Forced sale discount on property market – How to assess it?

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Abstract

The aim of the article is to reduce the gap in the lack of tools dedicated to determining forced sale value. It is focused on the development of a methodology that takes into account the procedures used in property valuation with the use of automated valuation methods based on rough set theory and fuzzy logic. The authors propose an alternative method serving as an alternative to individual/human valuation, which may not be objective and reliable for such purpose. The method gives more accurate results than hedonic models in the face of qualitatively and quantitatively ambiguous, imprecise and vague data, which is commonly found in the real estate market, especially regarding not typical/market sales. The methodology was verified on the basis of data collected on two property markets: city of Bari (Italy) and city of Olsztyn (Poland). The achieved results indicated that value regarding forced sale is about 22% lower in the case of Olsztyn when compared to average market value of the used sample, whereas in the case of Bari – about 45% lower.

Keywords: Real estate valuation; Forced sale value; AVM; Rough set theory; Valued tolerance relation

1. Introduction

The contemporary economy of every country involving, in particular, optimal land management, requires knowledge concerning real estate value. This is due to the fact that real estate (land, buildings) constitute a significant share in the total value of national assets. The value of real estate often forms the basis or is an instrument supporting decision-making processes by various public and private entities (Kucharska-Stasiak, 2017). Different purposes for which the value is applied require the adoption of particular assumptions under which it is determined. Knowledge of the real estate market value in its classic (standard) interpretation is most often needed. However, due to the significant development of the mortgage market, there is an increasing demand of creditors (primarily banks) for knowledge regarding the expected price that a property will achieve if the debtor does not repay the loan (Źróbek et al., 2013). This is seeing as how such a situation most often necessitates the selling of the debtor's real estate. The notion of real estate forced sale value functions as a special category of real estate values in the minds of market participants. Generally speaking "…The term "forced sale" is often used in circumstances where a seller is under compulsion to sell and that, as a consequence, a proper marketing period is not possible and buyers may not be able to undertake adequate due diligence. The price that could be obtained in these circumstances will depend upon the nature of the pressure on the seller and the reasons why proper marketing cannot be undertaken…" (IVS, 2017; General Standards IVS, 2016). The valuation problem is focused mainly on the determination of specific conditions of sales. The sale occurs in a hurried manner and there is no opportunity to carry out appropriate marketing activity by the seller. It is worth noticing that this kind of situation does not represent a new basis of value. The valuation standards state that "…A forced sale is a description of the situation under which the exchange takes place, not a distinct basis of value…" (IVS, 2017; General Standards IVS, 2016). When comparing the description of liquidation value with the basis of Market Value, there are three main differences (among others) that may be helpful to understanding these specific conditions of sale: "(a) consummation of sale within a short-term period …(d) the seller is under compulsion to sell … (g) a normal marketing effort is not possible due to the brief exposure time (IVS, 2017; General Standards IVS, 2016). In this specific case, the price is normally influenced not only by the interaction of supply and demand in the specific market segment but also by other important factors influencing the sale, such as the urgency to sell (Campbell et al., 2011). A further aspect of foreclosure is the spill over effect of foreclosures. In fact, foreclosure may have a potential impact on the value of properties in the surroundings, creating a disequilibrium between supply and demand. "…In some studies, foreclosure on a home has been found to reduce the prices of nearby homes by as much as 9 percent…" (United States Treasury, 2009). From the lenders point of view, a liquidating discount forecast is a way to address risk. The process of forced sell of a property is an aspect of real estate market risk. "Risk may be thought of as a measure of future uncertainties that may result in a fall

in the price or value of an asset" (Valuation Uncertainty, 2013). The aim of this paper is to develop a methodology that takes into account the procedures used in property valuation with the use of Automated Valuation Methods (further called AVMs) to determine the approximate (rough) change in property values at forced conditions, with particular emphasis placed on the time of market exposure. The aim of the article is to reduce the gap in the lack of tools dedicated to determining value under such special conditions. The research was based on the following two theses: 1. The price obtained from forced sales is lower than its market value and 2. The existing methods for determining the value for forced sales do not allow a reliable valuation result to be achieved. The authors of the article put forward a research hypothesis that: "Real estate valuation based on the assumptions of Automated Valuation Methods using rough set theory and valued tolerance relation enables the determination of property value for sales in shortened/ inadequate exposure time at a satisfactory level of credibility". The analyses were conducted on the basis of the residential real estate transactions concluded in 2016 on Olsztyn (Poland) and on Bari (Italy) markets, that constitute not only a common element for securing basic existential needs and capital location, but are also an important factor determining the conditions and investment potential of a given region. The research aims to present an analytical algorithm based on AVM that may provide a satisfactory level of credibility for establishing forced sale discount. The paper is structured in the following way. First, an explanation of the forced sale value definition is provided - Section 1. Section 2 presents an international point of view regarding forced sale value and problems with its interpretation. Section 3 presents research methodology, analytical procedures based on regression and rough set theory and empirical analyses/results. Section 4 presents final remarks, and finally, in Section 5, conclusions and future directions of research are formulated.

2. Literature review

In literature on the subject and in property valuation standards the types of property values are called "bases of value". They describe the fundamental premises on which the reported values will be based. It is essential that the basis (or bases) of value may influence or dictate a valuer's selection of methods, inputs and assumptions, and the ultimate opinion on value (IVS, 2017). Due to the fact that the most desirable basis for valuation is market value, this type of value has a uniform definition according to valuation standards (EVS, 2016; IVS, 2017; RICS, 2017) and the European Directive (Directive …, 2006). Also in Poland, this term has been harmonized and introduced by law (The Real Estate Management Act …, 1997). This "globally" agreed definition is as follows: Market Value is the estimated amount for which an asset or liability should exchange on the valuation date between a willing buyer and a willing seller in an arm's length transaction, after proper marketing and where the parties had each acted knowledgeably, prudently and without compulsion. The two fundamental assumptions (premises) where the

market value should be determined were explicitly articulated (EVS, 2016): 1. after proper marketing – this means that the property had been exposed to the market in the most appropriate manner to affect its disposal at the best price reasonably achievable in accordance with the Market Value definition. The length of exposure on the market must be sufficient to allow the property to be brought to the attention of an adequate number of potential purchasers. 2. without compulsion – this establishes that each party is motivated to undertake the transaction but is neither forced nor unduly coerced to complete it. Each freely enters into and completes the business. Evans and William (2007) distinguish two types of sales: forced sale - an involuntary sale resulting from foreclosure, execution on a judgment, partition action by a cotenant wanting to sell the property and divide the money, or a divorce proceeding; distressed sale – the sale of real property under circumstances creating great urgency on the part of the seller, such as impending foreclosure, divorce, relocation to another city, or any other such pressure. In valuation practice, the notion of "distressed property" also exists. This term means property that is under a foreclosure order or is advertised for sale by its mortgagee. Distressed property usually fetches a price that is much below its market value (Babineau, 2010). For example: according to data on house transactions in the state of Massachusetts over 20 years, foreclosure discount equaled on average 27% of the value of the house (Campbell et al., 2013). It should be noted that in most countries, including Poland, the basis for the valuation to secure the creditor's claims is, as a rule, the market value. However, on request of the creditor or the ordering party, other types of values can be determined (Professional Standard of Property Appraisers …, 2017) or a mortgage lending value, if this results from legal provisions (Directive …, 2006; Act of 29 August 1997 on Covered Bonds…, 1997Act of August, 1997bAct of 29 August 1997 on Covered Bonds…, 1997). However, the enforceability of debt in banking practice through the sale of real estate has caused these special circumstances of the transaction to be reflected in the valuation standards (EVS, 2016; IVS, 2017; RICS, 2017). EVS introduced the following definition of Forced Sale Value – "A sum that could be obtained for the property where, for whatever reason, the seller is under constraints that require the disposal of the property in conditions that do not conform with the definition of Market Value." However, these standards, at the same time, emphasize that forced sale value is not a basis of value but an example of Market Value under special assumption as to the conditions for marketing. Once all the relevant constraints are identified, it may be seen as a Market Value assessment under Special Assumption of a stated but limited period for marketing the property. Thus, the valuer should not undertake a valuation on a forced sale basis but rather on a Market Value basis under stated specific special assumptions relevant to the case in hand. IVS introduced the concept of forced transaction (forced sale) as premises which may affect the determined liquidation value (IVS, 2017). It should be emphasized here that liquidation value on the basis of European standards is one of the foundations of valuation as opposed to international standards, where liquidation value can

be determined under two different premises of value: (a) an orderly transaction with a typical marketing period, or (b) a forced transaction with a shortened marketing period. Interpretation of a forced transaction written in IVS and the Red Book is in accordance with the interpretation given in EVS. Everywhere, it is clearly emphasized that "a forced sale is a description of the situation under which the exchange takes place, not a distinct basis of value." Valid for the valuation practice is the fact that real estate sales transactions made on an inactive market should not be automatically considered as forced sales because the seller may conclude this transaction without force (RICS, 2017). The RICS has written down the recommendations for its members at the end of paragraph 170.1. "The price that a seller will accept in a forced sale will reflect its particular circumstances, rather than those of the hypothetical willing seller in the Market Value definition. In short, an RICS member can provide "forced sale" advice but must be very careful about the terms in which it is reported" (RICS, 2017). In spite of relatively unambiguous normalization of the definition issue in relation to forced sale, both in literature and in the professional experience of the authors of this paper as real estate appraisers, there is a lack of clear procedures or ways to determine value useful for this type of sale. Property appraisers often encounter barriers in its determination mainly due to the fact that the market of "forced" transactions is opaque and difficult to correctly detect and describe. The basic reason here is the difficulty in identifying the real reasons and conditions of the transactions (motives of the parties), allowing them to be clearly classified as "forced". As a consequence, the appraiser makes a number of hypothetical assumptions in the valuation process. Also, recipients of valuations (valuation reports) are often not fully aware of the interpretation of the received result and what assumptions its determination was based on. One of the ways to "evade" the problem of the lack of comparative forced transactions may be to determine coefficients (indicators) correcting market value. In this study, these were named as forced sale discount and liquidity.

3. Research methodology

The real estate market, including the housing market, is fundamentally different from the model - one could say the textbook model - of the liquid assets market with exogenous foundations (Trojanek, 2012; Źróbek-Różańska, 2016). This means that the sale price of a house or flat can be affected not only by the general conditions of supply and demand, but also by idiosyncratic factors, including the urgent need to sell them. Real estate mortgaged (foreclosures) by financial institutions as a result of the insolvency of their owners can be sold at low prices, both because they are subject to a natural destruction process over time and susceptible to damage by vandals, and also because financial institutions have motivation to sell them quickly. This is due to the fact that, e.g. "owning an empty house is equivalent to throwing away the dividend on a financial asset" (Campbell et al., 2011). Therefore, the basic problem in correctly defining forced sale values is the proper detection of the reasons for lower transaction prices. It is important to adopt

the real assumption that price decreases obtained in forced sales only reflect the sales conditions and not, for example, other unaddressed differences in the essential features of real estate, such as the technical condition of the building or a lower standard of the premises. Prices obtained in forced sales may also be the reason for the general reduction in prices obtained in typical market transactions, as they may, as cheaper alternatives, reduce the demand for those apartments that were exposed to the market later. As a consequence, they may constitute reference prices for the negotiations undertaken by the potential parties to the transaction. It is possible to say that the condition of forced sale will lead the seller to earn less money than in the situation of a typical market sale. Therefore, it is possible to write:

$$V_{FV} < V_{MV} \quad (1)$$

where:
VFV – forced sale value
VMV - market value
Therefore, as both of the values are measured in the same way, it is possible to write:

$$V_{MV} - V_{FV} = D \quad (2)$$

In Formula (2), D is the difference between the two values (further referred to as forced sale discount). It is possible to express the forced sale value as a percentage (Δ) of the market value, as in the Formula (3):

$$V_{MV} - V_{MV}\Delta = V_{FV} \quad (3)$$

Therefore, the difference between the two values will be:

$$V_{FV} = V_{MV}(1-\Delta) \quad (4)$$

and

$$D = V_{MV} - V_{MV}(1-\Delta) \quad (5)$$

The problem is, therefore, how to determine a forced sale discount (D), which can be considered

as the difference between the market value and as a percentage (Δ) of market value discount regarding forced sale.

3.1. Research methods

For the purposes of the practical analysis conducted in the paper the main assumptions of forced sales were adopted:

- the seller is under compulsion to sell the property,
- a normal marketing effort is not possible due to the not market exposure time,
- transaction takes place within a short-term period.

Due to these assumptions the authors established adequate methodology to investigate forced sales. Establishing forced sale value of real estate may be conducted in two ways. First are the traditional or manual methods of valuation (human valuers); second are methods based on advanced automated procedures called Automated Valuation Methods (computer valuers - AVM). The traditional methods based on human valuers are generally more detailed, but also more subjective. The latter usually cost less and are more time effective when it comes to processing bigger databases. In this case, we try to present, in our opinion, a fast and effective method as an alternative to individual/human valuation not undervaluing traditional methods which are indisputably one of the most reliable methods. The popularity of computerized systems for collecting and processing real estate market data has soared in recent years. From the analytical point of view, the biggest challenge in this domain is the selection and elaboration of appropriate methods for analyzing the available information characterized by a specific character, different from other markets (e.g.: heterogeneous nature of real estate, lack of information, unexpected changes, etc.). AVMs are based on mathematical models providing real estate valuation. The main role of these models is to make some generalizations using evidence (usually statistical data) about the real world (Backhouse, 2002; Bobkowska et al., 2017; Rao, 1994). The most common and well-known method used in this case are hedonic models using linear regression techniques that give the possibility of estimating the contribution of particular attributes of real estate to the overall value. In this case, the aggregation of observations into a form in which the dependent variable is a mathematical linear function of independent variables is assumed in a way that allows for statistical inference regarding the parameters of the function outside the specific sample (Ron, 1999). The main objections toward this method, raised, among others, by d'Amato and Kauko (2017), are that the assumptions in these models account for the average conditions of properties and lack intuition provided by human valuers, which may lead to misleading estimates in atypical situations. Additionally, valuations based on the regression method have to fulfill many conditions, for example: big databases of transactions, linear correlation between dependent and independent variables, or the minimal occurrence of and the lowest possible values of correlations between independent variables. Due to the simplicity of

use and popularity of the above-mentioned method, the authors calculated the influence of forced sale on the real estate prices with the use of regression models. However, in response to the raised doubts concerning the accuracy and efficiency of this method, the authors propose an alternative method to establish the influence of forced sale on the value of real estate. The alternative method is based on rough set theory and fuzzy logic. The classical rough set theory was developed (Pawlak, 1982, 1997) to analyze qualitatively and quantitatively ambiguous, imprecise and vague data, which is commonly found in the real estate market and accompanies decision making (fuzzy decision making) in that market. Rough set theory (RST) with a valued tolerance relation extension (VTR) is used in many sciences (Bello and Verdegay, 2012; Chi et al., 2011; Polkowski, 2010), with some of its applications being connected with the valuation of properties (e.g. d'Amato, 2008; Renigier-Biłozor, 2010). In this theory, an analyzed phenomenon is considered as an object characterized by features related to a specified piece of information. Conventional rough set theory is based on the crucial concept of the indiscernibility relation, which is a crisp equivalence relation (complete, reflexive, symmetric and transitive relation valued in {0,1}). Stefanowski and Tsoukiàs (2000) argue that, practically speaking, two objects described under a set of attributes are indiscernible if they have identical values, although objects may be practically indiscernible without having identical values. The concept of valued tolerance as an extension of the usual concept of indiscernibility (which is a crisp equivalence relation) in rough set theory gives such opportunity. During the analysis of a unique set of properties, the classic rough set theory has been enhanced with the valued tolerance relation formula. This formula was developed and discussed by Stefanowski and Tsoukias (2000), and deployed in real estate market analyses by Renigier-Biłozor (2011) and Renigier-Biłozor et al. (2017). In order to determine the value for forced sales by adjusting the market value, this value should be treated as a function of the sales period. At the same time, the built-in function should have certain features resulting from the economic content of the term "market value", "value for forced sales" and "exposure period" (Gnenny et al., 2013.) A property appraiser can also perform a sensitivity analysis of the valued property for changes in parameters. One such parameter may be the time of real estate exposure on the market of sales offers. In order to verify the research hypothesis regarding the valuation of real estate based on the assumptions of Automated Valuation Methods using rough set theory and valued tolerance allowing the value of property for sale in the terms of shortened/inadequate exposure time to be determined at a satisfactory level of reliability, first the authors applied cluster analysis. Cluster analysis is a set of methods of multidimensional statistical analysis, used to identify homogeneous subsets of objects within the studied population (Janowski et al., 2016; Janowski, 2018; Migdał-Najman and Najman, 2013; Miskiewicz et al., 2017). Cluster analysis methods are used when we do not have the a priori hypotheses and the research is in the exploratory phase. Finding groups (clusters) of objects takes place on the basis of variables characterizing the analyzed objects; therefore, an important element of cluster

analysis is the appropriate selection of variables used to extract coherent groups of objects. In order to justify and estimate the dependence of exposure length to forced sale discount, the authors of the paper determined the length of property exposure on the market according to the following formula:

$$E_L = D_S - D_F \quad (6)$$

where: EL - exposure length of property (in the number of days); DS – the first day of market exposure (date); DF – the day when the sale agreement was signed (date). The next step was to estimate the forced sale discount. The discount was estimated according to the formula:

$$D_P = \frac{V_{FV}}{V_{MV}} * 100\% \quad (7)$$

where: DP - forced sale discount in %; VFV – forced sale value (PLN/EUR/1sqm); VMV - market value (PLN/EUR/1sqm). The dependence of exposure length on forced discount (further referred to as liquidity force discount) was determined in the following way:

$$LD_P = \frac{D_P}{E_L} \quad (8)$$

where: LDP - liquidity force sale discount in %.

3.2. Analysis of data collected from the Olsztyn and Bari property markets

The reason for conducting analysis on European property markets has been predominately justified by the fact that "a vast majority of previous studies estimating the impact of a foreclosure on the real estate sales price have been based on U.S. data, whereas empirical studies on European data have received little attention" (Donner et al., 2016). The selected local real estate markets in Poland and Italy were analyzed to verify research assumptions and simulation results. The study was conducted on the basis of 74 real estate sales transactions from Olsztyn and 39 transactions from Bari that took place in 2016. The data was collected for residential real estate markets represented by residential apartments, taking into account free market and forced sales. The information on the properties (especially of forced sales conditions) used in the analysis in most cases were collected in the traditional way (during field inspections) considered to be the best data source in valuation practice. It is particularly important if one should collect

data on non-market terms of the transaction and exposure time. The number of comparable properties of market sale conditions was significantly decreased by the authors according to methodological suggestions proposed by Harding et al. (2012) and Mocking and Overvest (2017). In order to decrease the existence of other, in many situations unknown, factors influencing property value authors used comparable assumption to nearest neighborhood matching method. According to these amongst many transactions only the ones most comparable (of nearest neighborhood) to the forced sale ones were selected. Examples of data selection were presented in Figs. 1 and 2. These two markets account for difficult situations connected with social aspects of life. Both cities are main urban centers in terms of commercial, cultural and industrial activities of regions considered to be poor and undeveloped in comparison to other districts of the both countries. The cities constitute magnet that attract people from neighborhoods, due to the fact of lower unemployment rate and higher development and hope for better perspectives of life in comparison to the surrounding area and smaller cities. In recent years, a crisis has generally been observed on the Italian real estate market, especially in the part of Italy where Bari is located. Therefore, both markets are comparable regarding the overall classification of market condition (state) in these countries. The comparison of these two cities could be interesting and precious in terms of verification of the versatility of the proposed methodology to determine the difference between market value and forced sale value. It must be underlined that databases consist of two main categories of transactions. For the Olsztyn database, there are 25 transactions with forced sales and 49 with sales under market conditions. In the case of Bari database consists of 15 transactions deal with forced sell and 24 with market sale. The analyses were conducted on the basis of these two databases. Taking into consideration the efficiency and versatility assessment of the proposed method and procedure, two diverse sets of attributes were collected that represent specific conditions of the property markets and valuation assumptions in the two analyzed cities. The attribute domains for Olsztyn (Poland) residential properties were defined according to the following criteria (statistics of database attributes are presented in Table 1):

A_P1 – price of real estate in PLN (EUR - all values in PLN werecalculated according to rate 4.20 PLN per 1 EUR) per sqm.,

A_P2 – condition of transactions: 1 – forced; 0 – not forced,

A_P3 - date: in months, starting from 2016.01,

A_P4 – kind of ownership: 1 – full ownership; 2 – cooperative ownership,

A_P5 - usable area of apartment: in square meters

A_P6 - location on the floor: given as a number starting from 1 – ground floor,

A_P7 –year of construction: starting with 1950 – 1, followed by 20-year intervals,

A_P8 - technical condition: 1 - very good; 2 - good; 3 - mediumA_P9 - number of stories in the building

A_P10 - additional usable area (basement, balcony, etc.): 0 – lack of; in sqm - present

A_p11 – elevators: 0 – lack of; 1 - present

A_p12 - distance from the city center (km)

While the attribute domains for Bari (Italy) residential properties were defined as follows (statistics of database attributes are presented in Table 2):

A_I1 – price of real estate in EUR per sqm.

A_I2 – condition of transactions: 1 – forced; 0 – not forced,

A_I3 – date: in months, starting from 2016.01,

A_I4 – usable area of real estate: in square meters

A_I5 – location on the floor: given as a number starting with 1–basement,

A_I6 – technical condition: 1 – very good; 2 - good; 3 – medium; 4 -bad

A_I7 – number of sales attempts

A_I8 – number of bathrooms

A_I9 – square meters of balcony

A_I10 – elevator: 0 – lack of; 1 -present

A_I11 – distance from the center (km)

The analyses were conducted with the use of number methods/models. The first of these is single regression, which gives particular individual results of the influence of forced sales on markets, based on linear least squares assumptions. The second model is based on multivariate regression and takes into account a more complex, hedonic aspect of transaction diversity, also including attributes that comprise the condition/quality of real estate. The third model used in the analysis - log-linear regression is the most popular nowadays and allows the added value change proportionally to changes of the attributes. The fourth method that was applied is more robust regression represented by Quantile Regression (median). The fifth analyses took into account verification of the linearity of residuals to check validity of regression analyses. The sixth part of the analyses involves applying an alternative method of property market analyses based on the rough set method and fuzzy theory, which are resistant to the common specificity of data (defects) on the real estate market. The seventh method is based on the analysis of a linear time trend accounting for the time of exposition of the property on the market for what is defined by

the authors as a forced sale.

4. Empirical analyses

According to the hedonic price theory, the valuation of a property can be conducted on the basis of the following formula:

$$V_{MV} = \beta_0 + \sum_{i=1}^{j} \beta_i X_i + \varepsilon \qquad (9.1)$$

where:

VMV - market value,

β - regression coefficient,

X - attribute of property,

ε - random error.

The hedonic model in the form of linear regression was chosen due to the fact of obtaining results in currency that is comparison unit for model proposed by the authors. Before the regressions models analyses the pre-analysis on multicollinearity (Pearson correlation) were conducted. All of the variables were less corelated each other than with property value. The biggest linear correlation in both examples (Olsztyn and Bari property markets) was noticed between condition of transaction (AP2, AI2) and price of real estate (AP1, AI1). In case of Olsztyn example, it equalled - 0.69, in case of Bari example it equalled – 0.62. In this case, the D (Formula (5)), as the difference between the two values, can be determined by the analysis of the values of attribute parameters. As had been mentioned above, the first method applied was simple regression to check the linear relation between price (the dependent variable) and forced sales (independent variable). The achieved results indicated not the best accuracy for simple regression, with R2 - 0.48. These results make it possible to establish the difference between market value (VMV) and forced sale value (VFV), which is around 1.566 PLN (about 373 EUR) per sqm, assuming a standard error of 412 PLN (about 98 EUR) per sqm in the interpretation of the results. The same analysis were conducted in the case of Bari (Italian market). The results indicated relatively low reliability, with R2 - 0.37. The linear model has enables the difference between market value (VMV) and forced sale value (VFV) to be determined at the level of 741.34 EUR per sqm, assuming a standard error of 215 EUR per sqm. Both of the presented results indicated insufficient reliability, but an analysis was conducted to determine any linear relation and diversity of value in the case of forced sale, and to indicate the average level of the possible influence of forced sale on the prices. Accuracy was not satisfied, and thus multivariate linear regression was conducted. The results for the Polish case have been presented below (Table 3).

The results indicated quite good reliability, with R2 - 0.71. The multivariate model is presented as follows: y=2,810.45+(-1,406.60*XAP2)+(39.49*XAP3)+(-162.76*XAP4)+ (-18.21*XAP5) +(139.74*XAP6)+(278.82*XAP7)+(-690.08*XAP8)+(-5.49*XAP9)+ (34.63*XAP10)+ (-69.20* XAP11) +(-171.18* XAP12)

In this model, the independent variable referring to the relation of forced sale to market sale resulted in an decrease in the unit value of real estate by 1,406.60 PLN (334.90 EUR) per sqm, which is not much different than in the case of simple regression analyses. The accuracy of the achieved results could be satisfied when considering the relatively high R2, however the standard error is almost 50% of established unit value. This means that possible mistakes in valuation could lead to unreliable results. In the next part of the analyses, the Italian case (city of Bari) was calculated - Table 4. The results indicated reliability with R2- 0.77. The multivariate model is presented as follows: y=1,429.28+(-186.85*XAI1) + (-250.47*XAI2) + (40.14*XAI3) + (-10.77*XAI4) +(173.73*XAI5) +(5.04*XAI6)+(-2.22*XAI7)+(-173.21*XAI8)+(-900.90*XAI9) +(-17.13* XAI10) + (1.56* XAI11)

In this model, the independent variable regarding forced sale resulted in a unit value decrease of 900.90 EUR per sqm, which is different than in the case of simple regression analyses. The accuracy of the achieved results could be satisfied when considering the relatively high R2, however the standard error is almost 50% of unit value – the same as in the Polish case. Due to the fact that regression function in hedonic modeling, used quite often nowadays is in log-linear form, the model in this stream based on below function was prepared:

$$LogV_{MV} = \beta_0 + \sum_{i=1}^{j} \beta_i X_i + \varepsilon \qquad (9.2)$$

According to Malpezzi (2003), this kind of function give advantages compared to linear regression as follow: log-linear model allows the added value to change proportionally to changes of the attributes, estimated regression coefficients are easy to interpret, the log-linear function often eases problems connected with heteroscedasticity or with the variability of a random component. Taking into account log-linear regression, the log price and dummy variable as a forced sale was implemented. The results for the Polish and Italian, two investigated local markets have been presented below (Tables 5 and 6). The quality of results was comparably unsatisfactory to linear regression. For example, the R2 for Olsztyn market equaled 0.72 and for Bari market 0.77, while standard error 0.19 (21 percent) for Polish local market and 0.32 (38 percent) for Italian local market. Many of the established parameters are not statistically significant (p-value Tables 5 and 6). The model defines the impact of forced sale to market sale in percentage. The independent variable referring to the relation of forced sale to market sale

equaled 48 percent for Olsztyn and 141 percent for Bari decrease in the unit value of real estate. Furthermore, it was applied more robust regression represented by Quantile Regression (median). The quality of results compared to linear regression was even worse. The independent variable referring to the relation of forced sale to market sale equaled 63 percent for Olsztyn and 105 percent for Bari decrease in the unit value of real estate. The Pseudo R-squared (R2) for Polish local market equaled 0.55 and for Italian local market 0.58. Additionally, the linearity of residuals were conducted to check validity of regression analyses. The differences between the distribution of residuals and the normal distribution (value $p \leq \alpha$) may interfere the assessment of the individual model variables coefficients significance. For both markets two statistical tests of normal distribution (the Shapiro-Wilk and Kolmogorov-Smirnov with assumed p level 0.05) were conducted. For Polish market the results obtained from Shapiro-Wilk test was 0.960 (critical value 0.959) with p=0.0512 and Kolmogorov-Smirnov 0.097 (critical value 0.102) with $p < 0.05$. The results don't allow to draw unambiguous conclusions on normal distribution existence. For Italian market the result obtained from Shapiro-Wilk was 0.934 (critical value 0.940) with p=0.0259 and Kolmogorov-Smirnov 0.138 (critical value 0.149) with $p < 0.05$. On the basis of the results one can draw a certain conclusion that the residuals have no distribution close to the normal one. Thus, the efficiency and accuracy of the presented results based both on linear and log-linear statistical modelling are insufficient. The reason for this may be an insufficient amount of transactions for statistical analyses or nondeterministic relationships between attributes. As far as doubts regarding the accuracy of analyses achieved from the hedonic model resulting in low reliability of the analyses are concerned, the authors proposed an alternative method and procedure. In this case, the differences (D) between market value (VMV) and forced sale value (VFV) will be developed with the application of the rough set method, extended by value tolerance relations (fuzzy theory). The proposed method is perfect for both big and small databases, as well as data that is ambiguous, imprecise and varied. The analytical part of the research was prepared according to the procedure presented in Fig. 3. The authors proposed an analytical procedure based on RST which consisted of several stages listed below:

- collection of database consisting of transactions involving forced sales;
- division of database into two parts, i.e. transactions where the sellers were under compulsion to sell and sales transactions with a proper marketing period,
- identification of indiscernible (similar) properties with the use of RST and VTR for particular properties characterized by forced sales under the following assumptions:
- calculation of standard deviation for all attributes in the database,
- development of matrix with the use of the VTR formula:

$$R_j(x, y) = \frac{\max(0;\ min(c_j(x),\ c_j(y)) + k - max(c_j(x),\ c_j(y)))}{k} \quad (10)$$

where:

Rj (x, y) - relationship between two sets with a membership function [0,1]

cj (x), cj (y) - indicator of the analyzed real estate market

k - coefficient adopted as standard deviation for a given real estate market attribute.

• determination of the sum matrix for all attributes according the formula:

$$R_j(x, p) = max\left(\sum_{j=1}^{n} R_j(x, p)\right) \quad (11)$$

where: Rj is the valued tolerance relation, x is the analyzed property's attribute, p is the attribute in the conditional part, n is the number of property attributes,

• determination of the most similar property regarding market value to a given analyzed property sold with force; according to the rule, if $SIM_j(x) = \sum_{y=1}^{n} R_j(x, p)$ (12), then it is most indiscernible (similar) among other properties (without forced sales) when $SIM_j = max_{x=1:p} SIM_j(x)$ (13),

• calculation of the accuracy of similarities between selected properties expressed as a percentage between the number of attributes and the achieved results of $SIM_j(x) = \sum_{y=1}^{n} R_j(x, p)$ (12),

• calculation of the individual difference between forced and market sales Di, nominal and percentage,

• determination of average forced sale discount DA according to the formula:

$$D_A = \sum_{y=1}^{n} W(D_i) \quad (14)$$

where: W is the weight of Di calculated on the basis of accuracy of similarity (row V in Tables 5 and 6).

The presented procedure was applied for both the Polish and Italian local markets. The results have been presented in Table 7 (Olsztyn market) and Table 8 (Bari market). In Tables 7 and 8,

following results have been presented: I row - number of transaction in the database with forced sale; II - price of selected property in the database with forced sale; III - number of transactions in the database established on the basis of the RST procedure that are indiscernible to transactions with forced sale; IV - price of selected property in database with market sale; V – accuracy of similarity of two transactions with forced and market sale (percentage); VI – Di value of differences between market and forced sale - (percentage); VII - Di value of differences between market and forced sale - (currency). Summarizing the results achieved for the Polish and Italian local markets, the final values of differences were calculated. The final value of average DA was calculated with the use of Formula 14. The average forced discount sales were calculated taking into account the accuracy of the adjusted results as a weight of differences between the prices of selected properties with and without forced sale circumstances. The analyses indicated that average DA for the Polish market is equal to 999.95 PLN (238 EUR) per sqm, and it was calculated this makes for a 34.47% difference of averages, whereas for the Italian local market, DA is equal 508.94 EUR per sqm, thus a 64.34 % difference of averages. In order to estimate the liquidity forced sale discount, the main step was to identify, on a local property market (for both Olsztyn and Bari), the proper time of property exposure. On the basis of the collected data, the time was observed to differ significantly among property sales; nevertheless, the average length equaled 94 days in the case of Olsztyn, resulting in approximately a 3-month period of exposure (Fig. 4). In the case of the Bari market, the average exposure time was longer. Its average length equaled 143 days, which was caused mainly by the general market situation (crisis) (Fig. 5). The cluster analysis confirmed (on the example of Olsztyn transactions with a bigger representative group) the division into forced sale values and non-forced ones. Cluster analysis was used for the substantive verification of forced sale and non-forced sale transactions. In 85% of cases, it confirmed (assigned) forced transactions to the group with this feature. Thus, 15% of the transactions were not classified properly (Fig. 6). The determination of the percentage liquidity force discount LDP (Formula (8)) on the Bari real estate market made it possible to draw the conclusion that the property seller should account for an average discount of 5.91 EUR per square meter of his/her property if he/she wishes to sell the property one day faster; this amount equaled 0.003800 % of the market value (Fig. 7). The discount observed on the example of the Polish local real estate market was smaller, both in comparison to market value and the average price discount. It amounted to a 23.17 PLN (5.52 EUR) discount per square meter on average, equaling 0.000535% discount per day (Fig. 7).

5. Final remarks

The lack or unavailability of data poses one of the greatest obstacles impeding the investigation of real estate market specificity. If there is a small set of observations, the use of statistical methods is limited. That's why authors proposed Automated Valuation Methods with the use of

rough set theory and fuzzy logic. The rough set theory was developed to analyze qualitatively and quantitatively ambiguous, imprecise and vague data. The research results show the forced sale discount (D in Formula (5)) established on the basis of the regression model compared to the results obtained using the alternative model to be, on average, 40% lower in both countries. Moreover, the presented results achieved on the basis of the RST and VTR methods regarding the valuation of the forced discount are characterized by higher efficiency which can be explained on the basis of adjusting similar (indiscernible) properties. The precision of adjusting property is 84% and 79% for the Olsztyn and Bari residential real estate markets, respectively. Taking into account the rather diverse and small sample size of the dataset, the results proved to be quite satisfactory, more than in the case of hedonic models. Analyzing the average tendency from market and expert (valuers), more suitable results were achieved by the proposed alternative method. The achieved results indicated that value when it comes to forced sale is about 22% lower in the case of Olsztyn when compared to the average market value (of the used sample) of 4500 PLN (1071 EUR) per sqm; in the case of the Bari market, it is about 45% lower when compared to the average market value of 1150 EUR per sqm. In addition to practical information regarding discounted price in the case of the urgent need to sell property, the diagnosis of the condition of the market is also important. With such information we can diagnose the occurrence of a crisis on the analyzed market. The reason for higher discounts observed in forced sales is connected with lower possible demand. From this point of view, analyzing the achieved results, the Italian market faces a bigger crisis. The assumption that average discounts greater than 30–40 percent could be a warning that something worrisome is happening on the market. However, it must be stressed that this issue was not the subject of the present research, though the conclusion drawn may justify the need for further studies. Parallel determination of percentage liquidity forced discount confirmed the hypothesis. The results achieved with determination of exposure length, percentage forced discount and percentage liquidity forced discount were comparable with the conclusions presented above. Comparing the liquidity forced sale discount between the two cities from the two countries, the difference in its value was significant an equaled 14.08 This means that the same property sold within the same period of time under forced circumstances should be subjected to a 14 times higher discount on the Bari residential market than on the Olsztyn market.

6. Conclusions

Unfortunately, the term “forced sale value” is sometimes used loosely in practice. There is no recognized definition, however, “liquidation value” and “distress sale” are generally synonymous with the concept of forced sale, whereby all imply a reduced sale period and a compulsion to sell (Hyslop, 2013). Against this background (in Poland, there were such cases), negative consequences of unjustified reduction of the value of a debtor's property by an appraiser as part

of the debt collection process may occur. This is due to the fact that forced sale value is sometimes equated with the value determined for the needs of enforcement/bailiffs. It should be noted that Polish law explicitly regulates this issue, indicating only the market value of the property as the basis for the valuation (Act of 29 August 1997 on Court Bailiffs …, 1997Act of August, 1997aAct of 29 August 1997 on Court Bailiffs …, 1997). The valuation standards quoted in the article define the basic concepts important for the proper understanding of difficult issues in the field of work of a property appraiser. The problem considered in the paper, apart from defining the bases of value, involves the distinction between two concepts: 1. premises, assumptions, and 2. special premises, special assumptions. When accepting a property valuation order, one ought to remember that forced sale value is not separate from market value, but implies a set of marketing conditions that are less favorable to the seller than those defined in the market value definition. Hence, one of the basic duties of a real estate appraiser is to understand the client's needs and reasons why forced sales value is required. Despite all doubts or even misunderstandings regarding the proper comprehension of forced sale value and difficulty in estimating it, it should be expressed that it can be an additional effective tool for, among others, the analysis and reduction of risk related to real estate mortgages. In this article, the authors propose an alternative effective method aiding individual/human valuation, which may not always be objective or reliable. From an analytical perspective, the biggest challenge in this domain is the selection and development of appropriate methods for analyzing specific market information regarding forced sale discount. The authors propose a solution based on advanced automated procedures called Automated Valuation Methods with the use of rough set theory and fuzzy logic. This alternative method gives more accurate results than hedonic models, in the face of qualitatively and quantitatively ambiguous, imprecise and vague data, which is commonly found in the real estate market, especially regarding not typical/market sales. The proposed tools used to determine forced sale value discount can fill the gap in the methodological valuation approach, especially in terms of AVM adoption. The authors believe that the conducted research and drawn conclusions support the need for further analysis concerning:

- relationships between untypical phenomena occurring on the real estate market and the obtained transaction prices,
- the reasons for urgent/forced sales,
- forced sale discount on other property markets,
- methodology for determining comparable real estate markets and an accurate selection of sales for the needs of development of property valuation theory.

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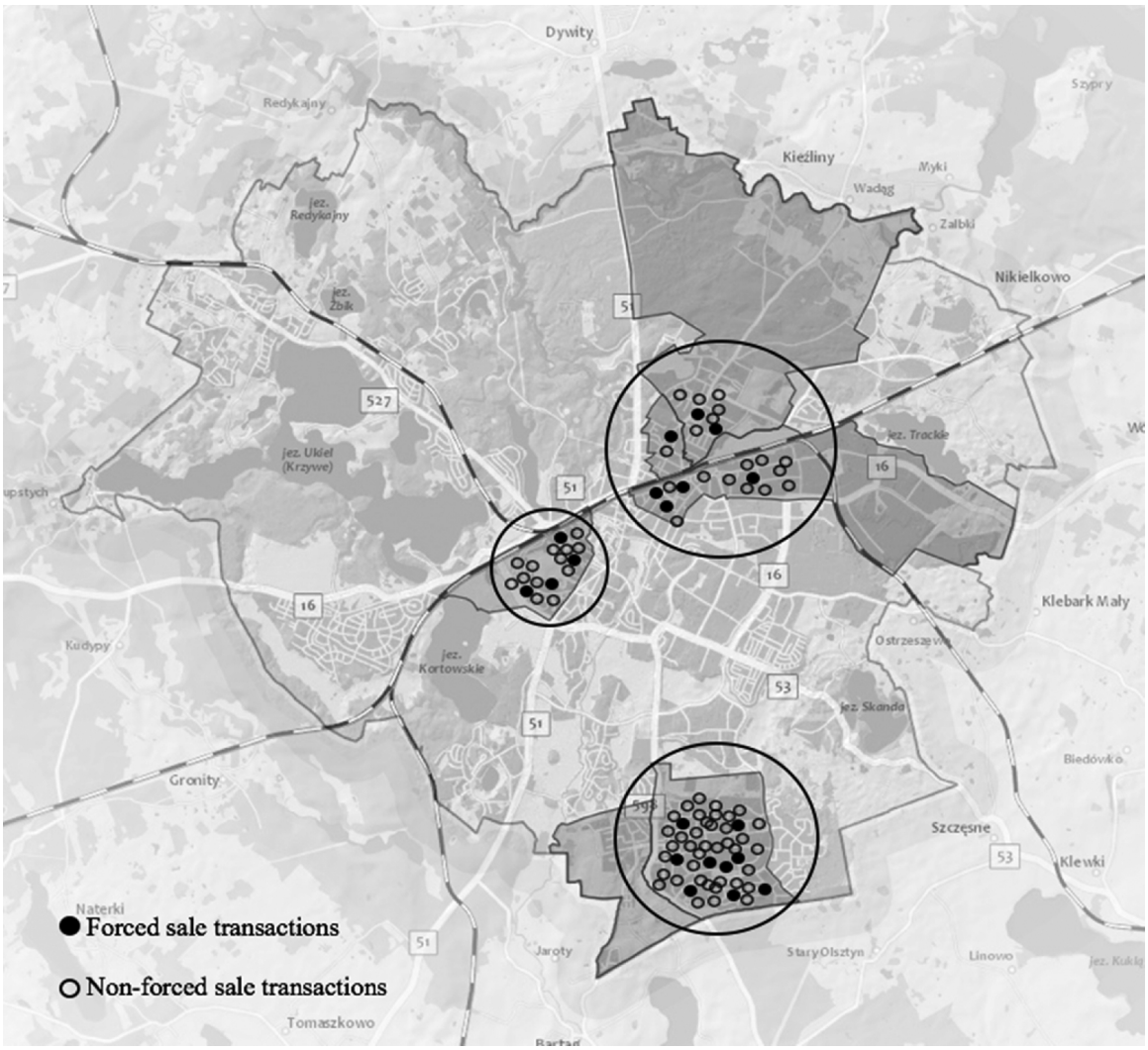

Fig. 1. Selection of forced sale transactions and most comparable non-forced sale ones on the example of Olsztyn City (Poland). *Source*: Own elaboration.

Fig. 2. Selection of forced sale transactions and most comparable non-forced sale ones on the example of Bari City (Italy). *Source*: Own elaboration.

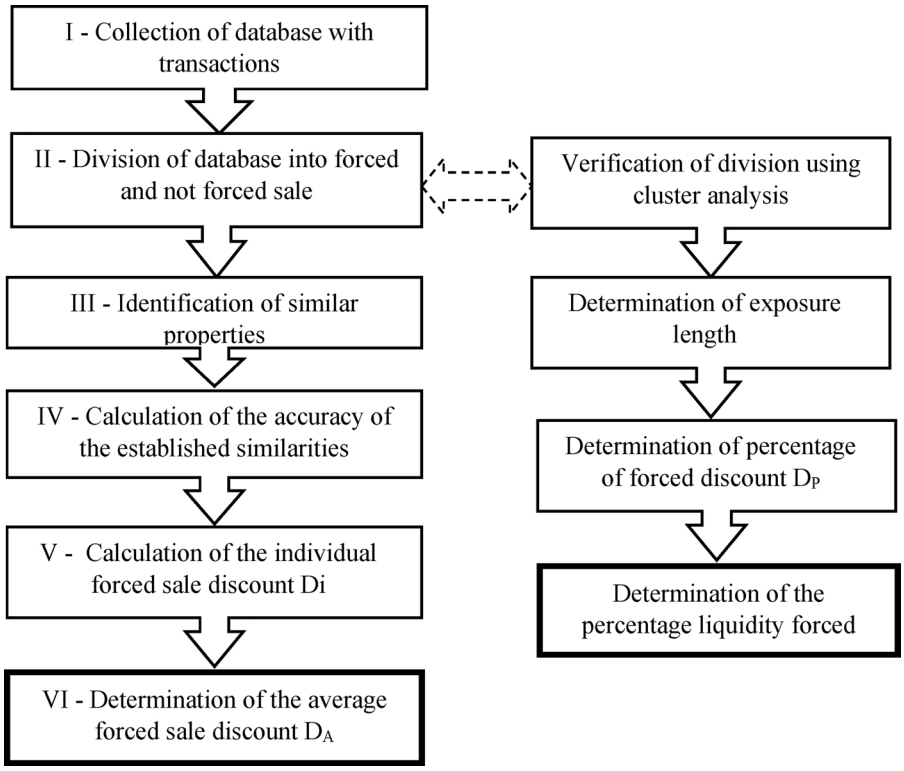

Fig. 3. Analytical procedure for determining the difference between market and forced sale value.
Source: Own elaboration.

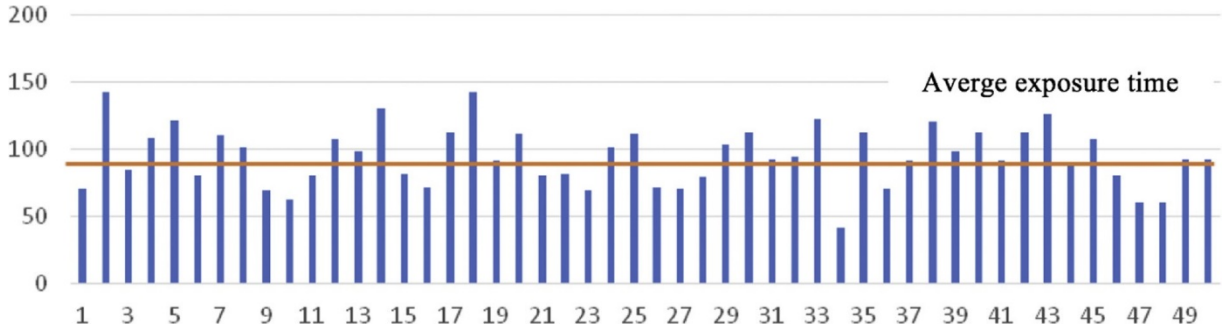

Fig. 4. Exposure time (residential properties) – Olsztyn.
Source: own elaboration.

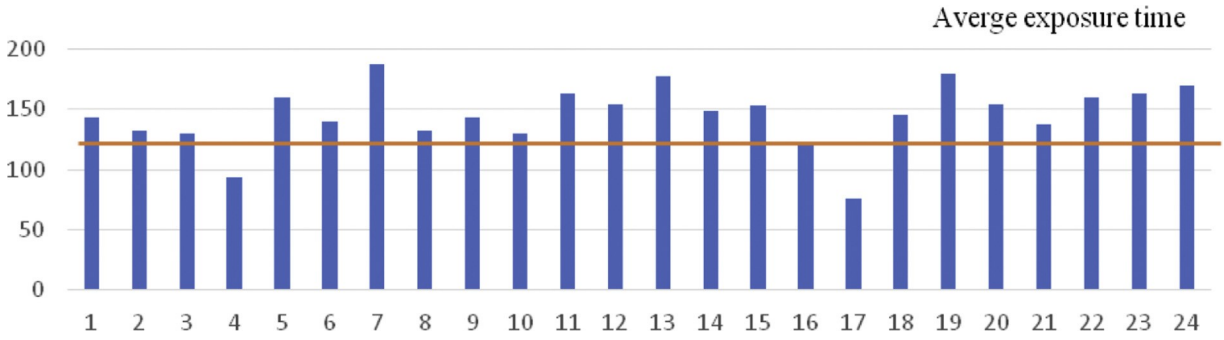

Fig. 5. Exposure time (residential properties) in Bari.

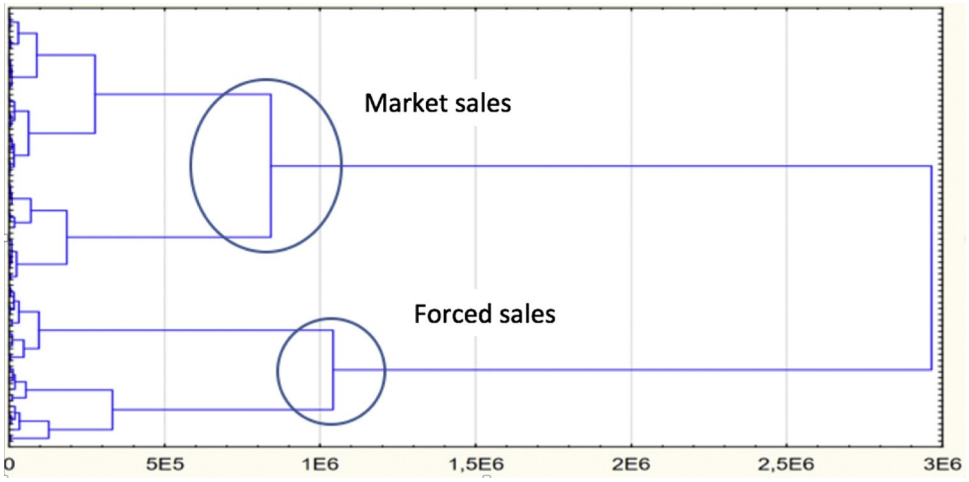

Fig. 6. Diagram Tree, Wards Method, Euclidean distance.
Source: Own elaboration.

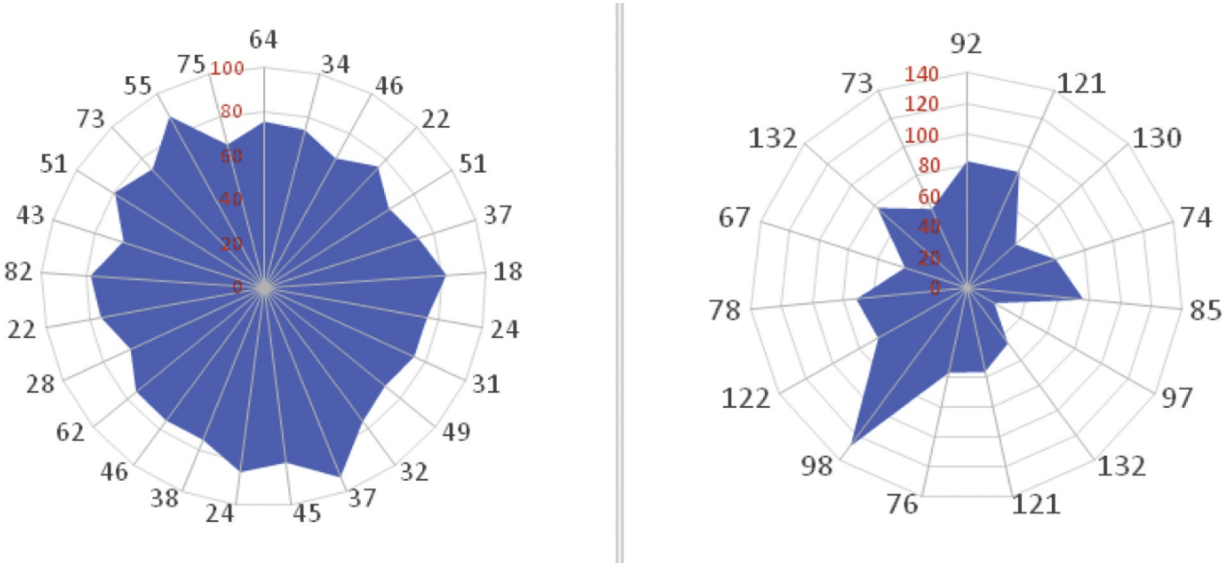

Fig. 7. Exposure time vs liquidity discount for Olsztyn - figure on left, and Bari – figure on right.
Source: Own elaboration.

Table 1
Statistics of real estate market attributes (Olsztyn).
Source: Own elaboration.

Statistics of attributes	A_p1	A_p2	A_p3	A_p4	A_p5	A_p6	A_p7	A_p8	A_p9	A_p10	A_p11	A_p12
min	2311 PLN (550EUR)	1	1	1	23	1	1	1	1	0	0	1
max	6495 PLN (1556 EUR)	0	17	2	90	11	6	3	12	17	1	9
average	3972 PLN (946 EUR)	0	6	1	46	3	4	2	6	3	0	3

Table 2
Statistics of real estate market attributes (Bari).
Source: Own elaboration.

Statistics of attributes	A_t1	A_t2	A_t3	A_t4	A_t5	A_t6	A_t7	A_t8	A_t9	A_t10	A_t11
min	322	0	1	23	1	1	1	1	0	0	1
max	2617	1	17	224	7	4	8	3	34	1	7
average	1122	0	4	95	3	2	4	1	10	0	3

Table 3
Results of multivariate regression – Olsztyn property market.
Source: Own elaboration.

Number of Observations	R^2	Standard Error	F	Parameters below 0.05 level of Significance	Parameters above 0.05 level of Significance
74	0.71	628.34 PLN (149.60 EUR)	1.5E-12	A_p2, A_p5, A_p6, A_p7, A_p8, A_p12	A_p3, A_p4, A_p9, A_p10, A_p11

Table 4
Results of multivariate regression - Bari property market.
Source: Own elaboration.

Number of observations	R^2	Standard error	F	Parameters below 0.05 level of significance	Parameters above 0.05 level of significance
39	0.77	458 EUR	8.13E-06	A_t2 A_t4 A_t5 A_t8 A_t9A_p10	A_t3 A_t6 A_t7 A_t11

Table 5
Results of log-linear regression – Olsztyn property market.
Source: Own elaboration.

Number of Observations	R^2	Standard Error	F	Parameters	*Value-p*
74	0.72	0.19	1.26E-13	Intercept - (8.64)	1.64105E-39
					1.6987E-06
				A_P2 - (- 0.39)	0.166981832
				A_P3 - 0.01	0.500639320
				A_P4 - (-0.05)	0.005717992
				A_P5 - (-0.004)	0.003305126
				A_P6 - 0.03	0.000944016
				A_P7 - 0.07	0.020510326
				A_P8 - (-0.17)	0.479298736
				A_P9 - (-0.01)	0.362025592
				A_P10 - 0.01	0.671088166
				A_p11 - (-0.03)	0.048176031
				A_p12 - (- 0.04)	

Table 6
Results of log-linear regression - Bari property market.
Source: Own elaboration.

Number of Observations	R^2	Standard Error	F	Parameters	*Value-p*
39	0.77	0.32	3.09E-07	Intercept - (8.85)	2,03E-20
					6,51E-08
				A_t2 - (- 0.88)	0,020429
				A_t3 - (-0.19)	0,004009
				A_t4 - (-0.01)	0,311497
				A_t5 - (0.04)	3,81E-05
				A_t6 - (-0.24)	0,618407
				A_t7 - (-0.02)	0,710016
				A_t8 - (0.06)	0,996461
				A_t9 - (0.00)	0,407021
				A_t10 - 0.01	0,020429
				A_t11 - (- 0.03)	

Table 7
Valuation of forced sale discount with the use of rough set assumptions for the Olsztyn property market.
Source: Own elaboration.

I	**1**	2	3	4	5	6	7	8	9	10	**11**	**12**
II	3310.87 PLN (788.30 EUR)	2900.66 PLN (690.63 EUR)	2310.68 PLN (550.16 EUR)	2858.27 PLN (680.54 EUR)	2801.82 PLN (667.10 EUR)	2767.12 PLN (658.84 EUR)	2529.46 PLN (602.25 EUR)	2737.62 PLN (651.81 EUR)	2402.39 PLN (572.00 EUR)	3152.40 PLN (750.57 EUR)	3256.41 PLN (775,34 EUR)	3381.74 PLN (805.18 EUR)
III	71	48	41	29	41	41	71	65	50	72	32	32
IV	3750.00 PLN (892.86 EUR)	3517.67 PLN (837.54 EUR)	3221.29 PLN (766.97 EUR)	3969.07 PLN (945.02 EUR)	3221.29 PLN (766.97 EUR)	3221.29 PLN (766.97 EUR)	3750.00 PLN (892.86 EUR)	3149.35 PLN (749.85 EUR)	4164.46 PLN (991.54 EUR)	4253.11 PLN (1012.65 EUR)	4398.34 PLN (1047.22 EUR)	4398.34 PLN (1047.22 EUR)
V	82.00	88.63	73.75	89.38	85.88	82.63	88.00	81.75	89.63	79.88	89.25	88.88
VI	13.26	21.27	39.41	38.86	14.97	16.41	48.25	15.04	73.35	34.92	35.07	30.06
VII	439.13 PLN (104.55 EUR)	617.01 PLN (146.91 EUR)	910.61 PLN (216.81 EUR)	1110.80 PLN (264.48 EUR)	419.47 PLN (99.87 EUR)	454.17 PLN (108.14 EUR)	1220.54 PLN (290.60 EUR)	411.73 PLN (98.03 EUR)	1762.07 PLN (419.54 EUR)	1100.71 PLN (262.07 EUR)	1141.93 PLN (271.89 EUR)	1016.60 PLN (242.05 EUR)
I	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
II	3579.64 PLN (852.30 EUR)	2776.09 PLN (660.97 EUR)	2725.63 PLN (648.96 EUR)	2971.21 PLN (707.43 EUR)	2527.69 PLN (601.83 EUR)	3207.09 PLN (763.59 EUR)	3367.82 PLN (801.86 EUR)	2529.68 PLN (602.30 EUR)	3009.36 PLN (716.51 EUR)	2746.73 PLN (653.98 EUR)	3393.39 PLN (807.95 EUR)	2686.73 PLN (639.70 EUR)
III	59	51	62	29	41	55	59	41	72	41	51	45
IV	4937.36 PLN (1175.56 EUR)	4230.77 PLN (1007.33 EUR)	4529.54 PLN (1078.46 EUR)	3969.07 PLN (945.02 EUR)	3221.29 PLN (766.97 EUR)	5162.26 PLN (1229.11 EUR)	4937.36 PLN (1175.56) EUR	3221.29 PLN (766.97 EUR)	4253.11 PLN (1012.65 EUR)	3221.29 PLN (766.97 EUR)	4230.77 PLN (1007.33 EUR)	3151.26 PLN (750.30 EUR)
V	90.38	80.63	69.25	83.50	94.25	68.63	90.38	94.25	79.88	82.63	82.88	67.63
VI	37.93	52.40	66.18	33.58	27.44	60.96	46.60	27.34	41.33	17.28	24.68	17.29
VII	1357.72 PLN (323.27 EUR)	1454.68 PLN (346.35 EUR)	1803.91 PLN (429.50 EUR)	997.86 PLN (237.59 EUR)	693.60 PLN (165.14 EUR)	1955.17 PLN (465.52 EUR)	1569.55 PLN (373.70 EUR)	691.61 PLN (164.67 EUR)	1243.76 PLN (296.13 EUR)	474.56 PLN (112.99 EUR)	837.38 PLN (199.38 EUR)	464.53 PLN (110.60 EUR)

Table 8
Valuation of forced sale discount with the use of rough set assumptions for the Bari property market.
Source: Own elaboration.

I	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
II	1200.00	1337.04	431.46	845.45	765.63	430.61	658.97	813.56	866.67	1007.75	1204.55	982.59	476.25	1476.19	1001.36
III	25	22	17	26	37	30	27	40	25	33	34	27	39	34	25
IV	1601.41	1780.33	652.91	1053.23	1515.99	1022.73	1572.92	1780.02	1601.41	1803.90	1896.83	1572.92	1053.23	1896.83	1601.41
V	92.53	93.47	83.49	84.25	75.20	64.22	66.36	75.22	69.77	69.33	85.64	87.40	57.45	95.64	86.41
VI	33.45	33.15	51.32	24.58	98.01	137.51	138.69	118.79	84.78	79.00	57.47	60.08	121.15	28.49	59.92
VII	401.41	443.29	221.45	207.77	750.37	592.12	913.94	966.46	734.74	796.15	692.28	590.33	576.98	420.63	600.05