Facing equity in transportation Network Design Problem: A flexible constraints based model

Facing Equity in Transportation Network Design Problem: A Flexible Constraints Based Model

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Abstract. In transportation planning, solutions designed to meet objectives of equity and social inclusion have to be achieved. From this standpoint, most of Network Design Problem (NDP) models aim at identifying the optimal layout of transportation networks by deterministic bi-level problems formulation to reflect the different goals of at least two decision makers (the network users and the planner).
Considering the societal function of transport systems, the NDP, even at operational level, should be addressed also to the research of solutions in which the outcomes of the design (costs and/or benefits) are distributed as much as possible among the potential users or classes of users.
Traditional approaches often neglect equity goals that, conversely, should play an important role or define some flows rebalancing problem. In this paper an Equity Based NDP is proposed, where the optimal layout of a road network is determined by minimizing the total system cost under flexible constraints, (jointly with rigid thresholds) by solving a single programming problem. It considers both horizontal and vertical equity criteria in the form of an equity constraint specified for uncertain variables or approximate reasoning environment: this results in a multi-objective fuzzy programming model that aims at maximizing user satisfaction according to all constraints, while taking into account the route choice behavior of network users.
A sensitivity analysis has been performed on a test network, and a comparison with a literature equity based model has been carried out; then, the proposed method has been applied to a real sized network, where an application of equity network design optimization is presented and the results discussed.

Keywords. Equity in transportation; fuzzy programming; network design problem; uncertainty; flexible constraints.

Highlights.

- A model of Equity based Network Design Problem is proposed.
- Equity concept is quantified
- A new horizontal equity performance indicator has been implemented.
- Three different optimizations have been performed in a sensitivity analysis.
- An application to a real size network validates the results.

1. Introduction

The Network Design Problem (NDP) is one of the most popular optimization problems with regard to transportation planning (Kim, Bae, and Chung, 2012). Conventionally, it can be classified into two types (Meng and Yang, 2002): the first type is the Discrete Network Design Problem, which defines an optimal set of locations for constructing some new facilities added to a current transportation network; the second one is the Continuous Network Design Problem (CNDP), which determines the optimal enhancements for some existing facilities.

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In the NDP, decisions are made based on an objective function, which can concern the minimization of costs or the maximization of benefits (Barbati, 2012). The objective function can be related to an efficiency measure (i.e. total network cost), or to an efficacy measure concerning demand satisfaction aspects (i.e. cost, accessibility).
The models able to define the transportation network layout are named Network Design models (Cascetta, 2009). Most of the NDP models in the literature have been specified as deterministic problems where all the relevant inputs are assumed to be known with certainty.
The NDP is generally formulated as a bi-level optimization problem to reflect the different aims of the two decision makers, who are the network users and the planner. The network users are free to choose their routes such that their individual travel costs are minimized, whereas the planner aims to make the best use of limited resources to optimize network performance (e.g., reducing congestion, minimizing environmental impact, and maximizing throughputs), taking into account users' route choice behavior. The upper-level subprogram describes the leader or planner problem, while the lower-level subprogram represents the follower or user's behavioral problem.
In addition to the mentioned criteria, a designing approach related to the concept of equity can be faced. This means that in the network design one could search for solutions in which some parameters (costs and/or benefits) are distributed as evenly as possible among the potential users (Barbati, 2012). Notwithstanding this, the integration of equity objectives in transportation planning is quite recent (Santos, Antunes, and Miller, 2008; Camporeale et al. 2017) and it has been drawing more and more attention. Traditional approaches often neglect equity goals that, conversely, should play an important role in transport NDP assuming the societal function of transport services.
In our opinion, it is necessary to think about alternative equity measures in road network design, and compare their implications and to quantitatively consider Equity goals also at operational planning level. In order to achieve this purpose, in this paper an Equity Based CNDP is presented where also flexible goals and constraints, jointly with rigid threshold, are considered. The CNDP is then specified as a fuzzy programming problem.
The remainder of the paper is organized as follows. The next section provides a review of the transport NDP under uncertainty, the introduction to the equity issue and a discussion about the incorporation of equity into transportation planning. After that, we describe the formulation of a general optimization model and present an outcome indicator for performance evaluation, represented in the form of an equity constraint. Finally, the results of a sensitivity analysis and a numerical application are provided, followed by some concluding remarks.

2. Literature review

2.1 Network Design under uncertainty.

The Network Design Problem has long been recognized to be one of the most difficult and challenging problems in transport.
A considerable amount of research effort has been devoted to road network design models over the last forty years. See Yang and Bell (1998) for a relatively recent review.
Most of this effort focused on formulations and solution procedures for the NDPs, which deal with the selection of either link improvements or link additions to an existing road network, assuming a given demand from each origin to each destination. The objective is to minimize the total network cost required to accommodate given traffic flows, assuming route choices to follow a user equilibrium pattern (Santos, Antunes, and Miller, 2008).
Actually, both data available to analysts and problem constraints can be affected by uncertainty; moreover within an interval, some values may better satisfy the purpose of the analysts (i.e. in a budget interval constraints the lower values are preferable to the others) (Caggiani and

Ottomanelli, 2011). Sources of uncertainty exist on supply side (that affects link travel times or roadway capacity variation), on demand side (that influences travel demand fluctuation) and on constraints-side (generally defined as decision-maker constraints) (Henn and Ottomanelli, 2006). Examples of supply-side uncertainty include weather conditions, traffic accidents, work zones and construction activities, and traffic management and control. Examples of demand-side uncertainty comprehend temporal variation (e.g. time of the day, day of the week, or seasonal effects), special events, population characteristics (e.g. age, car ownership, and household income), and traveler information as well as model related uncertainties (Caggiani and Ottomanelli, 2013). Examples of constraints-side uncertainty include available budget, level of reduction of the transport system externalities (e.g. threshold limit values of polluters) or equity thresholds as better explained in the next section.

Most of the practice of roadway network design does not take into account the uncertainty issue (Yang and Bell, 1998). The reason lies in the lack of suitable reliability and uncertainty analysis for road networks.

The design of a new network facility or the upgrading of existing facilities would require a good understanding of the uncertainty involved, the impact on the system-wide performance and the benefits derived from road improvements to the network users. Thus, it is important to study the uncertainty of road networks such that a cost-effective and equitable design can be implemented to improve its level of performance from the viewpoints of both the planner and the users.

Some recent studies have considered various sources of uncertainty in the transport NDP and proposed different criteria to hedge against the uncertainty. See Chen et al. (2011) for a more comprehensive state-of-the-art review.

It is possible, however, to highlight that these uncertain values can be better managed using a soft computing approach, in particular by fuzzy values/constraints (Zimmermann, 1996; Chen et al., 2011): fuzzy programming appears to be an ideal strategy for obtaining the optimal compromise solution to a multi-objective transportation problem.

Only a few authors have studied the opportunity to consider this knowledge together with the transportation Network Design problem. See Caggiani and Ottomanelli (2013; 2014) for a more extensive analysis.

Therefore, what it is meant to do with this paper is to carry on this trend, attempting to adopt a fuzzy approach to take into account equity goals and uncertain data/constraints in the solution of NDP.

2.2 Equity concepts.

Equity can be defined along many facets such as justice, rights, treatment of equals, capability, opportunities, resources, wealth, primary goods, income, welfare, utility and so on (Sen, 1992; Sen, 1997). However, there is a seemingly endless debate about the rules used to determine when equity is obtained (Marsh and Schilling, 1994).

Equity, like the related concepts of justice, fairness and right, is not a simple thing. Different people have different concepts of equity, but the aspect that matters will depend very much on the particular context and circumstances (Langmyhr, 1997). The importance of taking into account such aspects derives from various considerations. In general, if users perceive a substantial equity in their treatment in the fruition of a service, they will be more satisfied. In addition, when facilities are considered “undesirable”, an equitable distribution of the risk and/or disadvantage due to their locations can reduce conflicts among users and can help in accepting possible solutions (Barbati, 2012).

Equity refers to the distribution of impacts (benefits and costs) and whether that distribution is considered fair and appropriate (Litman, 2002). In the decision-making field, equity measures are commonly used to assess the economic and social impacts of different development

scenarios. Despite the increasing effort to incorporate equity in decision-making models, there is little agreement about the best way to assess equity (Santos, Antunes, and Miller, 2008). A large number of measures can be found in the literature, but we are still far from a general consensus on the best measure(s) to use in each case. Still, few attempts have been made until now to assemble these measures, compare them, and define appropriate measure(s) for each type of application. One of the rare exceptions is Marsh and Shilling (1994), where a detailed review of equity measures for public facility planning is presented.
There are three major categories of transportation equity.
1) Horizontal Equity: also called fairness and egalitarianism, it concerns the distribution of impacts between individuals and groups considered equal in ability and need. According to this definition, equal individuals and groups should receive equal shares of resources, bear equal costs, and in other ways be treated the same. It means that public policies should avoid favoring one individual or group over others, and that consumers should "get what they pay for and pay for what they get" from fees and taxes unless a subsidy is specifically justified. A series of studies related to this aspect of equity deal with the spatial distribution of transportation impacts (Spatial Equity).
2) Vertical Equity with Regard to Income and Social Class: also called social justice, environmental justice and *social inclusion*, this is concerned with the distribution of impacts between individuals and groups that differ in abilities and needs, in this case, by income or social class. By this definition, transport policies are equitable if they favor economically and socially disadvantaged groups, therefore compensating for overall inequities. Policies favoring disadvantaged groups are called progressive, while those that excessively burden disadvantaged people are called regressive. This definition is used to support affordable modes, discounts and special services for economically and socially disadvantaged groups, and efforts to insure that disadvantaged groups do not bear excessive external costs (pollution, accident risk, financial costs, etc.).
3) Vertical Equity with Regard to Mobility Need and Ability: This is concerned with the distribution of impacts between individuals and groups that differ in mobility ability and need, and therefore the degree to which the transportation system meets the needs of travelers with mobility impairments. This definition is used to support universal design (also called accessible and inclusive design), which means that transport facilities and services accommodate all users, including those with special needs.
These different types of equity often overlap or conflict. Therefore, transportation planning often involves making tradeoffs between different equity objectives (Litman, 2002).
It is however important to emphasis that different aspects of equity are important for different groups in society and it is essential to provide measures for the evaluation of their concerns and to reflect their views (Ramjerdi, 2005).

2.3 Incorporating equity analysis into transportation planning.

In the transportation field, until the end of the nineties, equity issues were generally limited to the evaluation of the economic impacts of transportation policies. In most cases, these studies regarded the distribution of policy impacts among different social groups in the case of the introduction of road prices in some links of the network (Yang and Zhang, 2002; Szeto and Lo, 2006).
Meng and Yang (2002) demonstrated that in the CNDP (Continuous Network Design Problem), with a total network cost reduction objective function, the benefits of a capacity enhancement in some selected links can lead to an increase in travel costs for some Origin-Destination (O-D) pairs; since then, the debate of equity issues in transportation network design became more intense. To this end, they introduced a parameter capable of measuring the degree of equitability of benefit distribution: it reflects the degree of an equitable reduction of the equilibrium O-D

travel costs before and after implementing a scenario. Meng and Yang proposed two models for CNDP. The parameter of the first model is a given value selected by a decision-maker; in the second model, instead, it can be treated as a decision variable: thus, the total system cost and the parameter have to be optimized simultaneously.

Yang and Zhang (2002) also observed that for the congestion-pricing problem there were significant differences among the benefits of some O-D pairs. Thus, in addition to the equity issues involving social groups they proposed the consideration of spatial equity in the road-pricing problem.

After these studies, some other authors suggested the inclusion of equity concerns in network design problems. Antunes, Seco and Pinto, (2003) considered the distribution of accessibility gains across population centers in an accessibility-maximization model. Cheng and Yang (2004) included spatial equity as a constraint in the link capacity improvement problem with demand uncertainty. Szeto and Lo (2006) propounded the integration of equity in a time-step network design problem. They considered social and user equity for different periods of time. Santos, Antunes, and Miller (2008) selected three different equity measures, which reflect different perspectives of equity, incorporating them into an accessibility-maximization road network design model.

More recently, Sumalee, Shepherd and May (2009) suggested an innovative approach for designing a road user charging scheme to meet multiple policy objectives; the objective functions or constraints taken into account include social welfare improvement, revenue generation, and distributional equity impact. Pricing policies is faced in Nahmias–Biran, Sharabi, and Shiftan, (2014) where the problem of the effects of transit fares on equity is considered.

Finally, Barbati (2012) proposed a multi-objective model in which balancing or equity aspects, i.e. measures of the distribution of distances of users from the path, are considered. In this case, she introduced an equity parameter representing the maximum cost or benefit difference between any pair of nodes. Adding this constraint to the model it was possible to obtain a formulation that combined efficiency (the minimization of path length), efficacy (the maximization of the accessibility), and the minimization of the inequity, in order to achieve a better distribution of costs or benefits among the users.

In conclusion, what we can assert is that transportation equity analysis is important and unavoidable; therefore, in the model discussed here we propose to include an outcome indicator for performance evaluation, represented in the form of an equity constraint, specified for imprecise variables or approximate reasoning environment. Differently from other NDP model we specify the model as one level problem formulation, so that the obtained solution respects simultaneously all the considered constraints, including equity.

3. The proposed equity based model

3.1 General formulation

For the general formulation of the supply design problem readers could refer to Cascetta (2009). In the classic CNDP, the optimal network enhancements are determined by minimizing the total system cost under a set of constraints, while taking into account the route choice behaviour of network users. However, the equilibrium travel costs between some O-D pairs may be increased or decreased after implementing an optimal network design scenario, leading to positive or negative results for network users. Therefore, the equity issue is raised and it becomes necessary to add equity constraints to the classic CNDP as shown by the following problem:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \; z(\mathbf{x}, \mathbf{f}^*) \tag{1}$$

s.t.

$$\mathrm{k}_u(\mathbf{x}) \in V \qquad \forall u = 1,2,\ldots, r \tag{1a}$$
$$\mathbf{f}^* = \mathbf{\Delta}(\mathbf{x})\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f}^*, \mathbf{x}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{x})) \tag{1b}$$
$$\mathbf{x}_e, \mathbf{f}^* \in E \qquad \forall e = 1,2,\ldots, p \tag{1c}$$
$$\mathbf{x}_i, \mathbf{f}^* \in T \qquad \forall t = 1,2,\ldots, q \tag{1d}$$

where:

- z is the function of the total cost of the network;
- $\mathbf{x}$ is the vector of the design variables;
- $\mathbf{f}^*$ is the vector of equilibrium assignment traffic flows;
- k_u are the equity performance indicators;
- r is the number of equity constraints;
- $\mathbf{\Delta}$ is the link-path incidence matrix;
- $\mathbf{P}$ is the path choice probability matrix;
- $\mathbf{C}$ is the vector of path costs;
- $\mathbf{d}$ is the vector of travel demand;
- $p+q$ is the number of certain design variables.

The eq. (1a) denotes the set of equity performance indicators, function of the design variables $\mathbf{x}$, satisfying equity constraints V. The eq. (1b) represents the consistency constraint among demand, flows and supply parameters (set of possible configurations of network flows), eqs. (1c) and (1d) express the sets of supply parameters satisfying external (E) and technical (T) constraints such as, respectively and for example, available budget and link flows-capacity ratio. The eq. (1a) can be specified as objective functions (bi or multi objective CNDP) or simple inequalities. For example, considering an equity performance indicator α that measures the degree of equitability of benefit distribution, the eq. (1a) can be stated by the eq. (2) where β_{max} is a fixed threshold (set by decision makers).

$$\alpha \leq \beta_{\max} \tag{2}$$

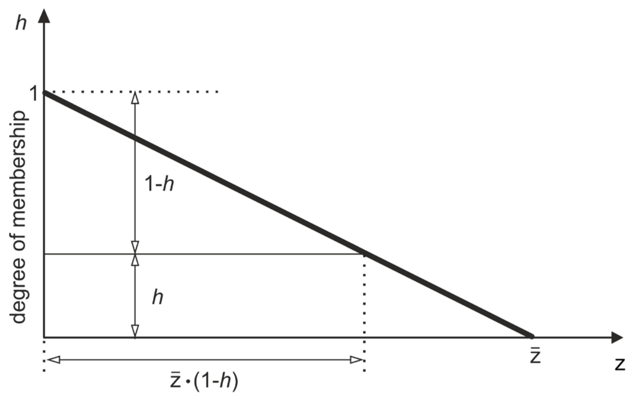
In this paper, we introduce uncertainty in the eq. (2) that is considering the equity performance indicator, expressed through incomplete information formalized quantitatively with linguistic/approximate expressions. Indeed, every category of transportation equity that we are going to quantify through α usually is affected by a certain degree of uncertainty, i.e. related to the zoning of the city, the population characteristics, or the share of impacts/resources to attribute to each individual or group. Consequently, it can be asserted that, for example, the (2) may be stated by decision makers with the following expression: "α must be approximately lower than or equal to β_{max}" (3).

$$\alpha \underset{\sim}{\leq} \beta_{\max} \tag{3}$$

With this approach also the optimization function (1) can be specified with an uncertain relation as depicted in the eq. (4) where $\bar{z}$ represents the maximum value admitted for the total cost of the network.

$$z \underset{\sim}{\leq} \bar{z} \tag{4}$$

In particular, we propose to specify these uncertain relations/expressions as a fuzzy set (Zadeh, 1965; Zimmermann, 1996). In fuzzy theory, a crisp number belongs to a set (fuzzy set) with a certain degree of membership, named also satisfaction $h \in [0,1]$. The degree of membership is defined by a "membership function". Choosing a triangular and a trapezoidal membership functions, a potential description of the (3) and the (4) is showed in Figure 1 (Teodorovic and Vukadinovic, 1998). In Figure 1(a) is represented the relation between total costs of the network (z) and h. Figure 1(b) shows the relation between the equity performance indicator α and h. This representation is obtained by setting two values, to be positioned on the horizontal axis. The first one is β_{min} (that is the minimum value of the threshold), up to which the satisfaction is maximum and equal to 1; the second one is the β_{max} threshold, namely the maximum admitted value of α selected by the policy maker, at which the satisfaction h is set equal to zero.
In this framework it results that the closer to one the degree of membership is, the more the optimization (4) and the equity constraint (3) are fulfilled. Therefore, in order to find the optimal solution to the problem (1) subject to certain and uncertain (i.e., fuzzy) constraints/relations, it is necessary to maximize the satisfaction h.



(a)

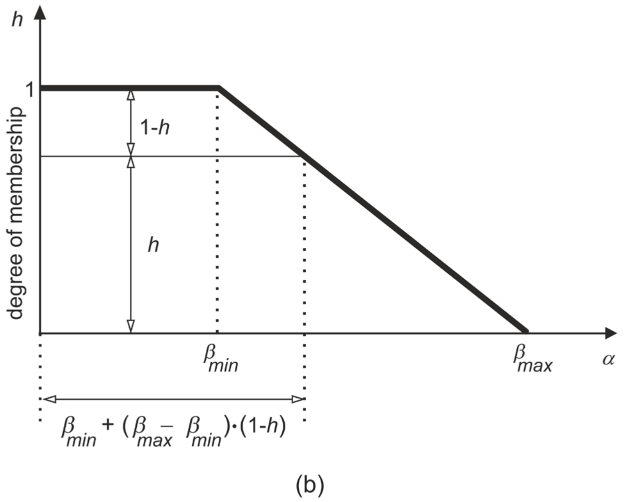


Figure 1. (a) Fuzzy set of O-D pairs cost differences; (b) Fuzzy set of equity performance indicator α.

For these reasons, the classic equity CNSP problem (1) turns in the proposed model (5):

$$\max h \qquad (5)$$

s.t.

$$z(\mathbf{x}(h), \mathbf{f^*}) \underset{\sim}{\leq} \bar{z} \qquad (5a)$$

$$\alpha_m(h) \underset{\sim}{\leq} \beta^m_{\max} \quad \forall\, m = 1,2,...v \qquad (5b)$$

$$\alpha_m \leq \beta^m_{\max} \quad \forall\, m = v+1, v+2,...u \qquad (5c)$$

$$\mathbf{f^*} = \mathbf{\Delta}(\mathbf{x})\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f^*}, \mathbf{x}))\mathbf{d}(\mathbf{C}(\mathbf{f^*}, \mathbf{x})) \qquad (5d)$$

$$\mathbf{x}_e, \mathbf{f^*} \in E \quad \forall\, e = 1,2,\ldots, p \qquad (5e)$$

$$\mathbf{x}_i, \mathbf{f^*} \in T \quad \forall\, t = 1,2,\ldots, q \qquad (5f)$$

where:

- α_m are the equity performance indicators with their maximum $\left(\beta^m_{\max}\right)$ values;
- v is the number of uncertain equity constraints;
- u is the number of certain (with fixed threshold) equity constraints;

The problem (5) is a fuzzy optimization problem (Teodorovic and Vukadinovic, 1998), where the total cost minimization of the (1) becomes the constraint (5a), expressed according to the value of satisfaction h. The set of equity performance indicators of the (1a) are transformed into uncertain (5b) and, if any, certain equity constraints. The assumed fuzzy constraints (5a) and (5b) will definitely depend on the same value of the satisfaction h. Therefore, the closer to one the value of h (maximization of satisfaction) is, the more the uncertain constraints are optimized. After the selection of the membership functions, the eq. (5a) and (5b) are turned into fuzzy sets. If we specified these constraints accordingly with the previous example (Figure 1) they become:

$$z(\mathbf{x}(h), \mathbf{f^*}) \leq \bar{z} \cdot (1-h) \qquad (6a)$$

$$\alpha_m \leq \beta_{\min}^m + \left(\beta_{\max}^m - \beta_{\min}^m\right) \cdot (1-h) \tag{6b}$$

Depending on the choice of the equity performance indicators, the (5b) constraints can be differently described. In the next subsection we present a specification of the proposed model.

3.2 An equity constraints specification

Among the different aspects related to equity, we will refer to the O-D travel costs imbalance due to the O-D travel costs increase or decrease after implementing an optimal network design scenario. In particular, we assume the equity performance indicator equal to the critical O-D travel cost ratio proposed in Meng, and Yang (2002). This indicator (eq. 5) is defined as the maximal ratio of the equilibrium O-D travel cost after implementing a network design scenario ($\mu_w(\mathbf{x})$), divided by the equilibrium O-D travel cost before scenario implementation ($\bar{\mu}_w$) for a set of specific O-D pairs W.

$$\alpha = \max_{w \in W} \left\{ \frac{\mu_w(\mathbf{x})}{\bar{\mu}_w} \right\} \tag{7}$$

If $\alpha < 1$ all users can benefit from the network design implementation, if $\alpha > 1$ there will exist users who suffer a travel cost increase induced by the design implementation. To deal with this equity issue, the equilibrium O-D travel cost reduction/increase for each O-D pair can be restricted beyond/below a given level by assuming α to be less than a desirable threshold β_{max}. The parameter β_{max} is a given appropriate positive constant, set by decision makers, which measures the degree of equitability of benefit distribution. A smaller value of this parameter means a more equitable distribution of benefits across network users. If β_{max} is set to be lower than 1, then each user will enjoy a travel cost reduction at least by $100 \cdot (1- \beta_{max})$ %. If, however, β_{max} is set to be greater than 1, it means that there may be users who suffer a travel cost increase induced by the improvement scheme, but travel cost increase cannot be more than $100 \cdot (\beta_{max}$ - 1)% (see Meng, and Yang (2002) for further details).

Unlike Meng and Yang (2002) that consider β_{max} a crisp threshold, in the proposed model specification we introduce uncertain and incomplete information about this threshold, in this case mostly related to the demand-side uncertainty that could affect the travel costs. In other words, using a single equity performance indicator and defining it according to the eq. (7), the (5b) becomes:

$$\max_{w \in W} \left\{ \frac{\mu_w(\mathbf{x}(h))}{\bar{\mu}_w} \right\} \underset{\sim}{\leq} \beta_{\max} \tag{8}$$

If we assume for the (8) a trapezoidal membership function, as depicted in Figure 1(b), and one uncertain equity constraint, the eq. (6b) becomes:

$$\max_{w \in W} \left\{ \frac{\mu_w(\mathbf{x}(h))}{\overline{\mu}_w} \right\} \leq \beta_{\min} + (\beta_{\max} - \beta_{\min}) \cdot (1 - h) \qquad (9)$$

4. Sensitivity Analysis and method comparison

The following application has a dual purpose. The first aim is to experimentally evaluate the performances of the proposed design model through a sensitivity analysis. The second aim is to compare our approach, which considers uncertainty in the equity constraints, with a classic equity based model that uses only crisp constraints such as the first equity based model proposed by Meng and Yang (2002). In this test, we propose a network design optimization considering signal settings parameters as supply design variables. The chosen approach to the problem is the global optimization of signal settings that consists in searching the vector of optimal effective green times (**x***) for all signalized intersection; these values are obtained through the minimization of the network total cost z depending on signal settings (**x**), on equilibrium flows (**f***) and on equity constraints. This analysis has been carried out on the test network and data proposed by Yang, Meng and Bell (2001) (Figure 2).

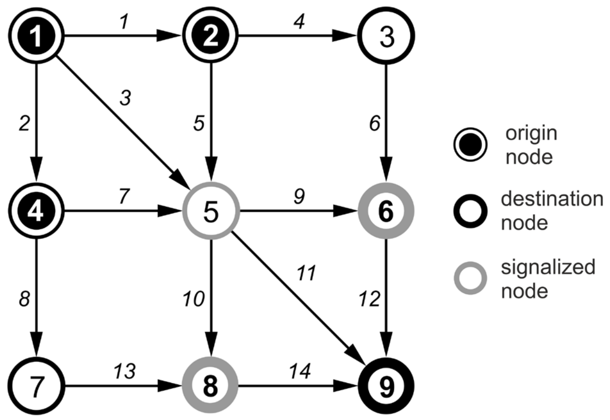


Figure 2. Test network.

The graph of the network is made up of 9 nodes (3 origins and 3 destinations), 14 links and 9 O-D pairs. We have set the signalized intersections in the node 5 with a three-phase regulation scheme and in the nodes 6 and 8 with a two-phase regulation scheme. If we assume g_{ph}^{nd} the effective green time for the node *nd* and for the phase *ph,* the vector of the design variables is $\mathbf{x} = [g_1^5;\ g_2^5;\ g_3^5; g_1^6;\ g_2^6;\ g_1^8;\ g_2^8]$. For all the signalized intersections the effective cycle time is fixed to $C_t = 90$ seconds.
The link cost values c_l for the numerical tests are the sum of the link travel time and the waiting time due to the signalized intersections. The link travel time tc_l, function of the link flow f_l and of the link capacity ca_l, is calculated using the well-known Bureau of Public Roads (BPR) link cost function (U.S. Bureau of Public Roads, 1964) (eq. 10) where the free flow travel time (*tr*) and capacity (*ca*) depend on the considered link *l*.

$$tc_l(f_l) = tr_l \left[1 + 0.15 \left(\frac{f_l}{ca_l} \right)^4 \right] \quad l = 1, 2, ..., 14 \qquad (10)$$

The waiting time is estimated using the Doherty's delay function (Doherty, 1977) (eq. 11):

$$t^l_{wa} = 0.5 \cdot C_t (1-\mu)^2 + \frac{1980}{\mu \cdot s} \cdot \frac{f_l}{\mu \cdot s - f_l} \qquad \text{if } f_l \leq 0.95 \cdot \mu \cdot s$$

$$t^l_{wa} = 0.5 \cdot C_t (1-\mu)^2 + \frac{198.55}{\mu \cdot s/3600} + \frac{220 \cdot f_l}{(\mu \cdot s)^2/3600} \qquad \text{if } f_l > 0.95 \cdot \mu \cdot s \tag{11}$$

where:

- t^l_{wa} is the waiting time at intersection on link l (s/veh);
- f_l is the traffic flow on link l (veh/h);
- s is the saturation flow (veh/h);
- g is the effective green time;
- μ is the effective green ratio (g/C_t).

The travel demand **d** (Table 1) has been assigned to the network using a Deterministic User Equilibrium (DUE) traffic assignment model. It is based on the assumptions that network travel times are deterministic for a given flow pattern and that all travelers are perfectly aware of the travel times on the network and always capable of identifying the shortest travel time route. The calculation of equilibrium link flow with rigid demand is based on different algorithms; in our model we adopt the Frank-Wolfe algorithm (LeBlanc, Morlok, and Pierskalla, 1975; Nguyen, 1976).

Table 1. O-D vector.

O-D	1-6	1-8	1-9	2-6	2-8	2-9	4-6	4-8	4-9
l	1	2	3	4	5	6	7	8	9
d	120	150	100	130	200	90	80	180	110

The sensitivity analysis has been carried out in two steps. During the first one, a series of Starting Configurations (SC) based on different effective green times, divided into different combinations among the traffic light phases and nodes, has been generated. In this way we have obtained, for each SC (i.e., combination of effective green times) and through the DUE assignment, an initial total cost of the network, denoted by $\bar{z}$ and starting values of equilibrium flows $\mathbf{f}^{*SC}$.

The second step implied the application of three different optimizations to each one of the SC. The optimizations performed, with the corresponding objective functions and constraints, are summarized in Table 2, in order to appreciate, at a glance, the differences existing among them.

- Crisp Optimization (CO)

This optimization is the classical CNDP with the minimization of network total cost. The only other requirements to be fulfilled are the flows on links 1-2 and 1-4, which must be reduced at least by 50% compared to the equilibrium flows of the corresponding starting configuration. It has been assumed hypothetically that these two selected links were characterized by specific conditions (e.g. the presence of schools). This reduction of flows, for that reason, appears to be a vertical equity constraint. Furthermore, the value of the equity performance indicator α_{CO} was calculated according to the eq. (7).

Table 2. Performed optimizations.

		Crisp Optimization (CO)	*Equity Crisp Optimization* (ECO)	*Equity Fuzzy Optimization* (EFO)
Objective functions	Satisfaction	-	-	$\max h$
	Network total cost	$\min z(\mathbf{x}, \mathbf{f}^*)$	$\min z(\mathbf{x}, \mathbf{f}^*)$	-
Problem constraints	Network total cost	-	-	$z(\mathbf{x}(h), \mathbf{f}^*) \leq \bar{z} \cdot (1-h)$
	Horizontal equity	-	$\alpha_{ECO} \leq 0.9 \cdot \alpha_{CO}$	$\alpha_{EFO} \leq \beta_{\min} + (0.9 \cdot \alpha_{CO} - \beta_{\min}) \cdot (1-h)$
	Satisfaction	-	-	$0.01 \leq h \leq 1$
	Vertical equity Flow (1-2)	$f^*_{1\text{-}2} \leq 0.5 \cdot f^{*SC}_{1\text{-}2}$		
	Vertical equity Flow (1-4)	$f^*_{1\text{-}4} \leq 0.5 \cdot f^{*SC}_{1\text{-}4}$		
	Demand – flows consistency	$\mathbf{f}^* = \mathbf{\Delta}(\mathbf{x})\mathbf{P}(\mathbf{x}, \mathbf{C}(\mathbf{f}^*, \mathbf{x}))\mathbf{d}(\mathbf{C}(\mathbf{f}^*, \mathbf{x}))$		
	Effective green time	$5 \leq g_{ph}^{nd} \leq 80 \quad \forall \; nd \in \{5,6,8\}$		
	Cycle time	$\sum_{ph} g_{ph}^{nd} = 90 \quad \forall \; nd \in \{5,6,8\}$		

- Equity Crisp Optimization (ECO)

A new constraint has been added to the previous optimization: the horizontal equity constraint. We assume that decision makers want to reduce, at least by 10% (such planning decision should reflect community needs and values), the equity performance indicator α_{CO}. In other words, the equity performance indicator calculated for the Equity Crisp Optimization (denoted by α_{ECO}) must be lower than or equal to the 90% of the corresponding α_{CO}. The ECO is a classic equity based optimization model (1) that does not consider uncertainty in the equity constraints. This optimization is equal to the first model proposed by Meng and Yang (2002) with the addition of vertical equity constraints explained by the reduction of flows on two specific links.

- Equity Fuzzy Optimization (EFO)

This optimization is based on our proposed model (5) with the specification of the equity constraints (5b) described in sub-section 3.2. In this optimization, the objective function to be maximized is the satisfaction h, and the network total cost minimization has become a further constraint of the problem expressed according to the value of satisfaction h. The horizontal equity constraint to this problem is the same as the one considered in ECO but with uncertainty in its definition. In other words, this equity constraint translates the following expression stated by decision makers: "α_{EFO} must be approximately lower than or equal to $0.9 \cdot \alpha_{CO}$". This constraint is equivalent to the fuzzy set of the Figure 1(b) where $\beta_{max} = 0.9 \cdot \alpha_{CO}$ and $\beta_{min} = 0.50$. This value should be set by decision makers and should be greater than or equal to the minimum value of the critical ratio $\mu_w(\mathbf{x}) / \overline{\mu}_w$ (in our case the local minimum value of this ratio is equal to 0.4886). The conditions about the reduction of flows on links 1-2 and 1-4 are unchanged.

All the proposed optimizations have been solved through an interior-point algorithm described in Waltz et al. (2006). Not all the generated starting configurations, under the optimization stopping criteria (tolerance on constraints violation equal to 10^{-3}, on function equal to 10^{-3} and on variables values equal to 10^{-10}), have led to a final feasible solution. Sixty initial setups, under these stopping criteria, allowed reaching a feasible solution at the same time for all the three described optimizations. These sixty configurations represent quite well the solution space since those effective green times imply starting values of network total costs ranging from a minimum to a value about eight times greater than it (Figure 3).

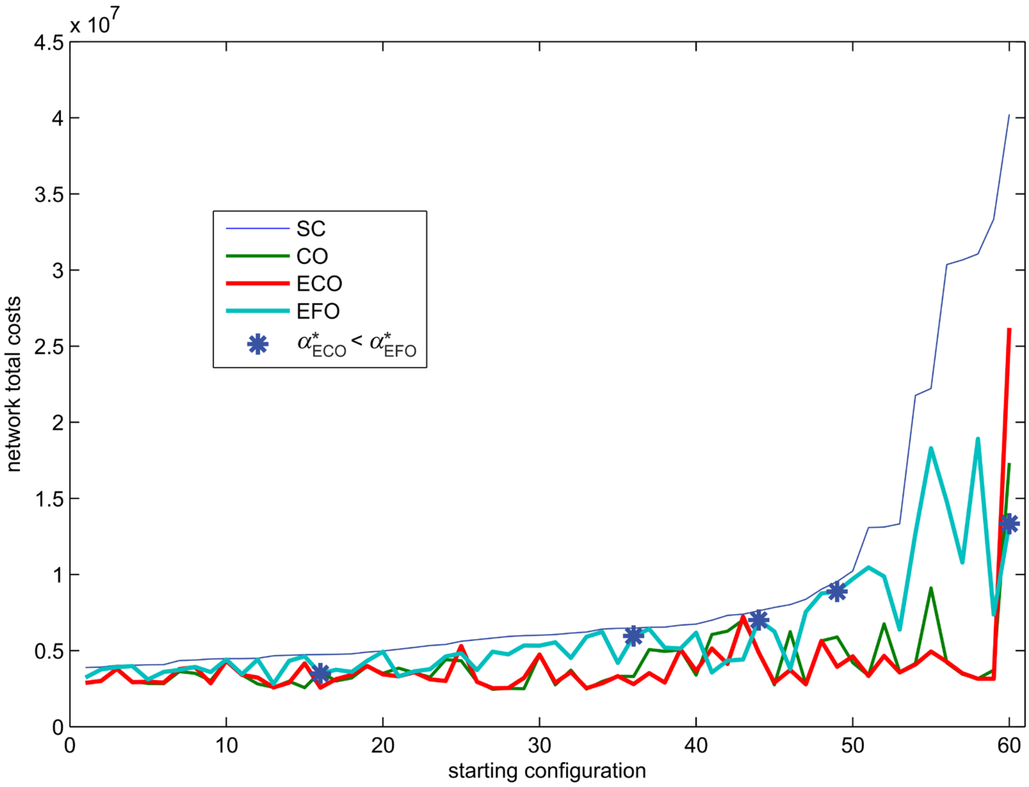


Figure 3. Network total costs comparison.

For each of these starting configurations and at the end of each optimization, equity performance indicators (α^*) and optimized values of total network costs (z^*), of flows ($f^*_{1\text{-}2}$, $f^*_{1\text{-}4}$) and of effective green times are obtained. In the Table 3 the results obtained starting from one SC are shown.

Table 3. Results excerpt.

	g_1^5[s]	g_2^5[s]	g_3^5[s]	g_1^6[s]	g_2^6[s]	g_1^8[s]	g_2^8[s]	$\alpha*$	$z*$	$f*_{1-2}$	$f*_{1-4}$
SC	35.0	20.0	35.0	80.0	10.0	10.0	80.0	-	5333852	120	150
CO	54.3	23.1	12.6	53.6	36.4	34.3	55.7	10.2	3254904	59	75
ECO	55.8	21.8	12.4	57.6	32.4	33.2	56.8	9.15	3116790	60	75
EFO	60.8	15.2	14.0	62.6	27.4	30.9	59.1	6.68	3826857	59	74

All the key results of the sensitivity analysis are shown in Figure 3. In this figure the SC have been sorted in view of their respective total cost of the network, arranged in ascending order. Examining the network total costs z obtained at the end of the three optimizations, we can see that the ECO has a better behaviour than the EFO, presenting optimized network total costs generally lower than the ECO case. This is only a partial point of view because it is also important to analyse the constraints satisfaction. In other words, it is essential to look what happens to the final values, at the end of the optimization, of the equity performance indicators (denoted by α^*_{ECO} and α^*_{EFO}) and of flows on links 1-2 and 1-4 (f_{1-2}^{*ECO} and f_{1-2}^{*EFO}).
In order to compare the equity performance indicators obtained from ECO and from EFO models we introduce the α_{red} percentage defined as follows (12):

$$\alpha_{red} = \frac{\alpha^*_{ECO} - \alpha^*_{EFO}}{\beta_{max}} \cdot 100 \qquad (12)$$

In Figure 4, we show the values of α_{red} for each starting configuration, sorted in the same way of the ones in Figure 3.

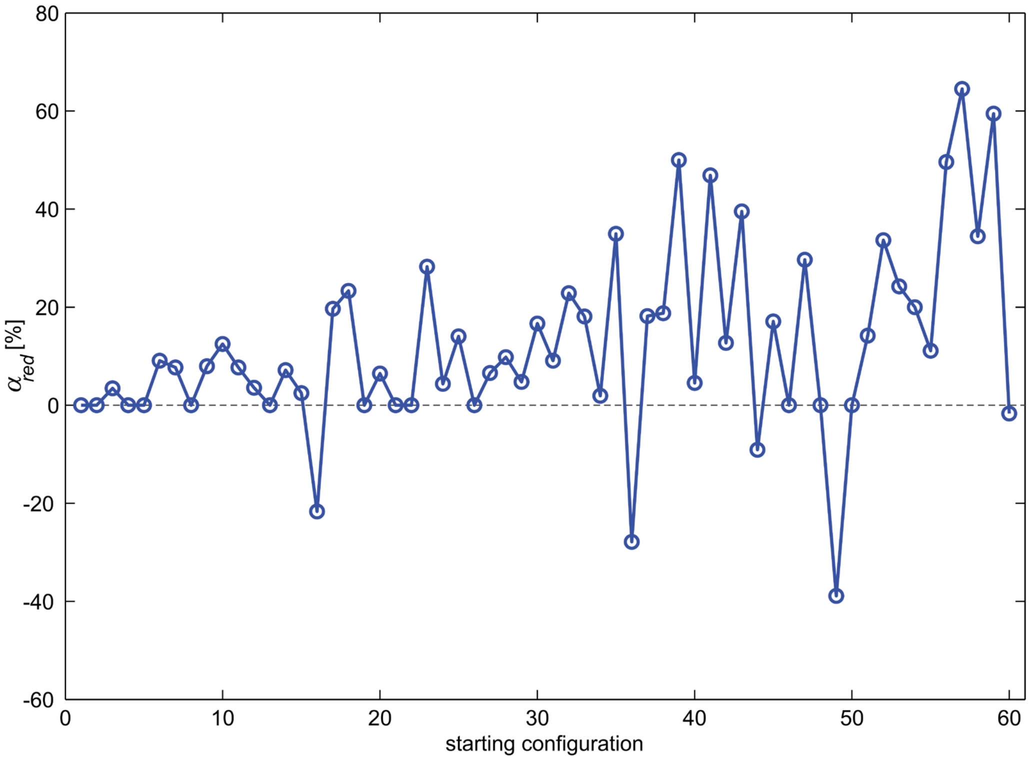


Figure 4. α_{red} values.

According to the specified definition of the equity performance indicator, the lower the α value is, the more all users can benefit from the network design implementation. Fuzzy optimization presents alpha values usually lower than those obtained by ECO; only in a few cases (five out of sixty) α^*_{ECO} is lower than α^*_{EFO}. These five optimized configurations, numerically described in Table 4, are highlighted with an asterisk in Figure 3 and with a negative value of α_{red} in Figure 4. However, in all of these just mentioned cases, except for the configuration number 36, the conditions regarding the reduction of the flows on the two links appear to be better satisfied in EFO (see Table 4).
Furthermore, it can be observed that as you move on the right side of the diagram in Figure 4, i.e. towards configurations having an initial higher total cost of the network, the value of α_{red} tends to increase. This means that the requirement of horizontal equity tends to be satisfied better and to a greater extent if we apply EFO starting from a very congested network.
Therefore, it is true that the costs of the network obtained with EFO are generally higher than those resulting from ECO, but in the face of an equity (both horizontal and vertical) considerably increased. The EFO allows spreading fairly 'disadvantages' on the network, better satisfying also the constraints imposed on the flows that have in proportion been further reduced. In conclusion, this sensitivity analysis allows to test the model considering both certain and uncertain constraints. According to the typology of variables involved in the formulation, we have proved that the model shows a different behaviour, leading to slightly different final remarks. If the ECO seems to better manage the reduction of the travel costs at the expense of the equity level reached on the network, the EFO's objective function aims at the maximization of the satisfaction h. In this case, travel costs represent a constraint for the model, such as the horizontal equity, and this latter is the one that benefits of a better room for improvement at the end of the optimization. We can assert that both the optimizations lead to valuable results, but the decision maker is the one that should choose one over the other (crisp or fuzzy) according to his final targets. Given that the purpose of this paper is to stress the importance of a fair level of equity that has to be reached on the network, we can say that EFO better satisfies our goals.

Table 4. The five optimized configurations with $\boldsymbol{\alpha^*_{ECO}}$ lower than $\boldsymbol{\alpha^*_{EFO}}$.

No. conf.	Opt.	z^{SC}	z^*	β_{max}	α^*	$f^{*SC}_{1\text{-}2}$	$f^*_{1\text{-}2}$	$f^{*SC}_{1\text{-}4}$	$f^*_{1\text{-}4}$
16	ECO	4735225	2568599	39.7	28.4	201	101	150	75
	EFO		3496947		37		97		65
36	ECO	6477105	2799069	7.9	4.1	356	145	0	0
	EFO		5957511		6.3		169		0
44	ECO	7609420	4854538	1.1	0.9	60	30	12	6
	EFO		7014479		1		26		5
49	ECO	9534159	3932306	1.8	1	33	17	138	69
	EFO		8887142		1.7		0		69
60	ECO	40237491	26191894	17.8	9	353	164	0	0
	EFO		13337529		9.3		150		0

5. Numerical application and results

In order to apply the proposed model to a larger network, more complex than the one used in the previous section, we use the Sioux Falls City network shown in Figure 5 as a real sized example. The network consists of 24 nodes, 76 links and 552 O-D pairs.
Signalized intersections are the nodes 4, 5, 6, 14 and 19 with a three-phases regulation scheme, and node 15 with a four-phases regulation scheme; for each of these six intersections, the starting effective cycle time is fixed to C_t = 90 seconds and the starting effective green time is equally divided for each phase.
The link cost values c_l are computed as in the sensitivity analysis, and the free flow travel times tr_l are numerically equal to those proposed by LeBlanc, Morlok, and Pierskalla (1975), but in minutes. Capacities have been set equal to 3600 veh/h for the peripheral links (i.e., links 1 and 3, 4 and 14, 16 and 19 …) and equal to 1800 veh/h for the remaining ones.
The travel demand **d** (the same matrix of trips shown in LeBlanc, Morlok, and Pierskalla (1975)) has been assigned to the network, using a Deterministic User Equilibrium traffic assignment model.
In this application, the vertical equity constraint that needs to be satisfied is the one related to flows on the arc 14-15, which have to be reduced by at least 10% compared to those on the same link of the corresponding starting configuration. The horizontal equity constraints are the same of the Table 2. The obtained results are summarized in Table 5 and Table 6.

Table 5. Optimized effective green time in seconds.

	g_1^4	g_2^4	g_3^4	g_1^5	g_2^5	g_3^5	g_1^6	g_2^6	g_3^6	g_1^{14}	g_2^{14}	g_3^{14}	g_1^{15}	g_2^{15}	g_3^{15}	g_5^{15}	g_1^{19}	g_2^{19}	g_3^{19}
SC	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	22.5	22.5	22.5	22.5	30.0	30.0	30.0
CO	23.9	13.2	66.2	16.1	40.8	49.2	19.7	14.8	58.7	37.8	15.5	79.8	79.4	12.2	15.0	13.9	13.4	48.2	80.0
ECO	45.5	9.3	69.2	15.6	5.0	71.5	9.4	10.2	71.4	36.9	12.6	80.0	80.0	8.6	12.8	19.7	13.2	48.2	8.4
EFO	29.9	28.9	31.4	29.9	29.2	31.0	29.9	30.2	30.1	34.5	30.8	30.3	29.4	10.9	22.0	26.5	27.2	32.0	30.2

Table 6. Results of the numerical application on the Sioux Falls City network.

Opt.	z^{SC}	$z*$	β_{max}	$\alpha*$	$f^{*SC}_{14\text{-}15}$	$f^*_{14\text{-}15}$
ECO	32010688	12696878	2.9710	1.4794	798	156
EFO		26177456		1.1072		303

Results in Table 6 go to validate what previously emerged from the sensitivity analysis performed on the smaller network. The EFO costs are higher than those of the ECO, while remaining well below the initial costs of the network (z^{SC}); the vertical equity constraint applied on flows is satisfied in both optimizations, but the $\alpha*$ value of the proposed fuzzy approach (α^*_{EFO}) is lower than the corresponding optimized value of the crisp case (α^*_{ECO}).

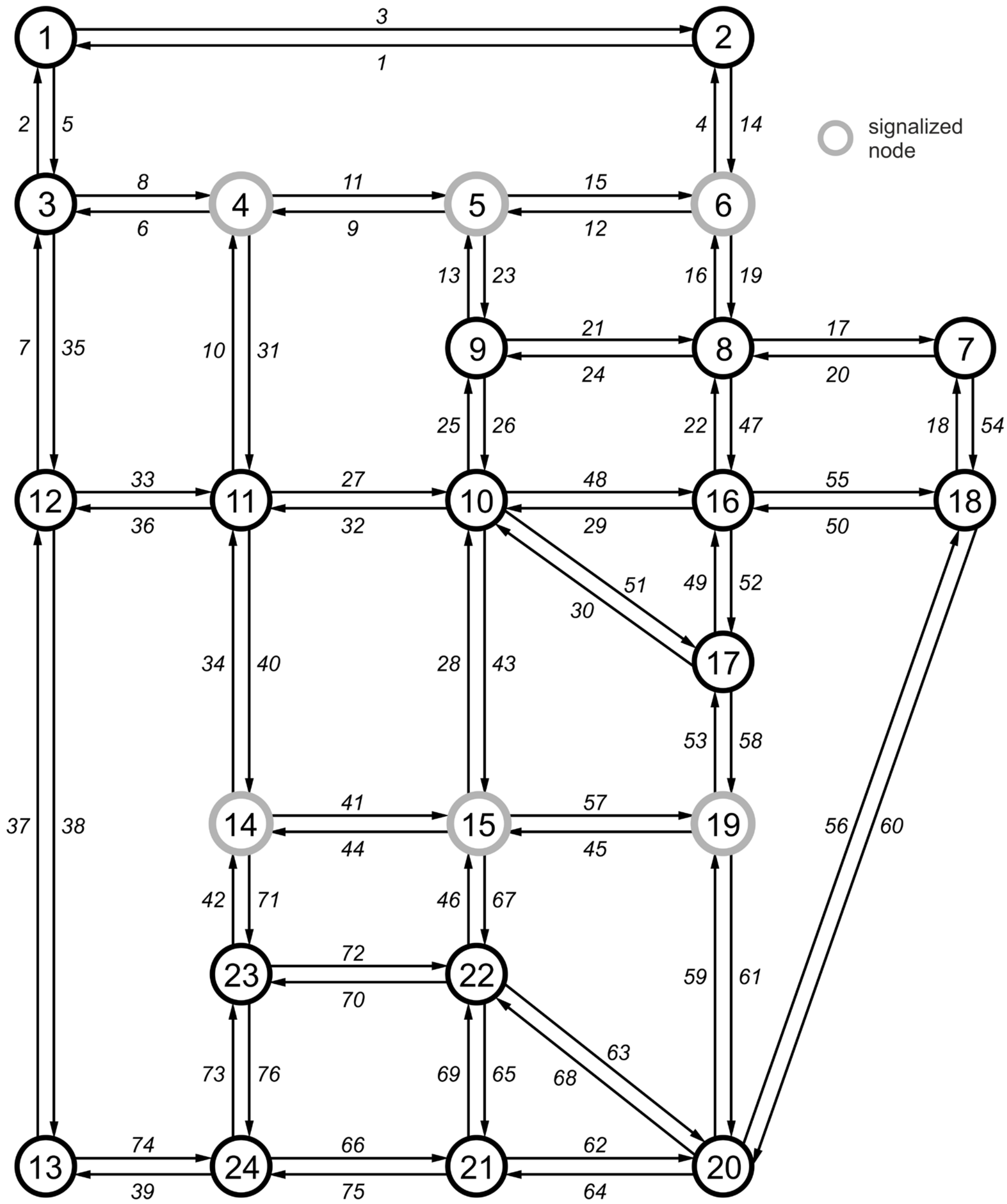


Figure 5. The Sioux Falls City network.

6. Conclusions

In literature, network design problems have been largely discussed. Traditional approaches often neglect equity goals that, conversely, should play an important role in transportation network design problem assuming the societal function of transport services. Equity constraints introduced by authors are mainly specified with rigid minimum and/or maximum thresholds. Actually, this constraints and other parameters of the network design problem can be affected by uncertainty.
In this paper, we suggest to quantitatively consider also flexible equity constraints in a network design problem explicitly represented by fuzzy sets. In order to include these

uncertain/imprecise values/linguistic expressions, the equity network design problem is then specified as a fuzzy programming problem.
In order to test the accuracy of the proposed model, it was first performed a sensitivity analysis starting from different initial configurations; after that, we compared our approach with a more traditional one (crisp), that even including equity constraints, does not take into account the presence of uncertainties in the problem.
It was found that, with a fuzzy approach, although the total costs of network result generally higher, the equity aspects (either horizontal or vertical) appear to be satisfied to a greater extent, enabling to spread more fairly the 'disadvantages' arising from a network redesign. These results were then confirmed by a numerical application to a larger network.
It may be concluded that it is important for decision makers to reach a compromise between costs and equity, not just focusing on the immediate feedback that an apparent saving can provide, but thinking about the real achievements (both spatial and social) that their actions will have on those who will actually take advantage of the transport network.
Further research activities are dealing with the inclusion of uncertainty level about other available vague information and test the formulation to transit network design problem.

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