leads this dissertation to deal with the concept of equity, proposing a conceptual framework that allows to incorporate it within the transportation planning stage. Sequentially, the problem has been discussed at first taking into account only the presence of private vehicles; after that, considering the public transit alongside the private transport, in a multimodal context; finally, shifting completely towards an equity-based transit network design problem. As a matter of fact, it has been largely recognized that the public transportation system is crucial in any society since it can provide people with mobility and access to employment, education, retail, health and recreational facilities, as well as community facilities, bridging the mobility gap between captive and choice riders. Consequently, the last section of this thesis remarks its importance, emphasizing that a good balance between the spatial and the social distribution of the service has to be achieved to grant a suitable access to everybody.

1.2 GOALS AND TASKS

Starting from the concept of transportation equity, and bearing in mind that it has many facets according to the involved aspects, the aim of this dissertation is to include it in the formulation of network design problems. Therefore, at first, an equity

indicator has to be defined, in order to be able to specify it in form of constraint and incorporate it into the optimization model that allows to efficiently and fairly plan a transport system.

Therefore, thanks to sensitivity analysis and applications to case studies, it could be possible to study which is the relation between the degree of transportation equity reached on a given network, and the other objectives – overall, the incurred costs that both users and operators have to face. In conclusion, it becomes feasible to actually understand if the integration of equity principles since the planning stage of a transportation system could be useful, and to what extent.

1.3 THESIS STRUCTURE

The present work consists of three main sections.

- *Chapter 2* – It offers an extensive literature review. The equity concept is introduced, defining the major categories of transportation equity and the variables involved. Following, the definition of Network Design Problem: its general formulation, the most common solution techniques, what happens when uncertainties are implied and how equity has been defined in this framework. Next, the transit planning process is described, along with its main objectives, constraints and solution techniques, and with the transit equity indicators that it has been possible to find in the past works.
- *Chapter 3* – Two model formulations of an equity-based network design problem are presented, accompanied by a different equity constraint specification. The first one focuses on private transport, proposes a method to deal with uncertainties and provides both a sensitivity analysis and a numerical application. The second multi-modal formulation takes into account the presence of public transit alongside private vehicles and compares two specifications of the problem using multi-criteria decision techniques.

- *Chapter 4* – A model able to incorporate equity in the Transit Network Design Problem is presented. A transit equity constraint is specified, and the suggested procedure is applied at first to a test network, to conduct a sensitivity analysis; afterward to a real case study, comparing the current status of the transit network with the one suggested by the provided methodology.

Conclusions and further researches end the dissertation, remarking the importance of the work that it has been done and suggesting new ways to continue and broaden the study.

2.0 LITERATURE REVIEW

This dissertation focuses on the incorporation of equity in transportation networks. The review starts with an overall introduction to the equity concept, going on with the explanation of the Network Design Problem (NDP) and the Transit Network Design Problem (TNDP), carefully examining how previous studies have embedded fairness considerations into planning and design of transport networks.

2.1 EQUITY CONCEPT

Equity is a complex and multiform subject, whose definition is not straightforward. It can be defined along many facets such as justice, rights, treatment of equals, capability, opportunities, resources, wealth, primary goods, income, welfare, utility and so on (Sen 1992 and 1997). However, there is a seemingly endless debate about the rules used to determine when equity is obtained (Marsh and Schilling 1994).

This subject area has been studied since at least the time of Aristotle, and it is relevant to many disciplines and fields, although it appears to be more developed in some of them (as health and education) than others (Rock et al. 2014).

Equity, like the related concepts of justice, fairness, and right, is not a simple thing. Different people have different concepts of equity, but the aspect that matters will depend very much on the particular context and circumstances (Langmyhr 1997). The

importance of taking into account such aspects derives from various considerations. In general, if users perceive a substantial equity in their treatment in the fruition of a service, they will be more satisfied. In addition, when facilities are considered “undesirable”, an equitable distribution of the risk and/or disadvantage due to their locations can reduce conflicts among users and can help in accepting possible solutions (Barbati 2012).

For the purposes of this dissertation, transport related equity refers to the *distribution of impacts (benefits and costs)* throughout all sectors of society (Martens 2011; Litman 2016) and whether that distribution is considered fair and appropriate (Litman 2002). However, contrary to “distribution”, “equity” entails a moral judgment: consequently, what constitutes an equitable distribution is difficult to define due to the existence of different social norms (Wee and Geurs 2011).

Although scholars in the area of social justice have historically paid little attention to the field of transport planning, there has been a move within the planning and transport fields in recent years to explore this relationship in greater detail and to provide a more solid theoretical basis for transportation focused equity (see, for example, Martens 2011; Beyazit 2011; Wee and Geurs 2011; Martens et al. 2012).

However, despite the increasing effort to incorporate equity in decision-making models, there is little agreement about the best way to assess equity (Santos et al. 2008). A large number of measures can be found in the literature, but we are still far from a general consensus on the best measure(s) to use in each case.

Early discussions of transport equity revolve around a more economic basis, considering how public transit changed consumer welfare and profit maximization (O’Sullivan and Ralston 1980; Hay 1993). Hodge (1995) provides a larger context for economically based transportation equity, arguing that transportation investments often affect different classes and races disproportionately, where the burden to pay for transportation investments is not always equal to the benefit enjoyed. This is something that should be considered in the planning process. Later, the focus turns towards a more socially oriented consideration of equity, with attention to how public transportation access was distributed amongst captive or low-income riders (Garrett

and Taylor 1999). Parallel to this, much work has been done to analyze how public transport might help in bridging the gap between welfare recipients and job locations (Sawicki and Moody 2000; Blumenberg and Ong 2001; Cervero et al. 2002).

Included in this type of equity analysis is how public transport can contribute to or reduce the incidence of social exclusion (Kenyon et al. 2002; Lucas 2006; Preston and Rajé 2007). Golub (2010) uses a utility function to model welfare impacts for service change scenarios. Bureau and Glachant (2011) measure the distributional effects of changes in transit fares and speed, finding fare reductions resulted in the greatest transit equity for low-income groups in Paris. Delmelle and Casas (2012) evaluate the distribution of transit access among different population groups in Cali, Colombia, finding that the addition of a bus rapid transit trunk line increased the equitable distribution of access to services. Only in a few papers equity has been applied to transit in the sense that it has been used in the rest of the literature (Delbosc and Currie 2011). Few attempts have been made until now to assemble these measures, compare them, and define an appropriate measure(s) for each type of application. One of the rare exceptions (although quite outdated) is Marsh and Shilling (1994), that provide a detailed review of equity measures for public facility planning. Recently, Litman (2002) surveys the available data and existing studies for transportation equity, finding a large variation in the measurement of transportation (mobility versus accessibility), the type of equity considered (horizontal versus vertical), and the measures of effectiveness used to calculate distribution (passenger miles, frequency, cost, etc.).

It is possible to single out three major categories of transportation equity.

- *Horizontal Equity*: also called fairness and egalitarianism, it concerns the distribution of impacts between individuals and groups considered equal in ability and need. According to this definition, equal individuals and groups should receive equal shares of resources, bear equal costs, and in other ways be treated the same. It means that public policies should avoid favoring one individual or group over others and that consumers should “get what they pay for and pay for what they get” from fees and taxes unless a subsidy is specifically justified.

- *Vertical Equity with Regard to Income and Social Class*: also called social justice, environmental justice and social inclusion, this is concerned with the distribution of impacts between individuals and groups that differ in abilities and needs, in this case, of income or social class. By this definition, transport policies are equitable if they favor economically and socially disadvantaged groups, therefore compensating for overall inequities. Policies favoring disadvantaged groups are called progressive, while those that excessively burden disadvantaged people are called regressive. This definition is used to support affordable modes, discounts and special services for economically and socially disadvantaged groups, and efforts to ensure that disadvantaged groups do not bear excessive external costs (pollution, accident risk, financial costs, etc.).

- *Vertical Equity with Regard to Mobility Need and Ability*: this concerns the distribution of impacts between individuals and groups that differ in mobility ability and need, and therefore the degree to which the transportation system meets the needs of travelers with mobility impairments. This definition is used to support universal design (also called accessible and inclusive design), which means that transport facilities and services accommodate all users, including those with special needs.

These different types of equity often overlap or conflict. Therefore, transportation planning often involves making tradeoffs between different equity objectives (Litman 2002). It is, however, important to emphasize that different aspects of equity are important for different groups in society and it is essential to provide measures for the evaluation of their concerns and to reflect their views (Ramjerdi 2005).

Table 1 summarizes the various types, impacts, measurement units and categories to consider in equity (Litman 2016).

Table 1 – Equity Evaluation Variables (Litman 2016).

Types of Equity	Impacts	Measurement Units	Categories of people
Horizontal Equal treatment of equals	Public Facilities and Services Facility planning and design Public funding and subsidies Road space allocation Public involvement	Per capita Per adult Per commuter or peak-period travel Per household Per Unit of Travel Per vehicle mile/km Per passenger-mile/km Per trip	Demographics Age and life cycle stage Household type Race and ethnic group Income class Quintiles Poverty line Lower-income areas Ability People with disabilities Licensed drivers
Vertical With-Respect-To Income And Social Class Transport affordability Housing affordability Impacts on low-income communities Fare structures and discounts Industry employment Service quality in lower income communities	User Costs and Benefits Mobility and accessibility Taxes, fees, and fares Service Quality Quality of various modes Congestion Universal design	Per dollar Per dollar user fees Per dollar of subsidy Cost recovery	Geographic location Jurisdictions Neighborhood and street Urban/suburban/rural Mode and Vehicle Type Pedestrians People with disabilities Cyclists & motorcyclists Motorists Public transit Industry Freight Public transport Auto and fuel industries Trip Type Emergency Commutes Commercial/freight Recreational/tourist
Vertical With-Respect-To Need And Ability Universal design Special mobility services Disabled parking Service quality for non-drivers	External Impacts Congestion Crash risk Pollution Barrier effect Hazardous material and waste Aesthetic impacts Community cohesion Economic Impacts Economic opportunities Employment and business activity Regulation and Enforcement Traffic regulation Regulations and enforcement Regulation of special risks		

Together with the concept of vertical equity, there exists a debate regarding the “*equity of opportunity*” and the “*equity of outcome*” (Wee and Geurs 2011; Litman 2016). There seems to be a general agreement that transport plays a role in achieving “equity of opportunity”: it is a form of vertical equity meaning that disadvantaged people have adequate access to education and employment opportunities. A study conducted by the UK Social Exclusion Unit concludes for example that transport is a significant contributing factor in the exclusion of many low-income groups and communities. It identifies lack of transport as a significant barrier to the take-up of employment for many job seekers and leads to failed health appointments and associated delays in the medical intervention (SEU 2002).

There is less evidence on the “equity of outcome” and the role of transport investments: it is a form of vertical equity implying that society ensures that disadvantaged people actually succeed in, for example, getting an education or a job. There have been, for example, evaluations of the contribution of improved transport services to these outcomes. Some evidence, however, does suggest that some targeted transport initiatives have been successful in enabling people to access new employment opportunities and facilitating other important activities, such as health visits, educational attendance and leisure and social activities (Lucas et al. 2009).

Each form of equity clearly corresponds to diverse objectives, despite all referring to a common term. It would be necessary to define different criteria in order to reflect the different principles and objectives underlying each form.

2.2 NDP: GENERAL FORMULATION AND SOLUTION TECHNIQUES

Planning, design and control issues are traditionally addressed in the Network Design Problem (NDP) (Faharani et al. 2013). The NDP is one of the most popular optimization problems with regard to transportation planning (Kim et al. 2012), extensively studied by engineers, mathematicians, operations research analysts, and plan-

ners. Conventionally, on the basis of its decision variables, NDP can be categorized into three types (Yang and Bell 1998):

- *Discrete Network Design Problem (DNDP)*, in which the decision variables are discrete variables - such as constructing new roads, adding new lanes, determining the directions of one-way streets, and determining the turning restrictions at intersections;
- *Continuous Network Design Problem (CNDP)*, in which the decision variables are continuous variables - such as expanding the capacity of streets, scheduling traffic lights, and determining tolls for some specific streets;
- *Mixed Network Design Problem (MNDP)*, which contains a combination of continuous and discrete decisions.

In the NDP decisions are made on the basis of an objective function, which can concern the minimization of costs and/or the maximization of benefits (Barbati 2012). The objective function can be related to an efficiency measure (i.e. total network cost), or to an efficacy measure concerning demand satisfaction aspects (i.e. cost, accessibility).

The models able to define the transportation network layout are named Network Design models (Cascetta 2009). Most of the NDP models in the literature have been specified as deterministic problems where all the relevant inputs are assumed to be known with certainty.

The NDP is generally formulated as a bi-level optimization problem to reflect the different aims of the two stakeholders, who are the network users and the planner. The network users are free to choose their routes such that their individual travel costs are minimized, whereas the planner aims to make the best use of limited resources to optimize network performance (e.g., reducing congestion, minimizing environmental impact, and maximizing throughputs), taking into account users' route choice behavior. The general stochastic bi-level mathematical program can be formulated as follows:

$$(\mathbf{UP}) \underset{\mathbf{u}}{\text{Minimize}} F(\mathbf{u}, \mathbf{v}(\mathbf{u}, \varepsilon)) \quad (1)$$

$$\text{subject to } G(\mathbf{u}, \mathbf{v}(\mathbf{u}, \varepsilon)) \leq 0 \quad (2)$$

where $\mathbf{v}(\mathbf{u}, \varepsilon)$ is implicitly defined by:

$$(\mathbf{LP}) \underset{\mathbf{v}}{\text{Minimize}} f(\mathbf{u}, \mathbf{v}(\mathbf{u}, \varepsilon)) \quad (3)$$

$$\text{subject to } g(\mathbf{u}, \mathbf{v}(\mathbf{u}, \varepsilon)) \leq 0 \quad (4)$$

where F is the objective function and $\mathbf{u}$ is the design vector of the upper-level subprogram (**UP**), G is the constraints set of UP, f is the objective function and $\mathbf{v}(\mathbf{u}, \varepsilon)$ is the decision variable vector of the lower-level subprogram (**LP**) as a function of the design vector $\mathbf{u}$ as well as a random vector ε , and g is the LP constraints set.

The upper-level subprogram describes the leader or planner problem, while the lower-level subprogram represents the follower or user's behavioral problem (Chen et al. 2009).

The *solution methods* for the NDP can be classified into three categories (Farahani et al. 2013):

- exact or mathematical methods;
- heuristics;
- meta-heuristics.

Exact methods such as the branch and bound method, branch-backtrack based algorithms or mathematical programming techniques rely on some mathematical properties to solve the problem to at least local optimality. Although some of them, i.e. mathematical programming techniques for solving linear and nonlinear problems, can be used to solve realistic, large networks efficiently, others for integer programming problems, such as the branch and bound method, are inapplicable for medium or large sized networks because of computational inefficiency.

Heuristics are usually developed from the insight of the problem but there may not be convergence. However, they are always more efficient than the branch and bound method and can be applied to large networks.

Meta-heuristics such as Simulated Annealing (SA) and Genetic Algorithm (GA) are originally based on analogies to physical, chemical, or biological processes. They do not require any mathematical property of the problem to solve and can be used for obtaining nearly global optimal solutions. The computation speed of these methods is much faster than exact methods for integer programming problems in general but at the expense of solution accuracy. GAs and SAs have been mostly used to solve urban transport network design problems. Some other common methods like Tabu Search (TS), Scatter Search (SS), Ant Colony (AC), Ant Systems (AS), and Particle Swarm Optimization (PSO) have been used in a few studies.

Although meta-heuristics approaches do not ensure a globally optimal solution, they could guarantee a local one. Figure 1 (Owais 2015) depicts how the meta-heuristics search space works.

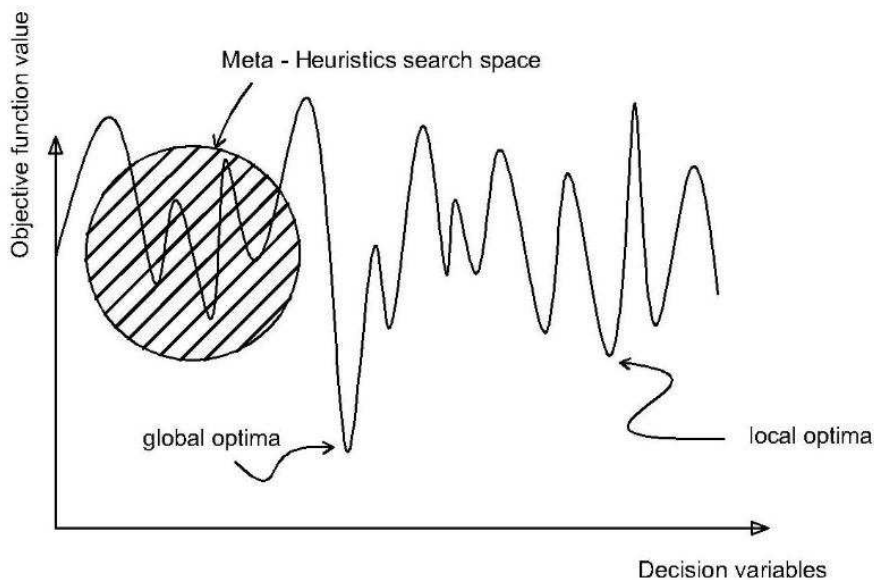


Fig. 1 – Meta-heuristic search space (Owais, 2015).

The literature on Continuous NDP encompasses a variety of solution techniques, given the differentiability of the continuous variables. This diversity is very noticeable in studies from the 1970s and 1980s (Farahani et al. 2013).

For Discrete and Mixed NDP, discrete variables make these problems NP-hard and non-convex so that exact methods such as the branch and bound method cannot solve these problems efficiently. Due to the intrinsic complexity of these problems, the diversity of solution algorithms is somewhat limited. A lot of solution algorithms belong to the meta-heuristics category and the few remaining algorithms are from exact and heuristic type methods.

Exact methods such as branch and bound can be found mostly in DNDP. Examples of the use of such methods include LeBlanc (1975), Drezner and Wesolowsky (1997), Chen and Alfa (1991), and Long et al. (2010). They develop the branch and bound algorithm to directly solve their (upper) problems. LeBlanc (1975) and Long et al. (2010) solve the 24-nodes and 76-links Sioux Falls network, and Chen and Alfa (1991) solve four networks, the largest of them being the 41-nodes and 73-links Winnipeg network. Drezner and Wesolowsky (1997) propose a solution to small and medium-sized networks, the largest being a 40-nodes and 99-links randomly made network.

Some studies transform Road NDP (RNDP) into a single level problem and use exact methods to solve the resultant problem. For example, Gao et al. (2005) transform DNDP to a nonlinear programming problem using the support function concept and solve the nonlinear problem by existing nonlinear programming techniques. Gao et al. (2007) develop a gradient-based method for CNDP using the lower-level problem's optimal value function. Zhang and Gao (2009) reformulate MNDP as CNDP and solve the bi-level model by the use of the optimal value function of the lower-level model in a gradient-based method.

Other studies directly formulate RNDP into a mathematical program with equilibrium constraints and then use exact methods to resolve the resultant problem. Not many studies have adopted this approach. For CNDP, Lo and Szeto (2004, 2009) and Szeto

and Lo (2005, 2006, 2008) adopt the generalized reduced gradient based method to solve the resultant nonlinear program, while for Multi-Modal NDP, Szeto et al. (2010) use the branch and bound algorithm to the mixed-integer problem and the generalized reduced gradient based method was employed to solve the relaxed problem.

The second category is *heuristics*. There exists a wide variety of heuristics, especially for CNDP. Before applying heuristics, a number of studies use the indirect approach, in which the proposed bi-level model is first transformed into a single level optimization model or a mathematical program with equilibrium constraints, and then the resultant model is solved. For instance, Steenbrink (1974b) approximates user optimal flows with system optimal flows and solved DNDP using the iterative decomposition method. Abdulaal and LeBlanc (1979) reformulate their problem as an unconstrained optimization and solve it by a direct search method. Poorzahedy and Turnquist (1982) approximate the bi-level problem to a single level problem, using the objective function of the lower-level problem as the objective function, and all constraints of upper and lower-level problems as the constraints set, and then use two branch-backtrack based algorithms to solve the Sioux Falls network. Marcotte (1983) transforms CNDP into a single level equivalent differentiable optimization problem and solves it using a constraint accumulation algorithm. LeBlanc and Boyce (1986) reformulate CNDP into a single level linear model with combined lower and upper-level objective functions and solved it using Bard's method. Marcotte (1986) and Marcotte and Marquis (1992) both transform the user equilibrium conditions into variational inequalities and develop heuristics for CNDP based on the iterative optimization assignment method. Davis (1994) transforms CNDP into a single level problem by including constraints to represent the stochastic user equilibrium (SUE) condition and solves it using sequential quadratic programming and generalized reduced gradient methods. Meng et al. (2001) develop an augmented Lagrangian method in which the deterministic user equilibrium (DUE) conditions are represented by a single constraint in terms of the marginal function. Finally, Wang and Lo (2010) transformed CNDP into a mixed integer linear program and solved it using CPLEX (an optimization software package).

The other group of NDP studies adopts heuristics to directly solve the bi-level model of the problem. They mostly use descent search methods and exploit the derivative information of the implicit response function of $\mathbf{v}(\mathbf{u}, \epsilon)$ that can be obtained from the lower-level problem. Most of these studies are of the CNDP type. For example, Suh and Kim (1992) use a bi-level descent algorithm based on variational inequality sensitivity analysis. Yang (1997) applies the sensitivity analysis based method to set road tolls. Ziyou and Yifan (2002) use the sensitivity analysis based method to determine signal setting and link expansion. To solve CNDP, Chiou (2005) employs four gradient-based methods, namely Rosen's gradient projection method, the conjugate gradient projection method, the quasi-Newton projection method, and Rosen's gradient projection method with the techniques of parallel tangents (PARTAN), with the use of derivative information. Chiou (2008) proposes a hybrid approach to iteratively solve the two bi-level models (one for reserved capacity maximization and one for delay minimization) and uses projected quasi-Newton in part of it. Again, the sensitivity analysis based method is also incorporated to find the gradient information.

The third category of solution methods belongs to *meta-heuristics*. In almost all of the proposed meta-heuristics, the lower level problem is solved for each newly generated solution to calculate the objective function value. The application of this type of solution method is prevalently found in DNDP and MNDP as mentioned earlier. Therefore, many studies available in the literature rely on meta-heuristics or their hybrids.

CNDP has the fewest applications of meta-heuristics, mainly adopting SA and GA. The budget limit as the primary constraint of CNDP is considered as the penalty for the objective function by Meng and Yang (2002) and Yang and Wang (2002) in SA and by Mathew and Sharma (2009) in GA.

In contrast, DNDP has a variety of meta-heuristics and their hybrids, and these solution techniques usually handle constraints directly within the algorithmic steps. For DNDP with strategic decisions, AS and its hybrids have been applied. For DNDP with tactical decisions, classical meta-heuristics such as SA, GA, and TS have been proposed for orientation of one-way streets and PSO has been used to solve the lane al-

location problem. In DNDP with strategic and tactical decisions, a number of classical meta-heuristics and two hybrid meta-heuristics of GA have been developed. The construction budget in DNDP is tackled inside the algorithmic steps rather than by penalties in the objective function. For instance, Poorzahedy and Abulghasemi (2005) and Poorzahedy and Rouhani (2007) consider budget feasibility after new solutions are generated by the ants. In Miandoabchi and Farahani (2010), budget feasibility is maintained when the perturbation is performed or it is repaired when generating new solutions (e.g. by a crossover operator in GA). When DNDP includes tactical decisions, some constraints are related to street orientations (e.g. connectivity between origin-destination pairs) or lanes configurations. These are either considered in the move generation phase (e.g. in SA) or are checked after generating new solutions (e.g. in GA) and the infeasible solutions are discarded. Only in the lane allocation problem by Zhang and Gao (2007), the out of bound lane allocations resulting from the solution update by PSO are repaired.

For MNDP, various meta-heuristics such as SA, TS, GA and SS have been used to solve the problems. Signal timing constraints are sometimes imposed in problems with signal setting decisions (e.g. Cantarella et al., 2006; Gallo et al., 2010). Signal timings are obtained before solving the lower level model for each network design scenario.

2.2.1 Network design under uncertainty

A considerable amount of research effort has been devoted to road network design models over the last forty years (see Farahani et al. 2013 recent review). Actually, both data available to analysts and problem constraints can be affected by uncertainty; moreover, within an interval, some values may better satisfy the purpose of the analysts (i.e. in a budget interval constraints the lower values are preferable to the others) (Caggiani and Ottomanelli 2011). Sources of uncertainty exist on the supply side (that affects link travel times or roadway capacity variation), on demand

side (that influences travel demand fluctuation) and on constraints-side (generally defined as decision-maker constraints) (Henn and Ottomanelli 2011). Examples of supply-side uncertainty include weather conditions, traffic accidents, work zones and construction activities, and traffic management and control. Examples of demand-side uncertainty comprehend temporal variation (e.g. time of the day, the day of the week, or seasonal effects), special events, population characteristics (e.g. age, car ownership, and household income), and traveler information as well as model related uncertainties (Caggiani et al. 2013). Examples of constraints-side uncertainty include available budget, the level of reduction of the transport system externalities (e.g. threshold limit values of polluters) or equity thresholds as better explained in the next section. All these factors cannot be measured accurately but can only be roughly estimated. Most of the practice of roadway network design does not take into account the uncertainty issue (Yang and Bell 1998). The reason lies in the lack of suitable reliability and uncertainty analysis for road networks.

The design of a new network facility or the upgrading of existing facilities would require a good understanding of the uncertainty involved, the impact on the system-wide performance and the benefits derived from road improvements to the network users. Thus, it is important to study the uncertainty of road networks such that a cost-effective and equitable design can be implemented to improve its level of performance from the viewpoints of both the planner and the users.

Some recent studies have considered various sources of uncertainty in the transport NDP and proposed different criteria to hedge against the uncertainty (see Chen et al. 2011 for a more comprehensive state-of-the-art review). Basically, it is possible to identify six main categories of models:

- *Expected Value Model*: this is maybe the most commonly used model for handling uncertainty. The main idea is to optimize the expected value of a linear (or additive) system-wide objective function subject to the budget constraint and the limit constraints on the decision variables.

- *Mean-Variance Model*: it is a classical model developed by Markowitz (1927) in the finance area. The basic assumption is that risk is measured by variance, and the decision criteria (or objectives) are used to maximize the expected return and to minimize the variance of return. In transportation, it has been applied to design a reliable/robust network under uncertainty. The mean–variance model involves maximizing the expected profit and minimizing the variance (or standard deviation) of profit. The variance associated with profit is considered as a risk.
- *Chance-constrained Model*: originally developed by Charnes and Cooper (1959), it deals with stochastic decision systems with the assumption that constraints will hold at least α times, where α is referred to the confidence level provided as an appropriate safety margin by the decision-maker. Its focus is on the system's ability to meet the chance constraints (risk measures) with certain reliability under uncertainty.
- *Probability Model*: also known as the dependent chance model in uncertain programming (Liu 1999), it has been recently proposed to solve the stochastic NDP model.
- *Min-Max Model*: optimization of the worst-case performance can be regarded as a min–max model, which is used to identify a robust design plan that minimizes the worst- case total travel time under different future demand scenarios. The min–max model is known to give a very conservative solution (i.e. trades off a significant amount of efficiency for reliability, since it has to consider the worst-case scenario).
- *Alpha-reliable Model*: the alpha-reliable NDP model, proposed by Chen et al. (2007), has the ability to specify a risk control measure through the confidence level α to identify a solution with an acceptable risk without sacrificing too much efficiency. It optimizes the total travel time by considering demand uncertainty for different risk aversion levels.

It is important, however, to highlight that uncertainty can be managed using a soft computing approach, in particular through the use of fuzzy values/constraints (Zimmermann 1996): fuzzy programming appears to be an ideal strategy for obtaining the optimal compromise solution to a multi-objective transportation problem.

Recently, great attention has been given to new paradigms developed in the theoretical framework of Fuzzy Set, in which Fuzzy Logic and Possibility Theory are the mathematical tools most used for solving transportation problems (Teodorovic and Vukadinovic 1998) like a fuzzy optimization (Kikuchi and Kronprasert 2007; Caggiani et al. 2013 and 2014). Only a few authors have studied the opportunity to consider this knowledge together with the transportation Network Design Problem (Das et al. 1999; Mudchanatongsuk et al. 2008; Selim and Ozkarahan 2008; Ghatee and Hashemi 2009a and 2009b, Caggiani and Ottomanelli 2011 and 2014).

2.2.2 Characterizing NDP equity

In the transportation field, until the end of the nineties, equity issues are generally limited to the evaluation of the economic impacts of transportation policies. In most cases, these studies regard the distribution of policy impacts among different social groups in the case of the introduction of road prices in some links of the network (Yang and Zhang 2002; Szeto and Lo 2006).

In 2002 Meng and Yang demonstrate that in the CNDP, with a total network cost reduction objective function, the benefits of a capacity enhancement in some selected links can lead to an increase in travel costs for some Origin-Destination (OD) pairs; since then, the debate of equity issues in transportation network design becomes more intense. To this end, they introduce a parameter capable of measuring the degree of equitability of benefit distribution: it reflects the degree of an equitable reduction of the equilibrium OD travel costs before and after implementing a scenario. Meng and Yang (2002) propose two models for CNDP. The parameter of the first model is a given value selected by a decision-maker; in the second model, instead, it can be treated as a decision variable: thus, the total system cost and the parameter have to be optimized simultaneously.

Yang and Zhang (2002) also observe that for the congestion pricing problem there are significant differences among the benefits of some OD pairs. Thus, in addition to the

equity issues involving social groups, they proposed the consideration of spatial equity in the road pricing problem.

After these studies, some other authors suggest the inclusion of equity concerns in network design problems. Antunes et al. (2003) consider the distribution of accessibility gains across population centers in an accessibility-maximization model. Chen and Yang (2004) include spatial equity as a constraint in the link capacity improvement problem with demand uncertainty. Szeto and Lo (2006) propound the integration of equity in a time-step network design problem. They consider social and user equity for different periods of time. Santos et al. (2008) select three different equity measures, that reflect different perspectives of equity, incorporating them into an accessibility-maximization road network design model.

More recently, Sumalee et al. (2009) suggest an innovative approach for designing a road user charging scheme to meet multiple policy objectives; the objective functions or constraints that are taken into account include social welfare improvement, revenue generation, and distributional equity impact.

Barbati (2012) proposes a multi-objective model in which balancing or equity aspects, i.e. measures of the distribution of distances of users from the path, are considered. In this case she introduces an equity parameter representing the maximum cost or benefit difference between any pair of nodes; adding this constraint to the model it is possible to obtain a formulation that combined efficiency (the minimization of path length), efficacy (the maximization of the accessibility), and the minimization of the inequity, in order to achieve a better distribution of costs or benefits among the users.

Furthermore, Farahani et al. (2013) note that minimizing the total travel time/cost, the most common form of objective function in NDP, may lead to unbalanced congestion levels throughout a network and hence some OD pairs may benefit more than others in term of travel time reduction. In some case, selected OD may even experience an increase in costs. Hence, it is necessary to introduce equity measures in the NDP formulation, to ensure a more equitable distribution of benefits.

As a result of the foregoing, it is possible to assert that in the decision-making process network planners should consider not only the efficiency of the network improvement but also the equity of the improvement. A network improvement plan should not lead any network users to be significantly worse off.

2.3 THE TRANSIT PLANNING PROCESS

The transit planning process encloses every decision taken before the system comes into operation, and it requires the solution of the Transit Network Planning problem (TNP). Due to its complexity, TNP is commonly divided into several sub-problems that embrace *strategical, tactical, and operational decisions* (Desaulniers and Hickman 2007; Ceder 2007), as shown in Figure 2. Strategical decisions are long-term decisions related to the infrastructures of transit networks, including the design of urban routes; tactical decisions are those concerned with the effective utilization of infrastructures and resources of existing transit networks; and operational decisions are short-term decisions, which are mostly related to frequency setting problem (Magnanti and Wong 1984; Guihaire and Hao 2008; Farahani 2013), but also to time schedules, fleet size and number of employees.

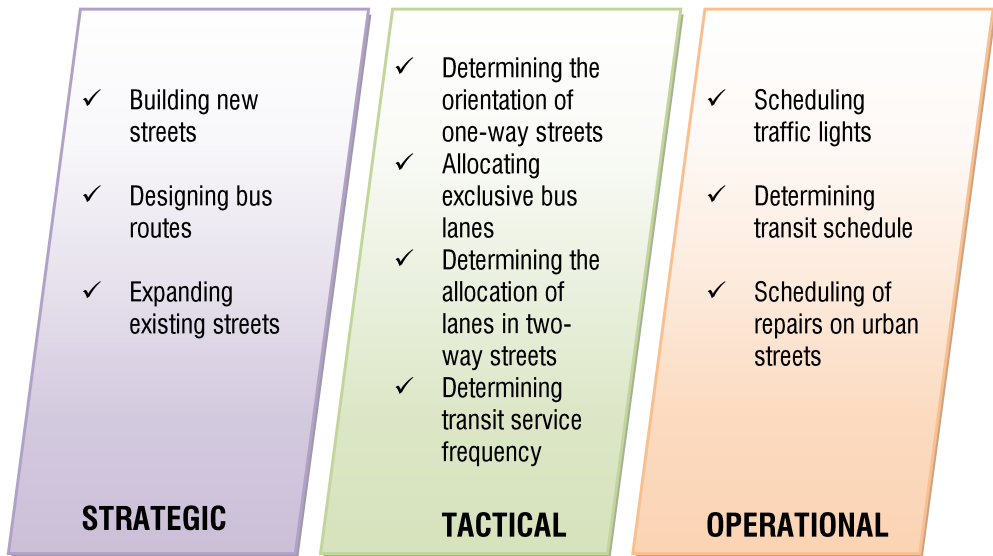


Fig. 2 – Example of decisions in Transit Network Planning problem.

As a result, various sub-problems (i.e., transit network design, frequencies setting, timetabling, vehicle scheduling, crew scheduling and rostering) have been defined over the years so as to solve the planning problem in a sequential manner, although it thereby loses any optimality guarantee. The concern of the dissertation relies on the first two steps of this framework: therefore, the main objective to achieve is the design of the bus lines layout and the determination of their associated operational characteristics. Given a trip demand distribution and an area's topologic characteristics, it involves:

- the definition/arrangement of lines and routes, each being formed of two terminals and a sequence of intermediate bus stops;
- the selection of the frequencies, to the time of the day and the synchronicity of transfers, fleet size, and resources, which should be assigned for each line.

2.3.1 TNDP: objectives, constraints and solution techniques

The Transit Network Design Problem (TNDP) is crucial because the overall costs of the system largely depend on it. As a matter of fact, the most common objective function concerns the *system generalized cost*, that is a weighted sum of user and operator costs: operator costs are usually measured in monetary units, whereas user costs are measured in time spent in the system (minutes). Weights reflect the assumptions on the relative value of operator and user costs and may include the conversion factor between time and money. The goal of TNDP is to define a set of bus routes in a particular area, each route being determined by a sequence of bus stops (Guihaire and Hao 2008). A transit stop is a location at which one enters a higher speed and/or mass transportation system (Murray and Wu 2003). The purposes of several stakeholders can be embedded in TNDP: the users of the transit system, i.e. the passengers; the authorities with responsibility for the system regulations; and the service operator (Ceder and Israeli 1998; Deb et al. 2002). These subjects usually have different goals, and a trade-off between their interests must be achieved.

Main input data are the road network and the origin-destination demand matrix. The *area's topology* can be defined by roads, possible areas for bus stops and transfer zones, and sometimes also the location of depots that serve as extreme terminals.

OD matrices are needed to define a transit network that satisfies as much as possible the community's demand. An OD matrix has the set of stop points as coordinates and contains the number of passengers willing to go from each origin to each destination in a given time period. Rows correspond to the origins and columns to the destinations of the users. The more precise the data, the more adequate the solution.

TNDP may involve multiple and conflicting *objectives and constraints*, that refer typically to resources availability and practical guidelines. Depending on the politics of the transit agency, constraints and objectives might intermingle:

- The *existing network*, if any, can play a role, in the sense that for some (political) reasons, it might be undesirable to disrupt service on already existing lines.

- The *area coverage* measures the percentage of the estimated demand that can be served by public transit. This ratio can be computed in several ways (Spasovic et al. 1993) but usually depends on characteristics such as route length, density, bus stop and route spacing (Murray 2003; Benn 1995). The Federal Transit Administration (1996) stipulates that typical spacing is 180 m in the CBD (Central Business District) and 230 m in urban areas, and increased further in suburban areas to 300 m. Stop spacing standards of 200–600 m are commonly observed by urban transit agencies, but could be as high as 800 m (Demetsky and Lin 1982; Ammons 2001). Some plans (Murray et al. 1998) aim for a 90% ratio.
- Limits are imposed on the distance that one user can cover in the transit network with consideration to one's trip demand. From the users' point of view, the bus network should indeed enable them to travel as directly as possible from their origin to their destination and to walk the shortest distance to reach the first and final bus stops. Different definitions can be used to evaluate this feature. *Route directness* can depend on the route's deviation from a linear path (Benn 1995) considering the additional mileage incurred by a bus trip compared to the same trip by car or another means of transportation. The number of transfers is also a recurrent criterion. Note that to compute *trip directness* for each user, it is necessary to go through a passenger trip assignment process. This consists in assigning routes and transfers to passengers with respect to some objectives such as shortest path or smallest number of transfers (Desaulniers and Hickman 2007).
- The *demand satisfaction* is obviously a crucial issue. When users' origin or destination are too distant from bus stops, or when trip directness is insufficient, the demand can be considered unsatisfied. Note that similarly to trip directness evaluation, computing demand satisfaction requires going through a transit trip assignment process. In a general manner, if a trip requires more than two transfers, it is assumed that the user will switch to another means of transportation.
- A general objective of the operator is to minimize the *total route length and/or cost* in the perspective of reducing the number of vehicle and crew resources needed to

sustain the global transit system. The *number of lines* can alternately be considered. Moreover, routes should neither be too short nor too long for profitability reasons.

- For some reason, transit agencies may want to develop a network with a *particular shape*. Radial, rectangular, grid and triangular (Van Nes 2002) are common shapes used.

Several objective functions, constraints, solution approaches and methodologies, and search algorithms are proposed in the literature (Pattnaik et al. 1998; Fan and Machemehl 2006; Cipriani et al. 2012; Cancela et al. 2015). Solution methodologies for TNDP can be roughly categorized into three categories, namely: mathematical optimization searching for exact solutions, heuristics, and meta-heuristics.

- *Mathematical optimization* methods are limited to idealized small networks (Schöbel 2009), whereas they can hardly deal with medium and large networks.

Murray (2003) studies two variations of the location and density of bus stops problem. In the first part, the relocation of bus stops in an existing network is considered with the objective of minimizing the number of bus stops. A location set covering problem and a maximal coverage location problem are used to model the issue. The second part deals with the optimal location of bus stops to create or extend the network. Given a fixed number of additional bus stops to locate, the objective is to maximize the extra service access provided to non-covered areas. A hybrid set covering problem formulation is proposed to model this second issue. The method is very interesting since it permits to expand both service access and accessibility.

The problem of simultaneous transit line configuration and passenger line assignment is dealt with by Guan et al. (2003). The focus is on large city railways and tests are carried out in Hong Kong city. Their work attempts to jointly model the transit line planning and the passenger transferring process through a linear binary integer program that can be solved by a standard branch and bound method. Given route length bounds, the maximum number of transfers and capacity constraints, the objective is to minimize a function of the total length of transit lines, the total number of transit

lines taken and the total length traveled by passengers. To keep the problem tractable under a mathematical solution method, a pre-computation of a restricted set of possible paths is achieved. However, since at this stage, frequencies and timetables are not known yet, passenger line assignments cannot be very accurate, questioning the whole process. In the previous approaches, it is acknowledged that the size of the instances becomes a serious limitation for real-world problems, where heuristic approaches are then usefully employed.

- *Heuristic approaches* provide feasible and reasonably good solutions (Baaj and Mahmassani 1995), but they do not guarantee global or even local optimal solution. They are largely based on planners' experience and previous knowledge.

Patz (1925) has been probably the first to tackle the TNDP using heuristics. He proposes an iterative procedure to generate a lines network using penalties. Initially, the network contains a line for each OD pair. For each one, a penalty is calculated based on the occupancy level and the number of passengers who need to transfer to complete their journey. Lines are iteratively deleted from the network based on this value and the capacity for passengers thereby freed is reassigned to other lines. The small instance (about ten nodes linked in a specific manner) on which Patz's tests are based permits the method to attain optimality. However, it is not extensible to larger networks.

In Sonntag (1977), the TNDP is approached with a heuristic procedure originally created for railway systems. Starting from a network containing a line for each OD pair (like Patz 1925), lines are iteratively deleted and passengers are reassigned to short paths in terms of travel time. This leads to a network of appropriate size with small average travel time and a small number of transfers.

In opposition to Patz (1925) and Sonntag (1977) and similar methods, Mandl (1979) tackles the TNDP starting with an empty routes network. He proposes a heuristic algorithm to define a transit network given a constant frequency on all bus lines. In a first part, routes on the shortest path that connects a pair of terminals and serve the greatest number of OD pairs are iteratively selected. They can then be adapted to fulfill service coverage and directness objectives. In a second part, the transit network is

also iteratively modified to minimize the total travel time of passengers. The method was applied to a real network in Switzerland, consisting of 15 nodes and 15,570 trip demands per day.

Following the objective of maximizing the number of direct travelers, Pape et al. (1992) present a heuristic method for the TNDP. This constructive heuristic is based on the concept of corelines, which describe lines or parts of lines with a large number of passengers. Corelines are combined in a complete enumeration scheme and the best partial line plan considering the number of lines and the number of direct passengers is generated. This line plan is then extended for service coverage purpose.

- *Meta-heuristics* are approximate methods that implement efficiently iterative mechanisms to explore a large part of the solution space aiming to find the global optimal solution or at least a local one. Examples of meta-heuristics approaches are Genetic Algorithm (GA) (Fan and Machemehl 2011; Szeto and Wu 2011; Cipriani et al. 2012; Chew et al. 2013; Amiripour et al. 2014; Nayeem et al. 2014), Simulated Annealing (Fan and Machemehl 2006; Yan et al. 2013) and Ant Colony Optimization (ACO) (Yang et al. 2007; Yu et al. 2012).

Xiong and Schneider (1992) present an innovative method to select additional routes for an existing network. Their method is based on an improvement from the ordinary genetic algorithm, called the cumulative GA. The principle is to collect all non-dominated solutions throughout the process and return this set instead of the last generation as is commonly done. The second proposal of this paper is to use neural networks instead of a passenger trip assignment algorithm to evaluate fitness functions (here waiting time and costs incurred). This proves to give accurate results and achieve a substantial time gain. The combination of these two improvements creates a very powerful tool, which can provide a set of interesting solutions to the human planner. This work is tested on a benchmark from LeBlanc et al. (1975).

Chakroborty and Dwivedi (2002) also propose a Genetic algorithm-based method. In a first part, an initial set of routes is determined heuristically. Then a process consisting of an evaluation and modification procedure is iterated. A solution is a set of routes and its fitness function is evaluated through five criteria: the average or total in-

vehicle time incurred (including transfer time), the percentage of passengers achieving a direct trip, a trip with one or two transfers and the percentage of unsatisfied demand. If the global quality is not good enough, routes or parts from routes are exchanged in such a way that new solutions are explored and eventually a solution of better quality will appear. Tests have been carried out on the Swiss benchmark from Mandl (1979).

An aggregated metaheuristic approach to the transit network design problem is considered by Zhao and Gan (2003), Zhao and Ubaka (2004) and Zhao and Zeng (2006) with the objective of minimizing the number of transfers and optimizing route directness while maximizing service coverage. The concept of key-node is defined to elaborate neighborhoods in the context of meta-heuristics solution methods. An Integrated Simulated annealing, Tabu, and Greedy search algorithm is proposed in Zhao and Gan (2003) and Zhao and Zeng (2006) while Basic greedy search and Fast hill climb search are implemented by Zhao and Ubaka (2004). These algorithms are tested on benchmark instances and on data from Miami-Dade County, Florida.

2.3.2 Transit Network Frequencies Setting Problem

The transit network frequencies setting problem (TNFSP) consists in determining adequate frequencies for each line of the network and each time period. A time period is defined according to the hour of the day, the day of the week or the time of the year. A line run corresponds to one scheduled service of the line. The inverse of the frequency over a determined period is called the *headway*. It corresponds to the time elapsing between consecutive line run departures.

A suitable frequency assignment should provide sufficiently regular service to satisfy the users and sufficiently sparse service to reduce the required fleet size, and thereby the operator's costs (Guihaire and Hao 2008).

The *transit routes network* constitutes the main inputs for the current matter, together with detailed *OD matrices* that are fundamental in this step. They should provide data

according to uniform demand time periods. These periods vary according to the following criteria: time of the day (peak/off-peak period), the day of the week (Monday–Friday/Saturday–Sunday), time of the year (seasons/vacation periods/others). Since demand is time-dependent and elastic, the survey should be led on extensive periods of time and regularly updated. Such a process is necessary to achieve an efficient network with satisfying service. However, it represents a real charge for the transit agency, since collecting this data is a very complex and expensive task. Such data is therefore rarely freely available.

In most approaches, line frequencies also depend on the *available fleet size and buses capacities*. In this case, a description of the vehicles used is needed, especially if the fleet is heterogeneous. The period-dependent bus running times associated with each route of the network must also be provided.

Among the main *constraints* to be satisfied and *objectives* to achieve, it is necessary to highlight the following:

- The *demand satisfaction*, which implies that the lines frequencies should match the demand at best so as to avoid overcrowding and excessively large headways, and thereby reduce waiting and transfer times.
- The *number of runs for each line* is an example of the multi-objective nature of the problem. While from the operator's point of view it is desirable to minimize this number for resource-related reasons, users for convenience wish to benefit from the widest offer of line runs.
- The operator can be imposed minimum and/or maximum *headways bounds* on some lines or areas by regulating authorities.
- *Historical line runs* can be imposed.

Solution methodologies, as previously clarified for TNDP, can be again categorized into three categories: mathematical optimization searching for exact solutions, heuristics, and meta-heuristics.

- *Mathematical approaches*: In a first study, Salzbom (1972) determines frequencies given passengers' arrival rate so as to minimize the required fleet size and passengers waiting time. In a later study, Salzbom (1980) analyzes the problem of a feeder bus system along with a single inter-town transfer route and determines requirements for the feasibility of such a system. The proposed model is highly simplified.

Scheele (1980) also deals with the TNFSP using mathematical resources. A non-linear model is proposed with the objective of minimizing the total generalized passenger travel time and the passenger trip assignment is solved simultaneously with the frequencies setting problem.

Furth and Wilson (1982) present another mathematical method: the objective is to maximize the net social benefit, consisting of ridership benefit and waiting time-saving. Constraints are imposed on fleet size, maximum headway, and total budget. The problem is solved by an algorithm using the Kuhn–Tucker conditions on a relaxation of a non-linear program where the maximum headway and fleet size constraints are relaxed. The result is an optimal allocation of buses to routes.

Constantin and Florian (1995) present a model and solution method for the TNFSP with the goal to minimize the passengers total expected to travel and waiting time under fleet size constraints. A non-linear non-convex mixed integer programming model is formulated. A projected subgradient algorithm is then used to find optimal line frequencies considering the passenger route choices. Tests are performed in an urban context in Stockholm, Winnipeg, and Portland.

- *Heuristic and meta-heuristic approaches*: Han and Wilson (1982) deal with the TNFSP using a heuristic method. Line frequencies are set given the total fleet size and a capacity for each route. They consider the problem as one of allocating vehicles among the routes of the network. They propose a two-stage heuristic to reach their objective, minimizing the maximum “occupancy level” at the maximum load point for each route. In the first phase, minimal frequencies are set so as to satisfy all the demand. In the second phase, frequencies are increased uniformly among lines so as to utilize all the available vehicles.

For the first time, Ceder and Wilson (1986) defined and presented a conceptual model for the whole bus planning process as a systematic decision sequence, which consists of five levels: network design, frequencies setting, timetable development, bus scheduling and driver scheduling and rostering. Minimum frequency and fleet size were given as constraints. A two level methodological approach was presented for the design of the bus routes network. The first level considers only the passengers' viewpoint and is handled by an enumeration method which generates routes whose length is within a certain factor from the length of the shortest path. The second level considers both the passengers and operator's viewpoints and relies on heuristic techniques. This algorithm aims at minimizing the difference between the number of users served directly and indirectly (i.e. using the shortest path or not).

Chowdhury and Chien (2001) consider transfer coordination for intermodal transit networks by optimizing both headways and slack times. Slack time is the additional time added to the schedule for a given trip in order to compensate potential bus delays and thus increase the probability of schedule adherence. A mathematical programming model is first developed and then a procedure is exposed that first, optimizes headways without taking coordination into account, and then optimizes slack times in the context of intermodal transit.

After that, Gao et al. (2003) proposed a bi-level programming technique to deal with the TNFSP. In the upper-level problem, the objective is to minimize the total deterrence of the transit system (consisting of in-vehicle and waiting time) and the cost caused by frequencies setting. The lower-level model is a transit equilibrium assignment model used to describe the path alternatives to transit users. A heuristic solution based on sensitivity analysis is designed to solve this model to optimize frequencies settings. The algorithm is designed to help the transit planners to adjust an existing transit network to evolutions in the demand and in various other parameters.

Park (2005) uses genetic algorithms and simulation to optimize bus schedules in an urban transit network. Two different cases are considered. When buses arrive following a deterministic process, a simple genetic algorithm is used in combination with problem-specific operators to determine optimized headways. On the opposite, when

buses arrive stochastically, a simulation-based genetic algorithm is used to optimize both headways and slack times. Problem-specific genetic operators include coordinated headway generator, crossover, and mutation.

2.3.3 Transit equity and related indicators

Transit network design problems have predominantly focused on minimizing user and operator cost without considering equity or access for disadvantaged population (Kepaptsoglou and Karlaftis 2009). The most common problem definitions and solution approaches that are possible to find in literature reflect the pervasive attention allocated to maximizing operating efficiencies to benefit users and transit agency. Although operating efficiencies are beneficial and critical to consider, *equity issues* are also pertinent to address in transit network design. This is particularly valid given the reduced mobility and accessibility experienced by the transportation disadvantaged and the resulting burden they experience meeting their basic needs (Ferguson et al. 2012).

Historically, some transit route and service network design problem formulations and solution approaches have included access to the transit system, but not the level of access the system provides to users. For example, Wu and Murray (2005) consider the quality of service provided based on transit travel time, the frequency of stops, and access to the transit system based on stop frequency; they focus on optimizing the balance between service quality and access to the transit system via stop placement. While such formulations address the trade-off between public transit service quality and access to the system, they do not explicitly consider access to basic amenities nor do they address equity in the levels of access provided.

Looking at a recent comprehensive literature review concerning the transit network design problem reported in Ibarra-Rojas et al. (2015), it is easily possible to see that, in most cases, in the works dated from 1975 to 2014 equity aspects are neglected. Two exceptions seem to be Chen and Yang (2004) and, more recently, Fan and

Machemehl (2011). These last propose a bi-level optimization model to solve the public transportation network redesign problem, in which the spatial equity issue is explicitly considered for the first time. To solve the problem, the authors develop a genetic algorithm (GA), and their numerical results show that the horizontal equity constraint leads to increasing total travel cost in the network.

The need for systematically incorporating spatial, temporal and socio-economic distributional effects in transport decision-making is treated in Jones and Lucas (2012). Equity and social inclusion have been initially discussed with respect to fare policies, concessionary fares, and transit subsidies. Recently the scope of research has been widened to include population groups with mobility limitations (Ferguson et al. 2012). The equitable distribution of transit services is a major concern of transportation planners and policymakers worldwide. As asserted by Krumholz and Forester (1990), promoting a wider variety of choices for people who have fewer ones is the first step towards an equity planning.

Aging population, rising fuel prices, increasing urbanization, improved mobility and accessibility options, growing health and environmental concerns, changing consumer preferences (particularly among younger people), and changing transport policies are all contributing to reduce automobile travel and increase demand for alternative modes (Litman 2005). An increasing portion of travelers prefers to drive less and rely more on alternative modes, provided they are comfortable, convenient and affordable (Litman 2006).

Moreover, the last decades are witnessing a gradual shift from a 'mass transit' planning (whose philosophy suggest that all people living within a community deserve equal access to public transportation), to the idea that social inclusion needs to become an integral part of the transit planning process (Kaplan et al. 2014). Some social groups are more likely to require public transport services, above all low-income and socially disadvantaged individuals that are transit-dependent (Denmark 1998; Pucher and Renne 2003; Sanchez et al. 2004; Dodson et al. 2007), as they cannot afford a car and therefore often cannot access their desired destinations (Lucas

2012). For these “transit dependent” (or “captives”) the continued availability of public mass transit is vital to access jobs, education, medical care, and other fundamental services (Garrett and Taylor 1999). Although there are good examples of social equity objectives in several urban transportation plans (Manaugh et al. 2015), often these aims are not adequately translated into specific objectives, and it seems to be a lack of appropriate indicators to assess the related achievements.

In the following, a brief review of some *indicators referred to the distribution of transit service* is presented: it is a starting point for the definition of the transit equity constraint suggested in Section 4 of this dissertation.

An early approach that it is possible to find in literature adopts the density of the transit service for a square kilometer as a measure of supply, comparing it with census social indicators (Currie and Wallis 1992). A subsequent refinement using a total generalized cost model of public transport and accessibility to a series of trip types has been undertaken for Hobart, Tasmania (Currie 2004).

The Public Transport Accessibility Level (PTAL) is another approach used to assess equity in transit supply used in the UK since the early 1990s (Wu and Hine 2003). In this method, transit accessibility is measured using an index made up of walking time to the transit stop, reliability of service, number of services within a catchment and average waiting time. This index does not measure whether this supply provides access to a desired range of destinations. Wu and Hine (2003) compare local PTAL index ratings to the spatial distribution of indices of deprivation, an assessment of vertical equity.

The above-mentioned indicators are basically examples of how to assess vertical equity, as they evaluate the quantity of service provided to disadvantaged social groups. The literature also contains examples of measures of horizontal equity in transit. Reports of this nature generally assess the distribution of transit in relation to either residential or employment distribution. As an example, Minocha et al. (2008) develop a “Transit Availability Index” made up of frequency, hours of service and service coverage and compared it to a “Transit Employment Accessibility Index” calculated us-

ing a gravity model of transit travel times and OD pairs. Although past approaches may highlight specific areas of inequality, a limitation of each approach shown is that they do not provide a single value assessing horizontal equity across an entire transit system.

A possible solution to overcome this drawback is proposed by Delbosc and Currie (2011): they suggest a single, system-wide measure able to reflect the horizontal (i.e. spatial) equity of transit service distribution throughout a metropolitan region, allowing also a visual representation of gaps in public transport supply relative to population and employment. This is basically a *Gini coefficient* (from now on, we call it D&C_Gini) able to measure *how well transit supply meets transit demand*. Actually, the Gini index has been used to measure equity in a number of studies (Karlström and Franklin 2009; Welch 2013; Kaplan et al. 2014). Traditionally, it compares a population’s distribution income (represented as a Lorenz curve), to a line representing perfect equality (Figure 3).

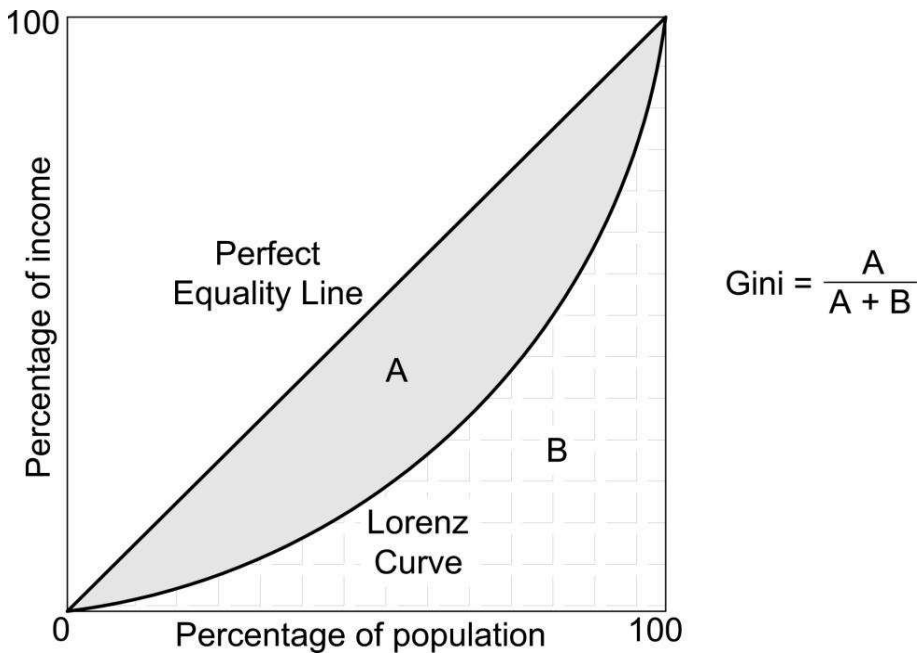


Fig. 3 – Lorenz curve and Gini coefficient.

Lorenz curves (Figure 3) are a representation of the cumulative distribution function of wealth across the population (Lorenz 1905): they can be applied not just to income but to any quantity that can be cumulated across a population. The Lorenz curve offers a representation of the whole distribution. An (X, Y) point on the Lorenz curve shows that X percent of the population receives Y percent of the supply. Therefore, a perfectly equitable situation is represented by the bisector of the first quadrant. To represent the distribution of transit supply, Lorenz curves are plotted as the cumulative proportion of population against the cumulative proportion of transit service supply. The Gini coefficient is a single mathematical metric summarizing the overall degree of inequality. It is the ratio of the area between the line of equality and the Lorenz curve (marked 'A' in Figure 3), to the total area below the line of equality (A+B in Figure 3) (Gini 1912). A perfectly even distribution of supply would result in a Gini coefficient of 0, while a perfectly unequal distribution would result in a coefficient of 1. As an example, two overall coefficients of the D&C_Gini index calculated by Delbosc and Currie (2011): for Melbourne, Australia was 0.68, i.e. around 70% of population shares only 19% of transit service; while Baltimore City has a slightly lower equity of transit services with a D&C_Gini index of 0.7083.

To be able to evaluate the D&C Gini coefficient, it is of paramount importance to define the *transit service supply*. A review of international approaches to measuring the relative supply of passenger transport and transport disadvantage has been undertaken by Currie and Wallis (1992). This shows an approach using available census and social data from sources such as the Australian Bureau of Statistics to identify relative transport needs at census collector district (CCD) level. Associated approaches to measuring supply of services for the transport disadvantaged are also described. Measures such as the relative bus vehicle kilometers per km² are used to illustrate public transport supply. Gaps are identified between public transport and social needs in Adelaide, particularly on the urban fringe.

A new variation of this approach has been developed in Hobart (Currie 2004), where a public transport network model utilizing generalized costs is used to represent public

transport supply quality across the network. This provides a more detailed refinement of the supply side modeling in previous applications of the approach; however, the transport needs measurement methods are similar to previous applications.

In Currie (2010), it is suggested to calculate the level of transit service supply in an area as:

$$SI_D = \sum_b \left(\frac{Area_{Bb}}{Area_D} \cdot SL_{Bb} \right) \quad (5)$$

where SI_D is the supply index for the district (traffic zone) D under analysis, b the number of walk access buffers to stops/stations in each district (see Delbosc and Currie 2011 for a more extensive examination), B_b the buffer b for each stop/station in each district, $Area$ is the square kilometer spatial area, SL_{Bb} is a service level measures (number of public vehicle arrivals for a given time interval).

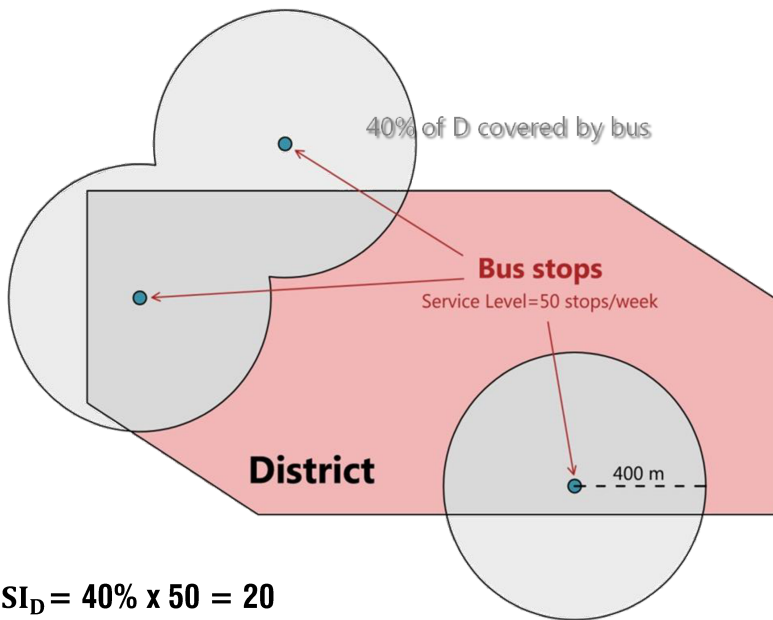


Fig. 4 – Simplified example of Supply Index calculation for a district D.

The transit supply index accounts for both the spatial coverage of a district by walk catchments to public transport and for the quality of the service itself. To calculate it, the next steps need to be followed:

- A database of bus stops (and/or tram stops – train stations, according to the public services considered) has to be obtained. This includes the location of each stop/station plus a listing of routes using the stops, and their service frequency (in the example shown in Figure 4, it is considered as the total number of service arrivals per week for each stop).
- Access distance to each stop has to be measured for every district, assuming a different threshold for each public transport mode (in Figure 4, the walk buffer around a bus stop is assumed equal to 400 m). Walk buffers are distances that 75% to 80% of people would walk to access a stop (Kittelson & Associates et al. 2003).
- Where two walk buffers are so close to overlapping, the buffers have to be merged together so as not to count the overlap area as “twice” the service. Note that service levels are calculated based on the overlap with a stop’s walk catchment, not the location of the catchment centroid (transit stop). That is, the actual transit stop does not have to be within a district in order for its service to contribute to the supply for that district.

A weakness of the proposed indicator is that it neglects the spatial distribution of people, assuming indeed an even spatial distribution of residents within the traffic zone. In addition, it does not consider how connected a particular stop is to the rest of the network or how transit supply varies throughout the day. Despite the limitations, the ease of calculation makes it a practical choice for practitioners to usefully characterize the level of supply in a certain reality.

Another important feature of transit services to be considered are *the social indicators*. As a matter of fact, one of the main functions of public transit is to provide accessibility to all members of a society, particularly to those with limited mobility

choices (Manaugh and El-Geneidy 2010). Since socially disadvantaged groups should receive some priority in public transportation planning, it is important to define correctly these groups; and a common way to do this is by means of a social indicator (Foth et al. 2013). Social indicators are instruments capable of identifying underprivileged groups lacking access to goods and resources, comparing them to the rest of society (Townsend et al. 1988).

Social indicators have been widely used to measure a variety of equity issues, including community deprivation (Sánchez-Cantalejo et al. 2008; Social Disadvantage Research Centre 2003). Researchers that use these indicators adopt several different methods. The cumulative method is common, due to its ease in interpretation and simplicity in calculations. Some social indicators are derived from sums of variables (Bauman et al. 2006), while others weight variables equally (Townsend et al. 1988). Meanwhile, some studies use the total of standardized values (Manaugh and El-Geneidy 2012). More complicated methods assign a various weighting to each category (Social Disadvantage Research Centre 2003). There are different methods used in the literature for the weighting of variables. One weights the variables by a number of survey responses (Jarman 1983), while another uses factorial analysis to determine weights (Sánchez-Cantalejo et al. 2008).

A number of factors contribute to transport disadvantage. For example, Morris (1981) identifies several groups which are particularly dependent on public transport: those that are too young or too old to drive, disabled, home workers, low-income earners, unemployed youngsters, and migrants. The result of Morris is confirmed by Starrs and Perrins (1989). A more extensive, although a not recent summary of the groups that potentially need public transport is given in Murray and Davis (2001). Choosing appropriately the *categories* to be included is the most important aspect that leads to the generation of social indicators: they must be selected to reflect the characteristics the research is trying to reveal.

Social indicators can be built from socio-demographic and economic information. It is possible to find an example in Ruiz et al. (2014), who obtain a social indicator of Public Transport Need (PTN) for each district by the Analytic Hierarchy Process (AHP)

multi-criteria analysis of a set of socioeconomic categories. In the AHP each category (i.e. adults without cars, persons aged over 65 years, persons on a disability pension, low-income households, students, etc.) is associated with a weight, representing its relative importance within the social framework of the study area/city. The final value for each district is calculated using the following expression:

$$PTN_D = \sum_y w_y x_y \quad (6)$$

where D is the district under analysis, y are the variables, w_y is the weight assigned to each category, and x_y is the value of the category y, ensuring that $PTN_D \leq Pop_D$ (i.e. the total population residing in the district D).

Certainly, each area city or region under investigation has its own social features to be studied and examined, and expertise and data are strictly required to identify the patterns of transport disadvantage that need to be included in the construction of the final index.

3.0 INCLUDING EQUITY IN NDP

This section focuses on the integration of equity concepts/constraints in the general formulation of the Network Design Problem: as a matter of fact, in transportation planning, it is important to achieve solutions designed to meet objectives of equity and social inclusion.

3.1 GENERAL OVERVIEW

Generally, NDP models aim at identifying the optimal layout of transportation networks by deterministic bi-level problems formulation to reflect the different goals of at least two decision makers (the network users and the planner). Considering the societal function of transport systems, Network Design Problems, even at the operational level, should be addressed also to the research of solutions in which the outcomes of the design (costs and/or benefits) are distributed as much as possible among the potential users or classes of users.

Traditional approaches often neglect equity goals that, conversely, should play an important role or define some flows rebalancing problem. In this chapter, *two different formulations of equity-based NDP* are proposed.

In the first one, the optimal layout of a road network is determined by minimizing the total system cost under flexible constraints, (jointly with rigid thresholds) by solving a

single programming problem. It considers *both horizontal and vertical equity criteria* in the form of an equity constraint specified for uncertain variables or approximate reasoning environment: this results in a multi-objective fuzzy programming model that aims at maximizing user satisfaction according to all constraints while taking into account the route choice behavior of network users.

On the other hand, the second formulation aims to support transport planners in dealing with *multimodal networks* in contexts characterized by different (and sometimes conflicting) interests and limited resources in a transparent way. It expands the scope of traditional NDP approaches: firstly, it takes public transit into account alongside private transport. Then, it considers the relevance of equity among other planning goals, enabling the achievement of solutions with a fair distribution of transport impacts (benefits and costs) among the users. Finally, it proposes the conjoint use of fuzzy and rigid goals and constraints to improve the quality of the solutions.

In the following, the two model formulations with the related equity constraint specifications are explained in greater details.

3.2 FIRST MODEL FORMULATION

There are different aspects of equity that could be taken into account and incorporated into the general formulation of NDP. Among them, in the first formulation proposed in this section equity refers to the OD travel costs imbalance due to the OD travel costs increase or decrease after implementing an optimal network design scenario.

3.2.1 General formulation

For the general formulation of the supply design problem readers could refer to Cascetta (2009). In the classic CNDP, the optimal network enhancements are de-

terminated by minimizing the total system cost under a set of constraints, while taking into account the route choice behavior of network users. However, the equilibrium travel costs between some OD pairs may increase or decrease after implementing an optimal network design scenario, leading to positive or negative results for network users. Therefore, the equity issue is raised and it becomes necessary to add equity constraints to the classic CNDP as shown by the following problem:

$$\mathbf{x}^* = \arg \min_{\mathbf{x}} z(\mathbf{x}, \mathbf{f}^*) \quad (7)$$

s.t.

$$k_u \in V \quad \forall u = 1, 2, \dots, r \quad (8)$$

$$\mathbf{f}^* = \Delta(\mathbf{x})\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f}^*, \mathbf{x}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{x})) \quad (9)$$

$$\mathbf{x}_e, \mathbf{f}^* \in E \quad \forall e = 1, 2, \dots, p \quad (10)$$

$$\mathbf{x}_t, \mathbf{f}^* \in T \quad \forall t = 1, 2, \dots, q \quad (11)$$

where:

- z is the function of the total cost of the network;
- $\mathbf{x}$ is the vector of the design variables;
- $\mathbf{f}^*$ is the vector of equilibrium assignment traffic flows;
- k_u are the equity performance indicators;
- r is the number of equity constraints;
- Δ is the link-path incidence matrix;
- $\mathbf{P}$ is the path choice probability matrix;
- $\mathbf{C}$ is the vector of path costs;
- $\mathbf{d}$ is the vector of travel demand;
- $p+q$ is the number of certain design variables.

The Equation (8) denotes the set of equity performance indicators satisfying equity constraints V . Equation (9) represents the consistency constraint among demand, flows and supply parameters (set of possible configurations of network flows). Equations (10) and (11) express the sets of supply parameters satisfying external (E) and technical (T) constraints such as, respectively and for example, available budget and link flows-capacity ratio.

Note that this formulation is suitable to Continuous NDP, that is problems in which the decision variables are continuous as in the case of road capacity expansion, timing traffic light, and determination of tolls for some specific streets.

3.2.2 Equity constraint specification

The above-mentioned Equation (8) can be specified as *objective functions* (bi or multi-objective CNDP) or simple *inequalities*. For example, considering an equity performance indicator α that measures the degree of equitability of benefit distribution, Equation (8) can be stated by Equation (12), where $\beta_{\max}$ is a fixed threshold (set by decision makers).

$$\alpha \leq \beta_{\max} \quad (12)$$

The equity performance indicator α , in this first attempt of re-formulation of the NDP in a 'fairer' way, has been supposed equal to the critical OD travel cost ratio proposed by Meng and Yang (2002). This indicator, as clarified in Equation (13), is defined as the maximal ratio of the equilibrium OD travel cost after implementing a network scenario ($z_w(\mathbf{x})$), divided by the equilibrium OD travel cost before scenario implementation ($\bar{z}_w$) for a set of specific OD pairs W .

$$\alpha = \max_{w \in W} \left\{ \frac{z_w(\mathbf{x})}{\bar{z}_w} \right\} \quad (13)$$

If $\alpha < 1$ all users can benefit from the network design implementation; conversely, if $\alpha > 1$ there will exist users who suffer a travel cost increase induced by the design implementation. To deal with this equity issue, the equilibrium OD travel cost reduction/increase for each OD pair can be restricted beyond/below a given level by assuming α to be less than a desirable threshold $\beta_{\max}$, as shown in Equation (12).

The parameter $\beta_{\max}$ is a given appropriate positive constant, set by decision makers, which measures the degree of equitability of benefit distribution. A smaller value of this parameter means a more equitable distribution of benefits across network users. If it is set to be lower than 1, then each user will enjoy a travel cost reduction at least by $100 \cdot (1 - \beta_{\max})\%$; if however, it is set to be greater than 1, it means that there may be users who suffer a travel cost increase induced by the improvement scheme, but this increase cannot be more than $100 \cdot (\beta_{\max} - 1)\%$ (see Meng and Yang 2002 for further details).

However, unlike Meng and Yang (2002) that consider $\beta_{\max}$ a crisp threshold, in the proposed model specification it has been introduced uncertain and incomplete information about this threshold. In the next subsection (3.2.3) a further explanation is provided.

3.2.3 Dealing with uncertainties

It is possible to add uncertainty to the inequality depicted in Equation (12), expressing it through incomplete information formalized quantitatively with linguistic/approximate expressions. Therefore, the same Equation (12) may be stated by decision makers with the following expression: “ α must be approximately lower than or equal to $\beta_{\max}$ ”. This concept is represented by Equation (14):

$$\alpha \leq \beta_{\max} \quad (14)$$

Using the same approach, also the optimization function (7) can be specified with an uncertain relation, as shown in Equation (15), where $\bar{z}$ represents the maximum value admitted for the total cost of the network.

$$\underset{\sim}{z} \leq \bar{z} \quad (15)$$

In particular, these uncertain relations/expressions can be specified as a fuzzy set (Zadeh 1965; Zimmermann 1996). In fuzzy theory, a crisp number belongs to a set (fuzzy set) with a certain degree of membership, named also *satisfaction* $h \in [0,1]$. The degree of membership is defined by a “membership function”. Choosing respectively a triangular and a trapezoidal membership function, a potential description of Equations (14) and (15) can be seen in Figure 5 and 6.

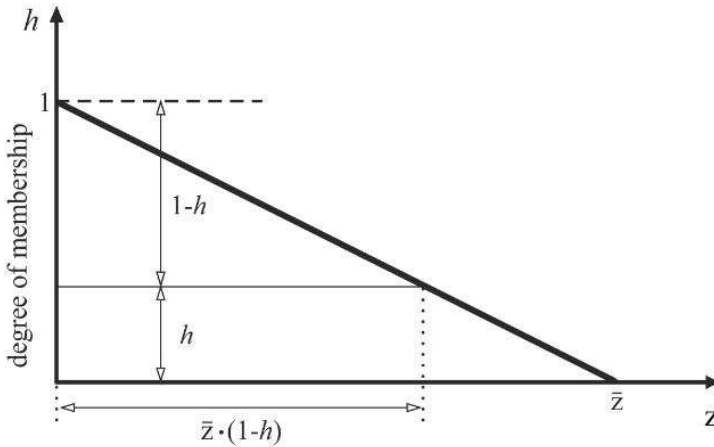


Fig. 5 – Fuzzy set of OD pairs cost differences: triangular membership function.

In Figure 5 is represented the relation between the total cost of the network (z) and h . Figure 6 shows the relation between the equity performance indicator α and h . This representation is obtained by setting two values, to be positioned on the horizontal axis: the first one is $\beta_{\min}$ (that is the minimum value of the threshold), below which the satisfaction is maximum and equal to 1; the second one is the $\beta_{\max}$ threshold, namely

the maximum admitted value of α selected by the policy maker, where the satisfaction h is set equal to zero.

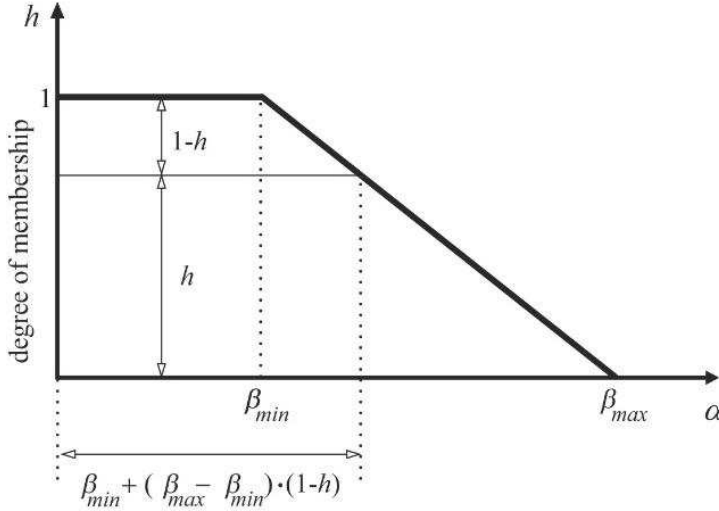


Fig. 6 – Fuzzy set of equity performance indicator α : trapezoidal membership function.

In this framework, it results that the closer to one the degree of membership is, the more the optimization (15) and the equity constraint (14) are fulfilled. Therefore, in order to find the optimal solution to the problem (7) subject to certain and uncertain (i.e., fuzzy) constraints/relations, it is necessary to maximize the satisfaction h .

3.2.4 Proposed NDP formulation

The classic equity CNSP problem expressed by Equations (7-11), to deal with the uncertainties above mentioned, turns in the proposed model:

$$\max h \quad (16)$$

s.t.

$$z(\mathbf{x}(h), \mathbf{f}^*) \leq \bar{z} \quad (17)$$

$$\alpha_m(h) \leq \beta_{\max}^m \quad \forall m = 1, 2, \dots, v \quad (18)$$

$$\alpha_m \leq \beta_{\max}^m \quad \forall m = v+1, v+2, \dots, v+u \quad (19)$$

$$\mathbf{f}^* = \Delta(\mathbf{x})\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f}^*, \mathbf{x}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{x})) \quad (20)$$

$$\mathbf{x}_e, \mathbf{f}^* \in E \quad \forall e = 1, 2, \dots, p \quad (21)$$

$$\mathbf{x}_i, \mathbf{f}^* \in T \quad \forall i = 1, 2, \dots, q \quad (22)$$

where:

- α_m are the equity performance indicators with their maximum values;
- v is the number of uncertain equity constraints;
- u is the number of certain (with fixed threshold) equity constraints.

The NDP expressed by Equations (16-22) is a fuzzy optimization problem (Teodorovic and Vukadinovic, 1998), where the total cost minimization of Equation (7) becomes the constraint (17), expressed according to the value of satisfaction h . The set of equity performance indicators of Equation (8) are transformed into uncertain (18) and, if any, certain equity constraints.

The assumed fuzzy constraints (17) and (18) definitely depend on the same value of the satisfaction h . Therefore, the closer to one the value of h (maximization of satisfaction) is, the more the uncertain constraints are optimized. After the selection of the membership functions, Equations (17) and (18) are turned into fuzzy sets. If these constraints are specified according to the membership functions in Figure 5 and 6, they become:

$$z(\mathbf{x}(h), \mathbf{f}^*) \leq \bar{z} \cdot (1 - h) \quad (23)$$

$$\alpha_m \leq \beta_{\min}^m + (\beta_{\max}^m - \beta_{\min}^m) \cdot (1 - h) \quad (24)$$

Depending on the choice of the equity performance indicators, constraints (19) (and accordingly, Equation (24)) can be differently described. Using a single equity performance indicator and defining it by means of Equation (13), Equation (24) becomes:

$$\max_{w \in W} \left\{ \frac{z_w(\mathbf{x}(h))}{\bar{z}_w} \right\} \leq \beta_{\min} + (\beta_{\max} - \beta_{\min}) \cdot (1 - h) \quad (25)$$

3.3 SENSITIVITY ANALYSIS AND METHOD COMPARISON

This subsection consists of an application having a dual purpose. The first aim is to experimentally evaluate the performances of the first proposed design model through a sensitivity analysis. The second goal is to compare this approach, which considers uncertainty in the equity constraints, with a classic equity-based model that uses only crisp constraints, like the equity-based model proposed by Meng and Yang (2002).

On urban networks most of the total travel time is spent at intersections: hence, an effective optimization of signalized intersections can significantly improve the transportation network performance. For this reason, in this test, it is suggested a network design optimization that considers signal settings parameters as supply design variables. The chosen approach to the problem consists in searching the vector of optimal effective green times ($\mathbf{x}^*$) for all signalized intersection; these values are obtained through the minimization of the network total cost z depending on signal settings ($\mathbf{x}$), on equilibrium flows ($\mathbf{f}^*$) and on equity constraints. This analysis has been carried out on the test network (Figure 7) and data proposed by Yang et al. (2001).

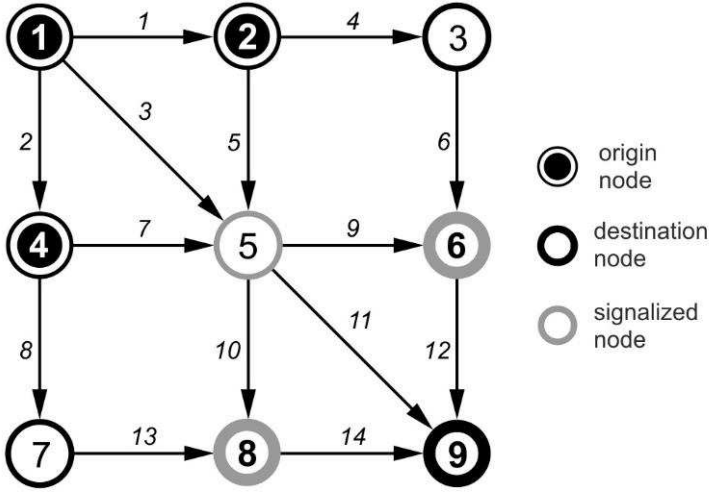


Fig. 7 – Test network for the first model formulation.

Usually, a transportation network is modeled by a weighted graph, where the nodes represent intersections (real nodes) and origin/destination points (centroid nodes); the links represent road connections between nodes (real arcs). In this example, the graph of the network is made up of 9 nodes (3 of which are origins -nodes 1, 2, 4- and 3 are destinations -nodes 6, 8, 9-), 14 links and 9 OD pairs. The signalized intersections have been set in the node 5 with a three-phases regulation scheme and in the nodes 6 and 8 with a two-phases regulation scheme. Assuming g_{ph}^{nd} the effective green time for the node nd and for the phase ph , the vector of the design variables is $\mathbf{x} = [g_1^5; g_2^5; g_3^5; g_1^6; g_2^6; g_1^8; g_2^8]$. For all the signalized intersections the effective cycle time is fixed to $C_t = 90$ seconds.

The link cost values c_l for the numerical tests are the sum of the link travel time and the waiting time due to the signalized intersections. The link travel time tc_l , function of the link flow f_l and of the link capacity ca_l , is calculated using the well-known Bureau of Public Roads (BPR) link cost function (1964), illustrates by Equation (26), where the free flow travel time (tr) and capacity (ca) depend on the considered link l .

$$tc_l(f_l) = tr_l \left[1 + 0.15 \cdot \left(\frac{f_l}{ca_l} \right)^4 \right] \quad l = 1, 2, \dots, 14 \quad (26)$$

The waiting time is estimated using the Doherty's delay function (Doherty 1977), shown in Equations (27) and (28):

$$t_{wa}^l = 0.5 \cdot C_l (1 - \mu)^2 + \frac{1980}{\mu \cdot s} \cdot \frac{f_l}{\mu \cdot s - f_l} \quad \text{if } f_l \leq 0.95 \cdot \mu \cdot s \quad (27)$$

$$t_{wa}^l = 0.5 \cdot C_l (1 - \mu)^2 + \frac{198.55}{\mu \cdot s / 3600} \cdot \frac{220 \cdot f_l}{(\mu \cdot s)^2 / 3600} \quad \text{if } f_l > 0.95 \cdot \mu \cdot s \quad (28)$$

where:

- t_{wa}^l is the waiting time at intersection on link l (s/veh);
- f_l is the traffic flow on link l (veh/h);
- s is the saturation flow (veh/h);
- g is the effective green time;
- μ is the effective green ratio (g/C_l).

The travel demand **d** (Table 2) has been assigned to the network using a Deterministic User Equilibrium (DUE) traffic assignment model; this is based on the assumptions that network travel times are deterministic for a given flow pattern and that all travelers are perfectly aware of the travel times on the network and always capable of identifying the shortest travel time route. The calculation of equilibrium link flow with rigid demand is based on different algorithms; in this model, the Frank-Wolfe algorithm has been adopted (LeBlanc et al. 1975; Nguyen 1976).

Table 2 – OD travel demand.

O-D	1-6	1-8	1-9	2-6	2-8	2-9	4-6	4-8	4-9
/	1	2	3	4	5	6	7	8	9
d	120	150	100	130	200	90	80	180	110

The sensitivity analysis has been carried out in two steps.

During the first one, a series of Starting Configurations (SC) based on different effective green times, divided into different combinations of the traffic light phases and nodes, has been generated. In this way, we have obtained, for each SC (i.e., combination of effective green times) and through the DUE assignment, an initial total cost of the network, denoted by $\bar{z}$ and starting values of equilibrium flows $\mathbf{f}^{*SC}$.

The second step implied the application of three different optimizations to each Starting Configuration. The optimizations performed, with their corresponding objective functions and constraints, are summarized in Table 3, in order to appreciate, at a glance, the differences existing among them. In the following, an explanation about the main features that characterize each optimization.

- Crisp Optimization (CO).

This optimization is the classical CNDP since it implies the minimization of the network total cost. The only other requirements to be fulfilled are the flows on links 1-2 and 1-4, which have to be reduced at least by 50% compared to the equilibrium flows of the corresponding starting configuration. It has been assumed hypothetically that these two selected links are characterized by specific conditions (e.g. the presence of schools), and so they have specific requirements to be fulfilled. Consequently, the flows reduction constitutes a vertical equity constraint applied to the network.

The value of the equity performance indicator α_{CO} associated to each SC has been calculated according to Equation (13).

Table 3 – Performed optimizations.

		Crisp Optimization (CO)	Equity Crisp Optimization (ECO)	Equity Fuzzy Optimization (EFO)
<i>Objective functions</i>	Satisfaction	-	-	$\max h$
	Network total cost	$\min z(\mathbf{x}, \mathbf{f}^*)$	$\min z(\mathbf{x}, \mathbf{f}^*)$	-
<i>Problem constraints</i>	Network total cost	-	-	$z(\mathbf{x}(h), \mathbf{f}^*) \leq \bar{z} \cdot (1 - h)$
	Horizontal equity	-	$\alpha_{ECO} \leq 0.9 \cdot \alpha_{CO}$	$\alpha_{EFO} \leq \beta_{\min} + (0.9 \cdot \alpha_{CO} - \beta_{\min}) \cdot (1 - h)$
	Satisfaction	-	-	$0.01 \leq h \leq 1$
	Vertical equity Flow (1-2)	$f_{1-2}^* \leq 0.5 \cdot f_{1-2}^{*SC}$		
	Vertical equity Flow (1-4)	$f_{1-4}^* \leq 0.5 \cdot f_{1-4}^{*SC}$		
	Demand – flows consistency	$\mathbf{f}^* = \Delta(\mathbf{x})\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f}^*, \mathbf{x}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{x}))$		
	Effective green time	$5 \leq g_{ph}^{nd} \leq 80 \quad \forall \quad nd \in \{5,6,8\}$		
	Cycle time	$\sum_{ph} g_{ph}^{nd} = 90 \quad \forall \quad nd \in \{5,6,8\}$		

- Equity Crisp Optimization (ECO).

A new constraint has been added to the previous optimization (CO): the horizontal equity constraint. It has been assumed that decision makers want to reduce, at least by 10% (such planning decision should reflect community needs and values), the equity performance indicator α_{CO} calculated as the output of the previous optimization. In other words, the equity performance indicator to be achieved at the

end of the Equity Crisp Optimization (denoted by α_{ECO}) must be lower than or equal to the 90% of the corresponding α_{CO} .

The ECO is a classic equity-based optimization model (see the general formulation, Equations (7-11)) that does not consider uncertainty in the equity constraints. Basically, this optimization is structured in the same way of the one proposed by Meng and Yang (2002), with the addition of the two vertical equity constraints represented by the reduction of flows on two specific links.

- Equity Fuzzy Optimization (EFO).

This optimization is based on the proposed fuzzy model (Equations (16-22)) with the specification of the equity constraints (18) as described in subsection 3.2.2.

In this optimization, the objective function to be maximized is the satisfaction h , and the network total cost minimization has become a further constraint of the NDP problem expressed according to the value of satisfaction h . The horizontal equity constraint to be satisfied is the same as the one considered in ECO, but with uncertainty in its definition. In other words, this equity constraint translates the following expression stated by decision makers: " α_{EFO} must be approximately lower than or equal to $0.9 \cdot \alpha_{CO}$ ". This constraint is equivalent to the fuzzy set displayed in Figure 6 where $\beta_{\max} = 0.9 \cdot \alpha_{CO}$ and $\beta_{\min} = 0.50$. The value of $\beta_{\min}$ should be set by decision makers, and it should be greater than or equal to the minimum value reached by the critical ratio $Z_w(x)/\bar{Z}_w$ (in the case under analysis the local minimum value of this ratio is equal to 0.4886). The conditions about the reduction of flows on links 1-2 and 1-4 are unchanged.

All the proposed optimizations have been solved through an interior-point algorithm described by Waltz et al. (2006), using the software MATLAB. Not all the generated starting configurations, under the optimization stopping criteria (tolerance on constraints violation equal to 10^{-3} , on function equal to 10^{-3} and on variables values equal to 10^{-10}), have led to a final feasible solution. Sixty initial setups, under these stopping criteria, allowed to reach a feasible solution at the same time for all the three

described optimizations. These sixty configurations represent quite well the solution space since those effective green times imply starting values of network total costs ranging from a minimum to a value about eight times greater than it (Figure 8). For each SC and at the end of each optimization, equity performance indicators (α^*) and optimized values of total network costs (z^*), flows (f_{1-2}^* , f_{1-4}^*) and effective green times are obtained.

Table 4 reports the results obtained starting from a chosen SC.

Table 4 – Results excerpt.

	$g_1^5[s]$	$g_2^5[s]$	$g_3^5[s]$	$g_1^6[s]$	$g_2^6[s]$	$g_1^8[s]$	$g_2^8[s]$	α^*	z^*	f_{1-2}^*	f_{1-4}^*
SC	35.0	20.0	35.0	80.0	10.0	10.0	80.0	-	5333852	120	150
CO	54.3	23.1	12.6	53.6	36.4	34.3	55.7	10.2	3254904	59	75
ECO	55.8	21.8	12.4	57.6	32.4	33.2	56.8	9.15	3116790	60	75
EFO	60.8	15.2	14.0	62.6	27.4	30.9	59.1	6.68	3826857	59	74

All the key results of the sensitivity analysis are shown in Figure 8. In this figure, the SC have been sorted in view of their respective total cost of the network, arranged in ascending order.

Examining the network total costs z obtained at the end of the three optimizations, it is possible to observe that the ECO shows a better behavior than the EFO, presenting optimized network total costs generally lower. This is only a partial point of view because it is also important to look at the fulfillment of the constraints. In other words, at the end of each optimization, it is essential to analyze what happens to the final values of the equity performance indicators (denoted by α_{ECO}^* and α_{EFO}^*) and of flows on links 1-2 and 1-4 (f_{1-2}^{*ECO} and f_{1-2}^{*EFO} , f_{1-4}^{*ECO} and f_{1-4}^{*EFO}).

In order to compare the equity performance indicators obtained from ECO and from EFO models, it has been introduced the α_{red} percentage, defined as follows:

$$\alpha_{red} = \frac{\alpha_{ECO}^* - \alpha_{EFO}^*}{\beta_{max}} \cdot 100 \quad (29)$$

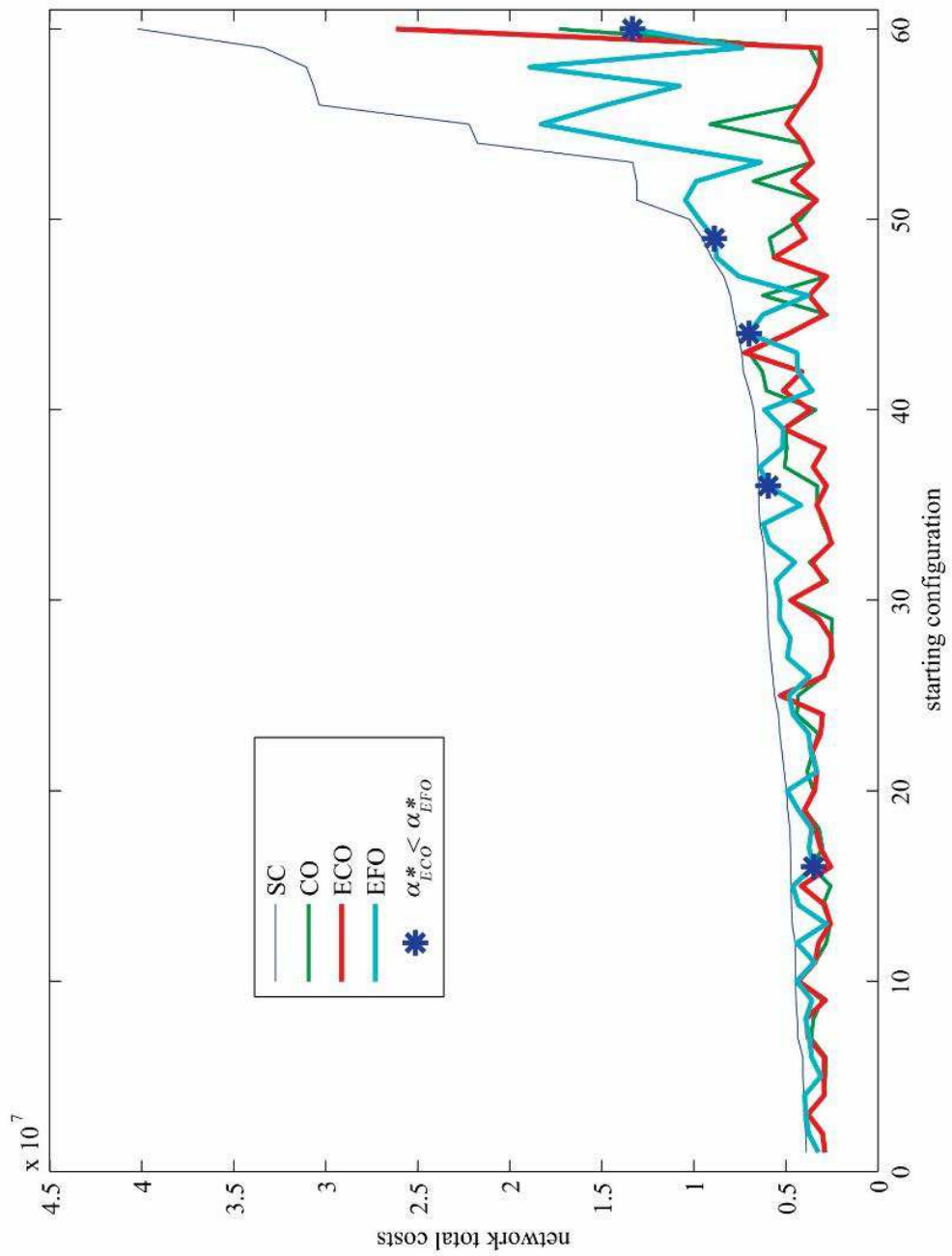


Fig. 8 – Network total costs comparison.

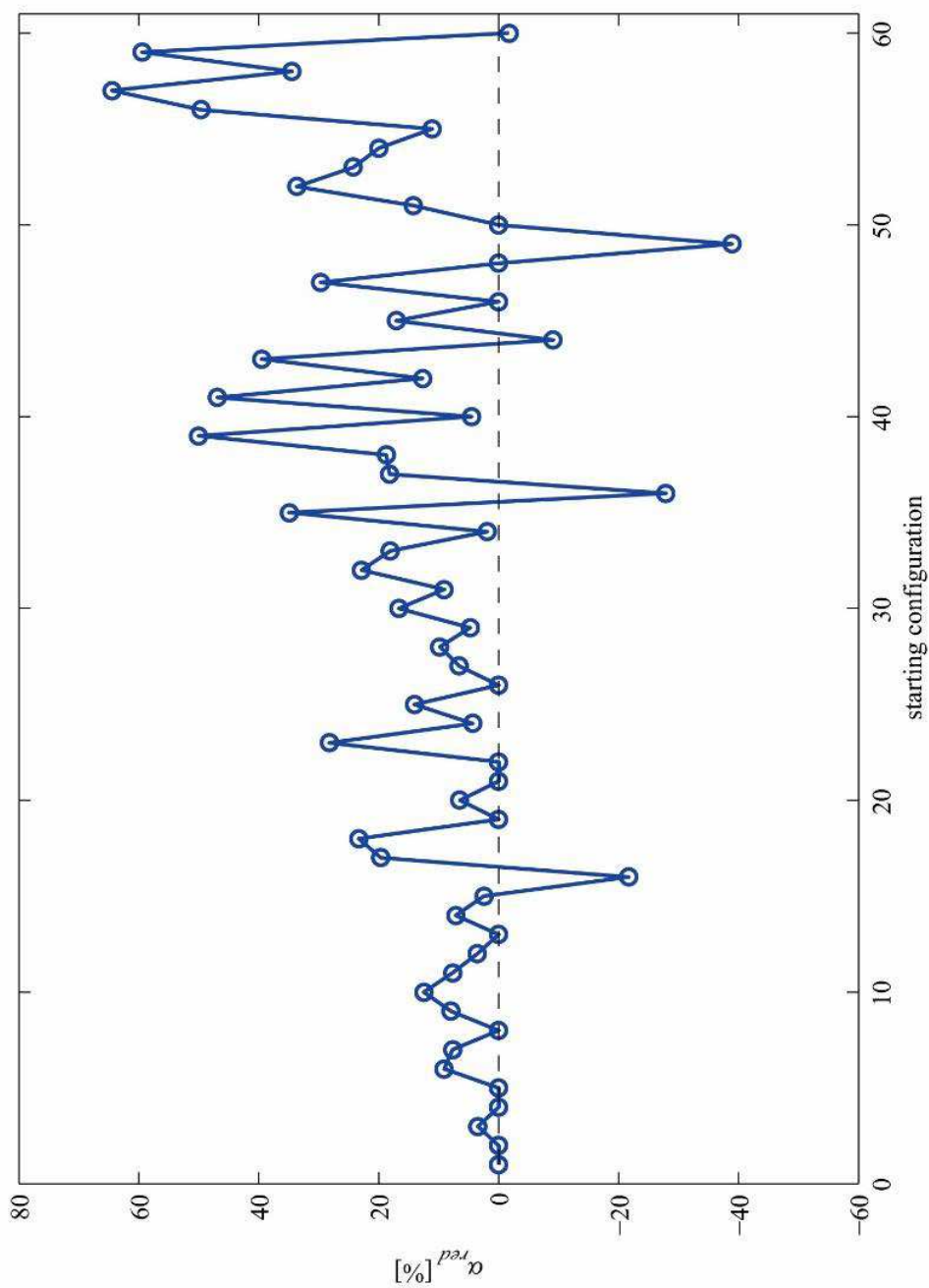


Fig. 9 – α_{red} values.

Figure 9 displays the values of α_{red} for each starting configuration, sorted in the same way on the ones in Figure 8.

According to the specified definition of the equity performance indicator, the lower the α value is, the more all users can benefit from the network design implementation. Fuzzy optimization presents alpha values almost always lower than those obtained by ECO; only in a few cases (five out of sixty) α_{ECO}^* is lower than α_{EFO}^* . These five optimized configurations, numerically described (highlighted in grey) in Table 5, are marked with an asterisk in Figure 8 and with a negative value of α_{red} in Figure 9. However, in all the above mentioned five cases (except for the configuration number 36) the conditions regarding the reduction of the flows on the two links appear to be better satisfied in EFO (see Table 5).

Furthermore, it can be observed that as you move on the right side of the diagram in Figure 9, i.e. towards configurations having an initial higher total cost of the network, the value of α_{red} tends to increase. This means that the requirement of horizontal equity tends to be satisfied better and to a greater extent if we apply EFO starting from a very congested network.

Therefore, it is true that the costs of the network obtained with EFO are generally higher than those resulting from ECO, but in the face of an equity (both horizontal and vertical) considerably increased. The EFO allows spreading fairly ‘disadvantages’ on the network, better satisfying also the constraints imposed on the flows that have in proportion been further reduced.

The following table (Table 5) reports all the values obtained at the end of the ECO and the EFO optimizations for each one of the chosen sixty configurations.

Table 5 – Results of the optimizations for the chosen sixty configurations. Highlighted in gray, the ones having α_{ECO}^* lower than α_{EFO}^* .

No. conf.	Opt.	z^{SC}	z^*	β_{max}	α^*	f^{*SC}_{1-2}	f^*_{1-2}	f^{*SC}_{1-4}	f^*_{1-4}
1	ECO	3891068	2880594	0.9	0.9	93	46	205	102
	EFO		3233549		0.9		30		102

2	ECO	3919778	3009603	1	1	190	95	85	42
	EFO		3764078		1		80		30
3	ECO	4028151	3797973	5.8	5.8	132	66	52	26
	EFO		3919448		5.6		65		24
4	ECO	4034557	2928557	1.6	1.5	258	129	9	4
	EFO		3981510		1.5		122		0
5	ECO	4064158	2949542	0.9	0.8	300	149	0	0
	EFO		3097219		0.8		149		0
6	ECO	4078857	2904477	1.1	1.1	274	137	8	4
	EFO		3601307		1		126		0
7	ECO	4350163	3785337	1.3	1.2	14	7	208	104
	EFO		3731920		1.1		7		104
8	ECO	4380266	3920247	1	1	78	23	142	69
	EFO		3923242		1		38		60
9	ECO	4454504	2856071	6.3	5.4	129	64	185	86
	EFO		3583431		4.9		41		88
10	ECO	4468298	4324106	1.6	1.6	78	39	38	19
	EFO		4419720		1.4		39		19
11	ECO	4471955	3407206	1.3	1.2	154	77	91	41
	EFO		3447981		1.1		66		45
12	ECO	4500790	3231339	2.8	2.7	342	148	0	0
	EFO		4413835		2.6		168		0
13	ECO	4663297	2582400	0.7	0.7	219	109	118	59
	EFO		2812793		0.7		107		54
14	ECO	4691023	2888481	1.4	1.4	305	146	0	0
	EFO		4321307		1.3		145		0
15	ECO	4726086	4158441	36.3	36.3	192	96	160	79
	EFO		4626472		35.4		96		77
16	ECO	4735225	2568599	39.7	28.4	201	100	150	75
	EFO		3496947		37		97		65

17	ECO	4747720	3127994	6.1	6.1	130	65	172	86
	EFO		3741673		4.9		64		81
18	ECO	4786645	3398355	6	6	84	38	167	83
	EFO		3595226		4.6		42		84
19	ECO	4905853	3982469	1.1	1	12	6	177	89
	EFO		4328804		1		0		83
20	ECO	4970970	3432250	3.1	3.1	52	25	183	90
	EFO		4924774		2.9		15		91
21	ECO	5091758	3305386	1.9	1.1	125	63	112	56
	EFO		3315071		1.1		63		56
22	ECO	5206278	3581302	2.7	2.6	22	9	210	105
	EFO		3636934		2.6		9		105
23	ECO	5333852	3116790	9.2	9.2	120	60	150	75
	EFO		3771626		6.6		60		75
24	ECO	5401364	3005453	2.3	2.1	43	21	245	122
	EFO		4610192		2		0		118
25	ECO	5615212	5294928	12.1	12.1	361	179	0	0
	EFO		4820150		10.4		176		0
26	ECO	5713988	2951759	0.7	0.7	251	123	16	8
	EFO		3717633		0.7		123		7
27	ECO	5821065	2534125	6.1	6.1	305	153	53	27
	EFO		4925244		5.7		150		21
28	ECO	5929503	2537339	5.1	5.1	303	151	54	27
	EFO		4747651		4.6		142		22
29	ECO	5988413	3206421	10.5	10.5	220	110	140	70
	EFO		5330526		10		107		67
30	ECO	6013714	4744853	1.2	1.2	0	0	89	41
	EFO		5325924		1		0		43
31	ECO	6053177	2889219	2.2	2.2	324	146	0	0
	EFO		5544889		2		152		0

32	ECO	6150387	3587715	7	6.9	327	161	0	0
	EFO		4515391		5.3		136		0
33	ECO	6222020	2528331	21	21	335	167	33	16
	EFO		5905425		17.2		167		15
34	ECO	6412473	2853485	5.2	5.1	357	142	0	0
	EFO		6233312		5		177		0
35	ECO	6465877	3318506	61.8	61.8	78	39	166	83
	EFO		4193097		40.2		39		83
36	ECO	6477105	2799069	7.9	4.1	356	145	0	0
	EFO		5957511		6.3		169		0
37	ECO	6527532	3518925	5.5	5.3	23	10	218	108
	EFO		6417863		4.3		10		82
38	ECO	6547718	2901956	3.2	3.2	357	147	0	0
	EFO		5194351		2.6		157		0
39	ECO	6671036	5090689	5	5	15	8	219	109
	EFO		5130848		2.5		7		62
40	ECO	6736105	3674116	2.2	2.2	59	22	175	85
	EFO		6170187		2.1		22		80
41	ECO	7001405	5133230	3.2	3.2	22	11	207	103
	EFO		3568698		1.7		9		101
42	ECO	7312401	4119712	6.3	4.7	87	44	51	25
	EFO		4342293		3.9		43		25
43	ECO	7388888	7224702	26.3	26.3	16	8	150	41
	EFO		4420140		15.9		8		74
44	ECO	7609420	4854538	1.1	0.9	60	30	12	6
	EFO		7014479		1		26		5
45	ECO	7840691	2892865	11.7	11.7	360	149	0	0
	EFO		6251319		9.7		174		0
46	ECO	8021737	3733203	2.8	1.6	16	7	192	95
	EFO		3794714		1.6		6		96

47	ECO	8370282	2791815	14.5	14.3	363	159	0	0
	EFO		7540204		10		146		0
48	ECO	9033182	5638245	1.6	1.6	25	13	158	76
	EFO		8756977		1.6		0		78
49	ECO	9534159	3932306	1.8	1	33	17	138	69
	EFO		8887142		1.7		0		69
50	ECO	10234974	4623504	1.2	1.1	0	0	179	88
	EFO		9707648		1.1		0		83
51	ECO	13083068	3337682	13.4	13.4	359	145	0	0
	EFO		10468485		11.5		173		0
52	ECO	13118268	4643577	11	11	359	92	0	0
	EFO		9876615		7.3		154		0
53	ECO	13325639	3570294	3.3	3.2	357	165	0	0
	EFO		6380495		2.4		171		0
54	ECO	21763382	4093040	2	1.9	0	0	210	99
	EFO		12709991		1.5		0		84
55	ECO	22219369	4935365	1.8	1.7	0	0	189	26
	EFO		18291431		1.5		0		77
56	ECO	30355938	4259242	13.1	13.1	86	43	45	22
	EFO		14791564		6.6		39		22
57	ECO	30662261	3503535	117.7	117.7	99	49	123	62
	EFO		10786780		41.8		49		61
58	ECO	31056605	3144955	176.4	176.4	120	60	148	74
	EFO		18911498		115.6		55		69
59	ECO	33349958	3147352	229.5	187.6	97	49	170	85
	EFO		7367501		51.1		48		85
60	ECO	40237491	26191894	17.8	9	353	164	0	0
	EFO		13337529		9.3		150		0

3.4 NUMERICAL APPLICATION

In order to apply the proposed model to a larger network, more complex than the one used in the previous section, it has been used the Sioux Falls City network shown in Figure 10 as a real sized example. The network consists of 24 nodes, 76 links, and 552 OD pairs.

Signalized intersections are the nodes 4, 5, 6, 14 and 19 with a three-phases regulation scheme, and node 15 with a four-phases regulation scheme; for each of these six intersections, the starting effective cycle time is fixed to $C_t = 90$ seconds and the starting effective green time is equally divided for each phase.

The link cost values c_l are computed as in the sensitivity analysis (see section 3.3), and the free flow travel times tr_l are numerically equal to those proposed by LeBlanc et al. (1975) but expressed in minutes. Capacities have been set equal to 3600 veh/h for the peripheral links (i.e., links 1 and 3, 4 and 14, 16 and 19 ...) and equal to 1800 veh/h for the remaining ones.

The travel demand $\mathbf{d}$ (the same matrix of trips shown in LeBlanc et al. 1975) has been assigned to the network, using a Deterministic User Equilibrium traffic assignment model.

In this application, the vertical equity constraint that needs to be satisfied is the one related to flows on the arcs 41 and 44 (the ones that connect nodes 14-15), which have to be reduced by at least 10% compared to those on the same link in the corresponding starting configuration. The horizontal equity constraints are the same displayed in Table 3. The achieved results are summarized in Table 6 and Table 7.

Table 6 – Results of the numerical application on the Sioux Falls City network.

Opt.	z^{SC}	z^*	β_{max}	α^*	f^{*SC}_{14-15}	f^*_{14-15}
ECO	32010688	12696878	2.9710	1.4794	798	156
EFO		26177456		1.1072		303

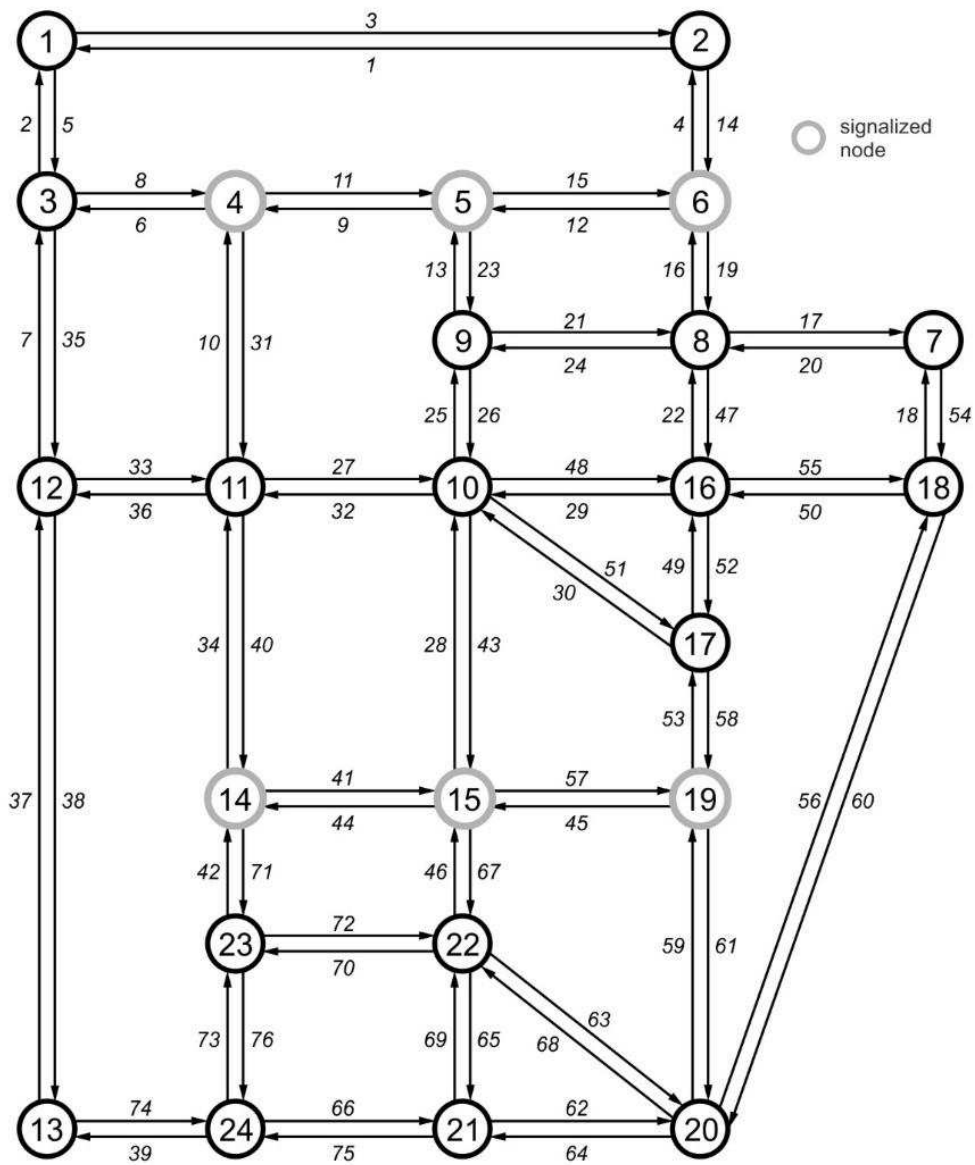


Fig. 10 – The Sioux Falls City network.

Table 7 – Optimized effective green times in seconds.

	g_1^4	g_2^4	g_3^4	g_1^5	g_2^5	g_3^5	g_1^6	g_2^6	g_3^6
SC	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0
CO	23.9	13.2	66.2	16.1	40.8	49.2	19.7	14.8	58.7
ECO	45.5	9.3	69.2	15.6	5.0	71.5	9.4	10.2	71.4
EFO	29.9	28.9	31.4	29.9	29.2	31.0	29.9	30.2	30.1

	g_1^{14}	g_2^{14}	g_3^{14}	g_1^{15}	g_2^{15}	g_3^{15}	g_4^{15}	g_1^{19}	g_2^{19}	g_3^{19}
SC	30.0	30.0	30.0	22.5	22.5	22.5	22.5	30.0	30.0	30.0
CO	37.8	15.5	79.8	79.4	12.2	15.0	13.9	13.4	48.2	80.0
ECO	36.9	12.6	80.0	80.0	8.6	12.8	19.7	13.2	48.2	8.4
EFO	34.5	30.8	30.3	29.4	10.9	22.0	26.5	27.2	32.0	30.2

Findings presented in Table 6 go basically to validate what is previously emerged from the sensitivity analysis performed on the smaller network. The EFO costs are higher than the ECO ones while remaining well below the initial costs of the network (z^{SC}). The vertical equity constraint applied on the selected link flows is satisfied in both optimizations, but the α^* value of the proposed fuzzy approach (α_{EFO}^*) proved to be lower than the corresponding optimized value of the crisp case (α_{ECO}^*).

Consequently, it is possible to assert that, with a fuzzy approach, although the total costs of network result generally higher, the equity aspects (either horizontal or vertical) appear to be satisfied to a greater extent, enabling to spread more fairly the ‘disadvantages’ arising from a network redesign.

Therefore, the preliminary conclusion that can be drawn at the end of this first formulation of the NDP model is that it seems to be important for decision makers to reach a compromise between costs and equity, not just focusing on the immediate feedback that an apparent saving can provide, but thinking about the real achievements (both spatial and social) that their actions will have on those who will actually take advantage of the transport network.

3.5 SECOND MODEL FORMULATION

The proposed second formulation of equity-based NDP expands the scope of the previous approach, taking public transit into account alongside private transport. Consequently, in this case, equity is defined as the mode-specific relative variation of the overall mobility between OD pairs. There is still a comparison between two specifications of multimodal equitable NDP (crisp and fuzzy) and the application of the approach in the case of a signal time planning problem in a small network.

3.5.1 General formulation

This second formulation addresses the shortcomings emerged by the literature review: namely, that multimodality has been largely ignored in the road and public transit NDP; and that, in addition, nearly all NDP are framed in a deterministic way. Accordingly, the idea to put forward a formulation that considers multimodal equity goals in a context of uncertain data/constraints.

Basically, the general formulation of the problem is the same described by Equations (7-11), where the objective function is to minimize the total system cost under a set of constraints, taking into account the user behavior.

It is worthy to highlight that nearly all road NDP (RNDP) studies deal with the improvement of existing networks. In contrast, public transit NDP (TNDP) studies mainly deal with the configuration of new transit networks, or the partial reconfiguration of the existing networks. A common characteristic of RNDP and TNDP is that they consider only a single mode. In reality, however, multiple modes coexist and interact with each other. Hence, this formulation is a case of a Multi-Modal Network Design Problem (MMNDP) with two modes (private transport and buses).

A multimodal problem arises when at least two modes are considered and simulated, even if design decisions are related to only one of the modes (Faharani et al., 2013). In this case, within the general formulation of the proposed model, the interactions

between flows of different modes are neglected: networks of different modes are not related to each other, and thus the flows of one mode do not have any effect on the flows of the other mode. Indeed, in the suggested car-bus problem, buses move in dedicated lanes, and therefore public transit flows are physically separated from private transport ones. Consequently, it is possible to evaluate z (the function of the total cost of the network) simply as the sum of the costs of private and public transport.

3.5.2 Multimodal equity constraint specification

In this second formulation, starting again from the definition given by Meng and Yang (2002), equity has been included in the NDP model considering the OD travel costs generated by the modification of a network. Therefore, restating Equation (13), it is possible to assert that:

$$\alpha_w = \frac{z_w(\mathbf{x})}{\bar{z}_w} \quad (30)$$

Equation (30) means that, in order to evaluate the equity in the network, it has to be considered the ratio between the equilibrium travel costs after ($z_w(\mathbf{x})$) and before ($\bar{z}_w$) changing the network for each OD pair w in the network. Then, observing that Equation (13) asserts that $\alpha = \max_w (\alpha_w)$, it is possible to say that if $\alpha < 1$ all users benefit from the network design implementation, if $\alpha > 1$ some users experience an increase of travel costs. As stated previously, to improve the equity of the solution a constraint can be added to the traditional NDP formulation, enforcing that the possible equilibrium OD travel cost increases are below a given threshold set by the decision makers. In other words, this threshold is the maximum permissible lack of equitability of the benefit distribution.

However, the formulation of Meng and Yang (2002) (and consequently the first formulation mentioned in Section 3.2 of the present dissertation) neglects the level of the

demand between the OD pairs. Therefore, the use of an equity constraint based on α may generate solutions with remarkable benefits (in terms of individual costs) for OD pairs with low demand and smaller negative consequences for busy OD connections. To avoid this problem, let:

$$\delta_w = \frac{d_w}{\sum d_w} (\alpha_w - 1) \quad (31)$$

i.e. the relative variation of OD pair cost brought about by the network modification, weighted by the ratio of the demand d_w associated to that OD to the total demand.

δ_w is an indicator of the variation of the overall mobility cost. Since 1 is subtracted from α , it is possible to state that the new discriminatory value (above which some users experience an increase of travel costs) is 0. Hence, only if $\delta < 0$ all users benefit from the network design implementation.

In this second multimodal NDP formulation, the proposal is to account for the equity issue in MMNDP by adding a constraint on the value of $\delta = \max_w (\delta_w)$.

3.6 NUMERICAL EXAMPLE

3.6.1 Comparison of two specifications of equity-based MMNDP

This subsection focuses on two different specifications of the multimodal NDP formulation explained in section 3.5: one based on crisp optimization, the other on fuzzy optimization. Basically, the framework is the same of section 3.3, as it has been used again the network of Yang et al. (2001) shown in Figure 7 to perform the application, and there is still a comparison between a certain and uncertain approach. The step forward is essentially represented by the addition of a bus lane along the route, and the refinement of the equity constraint that now takes into account the percentage of demand traveling between each OD pair.

During this test, supply design variables are the signal setting parameters; the aim is to search for the vector of optimal effective green times ($\mathbf{x}^*$) for all signalized intersections. These values are obtained through the minimization of the network total cost z depending on signal settings ($\mathbf{x}$), on equilibrium flows ($\mathbf{f}^*$) and on equity constraints. This analysis (as previously stated) has been carried out on the test network and data proposed by Yang et al. (2001), to which it has been added a bus rapid transit lane (Fig. 11).

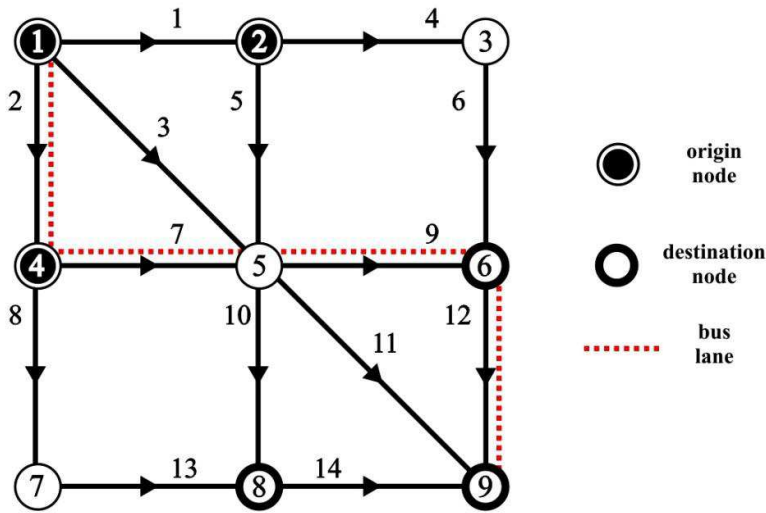


Fig. 11 – Test network for the second model formulation.

The description of the graph of the test network can be found in section 3.3. In Figure 11, the public transport line that it is supposed to operate in the network is displayed (with a red dashed line), assuming the presence of a bus lane along the route. Therefore, in this example, it has been neglected the interaction between flows of different modes (although the formulation presented above is general, and can be applied to fully multimodal situations). The bus stops coincide with the nodes traversed by the bus line (1,4,5,6,9). In this simulation, accurate data about the demand assigned to each bus stop are not provided, but it is known how many people globally moving

from a given origin to a given destination using public transport. The dwell time has been assumed equal to 10 seconds for all the stops.

The global travel demand $\mathbf{d}$ (see Table 2) is assigned to the network using again a Deterministic User Equilibrium traffic assignment model. It has been assumed that 30% of the demand generated by the OD pairs 1-6 and 1-9 is served by the public transport line.

It is worthy to specify different link cost functions for the two modes of transport. For the private transport system, the link cost c_i is the sum of the link travel time and the waiting time at the signalized intersections (see Equations 26-28).

In the public transport, the link cost values c_{bus} are the sum of the link travel time, the bus dwell time and the waiting time due to the signalized intersections. Travel time t'_i at every link i can be calculated by Equation (32):

$$t'_i = \frac{L_i}{v_i} \quad i = 1, 2, \dots \quad (32)$$

where L_i is the length of link i , while v_i is the bus speed at link i (in the test assumed to be 20 km/h). The model of dwell time t'_{dw} of buses at a bus stop is represented by Equation (33) (Liu and Sinha, 2007):

$$t'_{dw} = c_0 + \sum_j c_j p_j N \quad j = 1, 2, \dots \quad (33)$$

where c_j is the average boarding time of ticket type j passengers, p_j the proportion of type j passengers and N the total number of passengers waiting for the service. Parameter c_0 is a constant door opening and closing time and an observed value of 5 seconds (Clark and Pretty 1992) has been used in the model. In this study, only one ticket type has been considered.

The waiting time t'_{UD} is estimated using the Webster's Uniform Delay Model (Equation (34)), on the assumptions of stable flow and a simple uniform arrival function:

$$t'_{UD} = \frac{RV}{2} \quad (34)$$

where R is the duration of the red phase, V is the number of vehicles arriving during the time interval given by the difference between the instant in which they arrive at the traffic light and when the light has turned green. The length of red phase is given as the proportion of the cycle length which is not green.

The optimization problems have been solved using genetic algorithm (GA) meta-heuristic. The analysis entails two steps: in the first one 600 starting configurations have been generated, with different vectors of design variables; in the second step, GA has been applied using each of these starting configurations as the starting point of different runs.

Table 8 – MMNDP specifications.

		<i>Equity Crisp Specification (ECS)</i>	<i>Equity Fuzzy Specification (EFS)</i>
<i>Objective functions</i>	Satisfaction	-	$\max (g_1h_1 + g_2h_2 + g_3h_3)$
	Network total cost	$\min z(\mathbf{x}, \mathbf{f}^*)$	-
<i>Problem constraints</i>	Network total cost	-	$z(\mathbf{x}(h_i), \mathbf{f}^*) \leq (1 - h_i)$
	Car equity	$\delta_{ECS}^{car} \leq \delta_{max}^{car}$	$\delta_{EFS}^{car} \leq \delta_{l_car} + (\delta_{u_car} - \delta_{l_car}) \cdot (1 - h_2)$
	Bus equity	$\delta_{ECS}^{bus} \leq \delta_{max}^{bus}$	$\delta_{EFS}^{bus} \leq \delta_{l_bus} + (\delta_{u_bus} - \delta_{l_bus}) \cdot (1 - h_3)$
	Demand – flows consistency	$\mathbf{f}^* = \Delta(\mathbf{x})\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f}^*, \mathbf{x}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{x}))$	
	Cycle time consistency	$\sum_{nd} g_{ph}^{nd} = 90 \quad \forall \quad nd \in \{5,6,8\}$	

The crisp (ECS) and fuzzy (EFS) specifications of the MMNDP with equity constraints are shown in Table 8.

The objective in ECS is to minimize the total network cost, the sum of the costs of car and public transport users. In EFS aim is to maximize the overall satisfaction H , and a constraint on the admissible network total cost has been added. The total satisfaction H is the weighted sum of three components h_1 , h_2 , and h_3 , concerning the total network cost, the equity of the solution to private transport users, and the equity to public transport users respectively. The constraint on the total network cost requires the solution cost to be smaller than the one of the corresponding starting configuration. Each satisfaction index is a measure of how well the solution performs in terms of the related quantity. The weights (g_1, g_2, g_3) are introduced to let the decision makers rank the components of the overall satisfaction. They have been assumed equal to 1 in this application. The equity constraints of the fuzzy specification account for a certain degree of uncertainty in their definition. In the following, $\delta_{\max}$ and $\delta_{\min}$ in the ECS are used as the upper (δ_u) and lower (δ_l) bound of the fuzzy set of the equity constraints in EFS, but in general, they can be different.

Using different starting configuration, each GA run (potentially) generates a different solution. The results of the experiments are summarized in Figure 12 by means of box plots. In each box, the central mark indicates the median value, the edges are the 25th and 75th percentiles, the whiskers extend to the most extreme non-outlier values, and outliers are plotted individually.

Figures 12(a) and 12(b) show the private and public transport costs respectively. The fuzzy approach seems to work better in the case of public transport, although the values are more scattered. Figures 12(c) and 12(d) show the degree of equity achieved by the two approaches for car and bus users respectively. The values are comparable for private transport, whereas the fuzzy specification gives definitely better results for the public transport service (note that lower values of δ^{bus} correspond to more equitable solutions).

Overall, the test suggests that EFS should be preferred to ECS to promote the use of public transport because it generates more convenient and equitable solutions for bus users. Of course, this result cannot be considered conclusive but requires further confirmation by applications to more complex networks. In any case, the fuzzy ap-

proach is more suitable and theoretically consistent with contexts with approximate and/or uncertain data.

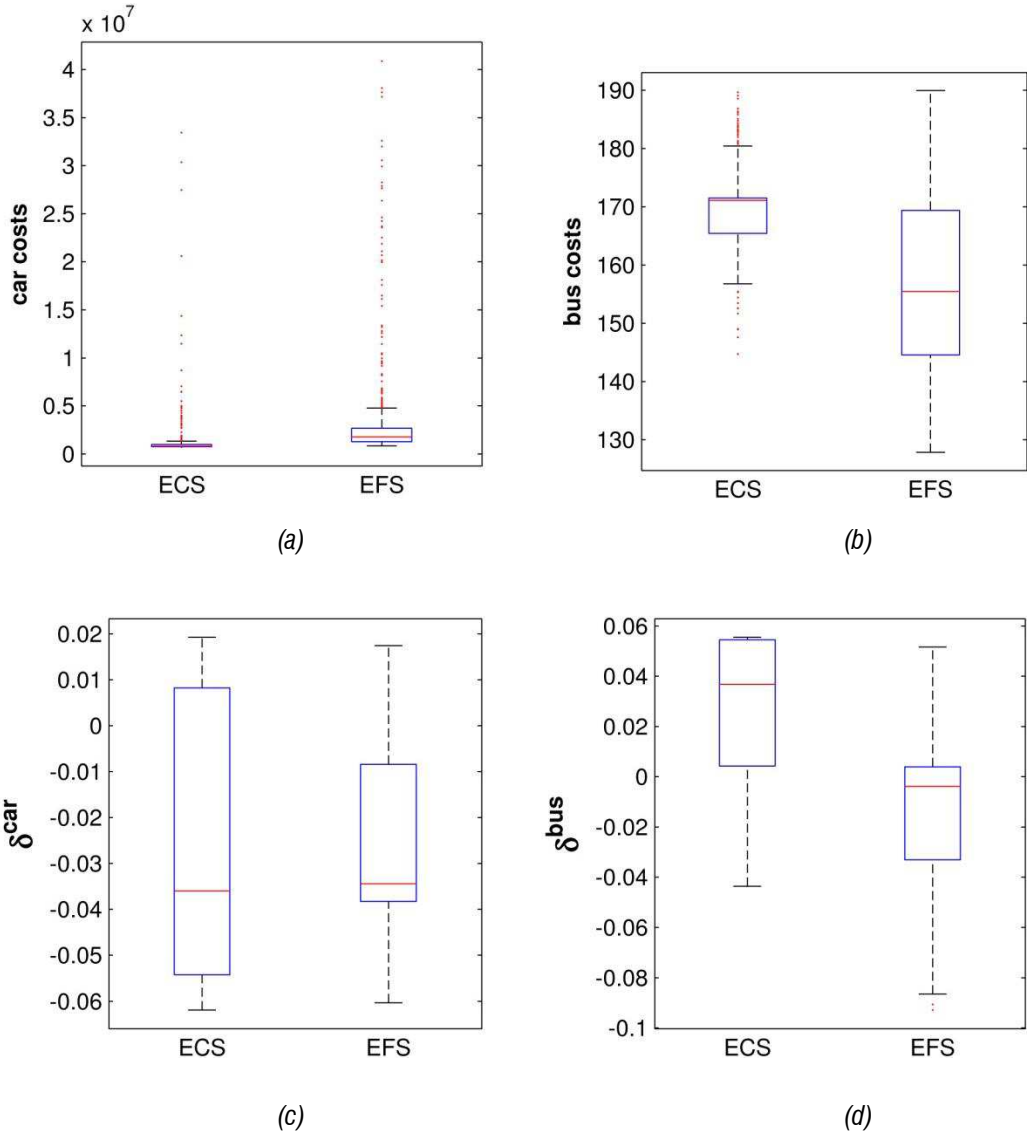


Fig. 12 – Optimizations results for the Equity Crisp Specification (ECS) and the Equity Fuzzy Specification (EFS): (a) private transport costs; (b) public transport costs; (c) private transport equity indicator; (d) public transport equity indicator.

In the next section (3.7), the solutions of EFS are analyzed in greater detail, focusing on the three components of the overall satisfaction H used as an objective function.

3.6.2 Multi-criteria decision in EFS

Solving a MMNDP taking into account not only the overall network cost but also the equity of the solution for each class of users means that the allocation problem has several concurrent (and possibly conflicting) goals. In order to find a proper solution, the above explained EFS has been formulated as a single objective maximization problem, in which the objective function is the sum of the degree at which the solution achieves each goal. Note that this presupposes the existence of a single network authority, with the power of imposing a design. Meta-heuristic methods like GA are not always able to identify global maxima and, in general, each run finds a different solution (for example, in the performed test, each one of the 600 runs has a different starting configuration). Therefore, the problem arises of selecting one of the detected local optima as a solution to the problem. It is not so obvious to choose the solution with the maximum overall satisfaction since it is also possible that the authority is confronted with solutions with similar values of the overall satisfaction, deriving from different combinations of satisfaction of the competing goals. In these cases, it could be useful to propose the analysis of the *Pareto front of the local optima* in the space of the specific satisfactions.

Pareto optimality is a state of allocation of resources in which it is impossible to make any subject better off without making at least one other subject worse off (for further explanations, readers could refer to Das and Dennis 1997). A Pareto Optimal Solution (POS) dominates non-optimal solutions and is non-dominated by other POS. A POS is non-better than another POS in at least one objective (Deb 2001). Therefore, any POS may be attractive to the network authority (especially those with high overall satisfaction). The network design can be selected by choosing a point on the Pareto front (the geometric locus of Pareto optima) according to the authority's priorities.

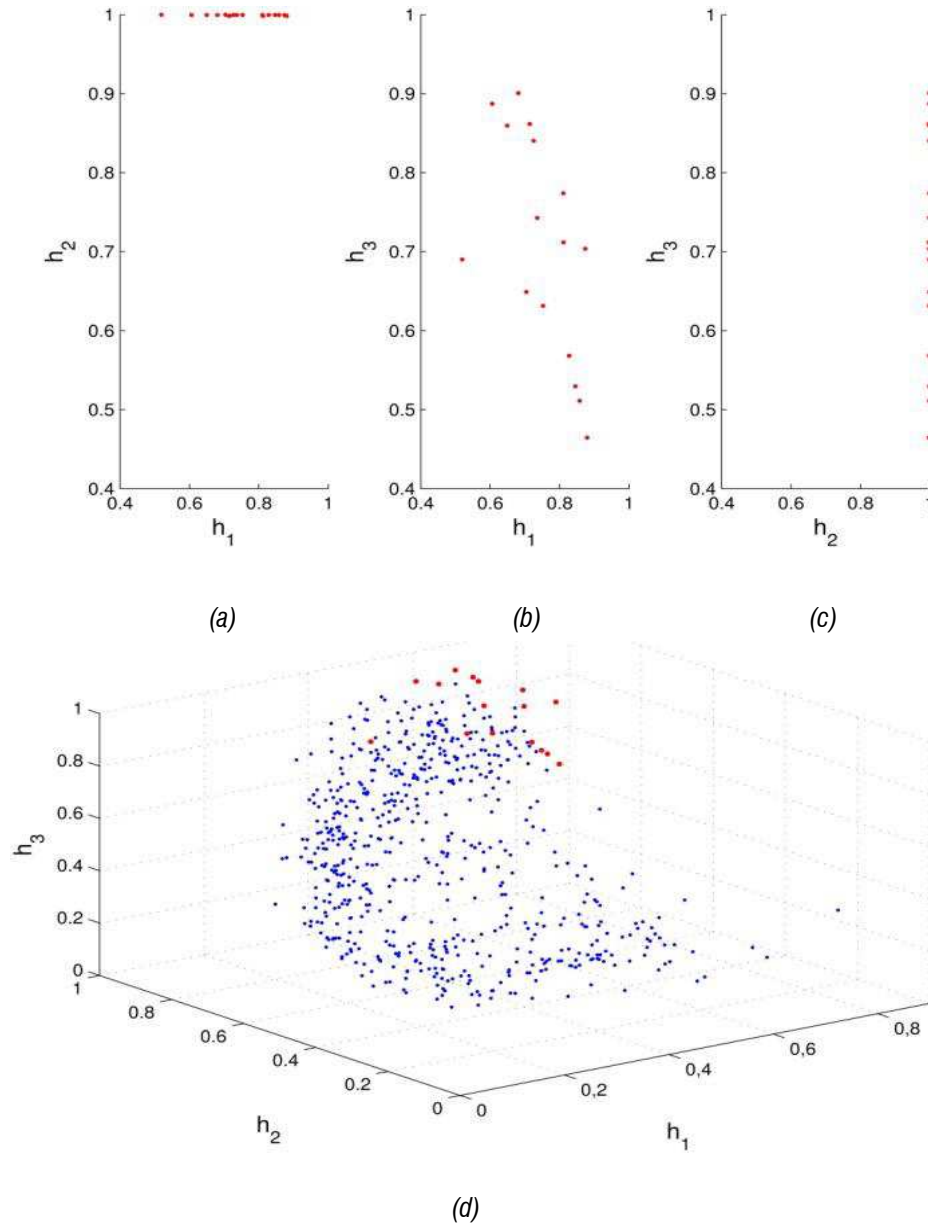


Fig. 13 – Graphical representation of Pareto fronts: (a) 2D - h_1 and h_2 ; (b) 2D - h_1 and h_3 ; (c) 2D - h_2 and h_3 ; (d) 3D - h_1, h_2 and h_3 .

Figures 13(a), 13(b) and 13(c) give a 2D representation of the Pareto front of satisfaction. Each point in Figure 13(d) represents the solution found by one run of GA (corresponding to a specific starting condition). The coordinates of the points are the values of the satisfaction of the three goals of the problem, namely overall cost, equity to car users, and equity to public transport users. The Pareto front is made up of the 16 non-dominated solutions in red. The coordinates of the 16 points on the front are given in Table 9, where solutions are listed for decreasing values of overall satisfaction.

Table 9 – Optimal Pareto solutions satisfaction.

Solution number	h_1 – Total costs	h_2 – Car equity	h_3 – Bus equity	$H = h_1 + h_2 + h_3$
S1	0.8107	0.9996	0.7738	2.5841
S2	0.6811	0.9996	0.9006	2.5813
S3	0.8742	0.9996	0.7035	2.5773
S4	0.7143	0.9984	0.8615	2.5741
S5	0.7256	0.9997	0.8403	2.5656
S6	0.8118	0.9983	0.7119	2.5219
S7	0.6496	0.9997	0.8596	2.5089
S8	0.6062	0.9997	0.8873	2.4932
S9	0.7363	0.9997	0.7428	2.4787
S10	0.8280	0.9996	0.5682	2.3958
S11	0.7527	0.9997	0.6313	2.3837
S12	0.8462	0.9997	0.5294	2.3753
S13	0.8580	0.9997	0.5112	2.3689
S14	0.7041	0.9999	0.6490	2.3530
S15	0.8800	0.9984	0.4645	2.3428
S16	0.5192	0.9999	0.6901	2.2092

It can be seen that all the POS have a level of satisfaction of the car equity constraint very close to 1. This means that it is easy to keep the fairness of the solution as to car drivers in the desired interval $[\delta_{min}^{car}, \delta_{max}^{car}]$; indeed, very likely the enhancement of the public transport equity does not affect to a great extent the private transport, as in the present case study buses move on a rapid transit (separate) lane. Probably allow-

ing cars and buses to stay in the same lane (neglecting the rapid transit bus lane), there would be more interaction between the two modes, and enhancements on public transport equity would affect more the private transport. Another possibility could be adopting a tolling strategy to have access to certain areas of the transport network. The satisfaction of the constraint concerning the network cost ranges from 0.5192 to 0.8580, and the one related to public transport equity takes values between 0.4605 and 0.9006. Note that the difference between the level of the overall satisfaction of S1 and S2 is very small (less than 0.11%). The two solutions have also the same level of satisfaction of the private transport equity constraint, but they clearly differ as to the satisfaction of the two other constraints: S1 performs better in terms of overall costs but is dominated by S2 as to the public transport equity.

These 16 solutions reach the highest level of satisfaction. This is indicative of how the optimization is able to improve the status of the corresponding starting configuration, namely how much it manages to reduce the overall costs and increase the achieved levels of equity. Nevertheless, objectively there is no clue about the actual level of equity and the actual costs to be incurred. These are displayed in Figures 14(a), 14(b), 14(c) and 14(d). Costs and equity values are normalized in the range between 0 and 1; the lowest overall cost scored at the end of the 600 runs is assumed to be the zero, the uppermost is set equal to 1. Same holds for the private and public transport indicators of equity shown on the two other axes. The Pareto front is made up of the 63 non-dominated solutions in red (see Table 10), the ones able to achieve the lowest overall cost and the greater level of car and bus equity (the closest to 0). Therefore, a network authority concerned with reducing the overall cost should opt for one of these 63 solutions, while taking into account the equity components. This would make the decision-making process more transparent, permitting the decision maker to identify, among all the optimal solutions, that with the set of goal-specific values which suits best his priorities.

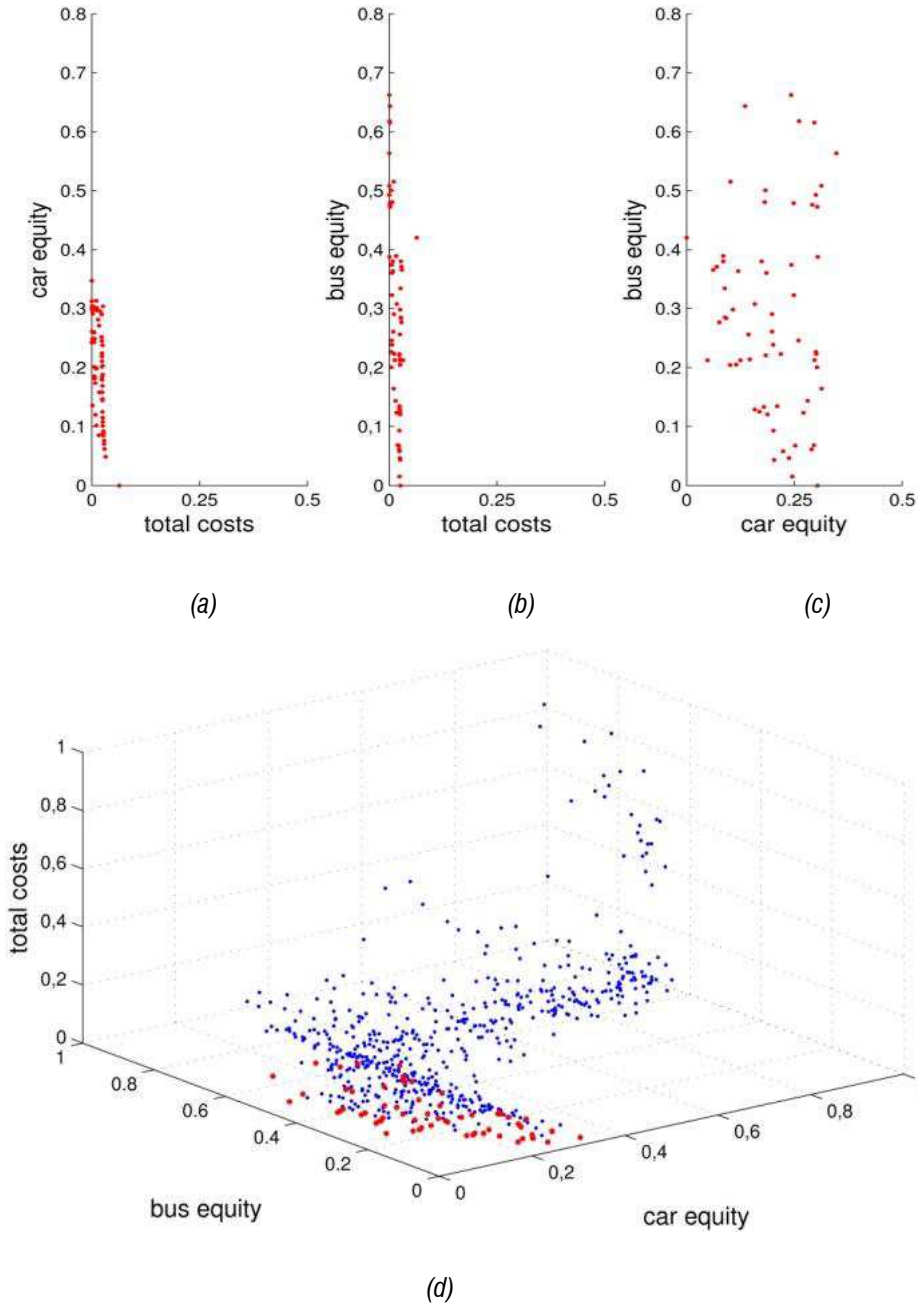


Fig. 14 – Graphical representation of Pareto fronts: (a) 2D - total costs and car equity; (b) 2D - total costs and bus equity; (c) 2D – car equity and bus equity; (d) 3D – car equity, bus equity, and total costs.

Table 10 – Optimal Pareto solutions according to normalized total costs, car equity, and bus equity.

Solution number	Normalized total costs	Normalized car equity	Normalized bus equity
<i>S1</i>	0,001817	0,135494	0,643487
<i>S2</i>	0,023742	0,218954	0,222952
<i>S3</i>	0,007293	0,181539	0,480636
<i>S4</i>	0,000204	0,242586	0,662216
<i>S5</i>	0,002452	0,303449	0,472696
<i>S6</i>	0	0,347331	0,563463
<i>S7</i>	0,002145	0,291458	0,476268
<i>S8</i>	0,000213	0,312779	0,508078
<i>S9</i>	0,001655	0,296206	0,615436
<i>S10</i>	0,026695	0,088666	0,334223
<i>S11</i>	0,02493	0,144361	0,256121
<i>S12</i>	0,000607	0,261284	0,618044
<i>S13</i>	0,005969	0,243089	0,37428
<i>S14</i>	0,006572	0,185123	0,360606
<i>S15</i>	0,001782	0,248595	0,478749
<i>S16</i>	0,001013	0,30417	0,387852
<i>S17</i>	0,029631	0,061864	0,365732
<i>S18</i>	0,026904	0,089517	0,285019
<i>S19</i>	0,010851	0,101878	0,515291
<i>S20</i>	0,024813	0,146699	0,213958
<i>S21</i>	0,025617	0,115107	0,205036
<i>S22</i>	0,024016	0,210628	0,134771
<i>S23</i>	0,024477	0,158466	0,128987
<i>S24</i>	0,023491	0,290262	0,0614
<i>S25</i>	0,006341	0,249078	0,322756
<i>S26</i>	0,063913	0	0,420272
<i>S27</i>	0,027279	0,085096	0,380145
<i>S28</i>	0,028717	0,07042	0,371031
<i>S29</i>	0,025373	0,107955	0,29854
<i>S30</i>	0,027362	0,092183	0,283621
<i>S31</i>	0,028502	0,076247	0,276671
<i>S32</i>	0,023902	0,183866	0,221001
<i>S33</i>	0,025004	0,124951	0,212623
<i>S34</i>	0,025682	0,101109	0,204615
<i>S35</i>	0,024155	0,179439	0,133283

S36	0,02511	0,169223	0,12529
S37	0,025732	0,188019	0,120487
S38	0,007974	0,173871	0,380091
S39	0,008617	0,119832	0,363672
S40	0,02305	0,252103	0,067663
S41	0,023754	0,224424	0,058224
S42	0,017147	0,27108	0,123581
S43	0,009529	0,198461	0,260954
S44	0,006131	0,259664	0,246151
S45	0,006434	0,201094	0,238542
S46	0,011022	0,313479	0,164133
S47	0,005576	0,299962	0,226484
S48	0,03239	0,048832	0,212333
S49	0,016742	0,085346	0,389304
S50	0,024117	0,20178	0,093194
S51	0,017393	0,158105	0,307873
S52	0,011161	0,19844	0,290809
S53	0,024038	0,245339	0,015067
S54	0,02602	0,303932	0
S55	0,012777	0,297109	0,212978
S56	0,004373	0,302766	0,200588
S57	0,024907	0,238045	0,04646
S58	0,025583	0,203222	0,043586
S59	0,012071	0,301339	0,223142
S60	0,019833	0,295455	0,068319
S61	0,014792	0,281253	0,143633
S62	0,000516	0,300215	0,493047
S63	0,004597	0,18334	0,500727

4.0 INCLUDING EQUITY IN TRANSIT NDP

This section puts forward a method to incorporate equity goals in a Transit Network Design Problem, formulated as a minimization problem with an objective function considering the cost of users, operators, and non-satisfied demand.

4.1 GENERAL OVERVIEW

As explained in previous chapters, in the transport literature equity has been and is still used with a variety of meanings and purposes. The aim of this section is to deal with transit service, looking at how to make it equitable from a spatial and a social point of view. Traditionally equity has been neglected in transit planning and, in the best cases, it has been an afterthought during service provision. Consequently low-income and socially disadvantaged individuals, who depend more on transit services, often face barriers to accessing to their desired destination due to insufficient/inefficient supply.

Clearly, there is a tradeoff that exists in making public transit accessible (Murray and Xu 2003). For example, one variable that determines whether transit is an appealing option for potential users is the balance between close and dense stop placement with far and sparse stop placement (Federal Transit Administration 1996): if the first shorten walks, the second can promote quicker trip travel times. Consequently, ac-

According to the urban area and orientation of the service, a planner needs to be sensitive to the degree to which stop frequency reflects the best compromise between faster travel speeds (and greater geographic coverage) and shorter access distances. Hence, the idea to propose a methodology to plan and design public transport routes, which meets the needs of communities fostering equitable accessibility, bridging the gap in the literature. As a matter of fact, it is important not only to analyze case studies defining the achieved level of equity but also to be aware of what it is wrong in their transit services, trying to incorporate a renewed set of principles in designing new realities.

Through a sensitivity analysis carried out on a test network, it has been possible to understand how the costs of the system change with the pursued level of equity (both horizontal and vertical). Furthermore, an application to a real case of study is presented to support the usefulness of the method, accompanied by a discussion about the results achieved.

4.2 SOLUTION METHODOLOGY

After a summary of the notation that is going to be used in this chapter, the transit equity constraint to be added to the classical formulation of TNDP is described. The proposed solution framework mainly consists of two components: an initial candidate route set generation procedure; and a social costs optimization that generates the optimal transit route set with the associated route frequencies.

4.2.1 Mathematical notation

Every network can be modeled by means of a directed graph $G = \{N, A\}$, establishing a finite number of nodes $n \in N$ to be connected by arcs $a \in A$. A 'route' is defined as a sequence of adjacent nodes in G , while a 'transfer path' is a cumulative

path using more than one route. Each arc has an associated cost c_a that represents the in-vehicle (or onboard) travel time, i.e. the time spent by vehicles to travel on it. The demand corresponding to a given zonal partition (traffic zone) is considered concentrated in centroid nodes (Cascetta 2009) and is represented by an origin-destination matrix $OD = \{d_{ij}, i, j \in [1 \dots n]\}$, where d_{ij} denotes the demand from node i to node j , expressed in trips per time unit in a given time period. In the rest of the chapter, it has been used the following notation.

Sets and indices

$n \in N$	Nodes.
$a \in A$	Arcs.
$t_k \in T$	Terminal pairs.
$\bar{t}$	Number of terminal pairs.
$m \in M$	Length intervals.
$\bar{m}$	Number of length intervals.
r_k	Generic route k .
$\mathfrak{R}$	Candidate set of routes.
$\mathfrak{R}'$	Optimal set of routes.
F'	Optimal set of frequencies associated with the optimal routes.
$a' \in A$	Arcs which constitute the optimal routes.
u	Number of optimal routes.
D	District (traffic or travel demand zone).
b	Number of walk access buffers to stops/stations in each district D .
B_b	Buffer b for each stop/station in each district D .
y	Socioeconomic categories.

Data and variables

Area	Square kilometer spatial area (km^2).
SL	Service level measure (bus/h).
w_y	Weight of the category y (%).

x_y	Value of the category y (number of people belonging to the category).
Pop_D	Total population in the district D .
$L_{\min}$	Minimum length of any route in the transit network (km).
$L_{\max}$	Maximum length of any route in the transit network (km).
α	Maximum allowed deviation from shortest path for any OD pair path (%).
k_{tr}	Maximum number of transfers in a path (number of vehicle changing).
$u_{\min}$	Minimum allowed number of routes in the network.
$u_{\max}$	Maximum allowed number of routes in the network.
$h_{\min}$	Minimum headway required for any route (min).
$h_{\max}$	Maximum headway required for any route (min)
W	Maximum bus fleet size available for operations on the route network.
P	Capacity of vehicles operating on network (pax/bus)
η	Desired vehicle occupancy (%).
δ	Minimum percentage of the total demand to cover (%).
β	Maximum value for the equity constraint.
ρ	Number of nodes constituting the shortest path.
Δt	Number of nodes for each length interval $m \in M$
d_{tot}	$\sum_{i=1}^n \sum_{j=1}^n d_{ij}$ Total demand on the transit network.
d_s	Share of d_{tot} covered by routes in $\mathfrak{R}$ directly (without transfers) or indirectly.
Q	Positive even number.
Z	Overall social cost of the final transit network (h)
$\gamma_1, \gamma_2, \gamma_3$	Weights reflecting the relative importance of user cost, operator cost, and unsatisfied total demand cost.
c_a	Cost associated to arc a (in-vehicle travel time).
v_a	Flow on arc a (person/h)
C_v	Bus operating cost per hour (currency/vehicle/h).
C_m	Value of time (currency/min).
C_d	Value of each unsatisfied transit demand (currency/person).
O_v	Operating hours for a bus running on any route (h).
T_{rk}	Round trip time of route r_k (h)
h_{rk}	Bus headway operating on route r_k (min/vehicle).
L_{rk}	Overall length of route r_k (km).

4.2.2 Transit equity constraint specification

The most innovative element introduced in the solution of the proposed TNDP consists, as stated before, in the attempt to achieve equity goals by means of a new constraint to be added to the problem. This constraint needs to be able to take into account both spatial/horizontal and social/vertical aspects, giving undoubtedly more thought to the planning process of a public transport network. Therefore, it is important to clarify how to summarize all these features into a single indicator capable of pushing toward the design of a fairer transit network.

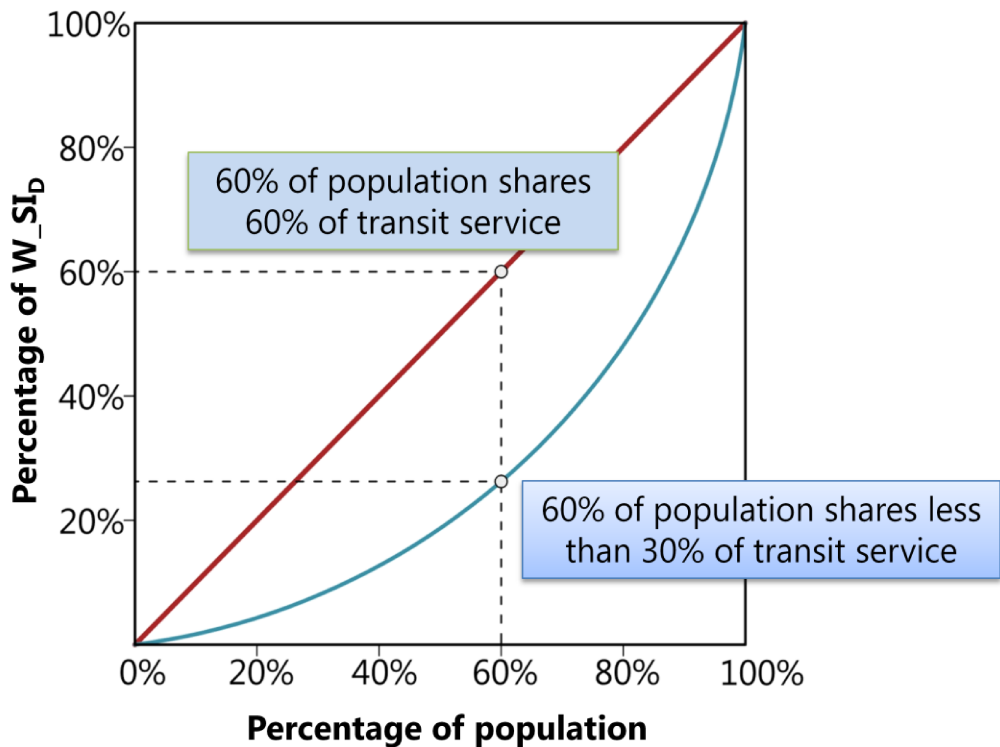


Fig. 15 – Representation of the line of equality (in red), and the Lorenz curve (in blue) related to the calculation of the revised Gini coefficient (R_Gini).

The proposed indicator can be defined as Revised Gini coefficient (from this moment on 'R_Gini') calculated on the entire network. The suggestion is to start from the original Gini's formulation (see subsection 2.3.3), where it corresponds with the ratio of the area between the line of equality and the Lorenz curve, to the total area below the line of equality (Figure 3).

However, there is a significant difference: if on the abscissa axis it is still possible to find the percentage of the resident population, on the y-axis there is the level of transit supply weighted according to the public transport need index (W_SI_D , specified in Equation (35)), in order to look also at the percentage of disadvantaged people living in each travel demand zone. In this way, the Lorenz curve associated with a given network represents the cumulative proportion of population against the cumulative proportion of the weighted transit service supply (Figure 15).

$$W_SI_D = \sum_b \left(\frac{Area_{Bb}}{Area_D} \cdot SL_{Bb} \right) \cdot \left(100 - \frac{(\sum_y w_y x_y) \cdot 100}{Pop_D} + 1 \right) \quad (35)$$

The ultimate aim is to guarantee that the final configuration of the transit service is the fairest possible compromise, according to both spatial distribution and social needs. Indeed, if it wants simply to be pursued a horizontal equity goal (i.e, SI_D in Equation (5)), it would be sufficient to ensure that each zone has an even number of bus stops and even frequencies, commensurate to the number of the residents in that district. Instead, using the W_SI_D in order to obtain the Lorenz curve, the goal is to provide a more massive presence of bus stops (so as to step up the sum of $Area_{Bb}$) and more frequent transit lines (increasing SL_{Bb}) in the areas with a larger presence of disadvantaged people.

It is worthy to explain the meaning of the expression given in the second round brackets of Equation (35). It is equal to the complement to 100 of the disadvantaged population deduced by the PTN_D (see Equation (6)), plus one (component added to prevent that the resultant product equals zero). As a matter of fact, the public transport need index is an input data of the problem, associated with the demographic composition

of each district and thus unchangeable for the purposes of the global optimization. Therefore, the larger is the number of disadvantaged people in a given demand zone, the more the value of the correlated W_Sl_D tends to decrease. Consequently, in order to guarantee to that penalized zone a level of transit supply able to compete with the one of the others districts in the network (in order to reach a global equity), the process of optimization intervenes on those 'editable' parameters related to the final configuration of the transit network (i.e., the number and the location of bus stops and their level of service). Accordingly, the optimal solution coincides with a transit network capable of serving in a more widespread manner those areas that need it most.

4.2.3 Proposed solution method

The proposed solution framework consists of two main components:

- (a) a starting candidate route set generation procedures;
- (b) a social costs minimization module with Genetic Algorithm (GA) solution approach to generate the optimal transit route set with the associated service frequencies, in compliance with all the constraints, including the equity one.

First of all, it is important to set the main inputs of the problem. Namely, having a directed graph made of a certain number of nodes and arcs, the traffic zones associated to the network have to be identified. In a real size application, they could coincide with a single census district or with an aggregation of more of them, depending mostly on their location (central or peripheral) within the city. Established a centroid for each zone, knowing the public transport Origin-Destination (OD) demand and the link costs associated with each arc in the network, the planner (or the transit agency) needs to set the remaining parameters. As a matter of fact, he/she has to determine, according to the size of the application and his/her expertise:

- the locations to be targeted as terminals, and a set of terminal pairs $t \in T$;

- the minimum and the maximum length of any route in the transit network, $L_{\min}$, and $L_{\max}$;
- the maximum percentage increase from the shortest path for any OD pair possible path, α ;
- the maximum number of transfers permitted in a path, k_{tr} (i.e., the number of the vehicle changing to reach a destination from a given origin).

With all these inputs, it is possible to generate all the existing routes between the terminals, to filter them according to a cost function (e.g. routes length) and their maximum deviation from the minimum cost (e.g. shortest) path, and to obtain a final set of feasible routes that represents one of the basic and fundamental input for the solution procedure.

Moving to the next step, the GA approach needs itself a series of parameters to be set, jointly with the above-mentioned set of feasible routes. Among them, it is essential to establish:

- the minimum and the maximum allowed number of routes in the network, $u_{\min}$ and $u_{\max}$;
- the minimum and maximum required headway, $h_{\min}$ and $h_{\max}$;
- the maximum bus fleet size available for operations on the route network, W ;
- the capacity of the vehicles operating on the network, P ;
- the desired vehicle occupancy, η ;
- the minimum percentage δ of the total demand to cover;
- the β value not to exceed for the equity constraint.

Applying the GA solution procedure, it is possible to detect the optimal transit route set (according to k_{tr}), the associated route frequencies and the related costs.

In the following subsections, the two components of this solution methodology are outlined in greater detail, to better understand the underlying elements of the proposed model.

4.2.4 Candidate route set generation procedure: model formulation

In order to generate all possible routes between each pair of terminals in the network, a way forward (Fan and Machemehl 2011) is the combined use of Dijkstra's shortest path algorithm (Abuja et al. 1993) and Yen's k-shortest path algorithm (Yen 1971).

Dijkstra's algorithm is an iterative method: at the n^{th} iteration, it is possible to find the n^{th} closest node to the start node (the origin) and the shortest route to that node. It stops iterating when the destination node is reached, even if not all the nodes in the network have been visited. By means of Dijkstra's, it is possible to know which is the shortest way (according to the predefined link costs) to connect a pair of nodes in the network.

Other attempts for constructing optimal transit routes are k-shortest path algorithms: they try to find alternatives able to increase the demand coverage, searching for quite longer routes. Yen is one of the first researchers to propose an easy-to-implement algorithm to solve the k-shortest path problem. The whole procedure of this algorithm can be summarized as follows: at first, he uses a standard shortest path algorithm (i.e., Dijkstra's) to find the shortest path from the source to the destination node, and store it; then, every node in the shortest path except the origin and destination ones are selected once and removed, the shortest path among the resulted paths would be considered the 2nd shortest path. This process could be repeated to generate the desired number of k-shortest paths. Therefore, through Yen's algorithm, a list of all the alternative paths that connect each pair of terminals is generated. All these potential paths have to be filtered according to some of the parameters set in the input stage (i.e., $L_{\min}$, $L_{\max}$, α), discarding the ones that are too long or too short, and those that deviate from the shortest path over a certain threshold. After stripping from the list these unsuitable paths, finally, it is possible to obtain only the feasible routes to use as input for the next step of the procedure.

In this subsection, however, it has been proposed an *alternative manner to generate the candidate route set*. In order to facilitate the understanding of the method, the following flowchart (Figure 16) summarizes the main steps of the suggested methodology.

Essentially, given a terminal pairs $t_k = (t'_k, t''_k) \in T$, $k \in [1 \dots \bar{t}]$, the main idea is to start from the shortest path between them. This shortest path is made by p nodes, belonging to the set $n \in N$ of the directed graph G . It is possible to find in the network a series of alternative paths able to connect the above mentioned two terminals, besides the shortest one. Through this method, the aim is to operate a selection of the best alternative paths able to satisfy to a greater extent the transit demand of the network. This selection is made establishing an interval (i.e. a certain number of nodes), called Δt . Adding progressively this Δt to the p nodes constituting the shortest path, it is possible to obtain different ranges of length ($m \in M$) for the alternative paths. For each range, by means of a GA process, the best path in terms of transit demand satisfaction can be found: as a matter of fact, the objective function to be minimized through the genetic algorithm is $(d_{\text{tot}} - d_s)$.

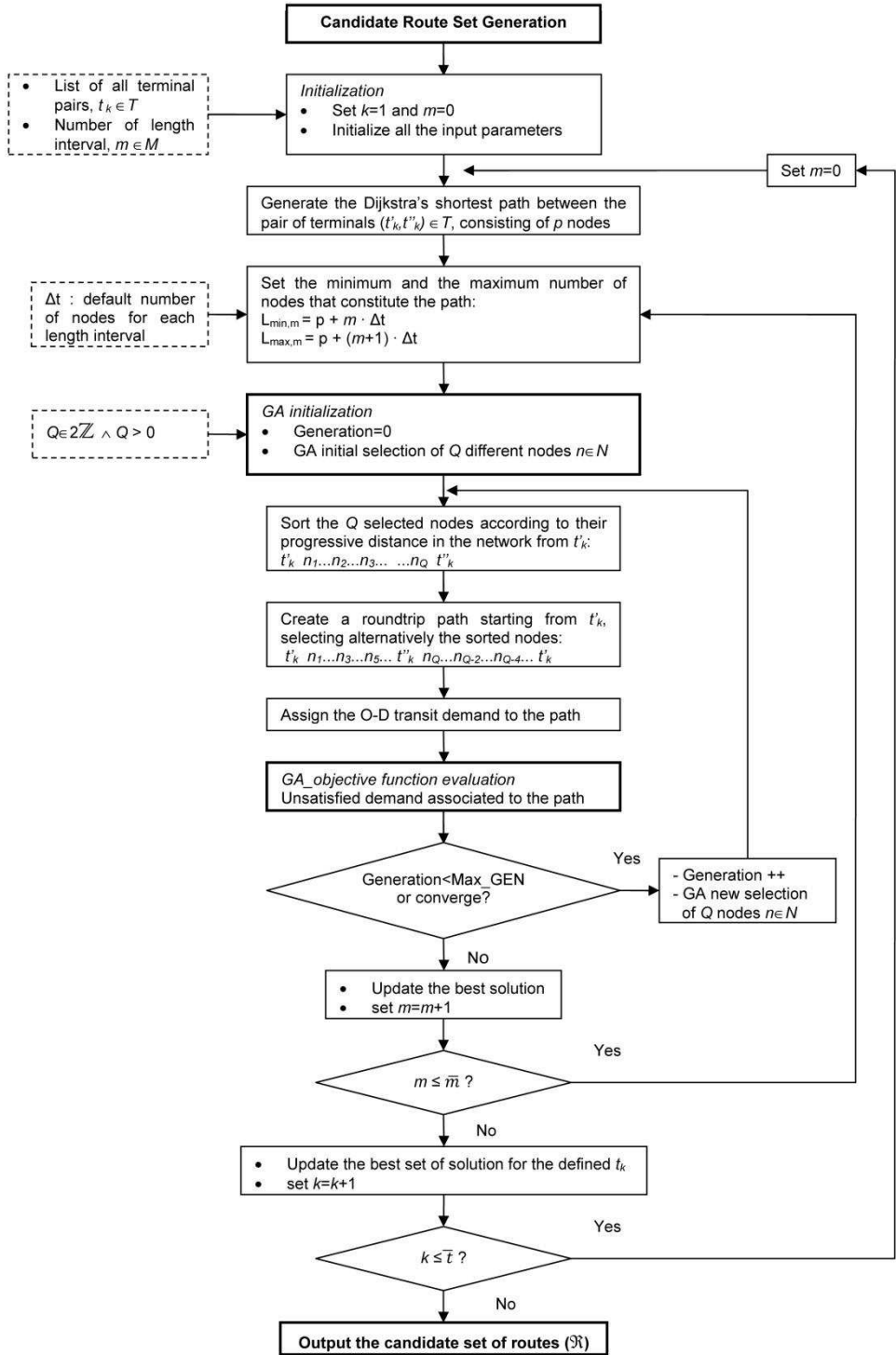


Fig. 16 – Flowchart of the proposed candidate route set generation procedure.

Looking at the flowchart in Figure 16, it is important to underline that the Q nodes selected by the GA must all be different from each other. The output is a set $\mathfrak{R}$ of candidate routes; the total number of routes is equal to the product of the number of terminal pairs $\bar{t}$ and the number of the chosen length intervals $\bar{m}$. Each route belonging the candidate set is the best in its length interval in satisfying the transit demand on the network.

4.2.5 Optimal transit route set generation procedure: model formulation

The goal of this subsection is to better explain the process that leads to the generation of an optimal route set, with associated costs and frequencies. In particular, it has been used a GA solution approach to solve this optimization problem.

Genetic Algorithms are a family of computational models inspired by evolution: they simulate the natural evolution of population and investigates the solution space by applying a probabilistic search process to all individuals representing a population of solutions simultaneously. GA searches by simulating evolution, starting from an initial set of hypothesis, and generating successive "generations" of solutions.

The objective function to be minimized corresponds to the overall social cost of the final transit network, assumed equal to the weighted sum of user, operator (the planner aims to make the best use of limited resources to optimize/improve the network performance) and unsatisfied demand (i.e., total travel demand excluding the transit demand served/satisfied by a specific network configuration) costs. They are represented respectively by the first, the second and third term in the sum given by Equation (36):

$$\min z = \gamma_1 \cdot \left(\sum_{a'} c_{a'} v_{a'} \right) + \gamma_2 \cdot \frac{C_v}{C_m} \cdot O_v \cdot \left(\sum_{k=1}^u \frac{T_{rk}}{h_{rk}} \right) + \gamma_3 \cdot \frac{C_d}{C_m} \cdot (d_{\text{tot}} - d_s) \quad (36)$$

subject to:

$$u_{\min} \leq u \leq u_{\max} \quad (\text{numbers of routes}) \quad (37)$$

$$h_{\min} \leq h_{rk} \leq h_{\max} \quad (\text{headway feasibility}) \quad (38)$$

$$\left(\sum_{k=1}^u \frac{T_{rk}}{h_{rk}} \right) \leq W \quad (\text{fleet size}) \quad (39)$$

$$d_s \geq \delta \cdot d_{\text{tot}} \quad (\text{demand coverage}) \quad (40)$$

$$R_{\text{Gini}} \leq \beta \quad (\text{equity}) \quad (41)$$

The input of the problem is the set of routes $\mathfrak{R}$ obtained at the end of the candidate route set generation procedure.

A solution of the problem is a pair $(\mathfrak{R}', F')$ where $\mathfrak{R}' = \{r_1 \dots r_u\}$, $\mathfrak{R}' \subset \mathfrak{R}$, and $F' = \{f_1 \dots f_u\}$ is the set of frequencies, where each f_k is a real value representing the inverse of the headway between subsequent vehicles on route r_k (headway).

The three weights $\gamma_1, \gamma_2, \gamma_3$ are introduced in the objective function to reflect the tradeoffs between user costs, operator costs, and unsatisfied travel trip costs (Fan and Machemehl 2011). They are dependent on the planners' experience and experts judgment. Different values of these weights may produce different optimal designs of the transit route network, still using the same proposed solution methodology.

The first constraint (Equation (37)) sets the minimum and the maximum number of routes, reflecting the fact that transit planners often set this range according to the fleet and the crew size. The second constraint (Equation (38)) on the headway feasibility reflects the usage of policy headways. The third in Equation (39) is the fleet size constraint, that represents the resource limits of the transit company and guarantees that the optimal network pattern never uses more vehicles than available. The fourth constraint (Equation (40)) specifies that d_s (the share of d_{tot} covered by routes directly or indirectly) has to be greater or equal to a specific percentage δ of the total transit demand. The last one (Equation (41)) is the equity constraint, which ensures that the proposed revised Gini coefficient R_{Gini} associated to a transit configuration does not exceed the β value specified by the transit network planner.

It is worthy to underline that the value of β should be selected carefully. On the one hand, if a lower value (close to 0, corresponding to a perfectly even distribution) is selected, the constraint may lead to infeasibility trying to reach the highest possible degree of equity in the network, and therefore there might be no solution to the problem. On the other hand, if a higher value is selected, the constraint may be too loose to be active (i.e., to influence the solution), and the equity aspects may be neglected.

4.3 SENSITIVITY ANALYSIS

The proposed solution methodology has been at first implemented on a test network, carrying out a sensitivity analysis to better clarify the correlation between equity and average costs on the network. The network, taken from Wan and Lo (2003), includes 5 traffic zones (from A to E, identified by different patterns), 10 nodes and 19 undirected arcs, as shown in Figure 17. The number on each link represents the link cost, c_a . There are nine OD pairs. The hourly OD demand is shown in Table 11.

Table 11 – OD travel demand (pax/h).

O-D	2-10	3-2	4-7	5-8	6-9	7-6	8-3	9-4	10-5
d	200	150	800	350	600	250	400	450	500

For sake of simplicity, in this sensitivity analysis, it has been assumed that all vertices of the infrastructure graph correspond to intersections of the network and could also be bus stops and centroids at the same time; this implies that the demand can be generated at any vertex and that walking arcs are then not considered. There is not a fixed set of terminals, consequently, each route generated at the end of the optimization could start and finish potentially in any node.

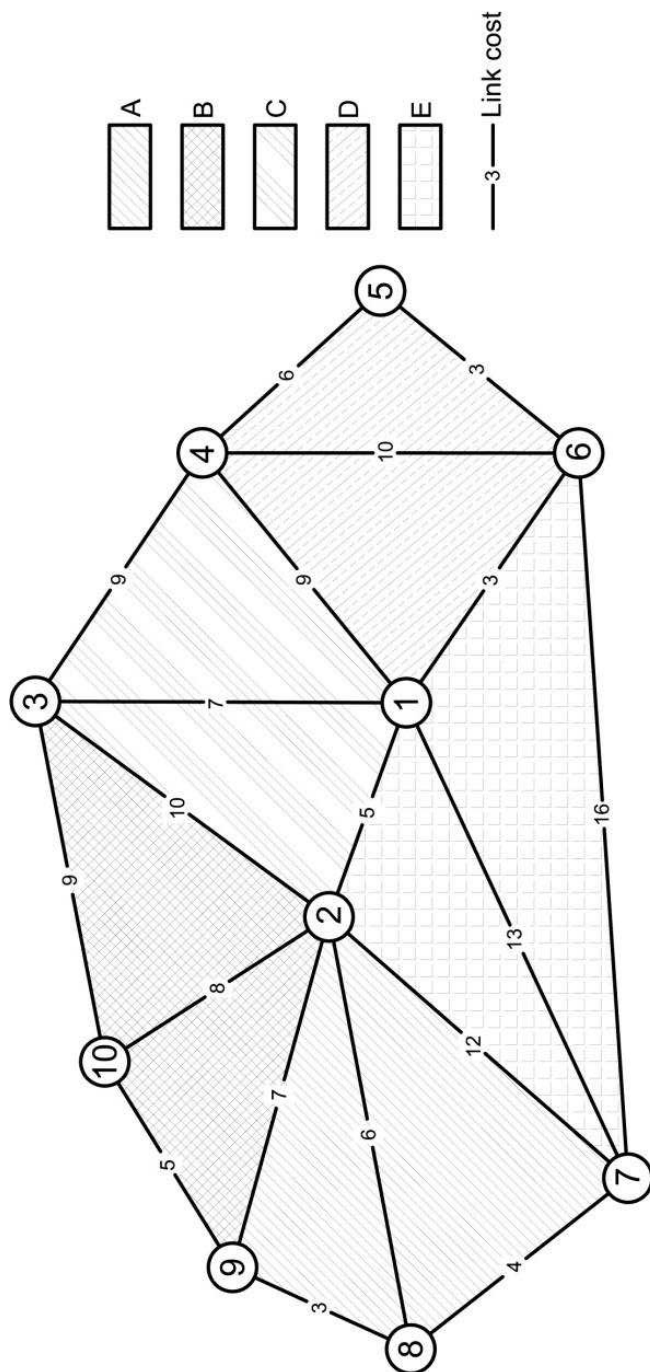


Fig. 17 – Sensitivity analysis network (Wan and Lo 2003)

In order to generate the candidate route set, during this sensitivity analysis it has been decided to follow the literature method previously mentioned, namely the combined use of Dijkstra's shortest path algorithm and Yen's k-shortest path algorithm, since the relatively small size of the network. In other words, given the limited number of nodes and arcs, indeed, it is possible to easily enumerate all possible paths that connect each pair of nodes, and then filter them according to their length and α . The minimum length $L_{\min}$ has been set equal to 3 km; the maximum deviation percentage allowed from the shortest path for any OD pair connection has been set equal to 50%, consequently $\alpha = 0.5$.

The three weights $\gamma_1, \gamma_2, \gamma_3$ introduced to reflect the tradeoffs between user costs, operator costs, and unsatisfied travel trip costs, have been assumed to be equal to 1: consequently, in this application, the hypothesis is to give the same significance to all these components. Moreover, C_v (i.e., per-hour operating cost of a bus [currency/vehicle/h]) has been set equal to 150, C_d (i.e., value of each unsatisfied transit demand [currency/person]) equal to 10, and C_m equal to 1 (value of time [currency/min]).

At most three transit lines are allowed in the network, i.e. $u_{\max} = 3$. Headway bounds are supposed equal to $h_{\max} = 6$ min and $h_{\min} = 20$ min (i.e., $f_{\max} = 10/h$ and $f_{\min} = 3/h$), and the transit vehicle capacity $P = 50\text{pax/bus}$ (Kittelson & Associates 1999). Finally, given the small size of the network, it has been decided to neglect paths with more than one transfer ($k_{tr}=1$), and not to impose the fleet size constraint, assuming that there is not a maximum threshold to the number of buses operating on the network. The minimum percentage δ of the total demand to cover has been set equal to 0.7, meaning that the aim is to satisfy at least 70% of users asking for public transport service.

In order to infer the value of the proposed R_Gini equity constraint corresponding to each possible routes configuration, it is necessary to calculate the weighted transit service supply W_Sl_D for each district D . The length of 6-7 (the longest link of the experimental network) has been supposed equal to 3 km. According to this assumption, each zone covers a total area ($Area_D$) roughly ranging from 1 km² to 2 km². Consist-

ently, it has been considered a population from about 2000 and 4000 inhabitants for each of them, guessing a population density coherent with the one of a medium size city center.

Often, a 5 minutes' walk to reach a bus stop is considered reasonable; in terms of physical distance, this corresponds to an access standard in urban areas of 400 meters (Demetsky and Lin 1982; Levinson 1992; Federal Transit Administration 1996; Ammons 2001). Then, a radius of 400 m around each bus stop identifies the circle that delimits its buffer area ($Area_{bb}$). As service level measure SL_{bb} , it has been considered the number of public transport vehicle arrivals per hour, assuming that the aim is to serve the hourly demand shown in Table 11. Time-dependent demand can be accounted for by considering time-dependent frequencies.

The sensitivity analysis has been accomplished performing three different optimizations: the first one without the equity constraint (No_EQ), the second one only with the horizontal (spatial) constraint (EQ_h), the last one considering both components of equity, the horizontal and the vertical one (EQ_hv). In the No_EQ optimization, the Delbosc and Currie (2011) Gini coefficient (D&C_Gini) has been calculated, placing on the x-axis of the Lorenz curve graph the percentage of population, and on the y-axis the percentage of the transit supply index SI_D ; in this way it is possible to have an idea of the level of equity achieved without imposing any equity constraint on the network. The same D&C_Gini is computed for the EQ_h optimization, reflecting the spatial distribution of the transit service among the population. Only in the last optimization (EQ_hv) that considers both the horizontal and the vertical equity components, the suggested R_Gini is calculated.

The key objective is to understand how much taking into account equity aspects since the planning stage of a public transportation network could affect the related global costs on it. Each set of optimization is run through for 30 times. Consequently, the final values summarized in Tables 12, 13, 14a, 15a, 16a, 17a and 18a show the minimum, the average and the maximum value of the overall cost and Gini coefficient obtained for each group of optimizations. More specifically, in the EQ_hv optimization, it has been made the assumption that only one district (A), has a certain percentage

of population belonging to a disadvantaged group; and this percentage has been gradually increased (10%-20%-30%-40%-50%) carrying out 5 different groups of EQ_hv optimizations (Tables 14, 15, 16, 17, 18). The values of SI_D and W_SI_D related to the columns of this district are highlighted in light gray. Through this gimmick (the progressive increase of disadvantaged in one district), the aim is to force solutions in which district A is guaranteed with a larger number of routes and/or higher service frequencies according to the given percentage of disadvantaged people.

Table 12 – Numerical results of the optimization without equity constraint (NO_Eq).

Overall cost			D&C_Gini		
Min	Med	Max	Min	Med	Max
16830	23548	31105	0,1931	0,3234	0,4861

Table 13 – Numerical results of the optimization with the horizontal equity constraint (EQ_h).

Horizontal e- quity constraint	Overall cost			D&C_Gini			Transit Service Supply (SI_D)				
	Min	Med	Max	Min	Med	Max	A	B	C	D	E
Pop _D	-	-	-	-	-	-	2300	350	410	3450	2800
D&C_Gini<0.05	3669	5383	5990	0,026	0,039	0,048	5,80	9,07	10,2	8,76	6,77
D&C_Gini<0.1	3891	4449	5291	0,054	0,085	0,099	4,51	7,25	7,63	7,42	5,25
D&C_Gini<0.15	2771	3490	4160	0,055	0,123	0,145	4,69	8,37	7,06	6,82	4,70
D&C_Gini<0.2	1995	3098	3940	0,113	0,166	0,197	5,83	10,7	6,53	8,13	6,03
D&C_Gini<0.3	1848	2556	3736	0,168	0,243	0,298	4,83	11,5	5,84	5,89	5,03
D&C_Gini<0.4	1813	2330	2989	0,168	0,272	0,350	4,88	10,1	4,74	4,38	3,95
D&C_Gini<0.6	1987	2339	3149	0,123	0,259	0,445	5,28	11,9	6,39	6,23	5,29
D&C_Gini<0.8	1813	2420	3166	0,086	0,354	0,721	4,90	9,03	3,99	4,85	3,88

Table 14a – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 10% of disadvantaged people in district A. Overall cost and R_Gini.

Horizontal and vertical equity constraint	Overall cost			R_Gini		
	Min	Med	Max	Min	Med	Max
R_Gini<0.05	37225	52727	62979	0,0327	0,0458	0,0495
R_Gini<0.1	30041	44980	56042	0,0497	0,0840	0,0999
R_Gini<0.15	25300	33344	40456	0,1011	0,1337	0,1497
R_Gini<0.2	19480	30202	38940	0,1103	0,1714	0,1989
R_Gini<0.3	19115	26667	34902	0,1256	0,2375	0,2984
R_Gini<0.4	18475	25021	31431	0,1369	0,2702	0,3984
R_Gini<0.6	17060	23219	29456	0,2115	0,3368	0,5840
R_Gini<0.8	18130	23794	31666	0,0930	0,3375	0,7219

Table 14b – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 10% of disadvantaged people in district A. Transit Service Supply and Weighted Transit Service Supply.

Horizontal and vertical equity constraint	Transit Service Supply (SI ₀)					Weighted Transit Service Supply (W_SI ₀)				
	A	B	C	D	E	A	B	C	D	E
Pop ₀	2300	3500	4100	3405	2800	2300	3500	4100	3450	2800
R_Gini<0.05	4,07	5,60	6,27	5,34	4,36	366,03	559,57	626,63	534,14	436,44
R_Gini<0.1	5,73	8,28	8,35	7,78	5,79	516,08	827,65	834,89	778,39	578,65
R_Gini<0.15	5,01	8,99	6,91	7,31	5,73	450,84	898,66	691,24	731,31	572,55
R_Gini<0.2	5,56	10,52	7,05	7,64	4,94	500,79	1051,89	705,21	764,32	493,69
R_Gini<0.3	4,79	10,75	5,44	6,63	5,02	431,55	1074,98	544,41	662,75	502,45
R_Gini<0.4	5,57	9,76	5,24	6,17	4,90	501,15	976,09	524,38	617,05	489,80
R_Gini<0.6	4,66	9,28	4,22	4,23	3,43	419,04	928,16	421,87	422,79	342,51
R_Gini<0.8	5,09	9,09	4,12	4,57	4,01	457,97	909,00	411,98	456,65	401,09

Table 15a – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 20% of disadvantaged people in district A. Overall cost and R_Gini.

Horizontal and vertical equity constraint	Overall cost			R_Gini		
	Min	Med	Max	Min	Med	Max
R_Gini<0.05	41301	45869	61168	0,0164	0,0409	0,0477
R_Gini<0.1	33405	45175	55797	0,0619	0,0827	0,0988
R_Gini<0.15	21225	35467	46546	0,0472	0,1243	0,1491
R_Gini<0.2	20375	31176	41391	0,0931	0,1713	0,1997
R_Gini<0.3	18475	26161	35216	0,1600	0,2449	0,2983
R_Gini<0.4	18130	23715	32130	0,1574	0,2770	0,3962
R_Gini<0.6	16830	21657	26685	0,2134	0,3133	0,5729
R_Gini<0.8	18130	23589	30652	0,1053	0,3296	0,7219

Table 15b – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 20% of disadvantaged people in district A. Transit Service Supply and Weighted Transit Service Supply.

Horizontal and vertical equity constraint	Transit Service Supply (SI _D)					Weighted Transit Service Supply (W_SI _D)				
	A	B	C	D	E	A	B	C	D	E
Pop _D	2300	3500	4100	3405	2800	2300	3500	4100	3450	2800
R_Gini<0.05	5,88	7,07	8,05	7,18	5,43	470,55	707,27	804,87	717,54	543,04
R_Gini<0.1	6,39	8,43	8,31	7,41	5,90	510,86	842,74	831,24	741,39	590,30
R_Gini<0.15	5,83	9,61	8,04	8,77	5,78	466,10	960,69	804,29	877,19	577,55
R_Gini<0.2	5,72	9,44	6,31	7,61	5,17	457,79	944,48	630,93	761,07	516,54
R_Gini<0.3	4,77	9,59	4,99	5,59	4,55	381,33	959,32	499,14	559,45	455,20
R_Gini<0.4	5,58	9,44	4,22	5,00	4,34	446,11	943,71	421,81	500,24	433,90
R_Gini<0.6	5,79	11,61	4,96	5,54	4,50	462,92	1160,98	496,08	553,56	450,10
R_Gini<0.8	5,49	10,59	4,79	5,06	4,37	439,29	1059,49	479,14	506,17	437,47

Table 16a – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 30% of disadvantaged people in district A. Overall cost and R_Gini.

Horizontal and vertical equity constraint	Overall cost			R_Gini		
	Min	Med	Max	Min	Med	Max
R_Gini<0.05	34746	51233	63528	0,0187	0,0433	0,0496
R_Gini<0.1	33660	44016	56577	0,0404	0,0824	0,9990
R_Gini<0.15	26865	34683	41700	0,0589	0,1210	0,1491
R_Gini<0.2	19576	31235	39441	0,0851	0,1669	0,1911
R_Gini<0.3	17515	26877	35326	0,1225	0,2396	0,2980
R_Gini<0.4	18900	23837	28500	0,1341	0,2794	0,3931
R_Gini<0.6	18130	22864	28656	0,1735	0,2857	0,5130
R_Gini<0.8	18130	21635	28210	0,1837	0,3106	0,6445

Table 16b – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 30% of disadvantaged people in district A. Transit Service Supply and Weighted Transit Service Supply.

Horizontal and vertical equity constraint	Transit Service Supply (SI ₀)					Weighted Transit Service Supply (W_SI ₀)				
	A	B	C	D	E	A	B	C	D	E
Pop ₀	2300	3500	4100	3405	2800	2300	3500	4100	3450	2800
R_Gini<0.05	5,27	5,69	6,38	5,44	4,28	368,75	569,19	637,92	544,27	428,43
R_Gini<0.1	6,84	7,75	7,75	7,06	5,45	479,14	774,91	775,36	705,70	544,82
R_Gini<0.15	6,37	8,33	7,21	7,72	5,21	445,85	832,53	720,77	772,48	521,19
R_Gini<0.2	5,95	9,52	6,19	7,07	5,05	416,45	951,56	618,98	707,19	505,11
R_Gini<0.3	5,14	9,47	5,12	6,41	4,66	359,85	946,56	511,96	641,17	466,29
R_Gini<0.4	5,91	9,89	4,25	5,15	4,61	413,71	989,00	425,41	515,47	461,41
R_Gini<0.6	5,39	10,86	5,27	6,39	5,14	377,39	1085,76	527,04	638,63	513,76
R_Gini<0.8	6,05	12,67	5,68	5,10	4,80	423,75	1266,58	567,88	509,81	479,85

Table 17a – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 40% of disadvantaged people in district A. Overall cost and R_Gini.

Horizontal and vertical equity constraint	Overall cost			R_Gini		
	Min	Med	Max	Min	Med	Max
R_Gini<0.05	33936	49453	62233	0,0255	0,0480	0,0499
R_Gini<0.1	23280	43175	56567	0,0539	0,0879	0,9910
R_Gini<0.15	26980	36743	44107	0,0890	0,1314	0,1499
R_Gini<0.2	20770	31892	38352	0,1167	0,1764	0,1999
R_Gini<0.3	18130	26208	36061	0,1088	0,2153	0,2885
R_Gini<0.4	19950	24415	30581	0,1866	0,2616	0,3897
R_Gini<0.6	18475	22012	28170	0,1979	0,3156	0,5497
R_Gini<0.8	18475	23163	27946	0,1913	0,3524	0,6445

Table 17b – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 40% of disadvantaged people in district A. Transit Service Supply and Weighted Transit Service Supply.

Horizontal and vertical equity constraint	Transit Service Supply (SI _D)					Weighted Transit Service Supply (W_SI _D)				
	A	B	C	D	E	A	B	C	D	E
Pop _D	2300	3500	4100	3405	2800	2300	3500	4100	3450	2800
R_Gini<0.05	5,46	5,59	5,83	5,27	4,12	327,53	558,79	582,99	526,78	412,31
R_Gini<0.1	7,09	7,58	7,69	6,92	5,10	425,33	757,77	769,05	691,89	510,21
R_Gini<0.15	6,57	8,25	6,74	6,88	5,32	394,26	825,37	674,13	687,66	531,73
R_Gini<0.2	6,26	9,22	6,39	7,30	6,03	375,75	922,36	638,72	730,04	603,38
R_Gini<0.3	5,16	10,30	5,82	6,70	5,38	309,62	1030,05	582,38	670,37	537,72
R_Gini<0.4	5,79	11,30	5,37	6,50	5,16	347,33	1130,26	537,15	650,06	516,42
R_Gini<0.6	5,78	11,74	5,02	5,05	4,79	347,03	1173,66	502,07	505,32	479,16
R_Gini<0.8	5,66	12,31	4,88	5,13	4,33	339,40	1230,77	487,92	513,04	432,63

Table 18a – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 50% of disadvantaged people in district A. Overall cost and R_Gini.

Horizontal and vertical equity constraint	Overall cost			R_Gini		
	Min	Med	Max	Min	Med	Max
R_Gini<0.05	37346	45729	62648	0,0394	0,0491	0,0499
R_Gini<0.1	37096	43883	58002	0,0565	0,0815	0,0995
R_Gini<0.15	25065	35238	44276	0,0856	0,1313	0,1480
R_Gini<0.2	20490	33019	38175	0,1385	0,1831	0,1999
R_Gini<0.3	18130	27807	37361	0,2112	0,2665	0,2968
R_Gini<0.4	19875	23526	30495	0,1543	0,2451	0,3809
R_Gini<0.6	18475	22015	28171	0,1974	0,3010	0,4343
R_Gini<0.8	16830	23488	30230	0,1737	0,3173	0,7094

Table 18b – Numerical results of the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 50% of disadvantaged people in district A. Transit Service Supply and Weighted Transit Service Supply.

Horizontal and vertical equity constraint	Transit Service Supply (SI ₀)					Weighted Transit Service Supply (W_SI ₀)				
	A	B	C	D	E	A	B	C	D	E
Pop ₀	2300	3500	4100	3405	2800	2300	3500	4100	3450	2800
R_Gini<0.05	7,63	6,52	6,93	5,87	5,12	381,29	652,13	693,12	587,22	511,69
R_Gini<0.1	6,95	6,10	6,15	5,54	4,60	347,49	610,26	615,43	554,36	459,64
R_Gini<0.15	8,41	8,59	6,97	7,29	5,40	420,58	859,34	697,30	728,77	540,27
R_Gini<0.2	8,17	9,83	6,67	7,41	5,69	408,42	983,46	666,72	741,47	568,77
R_Gini<0.3	5,70	10,99	5,47	5,71	5,26	284,92	1099,18	546,53	571,34	525,75
R_Gini<0.4	5,63	11,04	5,86	6,12	5,39	281,73	1104,11	585,71	611,97	539,49
R_Gini<0.6	5,78	11,67	5,53	5,55	5,04	288,89	1166,56	552,62	554,83	504,00
R_Gini<0.8	4,49	9,22	3,88	4,74	3,98	224,39	921,55	387,58	473,58	397,52

Note that the value assigned to β progressively increases from 0.05 to 0.8. Intervals are narrowed as they approach zero, that is the value of Gini coefficient representing the perfect equality. Looking at the tables, it is easily possible to observe that the more a higher level of equity wants to be achieved (both horizontal and horizontal & vertical) on the network, the more the overall cost rises. In a clear and immediate

way, Figures 18a and 18b show a surface with the relation among overall medium costs, Gini fixed threshold β and the percentage of the disadvantaged population. It can be assessed that, for the same value of β , it seems there is not an evident correlation between the percentage of disadvantaged people and costs, that fluctuate more or less around the same value. A possible explanation could be that the global amount of disadvantaged on the entire network is always too low compared to the total population living in all the districts, and it does not affect to a greater extent the final global costs.

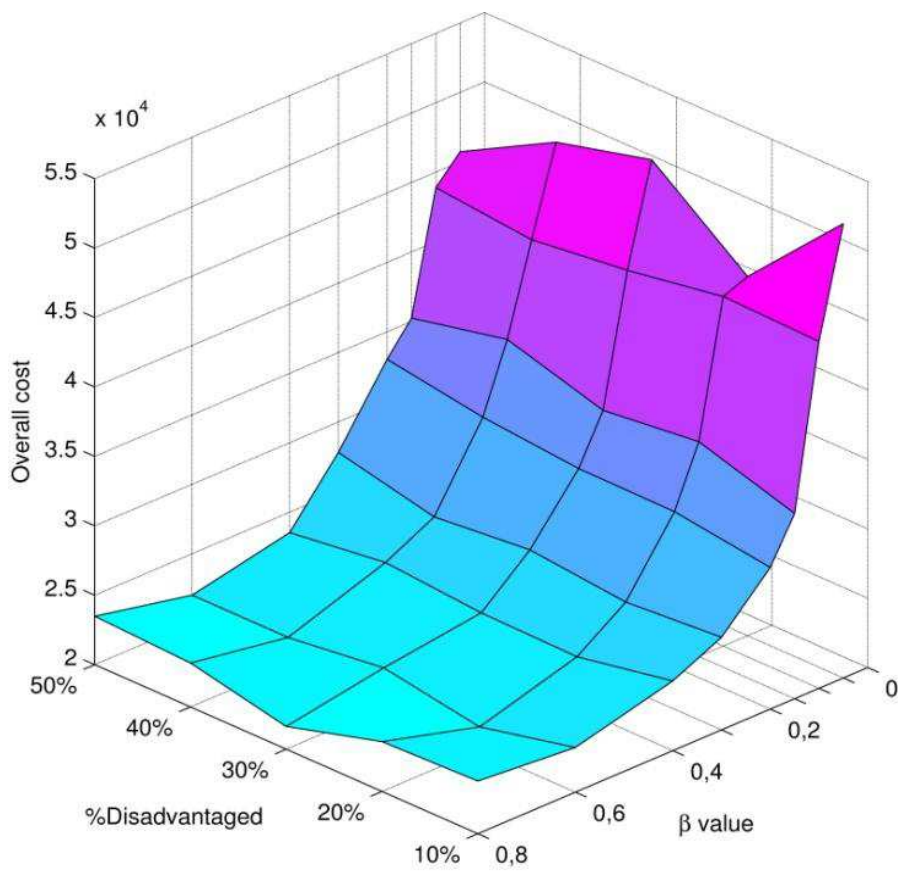


Fig. 18a – Relation among equity constraint ($R_Gini < \beta$), overall medium cost on the network and percentage of disadvantaged population living in the district A. Front view of the graph.

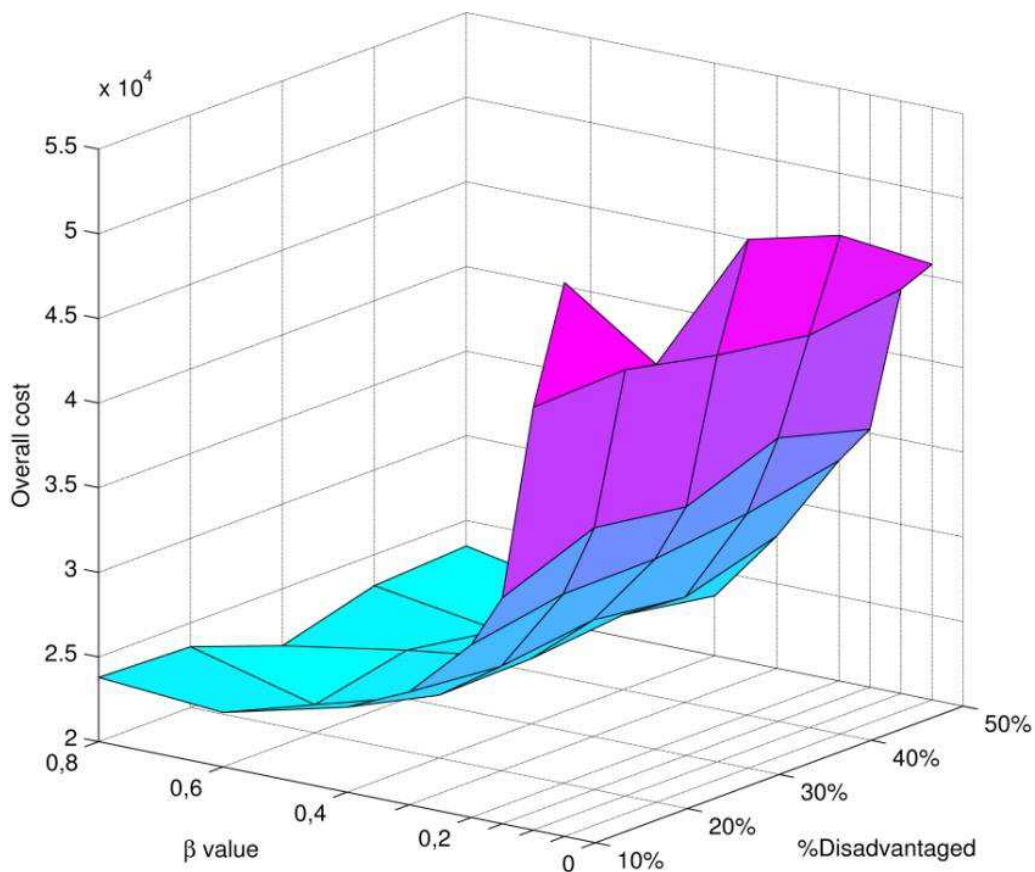


Fig. 18b – Relation among equity constraint ($R_Gini < \beta$), overall medium cost on the network and percentage of disadvantaged population living in the district A. Side view of the graph.

An additional remark can be done looking at the values of SI_D and W_SI_D in the previous Tables (13, 14b, 15b, 16b, 17b, 18b). The transit supply index SI_D reflects spatial/horizontal equity, so when it is in direct proportion to the number of people living in each district, it means that a fair degree of horizontal equity is achieved. As an example, in Table 13, for the sets of optimizations with $D\&C_Gini$ lower than 0.1 (where an optimal level of equity is reached), it is possible to observe that the greater SI_D values match with district C (4100 inhabitants), and gradually reduce up to reach the lowest value for district A, with less population among all (2300 inhabitants).

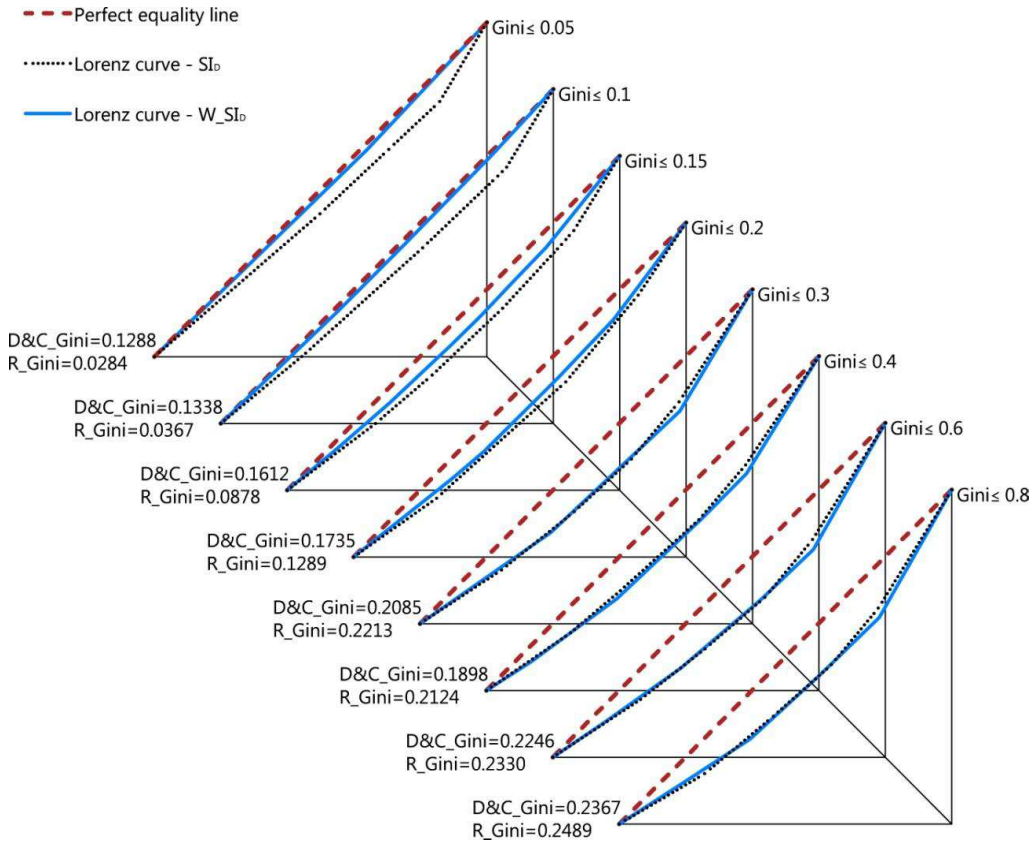


Fig. 19 – Comparison between SL_D and W_SL_D Lorenz curves - 50% of disadvantaged people living in district A.

This does not happen in the remaining tables, where the EQ_hv optimizations are performed trying to guarantee an amount of transit service proportional to both the spatial and social needs of the society. Therefore, although the values of W_SL_D are in proportion to the number of residents in each district, the corresponding SL_D show an unusually large value for the underpopulated district A. This occurs because district A has for sure fewer residents, but also a certain amount of disadvantaged people whose needs have to be taken into account granting them more bus stops and/or a greater frequency of the transit service, so requesting a bigger share of resources.

Consequently, the ‘apparent’ disproportion that it is observed is due to the vertical equity component included in W_SI_D .

This concept is also express in Figure 19, where it is possible to denote an imbalance in the SI_D Lorenz curve compared with the related W_SI_D Lorenz curve, closer to the line of perfect equity, at least for demanding values ($R_Gini < 0,2$) of the equity constraint. The difference between these two curves tends to be irrelevant as the constraint is loosened.

4.4 CASE STUDY

The methodology in this chapter is here applied to a case study, Molfetta, a city located in the South of Italy (Apulia region) of approximately 60,000 inhabitants, as reported in 2011 census. The real transport systems network is modeled by a graph G (consisting of selected arcs and intersections), a set of link cost functions, an Origin-Destination matrix, and a transit network operating in the city. This provides the basis for comparison and validation of the proposed model.

The graph is made by 519 directed arcs and 210 nodes. Part of the network model are only those streets of the city with an effective width sufficient to allow a bus to pass by easily, discarding the too narrow ones.

The city has been divided into 28 zones, deduced by aggregating different census districts, as shown in Figure 20. The radius of the walk access buffer around each bus stop has been considered equal to 200 m so that two stops are distant from each other approximately 400 m. This choice has been made trying to maintain the current distance between two following stops, as this is the level of accessibility required by the users to enjoy the transit service, according to recent surveys included in the Urban Master Plan of Sustainable Mobility of Molfetta. The demand produced (attracted) by a given zone is centered in centroids (black dots shown in Figure 20), that indicate balancing points of the population within a geographic area. This demand is

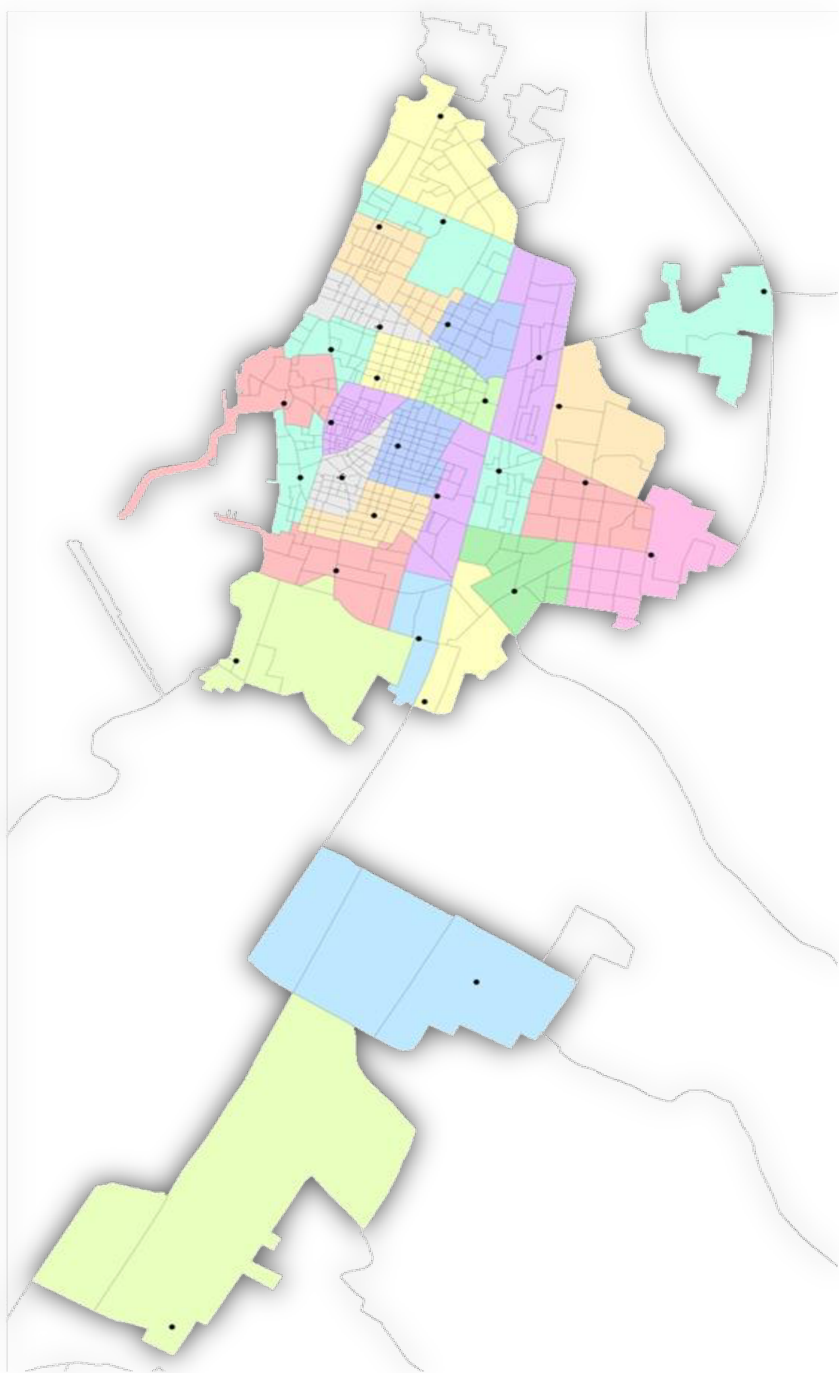


Fig. 20 – Molfetta case study: traffic zones and centroid nodes.

considered as covered when a transit line passes by any place in the street network both inside that zone and inside the destination (origin) zone, according to the capacity of the vehicles and their frequency.

Performing the proposed optimization, the purpose is to understand if it is possible to obtain a configuration of the transit system able to satisfy to a greater extent the public transport demand, reaching a higher level of equity, i.e. serving in a widespread manner those zones with a larger percentage of disadvantaged people. According to the available census data, this category includes unemployed, young (< 19 years old) and old (> 65 years old) people, so those that are most likely to make use of the public transport.

4.4.1 Description of the current status of the transit network

Currently, the public transportation system of Molfetta contemplates five bus lines (Table 19), having a path roughly circular, with quite low-frequency bounds (i.e., $f_{\max}=1,43/h$ and $f_{\min}=1/h$), covering an average of 84 km per hour of operation of the system (Figure 21).

Table 19 – Current public transport network in Molfetta.

Routes	Frequency (bus/h)	Length (km)	km per hour
1	1.20	11.00	13.20
2	1.20	12.26	14.71
3	1.15	13.74	15.80
4	1.43	13.75	19.66
5	1.00	20.67	20.67

In order to better understand the present state of the public transportation system in Molfetta, in terms of total costs supported by users and operator, unsatisfied demand and achieved level of equity, at first the model has been run giving as input the current five lines with their associated frequencies. As a matter of fact, the current

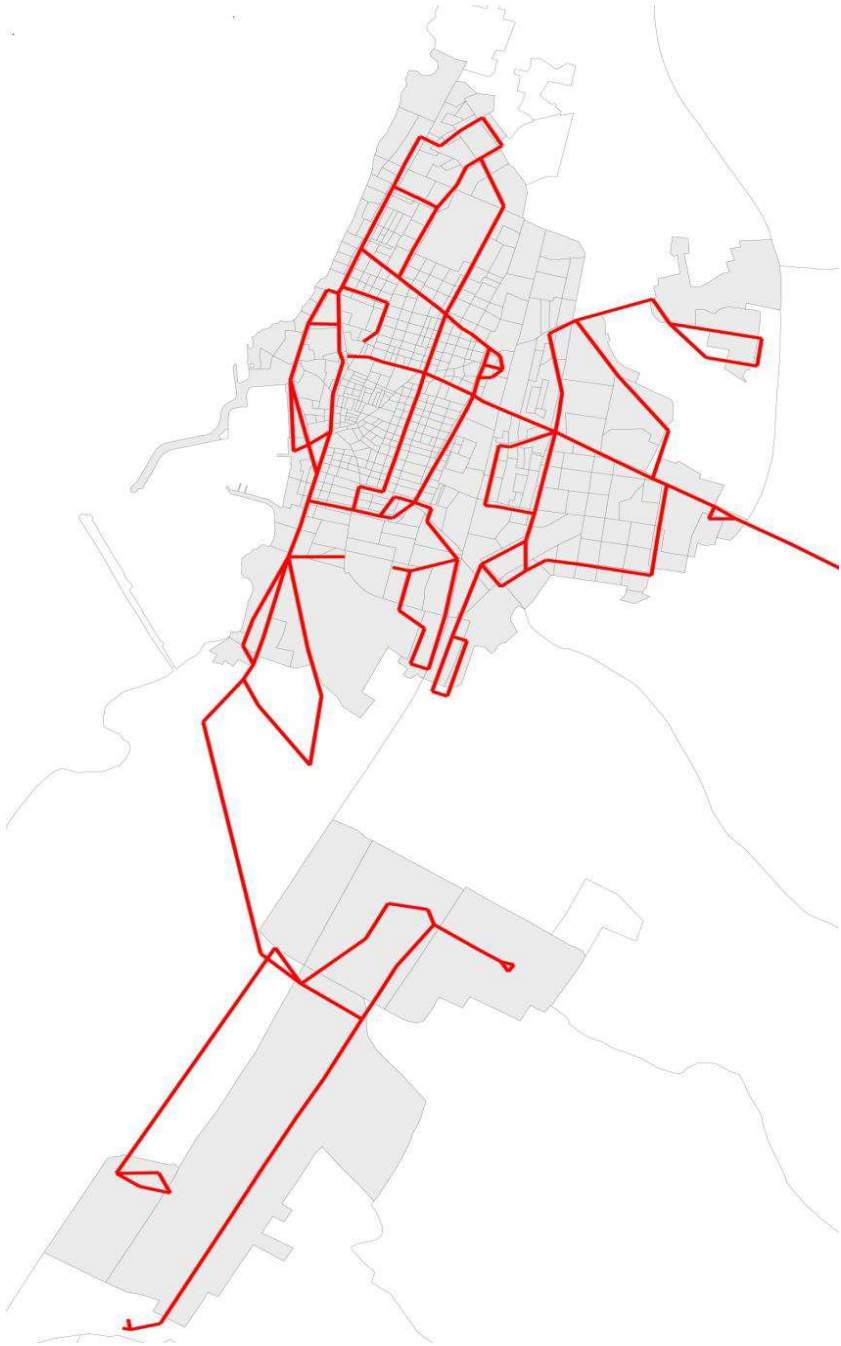


Fig. 21 – Path covered by the current set of routes.

configuration of the system is able to grant a R_Gini coefficient equals to 0,4064, and a percentage of unsatisfied demand of 6%. The values related to the overall costs on the network, the current unsatisfied demand and the R_Gini coefficient achieved having as input the present public transport lines are shown in the *Current status* column of Table 20.

Starting from these values, it is possible to define reasonable bounds associated with each constraint in the model, improving the actual public transport situation, without having excessive claims that would lead to the impossibility to converge to a feasible solution.

4.4.2 Proposed transit network

In order to apply the proposed solution methodology to the network of Molifetta, it is important to set in advance some parameters, defining also the bounds associated with each constraint (from Equation (37) to Equation (41)), summarized in the following:

$$4 \leq u \leq 6 \quad (\text{numbers of routes})$$

$$30 \leq h_{rk} \leq 60 \quad (\text{headway feasibility})$$

$$\left(\sum_{k=1}^u \frac{T_{rk}}{h_{rk}} \right) \leq 12 \quad (\text{fleet size})$$

$$d_s \geq 0,97 \cdot d_{tot} \quad (\text{demand coverage})$$

$$R_Gini \leq 0,38 \quad (\text{equity})$$

First of all, four terminals have been set, according to the locations suitable to allocate the stalled vehicles during the downtime of the service, and four terminal pairs $t_k \in T$. The method suggested in the flowchart displayed in Figure 16 has been implemented, considering as link cost in the calculation of the shortest paths the travel time on each arc of the network. At the end, a pool of 20 ($\bar{t} \cdot \bar{m}$) candidate routes has been ob-

tained, establishing five different length intervals $m \in M$ for each terminal pair $t_k \in T$. According to what explained previously, in this way it becomes possible to find the best paths able to satisfy the transit demand for each route length interval, obtaining a set of candidate routes $\mathfrak{R}$ that will be the input of the second part of the model.

It has been decided to neglect paths with more than one transfer ($k_{tr} = 1$), and to assume that the three weights $\gamma_1, \gamma_2, \gamma_3$ introduced to reflect the tradeoffs between user costs, operator costs, and unsatisfied travel trip costs, are equal to 1, giving the same significance to all these components as it has been done for the sensitivity analysis. In the network are allowed at most six transit lines, and at least 4 (i.e., $u_{min} = 4$ and $u_{max} = 6$), and the headway bounds have been set equal to $h_{max} = 60$ min and $h_{min} = 30$ min (i.e., $f_{max}=2/h$ and $f_{min}=1/h$). It has been assumed a maximum fleet size of 12 vehicles and transit vehicle capacity of $P = 50$ pax/bus, considering the vehicles currently used to perform the service.

The aim of the proposed model is to find (if it actually exists) a solution (i.e. routes configuration with associated frequencies) able to enhance the present situation of the public transportation network in Molfetta. Consequently, the remaining constraints to be satisfied have been set according to this purpose: the minimum percentage δ of the total demand to cover is fixed equal to 0.97 (allowing no more than 3% of users to be dissatisfied), while R_Gini has to be lower or equal to 0,38. Table 20 shows the optimal configuration values of the system obtained through the proposed methodology, compared with the values related the current status of the system.

Table 20 – Comparison of results: current status and proposed solution for the transit network service.

	Current status	Proposed solution
<i>Overall costs</i>	28015	20471
<i>User costs</i>	21283	17683
<i>Operator costs</i>	2283	2788
<i>Unsatisfied demand costs</i>	4449	0
<i>R_Gini coefficient</i>	0.4064	0.3113
<i>Unsatisfied demand</i>	6%	0%

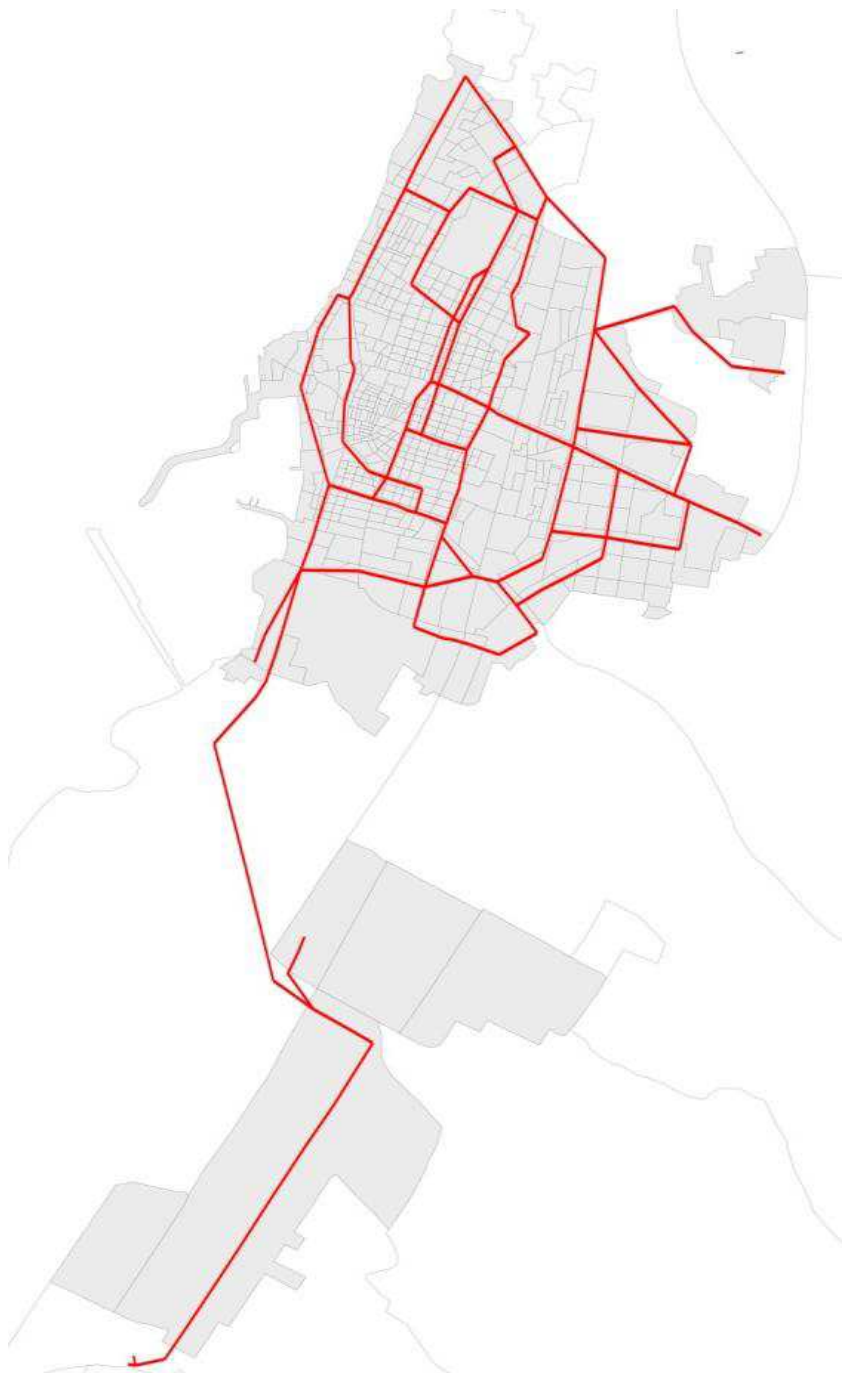


Fig. 22 – Path covered by the optimal transit route set.

The proposed solution for the public transportation system of Molfetta contemplates five bus lines (Table 21), covering an average of 80,88 km per hour of operation of the system (Figure 22).

It can be easily asserted that, following the proposed methodology, it is possible (at least in this case) to find a better compromise. It has been obtained a route configuration able not only to reduce the overall costs of the system, satisfying to a greater extent the transit demand on the network; but also to guarantee a better spatial and social distribution of the service, reaching an higher level of horizontal and vertical equity on the network, expressed by means of the value of R_Gini .

Table 21 – Optimal transit route set.

Routes	Frequency (bus/h)	Length (km)	km per hour
1	1.13	11.89	13.44
2	1.34	10.30	13.80
3	1.21	17.69	21.40
4	1.64	8.90	14.60
5	1.27	13.89	17.64

The attempt to integrate equity principles since the planning stage of a public transport network may be an added value to the design process that helps to ensure everyone a better service. It might be impossible or extremely expensive expect to reach an ‘ideal’ configuration, however, it has been shown that it is feasible to improve consistently the current status of the service.

Testing the same model on different realities could help in the understanding of the effective benefits of the method, depending on the different distributions of the vulnerable categories of people in the area served by the transit services.

5.0 CONCLUSIONS AND FURTHER RESEARCH

In literature, network design problems have been largely discussed. Traditional approaches often neglect equity goals that, conversely, should play an important role, assuming the societal function of transport service.

The present dissertation focuses on the application of horizontal/spatial and vertical/social equity concepts to transportation networks. It deals with the equitable network design problem, studying it from three different perspectives, progressively moving from the private transport to the public one.

At first, it has been proposed a general model formulation, that considers only the presence of private vehicles on the network, with equity constraints specified with rigid minimum and/or maximum thresholds. Actually, it has been stated that these constraints and other parameters of the network design problem can be affected by uncertainty. Hence, the suggestion to consider also flexible equity constraints, explicitly represented by fuzzy sets. In order to include these uncertain/imprecise values/linguistic expressions, the equity network design problem has been then specified as a fuzzy programming problem.

Aiming to test the accuracy of this first proposed model, it has been performed a sensitivity analysis starting from different initial configurations; after that, the flexible approach has been compared to the more traditional one (crisp), that even including equity constraints, does not take into account the presence of uncertainties in the problem.

It has been found that, through a fuzzy approach, although the total costs of network result generally higher, the equity aspects (either horizontal or vertical) appear to be satisfied to a greater extent, enabling to spread more fairly the 'disadvantages' arising from a network redesign. These results have been confirmed also by a numerical application to a larger network.

The second formulation of the problem that has been suggested is a multimodal one. It allows different typologies of vehicles (both private and public) on the network. Also, in this case, it has been presented two specifications of the equitable NDP, one formulated as crisp minimization problem (ECS), the other as fuzzy maximization problem (EFS). The two approaches have been illustrated by an implementation in a network already used in the literature. The test has revealed that the two methods can lead to different results. In the analyzed case, EFS seems to be more favorable to public transport; in any case, the fuzzy approach is more suitable and theoretically consistent with contexts with approximate and/or uncertain data.

In EFS, the objective function is a satisfaction index measuring the degree at which the solution overall fulfills the three goals to be achieved in the design problem: lowering network costs without excessively penalizing any group of car and bus users. This formulation provides solutions which can be implemented in situations where the network is regulated by an authority with a comprehensive responsibility for the transport system. Weights have been introduced in the problem specification to allow the possibility for the authority to prioritize some goals over the others.

EFS (and ECS) have been solved by GA. The application of GA does not guarantee the identification of the global optimum and different implementations may identify different local optima. Therefore, using the Pareto front, at first, it has been tested the ability of the optimization to improve the overall satisfaction starting from different initial configurations. Secondly it has been shown the Pareto front related to the final values assumed by costs and equity at the end of each optimization run. This would make the decision-making process more transparent, permitting the decision

maker to identify, among all the optimal solutions, that with the set of goal-specific values which suits best his priorities.

The last approach copes with the importance of applying the equity concept to a public transport network. As a matter of fact, although it is common to find in literature analysis ex-post regarding the pursued level of equity in a certain study area, or the socioeconomic characteristics that lead some categories of people to be excluded, it seems there are no attempts to incorporate both horizontal and vertical equity in the planning stage of a new public transportation system.

Therefore, this shortcoming has been addressed elaborating a method to incorporate equity concepts inside the TNDP, by means of a constraint to be added to the classical formulation, based on a novel comprehensive equity indicator. Starting from the definition of Gini coefficient applied to the economic field, and passing through the interpretation given by Delbosc and Currie (2011) with their definition of supply index and their attempt in defining an indicator of the horizontal level of equity reached by a transit network, it has been proposed and implemented a modified indicator. It has been called R_Gini - namely *Revised Gini* - and the TNDP has been reformulated taking into account this new constraint able to guarantee an optimal spatial distribution of the transit service, but also a social attention to those categories of people (called vulnerable or disadvantaged), i.e. the ones that more need to be served by an efficient and frequent service to reach their desired destination and improve the everyday quality of their life and opportunities.

The proposed model has been tested at first on a small network, carrying out a sensitivity analysis with the main purpose of understanding the correlation between the overall costs on the network and the pursued level of equity. This test has confirmed that a greater level of equity often means to bear more costs to be achieved. After this first analysis, the proposed model has been applied to a real case study, an Italian city with an operating transit service. The ultimate goal has been to verify if it is possible to achieve a better degree of horizontal and vertical equity on the

network and if the newly implemented configuration fits with the needs of both operator and users of the system. The conclusion is that it is possible, at least in the studied case, to find a better compromise, not only able to achieve the aforementioned equity aims, but also to allow a considerable saving of the associated costs.

It has been stated that the attempt to integrate equity principles since the planning stage of a public transport network may be an added value to the design process that helps to ensure everyone a better service. It might be impossible or extremely expensive expect to reach an 'ideal' configuration, however, it has been noted that it is feasible to improve consistently the current status of the service.

It may be concluded that it is important for decision makers to reach a compromise between costs and equity, not just focusing on the immediate feedback that an apparent saving can provide, but thinking about the real achievements (both spatial and social) that their actions will have on those who will actually take advantage of the transport network. Future researches may test the same models on different realities, to understand the effective benefits of each method.

Different network design models could be developed, considering the equity concept from a different perspective. As an example, it could be studied a pricing policy for the transport network, verifying if it is equitable and appropriate or not, identifying the most suitable urban areas to which apply the pricing scheme and defining their optimal perimeter and the amount of the toll.

Looking at the transit network design problem, it could be also possible to find the way to promote the shift of users toward the public transportation, making it more attractive in response to the need of the communities. The problem could be handled as a transit frequency setting one, recalibrating the frequencies of the urban public transport according to criteria related to social equity, which should facilitate the commuting for those vulnerable groups that more than others require an efficient public transport system.

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The main aim of the project is to map, analyze and evaluate the status of the transport sector in the transport network and propose new policies and strategies in infrastructure, processes, assets, ICT, legislation, norms and harmonization/ standardization

issues, in order to promote innovative green intermodal freight transport corridors. GIFT project drills down in three Pan-European Transport Corridors, namely IV, V and VII that cover almost the entire SEE region.

<http://www.gift-project.eu/index.php/en/>

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