



Politecnico
di Bari

Repository Istituzionale dei Prodotti della Ricerca del Politecnico di Bari

Multiscale modeling of complex flows in microfluidic systems

This is a PhD Thesis

Original Citation:

Multiscale modeling of complex flows in microfluidic systems / Signorile, S.. - ELETTRONICO. - (2026).

Availability:

This version is available at <http://hdl.handle.net/11589/307381> since: 2026-09-23

Published version

DOI:

Publisher: Politecnico di Bari

Terms of use:

(Article begins on next page)



Politecnico
di Bari

DEPARTMENT OF MECHANICS, MATHEMATICS AND
MANAGEMENT

AEROSPACE SCIENCE AND ENGINEERING
PH.D. PROGRAM

SSD: IIND-01/F - FLUID DYNAMICS

Final dissertation

Multiscale modeling of complex flows in microfluidic systems

by
Simona SIGNORILE

Supervisors:

Prof. Marco Donato DE TULLIO

Prof. Dario DE MARINIS

Coordinator of the Ph.D. Program:
Prof. Caterina CIMINELLI

Course XXXVIII, 01/11/2022 - 30/04/2026



LIBERATORIA PER L'ARCHIVIAZIONE DELLA TESI DI DOTTORATO

Al Magnifico Rettore
del Politecnico di Bari

La sottoscritta SIMONA SIGNORILE nata a BITRITTO (BA) il 28/08/1997

residente a BARI in via GIOVANNI BOVIO 20 e-mail simonasignorile97@gmail.com

iscritto al 3° anno di Corso di Dottorato di Ricerca in INGEGNERIA E SCIENZE AEROSPAZIALI ciclo 38°

ed essendo stato ammesso a sostenere l'esame finale con la prevista discussione della tesi dal titolo:

Multiscale modeling of complex flows in microfluidic systems

DICHIARA

- 1) di essere consapevole che, ai sensi del D.P.R. n. 445 del 28.12.2000, le dichiarazioni mendaci, la falsità negli atti e l'uso di atti falsi sono puniti ai sensi del codice penale e delle Leggi speciali in materia, e che nel caso ricorressero dette ipotesi, decade fin dall'inizio e senza necessità di nessuna formalità dai benefici conseguenti al provvedimento emanato sulla base di tali dichiarazioni;
- 2) di essere iscritto al Corso di Dottorato di ricerca INGEGNERIA E SCIENZE AEROSPAZIALI ciclo 38°, corso attivato ai sensi del "Regolamento dei Corsi di Dottorato di ricerca del Politecnico di Bari", emanato con D.R. n.286 del 01.07.2013;
- 3) di essere pienamente a conoscenza delle disposizioni contenute nel predetto Regolamento in merito alla procedura di deposito, pubblicazione e autoarchiviazione della tesi di dottorato nell'Archivio Istituzionale ad accesso aperto alla letteratura scientifica;
- 4) di essere consapevole che attraverso l'autoarchiviazione delle tesi nell'Archivio Istituzionale ad accesso aperto alla letteratura scientifica del Politecnico di Bari (IRIS-POLIBA), l'Ateneo archiverà e renderà consultabile in rete (nel rispetto della Policy di Ateneo di cui al D.R. 642 del 13.11.2015) il testo completo della tesi di dottorato, fatta salva la possibilità di sottoscrizione di apposite licenze per le relative condizioni di utilizzo (di cui al sito <http://www.creativecommons.it/Licenze>), e fatte salve, altresì, le eventuali esigenze di "embargo", legate a strette considerazioni sulla tutelabilità e sfruttamento industriale/commerciale dei contenuti della tesi, da rappresentarsi mediante compilazione e sottoscrizione del modulo in calce (Richiesta di embargo);
- 5) che la tesi da depositare in IRIS-POLIBA, in formato digitale (PDF/A) sarà del tutto identica a quelle **consegnate**/inviata/da inviarsi ai componenti della commissione per l'esame finale e a qualsiasi altra copia depositata presso gli Uffici del Politecnico di Bari in forma cartacea o digitale, ovvero a quella da discutere in sede di esame finale, a quella da depositare, a cura dell'Ateneo, presso le Biblioteche Nazionali Centrali di Roma e Firenze e presso tutti gli Uffici competenti per legge al momento del deposito stesso, e che di conseguenza va esclusa qualsiasi responsabilità del Politecnico di Bari per quanto riguarda eventuali errori, imprecisioni o omissioni nei contenuti della tesi;
- 6) che il contenuto e l'organizzazione della tesi è opera originale realizzata dal sottoscritto e non compromette in alcun modo i diritti di terzi, ivi compresi quelli relativi alla sicurezza dei dati personali; che pertanto il Politecnico di Bari ed i suoi funzionari sono in ogni caso esenti da responsabilità di qualsivoglia natura: civile, amministrativa e penale e saranno dal sottoscritto tenuti indenni da qualsiasi richiesta o rivendicazione da parte di terzi;
- 7) che il contenuto della tesi non infrange in alcun modo il diritto d'Autore né gli obblighi connessi alla salvaguardia di diritti morali ed economici di altri autori o di altri aventi diritto, sia per testi, immagini, foto, tabelle, o altre parti di cui la tesi è composta.

Luogo e data 23/09/2026

Firma

Il/La sottoscritto, con l'autoarchiviazione della propria tesi di dottorato nell'Archivio Istituzionale ad accesso aperto del Politecnico di Bari (POLIBA-IRIS), pur mantenendo su di essa tutti i diritti d'autore, morali ed economici, ai sensi della normativa vigente (Legge 633/1941 e ss.mm.ii.),

CONCEDE

- al Politecnico di Bari il permesso di trasferire l'opera su qualsiasi supporto e di convertirla in qualsiasi formato al fine di una corretta conservazione nel tempo. Il Politecnico di Bari garantisce che non verrà effettuata alcuna modifica al contenuto e alla struttura dell'opera.
- al Politecnico di Bari la possibilità di riprodurre l'opera in più di una copia per fini di sicurezza, back-up e conservazione.

Luogo e data 23/09/2026

Firma



Politecnico
di Bari

DEPARTMENT OF MECHANICS, MATHEMATICS AND
MANAGEMENT

AEROSPACE SCIENCE AND ENGINEERING
PH.D. PROGRAM

SSD: IIND-01/F - FLUID DYNAMICS

Final dissertation

Multiscale modeling of complex flows in microfluidic systems

by

Simona SIGNORILE

Referees:

Prof. Francesco PICANO

Prof. Paolo GUALTIERI

Supervisors:

Prof. Marco Donato DE TULLIO

Prof. Dario DE MARINIS

Coordinator of Ph.D. Program:

Prof. Caterina CIMINELLI

Course XXXVIII, 01/11/2022-30/04/2026

To the two greatest minds I have ever had the privilege of meeting...

To Dario, my mentor.

A brilliant engineer and an outstanding professor, deeply loved by his students, but above all, my one and only Tutor No.2.

A tireless debugger who, with patience and (at times) irony, guided me even through the most confusing moments.

To Michele, the friend everyone should meet at least once in their lifetime.

An eclectic and inquisitive researcher, a passionate soul distinguished by his multifaceted intellect, a rare example of integrity and consistency.

Your advice and unwavering support have always encouraged me to strive to do better and better.

To both of you, I owe more than I could ever express in these few lines...

Without you, this long journey would never have been the same.

Thank you for everything!

Ad maiora, semper.

Sommario

Negli ultimi anni, la microfluidica ha suscitato crescente interesse nel settore spaziale, in ambiti come l'astrobiologia, lo sviluppo di farmaci e la medicina spaziale, consentendo analisi precise dei parametri fisiologici degli astronauti e delle condizioni ambientali, quali qualità di acqua, aria e cibo, essenziali per missioni di lunga durata. Tuttavia, i tradizionali approcci sperimentali adottati per la progettazione e l'ottimizzazione dei microdispositivi non consentono di usufruire appieno dei vantaggi di tale disciplina, con conseguente incremento dei costi e dei tempi di sviluppo.

In questo contesto, la microfluidica computazionale si configura come uno strumento fondamentale per la modellazione e l'analisi quantitativa della complessità e della natura multifisica dei fluidi nei microcanali, essenziale per studiare le interazioni tra la fase continua, quale il fluido, e le fasi disperse, quali cellule e particelle. L'impiego di modelli numerici avanzati consente di investigare in modo sistematico l'influenza dei parametri fisici e geometrici del sistema, superando le principali limitazioni del classico approccio sperimentale, come l'accesso alle grandezze locali di interesse e il controllo e la riproducibilità delle condizioni operative.

La presente tesi si concentra sullo sviluppo e sulla validazione di metodologie computazionali basate sul metodo Lattice Boltzmann per lo studio di sistemi fluidi multicomponente, della dinamica di nanoparticelle e di fluidi viscoelastici in microcanali. L'approccio proposto è concepito per descrivere in modo sistematico il complesso comportamento reologico dei sistemi multicomponente, che costituiscono la base di un'ampia classe di dispositivi microfluidici, inclusi quelli a goccia quali i generatori di gocce core-shell e Janus. Questi possono essere impiegati per colture cellulari 3D, analisi a singola cellula e incapsulamento di sferoidi per la modellazione in vitro di organi, malattie e somministrazione di farmaci. Un elemento chiave del framework proposto è il trattamento esplicito degli effetti di bagnabilità delle superfici, che svolgono un ruolo cruciale nel governare la dinamica dei sistemi microfluidici multifase. In particolare, la bagnabilità influenza direttamente la morfologia, le dimensioni e la frequenza di produzione delle gocce. Il framework sviluppato consente la simulazione di sistemi fluidi immiscibili sia binari sia ternari su un ampio intervallo di rapporti di densità e/o viscosità, e supporta l'interazione con superfici di geometria arbitraria, gestendo condizioni al contorno sia di bagnabilità sia di non bagnabilità.

I dispositivi microfluidici manipolano fluidi di natura multifisica, come i fluidi biologici, costituiti da un'ampia varietà di cellule, vescicole e molecole di diversa dimensione, densità e forma, che interagiscono tra loro, con la matrice fluida e con il microcanale. In questo contesto, il solutore è stato esteso per modellare il trasporto di nanoparticelle immerse in fluidi Newtoniani mediante un approccio multiscala, in cui le nanoparticelle sono trattate come punti sottogriglia. Tale formulazione consente di descrivere accuratamente la dinamica del sistema, includendo le interazioni tra le diverse componenti e con

Abstract

In recent years, microfluidics has emerged as a key enabling technology in the space sector, with applications spanning astrobiology, pharmaceutical development, and space medicine. By enabling highly accurate analyses of astronauts' physiological parameters and critical environmental conditions, including water, air, and food quality, microfluidic systems play a pivotal role in supporting long-duration space missions. Nevertheless, conventional experimental approaches employed for the design and optimization of microdevices fail to fully exploit the potential of this technology, leading to increased development costs and extended development timelines.

Within this context, computational microfluidics emerges as a fundamental tool for the modeling and quantitative analysis of the inherent complexity and multiphysics nature of fluid behavior in microchannels. It is essential for investigating interactions between the continuous phase, i.e. fluid, and dispersed phases, including cells and particles. The adoption of advanced numerical models enables a systematic investigation of the influence of the system's physical and geometrical parameters, thereby overcoming the main limitations of conventional experimental approaches, such as limited access to local quantities of interest and insufficient control and reproducibility of operating conditions.

This thesis focuses on the development and validation of computational methodologies based on the Lattice Boltzmann method for the investigation of multicomponent fluid systems, nanoparticle dynamics, and viscoelastic fluid in microchannels. The proposed approach is designed to systematically capture the complex rheological behavior of multicomponent systems, which underpin a broad class of microfluidic devices, including droplet-based devices such as core-shell and Janus droplet generators. These systems can be employed for 3D cell cultures, single-cell analysis, and spheroid encapsulation for in vitro modeling of organs and diseases, as well as for drug delivery applications. A key aspect of the proposed framework is the explicit treatment of surface wettability effects, which play a critical role in governing the dynamics of multiphase microfluidic systems. In particular, wettability is shown to directly influence droplet morphology, size, and production frequency. The developed framework allows for the simulation of both binary and ternary immiscible fluid systems over a wide range of density and/or viscosity ratios, and supports interactions with surfaces of arbitrary geometry under both wetting and non-wetting boundary conditions.

Microfluidic devices manipulate multiphysics fluids, such as biological fluids, which consist of a wide variety of cells, vesicles, and molecules with different sizes, densities, and shapes, interacting with one another, with the fluid matrix, and with the microchannel walls. In this context, the solver has been extended to model the transport of nanoparticles suspended in Newtonian fluids through a multiscale approach, in which nanoparticles are represented as subgrid-scale point particles. This formulation

enables an accurate description of the system dynamics, including interactions among the different components and with solid boundaries.

Furthermore, many microfluidic devices exploit passive focusing techniques to separate and align cells and/or particles based on size, morphology, or density. However, as particle dimensions decrease, such as in the case of nanoparticles, isolation becomes increasingly challenging due to low inertia. Unlike Newtonian fluids, the use of viscoelastic fluids enables the separation of submicron particles without resorting to complex geometries, external forces, or high flow rates that could potentially damage embedded cells and vesicles. Accordingly, the developed framework allows for the simulation of viscoelastic fluid dynamics, supporting both viscoelastic and elasto-inertial focusing strategies.

Currently, commercially available software does not accurately capture the full spectrum of physical phenomena involved in microfluidic devices, either in terms of modeling flexibility or predictive capability. Accordingly, the present work aims to address these limitations by providing a quantitative, predictive, and generalizable approach for the investigation of complex and heterogeneous microfluidic systems, thereby partially bridging the existing gap.

Contents

List of Figures	xii
1 Introduction	1
1.1 Microfluidics	1
1.2 Human Physiology in the Space Environment	3
1.3 Microfluidics in Space Biomedical Applications	3
1.4 Extended Space Applications of Microfluidics	5
1.5 Future Perspectives in Space Microfluidics	6
1.6 Computational Microfluidics	7
1.6.1 Commercial CFD Software	8
1.6.2 Lattice Boltzmann Method and Mesoscopic Approaches	8
1.7 Objectives and Present Contributions	9
1.7.1 Conservative Phase-Field LBM for Multicomponent Systems . . .	10
1.7.2 Extension of the CPF-LBM to nonconformal geometries and droplet generation	11
1.7.3 Multiscale LBM–Langevin Coupling for Nanoparticle Dynamics . .	11
1.7.4 LBM for Viscoelastic Fluids	11
1.7.5 Summary of Original Contributions	12
1.8 Outline	12
2 Ternary fluid systems	15
2.1 Introduction	15
2.1.1 State of the Art of Numerical Models	17
2.1.1.1 Phase-Field Lattice Boltzmann methods and wettability .	19
2.1.2 Present Contribution	19
2.1.2.1 Three-dimensional ternary phase-field framework	19
2.2 Methods	20
2.2.1 Lattice Boltzmann method for ternary fluid systems	22
2.2.2 Wetting boundary scheme for lattice boundaries	25
2.2.3 Algorithm	26
2.2.4 Numerical setup and post-processing	27
2.3 Validation of the Lattice Boltzmann method for ternary fluid systems . .	28
2.3.1 Droplets in a quiescent fluid	28
2.3.2 Spinodal decomposition of a ternary fluid system	31
2.3.3 Spreading of a liquid lens	32
2.4 Validation of the wetting scheme for lattice boundaries	35
2.4.1 Droplets on a wettable substrate	36
2.4.2 A compound binary droplet on a wettable substrate	37

2.5	Multicomponent droplets under shear flows	41
2.5.1	A concentric binary droplet on a wettable substrate in a shear flow	42
2.5.2	The deformation of a three-layer modeled leukocyte under shear flow	44
3	Droplet generation in a flow-focusing microdevice	47
3.1	Introduction	47
3.1.1	State of the Art of Numerical Models	47
3.1.1.1	Lattice Boltzmann methods for droplet generation	48
3.1.2	Present Contribution	49
3.1.2.1	Extension to complex geometries and droplet generation	49
3.2	Methods	50
3.2.1	Wetting boundary scheme for arbitrarily shaped surfaces	50
3.2.2	Algorithm	52
3.3	Validation of the wetting scheme for arbitrarily shaped surfaces	52
3.3.1	The spreading of a compound droplet on wettable flat solid surfaces	53
3.3.2	The spreading of a compound droplet on a wettable circular cylinder	55
3.4	Monodispersed droplet generation in a cross-junction microdevice	58
3.4.1	Effect of flow conditions	59
3.4.2	Effect of contact angle	64
3.5	Multicomponent droplet generation in ternary fluid systems	65
4	Mesoscale modeling of nanoparticle dynamics in fluids	69
4.1	Introduction	69
4.1.1	State of the Art of Numerical Models	70
4.1.1.1	Forced lattice Boltzmann method for Newtonian fluids . .	73
4.1.2	Present Contribution	74
4.1.2.1	Coupling a forced lattice Boltzmann Method with Langevin Dynamics	74
4.2	Methods	75
4.2.1	Lattice Boltzmann method for Newtonian fluids	77
4.2.2	Langevin dynamics for nanoparticles	78
4.2.3	Algorithm	79
4.2.4	Numerical setup and post-processing	80
4.3	Validation of the two-way coupled LB-LD method	81
4.3.1	The velocity autocorrelation function of a single nanoparticle . . .	81
4.3.2	The Brownian diffusion of dilute nanoparticle solutions	83
4.4	Outlook	84
5	Viscoelastic fluids	87
5.1	Introduction	87
5.1.1	State of the Art of Numerical Models	88
5.1.1.1	Lattice Boltzmann methods for viscoelastic fluids	88
5.1.2	Present Contribution	90
5.1.2.1	Three-dimensional framework for viscoelastic fluids . . .	90
5.2	Methods	90
5.2.1	Lattice Boltzmann method for viscoelastic fluids	91
5.2.2	Algorithm	93

5.2.3	Numerical setup and post-processing	95
5.3	Validation of the Lattice Boltzmann method for viscoelastic fluids	96
5.3.1	Steady flow in planar Poiseuille channel	96
5.3.2	Transient flow in planar Poiseuille channel	98
5.3.3	Grid convergence tests	100
5.4	Outlook	100
6	Conclusions	103
6.1	Conservative Phase-Field LBM for Multicomponent Systems	103
6.2	Extension of the CPF-LBM to Nonconformal Geometries and Droplet Generation	105
6.3	Multiscale LBM–Langevin Coupling for Nanoparticle Dynamics	106
6.4	LBM for Viscoelastic Fluids	107
A	Non-equilibrium bounce-back method for ternary fluid systems	109
B	Non-equilibrium bounce-back method for viscoelastic fluids	111
	References	128
	List of related publications	129

List of Figures

1.1	Schematic representation of the impact of microgravity and ionizing radiation exposure on human health. Created with BioRender.com.	4
1.2	Schematic illustration of the application of microfluidic devices in space medicine. Created with BioRender.com.	6
2.1	Schematic of the wetting boundary scheme for a plane surface.	26
2.2	Schematic representation of the implemented algorithm.	28
2.3	Equilibrium configurations of the density fields of two droplets in a quiescent fluid (blue) for (a) low density ratio (red, $\rho_1/\rho_3 = 10$; green, $\rho_2/\rho_3 = 5$), and (b) high density ratio (red, $\rho_1/\rho_3 = 1000$; green, $\rho_2/\rho_3 = 500$).	29
2.4	Distribution of the index ϕ^* obtained for low- (a) and high- (b) density ratios.	30
2.5	The phase field of the fluid 3 at $y = L_0/2$ computed at steady state for Case 1 ($\rho_1 : \rho_2 : \rho_3 = 10 : 5 : 1$) and Case 2 ($\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$) compared to the analytical solution.	31
2.6	Time evolution of phase separation for initial concentrations equal to (a) $(\overline{\phi_1}, \overline{\phi_2}) = (2/5, 2/5)$; (b) $(\overline{\phi_1}, \overline{\phi_2}) = (1/3, 1/3)$; (c) $(\overline{\phi_1}, \overline{\phi_2}) = (1/4, 1/4)$. The time steps considered from left to right are $t = 10 \times 10^3, 100 \times 10^3, 1000 \times 10^3$ lu.	33
2.7	Schematic representation of a liquid lens after its spreading between two immiscible fluids.	34
2.8	Equilibrium configurations of density fields of the liquid lens with different surface tensions: (a) case I, (b) case II, and (c) case III.	35
2.9	Spreading of liquid lens with different surface tensions. Comparison between the analytical and numerical results of length d (a) and contact angles θ_i (b). θ_1 is reported in orange while θ_2 in green. The dashed lines represent the relative error of $\pm 5\%$	35
2.10	Spreading of liquid lens. Magnitude of the numerical parasitic currents $ u = \sqrt{u_x^2 + u_y^2 + u_z^2}$ in Case I (a), Case II (b), and Case III (c).	36
2.11	Density distribution of the equilibrium configuration of binary droplets (red, $\rho_1/\rho_3 = 1000$; green, $\rho_2/\rho_3 = 500$) in a quiescent fluid (blue) with different contact angles.	38
2.12	Temporal evolution of normalized mass of: (a) fluid 1, (b) fluid 2, (c) fluid 3 and (d) ternary fluid system of Case 1 ($(\theta_{13}, \theta_{23}) = (45^\circ, 60^\circ)$), Case 2 ($(\theta_{13}, \theta_{23}) = (90^\circ, 90^\circ)$), Case 3 ($(\theta_{13}, \theta_{23}) = (135^\circ, 120^\circ)$) and Case 4 ($(\theta_{13}, \theta_{23}) = (150^\circ, 160^\circ)$). M_0 is the initial mass at $t = 0$	39

2.13	Comparison between the analytical and numerical results of: (a) the contact angles, θ_{i3} ; (b) the lengths, d_i , and the heights, h_i , as a function of theoretical contact angles, θ_{i3}^{th} . The dashed lines represent the relative error of $\pm 5\%$	39
2.14	Density distribution of the equilibrium configuration of compound binary droplet (red, $\rho_1/\rho_3 = 1000$; green, $\rho_2/\rho_3 = 500$) in a quiescent fluid (blue) with different contact angles.	40
2.15	Compound binary droplet with different contact angles. Comparison between the analytical and numerical results of: (a) the contact angles, θ_{i3} ; (b) the lengths, d_i . The dashed lines represent the relative error of $\pm 5\%$	41
2.16	Evolution of a concentric binary droplet with unitary density ratio in a shear flow at time step: (a) $t = 0$, (b) $t = 15000$, (c) $t = 25000$, (d) $t = 35000$, (e) $t = 45000$, (f) $t = 100000$. Rescaled density, $\rho^* = 10\phi_1 + 5\phi_2 + \phi_3$, is shown for visualization purposes.	43
2.17	Evolution of a concentric binary droplet with high density ratio (red, $\rho_1/\rho_3 = 1000$; green, $\rho_2/\rho_3 = 500$; blue, $\rho_3/\rho_3 = 1$) in a shear flow at time step: (a) $t = 10000$, (b) $t = 45000$, (c) $t = 100000$, (d) $t = 250000$	43
2.18	Temporal evolution of normalized mass of: (a) fluid 1, (b) fluid 2, (c) fluid 3 and (d) ternary fluid system for both Case 1 ($\rho_1 : \rho_2 : \rho_3 = 1 : 1 : 1$) and Case 2 ($\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$). M_0 is the mass computed at $t = 0$	44
2.19	The shape of leukocyte at $t = 40000$ for: (a) Case 1, (b) Case 2 and (c) Case 3. Isosurfaces of rescaled density, $\rho^* = 10\phi_1 + 5\phi_2 + \phi_3$, are shown for visualization purposes.	46
3.1	Ternary fluid system in contact with a curved surface.	50
3.2	Schematic representation of the implemented algorithm.	52
3.3	Density distribution of the equilibrium configuration of a compound binary droplet on a flat surface with different wettability (red: fluid 1, green: fluid 2, and blue: fluid 3).	54
3.4	Comparison between the analytical θ^{th} and numerical θ^{num} contact angles of a compound droplet on flat surfaces. The relative error of $\pm 5\%$ is reported as dashed lines.	55
3.5	Density (left) and parasitic currents (right) distributions of a compound binary droplet with unitary density ratio at the equilibrium configuration with different contact angles. Rescaled density, $\rho^* = 10\phi_1 + 5\phi_2 + \phi_3$, is shown for visualization purposes.	56
3.6	Density (left) and parasitic currents (right) distributions of a compound binary droplet with high density ratio $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ at the equilibrium configuration with different contact angles.	57
3.7	Three-dimensional view of the flow-focusing microdevice for droplet generation. Here, w is the width of the two lateral channels, Q_d and Q_c are the volumetric flow rates of the dispersed and continuous phases, and L_D is the droplet length.	59
3.8	The influence of the flow rate ratio Q (a) and the viscosity ratio α (b) on droplet length L_D is analyzed by varying Ca . The dashed curves follow the power laws obtained by fitting the simulation data.	61

3.9	Droplet generation for viscosity ratio $\alpha = 4$ and different flow rate ratios Q as a function of Capillary number Ca . (a)-(f) $Ca = 0.004$, (b)-(g) $Ca = 0.006$, (c)-(h) $Ca = 0.012$, (d)-(i) $Ca = 0.03$, and (e)-(l) $Ca = 0.08$.	62
3.10	Droplet generation process for flow rate ratio $Q = 6$, Capillary number $Ca = 0.006$, and two different viscosity ratio α at (a)-(f) $t = 3.6$ s, (b)-(g) $t = 3.8$ s, (c)-(h) $t = 3.9$ s, (d)-(i) $t = 4$ s and (e)-(l) $t = 4.1$ s.	63
3.11	Droplet generation as a function of interfacial tension: (a)-(d) $\sigma = 0.002$ lu, (b)-(e) $\sigma = 0.006$ lu, and (c)-(f) $\sigma = 0.010$ lu.	64
3.12	The droplet length as a function of Ca for three different contact angle. The dashed curves are obtained by fitting the simulated data.	65
3.13	Droplet generation in the squeezing ($Ca < 0.01$) and dripping ($Ca > 0.01$) regimes for three different contact angles: (a)-(d) $\theta = 120^\circ$, (b)-(e) $\theta = 150^\circ$, and (c)-(f) $\theta = 180^\circ$	66
3.14	Multicomponent droplet generation in ternary fluid systems (red fluid 1 and green fluid 2) with different flow rate ratios $Q_1:Q_2:Q_3$ equal to (a) $0.25:0.5:1$, (b) $0.1:0.25:1$, and (c) $0.1:0.5:1$. The 3D view is obtained using isosurfaces with a threshold value of $\phi_1 = 0.5$ and $\phi_2 = 0.5$	68
4.1	The developed framework for simulating the transport of nanoscale particles in microchannels: the Lattice-Boltzmann method for the fluid phase dynamics and the Langevin model for nanoscale particle dynamics.	75
4.2	A 3D representation of lattice nodes (green) defined at site $\mathbf{x}$ and of sub-lattice NPs (yellow) at positions $\mathbf{x}_p$	77
4.3	Schematic representation of the implemented algorithm.	80
4.4	The normalized velocity relaxation of a single NP (100 nm in diameter) with initial velocity, $\mathbf{u}_p(0) = 0.01$ lu, released in three computational domains (50^3 , 70^3 , and 100^3 nodes) of quiescent water without stochastic forces.	82
4.5	The normalized velocity relaxation of a single NP (100 nm in diameter) with different mass, released, with the initial velocity, $\mathbf{u}_p(0) = 0.01$ lu, in quiescent water without stochastic forces.	82
4.6	The normalized velocity autocorrelation function of a single NP (100 nm in diameter) of mass $m_1 = 29.3$ lu with an initial velocity, $\mathbf{u}_p(0) = 0.01$ lu, released in quiescent water under deterministic (red curve) and stochastic condition (green curve).	83
4.7	The self-diffusivity of a 100 nm nanoparticle in quiescent water at room temperature. The black line represents the Stokes–Einstein prediction.	84
4.8	The trajectory of a 100 nm nanoparticle in quiescent water at room temperature at different time instants: (a) $36.964 \mu\text{s}$ and (b) $73.946 \mu\text{s}$	85
4.9	The self-diffusivity of dilute solutions of 100 NPs as a function of particle diameter. The black line is the Stokes-Einstein relation.	86
5.1	Schematic representation of the implemented algorithm.	95
5.2	(a) The normalized steady velocity profile u_y^* , (b) the normalized normal stress component τ_{yy}^* and (c) the normalized extra stress component τ_{xy}^* at the center of the domain for different Weissenberg number, i.e., $Wi = [0.05, 0.3, 0.6, 1]$. The solid black lines are the corresponding analytical solutions.	97

5.3	Normalized transient velocity at the center of the domain for different viscosity ratios, i.e., $\beta = [0.1, 0.3, 0.5, 0.7, 0.9]$. The solid black lines are the corresponding analytical solutions. The horizontal axis shows the dimensionless time $t\nu/L^2$, equivalently expressed as $(El\beta)t$	99
5.4	The L_2 errors of the normal stress component τ_{yy} , the extra-stress component τ_{xy} , and the velocity u_y as a function of the computational domain size for $Wi = 10$ and $\beta = 0.3$	101

Chapter 1

Introduction

1.1 Microfluidics

Microfluidics is the science and technology of manipulating small volumes of fluid, typically in the microliter to nanoliter range, within micrometer-scale geometries. The field encompasses a broad range of fluids, including Newtonian and non-Newtonian liquids, biological suspensions, and multiphase systems [1].

A central aspect of microfluidics concerns the characteristic length scale of the problem. Microchannels typically have dimensions ranging from tens to hundreds of micrometers. Despite this reduction in size, classical fluid mechanics remains valid in most practical situations. The continuum hypothesis applies because these dimensions are several orders of magnitude larger than the molecular mean free path in liquids and gases at moderate pressures, ensuring a sufficiently small Knudsen number to justify treating the fluid as a continuous medium. Consequently, the governing equations, including the conservation laws of mass, momentum, and energy, remain applicable without modification. Although microfluidics may appear to be merely a miniaturized form of conventional fluid mechanics, the reduction in characteristic length scale profoundly modifies the balance of forces governing transport and the relative importance of the mechanisms that control fluid behavior, giving rise to regimes that differ markedly from those observed at the macroscale.

Microfluidic systems typically operate in a low Reynolds number regime due to the small characteristic length and moderate flow velocities. Under these conditions, viscous forces dominate over inertial forces and the flow remains predominantly laminar. Transport phenomena are therefore governed by a different hierarchy of dimensionless parameters than in macroscopic systems. In particular, the Péclet number characterizes the relative importance of advection and molecular diffusion, which becomes a key factor in determining mixing efficiency and species transport at small scales. In the absence of turbulence, mixing occurs primarily through molecular diffusion, relying on the gradual interpenetration of species across fluid interfaces. At the same time, the high surface-to-volume ratio enhances heat and mass transfer and increases the influence of interfacial phenomena such as capillarity and surface tension. Heat transfer is particularly efficient because thermal diffusion occurs over short distances, enabling rapid temperature adjustments. This feature is especially beneficial in applications requiring precise thermal control, such as chemical synthesis or biochemical assays involving supported catalysts or enzymes. A distinctive strategy in microscale systems consists of transporting small volumes of fluid between predefined temperature zones on a chip rather than heating or

cooling the entire device, thereby achieving rapid and energy-efficient thermal cycling.

In addition to single-phase flows, multiphase systems play a central role in microfluidics. At the microscale, interfacial phenomena become dominant, and surface tension often governs the overall flow dynamics. The relative importance of viscous forces and interfacial tension is typically characterized by the Capillary number, which determines whether interfaces remain stable, deform, or break up into discrete droplets. Owing to the small characteristic dimensions and precise flow-rate control, microfluidic platforms enable the generation of highly monodisperse emulsions and segmented flows that are difficult to achieve at larger scales. Confinement within microchannels further enhances the stability and controllability of fluid–fluid interfaces, allowing accurate manipulation of droplet size, frequency, and transport. Such multiphase configurations are widely exploited for applications including compartmentalized chemical reactions, high-throughput screening, encapsulation of biological entities, and controlled mass transfer across interfaces. The dominance of interfacial forces at small scales therefore represents one of the most distinctive and powerful aspects of microfluidic systems.

Beyond passive approaches that rely solely on hydrodynamic and interfacial phenomena, microscale transport can also be actively controlled through the application of external fields. When electric fields are applied, as in electrophoretic or electroosmotic systems, fluid motion exhibits distinctive characteristics. Electroosmotic flow generates a nearly uniform, plug-like velocity profile that minimizes dispersion. This behavior is advantageous compared to pressure-driven laminar flows, which follows a parabolic velocity distribution described by the Hagen–Poiseuille equation and results in increased dispersive mixing effects.

The term microfluidic device can refer to either a system characterized by micrometer-scale dimensions, such as Micro-Electro-Mechanical Systems (MEMS), which typically have characteristic dimensions below 1 mm, or, more precisely, to a system designed for managing small fluid volumes, such as a micro-total analysis system (μ TAS), which generally processes volumes larger than 1 μ L. A microfluidic device therefore facilitates the manipulation, control, or analysis of minute fluid volumes within micrometer-scale channels or chambers. These systems leverage physical phenomena prevalent at small scales, including surface forces, capillarity, diffusion-driven transport, and field-induced flows. The defining criterion of a microfluidic system is not the overall size of the surrounding equipment, but rather the characteristic fluid volume and the dominance of microscale transport phenomena.

Such devices rely on microfabrication techniques originally developed for integrated circuit manufacturing in the semiconductor and microelectronics industries, later adapted and expanded to include micromachining, soft lithography, additive manufacturing, bioprinting, and paper-based technologies. Although the field emerged as a distinct research area in the early 1990s, its applications now extend well beyond academic laboratories. Numerous publications and patents across multiple domains demonstrate the widespread utilization and commercialization of microfluidic platforms.

These microscale transport advantages translate into practical benefits in analytical and biomedical applications, where precision, reduced sample volumes, and integration of multiple functions are key requirements. Microfluidic devices are revolutionizing diagnostics and laboratory analysis, particularly in the field of point-of-care testing (POCT). By integrating diverse functionalities onto a single chip, these platforms streamline analytical processes and enhance efficiency. Automation minimizes manual intervention and reduces human error, increasing reproducibility and ease of use. Furthermore, these sys-

tems promote more sustainable laboratory practices by lowering reagent consumption, energy demand, and material waste [2].

Beyond terrestrial laboratories, microfluidic systems are particularly attractive in environments where space, mass, energy consumption, and reagent availability are severely constrained. In microgravity conditions, the reduced influence of body forces further enhances the relative importance of surface forces and capillary phenomena, reinforcing the compatibility between microfluidic principles and space applications [1, 3].

1.2 Human Physiology in the Space Environment

The need for advanced diagnostic technologies in space arises from the profound physiological alterations induced by spaceflight. Spaceflight exposes astronauts to prolonged periods of microgravity and ionizing radiation, imposing significant physiological stress. The human body responds to the space environment through a range of direct and indirect adaptations, including sleep disturbances, bone density loss, muscle atrophy, and dysregulation of metabolic, endocrine, cardiovascular, and immune functions (see Fig. 1.1). These alterations ultimately lead to increased susceptibility to infections and an elevated risk of tumorigenesis [4, 5].

In microgravity, bodily fluids redistribute toward the upper body, resulting in a rapid reduction in blood plasma volume and a transient relative increase in red blood cell concentration. This initial hemoconcentration is followed by adaptive changes in erythropoiesis and a reduction in total red blood cell mass, a phenomenon commonly referred to as “space anemia”. Concurrent changes in the concentrations of dissolved gases, hormones, electrolytes, and other biomolecules trigger complex and interconnected systemic responses. Microgravity also induces structural modifications in tissues: cardiac geometry may be altered, skeletal muscles undergo atrophy due to the absence of mechanical loading, and bone mass in key weight-bearing regions such as the hip and lumbar spine can decrease by approximately $1.5 \div 2.5\%$ per month. Additional spaceflight-related factors include radiation exposure, disruption of circadian rhythms due to altered light–dark cycles, and psychological stress associated with prolonged confinement in high-risk environments. Ongoing biomedical research continues to expand our understanding of both the short- and long-term effects of spaceflight on human physiology [3, 6].

Several physiological alterations associated with microgravity exposure resemble those observed during natural aging on Earth, including sarcopenia-like muscle loss, osteopenia, and immune dysregulation. Consequently, investigating microgravity-induced changes is critical not only for monitoring astronaut health and developing countermeasures for long-duration missions, but also for advancing our understanding of the biological mechanisms underlying human aging and age-related diseases [5].

These physiological alterations highlight the critical need for compact, reliable, and autonomous diagnostic systems capable of operating under the severe environmental constraints of spaceflight.

1.3 Microfluidics in Space Biomedical Applications

Microfluidic technologies represent a promising solution for strengthening onboard diagnostic capabilities (see Fig. 1.2). Their intrinsic miniaturization reduces volume and mass requirements, which are critical parameters in spacecraft design. When appro-

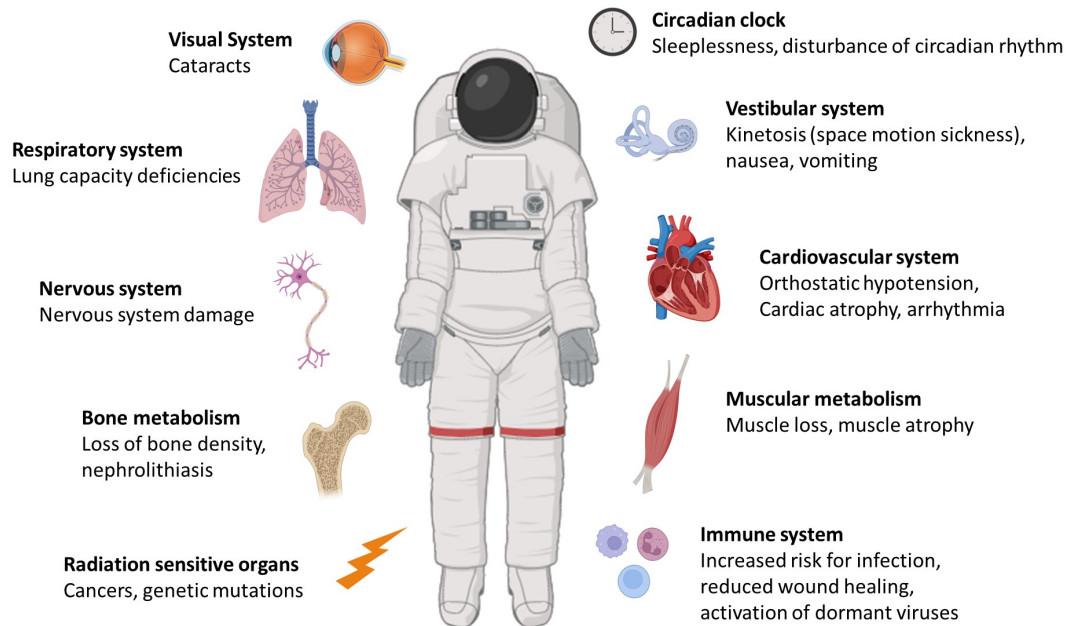


Figure 1.1: Schematic representation of the impact of microgravity and ionizing radiation exposure on human health. Created with BioRender.com.

privately engineered, such systems can operate reliably under harsh space conditions, including ionizing radiation, low humidity, and the absence of refrigeration.

The rapid and potentially reusable nature of microfluidic devices enables timely diagnostic assessments without requiring complex laboratory infrastructure. This efficiency is essential in space environments, where prompt medical responses may be decisive for safeguarding astronaut health and ensuring mission success. These platforms can be adapted to different biological fluids, including less invasive alternatives to blood such as urine and saliva, thereby enhancing the feasibility of regular health monitoring.

Onboard Diagnostics and Health Monitoring The integration of point-of-care testing (POCT) systems based on microfluidics enables real-time diagnostics, allowing astronauts to monitor physiological processes, pathological developments, and pharmacological responses directly onboard.

Recent advancements focus on miniaturized diagnostic tools, including compact microscopes, flow cytometers, and polymerase chain reaction (PCR) systems adapted for space constraints. Portable microfluidic flow cytometers enable in-flight leukocyte quantification, supporting immune monitoring. Wearable biosensors, such as epidermal patches and integrated smartwatch systems, allow continuous physiological monitoring through sweat analysis and other biomarkers.

Microfluidic culture platforms using *Caenorhabditis elegans* have provided insight into mechanobiological changes associated with spaceflight-induced muscle atrophy, supporting the development of countermeasures for long-duration missions.

Autonomous Platforms and CubeSat Systems Investigating biological systems in space is essential for astrobiology, space medicine, and drug development. Many

biomedical experiments conducted in orbit still depend on manual procedures, increasing mission costs and limiting crew time. The implementation of microfluidic technologies reduces experimental scale and decreases astronaut involvement, thereby enhancing feasibility.

The emergence of CubeSat platforms (Cube Satellite standard), compact satellites measuring approximately 1000 cm^3 and weighing less than 10 kg, has significantly expanded access to space experimentation. The integration of lab-on-a-chip (LOC) systems into CubeSats enables biological and chemical experiments within tightly constrained volumes [7, 8]. Autonomous CubeSat configurations such as the 12U platform, providing approximately 118W of power, highlight the advantages of microfluidic systems for low-energy operation.

Space missions such as GeneSat-1 (2006), PharmaSat (2009), O/OREOS (2010), EcAMSat (2017), and BioSentinel-1 (2022) have progressively demonstrated increasing biological complexity in microgravity research. These missions investigated bacterial growth, pharmaceutical responses, radiation tolerance, antibiotic resistance, and DNA damage under space conditions, providing critical data for long-duration exploration missions.

Engineering Constraints in Microgravity The integration of microfluidic systems into spacecraft presents significant engineering challenges. In microgravity, air bubbles do not rise naturally and may obstruct microchannels, requiring dedicated bubble management strategies. The vacuum of space demands complete encapsulation to prevent leakage. Extreme temperature fluctuations alter fluid properties and require advanced thermal management approaches [1, 9].

Reusable microfluidic architectures are being developed to minimize upmass and reduce dependence on disposable cartridges. Designs favor continuous-flow geometries and uniform channel heights to reduce stagnation and biofouling. Material selection is critical; silicon and glass are often preferred over polymers such as polydimethylsiloxane (PDMS) due to improved dimensional stability, reduced gas permeability, and lower molecular absorption.

Dry chemistry approaches based on on-demand reconstitution of lyophilized reagents, as well as the use of bioengineered molecules such as aptamers, enhance long-term storage stability [1, 3, 10].

1.4 Extended Space Applications of Microfluidics

Beyond biomedical monitoring, microfluidics plays an important role in planetary exploration, resource management, and advanced aerospace technologies.

Planetary Exploration and In Situ Resource Utilization Microanalytical systems can assess recycled water quality through ion-specific detection of chemical species. For missions targeting Mars and icy moons, integrated microfluidic platforms combining chromatographic separation and electrophoretic analysis are being developed to detect potential biosignatures such as nucleic acids and amino acids directly from regolith samples [11, 12].

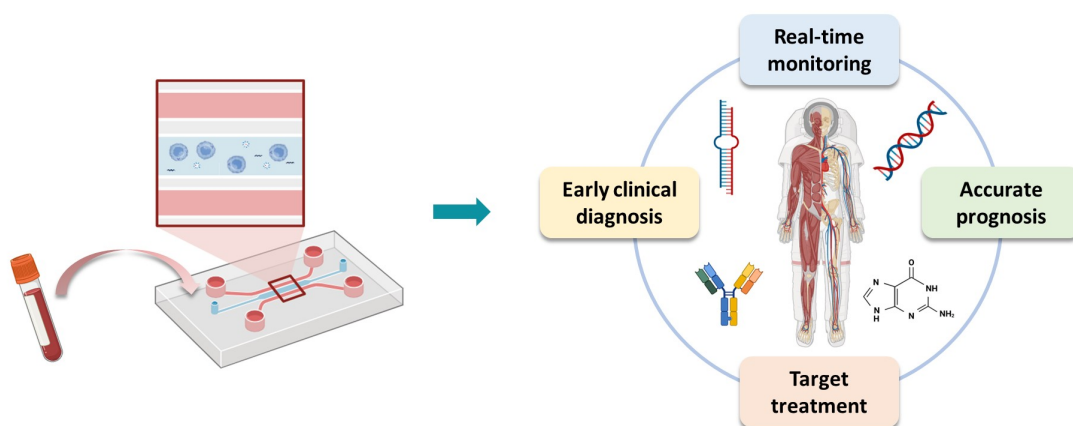


Figure 1.2: Schematic illustration of the application of microfluidic devices in space medicine. Created with BioRender.com.

Additive Manufacturing and Bioprinting Additive manufacturing enables on-demand fabrication of tools and components during long-duration missions. Three-dimensional printing allows production of complex geometries without clean-room facilities [13]. Bioprinting extends this capability to biological constructs, facilitating the formation of tissue analogs and organoids under microgravity for drug testing and mechanistic studies [14–17].

Microfluidics in Space Propulsion Systems Microfluidics also contributes to aerospace propulsion through micropropulsion systems capable of generating ultra-low thrust levels for CubeSat maneuvering and attitude control. Electro spray thrusters rely on microfluidic principles for precise propellant management [18–21]. High-precision missions such as the Laser Interferometer Space Antenna (LISA) require extremely fine thrust modulation achieved using micro-electro-mechanical systems (MEMS)-based proportional microvalves regulating propellant flow at nanoliter-per-second scales [22–24].

1.5 Future Perspectives in Space Microfluidics

Strategic initiatives led by major space agencies are accelerating the deployment of autonomous space laboratories. Extending microfluidic platforms to highly sensitive human-derived samples will require reduced payload integration times and advanced miniaturized life-support systems capable of sustaining delicate biological specimens under space conditions [9, 25]. Despite engineering challenges, microfluidics is establishing itself as a foundational technology for space medicine, astrobiology, environmental control, and propulsion systems in future long-duration missions. Taken together, the diversity of space-related applications and the stringent environmental constraints they impose highlight the necessity of predictive and physically consistent numerical tools. Accurate modeling of multiphase transport, particle–fluid interactions, and complex rheological behavior under microgravity-relevant conditions is essential for reducing experimental uncertainty, optimizing device design, and minimizing costly trial-and-error approaches in space-qualified systems.

1.6 Computational Microfluidics

In light of the multiphysical complexity and environmental constraints discussed in the previous sections, computational approaches emerge as fundamental tools for the rational design and optimization of microfluidic systems. As applications expand from terrestrial diagnostics to autonomous biomedical platforms, planetary exploration, and propulsion technologies, predictive numerical frameworks become essential for understanding and controlling microscale transport phenomena under both terrestrial and microgravity conditions.

While microfluidics was originally developed through predominantly experimental approaches, contemporary research integrates fluid mechanics, transport phenomena, materials science, and multiphysics analysis. Computational microfluidics provides a systematic framework to investigate microscale phenomena, complement experimental validation, and explore parameter spaces that would be impractical or prohibitively expensive to test physically.

Experimental approaches remain indispensable for validating microdevices; however, achieving high spatial and temporal resolution often requires costly materials such as polydimethylsiloxane (PDMS), glass, or silicon, along with fabrication techniques including photolithography and soft lithography. Each geometric modification typically necessitates new masks or molds, followed by fabrication, assembly, and testing cycles. Moreover, visualization of microscale flows requires high-resolution microscopy combined with advanced measurement techniques such as micro-particle image velocimetry (micro-PIV), fluorescence imaging, or particle tracking. Experimental results may be influenced by fabrication tolerances, surface–fluid interactions, environmental conditions, and operator-dependent procedures, making reproducibility and parametric exploration challenging.

Numerical modeling enables rigorous evaluation of microfluidic techniques and supports systematic parametric studies. It facilitates geometry optimization, reduces development time and cost, and provides deeper insight into microscale transport mechanisms and force balances. This capability is particularly valuable for applications ranging from blood component separation to droplet generation for drug delivery, as well as for the adaptation of terrestrial microdevices to microgravity environments.

At the microscale, fluid behavior is governed by mechanisms that differ substantially from those dominant at larger scales. Interfacial and line forces, including surface tension and capillary effects, become predominant, whereas body forces such as gravity are often negligible. Microscale flows are typically characterized by low Reynolds numbers and laminar regimes, with mixing primarily driven by diffusion. In microgravity environments, the suppression of buoyancy-induced convection further accentuates the role of diffusion and capillarity. Consequently, computational models are particularly well suited to isolate and quantify these effects under controlled conditions.

Although the governing equations of fluid dynamics remain unchanged in space, the relative importance of forces is altered. In terrestrial conditions, particles, cells, or droplets may undergo sedimentation due to gravity, whereas in microgravity they remain more uniformly suspended, which is advantageous for homogeneous cultures and suspension-based studies [6]. Similarly, on Earth, temperature or concentration gradients can induce natural convection, while in space transport occurs predominantly through diffusion or forced flow mechanisms. Bubble dynamics are also profoundly affected: buoyancy-driven bubble rise is suppressed in microgravity, leading to persistent

gas entrapment that may obstruct microchannels. Computational modeling enables systematic investigation of these altered regimes, including shifts in droplet formation maps and stability transitions in multiphase systems [25]. Under ideal bubble-free conditions, pressure-driven microflows exhibit dynamics comparable to terrestrial cases; however, predicting and mitigating gas entrapment remains a central modeling challenge for space microfluidics [1, 9].

1.6.1 Commercial CFD Software

A wide range of commercial computational fluid dynamics (CFD) software packages is available, yet no single platform can comprehensively capture the full spectrum of microscale phenomena, including multiphase flows, electrokinetics, particle transport, surface interactions, and microgravity effects. As a result, practical modeling typically requires selecting a solver whose core strengths match the dominant physics of the specific microfluidic problem, while introducing additional physical models to account for missing effects. Most solvers were originally developed for industrial-scale applications and must be adapted to microfluidic contexts through additional physical models and careful numerical treatment.

ANSYS Fluent and STAR-CCM+, both based on the Finite Volume Method (FVM), are widely used for modeling multiphase flows, species transport, diffusion, and chemical reactions. They are well suited for droplet-based devices, micromixers, and microreactors. However, accurate simulation of electrohydrodynamic phenomena or submicron particle dynamics requires additional modeling efforts.

FLOW-3D employs the Fractional Area/Volume Obstacle Representation (FAVOR) technique for geometry definition and incorporates specialized models for capillary and free-surface dynamics. It is effective for simulating complex interface evolution, including bubbles and jets, but is less suitable for coupled electrokinetic phenomena.

COMSOL Multiphysics, based on the Finite Element Method (FEM), offers strong multiphysics capabilities and is particularly appropriate for electrokinetic applications such as electro-osmosis, electrophoresis, and dielectrophoresis. Nevertheless, for high-fidelity CFD simulations focused exclusively on hydrodynamics, it may be computationally less efficient than FVM-based solvers and requires careful stabilization in the presence of steep gradients.

All these platforms require mesh generation and additional models to simulate suspended particle dynamics. Discrete Element Method (DEM) solvers such as Granuleworks can model solid particle interactions but must be coupled with CFD solvers to resolve fluid behavior. Particleworks, based on a meshless Moving Particle Simulation (MPS) method, can simulate complex fluid-particle interactions, including collisions and large interface deformations, yet it is not specifically designed for electrophoretic or colloidal microfluidics.

1.6.2 Lattice Boltzmann Method and Mesoscopic Approaches

In addition to classical continuum-based approaches such as FEM and FVM, the Lattice Boltzmann Method (LBM) has gained increasing attention in microfluidics due to its mesoscopic formulation, numerical efficiency, and flexibility in handling complex boundaries and multiphase phenomena. Derived from kinetic theory, LBM models the fluid as fictitious particles evolving on a discrete lattice through streaming and collision processes. The local collision operator relaxes distribution functions toward a discrete

Maxwell–Boltzmann equilibrium, enabling recovery of the Navier–Stokes equations in the appropriate limit without explicitly solving a Poisson equation.

The method’s local operations and structured lattice make it highly parallelizable and well suited for large-scale simulations on multicore and GPU architectures. Regular lattice grids simplify the implementation of complex microchannel geometries, porous media, and dynamic interfaces. Moreover, LBM can be extended to incorporate mass and heat transport, multiphase and multicomponent systems, non-Newtonian behavior, and electrokinetic effects. Its kinetic foundation allows a more direct representation of interfacial phenomena such as wettability, coalescence, and surface tension, avoiding some of the complexities associated with traditional interface-capturing or tracking methods [26, 27].

Several commercial solvers incorporate LBM-based formulations. XFlow and PowerFLOW are primarily targeted toward large-scale industrial applications and transient aerodynamic simulations. ProLB provides efficient parallel execution and turbulence modeling, while Pacefish offers cloud-based GPU acceleration for rapid prototyping. Although these tools can simulate certain microscale hydrodynamic problems, they are not specifically optimized for electrokinetic microfluidics or coupled particle transport at submicron scales.

Open-source frameworks such as OpenLB, Palabos, and Sailfish provide flexible alternatives for research-oriented applications. These platforms support multiphase and multicomponent flows, free-surface dynamics, droplet microfluidics, and microchannel networks in both two- and three-dimensional configurations. They also allow coupling with rigid particle dynamics and hydrodynamic fluid–particle interactions. However, advanced electrokinetic coupling, colloidal interactions, and complex biochemical reaction modeling typically require additional model development.

Overall, computational microfluidics constitutes a powerful design and analysis tool for both terrestrial and space applications. By complementing experimental investigations and enabling predictive modeling under microgravity conditions, it supports the rational development and optimization of next-generation microfluidic systems.

1.7 Objectives and Present Contributions

As discussed in Sections 1.6.1 and 1.6.2, despite the availability of numerous commercial CFD solvers and LBM-based frameworks, no existing numerical platform is capable of simultaneously capturing the full range of microscale phenomena relevant to advanced microfluidic systems.

In realistic microfluidic applications, several physical mechanisms often coexist: multicomponent interfacial dynamics with high density and viscosity ratios, wettability effects on complex geometries, transport and interaction of submicron particles, and non-Newtonian fluid behavior typical of biological or polymeric solutions. While individual numerical approaches may accurately describe one or two of these aspects, their simultaneous and robust integration within a unified framework remains a significant challenge.

Among mesoscopic approaches, the Lattice Boltzmann Method (LBM) is particularly attractive for microfluidic applications due to its capability to handle complex boundaries and interfacial phenomena. However, most multiphase LBM implementations rely on either the Shan–Chen pseudopotential model [28, 29] or the Color Gradient method [30]. Although effective for modeling immiscible fluids, these approaches present well-known

limitations. The Shan–Chen model restricts achievable density and viscosity ratios and may become unstable at high surface tensions. The Color Gradient method enables explicit interface reconstruction but requires more complex formulations and numerical stabilization, particularly for large property contrasts.

Beyond interfacial modeling, current LBM frameworks rarely incorporate submicron particle dynamics within a fully coupled fluid–particle formulation. Consequently, phenomena relevant to colloidal microfluidics and biomicrofluidics, including the transport of proteins, DNA filaments, vesicles, or nanoparticles, are typically not addressed. Likewise, non-Newtonian effects associated with biological fluids, e.g. hyaluronic acid solutions, or polymeric suspensions, e.g., polyethylene oxide (PEO), are often neglected or treated through simplified constitutive approximations.

Taken together, these limitations reveal a methodological gap between existing numerical tools and the multiphysical complexity of realistic microfluidic systems. Bridging this gap requires an integrative and mass-conservative framework capable of consistently treating multiphase, particulate, and viscoelastic phenomena within a single mesoscopic formulation. The present thesis addresses this need by extending an existing three-dimensional LBM solver [31] and developing three complementary methodological advances, described in the following subsections.

1.7.1 Conservative Phase-Field LBM for Multicomponent Systems

To provide accurate investigations of three-dimensional ternary fluid systems with high density and viscosity ratios in contact with flat surfaces, a D3Q19 LBM framework based on the conservative Allen–Cahn (A–C) equations is proposed. The two-dimensional conservative phase-field (CPF) model proposed by Haghani et al. [32] is adopted and extended to three-dimensional cases. The adopted model significantly reduces spurious currents compared to Cahn–Hilliard-based formulations [33,34], while strictly preserving mass conservation.

Extending the non-equilibrium bounce-back (NEBB) boundary scheme developed by Zou and He [35] for velocity and pressure imposition in two-dimensional single-phase flows, a novel NEBB formulation is derived to impose both thermodynamic and phase variables in three-dimensional ternary systems. This scheme enhances conservation of physical quantities at boundaries, even in the presence of solid walls.

To simulate droplets in contact with wettable flat surfaces, the system free energy is modified following the approaches in [36,37]. In contrast to previous works, walls are positioned directly on lattice boundary nodes, and wettability boundary conditions are derived through an alternative discretization scheme. Coupling these wettability conditions with the novel NEBB formulation improves accuracy and mass conservation compared to the half-way bounce-back method, particularly in dynamic cases with high density and viscosity ratios.

The framework is validated through benchmark tests of increasing complexity, including droplets in a quiescent fluid for low and high density ratios, spinodal decomposition of ternary systems, and liquid lens spreading. Static and dynamic droplet configurations on flat walls are examined to validate wettability conditions across a wide range of physical parameters. The deformation of a three-layer leukocyte under shear is also simulated to demonstrate the robustness and applicability of the model [38].

1.7.2 Extension of the CPF-LBM to nonconformal geometries and droplet generation

The CPF-LBM model is further extended to nonconformal geometries by introducing a treatment for rigid surfaces of arbitrary shape not aligned with the lattice grid. Mass conservation is preserved through NEBB boundary conditions at inlets and outlets, while standard bounce-back conditions are applied at nonconformal walls. Wettability on curved surfaces is modeled by adapting the approach of He et al. [39]. The method for nonconformal walls is first validated through spreading tests on straight/curved surfaces.

Following validation, the framework is employed in a parametric study of droplet generation in a three-dimensional flow-focusing microdevice, providing systematic insights into droplet formation dynamics. Sensitivity analyses are conducted with respect to flow-rate ratio, capillary number, viscosity ratio, and contact angle. Finally, a preliminary investigation of multicomponent droplet generation demonstrates the capability of the conservative Allen–Cahn formulation to simulate three immiscible fluids with wettability effects in three dimensions [40].

1.7.3 Multiscale LBM–Langevin Coupling for Nanoparticle Dynamics

Given the multiphysical nature of biological fluids, which contain cells ($2 \div 100 \mu m$), vesicles ($30 \div 150 nm$), and molecules ($< 30 nm$), the framework is extended to capture interactions between a Newtonian carrier fluid and submicron particles. A two-way coupled LBM and Langevin dynamics (LD) method is implemented following [41,42]. By representing nanoparticles as subgrid entities, the multiscale approach enables accurate simulation of micro- and nanoparticle dynamics without excessive grid refinement.

The framework is validated using single-particle and dilute multi-particle systems. Nanoparticles with different masses and sizes are considered, evaluating the hydrodynamic interactions with and without stochastic Brownian forcing. The simulations accurately reproduce mean square displacement, diffusion coefficients, long-range hydrodynamic interactions, and thermal fluctuations. This approach is applicable to a wide range of microfluidic processes, including blood component separation, cell sorting, polymer chain extraction, and colloidal suspension transport.

1.7.4 LBM for Viscoelastic Fluids

Biological fluids often exhibit non-Newtonian behavior due to the presence of polymeric chains, such as proteins, DNA, or RNA filaments. Moreover, biocompatible polymeric solutions, e.g., PEO solutions, can be used to tune elastic forces within microchannels, enabling the focusing and separation of submicron particles. To model viscoelastic effects at low ($Wi < 1$) and moderate ($Wi \leq 10$) Weissenberg numbers, the two-dimensional framework proposed by Su et al. [43] is extended here to a three-dimensional D3Q19 formulation. The use of two distribution functions allows efficient parallelization and ensures consistent mesoscopic coupling without the need for hybrid solvers.

A novel NEBB boundary scheme is implemented to impose hydrodynamic variables (velocity and pressure) while consistently reconstructing the polymeric stress tensor at the boundaries. This prevents unphysical compression and excessive polymer stretching at the walls. The framework is validated against the canonical planar Poiseuille flow, considering both transient and steady-state regimes. The effects of Weissenberg number

and viscosity ratio are systematically examined, and grid convergence tests confirm that the results are independent of the computational domain size.

Overall, this framework provides a robust and efficient tool for future investigations of suspended particles, including applications in particle focusing and separation within microchannels.

1.7.5 Summary of Original Contributions

Overall, the main contributions of this thesis can be summarized as follows:

- For multiphase flows, this work builds upon the conservative phase-field LBM formulation originally proposed by Haghani et al. [32] for two-dimensional systems and extends it to a three-dimensional D3Q19 framework for ternary fluids with high density and viscosity ratios. The proposed methodology introduces novel non-equilibrium bounce-back boundary conditions for both hydrodynamic and phase variables, new wettability treatments for conformal and nonconformal geometries, demonstrating the capability of the framework to accurately simulate realistic three-dimensional microfluidic devices.
- For nanoparticle transport, existing multiscale LBM–Langevin approaches [41, 42] are incorporated into the developed three-dimensional framework and extended to provide a fully coupled solver for submicron particle dynamics. The resulting methodology accurately captures Brownian motion, hydrodynamic interactions, and fluid–particle coupling, enabling the simulation of particulate transport within complex microfluidic configurations.
- For viscoelastic flows, the two-dimensional LBM formulation proposed by Su et al. [43] based on two distribution functions is extended to a three-dimensional D3Q19 implementation. In addition, a novel non-equilibrium bounce-back boundary treatment is developed to consistently reconstruct the polymeric stress tensor at solid boundaries, improving numerical stability and enabling accurate simulations of viscoelastic flows in microchannels.

Collectively, these developments result in a unified, mass-conservative, three-dimensional mesoscopic framework that integrates multiphase flows, nanoparticle transport, and viscoelastic fluid dynamics within a single numerical platform, significantly extending the capabilities currently available for the simulation of complex microfluidic systems.

1.8 Outline

The thesis is outlined as follows:

In Chapter 2, a three-dimensional conservative phase-field Lattice Boltzmann method (CPF-LBM) for ternary immiscible fluid systems is presented. Wettability boundary conditions, coupled with the novel NEBB scheme, are employed to accurately capture wetting phenomena on flat surfaces. The method is validated through a series of benchmark tests in both static and dynamic configurations. Finally, the CPF-LBM is applied to investigate the deformation of a three-layer leukocyte under shear flow, providing insights into the complex behavior of multilayered cells in flow environments.

In Chapter 3, the extension of the CPF-LBM model to arbitrary geometries is presented by introducing an algorithm to handle rigid surfaces not aligned with the lattice grid. Several validations and results are provided for both flat and curved surfaces. A parametric study of droplet generation in a flow-focusing microdevice is conducted, analyzing the sensitivity to flow-rate ratio, capillary number, viscosity ratio, and contact angle. Finally, a preliminary investigation of multicomponent droplet generation is presented.

In Chapter 4, a three-dimensional two-way coupled Lattice Boltzmann and Langevin dynamics (LD) method is presented to describe the dynamics of nanoparticles suspended in Newtonian fluids within microchannels. Validation tests are conducted for both single-particle and dilute multi-particle systems.

In Chapter 5, a three-dimensional viscoelastic Lattice Boltzmann method is presented to simulate the behavior of viscoelastic fluids in microchannel. A novel NEBB boundary condition scheme is employed to reconstruct both the unknown hydrodynamic variables and the polymeric stress tensor at the boundaries, ensuring accurate treatment of wall effects. The performance and accuracy of the framework are systematically validated by varying the Weissenberg number, viscosity ratio, and computational domain size, demonstrating its capability to capture viscoelastic effects in different flow regimes.

Finally, in Appendices A and B, the novel boundary condition equations for multicomponent systems and viscoelastic fluids, respectively, are briefly derived using the NEBB method.

Chapter 2

Ternary fluid systems

2.1 Introduction

Multiphase fluid systems, such as oil-in-water or water-in-oil mixtures, are fundamental to many scientific and engineering applications. Their behavior depends on properties like interfacial tension, viscosity ratios, and density differences, which govern droplet formation, coalescence, and stability. Controlling these parameters is essential for reproducible outcomes in chemical reactions, material synthesis, and biological studies. Microfluidic devices have been developed to handle these immiscible phases with high precision and control, enabling a wide range of applications such as particle focusing [44] and droplet-based lab-on-a-chip [45].

Droplet-based microfluidic devices generate small-volume droplets, typically within the picoliter to nanoliter range, dispersed in an immiscible continuous phase, such as aqueous droplets in oil. Each droplet functions as an independent microreactor, preventing interactions and thus contamination from the external environment, with precise control over composition and volume. This allows the parallel execution of thousands to millions of discrete experiments, reducing reagent consumption while ensuring excellent control over mass transfer, chemical reaction dynamics, and molecular exchange phenomena. Droplets can undergo operations such as merging, splitting, sorting, and incubation, enabling precise manipulation of their contents and dynamic behavior. Optical techniques, including fluorescence and Raman spectroscopy, spectrometric methods, and structural analyses, allow time-resolved observation of chemical, biological, and physical transformations within droplets [46–48].

Droplet-based devices are used in a wide range of engineering applications, spanning fields from chemistry to biology. In the food industry, simple and double emulsions are widely utilized, such as water-in-oil-in-water (W/O/W) and oil-in-water-in-oil (O/W/O) architectures. Unlike conventional emulsification methodologies, microfluidic devices enable the production of highly uniform droplets, i.e., with a high degree of monodispersity, which confers enhanced physical stability and improved predictability during storage. Furthermore, precise control over the number of encapsulated droplets is possible, which is essential for regulating molecular diffusion processes and degradation kinetics. The thickness and chemical composition of the external membrane can be finely tuned to achieve programmable release profiles, with release mechanisms triggered by specific environmental stimuli such as pH changes, temperature variations, or enzymatic activity. The external membrane protects sensitive bioactive compounds, including natural pigments and micronutrients, shielding them from oxidative or otherwise

adverse environmental conditions. This technology also enables *in vitro* gastrointestinal digestion studies for real-time monitoring of the release of lipophilic nutrients from lipid-based droplets directly on-chip. Beyond their use as delivery systems, double emulsions can serve as structural templates for the fabrication of alginate microcapsules, giant liposomes, and polymersomes, supporting applications ranging from nutraceutical formulations to regenerative medicine [49].

In biomedical applications, the compartmentalization of cells, biomolecules, or reactions into isolated microenvironments enables ultra-high-resolution analyses, e.g. 3D cell culture, single-cell analysis, and spheroid encapsulation for *in vitro* modeling of organs, diseases, and drug delivery [45, 50]. Among the numerous applications are single-cell assays, in which individual cells are encapsulated in discrete droplets for genomic, transcriptomic, or metabolomic analysis of cellular heterogeneity [51]; droplet digital PCR (ddPCR), in which diluted DNA samples are divided into thousands of microcompartments to achieve absolute quantification of nucleic acids and detection of rare genetic variants with sensitivities down to 0.001%; three-dimensional cell culture and organoid generation, whereby cells embedded in microgel droplets self-organize into spheroids or organotypic models suitable for toxicity evaluation and personalized drug screening; and high-throughput combinatorial screening, allowing rapid testing of extensive libraries of drug candidates or microbial consortia with minimal reagent usage relative to traditional microtiter plate formats. In this framework, droplets act as tightly regulated microenvironments that prevent cross-contamination and enable statistically robust, large-scale experimentation [45, 50].

In materials science, droplet-based methodologies constitute a substantial shift from traditional bulk-scale reactors. In these systems, each droplet acts as a spatially confined reaction chamber, enabling precise regulation of temperature, mixing dynamics, and residence time. This level of control has enabled the fabrication of a broad range of advanced materials. For instance, droplet microreactors are widely employed in nanomaterial synthesis, allowing the production of metallic nanocrystals, such as gold and silver, with narrow size distributions and high reproducibility across batches. They are equally suited for the preparation of lead halide perovskite nanocrystals and semiconductor nanoparticles, whose optical properties can be finely tuned through careful adjustment of reaction parameters [45]. In addition, droplets serve as versatile templates for the formation of microgels and microparticles, including polymeric microspheres and PLGA-based carriers designed for controlled drug delivery or as three-dimensional scaffolds for cell culture. By co-encapsulating distinct polymer phases within a single droplet prior to solidification, it is possible to generate Janus and other anisotropic particles characterized by chemically or physically differentiated domains. Beyond particle synthesis, controlled droplet deformation combined with the incorporation of magnetic or catalytic nanocomponents has facilitated the development of micromotors and other advanced functional materials [52].

As mentioned in the Chapter 1, droplet-based microfluidic devices are widely employed in space to investigate physical and chemical phenomena whose dynamics are profoundly altered under microgravity conditions, with particular emphasis on mixture stability and reaction kinetics. For example, in the DIDO-2 mission, launched in 2017 by SpacePharma, such devices were used as microreactors to observe the coalescence of suspensions, whose stability on Earth typically requires mechanical or chemical interventions. Droplets were also employed to study self-assembly processes, such as those involving phenylalanine peptides, which in the space environment tend to organize into

spherulite-like structures, facilitating the execution of complex and automated experiments without direct human intervention. Future missions aim to further investigate the effects of microgravity on antimicrobial agents and the binding properties of human proteins [1]. Furthermore, microgravity significantly influences cell growth, particularly in the case of cancer cells, which spontaneously assemble into three-dimensional spheroidal structures. Unlike typical two-dimensional cell cultures in terrestrial laboratories, these structures more faithfully replicate the tumor microenvironment and behavior *in vivo*, providing essential models for advanced studies of metastasis and innovative therapies, including those targeting drug-resistant cell lines such as breast and thyroid cancers. Such studies have already been conducted aboard the ISS and using CubeSats, including ESA-SPHEROIDS, an ESA project designed to analyze three-dimensional cell growth in orbit, assessing the impact of microgravity at both morphological and secretory levels. Through the integration of microfluidic technologies, it has been possible to culture sensitive cell lines in tightly controlled environments, automating nutrient delivery and real-time monitoring, thereby overcoming the limitations of conventional culture systems that often require post-flight analysis [9].

In summary, the ability of droplet-based microfluidic technologies to compartmentalize, protect, manipulate, and analyze complex systems at the microscale has profoundly influenced disciplines ranging from food science to molecular diagnostics and advanced materials engineering. The inherent suitability of these systems for space applications has revolutionized experiments and studies conducted in orbit, which are essential for monitoring astronauts' vital parameters, investigating new drugs, gaining a deeper understanding of biological phenomena and mechanisms, as well as synthesizing novel materials and performing chemical and biological analyses of samples from other planets.

In this context, computational fluid dynamics complements experimental investigations by enabling the modeling and analysis of multiphase flows in microchannels, as well as the accurate computation of relevant quantities such as pressure, velocity fields, and droplet size. However, ensuring numerical stability, efficiency, and accuracy, particularly for high density or viscosity ratios, remains a critical challenge.

2.1.1 State of the Art of Numerical Models

In the context of multiphase flows, computational fluid dynamics is a suitable tool for modeling and simulating related complex phenomena such as droplet coalescence and wettability. Several numerical methods are available to solve the Navier–Stokes equations while tracking the phase interface. These methods can be classified into sharp interface methods, e.g. front-tracking methods [53] and volume-of-fluid methods [54], and diffuse interface models, e.g. level-set methods [55] and phase-field methods [56]. In sharp interface methods, the interface is represented as a boundary that exhibits a jump in rheological properties while maintaining the continuity of the velocity field. These methods usually require remeshing of the immersed body, e.g. a droplet, at the end of each iteration, and interface reconstruction is needed to simulate droplet merging or separation. In contrast, diffuse interface methods treat the interface as a region of finite width, within which the rheological properties vary smoothly, thus avoiding numerical instabilities. The main challenge is represented by the need to solve the Poisson equation for pressure and determining the discrete form of Navier–Stokes equation terms using information from adjacent nodes. This nonlocal nature increases the complexity, particularly when parallelization is necessary.

The Lattice Boltzmann method (LBM) represents an established alternative to solve multiphase flow dynamics and capture the physics at the interface. In LBM, the non-linearity enters in a local collision process within each node, whereas the interactions between nodes are entirely linear. Due to its local nature, LBM are then suitable for high-performance computing on parallel architectures, since the heaviest computations are local. Many multiphase LBM models have been developed and reported in literature: color-gradient models, pseudopotential models, free-energy models, and phase-field models.

The color-gradient model was originally proposed by Gunstensen et al. [30] to simulate binary (immiscible) fluid systems. Each component is described by its own distribution function, and a forcing term is included in the collision step to take into account the surface tension effects. Starting from the original model, Latva-Kokko and Rothman [57] proposed an improved model to reduce spurious velocities, Liu et al. [58] extended the model to three-dimensional cases, and Leclaire et al. [59] and Huang et al. [60] to fluids with high density ratio. Although this method guarantees mass conservation with high numerical accuracy and low parasitic currents, the challenge was represented by simulating fluids with different viscosity. To overcome this limit, the multi-relaxation time collision operator was introduced to improve numerical stability [61], but the model was found to be not suitable for simulating fluids with a high density ratio. This model is also extended to multicomponent flows [62, 63].

In the pseudopotential model, Shan et al. [28, 29, 64] proposed a formulation based on the molecular interaction between the fluids that guarantees spontaneous separation at the interface. Thanks to its ease of implementation, this model has become widely used to simulate fluids with low viscosity ratio ($\mathcal{O}(10)$). It is usually used to investigate wetting phenomena, modeled by including an interaction potential between the fluids and the solid surface [65, 66]. The high density ratio version for multicomponent fluids was developed by Bao and Schaefer [67] and then modified to reduce spurious velocities and increase density and viscosity ratios between fluids [68, 69]. Nonetheless, its use is limited to a few dynamic cases, due to a lack of accuracy when high viscosity ratios are considered.

The free-energy model was introduced by Swift et al. [70, 71]. Its formulation is based on the achievement of thermodynamic equilibrium of isothermal binary fluid systems by minimizing the free-energy functional. Then, the hydrodynamic stress tensor redefined by Holdych et al. [72] improved the description of the interface forces, guaranteeing the Galilean invariance of the model. Inamuro et al. [73] improved the model to simulate fluids with high density ratio by solving the Poisson equation for pressure, thus failing the inherent simplicity of the LBM. Zheng et al. [74] developed a Galilean invariant version of such LBM without solving the Poisson equation, but later Fakhari et al. [75] demonstrated that it is accurate only for two immiscible fluids with the same density. Semperebon et al. [76] extended the model to simulate ternary fluid systems and wettability. Although the free-energy method effectively describes complex thermodynamic phenomena such as phase transitions and thermal fluctuations, the energy-based formulation requires careful treatment of the free energy and boundary conditions. In addition, it is sensitive to high gradients and the adopted discretisation. For multiphase systems with different densities, the pressure field is continuous, while the density is discontinuous at the interface. Therefore, this method suffers from numerical instability in the case of components with a high density ratio [77].

2.1.1.1 Phase-Field Lattice Boltzmann methods and wettability

The phase-field model was originally proposed by Jacqmin in the context of conventional methods [78, 79] and was later extended to LBM [80]. It can be considered as a subclass of the free-energy model, as it also relies on the minimization of free energy, but it is more stable because the phase field is treated as a state variable. A set of distribution functions is used to compute the hydrodynamic variables of the system, and another set is used to track the interface and compute the so-called order parameter, namely the phase-field variable. The evolution of the interface can be described by the Cahn–Hilliard (C-H) equation [81] or the Allen–Cahn (A-C) equation [82]. The C-H phase-field model is widely used as it preserves mass conservation with high accuracy [83]. However, it requires solving the fourth-order derivative of the chemical potential, rather than the second-order derivative that is solved in the A-C phase model. The use of the latter in the LBM was established in the literature when Chiu and Lin [84] developed its conservative form for binary fluid systems starting from the previous work of Sun et al. [85]. Several improvements have been made to the original method, such as the introduction of isotropic finite-difference schemes for computing the gradient and the Laplacian operators, which enhance the accuracy and symmetry of the model but reduce its local nature. Fakhari et al. [86] improved the efficiency of the work by Zu and He [87], using second-order moments to compute the velocity gradient. Geier et al. [88] and Fakhari et al. [89] computed the gradient of the phase-field variable using the first-order central moments for two- and three-dimensional simulations, respectively. It is optimal for parallel computing, but suffers from grid-scale oscillations [88, 90]. Recently, the conservative A-C phase-field model has also been extended to three-component systems [32] and to phase change [91].

The wetting boundary conditions in both C-H and A-C models can be integrated by introducing the wetting potential, as in the pseudopotential model. Starting from a modified free-energy of the system, Liang et al. [36] and Zhang et al. [37] proposed a wetting boundary scheme for ternary fluid systems based on the halfway bounce-back method [92] and integrated it in C-H and A-C models, respectively. The C-H wetting scheme [36] has been used for three-dimensional configurations with low density and/or viscosity ratios, $\mathcal{O}(10)$, while the A-C wetting scheme [37] for two-dimensional configurations with high density and/or viscosity ratios, $\mathcal{O}(1000)$. The models mentioned above can integrate wettability by describing fluids in contact with flat surfaces.

2.1.2 Present Contribution

In this thesis, the aim is to advance the modeling and simulation of three-dimensional multicomponent droplet systems in microfluidic environments.

2.1.2.1 Three-dimensional ternary phase-field framework

With the goal of providing accurate investigations of three-dimensional ternary fluid systems with high density and/or viscosity ratio in contact with flat surfaces, a D3Q19 LB framework based on the conservative A-C equations is proposed in the present work. The two-dimensional conservative phase-field (CPF) model proposed by Haghani et al. [32] is adopted and extended here for three-dimensional cases. The adopted model exhibits a significant reduction in spurious currents compared to models based on the C-H equation [33, 34], while strictly preserving mass conservation (as shown in Section 2.3). Fur-

thermore, starting from the idea of Zou and He [35] who developed a non-equilibrium bounce back (NEBB) boundary scheme for imposing velocity and/or pressure in two-dimensional single-phase flows, a novel NEBB boundary scheme is derived for imposing both thermodynamic and phase variables in the context of three-dimensional ternary fluid systems. The proposed NEBB scheme ensures better conservation of physical variables at the boundaries even in the presence of walls (as shown in Section 2.3.3). To simulate drops in contact with wettable flat surfaces, the free energy of the system is modified by adopting the same approach developed in [36, 37]. In contrast to previous works, the walls are placed directly on the lattice boundary nodes, and derived the wettability boundary conditions through a different discretization scheme. As a result, the use of such wettability boundary conditions coupled with the novel NEBB scheme improves the accuracy and mass-conserving properties of the overall solver compared to the half-way bounce-back method [36, 37], especially in dynamic cases with high density and viscosity ratios (as shown in Section 2.4).

The developed framework is validated through benchmark tests of increasing complexity. First, framework validation is performed without the wettability boundary conditions to observe the accuracy of the model in describing: droplets in a quiescent fluid for both low- and high- density ratios, spinodal decomposition of a ternary fluid system, and the spreading of a liquid lens. Then, both static and dynamic configurations of droplets on flat walls are investigated to validate the wettability boundary conditions for a wide range of viscosity- and density- ratios. Finally, the deformation of a three-layer leukocyte under shear is investigated by means of three-dimensional simulations to demonstrate the effectiveness of the proposed framework and to advance the state-of-the-art.

2.2 Methods

The continuity and momentum equations of an incompressible, immiscible and isothermal three-component multiphase system are respectively given by

$$\nabla \cdot \mathbf{u} = 0, \quad (2.1)$$

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \nabla \cdot [\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \mathbf{F}^s + \mathbf{F}^b \quad (2.2)$$

where ρ , μ , $\mathbf{u}$, p are the density, dynamic viscosity, velocity and pressure fields of the system, respectively; $\mathbf{F}^s$ is the force per unit volume deriving from the continuous surface tension field, and $\mathbf{F}^b$ is the external bulk force per unit volume. Following Zu and He [87], Eq. (4.2) can rewrite in the form of the velocity-based equation:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla \left(\frac{p}{\rho} \right) + \nabla \cdot [\nu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \frac{\mathbf{F}^s + \mathbf{F}^b + \mathbf{F}^p + \mathbf{F}^\eta}{\rho} \quad (2.3)$$

where ν is the kinematic viscosity of the system, $\nu = \mu/\rho$, the pressure correction force $\mathbf{F}^p$ is equal to $(-p/\rho)\nabla\rho$, and the viscous stress correction force $\mathbf{F}^\eta$ is equal to $[\nu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)] \cdot \nabla\rho$.

The general form of the advection equation for the evolution of each i -th ($i = \{1, 2, 3\}$) phase-field variable ϕ_i in an Eulerian framework is

$$\frac{\partial \phi_i}{\partial t} + \mathbf{v}_i \cdot \nabla \phi_i = 0 \quad (2.4)$$

where $\mathbf{v}_i$ is the total velocity that can be split into a component for external advection $\mathbf{u}$ and another component normal to the interface $u_i^\perp \mathbf{n}_i$, where $\mathbf{n}_i = \frac{\nabla \phi_i}{|\nabla \phi_i|}$ is the unit vector normal to the interface, and the normal velocity component $u_i^\perp$ is defined as $u_i^\perp = -M_i \kappa_i$, being M_i the mobility of the fluid and κ_i the interface curvature. The latter can be expressed as $\kappa_i = \nabla \cdot \mathbf{n}_i$ [84, 85], thus leading to the following form of Eq. (2.4):

$$\frac{\partial \phi_i}{\partial t} + \mathbf{u} \cdot \nabla \phi_i = M_i [\nabla^2 \phi_i - \mathbf{n}_i \cdot \nabla |\nabla \phi_i|]. \quad (2.5)$$

For multiphase flows there is no curvature-driven interface motion and this kind of motion must be eliminated from interface-tracking equation. Therefore, the counter-term approach by Folch et al. [93] is used to cancel out the curvature-driven term from the right-hand side of Eq. (2.5) such that

$$\frac{\partial \phi_i}{\partial t} + \mathbf{u} \cdot \nabla \phi_i = M_i [\nabla^2 \phi_i - \nabla \cdot (\mathbf{n}_i |\nabla \phi_i|)]. \quad (2.6)$$

Assuming that the divergence-free condition holds (the flow is incompressible and no phase-change phenomenon happens within the interfaces), the interface evolution is retrieved by solving the conservative Allen–Cahn (A-C) equation for each i -th ($i = \{1, 2, 3\}$) phase-field variable ϕ_i , i.e. volume fraction,

$$\frac{\partial \phi_i}{\partial t} + \nabla \cdot (\mathbf{u} \phi_i) = M_i \nabla \cdot \left[\nabla \phi_i - \mathbf{n}_i |\nabla \phi_i|_{eq} + \frac{1}{3} \sum_{j=1}^3 \mathbf{n}_j |\nabla \phi_j|_{eq} \right] \quad (2.7)$$

where the mobility, i.e. the diffusion coefficient, of each component M_i is assumed to be equal ($M_1 = M_2 = M_3 = M$) and the magnitude of the i -th phase-field gradient at the thermodynamic equilibrium $|\nabla \phi_i|_{eq}$ is obtained by minimizing the free energy of a fixed fluid domain Ω

$$F = \int_{\Omega} [\Phi(\phi_1, \phi_2, \phi_3) + \Psi(\nabla \phi_1, \nabla \phi_2, \nabla \phi_3)] d\Omega \quad (2.8)$$

where ξ is the chosen interface width, and Φ and Ψ are the bulk free energy and the capillary term, respectively, defined as

$$\Phi(\phi_1, \phi_2, \phi_3) = \frac{12}{\xi} \left[\frac{\gamma_1}{2} \phi_1^2 (1 - \phi_1)^2 + \frac{\gamma_2}{2} \phi_2^2 (1 - \phi_2)^2 + \frac{\gamma_3}{2} \phi_3^2 (1 - \phi_3)^2 \right], \quad (2.9)$$

$$\Psi(\nabla \phi_1, \nabla \phi_2, \nabla \phi_3) = \sum_{i=1}^3 \frac{3}{8} \xi \gamma_i |\nabla \phi_i|^2. \quad (2.10)$$

Each auxiliary coefficient γ_i is related to the surface tensions σ_{jk} :

$$\gamma_1 = \sigma_{12} + \sigma_{13} - \sigma_{23}, \quad (2.11a)$$

$$\gamma_2 = \sigma_{12} + \sigma_{23} - \sigma_{13}, \quad (2.11b)$$

$$\gamma_3 = \sigma_{13} + \sigma_{23} - \sigma_{12}. \quad (2.11c)$$

Thus, $|\nabla \phi_i|_{eq}$ is obtained as

$$|\nabla \phi_i|_{eq} = \frac{4}{\xi} |\phi_i (1 - \phi_i)|. \quad (2.12)$$

The mathematical derivation of Eq. (2.12) is available in [32]. The phase-field variable of the i -th fluid respects the partition of unity property, i.e.

$$\sum_{i=1}^3 \phi_i = 1. \quad (2.13)$$

Such a model satisfies the reduction-consistent properties: i) one fluid is absent if it does not appear at the initial time; ii) if only two phases appear at the initial time, the model allows to describe the binary fluid system.

In order to consider the wettability phenomena of ternary fluid systems in contact with a solid surface $\partial\Omega$, the total free energy of the domain Ω (2.8) is modified in the following way [36, 37]

$$F = \int_{\Omega} [\Phi(\phi_1, \phi_2, \phi_3) + \Psi(\nabla\phi_1, \nabla\phi_2, \nabla\phi_3)] d\Omega + \int_{\partial\Omega} \psi_s(\phi_1, \phi_2, \phi_3) ds \quad (2.14)$$

where ψ_s is a free energy density defined as

$$\psi_s(\phi_1, \phi_2, \phi_3) = \sum_{i=1}^3 \sigma_{wi} (3\phi_i^2 - 2\phi_i^3), \quad (2.15)$$

σ_{wi} being the surface tension between the solid surface and the i -th fluid. The Young's relations for the static contact angles reads:

$$\cos \theta_{ij} = \frac{\sigma_{wj} - \sigma_{wi}}{\sigma_{ij}}, \quad \text{with } 1 \leq i < j \leq 3 \quad (2.16)$$

leading to the following momentum balance

$$\sigma_{12} \cos \theta_{12} - \sigma_{13} \cos \theta_{13} + \sigma_{23} \cos \theta_{23} = 0. \quad (2.17)$$

The wetting boundary conditions will be derived minimizing the modified total free energy (2.14). This leads to

$$\mathbf{n}_w \cdot \nabla \phi_1 = -\frac{4}{\xi} (\phi_1 \phi_3 \cos \theta_{13} + \phi_1 \phi_2 \cos \theta_{12})_{\mathbf{x}_w} \quad (2.18a)$$

$$\mathbf{n}_w \cdot \nabla \phi_2 = -\frac{4}{\xi} (\phi_2 \phi_3 \cos \theta_{23} - \phi_1 \phi_2 \cos \theta_{12})_{\mathbf{x}_w} \quad (2.18b)$$

$$\mathbf{n}_w \cdot \nabla \phi_3 = -\frac{4}{\xi} (-\phi_2 \phi_3 \cos \theta_{23} - \phi_1 \phi_3 \cos \theta_{13})_{\mathbf{x}_w} \quad (2.18c)$$

where $\mathbf{n}_w$ is the unit vector normal to the surface. Hereafter, the terms on the right side will be called $C_{13} = \phi_1 \phi_3 \cos \theta_{13}$, $C_{12} = \phi_1 \phi_2 \cos \theta_{12}$, and $C_{23} = \phi_2 \phi_3 \cos \theta_{23}$ to lighten the notation of the equations.

2.2.1 Lattice Boltzmann method for ternary fluid systems

The CPF model proposed by Haghani et al. [32] is considered to investigate the wettability of ternary fluid systems with high density and viscosity ratios. Two sets of distribution functions are defined to describe the ternary fluid system. The mass and momentum equations (Eqs. (4.1) and (4.2)) are described by the hydrodynamic distribution functions, $f_{\alpha}(\mathbf{x}, t)$, being α the lattice direction. To solve the advection-diffusion

A-C equation (Eq. (2.7)), two of the three fluids are tracked defining two phase-field distribution functions, $h_\alpha^1(\mathbf{x}, t)$ and $h_\alpha^2(\mathbf{x}, t)$. The third phase-field variable is obtained by imposing the mass conservation (Eq. (2.13)).

First, the BGK scheme is solved adopting a three-dimensional computational lattice (D3Q19) with weighting factors $\omega_0 = 1/3$, $\omega_{1-6} = 1/18$, $\omega_{7-18} = 1/36$, and discrete velocity $\mathbf{e}_\alpha$

$$\mathbf{e}_\alpha = \frac{\Delta x}{\Delta t} \begin{cases} (0, 0, 0) & \alpha = 0 \\ (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1) & \alpha = 1, 2, \dots, 6 \\ (\pm 1, \pm 1, 0), (\pm 1, 0, \pm 1), (0, \pm 1, \pm 1) & \alpha = 7, 8, \dots, 18 \end{cases}, \quad (2.19)$$

where the lattice time step Δt and length $\Delta x = \Delta y = \Delta z$ is set equal to 1.

The conservative A-C equation (Eq. (2.7)) for the i -th fluid ($i = \{1, 2\}$), is described by phase-field distribution functions transported by using the Boltzmann equation

$$h_\alpha^i(\mathbf{x} + \Delta t \mathbf{e}_\alpha, t + \Delta t) - h_\alpha^i(\mathbf{x}, t) = -\frac{\Delta t}{\tau_{\phi_i} + \frac{\Delta t}{2}} [h_\alpha^i(\mathbf{x}, t) - h_\alpha^{i,eq}(\mathbf{x}, t)] + \Delta t F_\alpha^{\phi,i}(\mathbf{x}, t) \quad (2.20)$$

where the phase-field relaxation time τ_{ϕ_i} is related to the mobility M by

$$\tau_{\phi_i} = \frac{M}{c_s^2 \Delta t}. \quad (2.21)$$

The equilibrium phase-field distribution function is given by

$$h_\alpha^{i,eq}(\mathbf{x}, t) = \omega_\alpha \phi_i \left[1 + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})}{c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})^2}{2c_s^4} - \frac{(\mathbf{u} \cdot \mathbf{u})}{2c_s^2} \right], \quad (2.22)$$

where $\mathbf{u}$ is the fluid velocity of the position $\mathbf{x}$ at time t , while the source forcing term $F_\alpha^{\phi,i}(\mathbf{x}, t)$ is calculated as [94]

$$F_\alpha^{\phi,i}(\mathbf{x}, t) = \frac{\omega_\alpha}{c_s^2} \mathbf{e}_\alpha \cdot \mathbf{F}^{\phi,i}(\mathbf{x}, t), \quad (2.23)$$

where $\mathbf{F}^{\phi,i}$ is defined as

$$\mathbf{F}^{\phi,i} = \frac{c_s^2}{3} \left[2\mathbf{n}_i |\nabla \phi_i|_{eq} - \sum_{j \neq i} \mathbf{n}_j |\nabla \phi_j|_{eq} \right], \quad (2.24)$$

with $j = \{1, 2, 3\}$.

The zeroth moment of the i -th ($i = \{1, 2\}$) phase-field distribution functions is used to compute the phase-field variable $\phi_i(\mathbf{x}, t)$:

$$\phi_i(\mathbf{x}, t) = \sum_{\alpha=0}^{18} h_\alpha^i(\mathbf{x}, t) \quad (2.25)$$

and the phase-field variable of the third fluid is given by $\phi_3 = 1 - \phi_1 - \phi_2$ according to mass conservation (Eq. (2.13)). The density of each lattice node is updated using a linear interpolation

$$\rho(\mathbf{x}, t) = \sum_{i=1}^3 \phi_i(\mathbf{x}, t) \rho_i. \quad (2.26)$$

The hydrodynamic distribution functions are transported using the Boltzmann equation

$$f_\alpha(\mathbf{x} + \Delta t \mathbf{e}_\alpha, t + \Delta t) - f_\alpha(\mathbf{x}, t) = -\frac{\Delta t}{\tau(\mathbf{x}, t) + \frac{\Delta t}{2}} [f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t)] + \Delta t F_\alpha(\mathbf{x}, t) \quad (2.27)$$

where the hydrodynamic relaxation time $\tau(\mathbf{x}, t)$ is computed by a linear interpolation that approximates the Heaviside function

$$\tau(\mathbf{x}, t) = \sum_{i=1}^3 \phi_i(\mathbf{x}, t) \tau_i, \quad (2.28)$$

and τ_i is related to the i -th fluid density ρ_i and dynamic viscosity μ_i as

$$\tau_i = \frac{\mu_i}{\rho_i c_s^2 \Delta t}, \quad (2.29)$$

so that the kinematic viscosity $\nu(\mathbf{x}, t)$ and dynamic viscosity $\mu(\mathbf{x}, t)$ of the ternary fluid system are given by

$$\nu(\mathbf{x}, t) = \tau(\mathbf{x}, t) c_s^2 \Delta t, \quad (2.30a)$$

$$\mu(\mathbf{x}, t) = \rho(\mathbf{x}, t) \nu(\mathbf{x}, t). \quad (2.30b)$$

The equilibrium state $f_\alpha^{eq}(\mathbf{x}, t)$ is defined as

$$f_\alpha^{eq}(\mathbf{x}, t) = \omega_\alpha \left[\frac{p}{\rho c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})}{c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})^2}{2c_s^4} - \frac{(\mathbf{u} \cdot \mathbf{u})}{2c_s^2} \right], \quad (2.31)$$

where ρ , p and $\mathbf{u}$ are the fluid density, pressure, and velocity of the position $\mathbf{x}$ at time t . The source forcing term $F_\alpha(\mathbf{x}, t)$ is calculated as [94]

$$F_\alpha(\mathbf{x}, t) = \frac{\omega_\alpha}{\rho(\mathbf{x}, t) c_s^2} \mathbf{e}_\alpha \cdot \mathbf{F}(\mathbf{x}, t), \quad (2.32)$$

where the total force per unit volume is defined as

$$\mathbf{F} = \mathbf{F}^b + \mathbf{F}^s + \mathbf{F}^p + \mathbf{F}^\eta. \quad (2.33)$$

$\mathbf{F}^b$ is the external bulk force per unit volume; the continuous surface tension force per unit volume $\mathbf{F}^s$ is computed by

$$\mathbf{F}^s = \sum_{i=1}^3 \mu^{\phi_i} \nabla \phi_i \quad (2.34)$$

where the chemical potential μ^{ϕ_i} of the i -th fluid is given by

$$\mu^{\phi_i} = \frac{4\gamma_T}{\xi} \sum_{j \neq i} \left[\frac{1}{\gamma_j} \left(\frac{\partial \Phi}{\partial \phi_i} - \frac{\partial \Phi}{\partial \phi_j} \right) \right] - \frac{3}{4} \xi \gamma_i \nabla^2 \phi_i \quad j = \{1, 2, 3\} \quad (2.35)$$

and the total auxiliary coefficient, γ_T , is computed from $3\gamma_T^{-1} = \gamma_1^{-1} + \gamma_2^{-1} + \gamma_3^{-1}$; the pressure correction force $\mathbf{F}^p$ is defined as

$$\mathbf{F}^p = -\frac{p}{\rho} \nabla \rho; \quad (2.36)$$

and the viscous stress correction force $\mathbf{F}^\eta = [\nu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)] \cdot \nabla \rho$ is computed from the hydrodynamic distribution functions [86]:

$$\mathbf{F}_j^\eta = -\frac{\tau}{\tau + \frac{\Delta t}{2}} \left[\sum_{\alpha=1}^{18} e_{\alpha j} e_{\alpha k} (f_\alpha - f_\alpha^{eq}) \right] \frac{\partial \rho}{\partial x_k} \quad j, k = \{x, y, z\}. \quad (2.37)$$

The isotropic finite differences are used to compute the required gradient and Laplacian operators as [95]

$$\nabla \lambda = \frac{1}{2c_s^2 \Delta t} \sum_{\alpha=1}^{18} \omega_\alpha \mathbf{e}_\alpha [\lambda(\mathbf{x} + \mathbf{e}_\alpha \Delta t, t) - \lambda(\mathbf{x} - \mathbf{e}_\alpha \Delta t, t)] \quad (2.38)$$

$$\nabla^2 \lambda = \frac{c_s^2 \Delta t^2}{2\Delta x^2} \left[\sum_{\alpha=1}^{18} \lambda(\mathbf{x} + \mathbf{e}_\alpha \Delta t, t) + \sum_{\alpha=1}^6 \lambda(\mathbf{x} + \mathbf{e}_\alpha \Delta t, t) - 24\lambda(\mathbf{x}, t) \right] \quad (2.39)$$

where λ is the density ρ or the phase-field variable ϕ_i .

The fluid pressure and velocity are computed by the zeroth and first moments of the hydrodynamic distribution functions as:

$$p(\mathbf{x}, t) = \rho(\mathbf{x}, t) c_s^2 \sum_{\alpha=0}^{18} f_\alpha(\mathbf{x}, t), \quad (2.40)$$

$$\mathbf{u}(\mathbf{x}, t) = \sum_{\alpha=0}^{18} f_\alpha(\mathbf{x}, t) \mathbf{e}_\alpha + \frac{\Delta t}{2\rho(\mathbf{x}, t)} \mathbf{F}(\mathbf{x}, t). \quad (2.41)$$

Finally, the non-equilibrium bounce back (NEBB) method proposed by Zou and He [35] to enforce Dirichlet boundary condition on velocity or pressure is here adapted to compute the unknown distribution functions $f_\alpha(\mathbf{x}, t)$ and $h_a^i(\mathbf{x}, t)$ at the boundaries preserving mass and momentum conservation (details are reported in Appendix A).

2.2.2 Wetting boundary scheme for lattice boundaries

Here, a scheme to compute wetting boundary conditions on the lattice boundaries is presented. Considering a boundary node (x_w, y, z) lying on the yz -plane (as shown in Fig. 2.1), the wetting boundary condition is derived minimizing the total free energy as reported in Eq. (2.18).

Note that the phase-field variables ϕ_{i,x_w} at the boundary node (x_w, y, z) are known, as they are computed by using the NEBB method previously mentioned. Then, using a second order finite difference scheme for the evaluation of the gradient at the boundary node (x_w, y, z)

$$\mathbf{n}_w \cdot \nabla \phi_i = \frac{1}{2\Delta x} (-3\phi_{i,x_w} + 4\phi_{i,x_w+\Delta x} - \phi_{i,x_w+2\Delta x}), \quad (2.42)$$

and a second order interpolation for ϕ_{i,x_w}

$$\phi_{i,x_w} = \frac{\phi_{i,x_w-\Delta x} - \phi_{i,x_w+\Delta x}}{2}, \quad (2.43)$$

the following equation is obtained:

$$\mathbf{n}_w \cdot \nabla \phi_i = \frac{1}{2\Delta x} \left(-\frac{3}{2}\phi_{i,x_w-\Delta x} + \frac{5}{2}\phi_{i,x_w+\Delta x} - \phi_{i,x_w+2\Delta x} \right). \quad (2.44)$$

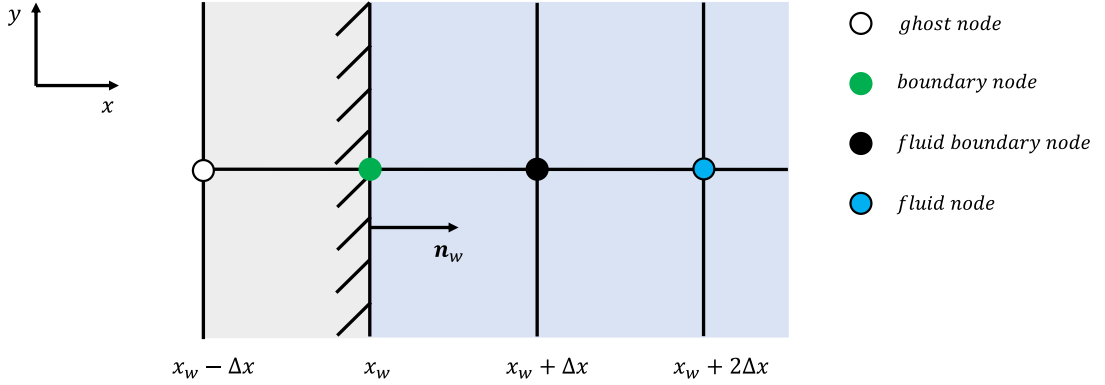


Figure 2.1: Schematic of the wetting boundary scheme for a plane surface.

Substituting Eq. (2.44) into Eq. (2.18), the wetting boundary conditions are imposed computing the phase-field variables at the ghost node $(x_w - \Delta x, y, z)$ as

$$\phi_{1,x_w-\Delta x} = \frac{5}{3}\phi_{1,x_w+\Delta x} - \frac{2}{3}\phi_{1,x_w+2\Delta x} + \frac{16\Delta x}{3\xi}(C_{13} + C_{12})_{x_w} \quad (2.45a)$$

$$\phi_{2,x_w-\Delta x} = \frac{5}{3}\phi_{2,x_w+\Delta x} - \frac{2}{3}\phi_{2,x_w+2\Delta x} + \frac{16\Delta x}{3\xi}(C_{23} - C_{12})_{x_w} \quad (2.45b)$$

$$\phi_{3,x_w-\Delta x} = \frac{5}{3}\phi_{3,x_w+\Delta x} - \frac{2}{3}\phi_{3,x_w+2\Delta x} + \frac{16\Delta x}{3\xi}(-C_{23} - C_{13})_{x_w} \quad (2.45c)$$

By analogy, the same procedure can be followed as described above to enforce such boundary conditions at all the boundaries aligned with the Cartesian directions.

2.2.3 Algorithm

The MPI parallel computing architecture is adopted to develop the multi-CPU framework. The overall data structure and communications features of the framework previously presented by De Marinis [31] have been extended here for multiphase flow applications. The implemented algorithm is described below and summarized in Fig. 2.2.

The initialization:

1. initialize velocity $\mathbf{u}$, pressure p , and phase-field variables ϕ_1, ϕ_2, ϕ_3 at all fluid (and boundary) nodes;
2. initialize the i -th phase-field distribution functions $h_a^i(\mathbf{x}, t) = h_a^{i,eq}(\mathbf{x}, t)$, by using Eq. (2.22);
3. update of the phase-field variables ϕ_1, ϕ_2, ϕ_3 by means of Eq. (2.25);
4. apply the wetting boundary conditions on ghost nodes for lattice boundaries as described in Section 2.2.2;
5. update of the density ρ by means of Eq. (2.26);
6. compute the relaxation time τ using Eq. (2.28);

7. calculate the continuous surface tension force $\mathbf{F}^s$ (Eq. (2.34)), pressure correction force term $\mathbf{F}^p$ (Eq. (2.36)), viscous stress correction force term $\mathbf{F}^\eta$ (Eq. (2.37)), and external body force $\mathbf{F}^b$;
8. initialize the hydrodynamic distribution functions $f_\alpha(\mathbf{x}, t) = f_\alpha^{eq}(\mathbf{x}, t)$ by using Eq. (2.31).

The iterative step:

9. update pressure p and velocity $\mathbf{u}$ using Eqs. (2.40) and (2.41), and phase-field variables ϕ_1 , ϕ_2 , and ϕ_3 and density ρ using Eqs. (2.25) and (2.26);
10. interpolate the relaxation time τ using Eq. (2.28);
11. calculate the continuous surface tension force $\mathbf{F}^s$ (Eq. (2.34)), pressure correction force term $\mathbf{F}^p$ (Eq. (2.36)), viscous stress correction force term $\mathbf{F}^\eta$ (Eq. (2.37)), and external body force $\mathbf{F}^b$;
12. calculate the forcing term $\mathbf{F}^\phi$ (Eq. (2.24));
13. perform collision and streaming processes of the hydrodynamic distribution functions $f_\alpha(\mathbf{x}, t)$ and the phase-field ones $h_\alpha^i(\mathbf{x}, t)$;
14. calculate the unknown hydrodynamic distribution functions $f_\alpha(\mathbf{x}, t)$ and the i -th phase-field distribution functions $h_\alpha^i(\mathbf{x}, t)$ onto the boundary nodes using the NEBB method (see Appendix A);
15. apply the wetting boundary conditions on ghost nodes for lattice boundaries as described in Section 2.2.2.

2.2.4 Numerical setup and post-processing

In all the numerical investigations reported in the following sections, the dimensionless quantities are defined as [32]

$$Cn = \frac{\xi}{L_0}, \quad Pe = \frac{U_0 L_0}{M}, \quad Ca_i = \frac{\mu_3 U_0}{\gamma_i}, \quad Re = \frac{\rho_3 U_0 (L_0/2)}{\mu_3}, \quad (2.46)$$

where L_0 is the reference length and U_0 is the reference velocity defined as $U_0 = \mu_3/(\rho_3 L_0)$ when quiescent fluids are considered, and $U_0 = \dot{\gamma} L_0/2$ when a shear rate $\dot{\gamma}$ is imposed. The Cahn number, Cn , defines the dimensionless tickness of the interface; the Péclet number, Pe , is used to compute the mobility, M ($M = M_1 = M_2 = M_3$); the Reynolds number, Re , is used to compute the dynamic viscosity μ_3 ; and the capillary number Ca_i of each i -th component is used to compute the i -th auxiliary coefficient γ_i . In all simulations, the effects of gravity fields are neglected, and the velocity and pressure fields are initialized to zero.

Two-dimensional cases are simulated with the proposed three-dimensional framework by defining only three nodes in the third (z -) direction and applying periodicity along it.

The geometric quantities characterizing the problem of droplets in contact with surfaces have been evaluated by means of a geometric approach, considering the threshold value $\phi_i = 0.5$ ($i = \{1, 2\}$). Finally, the physical quantities in the Lattice-Boltzmann simulations are presented in simulation units, defined hereafter as lu.

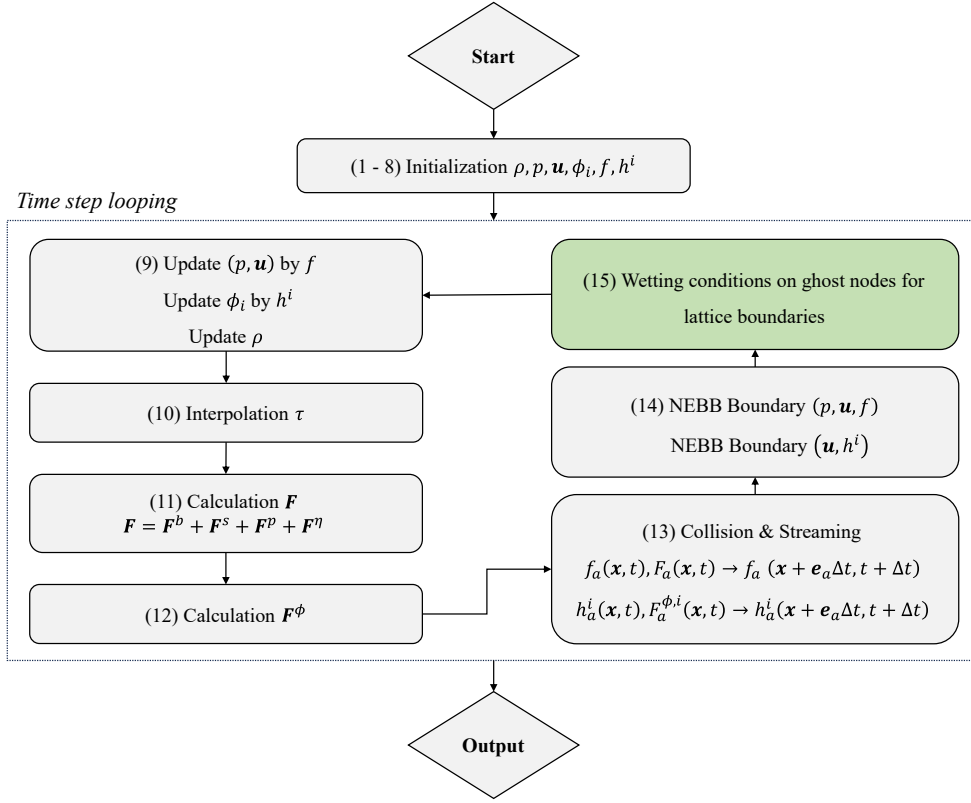


Figure 2.2: Schematic representation of the implemented algorithm.

2.3 Validation of the Lattice Boltzmann method for ternary fluid systems

In this section, several well-documented benchmark problems in the literature is investigated to validate the accuracy of the proposed D3Q19 lattice Boltzmann method without considering any wetting boundary conditions. The validation involves ternary fluid systems with low to high density- and viscosity- ratios.

2.3.1 Droplets in a quiescent fluid

To evaluate the accuracy of the model in tracking interfaces, the well-known case of two circular droplets with the same radius R placed in a quiescent fluid is investigated [32, 91, 96]. The investigation to both low- and high- density and viscosity ratios is extended. In the first case, the ternary fluid system presents a density ratio $\rho_1 : \rho_2 : \rho_3 = 10 : 5 : 1$ and a viscosity ratio $\mu_1 : \mu_2 : \mu_3 = 10 : 5 : 1$, while in the second case $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 1000 : 500 : 1$. The dimensionless governing parameters are set to $Ca_1 = Ca_2 = Ca_3 = 0.278$, $Pe = 8.33$, and $Cn = 4/100$.

A computational domain of size $2L_0 \times L_0$ ($L_0 = 100$ lu) is considered. The radius R is initially set to $L_0/4$. Periodic boundary conditions are enforced in all directions. In the numerical simulations, the relaxation time τ_3 is set to 0.5 lu, and the quiescent fluid presents a density $\rho_3 = 0.1$ lu and $\rho_3 = 0.01$ lu for Case 1 (low density ratio) and Case

2 (high density ratio), respectively. The phase-field variables are initialized as

$$\phi_1(x, y) = 0.5 + 0.5 \tanh \left[\frac{2}{\xi} (R - |\mathbf{x} - \mathbf{x}_{C_1}|) \right] \quad (2.47a)$$

$$\phi_2(x, y) = 0.5 + 0.5 \tanh \left[\frac{2}{\xi} (R - |\mathbf{x} - \mathbf{x}_{C_2}|) \right] \quad (2.47b)$$

$$\phi_3(x, y) = 1 - \phi_1(x, y) - \phi_2(x, y) \quad (2.47c)$$

where $\mathbf{x}_{C_1} = (L_0/2, L_0/2)$ and $\mathbf{x}_{C_2} = (3L_0/2, L_0/2)$ are the centroids of the two droplets.

Fig. 2.3 shows the steady configurations of the density fields reached for Case 1 and Case 2. The absence of displacement between the initially prescribed and final configurations highlights the ability of the method in ensuring mass conservation, independently of the density ratio.

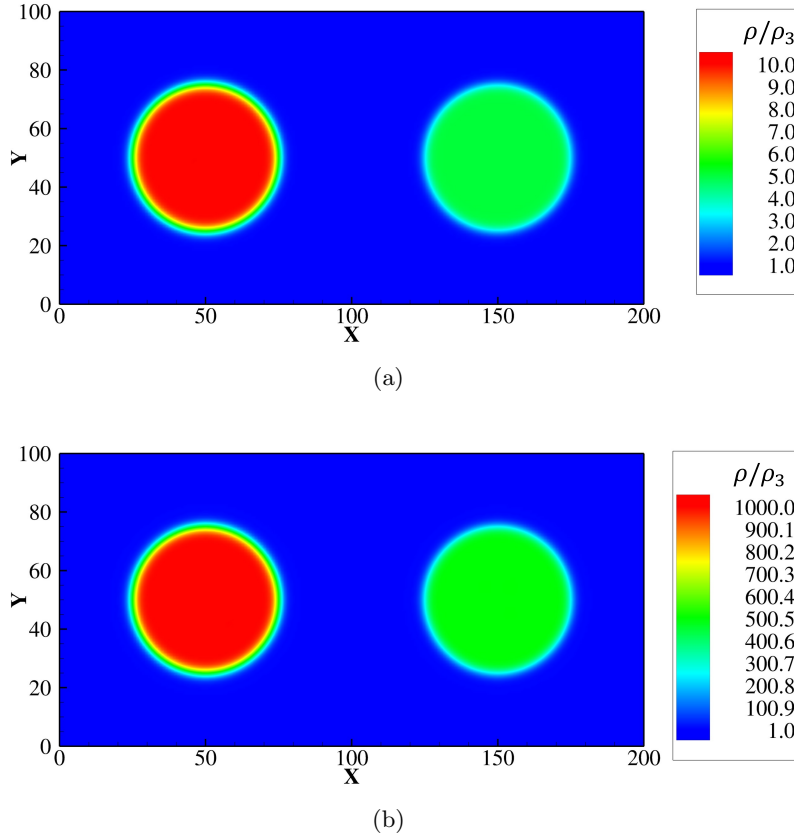
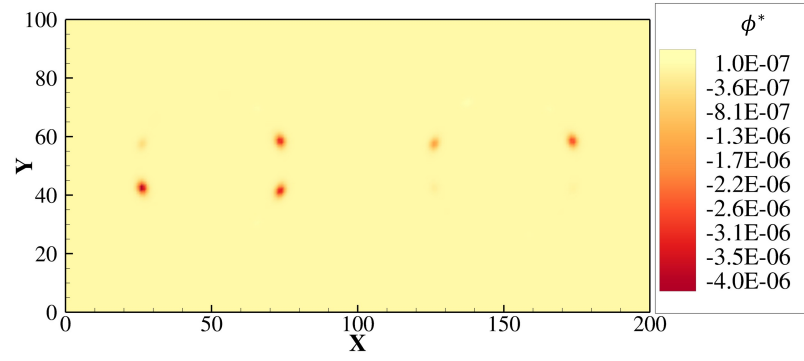


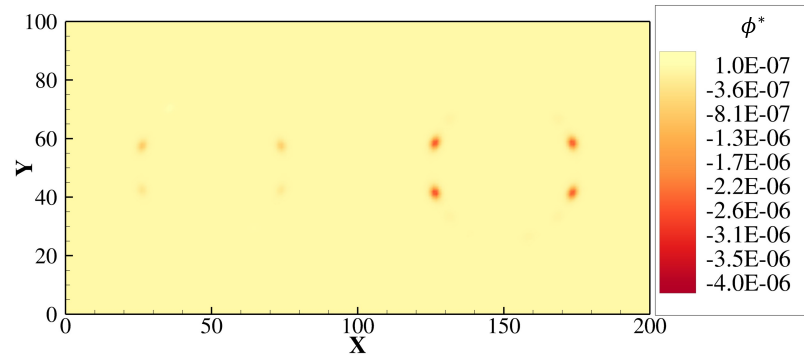
Figure 2.3: Equilibrium configurations of the density fields of two droplets in a quiescent fluid (blue) for (a) low density ratio (red, $\rho_1/\rho_3 = 10$; green, $\rho_2/\rho_3 = 5$), and (b) high density ratio (red, $\rho_1/\rho_3 = 1000$; green, $\rho_2/\rho_3 = 500$).

As in the reference work [32], the index $\phi^* = \phi_1\phi_2\phi_3$ is computed to measure the presence of nonphysical triple-point junctions. The distribution of ϕ^* is reported in Fig. 2.4. It is worth mentioning that the present framework returns values several order of magnitude (10^{-6}) lower than those (10^{-3}) reported in literature by [33, 34].

Fig. 2.5 shows the numerical results of the phase field of fluid 3 compared to the analytical solution given by Eqs. (2.47). The maximum absolute error has an order of



(a)



(b)

Figure 2.4: Distribution of the index ϕ^* obtained for low- (a) and high- (b) density ratios.

magnitude 10^{-2} at the interfaces between fluids 1 and 3, and 2 and 3, i.e. in the diffusion regions. The results demonstrate the accuracy of the model in capturing the interface.

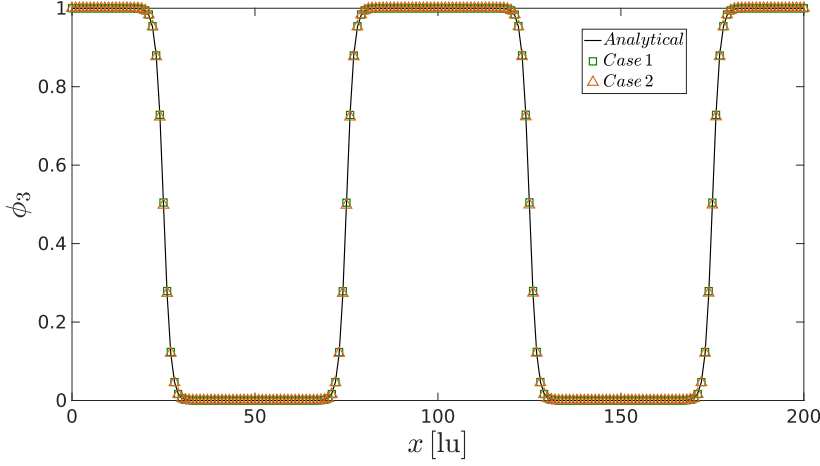


Figure 2.5: The phase field of the fluid 3 at $y = L_0/2$ computed at steady state for Case 1 ($\rho_1 : \rho_2 : \rho_3 = 10 : 5 : 1$) and Case 2 ($\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$) compared to the analytical solution.

2.3.2 Spinodal decomposition of a ternary fluid system

In a binary or ternary fluid system, when the temperature is below a particular critical value, a minor fluctuation in concentration within a homogeneous mixture raises the Gibbs energy, but it is still insufficient for complete phase separation. In this metastable state, the phases create a distinct interconnected region separated by interfaces. This continuous phase transformation is called spinodal decomposition, and is widely studied in literature [32, 37, 96]. The dimensionless quantities governing the ternary fluid system investigated here are $Ca_1 = Ca_2 = Ca_3 = 0.00139$, $Pe = 2.38$ and $Cn = 5/200$. The density and viscosity ratio are respectively $\rho_1 : \rho_2 : \rho_3 = 1000 : 100 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 100 : 100 : 1$.

A domain of size $L_0 \times L_0$ ($L_0 = 200$ lu) is defined and periodic boundary conditions are enforced in all Cartesian directions. The three phase-field variables are randomly initialized as

$$\phi_1(x, y) = \overline{\phi_1} + 0.01 \times rand(x, y) \quad (2.48a)$$

$$\phi_2(x, y) = \overline{\phi_2} + 0.01 \times rand(x, y) \quad (2.48b)$$

$$\phi_3(x, y) = 1 - \phi_1(x, y) - \phi_2(x, y) \quad (2.48c)$$

where $rand(x, y)$ is a random number between -1 and 1 with zero mean value, applied to mean phase variables $(\overline{\phi_1}, \overline{\phi_2})$. The density ρ_3 is set to 0.001 lu and the relaxation time τ_3 is set to 0.5 lu.

Three configurations with different initial concentrations are studied:

- case I: $(\overline{\phi_1}, \overline{\phi_2}) = (2/5, 2/5)$,
- case II: $(\overline{\phi_1}, \overline{\phi_2}) = (1/3, 1/3)$,

- case III: $(\overline{\phi_1}, \overline{\phi_2}) = (1/4, 1/4)$.

Fig. 2.6 shows the time evolution of phase separation of the ternary fluid system in the three cases. In all configurations, small droplets of the same component merge to form tiny domains that are not fully stable. The initial concentrations of each component determine the final minimum free energy distribution. When the initial concentrations of phases 1 and 2 are higher than that of phase 3, small droplets of phase 3 are surrounded by larger droplets of the other phases (Fig. 2.6(a)). In contrast, when all phases have the same initial concentration, the final distribution of each component is similar (Fig. 2.6(b)). In the third case, as the initial concentration of phase 3 is higher, the droplets of phases 1 and 2 will be smaller in the final distribution (Fig. 2.6(c)). All the results of the simulated cases are consistent with those available in the literature [32, 37, 96].

2.3.3 Spreading of a liquid lens

The classical problem of the spreading of a liquid lens with density ρ_1 between two immiscible fluids is considered here to investigate the accuracy of the model in the presence of a triple-point junction. The density and viscosity ratios are $\rho_1 : \rho_2 : \rho_3 = 1000 : 100 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 100 : 10 : 1$, respectively. The dimensionless quantities are $Pe = 4.17$ and $Cn = 5/250$. The initial circular shape of the droplet is deformed by the surface tension until the equilibrium state [32, 37, 96].

A circular droplet with radius $R = 3L_0/25$ is placed in a domain of size $L_0 \times 3L_0/5$ ($L_0 = 250$ lu). The density ρ_3 is set to 0.001 lu and the relaxation time τ_3 is set to 0.5 lu. No-slip boundary conditions are enforced at the walls placed at $y = 0$ and $y = 3L_0/5$ by means of the NEBB method, while periodic boundary conditions are enforced in x direction. The three phase-field variables are initialized as

$$\phi_1(x, y) = 0.5 + 0.5 \tanh \left[\frac{2}{\xi} (R - |\mathbf{x} - \mathbf{x}_c|) \right] \quad (2.49a)$$

$$\phi_2(x, y) = \max \left[0.5 + 0.5 \tanh \left(\frac{2}{\xi} (y - y_c) \right) - \phi_1(x, y), 0 \right] \quad (2.49b)$$

$$\phi_3(x, y) = 1 - \phi_1(x, y) - \phi_2(x, y) \quad (2.49c)$$

where $\mathbf{x}_c = (x_c, y_c) = (L_0/2, 3L_0/10)$ is the center of the lens.

Three cases have been investigated, and the related values of Capillary numbers and surface tensions are reported in Table 2.1.

Case	I	II	III
Ca_1	1.11×10^{-3}	6.17×10^{-4}	9.26×10^{-4}
Ca_2	1.11×10^{-3}	5.56×10^{-3}	2.77×10^{-3}
Ca_3	1.11×10^{-3}	5.56×10^{-3}	6.94×10^{-4}
σ_{12} [lu]	1.0×10^{-4}	1.0×10^{-4}	0.8×10^{-4}
σ_{13} [lu]	1.0×10^{-4}	1.0×10^{-4}	1.4×10^{-4}
σ_{23} [lu]	1.0×10^{-4}	0.2×10^{-4}	1.0×10^{-4}

Table 2.1: Spreading of a liquid lens. Capillary numbers, Ca_i , and surface tensions, σ_{ij} , for the three investigated cases.

The schematic representation of a liquid lens after its spreading is shown in Fig. 2.7. The theoretical equilibrium contact angles θ_1^{th} and θ_2^{th} can be analytically computed by

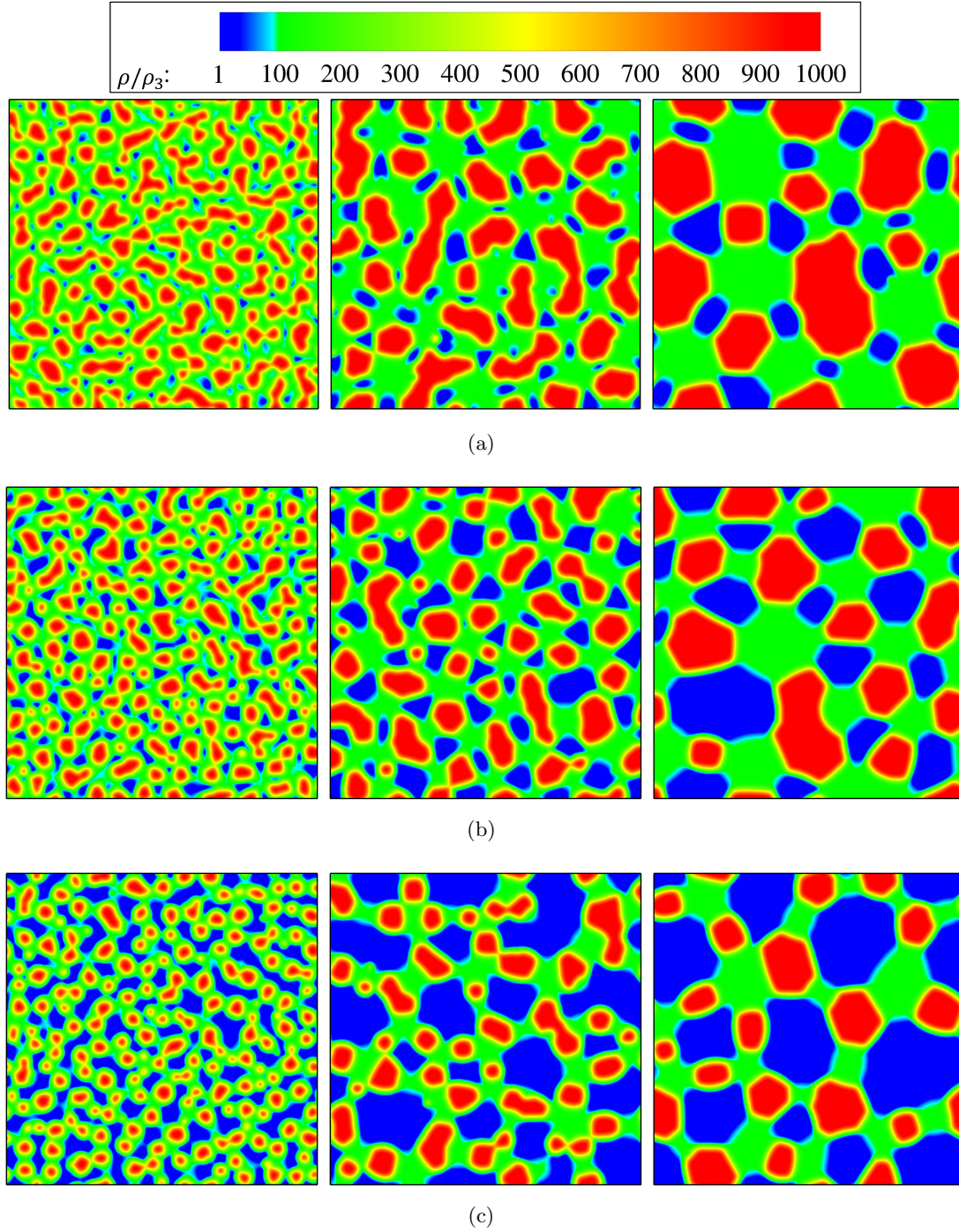


Figure 2.6: Time evolution of phase separation for initial concentrations equal to (a) $(\overline{\phi_1}, \overline{\phi_2}) = (2/5, 2/5)$; (b) $(\overline{\phi_1}, \overline{\phi_2}) = (1/3, 1/3)$; (c) $(\overline{\phi_1}, \overline{\phi_2}) = (1/4, 1/4)$. The time steps considered from left to right are $t = 10 \times 10^3, 100 \times 10^3, 1000 \times 10^3$ lu.

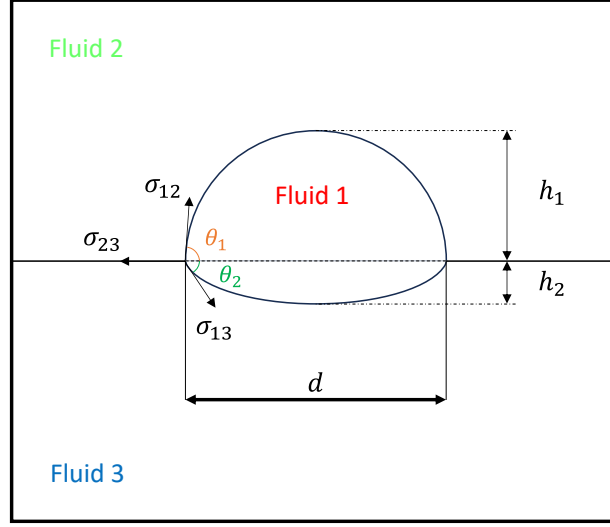


Figure 2.7: Schematic representation of a liquid lens after its spreading between two immiscible fluids.

the Neumann's law

$$\theta_1^{\text{th}} = \arccos \left(\frac{\sigma_{23}^2 + \sigma_{12}^2 - \sigma_{13}^2}{2\sigma_{23}\sigma_{12}} \right), \quad (2.50a)$$

$$\theta_2^{\text{th}} = \arccos \left(\frac{\sigma_{23}^2 + \sigma_{13}^2 - \sigma_{12}^2}{2\sigma_{23}\sigma_{13}} \right). \quad (2.50b)$$

The distance d^{th} between the two triple-point junctions is geometrically evaluated as

$$d^{\text{th}} = 2\sqrt{A} \left[\sum_{j=1}^2 \frac{\theta_j^{\text{th}} - \cos \theta_j^{\text{th}} \sin \theta_j^{\text{th}}}{\sin^2 \theta_j^{\text{th}}} \right]^{-\frac{1}{2}}, \quad (2.51)$$

with $A = \pi R^2$ being the area of the 2D lens. The heights h_j^{th} ($j = \{1, 2\}$) of the lens are given by

$$h_j^{\text{th}} = \frac{d^{\text{th}}}{2} \left[\frac{1 - \cos \theta_j^{\text{th}}}{\sin \theta_j^{\text{th}}} \right]. \quad (2.52)$$

The numerical contact angles θ_j^{num} are computed using the following geometrical relation

$$\theta_j^{\text{num}} = 2 \arctan \left(\frac{2h_j^{\text{num}}}{d^{\text{num}}} \right), \quad (2.53)$$

where the quantities d^{num} e h_j^{num} are extracted from the numerical results.

The contours of the lens at the equilibrium configurations are reported in Fig. 2.8 for the three different cases. In case I, the lens assumes an oval shape due to the equal value of all surface tensions involved. As the surface tensions between fluid 2 and 3 decrease (Case II), the lens shape becomes more spherical. Finally in case III the symmetry of the problem is disrupted due to the different values of σ_{12} and σ_{13} , resulting in the lens being forced up toward the fluid with which it exhibits a lower surface tension.

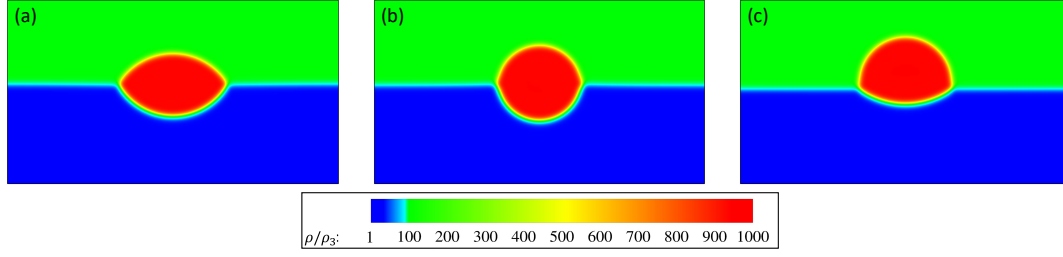


Figure 2.8: Equilibrium configurations of density fields of the liquid lens with different surface tensions: (a) case I, (b) case II, and (c) case III.

The length d^{num} and the contact angles θ_i^{num} are evaluated at the equilibrium configurations. Figs. 2.9(a) and 2.9(b) report the comparison with analytical solutions, showing a very good agreement. All results have an error less than 5% as in [32], and the case III has maximum error of 4.299% for θ_2 and 3.6329% for d .

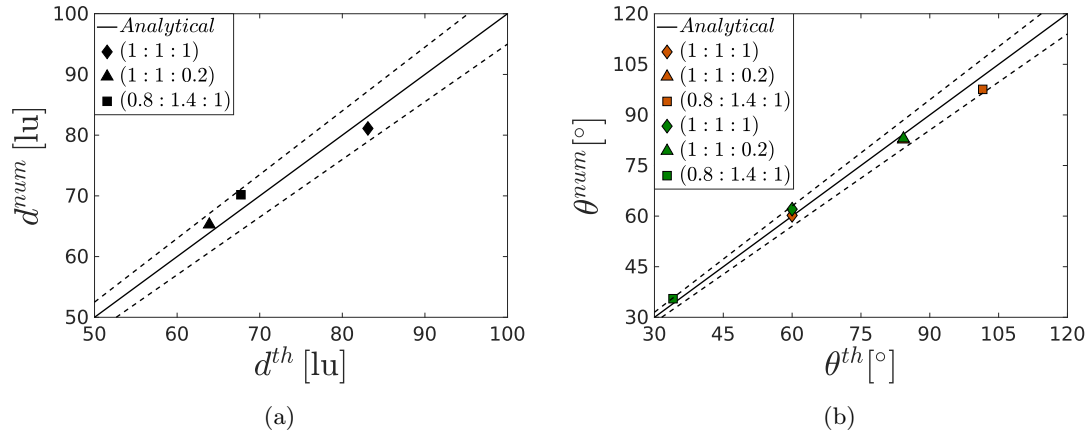


Figure 2.9: Spreading of liquid lens with different surface tensions. Comparison between the analytical and numerical results of length d (a) and contact angles θ_i (b). θ_1 is reported in orange while θ_2 in green. The dashed lines represent the relative error of $\pm 5\%$.

Additionally, the numerical parasitic currents of the model are evaluated, defined as $|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$, and reported in Fig. 2.10. The maximum values of nonphysical artifacts correspond to the triple-point junctions and is about $\mathcal{O}(10^{-4}\text{lu})$, while in the other regions the model returns undesired currents less than $\mathcal{O}(10^{-5}\text{lu})$.

2.4 Validation of the wetting scheme for lattice boundaries

In this section, three classical tests are reported to verify the accuracy of the wettability scheme and NEBB boundary conditions presented in Section 2.2.2. The numerical results with those available in the literature [36,37] are compared. The tests aim to observe the diffusion of droplets in contact with flat surfaces in static cases.

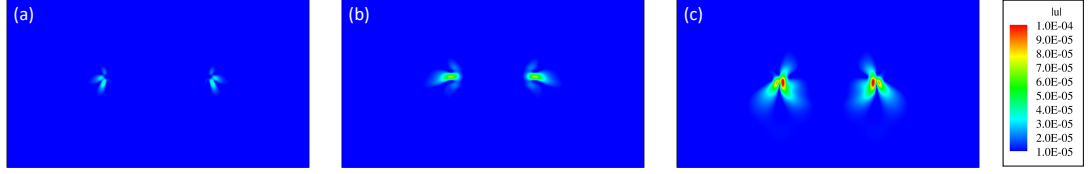


Figure 2.10: Spreading of liquid lens. Magnitude of the numerical parasitic currents $|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$ in Case I (a), Case II (b), and Case III (c).

2.4.1 Droplets on a wettable substrate

The spreading of two semicircular droplets is investigated as in [36, 37]. The density and viscosity ratios are $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 1000 : 500 : 1$, respectively. The dimensionless quantities are $Ca_1 = Ca_2 = Ca_3 = 1 \times 10^{-7}$, $Pe = 0.7143$, and $Cn = 4/100$.

In a domain of size $L_0 \times 3L_0$ ($L_0 = 100$ lu), two semicircular droplets with radius $R = 3L_0/10$ are placed on a flat wall located at $x = 0$. The density ρ_3 of the quiescent fluid is set to 0.01 lu and the relaxation time τ_3 is set to 0.3 lu. The three phase-field variables are initialized as Eq. (2.47), where $\mathbf{x}_{C_1} = (0, L_0/2)$ and $\mathbf{x}_{C_2} = (0, 3L_0/2)$ are the centers of the two droplets.

No-slip boundary conditions are enforced at the walls placed at $x = 0$ and $x = L_0$ using the NEBB method, while the periodic boundary condition is considered in x direction. The following cases with different contact angles are investigated:

- case I: $(\theta_{13}, \theta_{23}) = (45^\circ, 60^\circ)$,
- case II: $(\theta_{13}, \theta_{23}) = (90^\circ, 90^\circ)$,
- case III: $(\theta_{13}, \theta_{23}) = (135^\circ, 120^\circ)$,
- case IV: $(\theta_{13}, \theta_{23}) = (150^\circ, 160^\circ)$,

and θ_{12} is retrieved by Eq. (2.17). At the equilibrium, the analytical spreading length d_i^{th} and height h_i^{th} of the two droplets ($i = 1, 2$) are estimated by means of de Gennes theory [97] as

$$d_i^{\text{th}} = 2R \sqrt{\frac{\pi/2}{\theta_{i3}^{\text{th}} - \sin \theta_{i3}^{\text{th}} \cos \theta_{i3}^{\text{th}}}} \sin \theta_{i3}^{\text{th}} \quad (2.54a)$$

$$h_i^{\text{th}} = R \sqrt{\frac{\pi/2}{\theta_{i3}^{\text{th}} - \sin \theta_{i3}^{\text{th}} \cos \theta_{i3}^{\text{th}}}} (1 - \cos \theta_{i3}^{\text{th}}) \quad (2.54b)$$

where θ_{i3}^{th} is the theoretical contact angle. The numerical contact angles θ_{i3}^{num} are computed as

$$\theta_{i3}^{\text{num}} = 2 \arctan \left(\frac{2h_i^{\text{num}}}{d_i^{\text{num}}} \right), \quad (2.55)$$

where the quantities d_i^{num} e h_i^{num} are extracted from the numerical results.

The initial distance between two droplets is chosen large enough to minimize the interactions in the case of both hydrophilic and hydrophobic drops. As shown in Fig.

2.11, the droplets wetting the surface exhibit the classical hydrophilic behavior for contact angles lower than 90° , whereas the hydrophilic behavior is obtained for contact angles greater than 90° . Finally, in the case of neutral wettability, i.e. contact angle equal to 90° , initial configuration is maintained, independently of the density ratio with respect to the quiescent fluid.

The geometrical quantities θ_{i3}^{num} , d_i^{num} , and h_i^{num} are evaluated at the equilibrium configurations. The numerical results are compared with analytical solutions (Eq. (2.54)) in Figs. 2.13(a) and 2.13(b), showing a very good agreement. The maximum relative error (1.6381%) is observed for the droplet with contact angle $\theta_{13} = 90^\circ$, while the minimum one (0.3753%) with $\theta_{13} = 45^\circ$. In general, hydrophobic cases exhibit a smaller error than hydrophilic ones, and all numerical results have a relative error lower than 2%.

To demonstrate the accuracy of the CFP model and the NEBB boundary method in preserving mass, the temporal evolution of the mass of each i -th fluid is calculated as $M^i(t) = \int_{\Omega} \rho_i \phi_i(\mathbf{x}, t) d\Omega$, being Ω the computational domain. As shown in Fig. 2.12(d), Case 3 ($(\theta_{13}, \theta_{23}) = (135^\circ, 120^\circ)$) exhibits the highest error (0.7%), while Case 2 ($(\theta_{13}, \theta_{23}) = (90^\circ, 90^\circ)$) shows the lowest error (0.0077%). Furthermore, the mass of each fluid, as well as the total mass, presents an error below 1% (Figs. 2.12(a),(b) and (c)).

2.4.2 A compound binary droplet on a wettable substrate

The wetting boundary scheme for lattice boundaries is also validated considering the interactions between two droplets as done in [36, 37, 98]. The density- and viscosity-ratios are $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 1000 : 500 : 1$, respectively. The dimensionless quantities are $Ca_1 = Ca_2 = Ca_3 = 8.33 \times 10^{-6}$, $Pe = 0.7143$, and $Cn = 4/120$.

A semicircular compound binary droplet with radius $R = L_0/2$ is placed on a wall located at $x = 0$ of a domain of size $L_0 \times 5L_0/2$ ($L_0 = 120$ lu). The density ρ_3 of the quiescent fluid is set to 0.01 lu and the relaxation time τ_3 is set to 0.3 lu. The three phase-field variables are initialized as

$$\phi_1(x, y) = 0.5 + 0.5 \tanh \left[\frac{2}{\xi} (R - |\mathbf{x} - \mathbf{x}_C|) \right] \quad \left(y < \frac{L_0}{2} \right) \quad (2.56a)$$

$$\phi_2(x, y) = 0.5 + 0.5 \tanh \left[\frac{2}{\xi} (R - |\mathbf{x} - \mathbf{x}_C|) \right] \quad \left(y \geq \frac{L_0}{2} \right) \quad (2.56b)$$

$$\phi_3(x, y) = 1 - \phi_1(x, y) - \phi_2(x, y) \quad (2.56c)$$

where $\mathbf{x}_C = (0, L_0/2)$ is the center of the droplet.

The following configurations with different contact angles are investigated:

- case I: $(\theta_{13}, \theta_{23}) = (60^\circ, 90^\circ)$,
- case II: $(\theta_{13}, \theta_{23}) = (90^\circ, 90^\circ)$,
- case III: $(\theta_{13}, \theta_{23}) = (120^\circ, 90^\circ)$,
- case IV: $(\theta_{13}, \theta_{23}) = (150^\circ, 90^\circ)$,
- case V: $(\theta_{13}, \theta_{23}) = (150^\circ, 120^\circ)$.

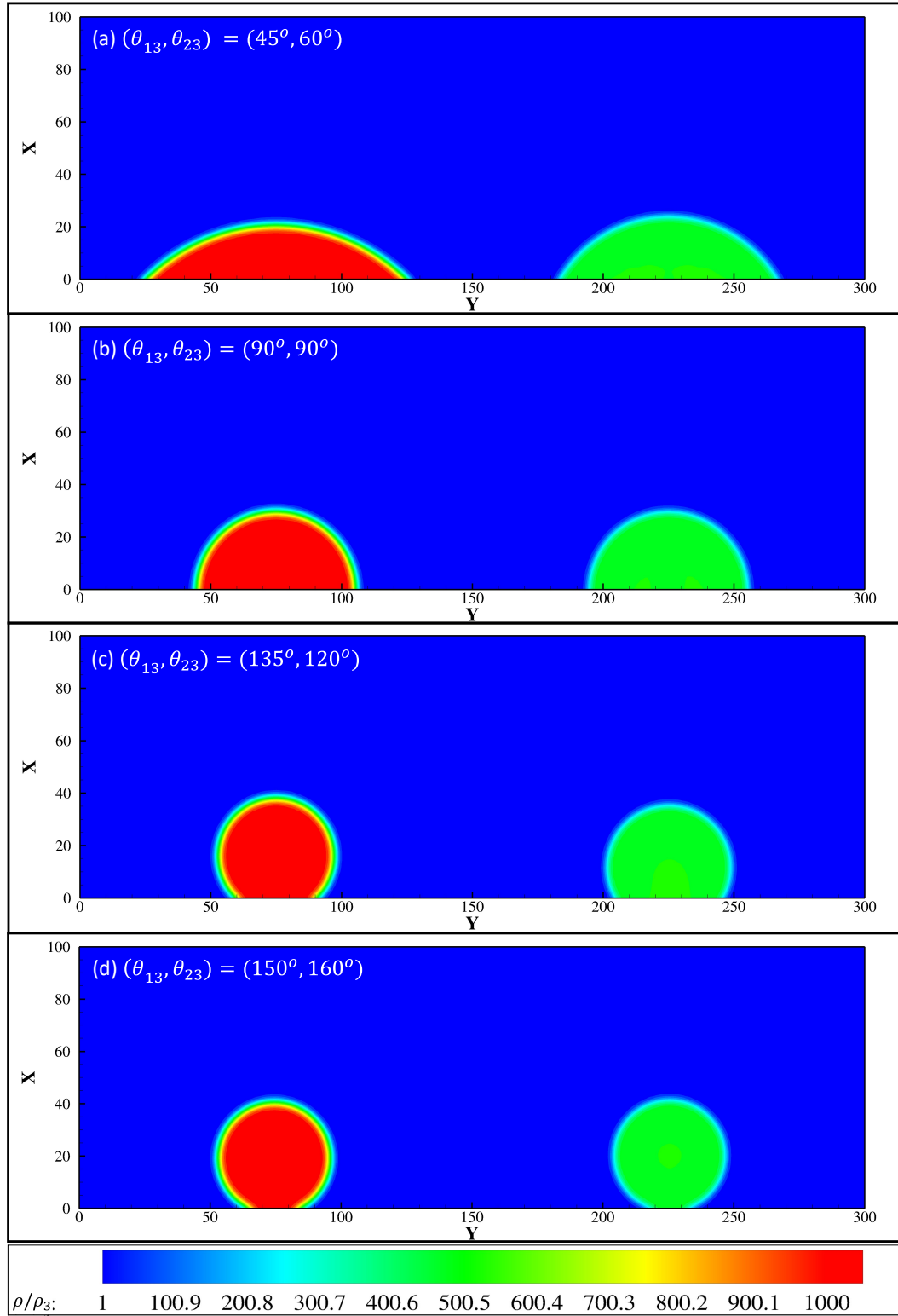


Figure 2.11: Density distribution of the equilibrium configuration of binary droplets (red, $\rho_1/\rho_3 = 1000$; green, $\rho_2/\rho_3 = 500$) in a quiescent fluid (blue) with different contact angles.

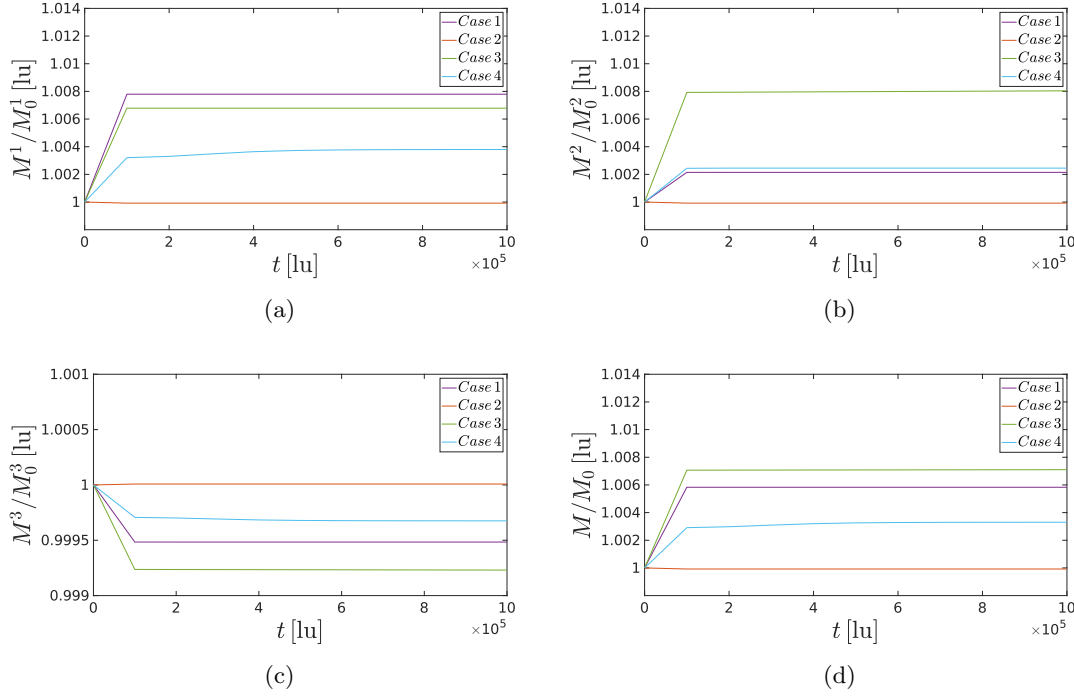


Figure 2.12: Temporal evolution of normalized mass of: (a) fluid 1, (b) fluid 2, (c) fluid 3 and (d) ternary fluid system of Case 1 ($(\theta_{13}, \theta_{23}) = (45^\circ, 60^\circ)$), Case 2 ($(\theta_{13}, \theta_{23}) = (90^\circ, 90^\circ)$), Case 3 ($(\theta_{13}, \theta_{23}) = (135^\circ, 120^\circ)$) and Case 4 ($(\theta_{13}, \theta_{23}) = (150^\circ, 160^\circ)$). M_0 is the initial mass at $t = 0$.

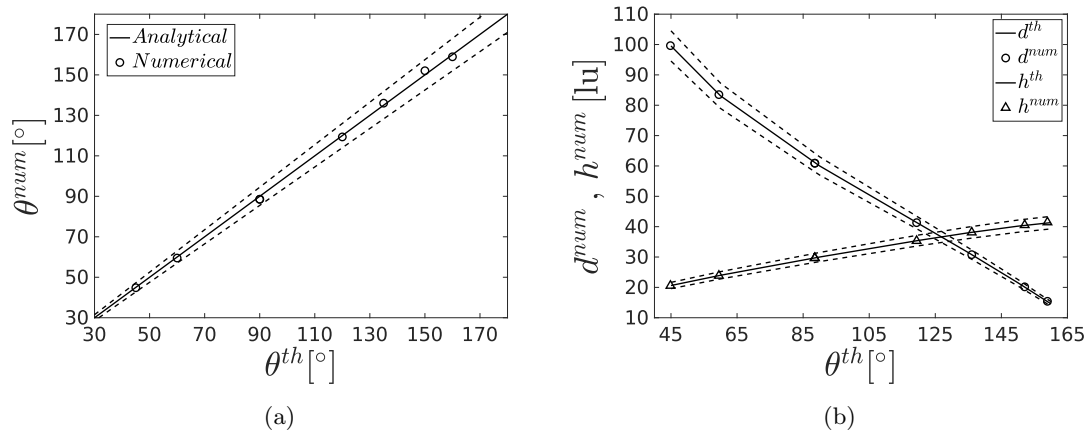


Figure 2.13: Comparison between the analytical and numerical results of: (a) the contact angles, θ_{i3} ; (b) the lengths, d_i , and the heights, h_i , as a function of theoretical contact angles, θ_{i3}^{th} . The dashed lines represent the relative error of $\pm 5\%$.

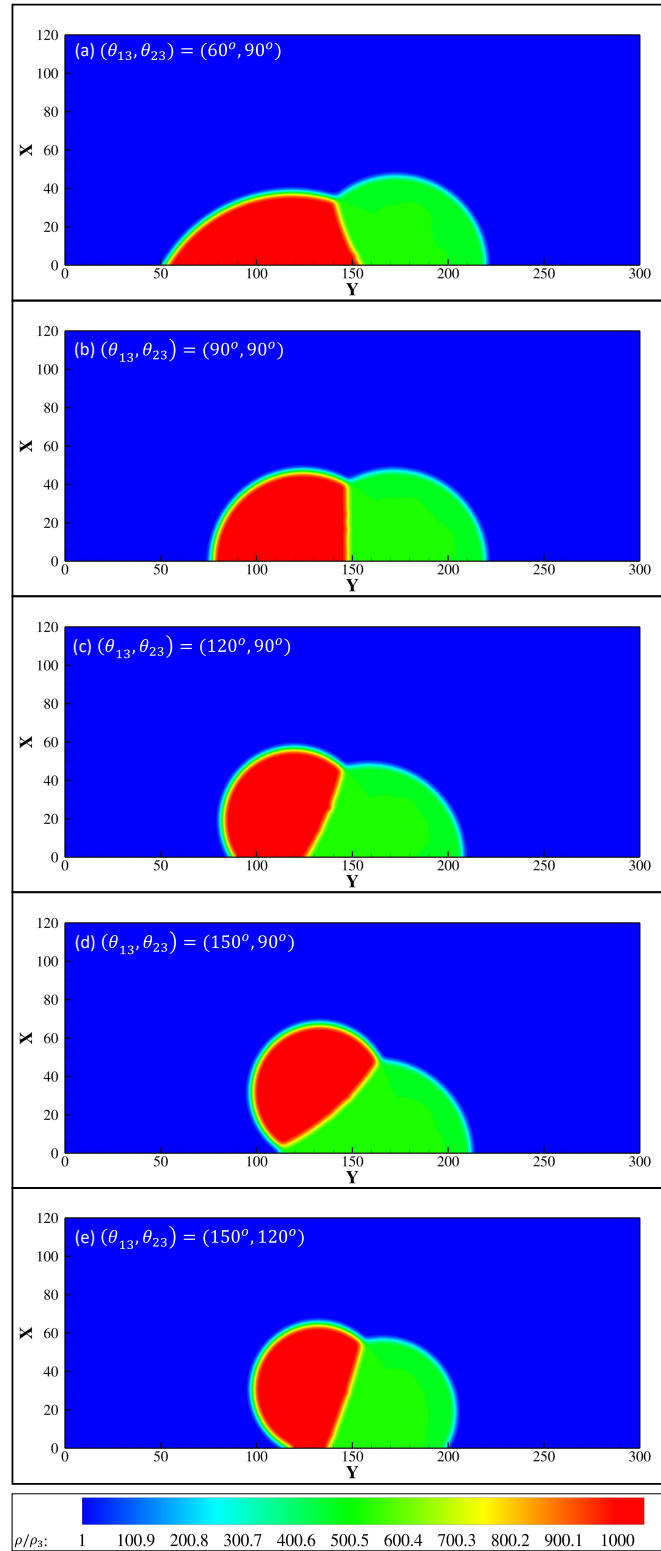


Figure 2.14: Density distribution of the equilibrium configuration of compound binary droplet (red, $\rho_1/\rho_3 = 1000$; green, $\rho_2/\rho_3 = 500$) in a quiescent fluid (blue) with different contact angles.

The joint interaction between the three fluids and the solid substrate results in different equilibrium configurations, as shown in Fig. 2.14. Figs. 2.14(a) and (c) show the spreading of the droplet 1, which is consistent with the hydrophilic and hydrophobic behavior, respectively. As shown in Fig. 2.14(b), when both the droplets are in the case of neutral wettability, their spreading lengths are the same. Increasing only the contact angle θ_{13} , the wall surface wetted by fluid 1 decreases until it does not touch the latter at all, as shown in Fig. 2.14(d). In contrast, when both droplets are in hydrophobic conditions, both remain in contact with the solid wall due to their counteracting hydrophobicity, see Fig. 2.14(e). In all cases, the spreading of the compound binary droplet is lower than in the case presented in the previous section as a result of the mutual interaction of the fluids 1 and 2.

To quantitatively compare the results with analytical solution [36,37,98], the spreading lengths of case I, II and III are computed. Figs. 2.15(a) and 2.15(b) show the comparison with the analytical solutions. The minimum relative error (0.6588%) is obtained for $\theta_{13} = 120^\circ$, while the maximum one (1.251%) for $\theta_{13} = 60^\circ$. In the case of neutral wettability, the maximum error 0.5159% is recorded in Case II for the fluid 2. All numerical results have a relative error lower than 2%.

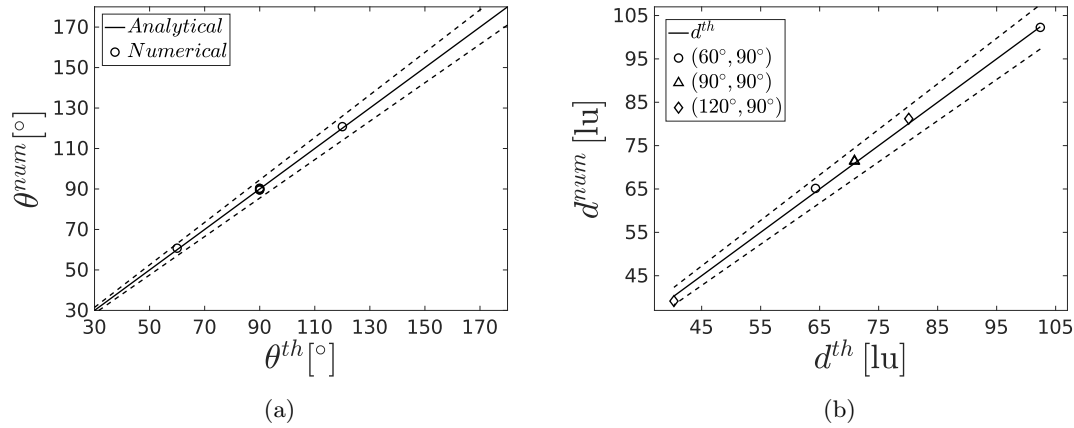


Figure 2.15: Compound binary droplet with different contact angles. Comparison between the analytical and numerical results of: (a) the contact angles, θ_{i3} ; (b) the lengths, d_i . The dashed lines represent the relative error of $\pm 5\%$.

2.5 Multicomponent droplets under shear flows

Following the extensive validation of the framework in static configurations, the dynamic results for multiphase droplets under shear flows is presented. First, a concentric binary droplet on a wettable substrate is examined, reproducing behaviors reported in the literature for low density and viscosity ratios and providing new insights for high ratios, while demonstrating that the framework preserves mass and maintains stability under challenging conditions. Then, the three-dimensional deformation of a three-layer leukocyte is studied, demonstrating the accuracy of the model in capturing complex dynamic interactions and advancing the understanding of multicomponent droplet deformation in biologically relevant scenarios.

2.5.1 A concentric binary droplet on a wettable substrate in a shear flow

To investigate the accuracy of the wetting scheme in a dynamic case, a shear flow is imposed to a concentric binary droplet as in [37]. The dimensionless quantities are $Ca_1 = Ca_2 = Ca_3 = 0.5$, $Re = 37.5$, $Pe = 75$ and $Cn = 4/150$ for both cases presented hereafter.

A computational domain of size $L_0 \times 8L_0/3$ ($L_0 = 150$ lu) with two walls located at $x = 0$ and $x = L_0$ is considered. Two concentric droplets with the radii $R_1 = 25.5$ lu and $R_2 = 60$ lu are placed on the wall at $x = 0$. The phase-field variables are initialized as

$$\phi_1(x, y) = 0.5 + 0.5 \tanh \left[\frac{2}{\xi} (R_1 - |\mathbf{x} - \mathbf{x}_{C_1}|) \right] \quad (2.57a)$$

$$\phi_2(x, y) = 0.5 + 0.5 \tanh \left[\frac{2}{\xi} (R_2 - |\mathbf{x} - \mathbf{x}_{C_2}|) \right] - \phi_1(x, y) \quad (2.57b)$$

$$\phi_3(x, y) = 1 - \phi_1(x, y) - \phi_2(x, y) \quad (2.57c)$$

where $\mathbf{x}_{C_1} = (37.5, 100)$ and $\mathbf{x}_{C_2} = (30, 100)$ are the centroids of the two droplets. The contact angles $\theta_{13} = 68.53^\circ$ and $\theta_{23} = 120^\circ$ are considered and a constant velocity $U = 0.1$ lu is applied to the upper wall along the x -direction.

First, the density- and viscosity- ratios are set to 1. The density ρ_3 is set to 1 lu and the relaxation time τ_3 to 0.3 lu.

Fig. 2.16 shows the temporal evolution of the system, matching the behavior obtained in the previous study by [37]. As the three fluids have the same density, a rescaled density $\rho^* = 10\phi_1 + 5\phi_2 + \phi_3$ is shown for visualization purposes. The initial configuration is shown in Fig. 2.16(a). In the beginning, fluid 2 deforms apically (Fig. 2.16(b)). As soon as the deformation involves the core region of fluid 2, fluid 1 also begins to deform resulting in the rupture of the interface between fluid 1 and 2 (2.16(c)). The low density ratio leads to large deformations of both droplets (2.16(d) - (e)). After 45000 time steps, the droplets break into smaller ones, their interfaces spinning around their axes as the droplets flow through fluid 3, assuming the usual tank-treading motion of red blood cells, as shown in Fig. 2.16(f).

Besides, the behavior of the concentric droplet under shear flow changes when high density- and viscosity- ratios are considered. The case of density and viscosity ratios equal to $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 1000 : 500 : 1$, respectively, has also been investigated. The density ρ_3 is set to 0.01 lu and the relaxation time τ_3 to 0.3 lu.

The imposed shear rate cannot generate large deformations of the concentric droplet, as shown in Fig. 2.17. The droplet adheres to the wall for more than 100000 time steps, maintaining the initially imposed shape almost unchanged, as shown in the previous study by Zhang et al. [37].

To demonstrate the properties of mass conservation even in dynamic cases, the time evolution of the mass normalized with respect to its initial value is evaluated and shown in Fig. 2.18 for each fluid and the overall ternary system. In the case of high density- and viscosity- ratios (Case 2), the mass of each fluid presents a relative error lower than 1% as the shear velocity is not high enough to break the concentric drop. In contrast, when the density and viscosity ratios are low (Case 1), fluid 1 and fluid 2 have a maximum mass change of about 2.8% and 1.5%, respectively, due to their separation and subsequent

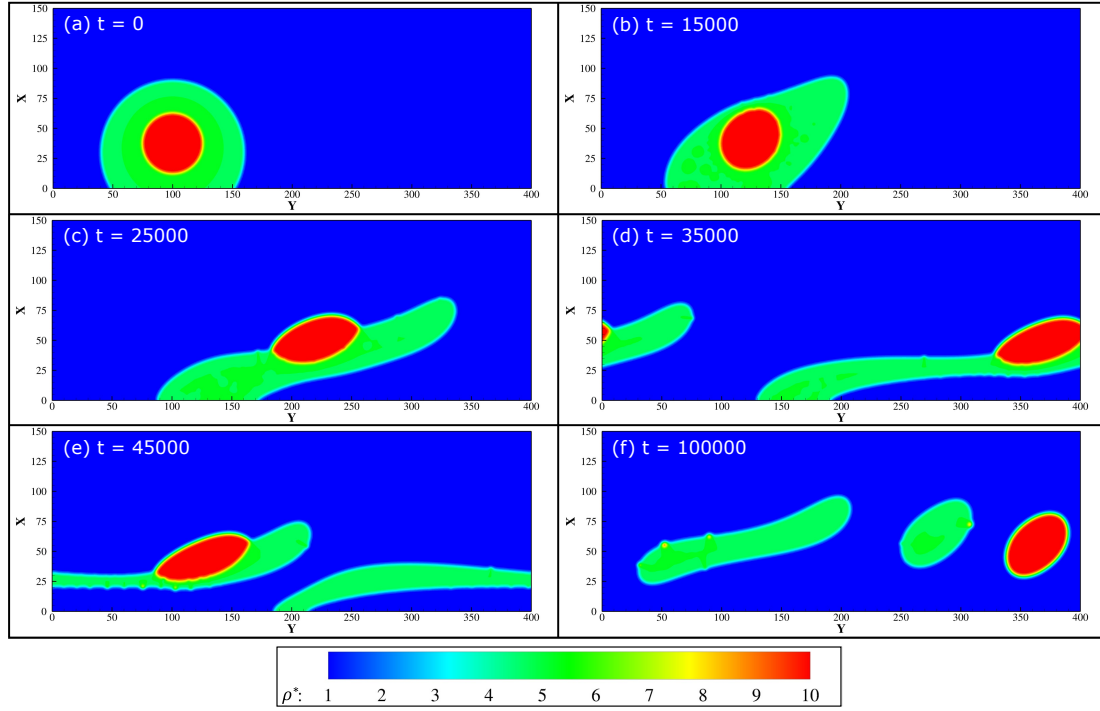


Figure 2.16: Evolution of a concentric binary droplet with unitary density ratio in a shear flow at time step: (a) $t = 0$, (b) $t = 15000$, (c) $t = 25000$, (d) $t = 35000$, (e) $t = 45000$, (f) $t = 100000$. Rescaled density, $\rho^* = 10\phi_1 + 5\phi_2 + \phi_3$, is shown for visualization purposes.

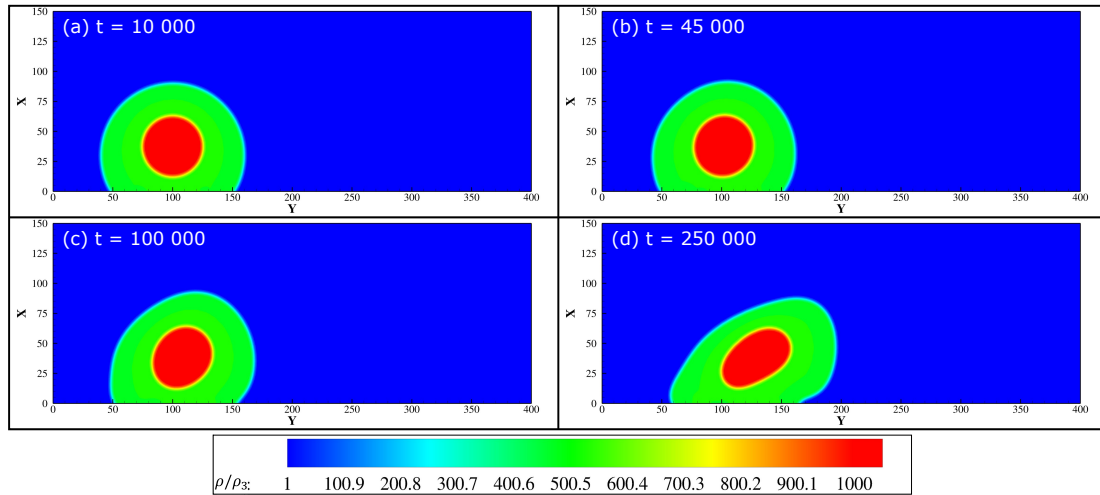


Figure 2.17: Evolution of a concentric binary droplet with high density ratio (red, $\rho_1/\rho_3 = 1000$; green, $\rho_2/\rho_3 = 500$; blue, $\rho_3/\rho_3 = 1$) in a shear flow at time step: (a) $t = 10000$, (b) $t = 45000$, (c) $t = 100000$, (d) $t = 250000$.

rupture into small droplets, see Fig. 2.18(a) and 2.18(b). Such small droplets may numerically vanish if their size is of the order of the interface width ξ , but the total mass still presents an acceptable error lower than 1%, as shown in Fig. 2.18(d).

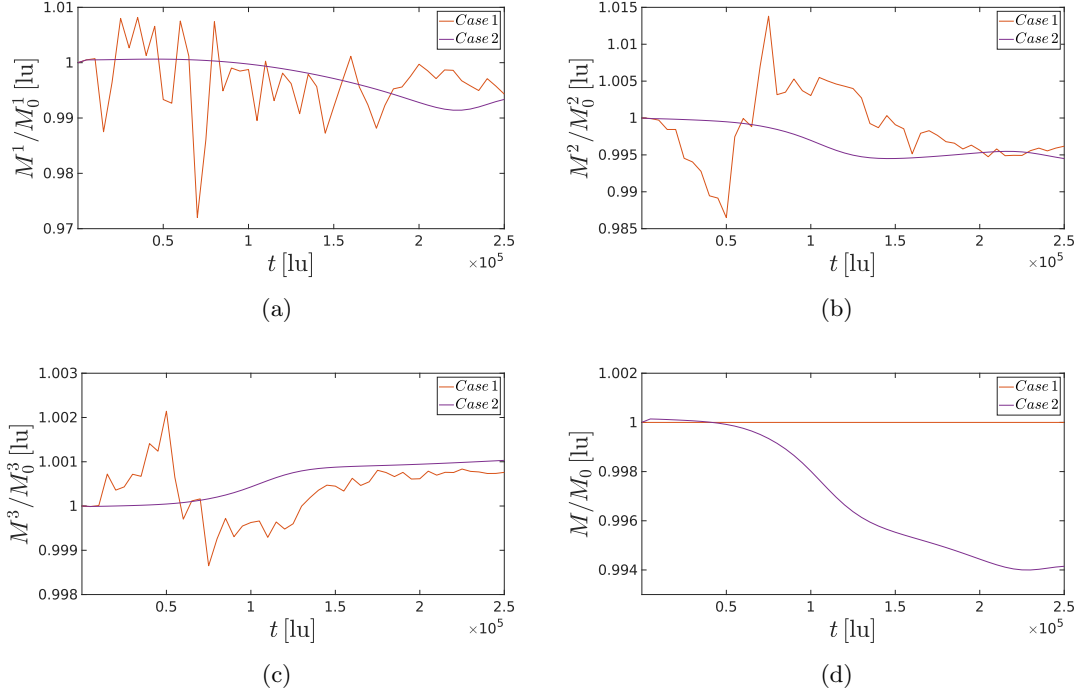


Figure 2.18: Temporal evolution of normalized mass of: (a) fluid 1, (b) fluid 2, (c) fluid 3 and (d) ternary fluid system for both Case 1 ($\rho_1 : \rho_2 : \rho_3 = 1 : 1 : 1$) and Case 2 ($\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$). M_0 is the mass computed at $t = 0$.

2.5.2 The deformation of a three-layer modeled leukocyte under shear flow

The three-layer drop model plays a fundamental role in biomedical applications. In fact, it can be used to study the deformation of leukocytes in the presence of external stimuli [99–102], which is crucial to a full understanding of their role in physiological conditions, e.g. hemostasis and chemotaxis, or pathological conditions, e.g. leukemia or autoimmune diseases [103, 104]. The leukocyte cytoplasm is a water-based solution, but the presence of biological polymers (e.g. microtubules and filaments) gives it a high viscosity ranging from 10 to 10000 times higher than that of the plasma. The volume of the neutrophil nucleus occupies 21% of the total cell volume. The viscosity and surface tension of the nucleus are typically 10 times higher than those of the cytoplasm. The density of the nucleus is approximately 1.5 to 2 times that of the plasma, while the ratio between the density of the cytoplasm and that of the plasma can be considered to be 1. Sick leukocytes can be up to 50 times stiffer than healthy ones, thus losing their deformability essential for extravasation and migration to the site of inflammation.

To study how the variation in the behavior of leukocytes is a function of rheological properties, i.e. cortex tension and viscosity ratio, 3D simulations are conducted. The

dimensionless quantities are $Re = 112.5$, $Pe = 75$ and $Cn = 4/150$. The density ratio is equal to $\rho_1 : \rho_2 : \rho_3 = 2 : 1 : 1$.

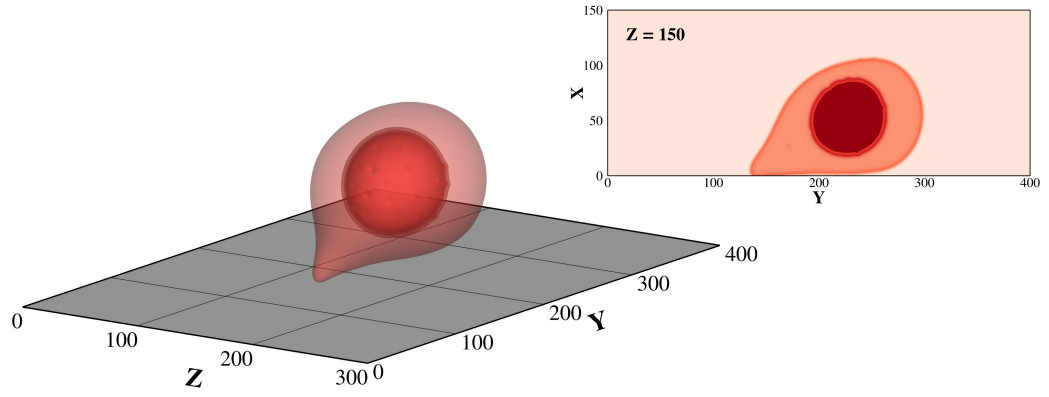
Two walls are located at $x = 0$ and $x = L_0$ in a computational domain of size $L_0 \times 8L_0/3 \times 2L_0$ ($L_0 = 150$ lu). The radii of the two droplets are $R_1 = 35$ lu and $R_2 = 60$ lu. ρ_3 is set to 1 lu, and the relaxation time τ_3 is fixed to 0.1 lu. A constant velocity $U = 0.1$ lu is applied to the upper wall along the x -direction. The three phase-field variables are initialized as in Eq. (2.57), where $\mathbf{x}_{C_1} = (45, 100, 150)$ and $\mathbf{x}_{C_2} = (45, 100, 150)$.

Three case studies are considered, and the related rheological properties are reported in Table 2.2. The contact angles are $(\theta_{13}, \theta_{23}) = (165^\circ, 165^\circ)$ and θ_{12} is retrieved by Eq. (2.17).

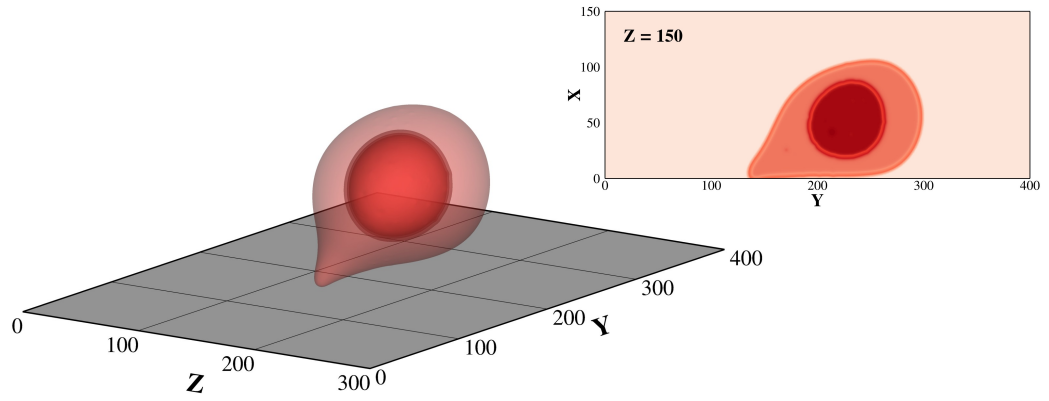
Case	I	II	III
$\mu_1 : \mu_2 : \mu_3$	1000 : 100 : 1	1000 : 100 : 1	100 : 10 : 1
Ca_1	0.0877	0.1667	0.0877
Ca_2	0.0877	0.1667	0.0877
Ca_3	1.667	0.1667	1.667
σ_{12} [lu]	0.01	0.01	0.01
σ_{13} [lu]	0.01	0.01	0.01
σ_{23} [lu]	0.001	0.01	0.001

Table 2.2: Three-layer modeled leukocyte under shear flow. Viscosity ratios, $\mu_1 : \mu_2 : \mu_3$, Capillary numbers, Ca_i , and surface tensions, σ_{ij} , for the three investigated cases.

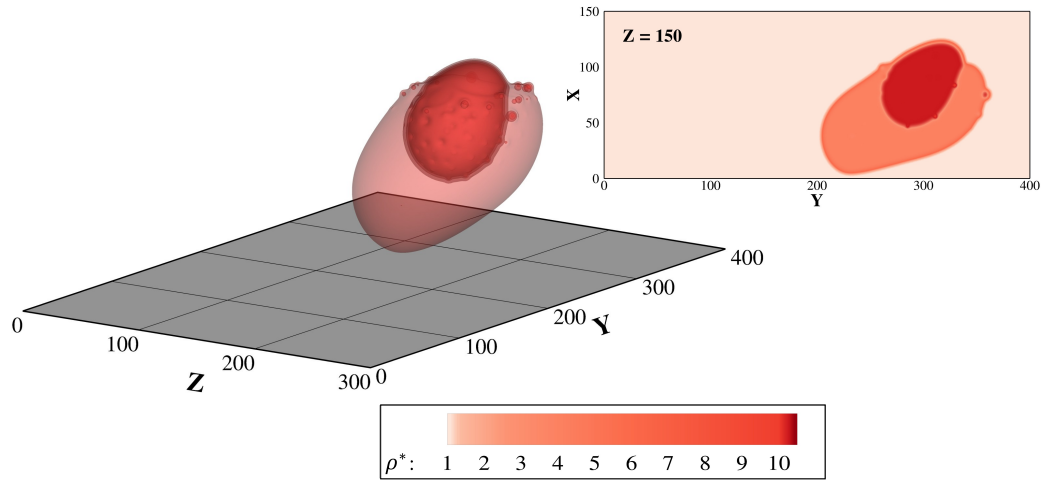
The results shown in Fig. 2.19 are extracted at the time step 40000, reconstructing the volumes using isosurfaces. As the density ratio is low, the latter is rescaled as $\rho^* = 10\phi_1 + 5\phi_2 + \phi_3$. For high viscosity ratios (Fig. 2.19(a)), the applied shear flow induces deformation in the leukocyte, causing it to align along y -axis. The contact area with the surface progressively decreases until complete detachment occurs, at which point the leukocyte is entrained by the flow. By keeping the same viscosity ratios and setting the cortex tension equal to that related to the nucleus (Fig. 2.19(b)), only a little effect on the change in the behavior of the leukocyte is observed. On the contrary, keeping the ratios between surface tensions constant and varying the viscosity ratio (Fig. 2.19(c)), the deformability of the leukocyte increases as the viscosity ratios decrease. Therefore, leukocyte detachment from the surface occurs at earlier stages for low viscosity ratios, and prolonged exposure to high shear flow conditions results in rupture of the cytoplasm membrane. The results obtained are consistent with previous studies [99–102]. The rheological properties vary smoothly across the interface, enhancing numerical stability in cases with high fluid-property ratios. Finally, leukocyte adhesion to the wall is modeled simply through the contact angle, without requiring additional adhesion models as in [99–102]. The study demonstrates that a thorough understanding of the rheological properties of leukocytes is crucial for a precise modeling and prediction of their behavior in practical applications, such as the separation of leukocytes from other blood components in microfluidic channels or their detachment from functionalized surfaces.



(a)



(b)



(c)

Figure 2.19: The shape of leukocyte at $t = 40000$ for: (a) Case 1, (b) Case 2 and (c) Case 3. Isosurfaces of rescaled density, $\rho^* = 10\phi_1 + 5\phi_2 + \phi_3$, are shown for visualization purposes.

Chapter 3

Droplet generation in a flow-focusing microdevice

3.1 Introduction

Fabrication and optimization of droplet-based microfluidic devices, such as core-shell or Janus droplet generators [45, 105], require considerable effort in terms of selecting and supplying tools and materials. Experimentally observing the interaction of multiple fluids with solid surfaces is laborious and time-consuming, requiring highly sensitive measurement systems. Optimizing channel geometry often demands extensive experimental campaigns due to the large number of parameters involved. Numerous devices have been developed for droplet generation, but T-junction [106, 107], co-flowing [108, 109], and flow-focusing [109, 110] geometries are among the most widely used and extensively studied experimentally. Droplet formation, size, shape, and production frequency are influenced by interfacial tension, flow rates, viscosity ratios, channel geometry, and wettability. Recent experimental studies have advanced understanding of these processes: Panwar and Roy [111] introduced a modified Capillary number that incorporates geometric effects, interfacial tension, viscosity, and flow velocity to predict transitions between droplet-generation regimes in suction-driven microfluidics, while Srikanth et al. [112] analyzed the influence of continuous-phase properties on droplet stability and breakup.

In this context, numerical methods enable *in silico* parametric studies by varying device geometry, fluid properties, and environmental conditions, providing access to data that would otherwise be difficult to obtain experimentally, such as force distributions within the channels.

3.1.1 State of the Art of Numerical Models

Several numerical studies have been conducted to simulate monodisperse droplet generation in T-junction and flow-focusing devices. Stevens et al. [113] performed numerical simulations to systematically examine the influence of interfacial tension in flow-focusing configurations, demonstrating how variations in interfacial tension modify necking dynamics, breakup location, and droplet size. Other studies have focused on cross-junction geometries: Jafari et al. [114] analyzed the effect of flow-rate conditions on the transition between squeezing and dripping regimes, while Asghari et al. [115] proposed an analytical and numerical model for droplet breakup in T-junction microchannels, showing how the Capillary number and channel width ratio control the droplet length at

which breakup occurs. Bourega et al. [116] reported the combined effects of flow-rate ratio, viscosity ratio, contact angle, and channel size ratio on droplet size and formation frequency, highlighting the role of wettability in determining detachment mechanisms. Similarly, Venkateshwarlu and Bharti [117] investigated the impact of surface wettability and flow-rate conditions on interface evolution and pinch-off in cross-flow microfluidic devices, demonstrating how contact-angle variations modify the breakup location and the degree of confinement. However, all the above works focus on binary fluid systems, and none of them perform three-dimensional phase-field simulations of multicomponent droplets with wettability in a flow-focusing cross-junction.

3.1.1.1 Lattice Boltzmann methods for droplet generation

Due to its local nature, the lattice Boltzmann (LB) method has proven effective in describing fluid-fluid interactions while ensuring mass conservation in the presence of complex geometries. Moreover, it is easy to implement and scalable on parallel computing architectures. Several multiphase LB models exist, such as color-gradient models, pseudopotential models, free-energy models, and phase-field models, as discussed in Section 2.1.1.

Recently, Poureslami et al. [118] applied a phase-field-based LB approach to study the deformation and breakup of a ferrofluid compound droplet migrating in a microchannel under a magnetic field, highlighting the remarkable capability of phase-field models to capture complex droplet dynamics. As diffusion is localized within the interfacial region and interfacial forces are expressed in a divergence-free and isotropic form, the conservative Allen–Cahn (AC) formulation allows the model to remain stable and accurate even for systems with very high density and viscosity ratios [32, 38]. Moreover, wetting boundary conditions can be naturally incorporated via a wetting potential [36–38], making the phase-field method highly flexible and reliable for simulating complex multiphase flows.

In the context of LB methods, several studies employing the color-gradient method [119, 120] or free-energy method [121] are reported in literature. Both methods are also employed to model more complex ternary systems such as Janus droplets [122, 123] or double emulsions [124, 125]. Although phase-field LB methods have been widely used for simulating multiphase flows, to the best of my knowledge, the work by Liu and Zhang [126] represents the only existing application of such models to droplet generation, and it is limited to two-dimensional simulations.

Moreover, accurately simulating the wettability of curved boundaries is challenging because the bounce-back method [92] approximates geometry using a staircase representation. Although the bounce-back method is straightforward to implement and provides second-order accuracy along with mass conservation, various alternative approaches have been developed to more precisely capture complex boundary geometries. Bouzidi et al. [127] combined linear or quadratic interpolation of the distribution functions at fluid nodes with bounce-back methods. Mei et al. [90] improved upon the work of Filippova and Hanel [128], enhancing its numerical stability. Tao et al. [129] and Zhao et al. [130] proposed a single-node boundary scheme with second-order accuracy. However, most curved boundary schemes applied to multiphase systems suffer from mass conservation issues. Typically, the quiescent distribution function is modified to compensate for mass leakage or gain at fluid nodes near the solid surface. This correction can result in negative values of the quiescent distribution function or large spurious velocities. Fakhari

and Bolster [131] adopted the Mei scheme for hydrodynamic equations and the halfway bounce-back scheme for the phase-field distribution. However, this approach is unsuitable for simulating phase changes. In contrast, Yu et al. [132] modified the Mei scheme to preserve mass conservation in a binary system by defining fictitious densities on ghost nodes. Furthermore, He et al. [39, 133] defined both sets of distribution functions at a fictitious node, from which they propagate and reflect back to the fluid boundary node after interacting with the wall.

3.1.2 Present Contribution

While droplet-based microfluidics has been extensively studied experimentally, there remains a critical need for accurate, mass-conserving numerical frameworks capable of handling high density and viscosity ratios, wettability effects, and complex geometries.

3.1.2.1 Extension to complex geometries and droplet generation

The framework previously developed for ternary fluid systems in contact with wettable lattice boundaries [38] is extended to handle rigid walls that are not conformal to the lattice grid. At the domain boundaries, non-equilibrium bounce-back (NEBB) boundary conditions are employed at both the inlet and outlet to ensure excellent mass conservation. For nonconformal walls, standard bounce-back conditions are applied, and the framework preserves mass even in complex geometrical configurations. Wettability on nonconformal walls is modeled by adapting the algorithm of He et al. [39], using a different discretization and bounce-back scheme, which ensures mass conservation even with staircase approximations on curved geometries.

First, the wettability scheme is validated by investigating the spreading behavior of a compound binary droplet in contact with both flat and curved surfaces. Spurious currents are evaluated to quantify nonphysical artifacts introduced by the model, especially near triple-point junctions and walls.

Next, a parametric study of droplet generation in a three-dimensional flow-focusing microdevice with a cross-junction geometry is conducted. The study investigates the sensitivity of droplet length, morphology, and production frequency to key parameters such as flow rate ratio, the capillary number, viscosity ratio, and contact angle. The parametric study not only enables accurate prediction of droplet behavior and access to quantities that are challenging to measure experimentally, but also offers deeper insight into the physical mechanisms governing droplet formation. Moreover, by addressing the inconsistencies across experimental studies that often vary in fluids, configurations, and operating conditions, this work helps consolidate current knowledge and build a more coherent understanding of multiphase flows in microfluidic devices. Finally, a preliminary study on the generation of multicomponent droplets is conducted to demonstrate the versatility and efficiency of the developed framework in generating both single-phase and double-phase droplets.

In the context of phase-field LB methods, this study addresses the lack of three-dimensional simulations of droplet generation in flow-focusing microfluidic devices involving three immiscible fluids and wettability effects. To the best of my knowledge, it presents the first conservative AC-based phase-field LB formulation capable of accurately capturing droplet formation in such configurations. The proposed model combines excellent mass conservation with robustness across a wide range of density and viscosity ratios, while effectively handling wettable walls nonconformal to the lattice grid. This

approach enables realistic multicomponent droplet dynamics, providing insights that complement experimental studies and supporting the design of advanced microfluidic systems for applications such as droplet-based cell encapsulation and material synthesis.

3.2 Methods

The CPF-LB model (see Section 2.2.1), introduced in Chapter 2, is extended to rigid walls of arbitrary shape in order to simulate realistic configurations. The scheme presented below enables the treatment of surfaces that are non-conforming with the lattice grid, either including or neglecting wettability effects.

3.2.1 Wetting boundary scheme for arbitrarily shaped surfaces

In the case of surfaces with arbitrary geometry that are nonconformal to the lattice domain (see Fig. 3.1), the ghost nodes, g , are the lattice nodes outside the fluid domain that have at least one fluid node nearby. In contrast, boundary fluid nodes are those inside the fluid domain that have at least one ghost node nearby.

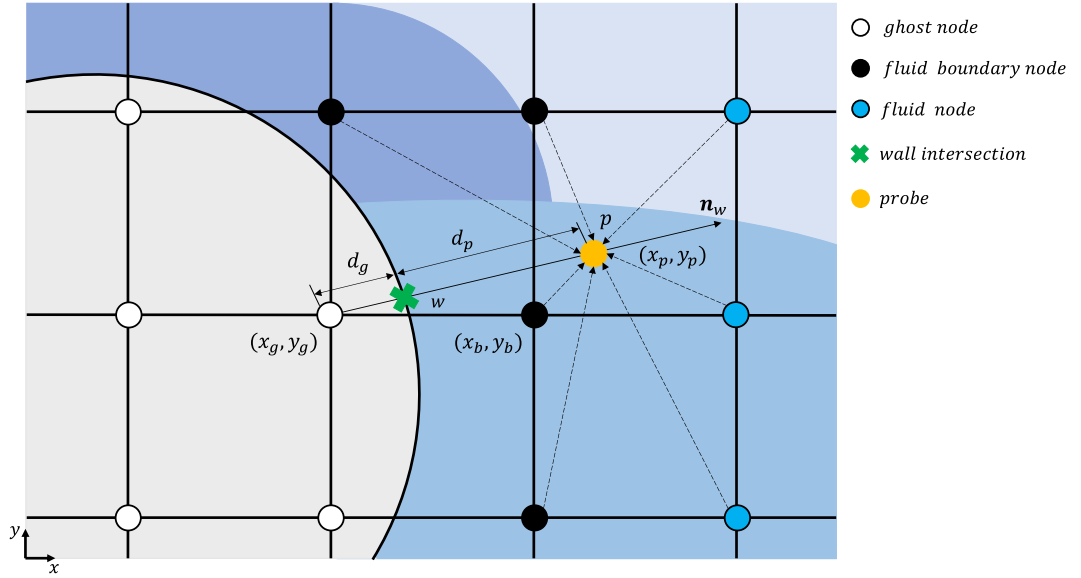


Figure 3.1: Ternary fluid system in contact with a curved surface.

An algorithm inspired by the work of He et al. [39] is developed and applied for each ghost node located at position $\mathbf{x}_g = (x_g, y_g)$ to determine the corresponding phase-field values $\phi_i(\mathbf{x}_g)$ satisfying Eq. (2.18), as described hereafter:

1. for each ghost node, the wall normal $\mathbf{n}_w$ and distance d_g to the intersection point w are determined as follows. The curved surface is discretized into triangular elements with known normals, all pointing toward the fluid. For each triangle, a line parallel to its normal and passing through the ghost node is traced, and its intersection with the triangle plane is computed. Intersections that lie inside their triangles are considered valid, and the one closest to the ghost node is selected, defining both the position $\mathbf{x}_w$ of the intersection w and $\mathbf{n}_w$;

2. calculate the position $\mathbf{x}_p$ of the probe, p , located at the distance d_p ($1.2\Delta x$) from the wall;
3. find the fluid boundary node, b , closest to the probe. For instance, in the schematic shown in Fig. 3.1, this corresponds to $(x_b, y_b) = (x_g + \Delta x, y_g)$;
4. estimate $\nabla\phi_i$ at the fluid boundary node ($\mathbf{x}_b$) using the finite differences as

$$\nabla\phi_i(\mathbf{x}_b) = \frac{\sum_{\alpha=1}^{18} g_\alpha \mathbf{e}_\alpha [\phi_i(\mathbf{x}_b + \mathbf{e}_\alpha \Delta t) - \phi_i(\mathbf{x}_b)]}{\sum_{\alpha=1}^{18} g_\alpha \mathbf{e}_\alpha \cdot \mathbf{e}_\alpha}. \quad (3.1)$$

In the above summation, the contributions from ghost nodes are excluded using a flag variable g_α , which is set to 0 if the node at $\mathbf{x}_b + \mathbf{e}_\alpha \Delta t$ is a ghost node, and to 1 otherwise.

5. extrapolate $\phi_i(\mathbf{x}_w)$ and $\phi_i(\mathbf{x}_p)$ along the normal $\mathbf{n}_w$ as

$$\phi_i(\mathbf{x}_w) = \phi_i(\mathbf{x}_b) + (\mathbf{x}_w - \mathbf{x}_b) \cdot \nabla\phi_i(\mathbf{x}_b) \quad (3.2)$$

$$\phi_i(\mathbf{x}_p) = \phi_i(\mathbf{x}_b) + (\mathbf{x}_p - \mathbf{x}_b) \cdot \nabla\phi_i(\mathbf{x}_b) \quad (3.3)$$

6. compute the phase-field values at the ghost node by first defining the normal component of the phase-field gradient at the wall as

$$\mathbf{n}_w \cdot \nabla\phi_i(\mathbf{x}_w) = \frac{\phi_i(\mathbf{x}_w) - \phi_i(\mathbf{x}_g)}{d_g}, \quad (3.4)$$

and then calculating $\phi_i(\mathbf{x}_g)$ according to Eq. (2.18) as

$$\phi_1(\mathbf{x}_g) = \phi_1(\mathbf{x}_w) + \frac{4d_g}{\xi} (C_{13} + C_{12})_{\mathbf{x}_w}, \quad (3.5a)$$

$$\phi_2(\mathbf{x}_g) = \phi_2(\mathbf{x}_w) + \frac{4d_g}{\xi} (C_{23} - C_{12})_{\mathbf{x}_w}, \quad (3.5b)$$

$$\phi_3(\mathbf{x}_g) = \phi_3(\mathbf{x}_w) + \frac{4d_g}{\xi} (-C_{23} - C_{13})_{\mathbf{x}_w}; \quad (3.5c)$$

7. update the phase-field values at the wall as

$$\phi_i(\mathbf{x}_w) = \frac{d_g}{d_g + d_p} (\phi_i(\mathbf{x}_p) - \phi_i(\mathbf{x}_g)) + \phi_i(\mathbf{x}_g); \quad (3.6)$$

8. combining Eqs. (3.5) and (3.6), recompute the final value of $\phi_i(\mathbf{x}_g)$ as

$$\phi_1(\mathbf{x}_g) = \phi_1(\mathbf{x}_p) + \frac{4(d_g + d_p)}{\xi} (C_{13} + C_{12})_{\mathbf{x}_w}, \quad (3.7a)$$

$$\phi_2(\mathbf{x}_g) = \phi_2(\mathbf{x}_p) + \frac{4(d_g + d_p)}{\xi} (C_{23} - C_{12})_{\mathbf{x}_w}, \quad (3.7b)$$

$$\phi_3(\mathbf{x}_g) = \phi_3(\mathbf{x}_p) + \frac{4(d_g + d_p)}{\xi} (-C_{23} - C_{13})_{\mathbf{x}_w}. \quad (3.7c)$$

Finally, the unknown phase-field distribution functions $h_{\bar{\alpha}}^i$ at the fluid boundary nodes are obtained using the halfway bounce-back boundary scheme [92]:

$$h_{\bar{\alpha}}^i(\mathbf{x}_b, t + \Delta t) = \tilde{h}_{\alpha}^i(\mathbf{x}_b, t) \quad (3.8)$$

where $\tilde{h}_{\alpha}^i$ is the post-collision distribution function and α is the opposite direction of $\bar{\alpha}$. The same boundary scheme is applied to the unknown $f_{\bar{\alpha}}(\mathbf{x}_b, t + \Delta t)$.

3.2.2 Algorithm

The scheme presented above is incorporated into the algorithm reported in Chapter 2 (see Section. 2.2.3), as shown in Fig. 3.2.

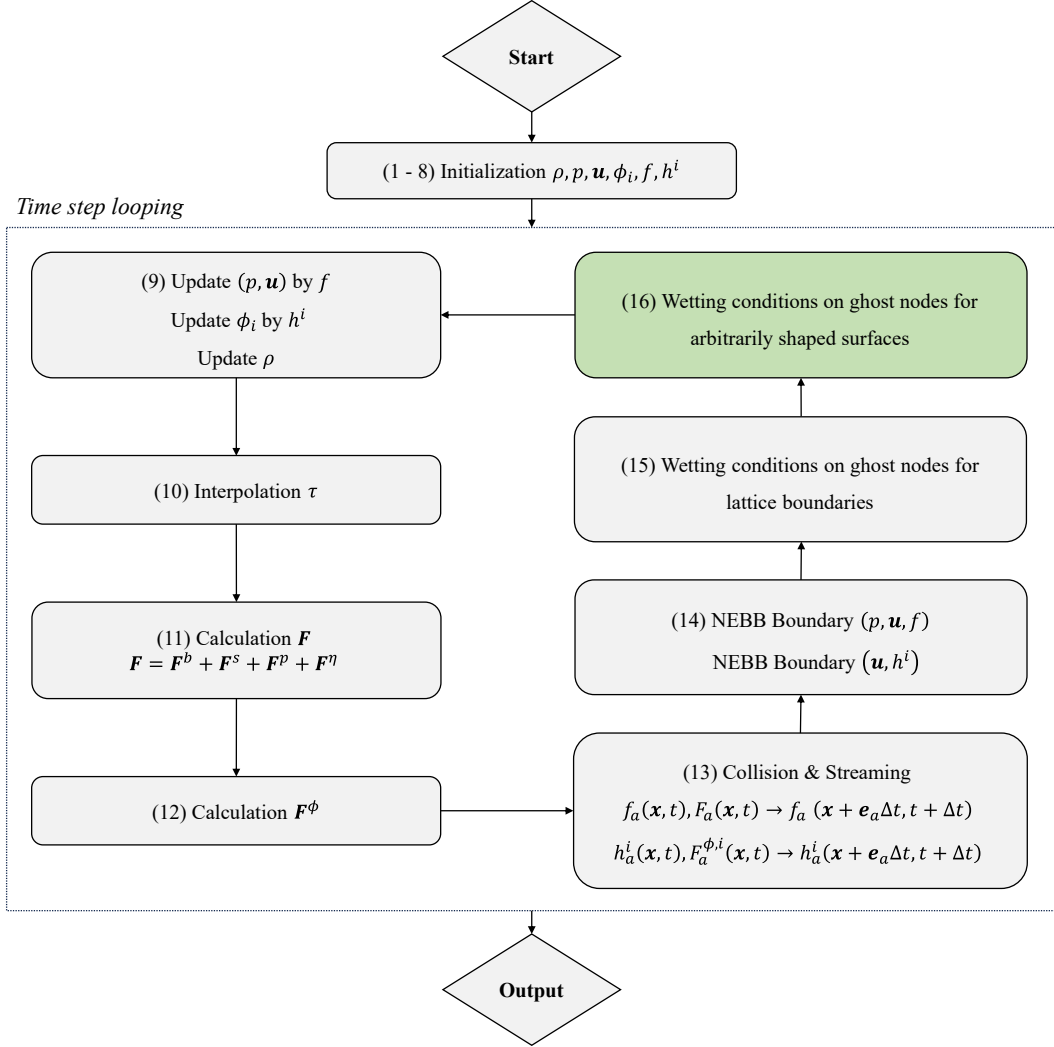


Figure 3.2: Schematic representation of the implemented algorithm.

3.3 Validation of the wetting scheme for arbitrarily shaped surfaces

In this section, the proposed wetting algorithm (Section 3.2.1) is validated using two well-established benchmark tests from the literature. First, the spreading behavior of a binary droplet in contact with flat surfaces is evaluated, comparing the results with those in the absence of nonconformal walls presented in Section 2.4.2 [38]. Subsequently, curved surfaces are considered, specifically cylindrical geometries, as proposed by He et al. [39]. Unlike their work, the present tests are conducted under conditions of low and high density and viscosity ratios.

The dimensionless quantities used in this section are given by Eq. 2.46. The pressure and velocity fields are initialized to zero and the gravity field is neglected.

All simulations are carried out in two dimensions by defining only three nodes in the third (y - or z -) direction and applying periodicity along it.

The geometric quantities describing the problem are evaluated by deploying the topographical strategy presented by Santeramo et al. [134], here aimed to provide an efficient description of the droplet contour. Specifically, once the phase-field ϕ_i is obtained, the interface of the i -th fluid droplet is detected at a threshold value $\tilde{\phi}_i$ equal to $\tilde{\phi}_i = 0.5$, as typically deployed [135–137]. Hence, the resulting contour is described via a Fourier series, based on the radial distance of each point of the boundary with respect to the droplet centroid [134]. This approach not only results in a clearer determination of the local tangents and, consequently, of the contact angles, but, crucially, it avoids the need to rely on global droplet geometrical features, so that it can be deployed to analyse both symmetric and asymmetric shapes.

3.3.1 The spreading of a compound droplet on wettable flat solid surfaces

The wetting boundary scheme is initially validated on flat surfaces with a prescribed set of static contact angles as reported in previous studies [36–38] and in Section 2.4.2. The following dimensionless quantities are considered: $Ca_1 = Ca_2 = Ca_3 = 8.33 \times 10^{-6}$, $Pe = 0.7143$, and $Cn = 4/120$. The density- and viscosity- ratios of three fluids are $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 1000 : 500 : 1$, respectively. The density of fluid 3 is $\rho_3 = 0.01$ lu and the relaxation time is fixed to $\tau_3 = 0.3$ lu.

In a domain of size $5L_0/2 \times L_0$ ($L_0 = 120$ lu), a compound droplet with radius $R = L_0/2$ is placed on a solid wall. A lattice wall is located at $z = L_0$ and no-slip boundary condition is applied using the NEBB method [38]. A solid wall is immersed equidistantly between two rows of lattice nodes and BB method is adopted. Three nodes are considered in the y -direction and periodic boundary conditions are applied in the x and y directions.

Three different contact angle configurations (θ_{13}, θ_{23}) are examined: case I corresponds to $(\theta_{13}, \theta_{23}) = (60^\circ, 90^\circ)$, case II to $(90^\circ, 90^\circ)$, and case III to $(120^\circ, 90^\circ)$. The angle θ_{12} is computed by Eq. (2.17).

Fig. 3.3 shows the equilibrium configurations of the three analyzed cases. The observed behavior is consistent with previous results obtained using the same solver for compound droplets on planar surfaces represented by the external lattice boundaries [38]. Indeed, in the case of neutral wettability (Fig. 3.3(b)), the droplets wet the surfaces equally. For contact angles smaller than the neutral angle (fluid 1 in Fig. 3.3(a)), the wetting length increases, whereas for larger angles (fluid 1 in Fig. 3.3(c)), it decreases.

Fig. 3.4 presents a quantitative comparison between numerical and analytical solutions. The results exhibit a relative error lower than 1%. In all cases, droplets with neutral wettability exhibit the smallest relative error (less than 0.3098%). The maximum error 0.5897% is observed for fluid 1 of the Case III and the minimum one 0.1089% for fluid 2 of the Case I.

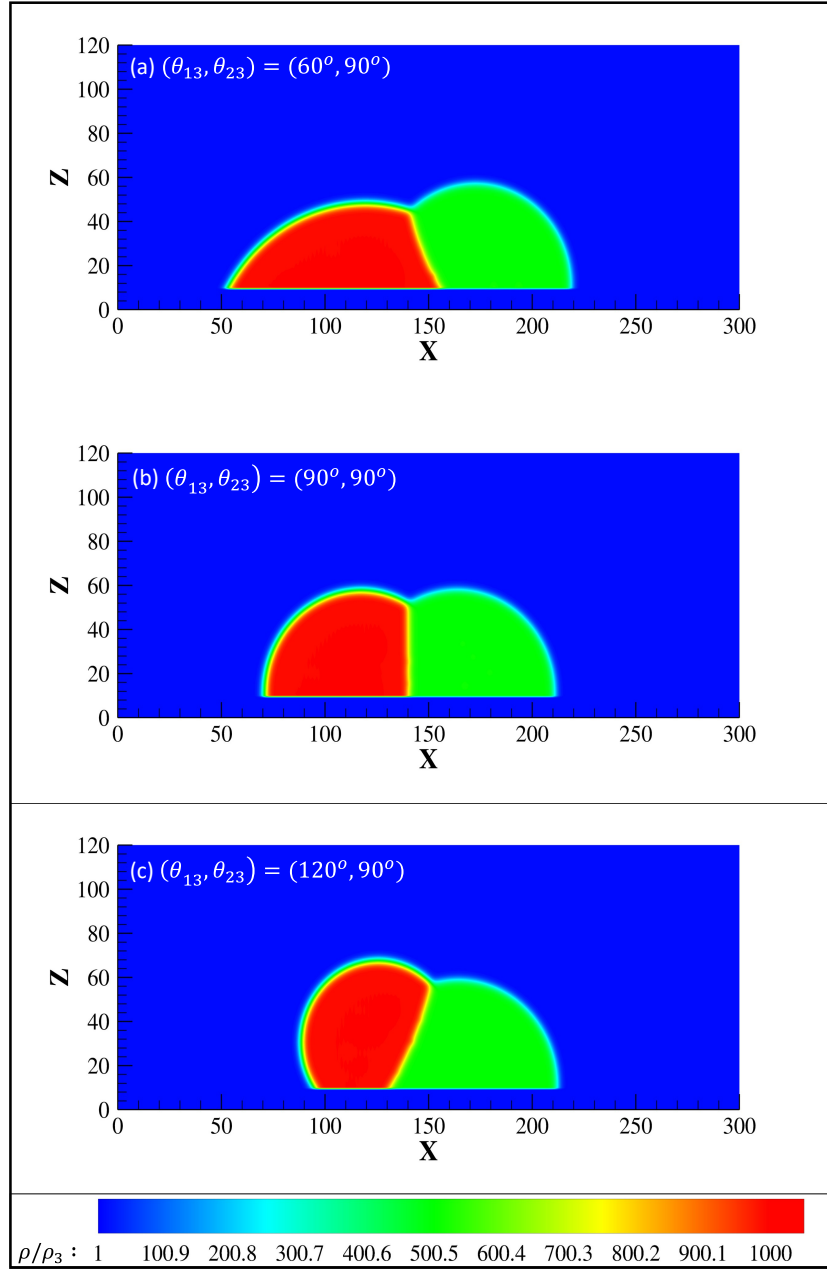


Figure 3.3: Density distribution of the equilibrium configuration of a compound binary droplet on a flat surface with different wettability (red: fluid 1, green: fluid 2, and blue: fluid 3).

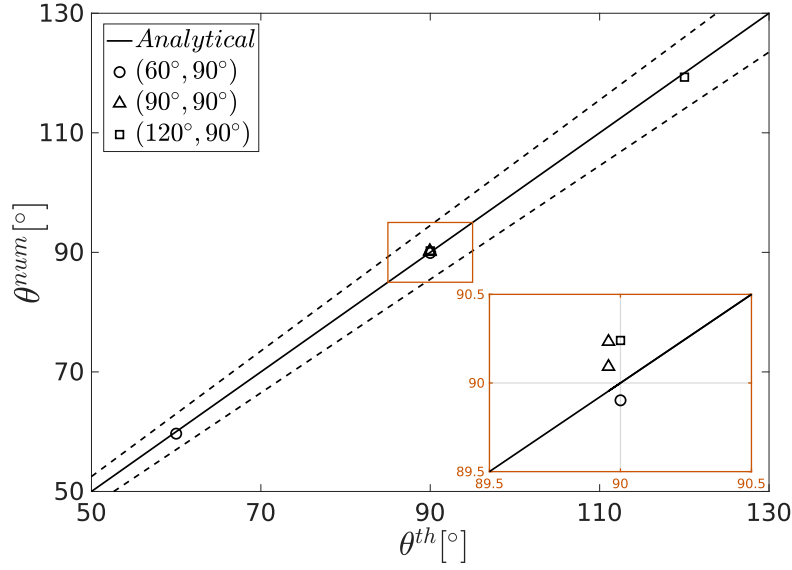


Figure 3.4: Comparison between the analytical θ^{th} and numerical θ^{num} contact angles of a compound droplet on flat surfaces. The relative error of $\pm 5\%$ is reported as dashed lines.

3.3.2 The spreading of a compound droplet on a wettable circular cylinder

To demonstrate the ability of the wettability scheme to handle droplets interacting with surfaces of arbitrary geometry, additional validation is performed using curved surfaces. The spreading of a compound binary droplet on a cylinder is investigated as in the study by He et al. [39]. In contrast to the reference, both low and high density and viscosity ratios are considered: $\rho_1 : \rho_2 : \rho_3 = 1 : 1 : 1$ ($\rho_3 = 1$ lu), $\mu_1 : \mu_2 : \mu_3 = 1 : 1 : 1$ and $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ ($\rho_3 = 0.01$ lu) and $\mu_1 : \mu_2 : \mu_3 = 1000 : 500 : 1$, respectively. The relaxation time τ_3 is equal to 0.3 lu and the dimensionless quantities are $Ca_1 = Ca_2 = Ca_3 = 0.0139$, $Pe = 10$, and $Cn = 5/360$. A cylinder of diameter $D = 160$ lu is placed at $(180, 170)$ in a domain of size $L_0 \times L_0$ ($L_0 = 360$ lu). The center and the radius of a compound droplet are fixed as $\mathbf{x}_C = (180, 90)$ and $R = 60$ lu, respectively. The following contact angle configurations $(\theta_{13}, \theta_{23})$ are investigated: case I corresponds to $(120^\circ, 60^\circ)$, case II to $(120^\circ, 120^\circ)$, and case III to $(90^\circ, 120^\circ)$. The angle θ_{12} is computed using Eq. (2.17).

Figs. 3.5(a)-(c) and 3.6(a)-(c) show how the equilibrium shape of a compound binary droplet depends on the contact angles, regardless of the density and viscosity ratios. Figs. 3.5(d)-(f) and 3.6(d)-(f) show the numerical parasitic currents of the model, computed as $|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$. At the triple-point junctions, the magnitude of spurious velocities is of the order $\mathcal{O}(10^{-5})$, while in the rest of the domain they decrease to $\mathcal{O}(10^{-6})$. Overall, the model produces errors an order of magnitude smaller than those reported in literature [39].

Table 3.1 reports the numerical values of the angles and the corresponding errors for the low- and high-density cases, respectively. For both cases, lower errors are recorded for hydrophobic surfaces, while the highest error is observed for hydrophilic ones. The relative errors obtained for droplets with low density and viscosity ratios are lower than

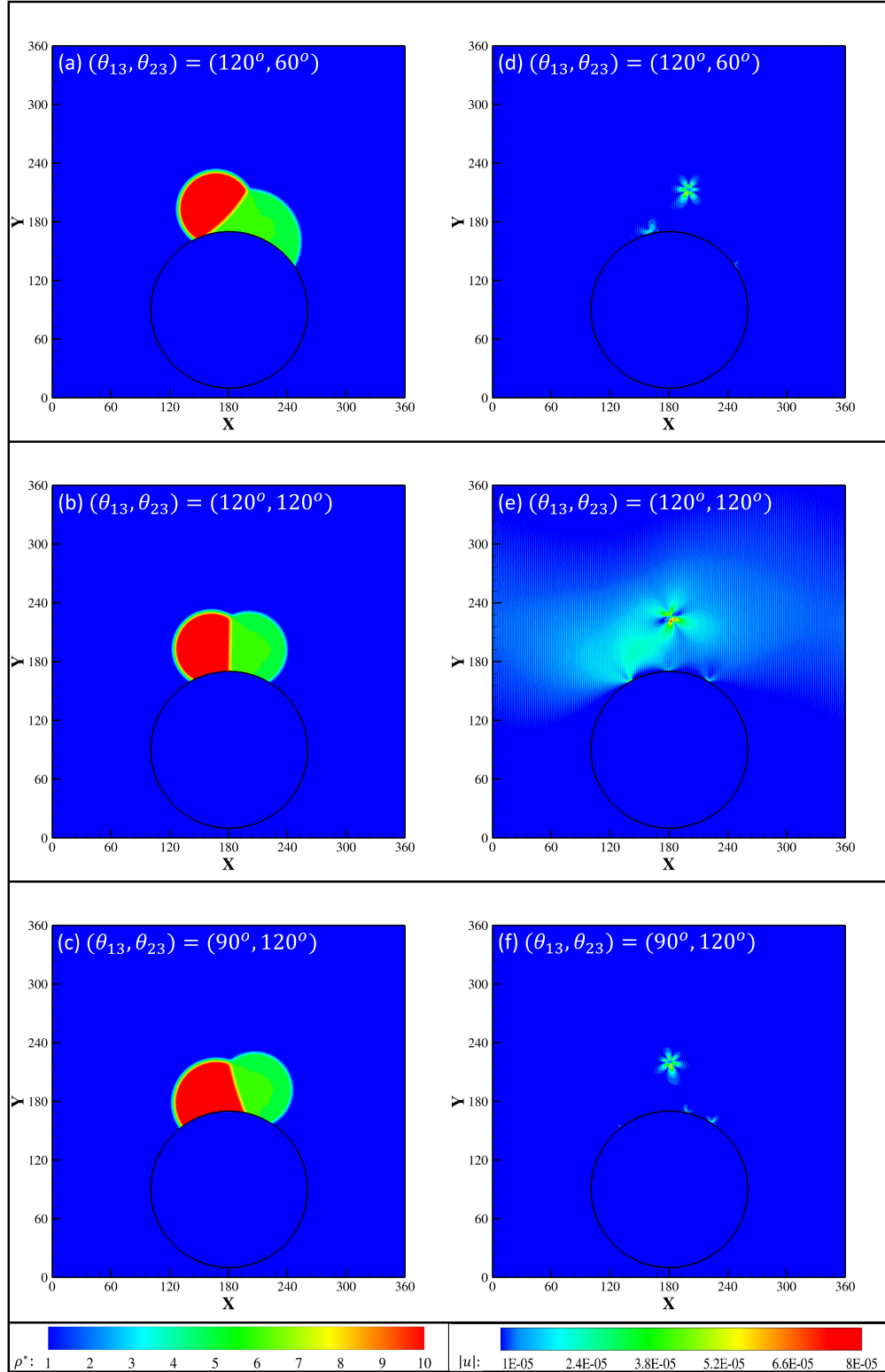


Figure 3.5: Density (left) and parasitic currents (right) distributions of a compound binary droplet with unitary density ratio at the equilibrium configuration with different contact angles. Rescaled density, $\rho^* = 10\phi_1 + 5\phi_2 + \phi_3$, is shown for visualization purposes.

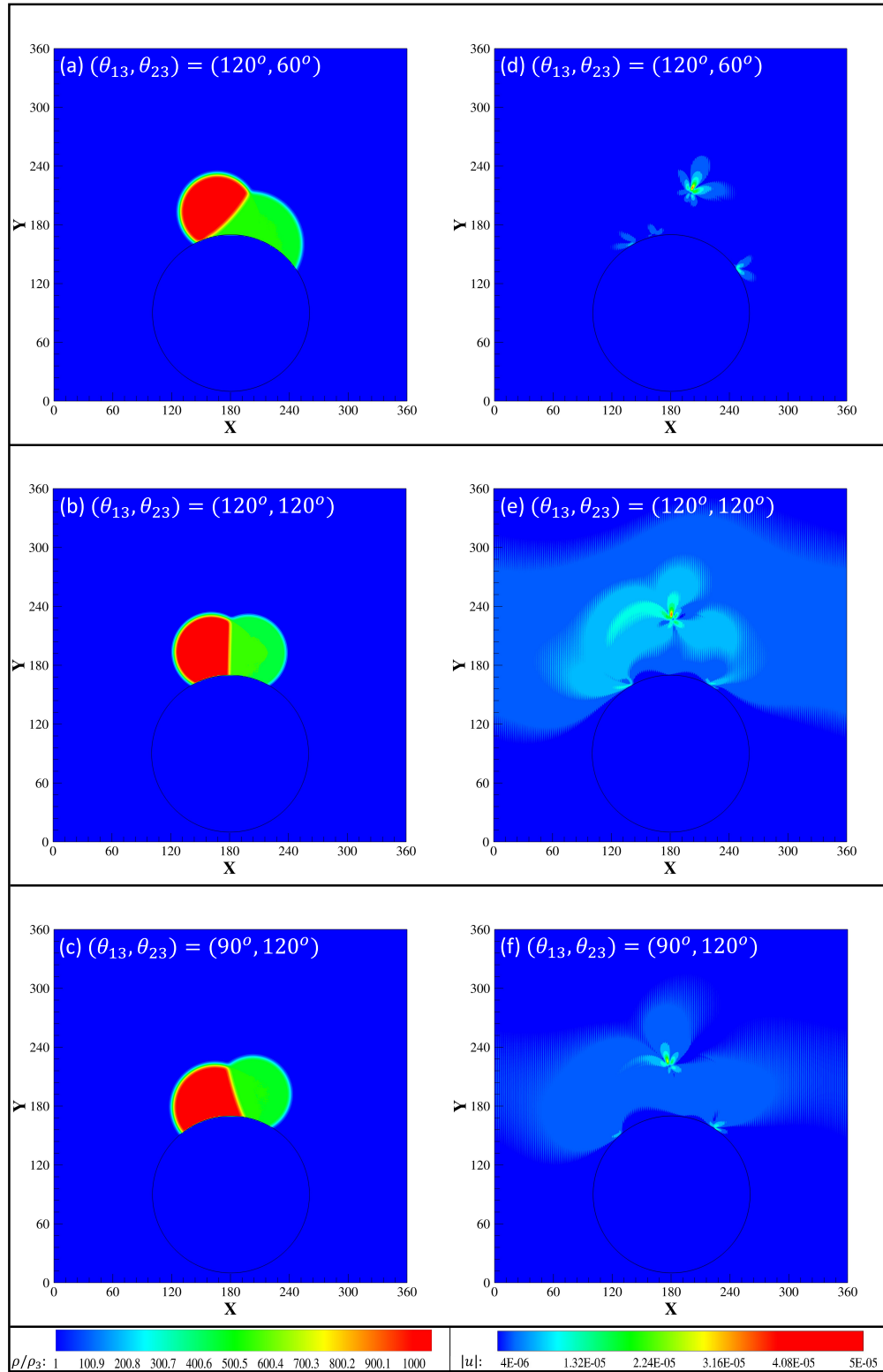


Figure 3.6: Density (left) and parasitic currents (right) distributions of a compound binary droplet with high density ratio $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ at the equilibrium configuration with different contact angles.

those reported in literature [39]. In general, relative errors lower than 3% are observed for both cases.

Case	Numerical		Analytical		Relative errors	
	θ_{13}	θ_{23}	θ_{13}	θ_{23}	θ_{13}	θ_{23}
I (LR)	119.81°	61.62°	120°	60°	0.16%	2.7%
II (LR)	120.96°	120.96°	120°	120°	0.8%	0.8%
III (LR)	89.04°	119.19°	90°	120°	1.07%	0.66%
I (HR)	120.56°	61.74°	120°	60°	0.47%	2.9%
II (HR)	121.01°	121.76°	120°	120°	0.84%	1.47%
III (HR)	89.53°	117.65°	90°	120°	0.52%	1.96%

Table 3.1: Equilibrium contact angles of a compound droplet on a cylinder for both low ratios (LR) $\rho_1 : \rho_2 : \rho_3 = 1 : 1 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 1 : 1 : 1$, and high ratios $\rho_1 : \rho_2 : \rho_3 = 1000 : 500 : 1$ and $\mu_1 : \mu_2 : \mu_3 = 1000 : 500 : 1$.

3.4 Monodispersed droplet generation in a cross-junction microdevice

Here, a study of monodispersed droplet generation in a flow-focusing microdevice with a cross-junction geometry is presented. The cross-junction comprises a main channel with a width, w , of $200 \mu\text{m}$ and two lateral channels, each $100 \mu\text{m}$ wide. The entire microdevice has a uniform height of $100 \mu\text{m}$. The dispersed phase, namely a water-based solution (density $\rho_d = 1000 \text{ kg/m}^3$ and viscosity $\mu_d = 0.01 \text{ Pa s}$), is injected through the main channel, while the continuous phase, an oil (density $\rho_c = 1000 \text{ kg/m}^3$ and viscosity μ_c), flows through the two lateral channels, as illustrated in Fig. 3.7. As the Reynolds number is $\mathcal{O}(10^{-2})$ in all the following tests, droplet formation in such a flow regime is governed solely by the flow rate ratio Q , the Capillary number Ca , and the viscosity ratio α , defined as follows:

$$Q = \frac{Q_c}{Q_d}, \quad Ca = \frac{u_c \mu_c}{\sigma}, \quad \alpha = \frac{\mu_c}{\mu_d}, \quad (3.9)$$

where Q_d and Q_c denote the volumetric flow rates of the dispersed (d) and continuous (c) phases, respectively, and $u_c = 2Q_c/w^2$.

The flow-focusing microdevice is simulated within a computational domain of $230 \times 40 \times 12$ nodes. This grid was selected by comparing the droplet length under different flow rate conditions with that obtained using a finer grid of $460 \times 80 \times 24$ nodes, as typically reported in the literature [119, 120]. The relative error between the two grids was found to be less than 1%, even for high inlet flow rates and large flow rate ratios.

To convert lattice parameters into physical units, the following reference values are adopted: $L_0 = 10^{-5} \text{ m}$, $t_0 = 10^{-6} \text{ s}$, and $m_0 = 10^{-12} \text{ kg}$. Accordingly, the density and viscosity in lattice units are set to $\rho_d = \rho_c = 1 \text{ lu}$ and $\mu_d = 0.1 \text{ lu}$, respectively; the interfacial tension between the dispersed and continuous phases is set to $\sigma = 0.01 \text{ lu}$; the interface thickness ξ and mobility M are set to 4 lu and 0.1 lu , respectively. Since the present tests involve a two-phase (binary) system, the volume fraction ϕ_2 is initialized to zero, thereby satisfying the reduction-consistency property [38].

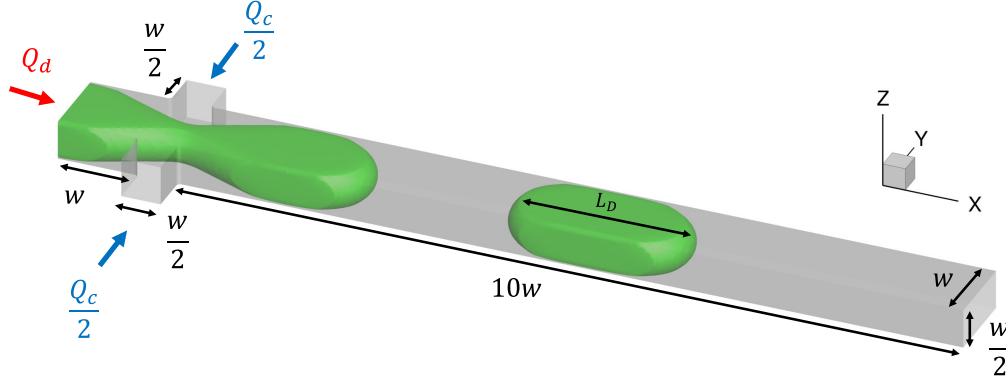


Figure 3.7: Three-dimensional view of the flow-focusing microdevice for droplet generation. Here, w is the width of the two lateral channels, Q_d and Q_c are the volumetric flow rates of the dispersed and continuous phases, and L_D is the droplet length.

All simulations are performed by imposing a velocity profile at the three inlets and using an outflow boundary condition at the outlet, implemented via the NEBB method [38]. To ensure mass conservation and enforce the no-slip condition, the halfway bounce-back method is applied at the solid walls. Wetting boundary conditions (cf. Section 3.2.1) are imposed on the channel walls.

The relevant quantities are evaluated using the image processing algorithm [134], based on a threshold value of $\phi_i = 0.5$ ($i = 1, 2$). Moreover, L_D is measured at a fixed streamwise position rather than at a fixed time, ensuring consistent comparison across simulations. The dispersed phase ϕ_d is monitored at the lattice node $(200, 20, 6)$, and the droplet is considered to have reached this location when $\phi_d(200, 20, 6)$ switches from $\phi_d < 0.5$ to $\phi_d > 0.5$. At that instant, the interface is extracted and L_D is evaluated. This point is located sufficiently far from the junction to avoid transient effects. Finally, all 3D views are obtained from the isosurface $\phi_1 = 0.5$, used to represent the droplet interface.

3.4.1 Effect of flow conditions

Here, the effect of variations in the Capillary number Ca on droplet formation is investigated for different values of the flow rate ratio Q and the viscosity ratio α , which are the main dimensionless parameters governing the problem. A contact angle of 150° is imposed at the channel walls to model wetting behavior. The results reported below show two distinct breakup regimes. At low Ca droplets form in the squeezing regime, dominated by geometric confinement and mechanical compression. At high Ca breakup shifts to the dripping regime, where viscous stresses control droplet detachment. These two behaviors are separated by a critical Capillary number.

First, the viscosity ratio is fixed at $\alpha = 4$, and the sensitivity of droplet breakup to the Capillary number Ca is evaluated for three flow rate ratios, namely $Q = [2, 3, 6]$. The flow rate of the continuous phase Q_c is varied corresponding to $Ca = [0.004, 0.006, 0.012, 0.03, 0.08]$. As shown in Fig. 3.8(a), for each value of Q , the droplet length L_D decreases as Ca increases. The change in droplet length is more pronounced for Ca lower than a critical value about 0.01, whereas for higher values this effect is negligible. At a fixed Ca , an increase in Q leads to smaller droplets. This effect becomes more significant for Ca lower than 0.01. Then, simulations are performed at fixed flow rate ratio $Q = 6$, while varying

the viscosity ratio α in the range $\alpha = [1, 2, 3, 4]$ for $Ca = [0.006, 0.012, 0.03]$. The results, shown in Fig. 3.8(b), indicate that the droplet length L_D decreases as α decreases. This behavior is due to the increase in shear forces acting on the droplet interface. For a given Ca , lowering α corresponds to reducing the continuous-phase viscosity μ_c . Since Q is fixed, this leads to a higher u_c . Although u_d also increases, the velocity difference $(u_c - u_d)$ becomes larger, enhancing the interfacial velocity gradient. As a result, the jump in shear stress across the interface increases, promoting earlier breakup and leading to shorter droplets.

In all cases, L_D exhibits a monotonic decrease with Ca . The dependence is well fitted by the power-law relation $L_D/w = c_1 Ca^{c_2}$, whose fitting coefficients are summarized in Table 3.2.

	$\alpha = 4$ $Q = 2$	$\alpha = 4$ $Q = 3$	$\alpha = 4$ $Q = 6$	$\alpha = 3$ $Q = 6$	$\alpha = 2$ $Q = 6$	$\alpha = 1$ $Q = 6$
c_1	0.430	0.449	0.457	0.422	0.419	0.371
c_2	-0.242	-0.242	-0.248	-0.233	-0.222	-0.229

Table 3.2: Coefficients c_1 and c_2 of the power law $L_D/w = c_1 Ca^{c_2}$ for different values of Q and α .

Fig. 3.9 reports the results obtained for viscosity ratio $\alpha = 4$ and different flow rate ratios Q . For $Ca \leq Ca_{critical} (\sim 0.01)$ the squeezing regime prevails, where droplet formation occurs due to mechanical compression of the dispersed phase, which completely occludes the downstream channel of the cross-junction. In this regime, interfacial tension forces dominate over viscous forces, resulting in the regular and slow formation of large droplets. In the dripping regime ($Ca > Ca_{critical}$), viscous forces dominate over interfacial forces, up to the point where droplets no longer touch the channel walls. The continuous phase drags the dispersed phase until it breaks up into smaller droplets with a high production frequency. Moreover, the higher the Ca , the more L_D becomes independent of Q . For a fixed Ca , an increase in Q induces the formation of a fluid film of the continuous phase between the channel wall and the droplet. This film facilitates the generation of droplets with a more spherical shape, but reduces the production frequency and increases the distance between droplets (see Fig. 3.9(a)-(f), (b)-(g), (c)-(h), (d)-(i), and (e)-(l)).

Fig. 3.10 shows the variation in the droplet formation process for two different viscosity ratios α at fixed $Q = 6$ and $Ca = 0.006$. At a fixed Ca , as the viscosity of the continuous phase decreases, the hydrodynamic focusing of the dispersed phase at the cross-junction is enhanced. The increase in shear forces thickens the fluid film of the continuous phase between the wall and the dispersed phase, leading to a reduction in both droplet volume and droplet formation time. The formation process can be divided into three stages: the expansion stage (Fig. 3.10(a)-(b) for $\alpha = 1$ and Fig. 3.10(f) for $\alpha = 4$), where the dispersed phase fills the cross-junction; the necking stage (Fig. 3.10(c) for $\alpha = 1$ and Fig. 3.10(g)-(h)-(i) for $\alpha = 4$), where the continuous phase starts to pinch off the dispersed phase leading to droplet formation; and the detachment stage (Fig. 3.10(d)-(e) for $\alpha = 1$ and Fig. 3.10(l) for $\alpha = 4$), where the generated droplet moves through the channel. Within the same time interval (from $t = 3.6$ s to $t = 4.1$ s), two droplets are released for $\alpha = 1$, whereas only a single droplet for $\alpha = 4$. When μ_c is lower ($\alpha = 1$), the stronger hydrodynamic focusing and the larger velocity contrast between the two phases accelerate the necking and detachment stages, leading to a

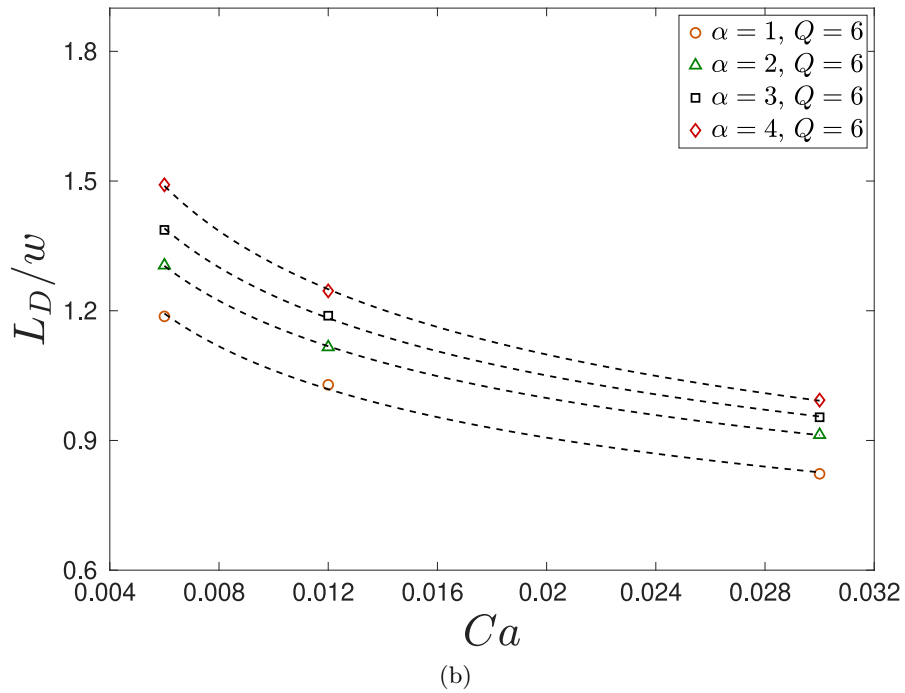
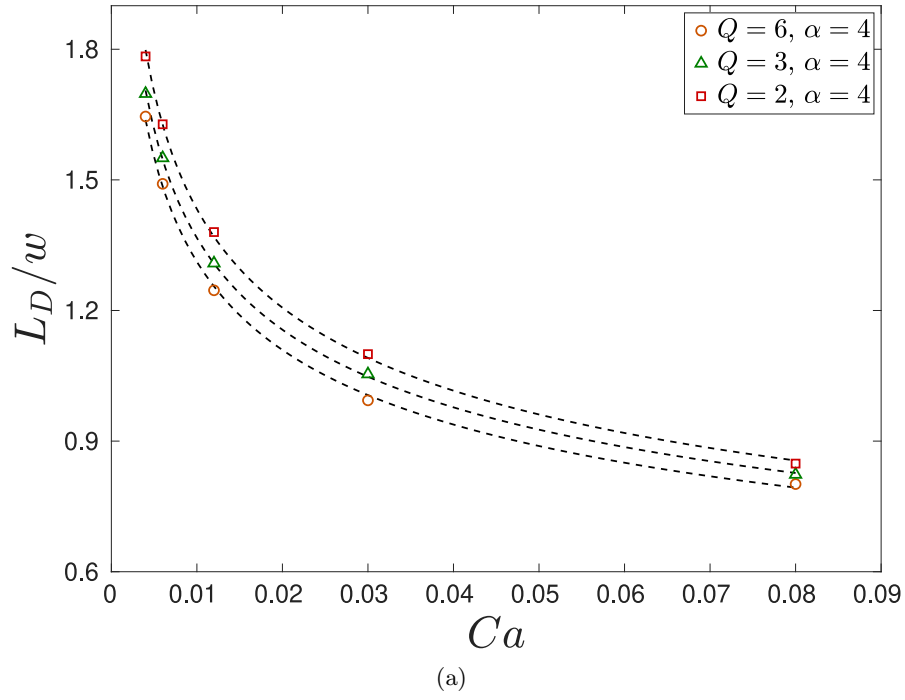


Figure 3.8: The influence of the flow rate ratio Q (a) and the viscosity ratio α (b) on droplet length L_D is analyzed by varying Ca . The dashed curves follow the power laws obtained by fitting the simulation data.

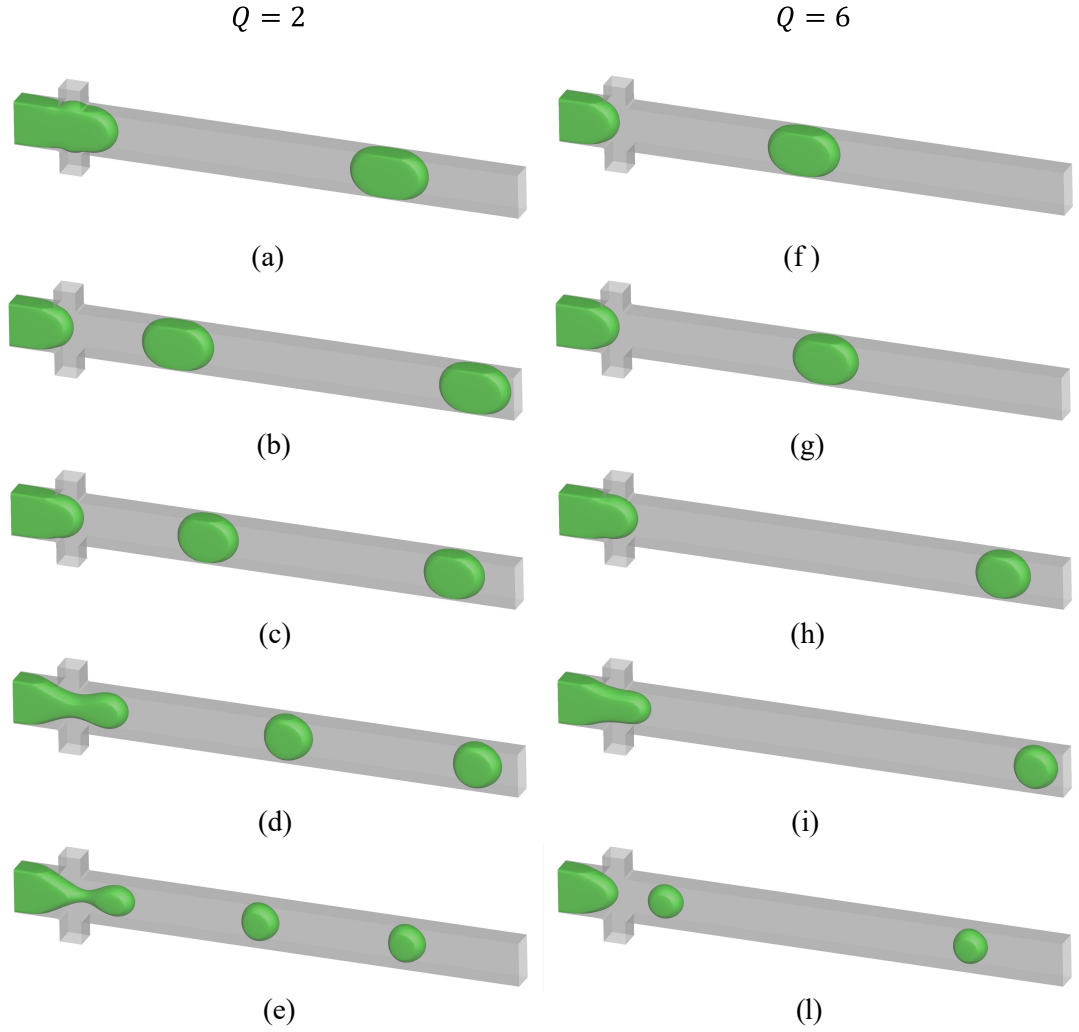


Figure 3.9: Droplet generation for viscosity ratio $\alpha = 4$ and different flow rate ratios Q as a function of Capillary number Ca . (a)-(f) $Ca = 0.004$, (b)-(g) $Ca = 0.006$, (c)-(h) $Ca = 0.012$, (d)-(i) $Ca = 0.03$, and (e)-(l) $Ca = 0.08$.

shorter generation period. In contrast, for $\alpha = 4$ the reduced shear forces slow down the thinning of the neck, resulting in a delayed breakup and a longer detachment time (approximately 0.1 s). This confirms that a higher α leads to a lower droplet production frequency, even at the same Ca .

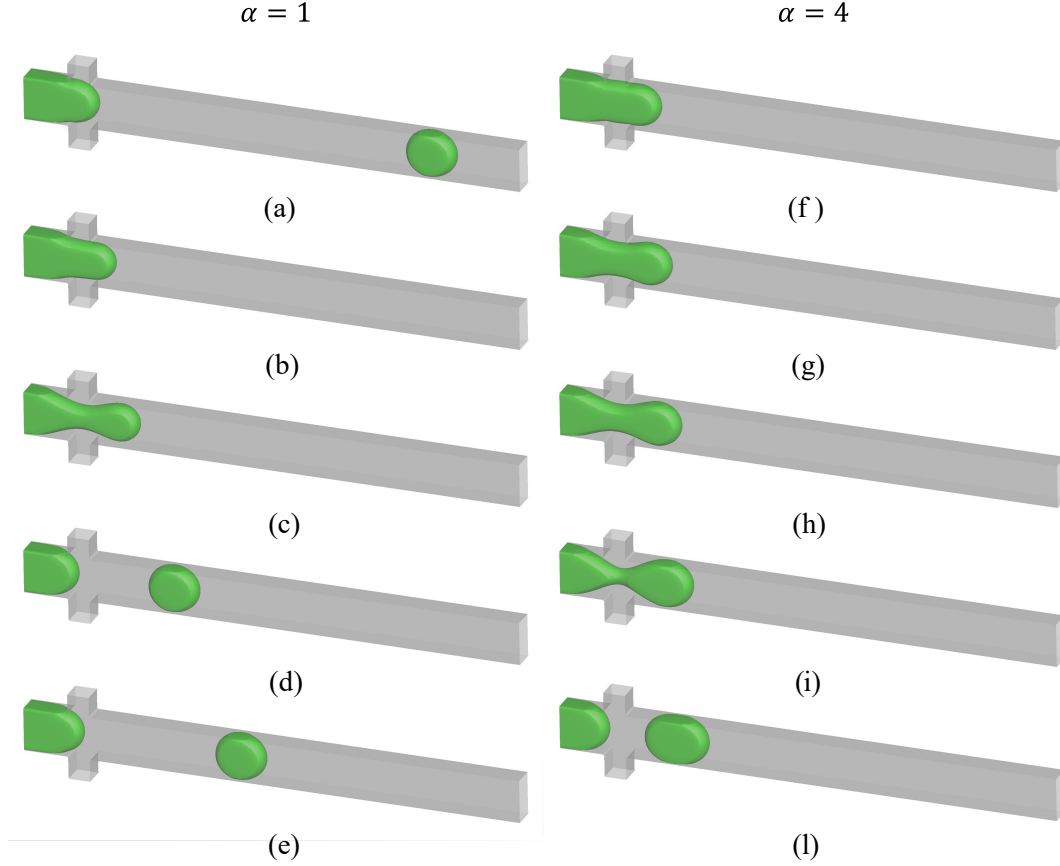


Figure 3.10: Droplet generation process for flow rate ratio $Q = 6$, Capillary number $Ca = 0.006$, and two different viscosity ratio α at (a)-(f) $t = 3.6$ s, (b)-(g) $t = 3.8$ s, (c)-(h) $t = 3.9$ s, (d)-(i) $t = 4$ s and (e)-(l) $t = 4.1$ s.

Finally, it is necessary to operate in the dripping regime to obtain very small and nearly spherical droplets. As previously shown, L_D becomes progressively less sensitive to changes in Ca in this regime, meaning that significantly higher Ca are required to further reduce droplet size. Numerically, however, increasing Ca by raising u_c or μ_c is limited by stability constraints. Therefore, an effective alternative is to decrease σ , which increases Ca while leaving the bulk flow field essentially unaffected, since σ acts only at the interface.

Two flow rate ratios, $Q = 6$ and $Q = 12$, are considered, while the flow rate of the dispersed phase is kept fixed at $Q_d = 0.02$ lu. Variations in σ in the dripping regime are simulated using $\sigma = [0.010, 0.006, 0.002]$ lu, which correspond to $Ca = [0.048, 0.08, 0.24]$ when $Q = 6$ and to $Ca = [0.096, 0.16, 0.48]$ when $Q = 12$. For $Q = 6$, the normalized droplet lengths are $L_D/w = 0.8859, 0.8088, \text{ and } 0.6668$ for $\sigma = 0.010, 0.006, \text{ and } 0.002$ lu, respectively. For $Q = 12$, the droplet size further decreases, yielding $L_D/w = 0.7008, 0.6184, \text{ and } 0.4966$ for the same values of σ , respectively.

Although the simulations are conducted in the dripping regime ($Ca > 0.01$) where viscous forces dominate, interfacial tension still affects both the necking dynamics and the location of breakup. As shown in Fig. 3.11, higher σ produces a longer neck that extends downstream of the cross-junction, delaying detachment, reducing the production frequency, and increasing the spacing between droplets. For any given σ , increasing Q results in smaller droplets, while reducing σ amplifies this effect due to the lower capillary resistance against deformation. In this regime, the combination of low σ and high Q promotes the formation of very small spherical droplets, as shown in Fig. 3.11(d).

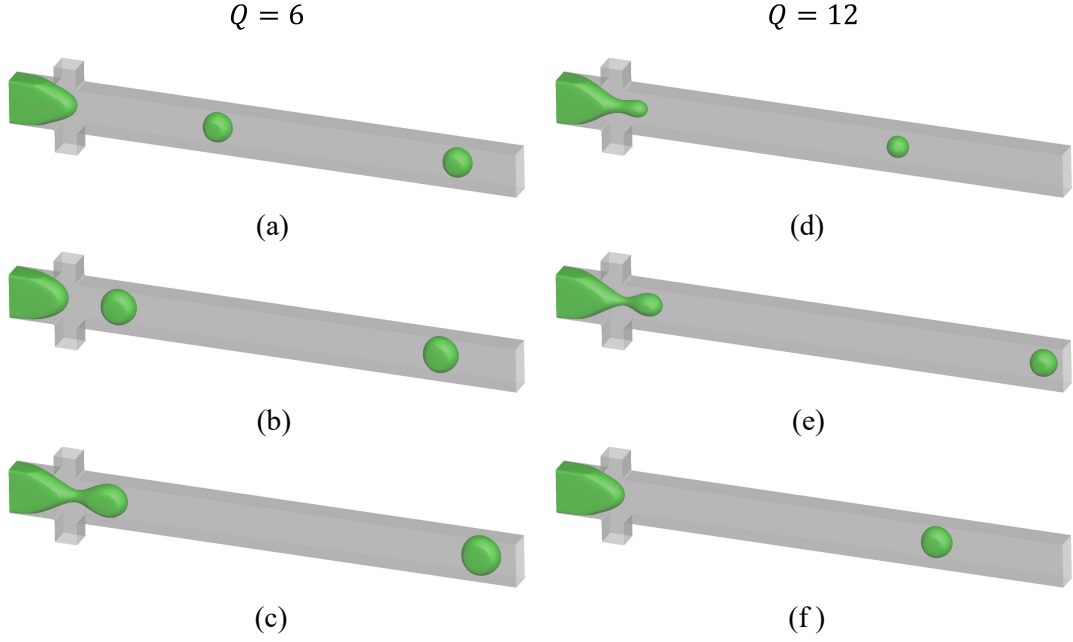


Figure 3.11: Droplet generation as a function of interfacial tension: (a)-(d) $\sigma = 0.002$ lu, (b)-(e) $\sigma = 0.006$ lu, and (c)-(f) $\sigma = 0.010$ lu.

3.4.2 Effect of contact angle

To understand the effect of contact angle on droplet dynamics and formation, as well as on droplet length, morphology and spacing, three sets of simulations are conducted using contact angles of 120° , 150° , and 180° . The viscosity ratio is fixed at $\alpha = 4$. The flow rate of the dispersed phase is fixed at $Q_d = 0.04$ lu, while the flow rate of the continuous phase Q_c is varied to increase the capillary number, with values of $Ca = [0.002, 0.004, 0.008, 0.012, 0.016, 0.02, 0.04]$. In the case $Ca = 0.002$ with a contact angle of 180° , the channel geometry is identical upstream of the junction, while the downstream part is twice as long as in the other simulations. This extension is necessary because the increase in L_D requires additional channel length for complete droplet formation.

As shown in Fig. 3.12, increasing Ca results in a decrease in L_D , which follows a trend fitted by $L_D/w = c_1 Ca^{c_2}$. For $\theta = 120^\circ$, the fit yields $c_1 = 0.101$ and $c_2 = -0.598$, while for $\theta = 150^\circ$ $c_1 = 0.151$ and $c_2 = -0.553$ are obtained. At $\theta = 180^\circ$, the coefficients are $c_1 = 0.082$ and $c_2 = -0.736$, indicating a stronger sensitivity of the droplet length to variations in Ca under fully hydrophobic conditions. These differences confirm that wettability plays a non-negligible role in droplet generation and breakup dynamics.

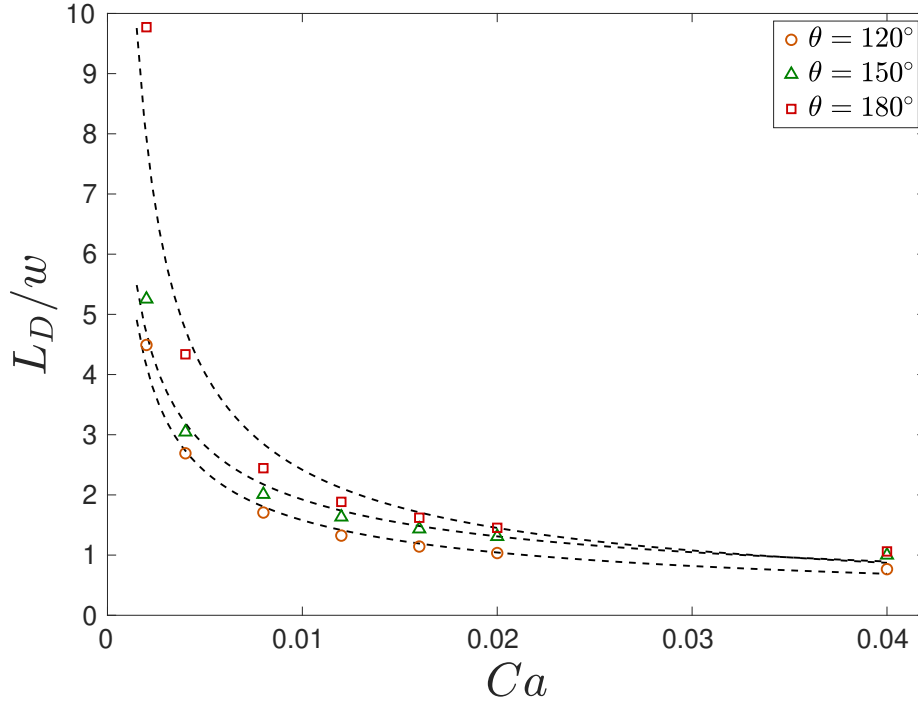


Figure 3.12: The droplet length as a function of Ca for three different contact angle. The dashed curves are obtained by fitting the simulated data.

Moreover, the contact angle influences not only the length and spacing of the droplets, but also their morphology, as shown in Fig. 3.13. For low Ca (squeezing regime), droplets in less hydrophobic channels come into contact with the channel walls over a small area, resulting in narrow droplets that span the channel width (Fig. 3.13(a)). As the contact angle increases, the presence of a stable fluid film between the droplet and the wall promotes the formation of longer droplets (Fig. 3.13(b)-(c)). However, as Ca increases, this difference becomes less pronounced. In the dripping regime, viscous forces dominate, leading to the formation of smaller droplets (Fig. 3.13(d)-(e)-(f)). At low contact angle values (Fig. 3.13(d)), the droplets come into contact with the walls, while at higher contact angles, the fluid film enhances the formation of more spherical droplets (3.13(e)-(f)).

Finally, it is noteworthy that for $\theta = 120^\circ$ and $Ca = 0.04$, the generated droplets have a length L_D of $0.765w$ (15.3 μ m). Given that the interface thickness is $0.25w$ (4 μ m), simulations at higher capillary numbers are not feasible in channels with this level of hydrophobicity due to insufficient resolution.

3.5 Multicomponent droplet generation in ternary fluid systems

Accurate control over ternary fluid systems, e.g. as required in alternating droplet generation, is crucial for producing droplets with distinct reagents or varying material compositions, useful for coupled or sequential chemical reactions, or for encapsulating

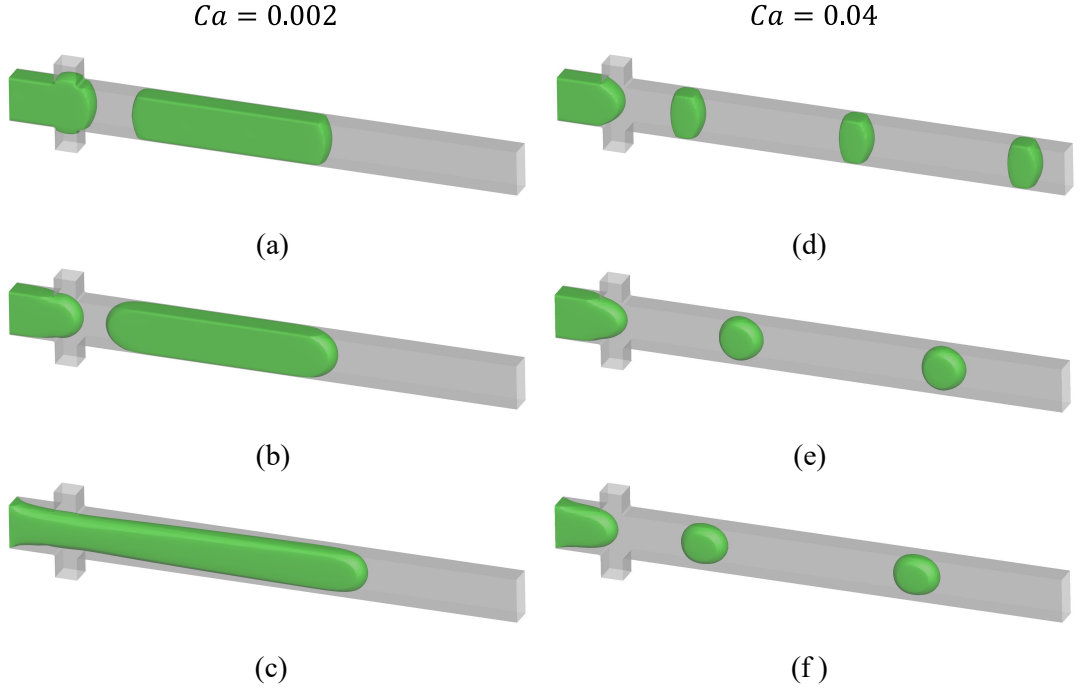


Figure 3.13: Droplet generation in the squeezing ($Ca < 0.01$) and dripping ($Ca > 0.01$) regimes for three different contact angles: (a)-(d) $\theta = 120^\circ$, (b)-(e) $\theta = 150^\circ$, and (c)-(f) $\theta = 180^\circ$.

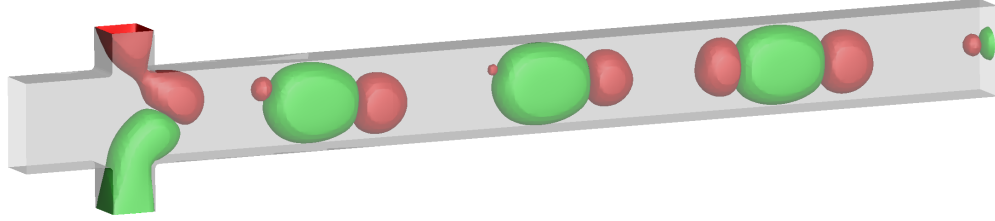
different cells in separate droplets, thereby creating independent microreactors ideal for rapidly and simultaneously testing multiple conditions. Moreover, the generation of different types of droplets, e.g. Janus or multicomponent droplets [122, 123], can be achieved by tuning the properties of fluids, microdevice geometry, and operating conditions.

In view of a more comprehensive study, a preliminary investigation into droplet generation in ternary immiscible fluid systems is proposed. Employing the same geometry described in Section 3.4, a flow rate $Q_3 = 12 \text{ mL/min}$ of the continuous phase, i.e. the oil (density $\rho_3 = 1000 \text{ kg/m}^3$ and viscosity $\mu_3 = 0.1 \text{ Pa s}$), is injected through the main channel, whereas two dispersed phases, i.e. the water-based solution (density $\rho_1 = 1000 \text{ kg/m}^3$ and viscosity $\mu_1 = 0.01 \text{ Pa s}$) and perfluorocarbons solution (density $\rho_2 = 2000 \text{ kg/m}^3$ and viscosity $\mu_2 = 0.02 \text{ Pa s}$), flow through the two lateral channels. The interfacial tensions between fluids are $\sigma_{12} = 0.04 \text{ N/m}$, $\sigma_{23} = 0.05 \text{ N/m}$ and $\sigma_{13} = 0.02 \text{ N/m}$, and the contact angles are $\theta_{13} = \theta_{23} = 180^\circ$.

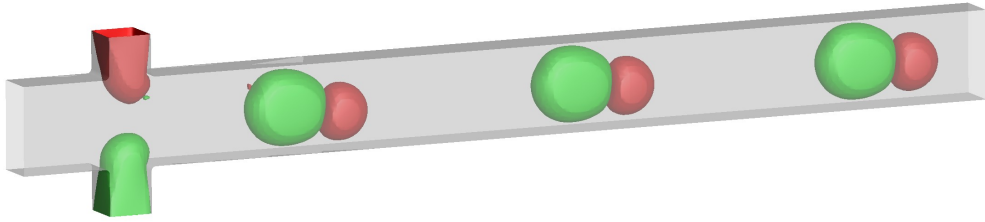
The physical parameters are converted into lattice units using the following reference values: $L_0 = 10^{-5} \text{ m}$, $t_0 = 10^{-6} \text{ s}$ and $m_0 = 10^{-12} \text{ kg}$. The density and viscosity in lattice units are set to $\rho_1 : \rho_2 : \rho_3 = 1 : 2 : 1$ ($\rho_1 = 1 \text{ lu}$) and $\mu_1 : \mu_2 : \mu_3 = 1 : 2 : 10$ ($\mu_1 = 0.1 \text{ lu}$), respectively. The interfacial tensions between phases are set to $\sigma_{12} = 0.04 \text{ lu}$, $\sigma_{23} = 0.05 \text{ lu}$ and $\sigma_{13} = 0.02 \text{ lu}$; the interface thickness ξ and mobility M for all fluids are set to 4 lu and 0.1 lu , respectively. The flow rate of the continuous phase Q_3 is set to 0.2 lu .

Three different flow rate ratios are considered: case I corresponds to $Q_1 : Q_2 : Q_3 = 0.25 : 0.5 : 1$, case II to $0.1 : 0.25 : 1$, and case III to $0.1 : 0.5 : 1$.

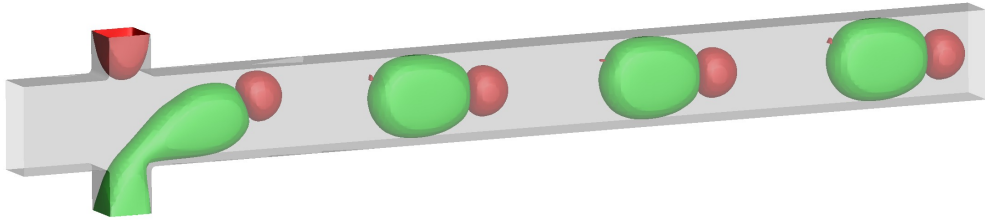
The results show each lateral channel behaving as a T-junction, generating single droplets of phase 1 and phase 2. However, by appropriately selecting the flow rate ratios among the three fluids, the two dispersed phases can come into contact at the junction, leading to the formation of multicomponent droplets (Fig. 3.14). In all cases, it can be observed that, given the selected flow rate ratios, phase 1, upon coming into contact with phase 2, breaks up earlier than phase 2. This leads to the formation of droplets that are not aligned with the channel axis. Subsequently, these droplets rotate until reaching an equilibrium position. The flow rate ratio in Case I produces a repetitive pattern consisting of two asymmetrical droplets followed by a third droplet with a symmetrical morphology, as shown in Fig. 3.14(a). The distance between the droplets has a mean value of $160\text{ }\mu\text{m}$, with a new pattern generated approximately every 0.176 s . By reducing the flow rates of both phase 1 and phase 2, i.e. as in the case II, identical droplets are generated. As a result, the production frequency decreases, and the average droplet spacing rises to $395\text{ }\mu\text{m}$ (Fig. 3.14(b)), with a new droplet generated approximately every 0.087 s . By keeping the flow rate of phase 1 constant and doubling the flow rate of phase 2 compared to case II, it is possible to generate identical droplets while reducing the spacing between them to a mean value of approximately $215\text{ }\mu\text{m}$, with a new droplet generated approximately every 0.0653 s . The increase in the flow rate of phase 2 also leads to a reduction in the droplet length of the phase 1 and an increase in that of phase 2, as the latter comes into contact with phase 1 earlier at the junction, facilitating the breakup of the phase 1 (as shown in Fig. 3.14(c)).



(a)



(b)



(c)

Figure 3.14: Multicomponent droplet generation in ternary fluid systems (red fluid 1 and green fluid 2) with different flow rate ratios $Q_1 : Q_2 : Q_3$ equal to (a) 0.25 : 0.5 : 1, (b) 0.1 : 0.25 : 1, and (c) 0.1 : 0.5 : 1. The 3D view is obtained using isosurfaces with a threshold value of $\phi_1 = 0.5$ and $\phi_2 = 0.5$.

Chapter 4

Mesoscale modeling of nanoparticle dynamics in fluids

4.1 Introduction

Numerous epidemiological studies conducted on astronauts from NASA and the ISS have analyzed the oncological risks and mortality associated with exposure to space radiation. By comparing their health status with that of the general population, it has emerged that astronauts generally enjoy excellent health due to rigorous medical selection criteria, outstanding physical fitness, and access to high-quality medical care. Nevertheless, galactic cosmic rays have been shown to increase the probability of developing fatal cancers. In particular, missions lasting longer than 18–24 months, such as future missions to Mars or prolonged stays on the ISS, may increase the Risk of Exposure-Induced Death from cancer by up to 50%, especially during solar minimum, when the flux of these rays is more intense [138]. Moreover, there is significant concern that space radiation, particularly heavy HZE ions, may induce more lethal and aggressive tumors than those typically observed on Earth, a factor not yet fully incorporated into current risk models. Indeed, the combined exposure to radiation and microgravity appears to play a synergistic role: microgravity slows the repair of radiation-induced DNA damage, thereby increasing genomic instability and the frequency of chromosomal aberrations.

Therefore, current research is focused on elucidating these effects and developing pharmacological countermeasures to protect astronauts during deep-space exploration [139, 140]. Such studies are also essential for improving our understanding of tumorigenesis, and the therapies developed can be translated to Earth. Indeed, to date, cancer remains the second leading cause of non-accidental death worldwide and is diagnosed at an advanced stage, as it is usually asymptomatic in its early stages. Early diagnosis significantly reduces morbidity and mortality, as well as healthcare costs.

The gold standard methods for diagnosing cancer include imaging techniques and tissue biopsy. The latter requires the removal of a small portion of tumor tissue and therefore involves an invasive surgical procedure performed by specialized personnel, with risks of hemorrhage, inflammation, and even the dissemination of tumor cells, providing alternative routes for metastasis.

In recent decades, liquid biopsy has emerged as an early and non-invasive diagnostic method for cancer [141, 142], particularly when tumor samples are inaccessible or difficult to obtain. Unlike tissue biopsy, it can provide histological and molecular information

related to changes occurring during disease progression and treatment. Liquid biopsies can be performed using blood or several other types of biological fluids, such as urine, saliva, sweat, cerebrospinal fluid, ascitic fluid, and breast milk. Some biological fluids, such as urine, provide information about specific types of tumors, such as bladder or colon, whereas blood can potentially provide information about tumors throughout the body. However, blood contains a wide range of cells, vesicles, and molecules with different sizes (from microscale to nanoscale), densities, and shapes, which makes their isolation and detection more challenging.

The most promising line of research in this field is the detection and analysis of exosomes [143–146], nanoscale vesicles (30–150 nm) suspended in numerous biological fluids. These vesicles carry important biological signals to target cells, influencing the body's physiological state, cell communication, immunoregulation, angiogenesis, tumorigenesis, and metastasis [147, 148]. Exosome isolation allows researchers to study physiological processes, pathological developments, or pharmacological responses to therapeutic interventions [149, 150]. Moreover, it enables the early diagnosis of cancer [148, 151, 152] and numerous other diseases, as well as applications in personalized medicine, where exosomes can be used as vehicles for drug delivery [153–155]. Current exosome isolation techniques, such as ultracentrifugation require large sample volumes and pre-treatment steps, and often suffer from low efficiency, poor purity, and long processing times [141].

As a result, there is considerable interest in the use of microfluidic devices [154], which enable multiple analyses using small sample volumes, reduce processing time, and improve throughput. Microfluidic applications employing nanoparticles span fields ranging from biomedicine to materials science. Gold, magnetic, polymeric, and semiconductor nanoparticles are utilized as functionalized probes for on-chip immunoassays, nucleic acid detection, and the ultrasensitive identification of biomarkers or pathogens. Microfluidic systems also enable the synthesis of lipid nanoparticles for mRNA delivery and polymeric nanoparticles for targeted drug release [156]. Microfluidic platforms are particularly suited to study colloidal suspensions, providing a versatile tool to investigate the behavior of nanoparticles and other suspended particles in a fluid. In many biological and material systems, these colloidal suspensions consist of particles interacting among themselves and with the surrounding fluid, which critically influences system behavior. Advanced materials can be fabricated through colloidal self-assembly and the controlled synthesis of nanocomposites, as well as through the monitoring of nanoparticles in solution or the integration of nanoparticulate catalysts into micro-electrochemical systems for environmental and energy applications [157].

Moreover, microfluidic studies of colloidal suspensions allow the investigation of fundamental phenomena such as Brownian diffusion, electrostatic interactions, aggregation, inertial migration, and non-Newtonian rheology, with direct implications for materials engineering, industrial processing, and complex biological fluids [158, 159]. Experimental techniques, including microscopy and scattering methods, often suffer from limitations in both temporal and spatial resolution. For this reason, mesoscale numerical simulations have become an essential tool for gaining deeper insight into particle dynamics and enabling predictive modeling of complex particulate systems.

4.1.1 State of the Art of Numerical Models

Depending on the level of complexity of the suspensions under investigation, different theoretical and computational frameworks can be adopted. These range from the funda-

mental description of Brownian motion established by A. Einstein and M. Smoluchowski, to the classical Derjaguin-Landau-Verwey-Overbeek (DLVO) theory and further to advanced computational approaches.

The dynamics of nanoparticles suspended in a fluid arises from the interplay between deterministic interactions and stochastic thermal fluctuations. At the microscale, continuous collisions with solvent molecules induce random motion, known as Brownian motion, which plays a fundamental role in transport, diffusion, and particle interactions in colloidal systems. A widely adopted theoretical framework to describe such dynamics is provided by the Langevin equation, in which particle dynamics results from the balance between conservative forces, hydrodynamic drag, and a stochastic force accounting for thermal fluctuations. In the overdamped limit, where particle inertia is negligible, this description reduces to Brownian Dynamics (BD), while retaining inertia leads to Langevin Dynamics (LD). In both cases, the stochastic and dissipative forces are related through the fluctuation-dissipation theorem, ensuring that the system reaches the correct thermodynamic equilibrium.

In the Langevin framework, the dynamics of a Brownian particle is described as a stochastic process. The motion is random because it results from continuous collisions with the surrounding fluid molecules. Under equilibrium conditions, the process is assumed to be stationary, so that statistical quantities such as the mean and variance of the particle velocity do not depend on time. Moreover, the dynamics is commonly treated as ergodic, meaning that macroscopic properties, such as the diffusion coefficient, can be obtained either by averaging over an ensemble of particles or by following the time evolution of a single particle for a sufficiently long time. Finally, the process is assumed to be Markovian, i.e., memoryless, since the correlation time of the stochastic force is much shorter than the characteristic relaxation time of the particle.

Modeling such systems requires accounting for the coupling between particle dynamics and the hydrodynamic behavior of the surrounding fluid. In this context, two main modeling strategies can be identified, which differ in the way the solvent is represented and in how hydrodynamic interactions are accounted for. In implicit-solvent approaches, the fluid is not modeled explicitly; instead, its effect on the suspended particles is incorporated through effective forces or mobility relations. In these approaches, hydrodynamic interactions, when included, are introduced in a prescribed manner rather than arising from the underlying dynamics. In contrast, explicit-solvent approaches retain additional degrees of freedom to represent the fluid itself, either through discrete particles or a discretized continuum description. As a result, hydrodynamic interactions are not imposed a priori but emerge naturally from the coupled evolution of the fluid and the suspended particles.

Fast Lubrication Dynamics (FLD) is widely used as an efficient approximation of Stokesian Dynamics for suspensions in a continuum solvent [160]. In this framework, only the equations of motion of the colloidal particles are integrated, while the solvent is treated implicitly through hydrodynamic resistance and thermal fluctuations. The hydrodynamic resistance is decomposed into an isotropic contribution accounting for long-range many-body interactions and a pairwise lubrication term describing short-range interactions when particles approach near contact. Two main formulations are commonly used. The inertial FLD (iFLD) retains particle inertia and corresponds to a Langevin description including hydrodynamic interactions. The equations of motion of the colloids sum Brownian, hydrodynamic, and conservative forces, and particle positions and velocities are updated using standard molecular-dynamics integration schemes.

This formulation can resolve time scales smaller than the Brownian relaxation time τ_r , but numerical stability and accuracy require time steps significantly smaller than τ_r . In contrast, the non-inertial FLD (nFLD) neglects particle inertia and corresponds to the overdamped, Brownian-dynamics limit with hydrodynamic interactions. The net force on each particle vanishes instantaneously, and velocities are obtained from a linear system involving the hydrodynamic resistance matrix, typically solved using a conjugate-gradient algorithm. This allows time steps larger than τ_r , although each step is computationally more expensive. Both formulations assume instantaneous momentum relaxation in the solvent, corresponding to the Stokes regime. In practice, the choice between the two approaches often depends on the conservative interaction potential. For very steep potentials (e.g., hard-sphere interactions), the time step is already limited by the need to prevent particle overlap, making iFLD often more efficient. Conversely, for soft potentials, nFLD can exploit much larger time steps to reach longer physical time scales. In the absence of hydrodynamic interactions, the corresponding implicit-solvent descriptions reduce to Brownian Dynamics in the overdamped limit and to Langevin Dynamics (LD) when particle inertia is retained [161, 162].

The Multi-Particle Collision Dynamics (MPCD) method, originally known as Stochastic Rotation Dynamics (SRD) [163], is an explicit-solvent, particle-based simulation technique derived from models of rarefied gas flows. In this approach, the solvent is represented by point-like particles that do not interact directly over long range. Fluid behavior emerges from a simple algorithm alternating between propagation and collision steps. During propagation, solvent particles move ballistically with constant velocity until the next collision stage. In the collision step, the simulation space is divided into cubic cells, or bins, where all particles within a cell exchange momentum collectively. This procedure conserves linear momentum and total energy, reproducing microscopic collisions and generating correct macroscopic hydrodynamic behavior. Several schemes exist for implementing collisions. In the SRD approach, particle velocities relative to the cell's center-of-mass velocity are rotated around a randomly chosen orthogonal axis, preserving their magnitude while exchanging momentum. Alternatively, the MPC-AT (Andersen Thermostat) [164] scheme samples particle velocities from a Maxwell-Boltzmann distribution at the system temperature, with a correction to ensure total linear momentum conservation within the cell. When simulating colloidal suspensions, solvent and solid particles are coupled via collision detection. During streaming, if a solvent particle intersects a colloid surface, a perfectly elastic collision enforces the no-slip boundary condition. To prevent solid particles from artificially reducing local solvent density and viscosity, virtual particles are introduced inside the solids. These particles participate in collisions within overlapping bins, preserving hydrodynamic properties without representing real solvent molecules. MPCD offers flexibility for complex geometries, such as microchannels or confined systems, and is relatively straightforward to implement compared with other mesoscale fluid dynamics methods. Extensions allow simulation of non-Newtonian solvents by introducing elastic interactions among solvent particles, e.g., via harmonic springs to reproduce viscoelastic effects. Challenges include mapping simulation parameters onto real physical properties, since diffusion and viscosity are highly sensitive to algorithmic details, such as the collision scheme and cell size. Additionally, the Schmidt number, defined as momentum diffusion over mass diffusion, in MPCD is typically lower than in real liquids, though it can be increased by reducing the collision time step [161, 162].

Dissipative Particle Dynamics (DPD) is an explicit-solvent simulation technique in

which the fluid is modeled as particles interacting through soft potentials [165–167]. Unlike classical molecular dynamics with steep short-range forces, DPD interactions are smoother, allowing significantly larger time steps, which makes the method well suited for mesoscale studies of colloids and complex fluids. In DPD, the force between two particles is the sum of three contributions: a conservative force providing soft repulsion and determining fluid compressibility; a dissipative force acting as friction and reducing relative velocities to model solvent viscosity; and a random force introducing thermal fluctuations. Dissipative and random forces are correlated, and together with the conservative force, they preserve system temperature and total linear momentum, ensuring correct macroscopic hydrodynamics. For colloidal suspensions, solvent-colloid coupling is introduced. Early models represented colloids as rigid clusters of DPD particles, while more recent approaches treat each colloid as a single solid sphere. A shifted Lennard-Jones potential prevents solvent penetration and enforces proper interaction at the colloid surface. Despite its versatility, DPD presents challenges. Mapping simulation parameters onto real physical properties is nontrivial, as macroscopic quantities like viscosity cannot be directly predicted. Parameters such as the friction coefficient require calibration. At high volume fractions (i.e., $\phi > 0.3$), the finite size of DPD particles can induce artificial gel-like structures, restricting colloid motion and affecting dynamic properties. Standard DPD also primarily transfers linear momentum, so colloid rotation and accurate shear viscosity require advanced variants like the Fluid Particle Model [168, 169]. Nevertheless, DPD remains a powerful tool for complex and non-Newtonian fluids. Viscoelastic solvents can be modeled by connecting particles via harmonic potentials, reproducing polymeric or additive behavior. While less precise than specialized methods such as FLD for simple spherical colloids, DPD is highly effective for studying nonlinear rheology, phase transitions, and mesoscale phenomena in colloidal suspensions when solvent-colloid interactions are properly calibrated [161, 162].

4.1.1.1 Forced lattice Boltzmann method for Newtonian fluids

In contrast to the particle-based approaches discussed above, the Lattice Boltzmann (LB) method treats the solvent as a discretized continuum and provides an efficient mesoscale framework for coupling fluid flow with nanoparticle dynamics. Hydrodynamic interactions with nanoparticles are incorporated through a forcing term in the Boltzmann equation. The implementation of this forcing term within the governing equations remains a critical issue, which has led to the development of numerous approaches over the years. These approaches can be grouped into three main categories based on how the force is incorporated: via the equilibrium distribution function, as an independent source term, or through a combination of both [170].

Among the early approaches, Shan and Chen's [28] method was developed for multiphase and multicomponent systems. It incorporates the force by shifting the macroscopic velocity in the equilibrium function. While straightforward, this approach does not conserve local momentum and requires defining a physical velocity to correct the difference between the momentum before and after the collision. In contrast, Luo's [171, 172] method adds the force solely as a source term without modifying the velocity field. This approach is effective for spatially constant forces; however, when the force varies, Galilean invariance is lost and the resulting energy equation becomes inaccurate.

Subsequent developments led to the work of He et al., initially in 1997 [173], which introduced the conceptual formulation of the forcing term in the LB BGK model and

provided a basis for understanding how forces affect the macroscopic flow. This formulation was sufficient for problems such as simple flows and early multiscale coupling with particle methods. In 1998, He et al. [174] further refined the scheme to account for discrete lattice effects, ensuring the exact recovery of the Navier–Stokes equations, preserving Galilean invariance for spatially varying forces, and maintaining consistency between microscopic and macroscopic velocities.

Other methods, including those of Martys et al. [175] and Luo [176], employ Taylor series expansions or Hermite polynomials to handle both constant and variable forces but introduce additional error terms in the continuity equation. Guo et al.’s [177] method is currently the most widely used and is considered the most accurate for unstable and variable flows, as it corrects both the continuity and Navier–Stokes equations through a combination of velocity shifting and source-term implementation.

The Exact Difference Method (EDM) proposed by Kupershtokh et al. [178] computes the force as the exact difference between equilibrium distribution functions, allowing strong forcing to be accurately treated, such as in liquid–vapor transition layers. Building on this approach, Khazaeli et al. [179] modified the EDM to ensure the correct recovery of the Navier–Stokes equations. This scheme addresses the limitations of the Kupershtokh method when dealing with non-uniform forces.

Despite differing theoretical foundations and claims of superiority by various authors, tests on natural convection in a heated cavity, a problem characterized by a spatially varying force field, show that for single-phase flows, all approaches produce similar results consistent with reference solutions. Therefore, the choice of the optimal forcing scheme primarily depends on the nature of the problem, such as whether the forces are constant or variable, single-phase or multiphase, and on the requirement for momentum conservation [170].

4.1.2 Present Contribution

Here, the aim is to extend an existing LB framework for Newtonian fluids [31] to colloidal suspensions where the presence of nanoparticles is taken into account by means of a forcing term in the Boltzmann equation and the nanoparticle dynamics is described using the Langevin equation.

4.1.2.1 Coupling a forced lattice Boltzmann Method with Langevin Dynamics

In this context, a multiscale framework obtained by coupling the lattice Boltzmann (LB) method and the Langevin dynamics (LD) [41, 42] has proven to be a suitable three-dimensional multiscale approach for performing parametric studies. A two-way coupled LB-LD method is adopted to investigate the transport of nanoparticles (NPs) in microchannels (Fig. 4.1). In this hybrid approach, the solvent is discretized on a lattice via LB method, while colloidal particles are propagated using Langevin dynamics. Hydrodynamic interactions naturally arise from the coupling between particle motion and the fluid field, offering an effective framework for simulating complex nanoparticle–fluid systems.

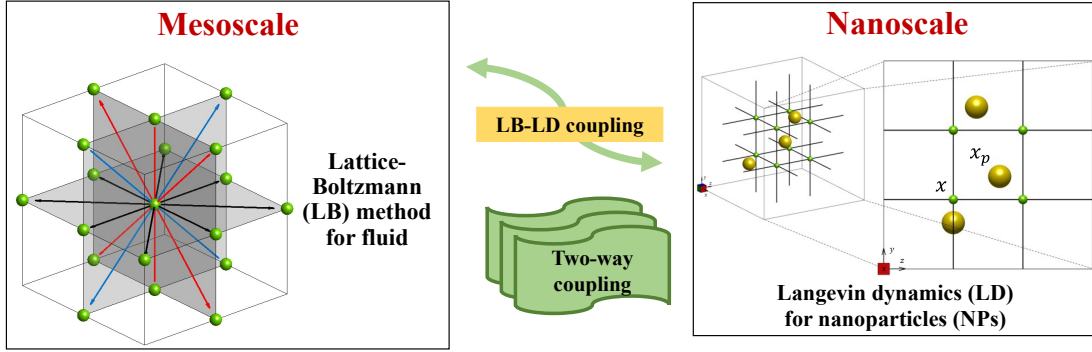


Figure 4.1: The developed framework for simulating the transport of nanoscale particles in microchannels: the Lattice-Boltzmann method for the fluid phase dynamics and the Langevin model for nanoscale particle dynamics.

4.2 Methods

The dynamics of the coupled nanoparticle–fluid system is described by the Navier–Stokes equations for an incompressible and isothermal Newtonian fluid,

$$\nabla \cdot \mathbf{u}(\mathbf{x}, t) = 0, \quad (4.1)$$

$$\rho \left[\frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial t} + (\mathbf{u}(\mathbf{x}, t) \cdot \nabla) \mathbf{u}(\mathbf{x}, t) \right] = -\nabla p(\mathbf{x}, t) + \mu \nabla^2 \mathbf{u}(\mathbf{x}, t) + \mathbf{F}(\mathbf{x}, t), \quad (4.2)$$

where ρ , μ , $\mathbf{u}$, and p denote the fluid density, dynamic viscosity, velocity field, and pressure, respectively; the term $\mathbf{F}(\mathbf{x}, t)$ represents the force per unit volume exerted on the fluid by the nanoparticles. The dynamics of an individual nanoparticle p is described by the Langevin equation

$$m_p \frac{d\mathbf{u}_p(t)}{dt} = \tilde{\mathbf{F}}_p(t) + \tilde{\mathbf{S}}_p(t) + \tilde{\mathbf{C}}_p(t), \quad (4.3)$$

where m_p and $\mathbf{u}_p$ are the mass and velocity of a nanoparticle p , respectively, which is considered neutrally buoyant. The friction force $\tilde{\mathbf{F}}_p(t)$, exerted by the fluid, is described by Stokes' Law. The random force $\tilde{\mathbf{S}}_p(t)$ is modeled as white noise with zero mean value and an auto-correlation function defined by the fluctuation-dissipation theorem. The conservative force, $\tilde{\mathbf{C}}_p(t)$, describes the particle-particle and particle-wall short-distance interactions. Accordingly, the forcing term acting on the fluid is given by

$$\mathbf{F}(\mathbf{x}, t) = - \sum_{p=1}^{n_{tot}} \tilde{\mathbf{F}}_p(t) \delta(\mathbf{x} - \mathbf{x}_p(t)), \quad (4.4)$$

where n_{tot} is the total number of nanoparticles suspended in the fluid, and δ is the three-dimensional Dirac-delta function, which inherently carries the dimension $[\text{m}^{-3}]$ as its argument is a spatial variable.

The friction force exerted by the fluid, $\tilde{\mathbf{F}}_p$, follows the Stokes' law,

$$\tilde{\mathbf{F}}_p(t) = -\zeta [\mathbf{u}_p(t) - \mathbf{u}(\mathbf{x}_p, t)], \quad (4.5)$$

where $\mathbf{u}_p(t)$ is the velocity of the p -th nanoparticle and $\mathbf{u}(\mathbf{x}_p, t)$ is the fluid velocity at the position $\mathbf{x}_p$ of the p -th nanoparticle at time t (given by Eq. 4.25). For a Brownian

nanoparticle at low Reynolds number, the friction coefficient is assumed to follow Stokes' drag law,

$$\zeta = 3\pi\mu d_p\psi, \quad (4.6)$$

where μ is the dynamic viscosity of the fluid, d_p is the diameter of the nanoparticle and ψ is the shape factor, e.g. equal to 1 for a spherical particle.

In the Langevin equation, the stochastic force $\tilde{\mathbf{S}}_p(t)$ accounts for thermal fluctuations arising from molecular interactions with the surrounding fluid. It is modeled as a Gaussian white-noise process with zero mean and a correlation time much shorter than the Brownian relaxation time m_p/ζ . Under this Markovian approximation, $\tilde{\mathbf{S}}_p(t)$ and $\tilde{\mathbf{S}}_p(t')$ are treated as independent random variables for $t \neq t'$, and their autocorrelation is given by $\langle \tilde{\mathbf{S}}_p(t) \tilde{\mathbf{S}}_p(t') \rangle = a\delta(t - t')$, where a is a constant determined by the fluctuation-dissipation theorem and δ is the Dirac-delta function. Each component of the stochastic force, $\tilde{S}_{p,i}$, exhibits a Gaussian distribution with zero mean and autocorrelation function defined as

$$\langle \tilde{S}_{p,i}(t) \rangle = 0, \quad (4.7)$$

$$\langle \tilde{S}_{p,i}(t) \tilde{S}_{p,i}(t') \rangle = 2k_B T \zeta \delta(t - t'), \quad (4.8)$$

where $i \in \{x, y, z\}$, k_B is the Boltzmann constant, T is the fluid temperature and $\langle \rangle$ is the average over the ensemble of the realizations of the random variables.

The conservative force, $\tilde{\mathbf{C}}_p$, accounts for short-range interactions between nanoparticles as well as interactions with the confining walls. It is decomposed as

$$\tilde{\mathbf{C}}_p = \tilde{\mathbf{C}}_p^{NP-NP} + \tilde{\mathbf{C}}_p^{NP-wall} + \tilde{\mathbf{C}}_p^{other}, \quad (4.9)$$

where the first term represents particle-particle interactions, the second term accounts for particle-wall interactions, and the third term includes additional contributions such as electrostatic or bond forces. The particle-particle interaction force is derived from a pairwise potential as

$$\tilde{\mathbf{C}}_p^{NP-NP} = - \sum_{\substack{j=1 \\ j \neq p}}^{n_{tot}} \frac{dU_{NP-NP}(r_{pj})}{dr_{pj}} \frac{\mathbf{r}_{pj}}{r_{pj}}, \quad (4.10)$$

where $\mathbf{r}_{pj} = \mathbf{x}_p - \mathbf{x}_j$ is the vector connecting particle j to particle p , and $r_{pj} = |\mathbf{r}_{pj}|$ is the corresponding distance. Similarly, the particle-wall interaction force is obtained from

$$\tilde{\mathbf{C}}_p^{NP-wall} = - \frac{dU_{NP-wall}(r_{pw})}{dr_{pw}} \mathbf{n}_w, \quad (4.11)$$

where r_{pw} is the minimum distance between the particle and the wall, and $\mathbf{n}_w$ is the unit vector normal to the wall surface pointing toward the particle. In this work, both particle-particle and particle-wall interactions are modeled using Lennard-Jones-type potentials. The particle-particle interaction potential is defined as

$$U_{NP-NP}(r_{pj}) = 4\epsilon \left[\left(\frac{\sigma}{r_{pj}} \right)^{12} - \left(\frac{\sigma}{r_{pj}} \right)^6 \right], \quad (4.12)$$

while the particle-wall interaction is modeled as a purely repulsive potential,

$$U_{NP-wall}(r_{pw}) = 4\epsilon \left[\left(\frac{\sigma}{r_{pw}} \right)^{12} \right]. \quad (4.13)$$

Here, ϵ represents the potential well depth, set to $\epsilon = 2k_B T$, and σ is the characteristic interaction length, chosen as $\sigma = 2.5d_p$. The term $\tilde{\mathbf{C}}_p^{other}$ can be used to incorporate additional interactions, such as electrostatic forces or bonding potentials, depending on the specific application.

4.2.1 Lattice Boltzmann method for Newtonian fluids

The multiscale approach proposed by Liu et al. [41, 42] is adopted and implemented to investigate the dynamics of nanoparticles suspended in a Newtonian fluid. The p -th nanoparticle is treated as a subgrid point particle (see Fig. 4.2), while the Navier–Stokes equations are solved on a three-dimensional computational domain (D3Q19), where 19 scalar distribution functions $f_\alpha(\mathbf{x}, t)$ ($\alpha = 0, \dots, 18$) represent particles of position $\mathbf{x}$ at time t .

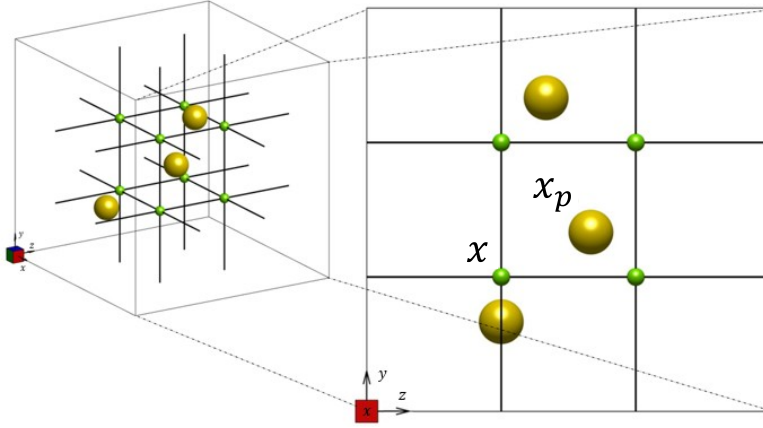


Figure 4.2: A 3D representation of lattice nodes (green) defined at site $\mathbf{x}$ and of sub-lattice NPs (yellow) at positions $\mathbf{x}_p$.

At each time step Δt , $f_\alpha(\mathbf{x}, t)$ stream to neighboring nodes with discrete velocity $\mathbf{e}_\alpha$,

$$\mathbf{e}_\alpha = \frac{\Delta \mathbf{x}}{\Delta t} \begin{cases} (0, 0, 0) & \alpha = 0 \\ (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1) & \alpha = 1, 2, \dots, 6 \\ (\pm 1, \pm 1, 0), (\pm 1, 0, \pm 1), (0, \pm 1, \pm 1) & \alpha = 7, 8, \dots, 18 \end{cases}, \quad (4.14)$$

where the lattice time step Δt and length $\Delta x = \Delta y = \Delta z$ is set equal to 1, and then collide, according to the Boltzmann equation as

$$f_\alpha(\mathbf{x} + \Delta t \mathbf{e}_\alpha, t + \Delta t) - f_\alpha(\mathbf{x}, t) = -\frac{\Delta t}{\tau} [f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t)] + \mathcal{F}_\alpha(\mathbf{x}, t). \quad (4.15)$$

The equilibrium state $f_\alpha^{eq}(\mathbf{x}, t)$ is reached over the unique relaxation time τ (here set equal to 1),

$$f_\alpha^{eq}(\mathbf{x}, t) = \omega_\alpha \rho \left[1 + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})}{c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})^2}{2c_s^4} - \frac{(\mathbf{u} \cdot \mathbf{u})}{2c_s^2} \right], \quad (4.16)$$

where ρ and $\mathbf{u}$ are the fluid density and velocity evaluated at the position $\mathbf{x}$ and time t , $\omega_0 = 1/3$, $\omega_{1-6} = 1/18$, $\omega_{7-18} = 1/36$ are the weighting factors. The relaxation time τ is related to the solvent kinematic viscosity according to $\nu = c_s^2 (\tau - 0.5\Delta t)$, where

$c_s = \Delta x / (\sqrt{3}\Delta t)$. The forcing term accounting for the presence of NPs, $\mathcal{F}_\alpha(\mathbf{x}, t)$, based on the He et al. [173] method, is defined as

$$\mathcal{F}_\alpha(\mathbf{x}, t) = \frac{\omega_\alpha \mathbf{J}(\mathbf{x}, t) \cdot \mathbf{e}_\alpha}{c_s^2}. \quad (4.17)$$

The impulse density, $\mathbf{J}(\mathbf{x}, t)$, arises from the interactions between the fluid and the nanoparticles. A cubic support domain of 27 Eulerian lattice nodes is defined and centered on the lattice node closest to the nanoparticle p . Denoting by n_p the total number of nanoparticles whose support domain contains the lattice node at position $\mathbf{x}$, and by p the index of a generic particle located at position $\mathbf{x}_p$, the impulse density is defined as

$$\mathbf{J}(\mathbf{x}, t) = - \sum_{p=1}^{n_p} w(\mathbf{x}, \mathbf{x}_p) \frac{\tilde{\mathbf{F}}_p(t) + \tilde{\mathbf{S}}_p(t)}{(\Delta x)^3} \Delta t, \quad (4.18)$$

where the weight function is defined by Peskin's stencil [180] to have second-order accuracy:

$$w(\mathbf{x}_j, \mathbf{x}_p) = \frac{1 + \cos\left[\frac{\pi(x_j - x_p)}{2\Delta x}\right]}{4\Delta x} \frac{1 + \cos\left[\frac{\pi(y_j - y_p)}{2\Delta y}\right]}{4\Delta y} \frac{1 + \cos\left[\frac{\pi(z_j - z_p)}{2\Delta z}\right]}{4\Delta z}. \quad (4.19)$$

The Chapman–Enskog procedure leads to the Navier–Stokes equations where the fluid density and velocity are computed as the moments of the distribution functions:

$$\rho(\mathbf{x}, t) = \sum_{\alpha=0}^{18} f_\alpha(\mathbf{x}, t) \quad (4.20)$$

$$\rho(\mathbf{x}, t) \mathbf{u}(\mathbf{x}, t) = \sum_{\alpha=0}^{18} f_\alpha(\mathbf{x}, t) \mathbf{e}_\alpha \quad (4.21)$$

and the pressure p is recovered from the density through the equation of state

$$p(\mathbf{x}, t) = c_s^2 \rho(\mathbf{x}, t). \quad (4.22)$$

Finally, the non-equilibrium bounce back (NEBB) method proposed by Zou and He [35] to enforce Dirichlet boundary condition on velocity or pressure can here be adapted to compute the unknown distribution functions $f_\alpha(\mathbf{x}, t)$ at the boundaries (details are reported in Appendix B).

4.2.2 Langevin dynamics for nanoparticles

To limit the computational cost, the fluid and particle dynamics are advanced with the same time step Δt , the Gaussian distribution of the stochastic force is implemented via a random number generator using the Box-Muller transformation [181], and the eq. (4.8) and the eq. (4.9) are truncated at an inter-particle distance equal to 3σ .

The Brownian particle velocity and displacement are updated at each time step by

$$\mathbf{u}_p(t) = \begin{cases} \mathbf{u}(\mathbf{x}_p, t) + \frac{1}{\zeta} [\tilde{\mathbf{C}}_p(t) + \tilde{\mathbf{S}}_p(t)] & \text{for } \Delta t > \tau_r \\ \mathbf{u}_p(t) + \frac{\Delta t}{m_p} [\tilde{\mathbf{C}}_p(t) + \tilde{\mathbf{S}}_p(t) + \tilde{\mathbf{F}}_p(t)] & \text{for } \Delta t \leq \tau_r \end{cases} \quad (4.23)$$

and

$$\mathbf{x}_p(t + \Delta t) = \mathbf{x}_p(t) + \Delta t \mathbf{u}_p(t + \Delta t) \quad (4.24)$$

where the fluid velocity at the position $\mathbf{x}_p$ of the p -th nanoparticle at time t , $\mathbf{u}(\mathbf{x}_p, t)$, is obtained as the interpolation of 27 neighbouring lattice nodes

$$\mathbf{u}(\mathbf{x}_p, t) = \sum_{j=1}^{27} w(\mathbf{x}_j, \mathbf{x}_p) \mathbf{u}(\mathbf{x}_j, t), \quad (4.25)$$

and the Brownian relaxation time, τ_r , over which the particle velocity decays to the algebraic long-time tail regime is

$$\tau_r = \frac{m_p}{\zeta}. \quad (4.26)$$

The Stokes number, $St = \frac{\tau_r}{\Delta t}$, is useful to characterize the short-time particle inertial effect. When $St < 1$, the particle has no inertia and follows the fluid streamline, so the over-damped Langevin equation is solved without using sub-time steps. When $St \geq 1$, the particle could not strictly follow the fluid streamline and the under-damped Langevin equation is solved. To ensure numerical stability, $|1 - \Delta t \frac{\zeta}{m_p}| \leq 1$, i.e. $St \geq 0.5$.

4.2.3 Algorithm

The multi-CPU framework is developed by extending the framework previously presented in the works of De Marinis et al. [31] to applications involving nanoparticle dynamics. The implemented algorithm is described below and summarized in Fig. 4.3.

The initialization:

1. initialize velocity $\mathbf{u}$, pressure p and density ρ at all fluid domain nodes;
2. initialize the density distribution function $f_a(\mathbf{x}, t) = f_a^{eq}(\mathbf{x}, t)$, by using Eq. (4.16);
3. update velocity $\mathbf{u}$, density ρ and pressure p (Eqs. (4.20), (4.21) and (4.22));
4. initialize position $\mathbf{x}_p$ and velocity $\mathbf{u}_p$ of the p -th nanoparticle using the random number generator and the interpolation of 27 neighboring lattice nodes with Peskin's weight function (Eqs. (4.25) and (4.19)), respectively.

The iterative step:

5. calculate the friction force $\tilde{\mathbf{F}}_p$ (Eq. (4.5)), stochastic force $\tilde{\mathbf{S}}_p$ (Eq. (4.8)) and conservative force $\tilde{\mathbf{C}}_p$ (Eq. (4.9)) of the p -th nanoparticle;
6. update position $\mathbf{x}_p$ and velocity $\mathbf{u}_p$ of the p -th nanoparticle using Eqs. (4.24) and (4.23), respectively.
7. calculate the impulse density $\mathbf{J}$ using Eq. (4.18);
8. perform collision and streaming processes of the density distribution functions $f_\alpha(\mathbf{x}, t)$;
9. calculate the unknown density distribution functions $f_\alpha(\mathbf{x}, t)$ onto the boundary nodes using the NEBB method (see Appendix B);
10. update velocity $\mathbf{u}$, density ρ and pressure p (Eqs. (4.20), (4.21) and (4.22)).

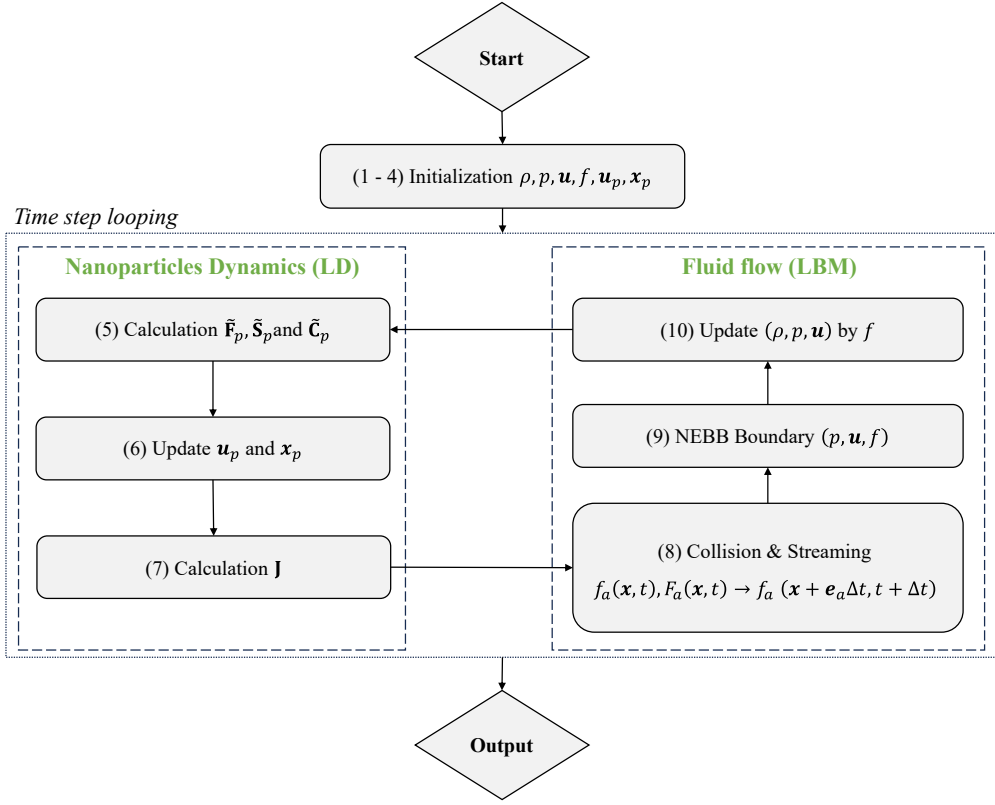


Figure 4.3: Schematic representation of the implemented algorithm.

4.2.4 Numerical setup and post-processing

In all the numerical investigations reported in the following sections, all fundamental lattice units, e.g., time, length, mass, temperature, and charge, are set equal to 1 in lattice unit system [182, 183] and the lattice length, $\Delta x = \Delta y = \Delta z$, is chosen to be equal to 333 nm in physical unit system.

For nano or micro scale spherical particles suspended in a solvent, e.g., the colloids, the Brownian diffusive time scale, τ_b , is defined as a time interval over which the particles diffuse over a distance comparable to their radius,

$$\tau_b = \frac{d_p^2 \zeta}{4k_B T} \quad (4.27)$$

and the viscous diffusion time scale, τ_v , is defined as the hydrodynamic time scale for momentum to diffuse over a distance equal to the radius of particles,

$$\tau_v = \frac{d_p^2}{4\nu}. \quad (4.28)$$

If the simulated time satisfies $N \cdot \Delta t = t \gg \tau_r$ (diffusive regime), the translational mean-square displacement increases linearly in time following the Stokes–Einstein relation

$$D = \frac{k_B T}{3\pi\mu d_p}. \quad (4.29)$$

Moreover, since $\tau_r \gg \tau_v$, it is also possible to capture the particle velocity relaxation in the short-time regime ($t < \tau_v$, Eq. (4.30)), i.e. the Stokes decay, and in the long-time regime ($t > \tau_v$, Eq. (4.31)), i.e. the algebraic decay.

4.3 Validation of the two-way coupled LB-LD method

In this section, the two-way coupled LB-LD method is validated using well-documented benchmark problems from the literature to assess the accuracy of the proposed framework in describing the dynamics of both single NPs and dilute solutions.

4.3.1 The velocity autocorrelation function of a single nanoparticle

From a theoretical standpoint, the dynamics of a Brownian particle can be characterized through the velocity autocorrelation function (VACF). In the short-time regime, the VACF decays exponentially as

$$\langle \mathbf{u}_p(\tau) \cdot \mathbf{u}_p(0) \rangle = (\mathbf{u}_p(0) \cdot \mathbf{u}_p(0)) e^{-\frac{\zeta|\tau|}{m_p}}, \quad (4.30)$$

where ζ is the friction coefficient. However, Alder and Wainwright [184] discovered that in the long-time regime the VACF decays as a power-law,

$$\langle \mathbf{u}_p(\tau) \cdot \mathbf{u}_p(0) \rangle \sim \tau^{-\frac{d}{2}}, \quad \text{for } \tau \rightarrow \infty, \quad (4.31)$$

where d is the dimension of space. When the particle density is closer to fluid density, the VACF does not decay as the exponential law in the short-time regime, but only as a power law [185].

First, by setting the stochastic forces to zero, a single NP (100 nm in diameter, i.e. 0.3003 lu [182, 183]) with initial nonzero velocity ($\mathbf{u}_p(0) = 0.01$ lu) in the z-direction is released at the center of quiescent water ($Re = 0$) at room temperature ($T = 298K$, i.e. 1 lu) to observe the deterministic particle–fluid interaction and its velocity relaxation process. The NP mass is chosen to be $m_1 = 29.3$ lu to solve the under-damped Langevin equation and, given the chosen parameters, the friction coefficient is $\zeta = 0.48$. To assess the domain-size dependency, three computational domains of 50^3 , 70^3 , and 100^3 nodes are simulated with periodic boundary conditions in all directions.

Figure 4.4 shows that, in the short-time regime, $t < \tau_v$ (Eq. (4.28)), the VACF normalized by $\mathbf{u}_p(0) \cdot \mathbf{u}_p(0)$, decays exponentially as $e^{-\zeta t/m_1}$, consistently with the analytical prediction (Eq. (4.30)). In the long-time regime, $t > \tau_v$, the VACF follows the power-law behavior $t^{-3/2}$ predicted by Eq. (4.31), due to hydrodynamic interactions between the nanoparticle and the surrounding fluid. This behavior, observed for all computational domains, indicates that the hydrodynamic and particle dynamics are correctly coupled through the LB-LD framework.

Considering three nanoparticles of 100 nm in diameter with different masses, $m_1 = 29.3$ lu, $m_2 = 2.93$ lu and $m_3 = 0.016$ lu, the over-damped LE is solved only for NP of mass m_3 . Fig. 4.5 shows that, for inertial NPs of mass m_1 and m_2 , the velocity decays exponentially in the short-time and as a power law in the long-time. However, the velocity of the exosomes, nanoparticles with negligible inertia, directly exhibits the long-time behavior without the initial exponential decay and closely follows the fluid streamline.

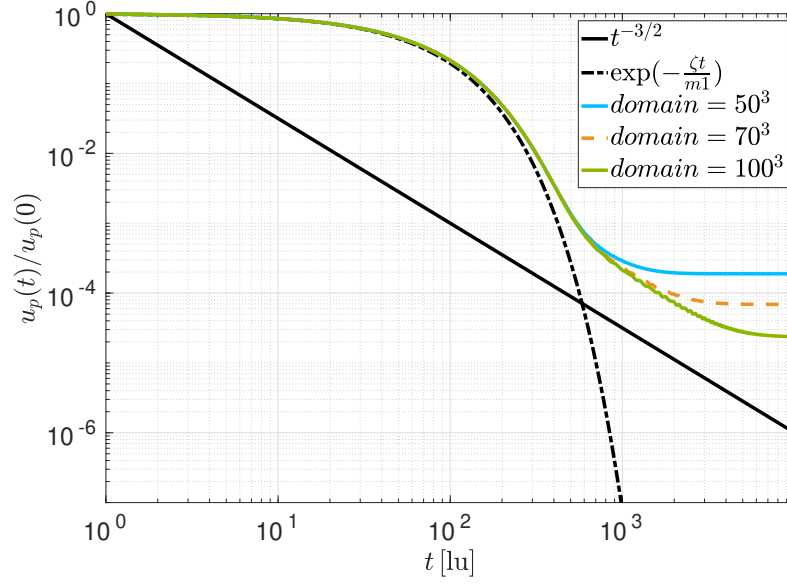


Figure 4.4: The normalized velocity relaxation of a single NP (100 nm in diameter) with initial velocity, $\mathbf{u}_p(0) = 0.01$ lu, released in three computational domains (50^3 , 70^3 , and 100^3 nodes) of quiescent water without stochastic forces.

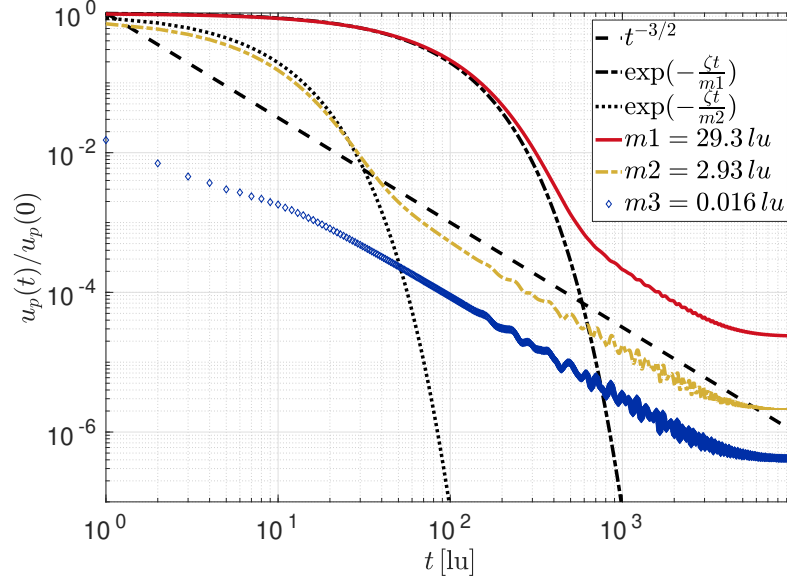


Figure 4.5: The normalized velocity relaxation of a single NP (100 nm in diameter) with different mass, released, with the initial velocity, $\mathbf{u}_p(0) = 0.01$ lu, in quiescent water without stochastic forces.

The contribution of additional stochastic forces is then evaluated. Brownian noise is then added to the previous deterministic case of the NP of mass m_1 , and the stochastic curve is averaged over an ensemble of ten independent runs. Figure 4.6 shows that the stochastic curve agrees with the deterministic one, showing that the fluctuation-dissipation theorem is correctly captured by the two-way coupled LB-LD method.

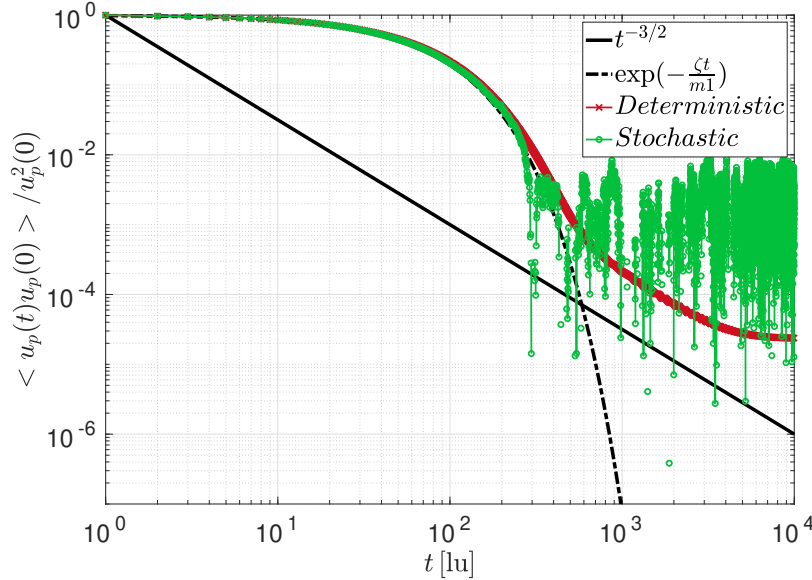


Figure 4.6: The normalized velocity autocorrelation function of a single NP (100 nm in diameter) of mass $m_1 = 29.3$ lu with an initial velocity, $\mathbf{u}_p(0) = 0.01$ lu, released in quiescent water under deterministic (red curve) and stochastic condition (green curve).

4.3.2 The Brownian diffusion of dilute nanoparticle solutions

To evaluate the long-time Brownian diffusivity, an isolated nanoparticle (100 nm in diameter) of mass m_3 is simulated in a 100^3 fluid lattice domain at room temperature ($T = 298K$, i.e. 1 lu). Given the parameters of the problem, the temporal resolution is equal to $0.018482 \mu s$ per LB time step, satisfying the condition $\Delta t \gg \tau_r$ to ensure that the system is in the diffusive regime. The mean-square displacement (MSD) is computed as $\langle \Delta \mathbf{x}(t)^2 \rangle = 6Dt$, and sampled every 100 LB time steps (i.e., $1.8482 \mu s$). The simulated diffusivity asymptotically follows the Stokes–Einstein relation (Eq. (4.29)), as shown in Fig. 4.7. Figure 4.8 illustrates the typical Brownian motion of a nanoscale particle at different time instants.

Then, a dilute concentration of 100 nanoparticles with five different diameters (10, 50, 100, 500, and 1000 nm) is simulated by solving the over-damped Langevin equation. Since each case has a different temporal resolution, the MSD is sampled at different LB time intervals to ensure a consistent physical sampling on the order of 10^{-6} s and averaged over all nanoparticles. Figure 4.9 shows that the long-time diffusivity follows the Stokes–Einstein relation and decreases with increasing particle diameter, as expected from theoretical predictions.

This result further confirms that the proposed LB-LD framework correctly captures long-time diffusive behavior.

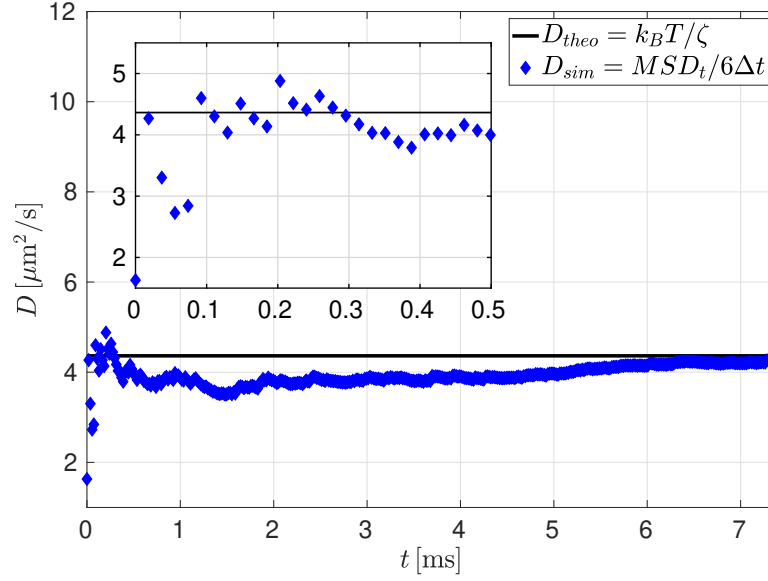


Figure 4.7: The self-diffusivity of a 100 *nm* nanoparticle in quiescent water at room temperature. The black line represents the Stokes–Einstein prediction.

4.4 Outlook

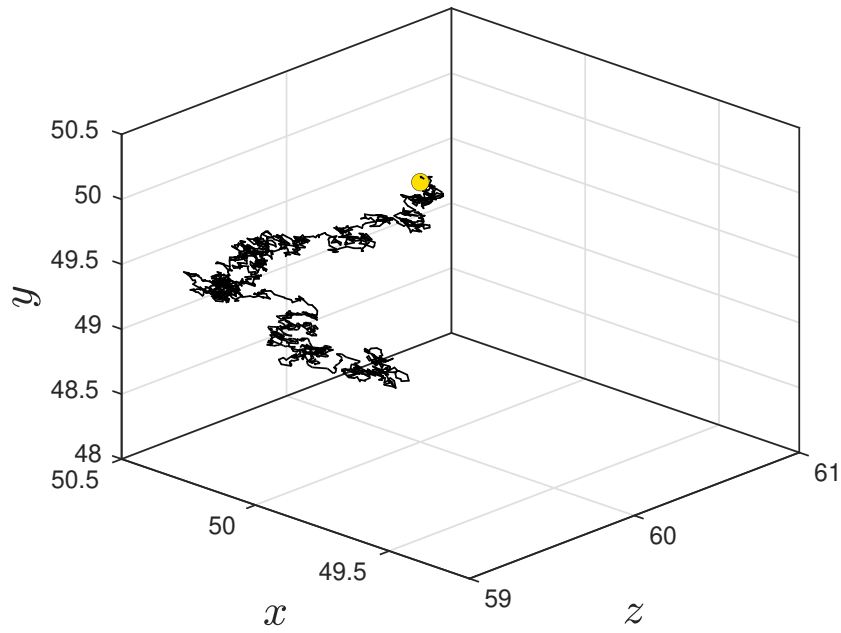
The results demonstrate that the proposed LB-LD framework accurately captures the complex dynamics of nanoparticles, including long-range hydrodynamic interactions, thermal fluctuations, and particle–particle interactions. This approach provides a robust and flexible tool for investigating the transport and diffusivity of nanoparticles in complex fluids.

Future developments may extend the present model to a wider range of interactions and applications. In particular, conservative forces can be generalized to account for multiple physical contributions. Interparticle interactions are commonly described within the DLVO framework, which considers the competition between attractive van der Waals forces and electrostatic repulsion [41]. Within this context, the repulsive component can be represented using a Lennard–Jones-type potential [186], while the attractive contribution may be modeled through the Hamaker formulation [187].

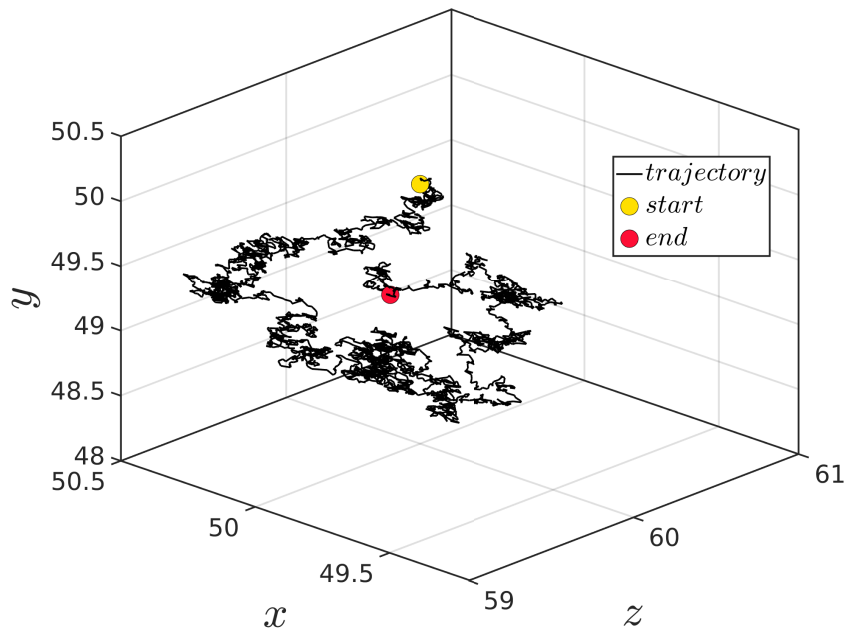
Electrostatic interactions can also be explicitly incorporated to describe charged nanoparticles, for instance through Coulomb forces [162]. Moreover, interactions involving deformable biological entities, such as cells, may be modeled using alternative potentials, e.g., the Morse potential [42].

At larger structural scales, the framework can be extended to model biological and synthetic polymeric systems using coarse-grained bead–spring representations. In such models, nanoparticles act as effective beads connected by elastic springs, enabling the investigation of complex conformational dynamics. A relevant example is the von Willebrand factor (vWF), which undergoes a shear-induced transition from a compact globular state to an elongated configuration that promotes platelet adhesion [188].

Within this context, the total interaction potential must account for both cohesive interactions and excluded-volume effects. The depth of the Lennard–Jones potential is typically tuned to reproduce intermolecular cohesion, while monomer connectivity is



(a)



(b)

Figure 4.8: The trajectory of a 100 nm nanoparticle in quiescent water at room temperature at different time instants: (a) 36.964 μs and (b) 73.946 μs .

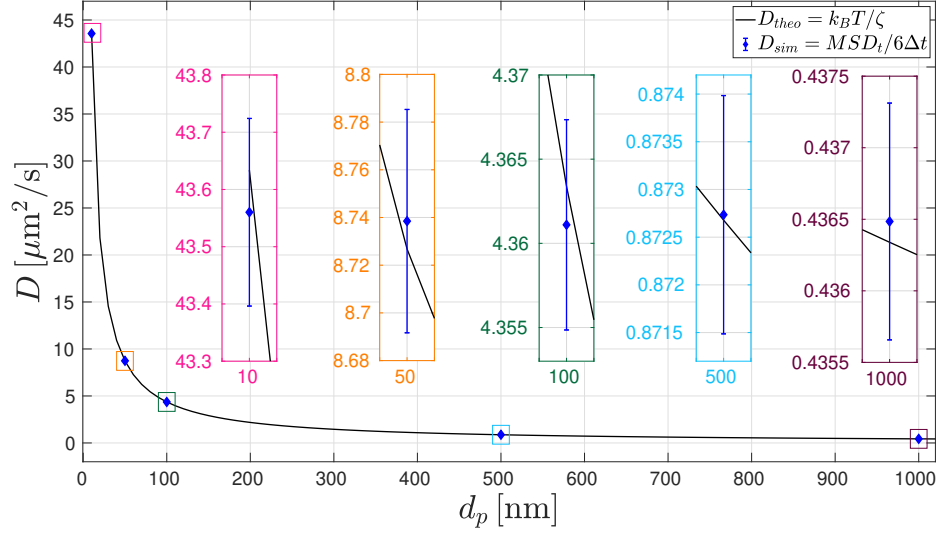


Figure 4.9: The self-diffusivity of dilute solutions of 100 NPs as a function of particle diameter. The black line is the Stokes-Einstein relation.

modeled through a Finitely Extensible Nonlinear Elastic (FENE) potential, capturing the nonlinear elastic response of polymer chains near full extension [41].

Chapter 5

Viscoelastic fluids

5.1 Introduction

In the last two decades, microfluidic devices have been shown to reduce invasiveness, cost, and the volume of sample needed to perform biological analyses, improving the efficiency of diagnostic tests and personalized therapies. Typically, microfluidic devices are tested using particles suspended in aqueous buffer solutions, i.e., fluids that exhibit Newtonian behavior with constant viscosity. However, many biological fluids contain polymeric chains, such as proteins, DNA filaments, or RNA filaments, which give rise to non-Newtonian viscoelastic and shear-thinning behavior. As a result, fluids such as blood, saliva, and hyaluronic acid alter the dynamics of suspended cells and vesicles because the fluid stress depends on its deformation history [189–191].

Due to the complex nature and dynamics of biological fluids, sample preparation and the separation of target cells or molecules represent fundamental steps in biological research and in most diagnostic tests. Inertial microfluidics allows particles to be focused and separated based on their size, shape, and deformability; however, submicron or nanoscale particles, such as extracellular vesicles, are difficult to isolate due to their low inertia [192]. For this reason, many studies have demonstrated that the elastic forces arising in non-Newtonian fluids can be exploited to enhance the manipulation of small particles, such as exosomes [144, 193] and DNA filaments [193, 194]. In addition, biocompatible, low-cost, and stable polymeric solutions, e.g. polyvinylpyrrolidone (PVP), polyethylene oxide (PEO), or polyacrylamide (PAA), can be used to tune the elastic forces within the microchannel [195, 196].

Indeed, Yang et al. [197] demonstrated that single-line focusing in a square channel, referred to as elasto-inertial focusing, can be achieved. In the inertial flow regime, particles focus at the center of each wall of the square channel, whereas in the elastic flow regime they focus at both the center and the corners of the channel. Thus, in viscoelastic fluids, the equilibrium positions and trajectories of particles depend on the balance between inertial and elastic forces.

These features provide functionalities and levels of control that cannot be achieved with Newtonian fluids alone. In particular, viscoelastic fluids such as blood, plasma, saliva, and mucus are employed to separate cells from plasma, assess erythrocyte deformability, and develop diagnostic devices that operate without centrifugation through controlled lateral particle migration [198]. More broadly, viscoelastic microfluidics is emerging as a promising area not only in biomedicine but also in the space sector, with applications in biofabrication and the study of complex fluids.

Under microgravity conditions, precise control over the transport and stability of cell suspensions, bioinks, and shear-thinning hydrogels can be achieved owing to the near absence of sedimentation and the predominance of viscoelastic effects [199]. These conditions are exploited in space for the 3D printing of tissues and organoids, as well as for investigating the behavior of complex fluids, including elastic instabilities, elastic turbulence, and non-Newtonian rheology at low Reynolds numbers [198]. Furthermore, the synthesis of microgels or polymeric nanoparticles may enable the *in situ* production of drug delivery systems directly in space [159].

Although numerous experiments involving non-Newtonian fluids, such as those related to viscoelastic focusing, have been conducted, understanding how particles interact with each other and with the surrounding fluid, as well as the mechanisms governing their lateral migration and trajectories within the channel, remains challenging. Numerical simulations can therefore play a key role in elucidating these phenomena, predicting equilibrium positions, and enabling *in silico* parametric studies.

5.1.1 State of the Art of Numerical Models

Numerous models can be used to describe the behavior of viscoelastic fluids, ranging from molecular-level approaches, such as molecular dynamics, e.g., the Langevin dynamics method (cf. Chapter 4), which capture the interactions between polymeric particles, to continuum mechanics approaches that describe their macroscopic behavior through constitutive equations, such as the Oldroyd-B [200], Finitely Extensible Nonlinear Elastic (FENE) [201], Giesekus [202], or Phan-Thien-Tanner (PTT) [203] models. The latter have been widely implemented using numerical methods such as finite difference [204], finite volume [205], finite element [206], and spectral element [207] methods.

Oldroyd-B is the most commonly used model as a benchmark test, as it incorporates memory effects through the upper-convected derivative. Nevertheless, it exhibits unbounded growth in extensional flows, which limits its applicability to low Weissenberg (Wi) numbers. To overcome these limitations, constitutive equations have been improved, for example through the Giesekus model, which accounts for a friction coefficient between polymer chains, or through finite-extensibility models such as FENE-P and FENE-CR [208, 209]. Alternatively, numerical discretization schemes have been enhanced to mitigate instabilities in regions characterized by strong spatial and temporal variations. For example, the Streamline Upwind Petrov-Galerkin (SUPG) [210, 211] method reduces directional errors by applying weighting to the convective terms, while the Elastic-Viscous Stress Splitting (EVSS) [212, 213] method reduces coupling oscillations by splitting the stress into elastic and viscous components [214].

Despite decades of numerical research in this field, the simulation of viscoelastic fluids remains one of the most challenging problems in computational fluid dynamics, particularly at high Weissenberg numbers. Due to the hyperbolic nature of the constitutive equations, regions where stresses undergo rapid spatial and temporal variations are prone to exponential growth of stress components, numerical instabilities, loss of solution convergence, and mesh-dependent solutions.

5.1.1.1 Lattice Boltzmann methods for viscoelastic fluids

Recently, many hybrid approaches have been developed in which the Lattice Boltzmann Method (LBM) is used to solve the Newtonian part of the momentum equation, while

other numerical techniques, such as the finite difference method [215] or the finite volume method [216], are employed to solve the constitutive equation. However, these approaches generally require two different time steps, which increases the computational cost, especially for unsteady flows. In addition, the transformation of physical variables into lattice units and vice versa requires special attention when choosing the grid and time step.

To exploit the mesoscopic nature of the LBM, which allows the description of complex phenomena, e.g., suspensions, and geometries, e.g., porous media, many efforts have been made to incorporate elastic effects directly into the method. Qian and Deng [217] modified the equilibrium distribution function, while Ispolatov and Grant [218] introduced a body force based on the Maxwell elastic stress. Giraud et al. [219, 220] and Lallemand et al. [221] solved the Jeffreys model assuming linearity of the stress tensor; however, none of these approaches solved the full constitutive equation.

Onishi et al. [222] correctly integrated the Oldroyd-B model, but only for simple shear flows, by discretizing the Fokker-Planck equation. Denniston et al. [223] and Marenduzzo et al. [224, 225] simulated liquid crystal hydrodynamics by introducing a second distribution function to solve the constitutive equation, although this approach is difficult to extend to complex geometries. A similar method was applied by Malaspinas et al. [226], who expressed the constitutive equation in terms of the conformation tensor and resolved each component with its own distribution function. However, this scheme introduces two additional terms in the constitutive equation that lack physical meaning and cannot accurately simulate flows at moderate Wi numbers or in complex geometries. Su et al. [43] removed one of these extra terms, reformulating the source term of the stress tensor distribution function, and successfully simulated viscoelastic fluids at high Wi numbers.

Although the intrinsic nature of the LBM alleviates several issues commonly encountered in finite difference and finite element methods, such as those related to convective terms and stress tensor discretization, the implementation of boundary conditions remains an open research problem. Boundary conditions are essential for ensuring stability and consistency, particularly in strongly extensional flows. Incorrect imposition of boundary conditions on the stress tensor can lead to artificial stretching or compression of polymer chains, resulting in spurious stresses and erroneous solutions that propagate throughout the domain and generate numerical instabilities at high Wi numbers.

To prevent such phenomena, the non-equilibrium extrapolation method (NEEM), proposed by Guo et al. [227], is typically used [43, 228]. It extrapolates the non-equilibrium part of the unknown distribution functions at the wall from neighboring nodes, maintaining continuity with the internal flow. This method is valid both for imposed velocities, i.e., Dirichlet boundary conditions, and for imposed fluxes, i.e., Neumann boundary conditions. Moreover, it reduces spurious stresses caused by poorly defined boundary conditions, which is essential for maintaining stability at high Wi numbers. It is also straightforward to implement, as it does not require the implicit solution of linear systems or additional iterations. However, the extrapolation of the non-equilibrium part of the distribution functions from neighboring nodes relies on the assumption that the flow is sufficiently smooth. In highly irregular geometries or in regions with strong curvature, this approximation may introduce local errors in velocity or stress. For flows with large gradients or extreme transients, the extrapolation can produce numerical spikes or localized instabilities, particularly at high Wi numbers. While NEEM improves stability compared to simpler methods, at very high Wi these extrap-

olation errors can still accumulate and lead to numerical instabilities. The accuracy of the method also depends on the resolution of the neighboring nodes: coarse grids may generate spurious stresses or deviations in the elastic stress tensor. Finally, for moving walls, porous surfaces, or complex 3D geometries, the extrapolation may become less reliable and require specific adjustments [26].

5.1.2 Present Contribution

In this thesis, the aim is to advance the modeling and simulation of three-dimensional viscoelastic fluids in microfluidic environments.

5.1.2.1 Three-dimensional framework for viscoelastic fluids

To provide an accurate analysis of viscoelastic fluids at moderate Weissenberg numbers ($Wi < 10$), the two-dimensional framework proposed by Su et al. [43] is adopted and extended here to three-dimensional cases (D3Q19). The range of Weissenberg numbers considered is selected with a view to future investigations involving suspended particles, such as cells and nanoparticles, in microchannels. Although the adopted model includes a non-physical term that increases with Wi , the use of two distribution functions is particularly well suited for parallelization, which is essential for simulating particle dynamics and long channels. This approach avoids compatibility issues associated with hybrid methods and fully exploits the mesoscopic nature of the lattice Boltzmann method. In contrast to Su et al., a different formulation of the density distribution function is employed here to describe weakly compressible viscoelastic fluids.

Building upon the work of Zou and He [35], who introduced a non-equilibrium bounce-back (NEBB) scheme for enforcing velocity and/or pressure in two-dimensional Newtonian fluids, this study develops a novel NEBB boundary scheme for three-dimensional viscoelastic fluids (see Appendix B). The proposed scheme enhances the conservation of physical quantities at solid boundaries, preventing unphysical compression and excessive stretching of polymer chains, while rigorously preserving wall quantities such as velocity and the polymeric stress tensor. To the best of my knowledge, this is the first application of the NEBB method to viscoelastic fluids.

The framework is validated against the well-established planar Poiseuille flow benchmark, considering both transient and steady-state regimes. The influence of varying the Weissenberg number and the viscosity ratio is systematically assessed, and a grid convergence study is performed to ensure that the solution is independent of the computational domain discretization.

Overall, the model ensures excellent mass conservation and robustness across a wide range of Weissenberg numbers, enabling realistic simulations of viscoelastic flows. These results can complement experimental observations and support the design and optimization of microfluidic systems involving complex fluids, including applications such as viscoelastic focusing.

5.2 Methods

The continuity and momentum equations of an incompressible and isothermal viscoelastic fluid are, respectively, given by

$$\nabla \cdot \mathbf{u} = 0, \tag{5.1}$$

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \mu_S \nabla^2 \mathbf{u} + \nabla \cdot \boldsymbol{\tau} + \mathbf{F}_B \quad (5.2)$$

where ρ , $\mathbf{u}$, and p are the density, velocity, and pressure of the system, respectively, μ_S is the dynamic viscosity of the Newtonian solvent, $\boldsymbol{\tau}$ is the polymeric stress tensor, and $\mathbf{F}_B$ denotes a generic external or body force per unit volume (e.g. an imposed pressure-gradient surrogate, gravity forcing, or Immersed Boundary forcing).

The polymeric stress tensor $\boldsymbol{\tau}$ evolves according to the Oldroyd-B constitutive equation which combines stress advection by the flow, stress stretching and rotation due to velocity gradients, and a relaxation term driving the stress toward its equilibrium value over a characteristic polymer relaxation time λ_P

$$\left[\frac{\partial \boldsymbol{\tau}}{\partial t} + (\mathbf{u} \cdot \nabla) \boldsymbol{\tau} \right] = \boldsymbol{\tau} \cdot \nabla \mathbf{u} + (\nabla \mathbf{u})^T \cdot \boldsymbol{\tau} + \frac{1}{\lambda_P} [\mu_P (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \boldsymbol{\tau}], \quad (5.3)$$

where μ_P is the polymer dynamic viscosity.

5.2.1 Lattice Boltzmann method for viscoelastic fluids

To solve the velocity and polymeric stress fields, two types of distribution functions are introduced. The evolution of mass and momentum (Eqs. 5.1 and 5.2) is described by the scalar density distribution function $f_\alpha(\mathbf{x}, t)$, with the divergence of the polymeric stress tensor treated as an external force. The advection-diffusion LBM scheme proposed by Su et al. [43] is adopted to solve the constitutive equation (Eq. 5.3). The stress tensor distribution function $G_{ij\alpha}(\mathbf{x}, t)$ is used to describe the evolution of the polymeric stress tensor once the velocity field is known.

The Navier–Stokes equations are solved on a three-dimensional computational lattice (D3Q19), where 19 scalar density distribution functions $f_\alpha(\mathbf{x}, t)$ ($\alpha = 0, \dots, 18$) are defined at position $\mathbf{x}$ and time t . At each time step Δt , they stream to neighboring nodes with discrete velocity $\mathbf{e}_\alpha$,

$$\mathbf{e}_\alpha = \frac{\Delta \mathbf{x}}{\Delta t} \begin{cases} (0, 0, 0) & \alpha = 0 \\ (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1) & \alpha = 1, 2, \dots, 6 \\ (\pm 1, \pm 1, 0), (\pm 1, 0, \pm 1), (0, \pm 1, \pm 1) & \alpha = 7, 8, \dots, 18 \end{cases}, \quad (5.4)$$

where the lattice time step Δt and the lattice spacing $\Delta x = \Delta y = \Delta z$ are set equal to 1. The distribution functions then collide according to the forced lattice Boltzmann Bhatnagar-Gross-Krook (LBGK) scheme [229]

$$f_\alpha(\mathbf{x} + \Delta t \mathbf{e}_\alpha, t + \Delta t) - f_\alpha(\mathbf{x}, t) = -\frac{\Delta t}{\tau} [f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t)] + \Delta t F_\alpha(\mathbf{x}, t), \quad (5.5)$$

where the equilibrium state $f_\alpha^{eq}(\mathbf{x}, t)$ is defined as

$$f_\alpha^{eq}(\mathbf{x}, t) = \omega_\alpha \rho \left[1 + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})}{c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})^2}{2c_s^4} - \frac{(\mathbf{u} \cdot \mathbf{u})}{2c_s^2} \right], \quad (5.6)$$

with ρ and $\mathbf{u}$ denoting the fluid density and velocity at position $\mathbf{x}$ and time t , respectively. The weighting factors are $\omega_0 = 1/3$, $\omega_{1-6} = 1/18$, and $\omega_{7-18} = 1/36$. The relaxation time τ is related to the solvent kinematic viscosity according to $\nu = c_s^2 (\tau - \frac{\Delta t}{2})$, where

$c_s = \frac{\Delta x}{\sqrt{3}\Delta t}$. The external force term $F_\alpha(\mathbf{x}, t)$ is calculated according to Guo et al. scheme [177]

$$F_\alpha(\mathbf{x}, t) = \left(1 - \frac{1}{2\tau}\right) \omega_\alpha \left[\frac{(\mathbf{e}_\alpha - \mathbf{u})}{c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})}{c_s^4} \mathbf{e}_\alpha \right] \cdot \mathbf{F}(\mathbf{x}, t), \quad (5.7)$$

where $\mathbf{F} = \mathbf{F}_E + \mathbf{F}_B$ is the total force acting on the solvent and $\mathbf{F}_E = \nabla \cdot \boldsymbol{\tau}$ is the elastic force. The macroscopic density and velocity of the viscoelastic fluid are computed as

$$\rho(\mathbf{x}, t) = \sum_{\alpha=0}^{18} f_\alpha(\mathbf{x}, t) \quad (5.8)$$

$$\rho(\mathbf{x}, t) \mathbf{u}(\mathbf{x}, t) = \sum_{\alpha=0}^{18} f_\alpha(\mathbf{x}, t) \mathbf{e}_\alpha + \frac{\Delta t}{2} \mathbf{F}(\mathbf{x}, t) \quad (5.9)$$

and the pressure is recovered from the equation of state

$$p(\mathbf{x}, t) = c_s^2 \rho(\mathbf{x}, t). \quad (5.10)$$

The evolution of each component of the polymeric stress tensor τ_{ij} is modeled by 19 scalar distribution functions $G_{ij\alpha}(\mathbf{x}, t)$ ($\alpha = 0, \dots, 18$), defined at the same lattice nodes as $f_\alpha(\mathbf{x}, t)$. The D3Q19 scheme is adopted for the LBGK evolution of $G_{ij\alpha}(\mathbf{x}, t)$

$$G_{ij\alpha}(\mathbf{x} + \Delta \tilde{t} \tilde{\mathbf{e}}_\alpha, t + \Delta \tilde{t}) = G_{ij\alpha}(\mathbf{x}, t) - \frac{\Delta \tilde{t}}{\tilde{\lambda}} \left[G_{ij\alpha}(\mathbf{x}, t) - G_{ij\alpha}^{eq}(\mathbf{x}, t) \right] \\ + \Delta \tilde{t} \chi_{ij\alpha}(\mathbf{x}, t) + 0.5 \Delta \tilde{t}^2 \partial_t \chi_{ij\alpha}(\mathbf{x}, t) \quad (5.11)$$

where $\tilde{\lambda}$ is the relaxation time, $\Delta \tilde{t}$ is the time step, and $\tilde{\mathbf{e}}_\alpha$ is the lattice velocity associated with $G_{ij\alpha}(\mathbf{x}, t)$. To distinguish the time scale of the stress-tensor evolution, the velocity $\tilde{\mathbf{e}}_\alpha$ is defined as

$$\tilde{\mathbf{e}}_\alpha = \frac{\Delta \mathbf{x}}{\Delta \tilde{t}} \begin{cases} (0, 0, 0) & \alpha = 0 \\ (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1) & \alpha = 1, 2, \dots, 6 \\ (\pm 1, \pm 1, 0), (\pm 1, 0, \pm 1), (0, \pm 1, \pm 1) & \alpha = 7, 8, \dots, 18 \end{cases}, \quad (5.12)$$

where $\Delta \mathbf{x}$ is the same lattice spacing adopted for the momentum evolution. The equilibrium distribution functions $G_{ij\alpha}^{eq}(\mathbf{x}, t)$ are computed as

$$G_{ij\alpha}^{eq}(\mathbf{x}, t) = \omega_\alpha \tau_{ij} \left[1 + \frac{(\tilde{\mathbf{e}}_\alpha \cdot \mathbf{u})}{\tilde{c}_s^2} + \frac{(\tilde{\mathbf{e}}_\alpha \cdot \mathbf{u})^2}{2\tilde{c}_s^4} - \frac{(\mathbf{u} \cdot \mathbf{u})}{2\tilde{c}_s^2} \right], \quad (5.13)$$

where $\tilde{c}_s = \frac{\Delta \mathbf{x}}{\sqrt{3}\Delta \tilde{t}}$ is the lattice sound speed associated with $G_{ij\alpha}(\mathbf{x}, t)$ and ω_α are the same weighting factors used for the momentum evolution. The source term $\chi_{ij\alpha}$ is given by

$$\chi_{ij\alpha} = \omega_\alpha \chi_{ij} \left[1 + \left(\frac{\tilde{\lambda} - \frac{\Delta \tilde{t}}{2}}{\tilde{\lambda}} \right) \frac{\tilde{\mathbf{e}}_\alpha \cdot \mathbf{u}}{\tilde{c}_s^2} \right] \quad (5.14)$$

where χ_{ij} is the (i, j) component of the tensor $\boldsymbol{\chi}$, determined according to the constitutive equation (Eq. 5.3) as

$$\boldsymbol{\chi} = \boldsymbol{\tau} \cdot \nabla \mathbf{u} + (\nabla \mathbf{u})^T \cdot \boldsymbol{\tau} + \frac{1}{\lambda_P} [\mu_P (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \boldsymbol{\tau}]. \quad (5.15)$$

The velocity gradient is defined as

$$\nabla \mathbf{u} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_y}{\partial x} & \frac{\partial u_z}{\partial x} \\ \frac{\partial u_x}{\partial y} & \frac{\partial u_y}{\partial y} & \frac{\partial u_z}{\partial y} \\ \frac{\partial u_x}{\partial z} & \frac{\partial u_y}{\partial z} & \frac{\partial u_z}{\partial z} \end{bmatrix}. \quad (5.16)$$

The moments of the stress tensor distribution function $G_{ij\alpha}(\mathbf{x}, t)$ are computed as

$$\tau_{ij}(\mathbf{x}, t) = \sum_{\alpha=0}^{18} G_{ij\alpha}(\mathbf{x}, t) = \sum_{\alpha=0}^{18} G_{ij\alpha}^{eq}(\mathbf{x}, t) \quad (5.17)$$

$$\tau_{ij}(\mathbf{x}, t) \mathbf{u}(\mathbf{x}, t) = \sum_{\alpha=0}^{18} G_{ij\alpha}^{eq}(\mathbf{x}, t) \tilde{\mathbf{e}}_{\alpha} = \sum_{\alpha=0}^{18} G_{ij\alpha}(\mathbf{x}, t) \tilde{\mathbf{e}}_{\alpha} + \frac{\Delta \tilde{t}}{2} \chi_{ij}(\mathbf{x}, t) \mathbf{u}(\mathbf{x}, t). \quad (5.18)$$

The Chapman–Enskog expansion leads to the following equation

$$\frac{D\tau_{ij}}{Dt} = \chi_{ij} + \tilde{\nu} \nabla^2 \tau_{ij} \quad (5.19)$$

which differs from the constitutive equation (Eq. 5.3) by the presence of the polymer stress diffusion term $\tilde{\nu} \nabla^2 \tau_{ij}$. The diffusivity constant $\tilde{\nu} = \tilde{c}_s^2 \left(\tilde{\lambda} - \frac{\Delta \tilde{t}}{2} \right)$ controls the magnitude of this numerical diffusion and therefore affects both the smoothness of polymer stress gradients and the stability of the simulations.

Finally, the non-equilibrium bounce back (NEBB) method proposed by Zou and He [35] is adopted to impose velocity, pressure, and no-slip conditions on the boundaries (see Appendix B). This method allows the unknown distribution functions $f_{\alpha}(\mathbf{x}, t)$ and $G_{ij\alpha}(\mathbf{x}, t)$ to be computed at the boundaries while maintaining high accuracy over the range of relaxation times considered here.

5.2.2 Algorithm

The adopted approach requires a large amount of memory to store the multiple distribution functions. Therefore, the multi-CPU framework is developed using an MPI-based parallel computing architecture, extending the framework previously introduced by De Marinis [31] to applications involving viscoelastic fluids. The implemented algorithm is described below and summarized in Fig. 5.1.

The initialization step:

1. initialize velocity $\mathbf{u}$, pressure p , density ρ , and stress tensor $\boldsymbol{\tau}$ of the viscoelastic fluid at all fluid-domain nodes;
2. initialize the density distribution function $f_a(\mathbf{x}, t) = f_a^{eq}(\mathbf{x}, t)$ using Eq. (5.6);

3. calculate the elastic force $\mathbf{F}_E = \nabla \cdot \boldsymbol{\tau}$ using a centered finite-difference scheme

$$\begin{aligned}\partial_x \boldsymbol{\tau}(x, y, z) &= \frac{\boldsymbol{\tau}(x+1, y, z) - \boldsymbol{\tau}(x-1, y, z)}{2\Delta x}, \\ \partial_y \boldsymbol{\tau}(x, y, z) &= \frac{\boldsymbol{\tau}(x, y+1, z) - \boldsymbol{\tau}(x, y-1, z)}{2\Delta y}, \\ \partial_z \boldsymbol{\tau}(x, y, z) &= \frac{\boldsymbol{\tau}(x, y, z+1) - \boldsymbol{\tau}(x, y, z-1)}{2\Delta z};\end{aligned}\tag{5.20}$$

at the walls, three-point backward and forward difference schemes are adopted to calculate the derivative of the stress tensor in the direction perpendicular to the wall;

4. update density ρ , velocity $\mathbf{u}$, and pressure p using Eqs. (5.8), (5.9), and (5.10);
5. initialize the stress tensor distribution function $G_{ij\alpha}(\mathbf{x}, t) = G_{ij\alpha}^{eq}(\mathbf{x}, t)$ using Eq. (5.13).

The iterative step:

6. update the stress tensor $\boldsymbol{\tau}$ using Eq. (5.17);
7. calculate the elastic force $\mathbf{F}_E$ (see point 3);
8. update density ρ , velocity $\mathbf{u}$, and pressure p using Eqs. (5.8), (5.9), and (5.10);
9. calculate the source term $\chi_{ij\alpha}$ (Eq. (5.14)) using a centered finite-difference scheme to compute the velocity gradient

$$\begin{aligned}\partial_x \mathbf{u}(x, y, z) &= \frac{\mathbf{u}(x+1, y, z) - \mathbf{u}(x-1, y, z)}{2\Delta x}, \\ \partial_y \mathbf{u}(x, y, z) &= \frac{\mathbf{u}(x, y+1, z) - \mathbf{u}(x, y-1, z)}{2\Delta y}, \\ \partial_z \mathbf{u}(x, y, z) &= \frac{\mathbf{u}(x, y, z+1) - \mathbf{u}(x, y, z-1)}{2\Delta z};\end{aligned}\tag{5.21}$$

at the walls, three-point backward and forward difference schemes are adopted to calculate the derivative of the velocity in the direction perpendicular to the wall;

10. perform collision and streaming processes for the density distribution function $f_\alpha(\mathbf{x}, t)$ and the stress tensor distribution function $G_{ij\alpha}(\mathbf{x}, t)$, using the backward finite-difference scheme for the time derivative of the source term

$$\partial_t \chi_{ij\alpha}(\mathbf{x}, t) = \frac{\chi_{ij\alpha}(\mathbf{x}, t) - \chi_{ij\alpha}(\mathbf{x}, t - \Delta \tilde{t})}{\Delta \tilde{t}};\tag{5.22}$$

11. calculate the density distribution function $f_\alpha(\mathbf{x}, t)$ and the stress tensor distribution function $G_{ij\alpha}(\mathbf{x}, t)$ at the boundary nodes using the NEBB method (see Appendix B).

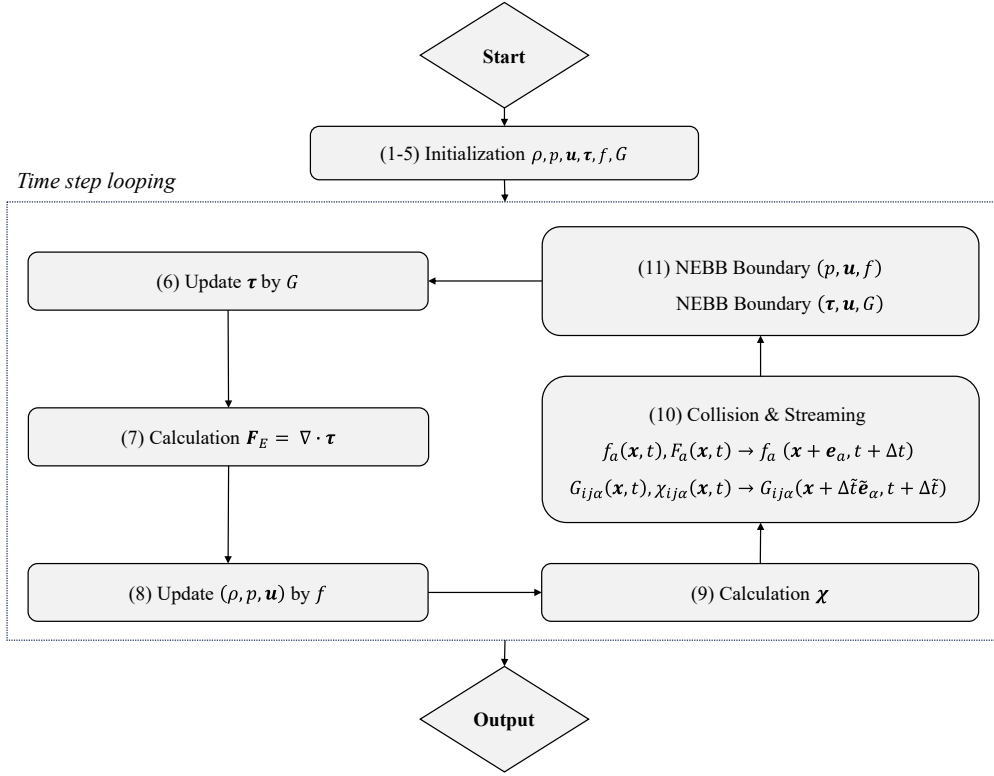


Figure 5.1: Schematic representation of the implemented algorithm.

5.2.3 Numerical setup and post-processing

Viscoelastic flows are commonly characterized by three dimensionless numbers,

$$Re = \frac{\rho U L}{\mu_{tot}}, \quad Wi = \frac{\lambda_P U}{L}, \quad \beta = \frac{\mu_S}{\mu_{tot}}, \quad (5.23)$$

where ρ is the fluid density, U and L are characteristic velocity and length scales, λ_P is the polymer relaxation time, μ_S is the solvent dynamic viscosity, μ_P is the polymer dynamic viscosity, and $\mu_{tot} = \mu_S + \mu_P$ is the total dynamic viscosity. The Reynolds number Re and the Weissenberg number Wi represent the inertial and elastic effects of the fluid, respectively, while β defines the viscosity ratio between the solvent dynamic viscosity and the total dynamic viscosity. The ratio of the polymer relaxation time to the momentum diffusion time is represented by the elasticity number $El = Wi/Re$, which is independent of the flow kinematics.

In all simulations, the velocity, pressure, and stress tensor fields are initialized to zero at all nodes of the domain. To minimize the error associated with the non-physical term in Eq. 5.19 while maintaining numerical stability, $\tilde{\lambda}$ is set to 0.500001. Furthermore, $\Delta \tilde{t}$ is set equal to Δt to achieve an accurate solution at a limited computational cost.

Two-dimensional cases are simulated with the proposed three-dimensional framework by defining only three nodes in the third (z -) direction and applying periodic boundary conditions along it.

5.3 Validation of the Lattice Boltzmann method for viscoelastic fluids

In this section, the well-documented planar Poiseuille flow problem is investigated to validate the proposed D3Q19 lattice Boltzmann method. The validation considers both transient and steady-state regimes. The numerical components of the stress tensor and the velocity profile are compared with the corresponding analytical solutions for Oldroyd-B fluids [230, 231].

In the simulations, the Reynolds number is fixed at $Re = 1$, while the Weissenberg number Wi , the viscosity ratio β , and the domain size are varied. Two flat walls are placed at $x = 0$ and $x = L$, where no-slip boundary conditions are imposed using the NEBB method to reconstruct the unknown distribution functions (see Appendix B). Periodic boundary conditions are applied in the y and z directions, and a constant driving force $\mathbf{F}_B = 8\mu_{tot}U/L^2$ is applied in the y direction, where U denotes the centerline velocity calculated from Re (Eq. 5.23).

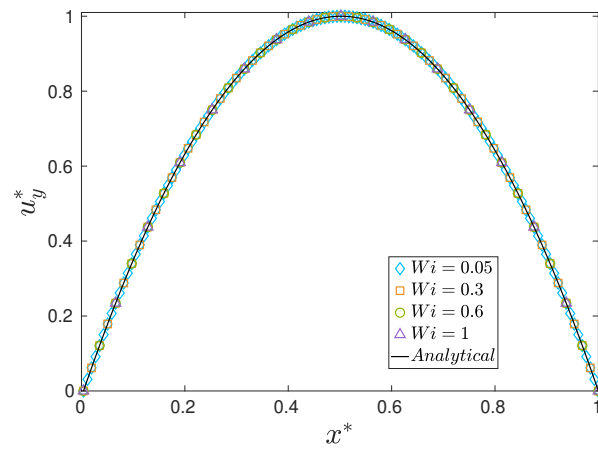
5.3.1 Steady flow in planar Poiseuille channel

The steady-state regime is evaluated in a domain of size $L \times L$ ($L = 256$ lu). Four simulations are performed by varying the Weissenberg number as $Wi = [0.05, 0.3, 0.6, 1]$ and fixing the viscosity ratio at $\beta = 0.3$.

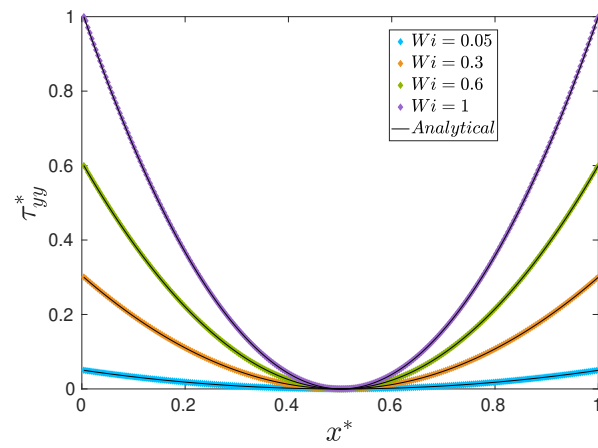
The analytical solutions for the equilibrium velocity and stress tensor profiles are given by [230, 231]

$$\begin{aligned}
 u_x &= 0, \\
 u_y(x) &= -\frac{4U}{L^2} (x^2 - Lx), \\
 \tau_{yy}(x) &= 2\lambda_p\mu_{tot} (1 - \beta) \left(\frac{\partial u_y}{\partial x} \right)^2, \\
 \tau_{xy}(x) &= \mu_{tot} (1 - \beta) \left(\frac{\partial u_y}{\partial x} \right), \\
 \tau_{xx}(x) &= 0.
 \end{aligned} \tag{5.24}$$

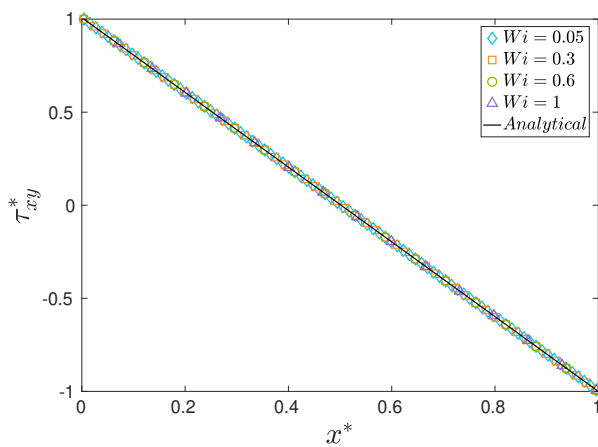
The numerical results presented in Fig. 5.2 are extracted at $y = L/2$, and the dimensionless variables u_y^* , x^* , and τ_{ij}^* are scaled by the reference values U , L , and τ_{ij}^{max} for $Wi = 1$, respectively. As shown in Fig. 5.2(a), the numerical velocity profiles for all Weissenberg numbers are in excellent agreement with the corresponding analytical solutions, with a relative error below 0.05%. Furthermore, Figs. 5.2(b) and (c) show the profiles of the normalized normal stress component τ_{yy}^* and the normalized shear stress component τ_{xy}^* , respectively. The numerical results closely match the analytical solutions, confirming the accuracy of the simulations. The maximum relative error is observed for the normal component of the polymeric stress tensor at the wall, reaching a value of 0.25% for $Wi = 1$. Overall, the relative errors of the stress tensor components remain below 0.5%. Therefore, the NEBB method enables an accurate reconstruction of both the velocity and the polymer stress tensor at the wall, preventing undesired stretching or compression of the polymer chains and ensuring numerical stability.



(a)



(b)



(c)

Figure 5.2: (a) The normalized steady velocity profile u_y^* , (b) the normalized normal stress component τ_{yy}^* and (c) the normalized extra stress component τ_{xy}^* at the center of the domain for different Weissenberg number, i.e., $Wi = [0.05, 0.3, 0.6, 1]$. The solid black lines are the corresponding analytical solutions.

5.3.2 Transient flow in planar Poiseuille channel

Here, the transient velocity at the center of the domain is evaluated as a function of time for five different values of the viscosity ratio, i.e., $\beta = [0.1, 0.3, 0.5, 0.7, 0.9]$, at a fixed Weissenberg number $Wi = 1$ in a domain of size $L \times L$ ($L = 256$ lu).

The exact solution of the transient velocity profile is computed as [230, 231]

$$u_y^*(x^*, t^*) = 4x^*(1 - x^*) - 32 \sum_{n=1}^{\infty} \frac{\sin(Nx^*)}{N^3} G_N(El, t^*), \quad (5.25)$$

where u_y^* , x^* , and t^* are the dimensionless variables scaled by U , L , and λ_P , respectively, and

$$G_N(El, t^*) = \begin{cases} \exp(-\alpha_N^* t^*) \left(\cos(\beta_N^* t^*) + \frac{S_N}{\beta_N} \sin(\beta_N^* t^*) \right) & \text{if } \alpha_N^2 < 4N^2 El, \\ \frac{1}{2} (a_N \exp(p_N t^*) + b_N \exp(q_N t^*)) & \text{if } \alpha_N^2 \geq 4N^2 El, \end{cases} \quad (5.26)$$

where

$$\begin{aligned} N &= (2n - 1)\pi \\ \alpha_N &= 1 + \beta El N^2 \\ \beta_N &= \sqrt{|\alpha_N^2 - 4N^2 El|} \\ \alpha_N^* &= \frac{\alpha_N}{2El} \\ \beta_N^* &= \frac{\beta_N}{2El} \\ S_N &= 1 - (2 - \beta) N^2 El \\ a_N &= 1 + \frac{S_N}{\beta_N} \\ b_N &= 1 - \frac{S_N}{\beta_N} \\ p_N &= -\alpha_N^* + \beta_N^* \\ q_N &= -\alpha_N^* - \beta_N^*. \end{aligned} \quad (5.27)$$

After initializing both the velocity field and the viscoelastic stress tensor to zero, the flow begins to accelerate due to the applied driving force, resulting in fluid deformation and, consequently, an increase in the stress tensor, as evidenced by the peaks shown in Fig. 5.3. This, in turn, leads to a reduction in velocity, visible in the troughs of Fig. 5.3. As a result, a transient cyclical dominance between viscous and viscoelastic effects emerges, reflected by the alternating troughs and peaks in Fig. 5.3, which eventually vanishes as the flow reaches equilibrium. Moreover, as the viscosity ratio β increases, the number of peaks and troughs in the velocity profile decreases, as does the time required to reach the steady state. In fact, as β decreases, the fluid exhibits increasingly non-Newtonian behavior due to the growing dominance of viscoelastic effects. Table 5.1 reports the numerical and analytical values of the first peak for different viscosity ratios. The maximum relative error is observed for $\beta = 0.1$, while the minimum occurs at $\beta = 0.9$. Overall, the relative error remains below 0.5%.

β	Numerical	Analytical	Relative error
0.1	2.8384	2.8454	0.2430%
0.3	2.0155	2.0164	0.0439%
0.5	1.5577	1.5580	0.0148%
0.7	1.2664	1.2665	0.0052%
0.9	1.0695	1.0695	0.0011%

Table 5.1: Comparison between the numerical and analytical values of the first peak for different viscosity ratios β .

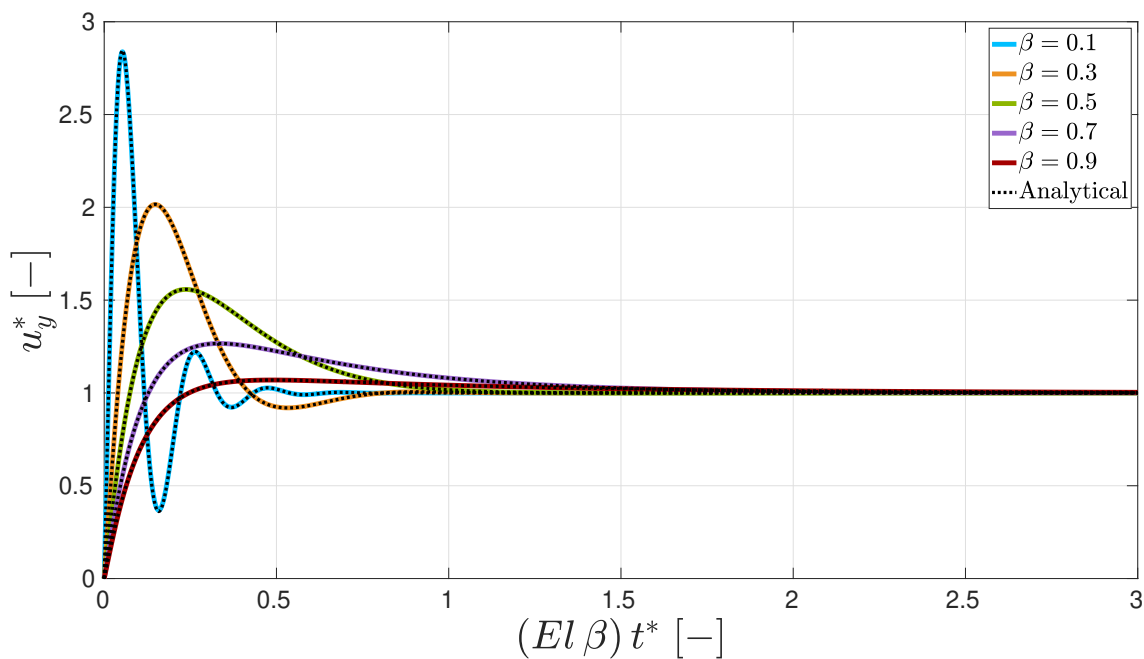


Figure 5.3: Normalized transient velocity at the center of the domain for different viscosity ratios, i.e., $\beta = [0.1, 0.3, 0.5, 0.7, 0.9]$. The solid black lines are the corresponding analytical solutions. The horizontal axis shows the dimensionless time $t\nu/L^2$, equivalently expressed as $(El\beta)t$.

5.3.3 Grid convergence tests

To assess the convergence rate of the numerical solution with respect to the analytical reference, four computational domains of size $L \times L$, with $L = [64, 128, 256, 512]$ lu, are considered. The Weissenberg number and the viscosity ratio are fixed at $Wi = 10$ and $\beta = 0.3$, respectively.

The relative error for each computational domain is computed to evaluate the numerical accuracy of the model. The L_2 norms of the velocity, the normal stress component, and the extra-stress component of the polymeric stress tensor are defined as

$$E_{u_y} = \sqrt{\frac{1}{m} \sum_{k=1}^m |u_y^{num}(\mathbf{x}_k) - u_y^{an}(\mathbf{x}_k)|^2}, \quad (5.28)$$

$$E_{\tau_{ij}} = \sqrt{\frac{1}{m} \sum_{k=1}^m |\tau_{ij}^{num}(\mathbf{x}_k) - \tau_{ij}^{an}(\mathbf{x}_k)|^2},$$

where $m = (N + 1)^2$ and N denotes the grid resolution (e.g., $N = 64$), while the superscripts *num* and *an* indicate the numerical and analytical solutions, respectively.

Once the steady state is reached, the numerical results are evaluated at every point in the domain. The results presented in Fig. 5.4 demonstrate that, as the size of the computational domain increases, the numerical solution steadily improves. The L_2 errors of the normal stress component τ_{yy} decrease from 10^{-6} to 10^{-8} , while those of the extra-stress component τ_{xy} decrease from 10^{-8} to 10^{-11} . The L_2 error of the velocity u_y decrease from 10^{-6} to 10^{-7} .

The regression line shown in the Fig. 5.4 was obtained by excluding the coarsest grid resolution, i.e., $N = 64$. The velocity field exhibits a reduced convergence rate because the error is dominated by a combination of first-order effects introduced by the non-equilibrium bounce-back boundary condition, the imposition of velocity at the wall, and the use of one-sided finite-difference stencils for spatial gradients without ghost nodes near the boundary. These factors generate boundary-dominated truncation errors that prevent the recovery of the asymptotic bulk accuracy. In contrast, the stress components are computed from velocity gradients evaluated mainly in the interior of the domain, where the solution is smoother and less affected by boundary discretization. Consequently, they benefit from error cancellation effects and reduced sensitivity to the boundary treatment, which results in a faster decay of the numerical error.

Therefore, the NEBB formulation can accurately reconstruct both the hydrodynamic variables and the polymeric stress tensor at the walls, even at moderate Weissenberg numbers, while effectively preventing non-physical effects that would otherwise contaminate the solution.

5.4 Outlook

The results demonstrate that the proposed framework accurately captures the behavior of viscoelastic fluids in microchannels. The proposed boundary conditions within the lattice Boltzmann framework enhance numerical accuracy while rigorously preserving

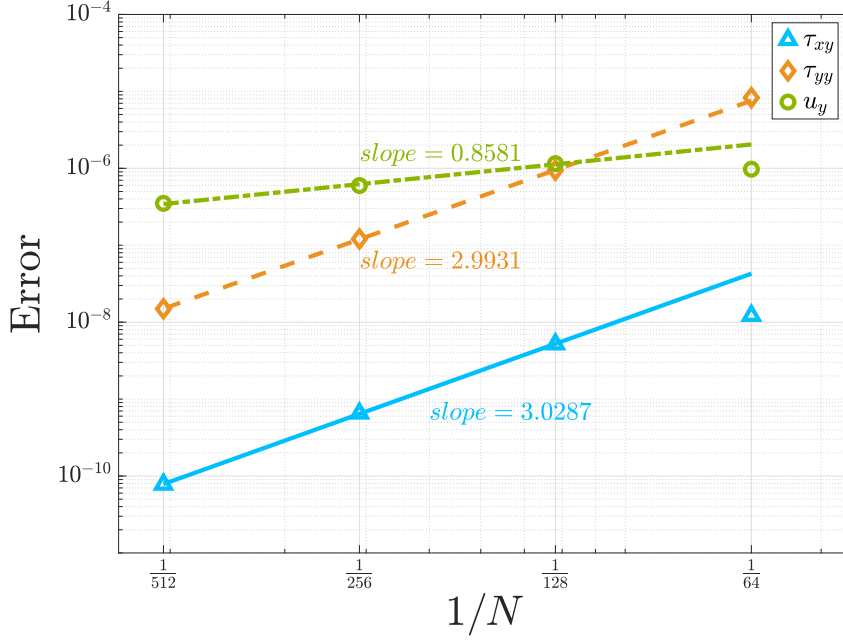


Figure 5.4: The L_2 errors of the normal stress component τ_{yy} , the extra-stress component τ_{xy} , and the velocity u_y as a function of the computational domain size for $Wi = 10$ and $\beta = 0.3$.

key physical quantities, including velocity and the polymeric stress tensor at the boundaries. This prevents unphysical compression and excessive polymer stretching, ensuring physically consistent behavior, as shown in Section 5.3.

The selected range of Weissenberg numbers supports future investigations of suspended particles. Biomicrofluidic applications, such as particle focusing and separation, typically operate at low ($Wi < 1$) or moderate ($1 \leq Wi \leq 10$) Weissenberg numbers. Viscoelastic focusing, occurring at low Reynolds numbers, drives particles toward well-defined equilibrium positions, enabling the alignment and even separation of sub-micrometric particles based on size or deformability. Viscoelastic focusing is generally observed for $1 \leq Wi \leq 10$ and $Re \ll 1$, whereas elasto-inertial focusing arises for $0.1 \leq Wi \leq 10$ and $1 \leq Re \leq 100$. In practice, most operations occur in the elasto-inertial regime, as dilute polymer solutions are employed and excessively high Wi values are avoided to prevent damage to cells or vesicles [144, 193].

Three-dimensional simulations of particle migration in viscoelastic fluids remain limited in the literature due to the inherent complexity and computational cost of the problem. In this context, the proposed framework offers significant advantages: its fully mesoscopic formulation naturally incorporates viscoelastic stresses, avoids compatibility issues typical of hybrid methods, and is highly amenable to parallelization, which is crucial for simulating particle-laden flows in long or complex channels. Additionally, the framework can be extended to account for solid walls misaligned with the lattice nodes, enabling the study of realistic microchannel geometries for enhanced particle focusing and precise separation.

Chapter 6

Conclusions

Beyond terrestrial applications, microfluidics is emerging as a critical technology for space exploration, with implications in astrobiology, pharmaceutical development, and space medicine. Traditional experimental approaches for the design and optimization of microdevices, however, are inherently constrained, limiting the realization of their full potential and leading to elevated costs and extended development timelines. By enabling rigorous simulation of multiphysics and multiscale fluid behavior, computational microfluidics provides an essential avenue for investigating both biological and non-biological fluids under conditions that are difficult, or even impossible, to reproduce experimentally.

This thesis introduces novel Lattice Boltzmann-based methodologies for the investigation of ternary fluid systems, nanoparticle dynamics, and viscoelastic fluids in microchannels. These phenomena are central to a wide spectrum of microfluidic applications, ranging from chemical analyses to advanced biological diagnostics. Despite the increasing importance of such systems, existing commercial software remains limited in both modeling flexibility and predictive capability. The computational frameworks developed in this work address these shortcomings, offering quantitative, predictive, and generalizable tools for the systematic study of complex microfluidic systems.

6.1 Conservative Phase-Field LBM for Multicomponent Systems

First, a conservative phase-field lattice Boltzmann (CPF-LB) framework is developed for describing the behavior of ternary fluid systems in contact with wettable flat surfaces. The CPF model proposed by Haghani et al. [32] is extended to the three-dimensional form (D3Q19).

As shown in Section 2.3, the framework is able to accurately track the interfaces of ternary fluid systems with high density and viscosity ratios when no boundary conditions for wettability are needed. The proposed method does not require the use of a time derivative term in the source term to enhance accuracy and stability, while preserving the local nature of the LBM and limiting the computational cost. This represents an enhancement with respect to the available literature, e.g. [36, 37], especially in the case of high density and/or high viscosity ratios. The presented results demonstrate that the model conserves mass and simultaneously reduces spurious velocities. In the presence of triple-point junctions, the intensity of spurious velocities increases, but still exhibits a

lower order of magnitude if compared with the results available in the literature.

Wettability is integrated by modifying the total free energy, following the approach adopted by Liang et al. [36] and Zhang et al. [37]. A novel non-equilibrium bounce-back (NEBB) method based on the previous work by Zou and He [35] is derived to reconstruct both the hydrodynamic and phase-field unknown distribution functions at the lattice boundaries. Since the surfaces lie on lattice nodes, a different discretization scheme with respect to those using half-way bounce-back methods is also used to reconstruct the desired quantities onto the ghost nodes, in order to evaluate accurately the phase-field and density gradients at the domain boundaries. The proposed boundary conditions improve accuracy and mass conservation at the boundary, as shown in Section 2.4, especially in the presence of an external driving force, e.g. shear rate, applied to ternary fluid systems with high density and/or high viscosity ratios (cf. Section 2.5.1). The framework allows the simulation of hydrophobic and hydrophilic surfaces with an error of less than 2% in the measurement of contact angles, recording smaller errors for hydrophobic cases and a maximum error in the case of neutral wettability. The results also show that when two droplets are in contact with each other, the maximum error remains below 2%. The wettability scheme has also been validated for the dynamic case of concentric droplets under shear in contact with wettable walls. The results for both low and high density ratios are consistent with previous studies [36, 37].

Finally, the ability to achieve high viscosity ratios even in 3D cases with the CPF model based on the Allen–Cahn equation enables the investigation of leukocyte deformation across a wide range of rheological parameters (see Section 2.5.2). While some of the authors have previously investigated the transport dynamics of deformable cells using surface-based Lagrangian models coupled with Immersed Boundary Methods [?, 31], those approaches are inherently limited when applied to complex biological cells such as leukocytes, which contain an internal nucleus. The fully Eulerian framework presented in this work overcomes these limitations. By avoiding surface tracking and immersed boundary coupling, the proposed method offers improved computational efficiency and enhanced numerical stability through smooth rheological transitions. In addition, leukocyte adhesion is captured through a simple yet effective contact angle condition. This formulation enables robust simulations of biologically realistic scenarios that are difficult to handle with existing surface-based methods.

Three-dimensional simulations are carried out to observe leukocyte deformation under shear as a function of rheological properties. The leukocyte is modeled as a three-layer drop, each layer having different rheological properties. As the deformability of the leukocyte increases when the viscosity ratios decrease, leukocyte detachment from the surface occurs at earlier stages for low viscosity ratios, and prolonged exposure to high shear flow conditions results in rupture of the cytoplasm membrane. On the other hand, for given viscosity ratios the sensitivity of leukocyte behavior to cortex tension is almost negligible. These results show that the developed framework is able to correctly describe leukocyte behavior, which is essential for a better understanding of its dynamics in microchannels, e.g. to improve binding efficiency on functionalized surfaces, or in blood vessels, e.g. to study physiological functions such as extravasation or pathological conditions such as immune diseases.

6.2 Extension of the CPF-LBM to Nonconformal Geometries and Droplet Generation

Then, a treatment for rigid walls that are not conformal to the lattice grid is incorporated, enabling the simulation of multicomponent systems in complex geometries. This study addresses the lack of three-dimensional simulations of droplet generation in flow-focusing microfluidic devices involving three immiscible fluids and wettability effects. To the best of my knowledge, it presents the first conservative Allen–Cahn-based phase-field LB formulation capable of accurately capturing droplet formation in such configurations.

Inspired by the algorithm proposed by He et al. [39], wettability is incorporated using a different discretization scheme and boundary conditions (cf. Section 3.2.1). The results further confirm that the Allen–Cahn approach exhibits low spurious currents even in systems with high density and viscosity ratios in contact with solid structures (cf. Section 3.3), reinforcing the suitability of the approach for the present simulations. The framework is capable of simulating both hydrophilic and hydrophobic solid surfaces, whether flat or curved, as illustrated in Sections 3.3.1 and 3.3.2. In the case of flat surfaces, relative errors are below 1%, with the largest errors occurring for hydrophilic surfaces and smaller errors observed for neutrally wetting conditions. For curved surfaces, relative errors in both low and high density and viscosity ratio scenarios remain below 3%, with the smallest errors for hydrophobic surfaces and the largest again for hydrophilic ones. Moreover, the spurious velocities introduced by the model and by the presence of nonconformal walls are of the order of $\mathcal{O}(10^{-6})$, with a maximum value observed at the triple junction point ($\mathcal{O}(10^{-5})$), and are one order of magnitude lower than the reference ones [39]. Despite the use of the bounce-back method, which approximates curved surfaces with a staircase representation, the developed framework enables the simulation of ternary systems in contact with wettable surfaces of arbitrary geometry, without exhibiting any mass gain or loss.

Then, monodispersed water-based droplet generation and dynamics are simulated in a cross-junction flow-focusing microdevice. The influence of the flow rate ratio Q and the viscosity ratio α on the droplet length L_D is first investigated in Section 3.4.1. The results are well described by the power-law relation $L_D/w = c_1 Ca^{c_2}$, with fitting coefficients reported in Table 3.2. This scaling allows the identification of a critical Capillary number Ca_{critical} separating two distinct breakup regimes. In the squeezing regime ($Ca \leq Ca_{\text{critical}}$), L_D/w decreases sharply with Ca , resulting in a large negative slope of the power-law relation. In the dripping regime ($Ca > Ca_{\text{critical}}$), the slope approaches zero, indicating that L_D becomes nearly insensitive to Ca . The transition between these two behaviors marks Ca_{critical} , which in the present study is approximately 0.01.

Experimental observation of the transition between these regimes is expected to be challenging due to the high throughput of the microdevice. The simulations, instead, are able to capture droplet breakup through mechanical compression in the squeezing regime and through the dominance of viscous forces in the dripping regime. Such a transition and its underlying mechanisms are consistent with classical scaling arguments and with experimental observations reported in the microfluidic literature. Moreover, L_D decreases as Q increases, facilitating the formation of more spherical droplets. Therefore, to obtain very small spherical droplets, it is necessary to have very high Ca , e.g. by reducing the interfacial tension through the addition of surfactants, combined with high Q . As the viscosity ratio decreases, a slight reduction in droplet length and a significantly

earlier onset of droplet breakup are also observed.

The wettability properties of the channel play a crucial role in the droplet formation process. In particular, as the contact angle increases, that is, in the case of highly hydrophobic walls, the resulting fluid film promotes the formation of small and spherical droplets in the dripping regime (high Ca). Conversely, in the squeezing regime (low Ca), breakup due to mechanical compression is favored by less hydrophobic walls, since the dispersed phase exhibits a larger contact area with the channel surface and adopts a more slug-like shape (cf. Section 3.4.2). In general, the formation of a fluid film of the continuous phase between the wall and the dispersed phase, resulting from an increase in the continuous-phase velocity, flow rate ratio, viscosity ratio, or wall contact angle, promotes the formation of small and spherical droplets in the dripping regime. However, this does not necessarily lead to an increase in droplet production frequency or a reduction in the spacing between droplets, as these outcomes depend on the balance among viscous and surface forces. Therefore, the developed framework proves capable of predicting the shape, size, and production frequency of droplets under various operating conditions and wettability properties of the microdevice.

Finally, the adopted conservative phase-field LB method enables the simulation of ternary systems, allowing the study of Janus or multicomponent droplet formation, as shown in Section 3.5. The results demonstrate that, by keeping the fluid properties constant and varying only the flow rate ratios among the three fluids, patterns of multicomponent droplets with different sizes of phase 1 and phase 2 are generated. This enables modulation of both the distance between droplets and their production frequency. Thus, this study employs a realistic three-dimensional cross-junction, which already represents a challenging geometry for diffuse-interface multiphase simulations.

6.3 Multiscale LBM–Langevin Coupling for Nanoparticle Dynamics

Subsequently, Langevin dynamics (LD) is integrated into the LB method [41,42] through a two-way coupling, providing a multiscale framework for the study of nanoparticle dynamics suspended in Newtonian fluids (cf. Chapter 4). By representing nanoparticles as subgrid-scale points, the approach captures the combined influence of many-body hydrodynamic interactions, thermal fluctuations, and inter-particle effects.

The results show that the velocity autocorrelation function is correctly reproduced in both the short-time and long-time regimes, and that the framework is able to recover the expected Brownian diffusivity. In particular, the developed method captures the exponential decay of the velocity autocorrelation function in the short-time regime and the algebraic decay in the long-time regime, as well as the Stokes–Einstein behavior for the translational diffusivity. The framework also distinguishes between overdamped and underdamped particle dynamics, depending on the particle mass and the Brownian relaxation time.

The results demonstrate that the methodology reliably reproduces the complex dynamics of nanoparticles and offers a versatile multiscale tool for investigating a wide range of microfluidic processes, including blood component separation, cell sorting, polymer chain extraction, and colloidal suspension transport.

6.4 LBM for Viscoelastic Fluids

Ultimately, a three-dimensional viscoelastic Lattice Boltzmann framework with a novel NEBB boundary-condition scheme is presented, enabling accurate reconstruction of both hydrodynamic variables and the polymeric stress tensor at the boundaries (cf. Chapter 5). To the best of my knowledge, these NEBB formulations represent the first complete derivation of this type and prevent non-physical wall artifacts, such as polymer-chain compression and stretching, which propagate into the interior domain and compromise solution accuracy.

By employing two distribution functions for the hydrodynamic variables and the constitutive equation, hybrid approaches are avoided and the parallelization capabilities of the Lattice Boltzmann method are fully exploited, which is an essential advantage for realistic microfluidic applications. Systematic validation across key parameters, including Weissenberg number, viscosity ratio, and domain size, demonstrates the framework's ability to capture essential viscoelastic effects. The developed method therefore provides a reliable tool for the study of wall-bounded viscoelastic flows and lays the foundation for future investigations of complex geometries and microfluidic processes, such as viscoelastic particle focusing.

Final Remarks and Outlook

The methodologies developed in this thesis provide a computational framework for the study of complex microfluidic systems involving multicomponent fluids, nanoparticle suspensions, and viscoelastic effects. By combining different Lattice Boltzmann-based formulations with appropriate mesoscale models and boundary treatments, the present work contributes to the predictive simulation of microscale transport phenomena and provides tools that complement experimental investigations in both terrestrial and space-related applications.

Despite the breadth of the proposed framework, several limitations can be identified. In particular, model validation remains predominantly numerical, with limited direct comparison against systematic experimental datasets. This aspect is particularly relevant for the nanoparticle transport and viscoelastic fluid models, which would benefit from further quantitative validation in well-controlled microfluidic configurations. Furthermore, biomedical applications have been addressed primarily in terms of physical consistency and feasibility, rather than through fully developed application-oriented case studies, indicating that additional work is required to strengthen the connection with experimental and technological implementations.

From a modelling perspective, the focus of this thesis has been primarily on the formulation and numerical implementation of advanced Lattice Boltzmann-based methods, while more extensive comparisons with alternative computational approaches and a more detailed exploration of specific application scenarios remain open issues. In particular, the nanoparticle dynamics and viscoelastic extensions constitute initial developments that could be further refined through improved boundary treatments for more complex geometrical configurations, as well as through more comprehensive validation strategies.

From a computational standpoint, the proposed framework is naturally suited to parallel implementation due to the local nature of the Lattice Boltzmann method. However, large-scale three-dimensional simulations, especially in multicomponent and viscoelastic regimes, remain computationally demanding. In this context, further optimisation and

extension towards massively parallel architectures, including GPU-based implementations, would be beneficial to enhance the applicability of the models to realistic aerospace and biomedical problems.

Overall, the methodologies developed herein provide a consistent and flexible framework for the simulation of complex microfluidic systems. Further developments in terms of experimental validation, application-oriented case studies, and computational scalability are considered necessary in order to fully exploit their potential in both academic and industrial contexts.

Appendix A

Non-equilibrium bounce-back method for multicomponent systems

The novel non-equilibrium bounce back (NEBB) scheme is derived to reconstruct both the hydrodynamic and phase-field unknown distribution functions at the lattice boundaries. The NEBB method proposed by Zou and He [35] to enforce Dirichlet boundary condition on velocity or pressure in a single-phase flow is here adapted to compute the unknown distribution functions $f_\alpha(\mathbf{x}, t)$ and $h_a^i(\mathbf{x}, t)$ at the boundaries preserving mass and momentum conservation for the general case of non-zero forcing term.

At the boundaries, three of the four thermodynamic variables are known. As an example, take the case of a node of coordinates (x_{min}, y, z) . After streaming, the distribution functions $f_1, f_7, f_9, f_{13}, f_{15}$ are unknown, while the other distribution functions are known. Setting the lattice time step Δt and the lengths $\Delta x, \Delta y, \Delta z$ equal to 1, the macroscopic moments (Eqs. (2.40) and (2.41)) are

$$f_1 + f_7 + f_9 + f_{13} + f_{15} = \frac{p}{\rho c_s^2} - (f_0 + f_2 + f_3 + f_4 + f_5 + f_6 + f_8 + f_{10} + f_{11} + f_{12} + f_{14} + f_{16} + f_{17} + f_{18}) \quad (\text{A.1})$$

$$f_1 + f_7 + f_9 + f_{13} + f_{15} = u_x + (f_2 + f_8 + f_{14} + f_{10} + f_{16}) - \frac{F_x \Delta t}{2\rho} \quad (\text{A.2})$$

$$f_7 - f_{13} = u_y - (f_3 - f_4 - f_8 + f_{11} - f_{12} + f_{14} + f_{17} - f_{18}) - \frac{F_y \Delta t}{2\rho} \quad (\text{A.3})$$

$$f_9 - f_{15} = u_z - (f_5 - f_6 - f_{10} + f_{11} - f_{12} + f_{16} - f_{17} + f_{18}) - \frac{F_z \Delta t}{2\rho} \quad (\text{A.4})$$

Thus, the unknown thermodynamic variable (pressure or velocity component) is computed combining Eqs. (A.1) and (A.2):

$$u_x - \frac{p}{\rho c_s^2} + (f_0 + f_3 + f_4 + f_5 + f_6 + f_{11} + f_{12} + f_{17} + f_{18}) + 2(f_2 + f_8 + f_{14} + f_{10} + f_{16}) - \frac{F_x \Delta t}{2\rho} = 0 \quad (\text{A.5})$$

The unknown distribution functions are determined by the bounce-back of their non-

equilibrium part, considering the tangential, N_y and N_z , and the normal, N_x , corrections

$$f_1 = f_2 + (f_1^{eq} - f_2^{eq}) + N_x, \quad (\text{A.6})$$

$$f_7 = f_8 + (f_7^{eq} - f_8^{eq}) + N_y + N_x, \quad (\text{A.7})$$

$$f_{13} = f_{14} + (f_{13}^{eq} - f_{14}^{eq}) - N_y + N_x, \quad (\text{A.8})$$

$$f_9 = f_{10} + (f_9^{eq} - f_{10}^{eq}) + N_z + N_x, \quad (\text{A.9})$$

$$f_{15} = f_{16} + (f_{15}^{eq} - f_{16}^{eq}) - N_z + N_x. \quad (\text{A.10})$$

Substituting such relations in Eqs. (A.2), (A.3), (A.4) and computing f_i^{eq} from Eq. (2.31), the tangential and normal corrections are obtained

$$N_x = -\frac{F_x \Delta t}{10\rho} \quad (\text{A.11})$$

$$N_y = \frac{1}{3}u_y - \frac{F_y \Delta t}{4\rho} + \frac{1}{2}(f_4 + f_{12} + f_{18} - f_3 - f_{11} - f_{17}) \quad (\text{A.12})$$

$$N_z = \frac{1}{3}u_z - \frac{F_z \Delta t}{4\rho} + \frac{1}{2}(f_6 + f_{12} + f_{17} - f_5 - f_{11} - f_{18}) \quad (\text{A.13})$$

Finally, the unknown distribution functions are computed by Eqs. (A.6)–(A.10).

Considering the same approach described before, the phase-field variable ϕ_i is computed by

$$\begin{aligned} \phi_i = \frac{1}{(1 - u_x)} & \left[(h_0^i + h_3^i + h_4^i + h_5^i + h_6^i + h_7^i + h_{12}^i + h_{17}^i + h_{18}^i) + \right. \\ & \left. + 2(h_2^i + h_8^i + h_{14}^i + h_{10}^i + h_{16}^i) - \frac{F_x^{\phi,i} \Delta t}{2} \right] \end{aligned} \quad (\text{A.14})$$

and the unknown distribution functions by

$$h_1^i = h_2^i + (h_1^{i,eq} - h_2^{i,eq}) + N_x^i \quad (\text{A.15})$$

$$h_7^i = h_8^i + (h_7^{i,eq} - h_8^{i,eq}) + N_y^i + N_x^i \quad (\text{A.16})$$

$$h_{13}^i = h_{14}^i + (h_{13}^{i,eq} - h_{14}^{i,eq}) - N_y^i + N_x^i \quad (\text{A.17})$$

$$h_9^i = h_{10}^i + (h_9^{i,eq} - h_{10}^{i,eq}) + N_z^i + N_x^i \quad (\text{A.18})$$

$$h_{15}^i = h_{16}^i + (h_{15}^{i,eq} - h_{16}^{i,eq}) - N_z^i + N_x^i \quad (\text{A.19})$$

where the normal and tangential corrections are given by

$$N_x^i = -\frac{F_x^{\phi,i} \Delta t}{10} \quad (\text{A.20})$$

$$N_y^i = \frac{1}{3}\phi_i u_y - \frac{F_y^{\phi,i} \Delta t}{4} + \frac{1}{2}(h_4^i + h_{12}^i + h_{18}^i - h_3^i - h_{11}^i - h_{17}^i) \quad (\text{A.21})$$

$$N_z^i = \frac{1}{3}\phi_i u_z - \frac{F_z^{\phi,i} \Delta t}{4} + \frac{1}{2}(h_6^i + h_{12}^i + h_{17}^i - h_5^i - h_{11}^i - h_{18}^i) \quad (\text{A.22})$$

By analogy, the same procedure described above can be applied to enforce these boundary conditions at all nodes located on boundaries aligned with any of the Cartesian directions.

Appendix B

Non-equilibrium bounce-back method for viscoelastic fluids

The novel non-equilibrium bounce back (NEBB) scheme is derived to reconstruct both the density and stress tensor unknown distribution functions at the lattice boundaries. The NEBB method proposed by Zou and He [35] to enforce Dirichlet boundary condition on velocity or pressure in a newtonian flow is here adapted to compute the unknown distribution functions $f_\alpha(\mathbf{x}, t)$ and $G_{ij\alpha}(\mathbf{x}, t)$ at the boundaries preserving mass and momentum conservation for the general case of non-zero forcing term.

At the boundaries, three of the four thermodynamic variables are known. As an example, take the case of a node of coordinates (x_{min}, y, z) . After streaming, the distribution functions $f_1, f_7, f_9, f_{13}, f_{15}$ are unknown, while the other distribution functions are known. Setting the lattice time step Δt and the lengths $\Delta x, \Delta y, \Delta z$ equal to 1, the macroscopic moments (Eqs. (5.8) and (5.9)) are

$$f_1 + f_7 + f_9 + f_{13} + f_{15} = \rho - (f_0 + f_2 + f_3 + f_4 + f_5 + f_6 + f_8 + f_{10} + f_{11} + f_{12} + f_{14} + f_{16} + f_{17} + f_{18}) \quad (\text{B.1})$$

$$f_1 + f_7 + f_9 + f_{13} + f_{15} = \rho u_x + (f_2 + f_8 + f_{14} + f_{10} + f_{16}) - \frac{F_x \Delta t}{2} \quad (\text{B.2})$$

$$f_7 - f_{13} = \rho u_y - (f_3 - f_4 - f_8 + f_{11} - f_{12} + f_{14} + f_{17} - f_{18}) - \frac{F_y \Delta t}{2} \quad (\text{B.3})$$

$$f_9 - f_{15} = \rho u_z - (f_5 - f_6 - f_{10} + f_{11} - f_{12} + f_{16} - f_{17} + f_{18}) - \frac{F_z \Delta t}{2} \quad (\text{B.4})$$

Thus, the unknown thermodynamic variable (density or velocity component) is computed combining Eqs. (B.1) and (B.2):

$$(u_x - 1)\rho + (f_0 + f_3 + f_4 + f_5 + f_6 + f_{11} + f_{12} + f_{17} + f_{18}) + 2(f_2 + f_8 + f_{14} + f_{10} + f_{16}) - \frac{F_x \Delta t}{2} = 0 \quad (\text{B.5})$$

The unknown distribution functions are determined by the bounce-back of their non-

equilibrium part, considering the tangential, N_y and N_z , and the normal, N_x , corrections

$$f_1 = f_2 + (f_1^{eq} - f_2^{eq}) + N_x, \quad (\text{B.6})$$

$$f_7 = f_8 + (f_7^{eq} - f_8^{eq}) + N_y + N_x, \quad (\text{B.7})$$

$$f_{13} = f_{14} + (f_{13}^{eq} - f_{14}^{eq}) - N_y + N_x, \quad (\text{B.8})$$

$$f_9 = f_{10} + (f_9^{eq} - f_{10}^{eq}) + N_z + N_x, \quad (\text{B.9})$$

$$f_{15} = f_{16} + (f_{15}^{eq} - f_{16}^{eq}) - N_z + N_x. \quad (\text{B.10})$$

Substituting such relations in Eqs. (B.2), (B.3), (B.4) and computing f_i^{eq} from Eq. (5.6), the tangential and normal corrections are obtained

$$N_x = -\frac{F_x \Delta t}{10} \quad (\text{B.11})$$

$$N_y = \frac{1}{3} \rho u_y - \frac{F_y \Delta t}{4} + \frac{1}{2} (f_4 + f_{12} + f_{18} - f_3 - f_{11} - f_{17}) \quad (\text{B.12})$$

$$N_z = \frac{1}{3} \rho u_z - \frac{F_z \Delta t}{4} + \frac{1}{2} (f_6 + f_{12} + f_{17} - f_5 - f_{11} - f_{18}) \quad (\text{B.13})$$

Finally, the unknown distribution functions are computed by Eqs. (B.6)–(B.10).

Considering the same approach described before, the stress tensor component τ_{ij} is computed by

$$\begin{aligned} \tau_{ij} = \frac{1}{(1 - u_x)} \left[(G_{ij0} + G_{ij3} + G_{ij4} + G_{ij5} + G_{ij6} + G_{ij7} + G_{ij12} + G_{ij17} + G_{ij18}) + \right. \\ \left. + 2(G_{ij2} + G_{ij8} + G_{ij14} + G_{ij10} + G_{ij16}) - \frac{\Delta \tilde{t}}{2} \chi_{ij} u_x \right] \end{aligned} \quad (\text{B.14})$$

and the unknown distribution functions by

$$G_{ij1} = G_{ij2} + (G_{ij1}^{eq} - G_{ij2}^{eq}) + N_x \quad (\text{B.15})$$

$$G_{ij7} = G_{ij8} + (G_{ij7}^{eq} - G_{ij8}^{eq}) + N_y + N_x \quad (\text{B.16})$$

$$G_{ij13} = G_{ij14} + (G_{ij13}^{eq} - G_{ij14}^{eq}) - N_y + N_x \quad (\text{B.17})$$

$$G_{ij9} = G_{ij10} + (G_{ij9}^{eq} - G_{ij10}^{eq}) + N_z + N_x \quad (\text{B.18})$$

$$G_{ij15} = G_{ij16} + (G_{ij15}^{eq} - G_{ij16}^{eq}) - N_z + N_x \quad (\text{B.19})$$

where the normal and tangential corrections are given by

$$N_x = -\frac{\Delta \tilde{t}}{10} \chi_{ij} u_x \quad (\text{B.20})$$

$$N_y = \frac{1}{3} \tau_{ij} u_y + \frac{1}{2} (G_{ij4} + G_{ij12} + G_{ij18} - G_{ij3} - G_{ij11} - G_{ij17}) - \frac{\Delta \tilde{t}}{4} \chi_{ij} u_y \quad (\text{B.21})$$

$$N_z = \frac{1}{3} \tau_{ij} u_z + \frac{1}{2} (G_{ij6} + G_{ij12} + G_{ij17} - G_{ij5} - G_{ij11} - G_{ij18}) - \frac{\Delta \tilde{t}}{4} \chi_{ij} u_z \quad (\text{B.22})$$

By analogy, the same procedure described above can be applied to enforce these boundary conditions at all nodes located on boundaries aligned with any of the Cartesian directions.

References

- [1] S. Kuang, N. M. Singh, Y. Wu, Y. Shen, W. Ren, L. Tu, K.-T. Yong, P. Song, Role of microfluidics in accelerating new space missions, *Biomicrofluidics* 16 (2) (2022).
- [2] P. K. Panigrahi, *Transport phenomena in microfluidic systems*, John Wiley & Sons, 2016.
- [3] E. S. Nelson, Design principles for microfluidic biomedical diagnostics in space, *Biomedical Engineering—From Theory to Applications*. InTech, London (UK) (2011) 131–156.
- [4] P. W. Taylor, Impact of space flight on bacterial virulence and antibiotic susceptibility, *Infection and drug resistance* (2015) 249–262.
- [5] N. Nguyen, G. Kim, K.-S. Kim, Effects of microgravity on human physiology, *Journal of Aerospace Medicine* 30 (1) (2020) 25–29.
- [6] A. Jogdand, M. Landolina, Y. Chen, Organs in orbit: how tissue chip technology benefits from microgravity, a perspective, *Frontiers in lab on a Chip Technologies* 3 (2024) 1356688.
- [7] A. Krakos, J. Jarosz, P. Śniadek, M. Psurski, A. Graja, M. Białas, E. Oliszewska, J. Wietrzyk, R. Walczak, J. Dziuban, Microfluidic-assisted human cancer cells culturing platform for space biology applications, *Sensors* 22 (16) (2022) 6183.
- [8] D. J. Robson, C. Cappelletti, Biomedical payloads: A maturing application for cubesats, *Acta Astronautica* 191 (2022) 394–403.
- [9] A. Krakos, Lab-on-chip technologies for space research—current trends and prospects, *Microchimica Acta* 191 (1) (2024) 31.
- [10] Y. Chen, B. Wu, C. Zhang, Z. Fan, Y. Chen, B. Xin, Q. Xie, Current progression: Application of high-throughput sequencing technique in space microbiology, *BioMed Research International* 2020 (1) (2020) 4094191.
- [11] A. Kanapskyte, E. M. Hawkins, L. C. Liddell, S. R. Bhardwaj, D. Gentry, S. R. Santa Maria, Space biology research and biosensor technologies: past, present, and future, *Biosensors* 11 (2) (2021) 38.
- [12] L. Zea, S. R. Santa Maria, A. J. Ricco, Cubesats for microbiology and astrobiology research, in: *Cubesat Handbook*, Elsevier, 2021, pp. 147–162.

- [13] C. Chen, B. T. Mehl, A. S. Munshi, A. D. Townsend, D. M. Spence, R. S. Martin, 3d-printed microfluidic devices: fabrication, advantages and limitations—a mini review, *Analytical Methods* 8 (31) (2016) 6005–6012.
- [14] M. Mobaraki, M. Ghaffari, A. Yazdanpanah, Y. Luo, D. Mills, Bioinks and bioprinting: A focused review, *Bioprinting* 18 (2020) e00080.
- [15] A. C. Daly, M. E. Prendergast, A. J. Hughes, J. A. Burdick, Bioprinting for the biologist, *Cell* 184 (1) (2021) 18–32.
- [16] P. S. Gungor-Ozkerim, I. Inci, Y. S. Zhang, A. Khademhosseini, M. R. Dokmeci, Bioinks for 3d bioprinting: an overview, *Biomaterials science* 6 (5) (2018) 915–946.
- [17] A. B. Dababneh, I. T. Ozbolat, Bioprinting technology: a current state-of-the-art review, *Journal of Manufacturing Science and Engineering* 136 (6) (2014) 061016.
- [18] E. Stuhlinger, *Electric Propulsion Development*, Academic Press, 1963.
- [19] Y.-H. Chiu, G. Gaeta, T. Heine, R. Dressler, D. Levandier, Analysis of the electro-spray plume from the emi-im propellant externally wetted on a tungsten needle, in: 42nd AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, 2006, p. 5010.
- [20] J. Zeleny, The electrical discharge from liquid points, and a hydrostatic method of measuring the surface tension, *Phys. Rev.* 3 (1914) 69.
- [21] M. Armano, H. Audley, G. Auger, et al., Sub-femto-g free fall for space-based gravitational wave observatories: Lisa pathfinder results, *Phys. Rev. Lett.* 116 (2016) 231101.
- [22] B. Lange, The drag-free satellite, *AIAA J.* 2 (2011) 1590.
- [23] J. Zwahlen, V. Hruby, C. Campbell, N. Demmons, E. Ehrbar, C. Freeman, R. Martin, T. Roy, D. Spence, T. Randolph, et al., Flow control micro-valve for the st7-drs colloid thruster, in: 44th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, 2013, p. 4930.
- [24] S. Dandavino, C. Ataman, C. N. Ryan, S. Chakraborty, D. Courtney, J. P. W. Stark, H. Shea, Microfabricated electrospray thrusters for spacecraft propulsion, *J. Micromech. Microeng.* 24 (2014) 075011.
- [25] Y. Zhao, X. Lv, X. Li, N. Rcheulishvili, Y. Chen, Z. Li, Y. Deng, Microfluidic actuated and controlled systems and application for lab-on-chip in space life science, *Space: Science & Technology* 3 (2023) 0008.
- [26] K. Timm, H. Kusumaatmaja, A. Kuzmin, O. Shardt, G. Silva, E. Viggen, *The lattice boltzmann method: principles and practice*, Cham, Switzerland: Springer International Publishing AG (2016).
- [27] A. Mohamad, *Lattice boltzmann method*, Vol. 70, Springer, 2011.
- [28] X. Shan, H. Chen, Lattice boltzmann model for simulating flows with multiple phases and components, *Physical review E* 47 (3) (1993) 1815.

- [29] X. Shan, H. Chen, Simulation of nonideal gases and liquid-gas phase transitions by the lattice boltzmann equation, *Physical Review E* 49 (4) (1994) 2941.
- [30] A. K. Gunstensen, D. H. Rothman, S. Zaleski, G. Zanetti, Lattice boltzmann model of immiscible fluids, *Physical review A* 43 (8) (1991) 4320.
- [31] D. De Marinis, A. Mantegazza, A. Coclite, M. D. de Tullio, A fluid-structure interaction method for soft particle transport in curved microchannels, *Computer Methods in Applied Mechanics and Engineering* 418 (2024) 116592.
- [32] R. Haghani-Hassan-Abadi, M. H. Rahimian, A. Fakhari, Conservative phase-field lattice-boltzmann model for ternary fluids, *Journal of Computational Physics* 374 (2018) 668–691.
- [33] R. Haghani-Hassan-Abadi, A. Fakhari, M. H. Rahimian, Numerical simulation of three-component multiphase flows at high density and viscosity ratios using lattice boltzmann methods, *Physical Review E* 97 (3) (2018) 033312.
- [34] H. Liang, B. Shi, Z. Chai, Lattice boltzmann modeling of three-phase incompressible flows, *Physical Review E* 93 (1) (2016) 013308.
- [35] Q. Zou, X. He, On pressure and velocity boundary conditions for the lattice boltzmann bgk model, *Physics of fluids* 9 (6) (1997) 1591–1598.
- [36] H. Liang, J. Xu, J. Chen, Z. Chai, B. Shi, Lattice boltzmann modeling of wall-bounded ternary fluid flows, *Applied Mathematical Modelling* 73 (2019) 487–513.
- [37] Y. Zhang, B. Dong, X. An, Y. Wang, X. Zhou, W. Li, Phase-field-based lattice boltzmann model for ternary fluid flows considering the wettability effect, *Applied Mathematical Modelling* 103 (2022) 195–220.
- [38] S. Signorile, D. De Marinis, M. D. de Tullio, A 3D lattice Boltzmann method for accurate wetting of ternary fluids with broad rheological variability, *Computer Methods in Applied Mechanics and Engineering* 446 (2025) 118246.
- [39] Q. He, Y. Li, W. Huang, Y. Hu, Y. Wang, Lattice boltzmann model for ternary fluids with solid particles, *Physical Review E* 101 (3) (2020) 033307.
- [40] D. De Marinis, S. Signorile, M. Santeramo, M. D. de Tullio, Three-dimensional conservative phase-field lattice boltzmann simulations of droplet generation in a flow-focusing microdevice, *Physics of Fluids* 37 (12) (2025).
- [41] Z. Liu, Y. Zhu, J. R. Clausen, J. B. Lechman, R. R. Rao, C. K. Aidun, Multiscale method based on coupled lattice-boltzmann and langevin-dynamics for direct simulation of nanoscale particle/polymer suspensions in complex flows, *International Journal for Numerical Methods in Fluids* 91 (5) (2019) 228–246.
- [42] Z. Liu, Y. Zhu, R. R. Rao, J. R. Clausen, C. K. Aidun, Nanoparticle transport in cellular blood flow, *Computers & Fluids* 172 (2018) 609–620.
- [43] J. Su, J. Ouyang, X. Wang, B. Yang, W. Zhou, Lattice boltzmann method for the simulation of viscoelastic fluid flows over a large range of weissenberg numbers, *Journal of Non-Newtonian Fluid Mechanics* 194 (2013) 42–59.

- [44] T. Zhang, Z.-Y. Hong, S.-Y. Tang, W. Li, D. W. Inglis, Y. Hosokawa, Y. Yalikun, M. Li, Focusing of sub-micrometer particles in microfluidic devices, *Lab on a Chip* 20 (1) (2020) 35–53.
- [45] Y. Ding, P. D. Howes, A. J. deMello, Recent advances in droplet microfluidics, *Analytical chemistry* 92 (1) (2019) 132–149.
- [46] X. C. i Solvas, A. DeMello, Droplet microfluidics: recent developments and future applications, *Chemical Communications* 47 (7) (2011) 1936–1942.
- [47] H. Miwa, R. Dimatteo, J. de Rutte, R. Ghosh, D. Di Carlo, Single-cell sorting based on secreted products for functionally defined cell therapies, *Microsystems & Nanoengineering* 8 (1) (2022) 84.
- [48] H. Lin, W. Yu, K. A. Sabet, M. Bogumil, Y. Zhao, J. Hambalek, S. Lin, S. Chandrasekaran, O. Garner, D. Di Carlo, et al., Ferrobotic swarms enable accessible and adaptable automated viral testing, *Nature* 611 (7936) (2022) 570–577.
- [49] K. Schroen, C. Berton-Carabin, D. Renard, M. Marquis, A. Boire, R. Cochereau, C. Amine, S. Marze, Droplet microfluidics for food and nutrition applications, *Micromachines* 12 (8) (2021) 863.
- [50] Z. Chen, S. Kheiri, E. W. Young, E. Kumacheva, Trends in droplet microfluidics: From droplet generation to biomedical applications, *Langmuir* 38 (20) (2022) 6233–6248.
- [51] H. N. Joensson, H. Andersson Svahn, Droplet microfluidics—a tool for single-cell analysis, *Angewandte Chemie International Edition* 51 (49) (2012) 12176–12192.
- [52] H. Jia, Y. Liu, Y. Zhao, X. Wang, X. Yu, One-to-one alternating droplet generation in a microfluidic double t-junction at low interfacial tension, *International Journal of Multiphase Flow* 189 (2025) 105247.
- [53] S. O. Unverdi, G. Tryggvason, A front-tracking method for viscous, incompressible, multi-fluid flows, *Journal of computational physics* 100 (1) (1992) 25–37.
- [54] C. W. Hirt, B. D. Nichols, Volume of fluid (vof) method for the dynamics of free boundaries, *Journal of computational physics* 39 (1) (1981) 201–225.
- [55] M. Sussman, P. Smereka, S. Osher, A level set approach for computing solutions to incompressible two-phase flow, *Journal of Computational physics* 114 (1) (1994) 146–159.
- [56] H. Garcke, B. Nestler, B. Stoth, A multiphase field concept: numerical simulations of moving phase boundaries and multiple junctions, *SIAM Journal on Applied Mathematics* 60 (1) (1999) 295–315.
- [57] M. Latva-Kokko, D. H. Rothman, Diffusion properties of gradient-based lattice boltzmann models of immiscible fluids, *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics* 71 (5) (2005) 056702.
- [58] H. Liu, A. J. Valocchi, Q. Kang, Three-dimensional lattice boltzmann model for immiscible two-phase flow simulations, *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics* 85 (4) (2012) 046309.

- [59] S. Leclaire, M. Reggio, J.-Y. Trépanier, Numerical evaluation of two recoloring operators for an immiscible two-phase flow lattice boltzmann model, *Applied Mathematical Modelling* 36 (5) (2012) 2237–2252.
- [60] H. Huang, J.-J. Huang, X.-Y. Lu, M. C. Sukop, On simulations of high-density ratio flows using color-gradient multiphase lattice boltzmann models, *International Journal of Modern Physics C* 24 (04) (2013) 1350021.
- [61] S. Leclaire, N. Pellerin, M. Reggio, J.-Y. Trépanier, Unsteady immiscible multiphase flow validation of a multiple-relaxation-time lattice boltzmann method, *Journal of Physics A: Mathematical and Theoretical* 47 (10) (2014) 105501.
- [62] I. Halliday, A. Hollis, C. Care, Lattice boltzmann algorithm for continuum multicomponent flow, *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics* 76 (2) (2007) 026708.
- [63] Y. Yu, H. Liu, D. Liang, Y. Zhang, A versatile lattice boltzmann model for immiscible ternary fluid flows, *Physics of Fluids* 31 (1) (2019).
- [64] X. Shan, G. Doolen, Multicomponent lattice-boltzmann model with interparticle interaction, *Journal of statistical physics* 81 (1995) 379–393.
- [65] W. Wang, Z. Liu, Y. Jin, Y. Cheng, Lbm simulation of droplet formation in microchannels, *Chemical engineering journal* 173 (3) (2011) 828–836.
- [66] F. Pelusi, M. Sega, J. Harting, Liquid film rupture beyond the thin-film equation: A multi-component lattice boltzmann study, *Physics of Fluids* 34 (6) (2022).
- [67] J. Bao, L. Schaefer, Lattice boltzmann equation model for multi-component multiphase flow with high density ratios, *Applied Mathematical Modelling* 37 (4) (2013) 1860–1871.
- [68] A. Montessori, G. Falcucci, M. La Rocca, S. Ansumali, S. Succi, Three-dimensional lattice pseudo-potentials for multiphase flow simulations at high density ratios, *Journal of Statistical Physics* 161 (2015) 1404–1419.
- [69] D. Lycett-Brown, K. H. Luo, Cascaded lattice boltzmann method with improved forcing scheme for large-density-ratio multiphase flow at high reynolds and weber numbers, *Physical Review E* 94 (5) (2016) 053313.
- [70] M. R. Swift, W. Osborn, J. M. Yeomans, Lattice boltzmann simulation of nonideal fluids, *Physical review letters* 75 (5) (1995) 830.
- [71] M. R. Swift, E. Orlandini, W. Osborn, J. Yeomans, Lattice boltzmann simulations of liquid-gas and binary fluid systems, *Physical Review E* 54 (5) (1996) 5041.
- [72] D. J. Holdych, D. Rovas, J. G. Georgiadis, R. O. Buckius, An improved hydrodynamics formulation for multiphase flow lattice-boltzmann models, *International Journal of Modern Physics C* 9 (08) (1998) 1393–1404.
- [73] T. Inamuro, T. Ogata, S. Tajima, N. Konishi, A lattice boltzmann method for incompressible two-phase flows with large density differences, *Journal of Computational physics* 198 (2) (2004) 628–644.

- [74] H. Zheng, C. Shu, Y.-T. Chew, A lattice boltzmann model for multiphase flows with large density ratio, *Journal of computational physics* 218 (1) (2006) 353–371.
- [75] A. Fakhari, M. H. Rahimian, Phase-field modeling by the method of lattice boltzmann equations, *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics* 81 (3) (2010) 036707.
- [76] C. Semperebon, T. Krüger, H. Kusumaatmaja, Ternary free-energy lattice boltzmann model with tunable surface tensions and contact angles, *Physical Review E* 93 (3) (2016) 033305.
- [77] E. Dinesh Kumar, S. A. Sannasiraj, V. Sundar, Phase field lattice boltzmann model for air-water two phase flows, *Physics of Fluids* 31 (7) (2019).
- [78] D. Jacqmin, Calculation of two-phase navier–stokes flows using phase-field modeling, *Journal of computational physics* 155 (1) (1999) 96–127.
- [79] D. Jacqmin, Contact-line dynamics of a diffuse fluid interface, *Journal of Fluid Mechanics* 402 (2000) 57–88.
- [80] X. He, S. Chen, R. Zhang, A lattice boltzmann scheme for incompressible multiphase flow and its application in simulation of rayleigh–taylor instability, *Journal of computational physics* 152 (2) (1999) 642–663.
- [81] J. W. Cahn, J. E. Hilliard, Free energy of a nonuniform system. iii. nucleation in a two-component incompressible fluid, *The Journal of chemical physics* 31 (3) (1959) 688–699.
- [82] S. M. Allen, J. W. Cahn, Mechanisms of phase transformations within the miscibility gap of fe-rich fe-al alloys, *Acta Metallurgica* 24 (5) (1976) 425–437.
- [83] D. Sun, H. Xing, X. Dong, Y. Han, An anisotropic lattice boltzmann–phase field scheme for numerical simulations of dendritic growth with melt convection, *International Journal of Heat and Mass Transfer* 133 (2019) 1240–1250.
- [84] P.-H. Chiu, Y.-T. Lin, A conservative phase field method for solving incompressible two-phase flows, *Journal of Computational Physics* 230 (1) (2011) 185–204.
- [85] Y. Sun, C. Beckermann, Sharp interface tracking using the phase-field equation, *Journal of Computational Physics* 220 (2) (2007) 626–653.
- [86] A. Fakhari, T. Mitchell, C. Leonardi, D. Bolster, Improved locality of the phase-field lattice-boltzmann model for immiscible fluids at high density ratios, *Physical Review E* 96 (5) (2017) 053301.
- [87] Y. Zu, S. He, Phase-field-based lattice boltzmann model for incompressible binary fluid systems with density and viscosity contrasts, *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics* 87 (4) (2013) 043301.
- [88] M. Geier, A. Fakhari, T. Lee, Conservative phase-field lattice boltzmann model for interface tracking equation, *Physical Review E* 91 (6) (2015) 063309.

- [89] A. Fakhari, M. Geier, D. Bolster, A simple phase-field model for interface tracking in three dimensions, *Computers & Mathematics with Applications* 78 (4) (2019) 1154–1165.
- [90] R. Mei, L.-S. Luo, W. Shyy, An accurate curved boundary treatment in the lattice boltzmann method, *Journal of computational physics* 155 (2) (1999) 307–330.
- [91] R. Haghani-Hassan-Abadi, A. Fakhari, M.-H. Rahimian, Phase-change modeling based on a novel conservative phase-field method, *Journal of Computational Physics* 432 (2021) 110111.
- [92] A. J. Ladd, Numerical simulations of particulate suspensions via a discretized boltzmann equation. part 1. theoretical foundation, *Journal of fluid mechanics* 271 (1994) 285–309.
- [93] R. Folch, J. Casademunt, A. Hernández-Machado, L. Ramirez-Piscina, Phase-field model for hele-shaw flows with arbitrary viscosity contrast. i. theoretical approach, *Physical Review E* 60 (2) (1999) 1724.
- [94] J. M. Buick, C. Greated, Gravity in a lattice boltzmann model, *Physical Review E* 61 (5) (2000) 5307.
- [95] M. Cheng, J. Hua, J. Lou, Simulation of bubble–bubble interaction using a lattice boltzmann method, *Computers & Fluids* 39 (2) (2010) 260–270.
- [96] R. Kalantarpour, A. Ebadi, S. M. Hosseinalipour, H. Liang, Three-component phase-field lattice boltzmann method with high density ratio and ability to simulate total spreading states, *Computers & Fluids* 204 (2020) 104480.
- [97] P.-G. De Gennes, F. Brochard-Wyart, D. Quéré, *Capillarity and wetting phenomena: drops, bubbles, pearls, waves*, Springer Science & Business Media, 2003.
- [98] C.-Y. Zhang, H. Ding, P. Gao, Y.-L. Wu, Diffuse interface simulation of ternary fluids in contact with solid, *Journal of Computational Physics* 309 (2016) 37–51.
- [99] H.-C. Kan, H. S. Udaykumar, W. Shyy, R. Tran-Son-Tay, Hydrodynamics of a compound drop with application to leukocyte modeling, *Physics of fluids* 10 (4) (1998) 760–774.
- [100] H.-C. Kan, H. S. Udaykumar, W. Shyy, R. Tran-Son-Tay, Numerical analysis of the deformation of an adherent drop under shear flow, *Journal of Biomechanical Engineering* 121 (2) (1999) 160–9.
- [101] Z. Luo, F. XU, T. Lu, B. Bai, Direct numerical simulation of detachment of single captured leukocyte under different flow conditions, *Journal of Mechanics in Medicine and Biology* 11 (02) (2011) 273–284.
- [102] X. Liu, X. Wang, The deformation of an adherent leukocyte under steady shear flow: a numerical study, *Journal of biomechanics* 37 (7) (2004) 1079–1085.
- [103] H. Y. Shin, S. Fukuda, G. W. Schmid-Schönbein, Fluid shear stress-mediated mechanotransduction in circulating leukocytes and its defect in microvascular dysfunction, *Journal of Biomechanics* 120 (2021) 110394.

- [104] F. Moazzam, F. A. DeLano, B. W. Zweifach, G. W. Schmid-Schönbein, The leukocyte response to fluid stress, *Proceedings of the National Academy of Sciences* 94 (10) (1997) 5338–5343.
- [105] H. Feng, T. Zheng, M. Li, J. Wu, H. Ji, J. Zhang, W. Zhao, J. Guo, Droplet-based microfluidics systems in biomedical applications, *Electrophoresis* 40 (11) (2019) 1580–1590.
- [106] S. K. Jena, T. Srivastava, S. S. Bahga, S. Kondaraju, Effect of channel width on droplet generation inside T-junction microchannel, *Physics of Fluids* 35 (2) (2023).
- [107] W. Zeng, Z. Tong, X. Shan, H. Fu, T. Yang, Monodisperse droplet formation for both low and high capillary numbers in a T-junction microdroplet generator, *Chemical Engineering Science* 243 (2021) 116799.
- [108] A. Sattari, S. Janfaza, M. Mashhadi Keshtiban, N. Tasnim, P. Hanafizadeh, M. Hoorfar, Microfluidic on-chip production of alginate hydrogels using double coflow geometry, *ACS omega* 6 (40) (2021) 25964–25971.
- [109] A. Kalantarifard, E. Alizadeh-Haghighi, C. Elbuken, A microfluidic droplet system for ultra-monodisperse droplet generation: a universal approach, *Chemical Engineering Science* 261 (2022) 117947.
- [110] T. Deydier, G. Bolognesi, G. T. Vladislavljević, Scaled-up droplet generation in parallelised 3D flow focusing junctions, *Colloids and Surfaces A: Physicochemical and Engineering Aspects* 641 (2022) 128439.
- [111] J. Panwar, R. Roy, Modified capillary number to standardize droplet generation in suction-driven microfluidics, *Microfluidics and Nanofluidics* 28 (4) (2024) 23.
- [112] S. Srikanth, S. Raut, S. K. Dubey, I. Ishii, A. Javed, S. Goel, Experimental studies on droplet characteristics in a microfluidic flow focusing droplet generator: effect of continuous phase on droplet encapsulation, *The European Physical Journal E* 44 (8) (2021) 108.
- [113] M. Stevens, E. Balaur, B. Abbey, Computational simulation of the effects of interfacial tension in microfluidic flow focusing droplet generators, *Frontiers in Physics* 11 (2023) 1060780.
- [114] A. Jafari, A. Shamloo, CFD study of droplet formation in a cross-junction microfluidic device: investigating the impact of outflow channel design and viscosity ratio, *Engineering Applications of Computational Fluid Mechanics* 17 (1) (2023) 2243091.
- [115] E. Asghari, A. Moosavi, S. K. Hannani, Analytical and numerical study on droplet breakup in microfluidic t-junction, *Chemical Engineering and Processing-Process Intensification* 177 (2022) 108995.
- [116] M. Bourega, I. Kromba, K. Arbi, S. Soulimane, Effect of velocity ratio, viscosity ratio, contact angle, and channel size ratio on droplet formation, *Fluid Dynamics & Materials Processing* 19 (10) (2023) 2471.

- [117] A. Venkateshwarlu, R. P. Bharti, Effects of surface wettability and flow rates on the interface evolution and droplet pinch-off mechanism in the cross-flow microfluidic systems, *Chemical Engineering Science* 267 (2023) 118279.
- [118] P. Poureslami, M. Majidi, J. R. Kermani, M. A. Bijarchi, Deformation and breakup of a ferrofluid compound droplet migrating in a microchannel under a magnetic field: A phase-field-based multiple-relaxation time lattice boltzmann study, *Journal of Fluid Mechanics* 1013 (2025) A37.
- [119] Y. Ba, H. Liu, J. Sun, R. Zheng, Three dimensional simulations of droplet formation in symmetric and asymmetric T-junctions using the color-gradient lattice Boltzmann model, *International Journal of Heat and Mass Transfer* 90 (2015) 931–947.
- [120] L. Wu, M. Tsutahara, L. S. Kim, M. Ha, Three-dimensional lattice Boltzmann simulations of droplet formation in a cross-junction microchannel, *International Journal of multiphase flow* 34 (9) (2008) 852–864.
- [121] Y. Shi, G. Tang, H. Xia, Lattice Boltzmann simulation of droplet formation in T-junction and flow focusing devices, *Computers & Fluids* 90 (2014) 155–163.
- [122] Y. Fu, L. Bai, K. Bi, S. Zhao, Y. Jin, Y. Cheng, Numerical study of Janus droplet formation in microchannels by a lattice Boltzmann method, *Chemical Engineering and Processing: Process Intensification* 119 (2017) 34–43.
- [123] H. Wang, Y. Fu, Y. Wang, L. Yan, Y. Cheng, Three-dimensional lattice Boltzmann simulation of Janus droplet formation in y-shaped co-flowing microchannel, *Chemical Engineering Science* 225 (2020) 115819.
- [124] N. Wang, C. Semperebon, H. Liu, C. Zhang, H. Kusumaatmaja, Modelling double emulsion formation in planar flow-focusing microchannels, *Journal of Fluid Mechanics* 895 (2020) A22.
- [125] Y. Fu, S. Zhao, L. Bai, Y. Jin, Y. Cheng, Numerical study of double emulsion formation in microchannels by a ternary lattice Boltzmann method, *Chemical Engineering Science* 146 (2016) 126–134.
- [126] H. Liu, Y. Zhang, Lattice Boltzmann simulation of droplet generation in a microfluidic cross-junction, *Communications in Computational Physics* 9 (5) (2011) 1235–1256.
- [127] M. Bouzidi, M. Firdaouss, P. Lallemand, Momentum transfer of a boltzmann-lattice fluid with boundaries, *Physics of fluids* 13 (11) (2001) 3452–3459.
- [128] O. Filippova, D. Hänel, Grid refinement for lattice-bgk models, *Journal of Computational physics* 147 (1) (1998) 219–228.
- [129] S. Tao, Q. He, B. Chen, X. Yang, S. Huang, One-point second-order curved boundary condition for lattice boltzmann simulation of suspended particles, *Computers & Mathematics with Applications* 76 (7) (2018) 1593–1607.

- [130] W. Zhao, J. Huang, W.-A. Yong, Boundary conditions for kinetic theory based models i: Lattice boltzmann models, *Multiscale Modeling & Simulation* 17 (2) (2019) 854–872.
- [131] A. Fakhari, D. Bolster, Diffuse interface modeling of three-phase contact line dynamics on curved boundaries: A lattice boltzmann model for large density and viscosity ratios, *Journal of Computational Physics* 334 (2017) 620–638.
- [132] Y. Yu, Q. Li, Z. Wen, Modified curved boundary scheme for two-phase lattice boltzmann simulations, *Computers & Fluids* 208 (2020) 104638.
- [133] Q. He, Y. Li, W. Huang, Y. Hu, Y. Wang, Phase-field-based lattice boltzmann model for liquid-gas-solid flow, *Physical Review E* 100 (3) (2019) 033314.
- [134] M. Santeramo, G. Carbone, S. Krenn, C. Putignano, A novel energy-based numerical approach for adhesive contact mechanics of rough surfaces, *Journal of the Mechanics and Physics of Solids* 203 (2025) 106217.
- [135] T. Lee, L. Liu, Lattice boltzmann simulations of micron-scale drop impact on dry surfaces, *Journal of Computational Physics* 229 (20) (2010) 8045–8063.
- [136] Q. He, Y. Li, W. Huang, Y. Hu, D. Li, Y. Wang, A unified lattice boltzmann model for immiscible and miscible ternary fluids, *Computers & Mathematics with Applications* 80 (12) (2020) 2830–2859.
- [137] N. Younes, Z. Benseghier, O. Millet, A. Wautier, F. Nicot, R. Wan, Phase-field lattice boltzmann model for liquid bridges and coalescence in wet granular media, *Powder Technology* 411 (2022) 117942.
- [138] S. R. Elgart, M. P. Little, L. J. Chappell, C. M. Milder, M. R. Shavers, J. L. Huff, Z. S. Patel, Radiation exposure and mortality from cardiovascular disease and cancer in early nasa astronauts, *Scientific reports* 8 (1) (2018) 8480.
- [139] J. L. Cortés-Sánchez, J. Callant, M. Krüger, J. Sahana, A. Kraus, B. Baselet, M. Infanger, S. Baatout, D. Grimm, Cancer studies under space conditions: finding answers abroad, *Biomedicines* 10 (1) (2021) 25.
- [140] F. A. Cucinotta, Space radiation risks for astronauts on multiple international space station missions, *PloS one* 9 (4) (2014) e96099.
- [141] J. Wang, P. Ma, D. H. Kim, B.-F. Liu, U. Demirci, Towards microfluidic-based exosome isolation and detection for tumor therapy, *Nano Today* 37 (2021) 101066.
- [142] A. Kulasinghe, H. Wu, C. Punyadeera, M. E. Warkiani, The use of microfluidic technology for cancer applications and liquid biopsy, *Micromachines* 9 (8) (2018) 397.
- [143] Y. Jia, Z. Ni, H. Sun, C. Wang, Microfluidic approaches toward the isolation and detection of exosome nanovesicles, *Ieee Access* 7 (2019) 45080–45098.
- [144] M. Asghari, X. Cao, B. Mateescu, D. van Leeuwen, M. K. Aslan, S. Stavarakis, A. J. deMello, Oscillatory viscoelastic microfluidics for efficient focusing and separation of nanoscale species, *ACS nano* 14 (1) (2019) 422–433.

- [145] J. Hur, A. J. Chung, Microfluidic and nanofluidic intracellular delivery, *Advanced Science* 8 (15) (2021) 2004595.
- [146] L. Miccio, F. Cimmino, I. Kurelac, M. M. Villone, V. Bianco, P. Memmolo, F. Merola, M. Mugnano, M. Capasso, A. Iolascon, et al., Perspectives on liquid biopsy for label-free detection of “circulating tumor cells” through intelligent lab-on-chips, *View* 1 (3) (2020) 20200034.
- [147] J. Chen, P. Li, T. Zhang, Z. Xu, X. Huang, R. Wang, L. Du, Review on strategies and technologies for exosome isolation and purification, *Frontiers in bioengineering and biotechnology* 9 (2022) 811971.
- [148] J. DeCastro, J. Littig, P. P. Chou, J. Mack-Onyeike, A. Srinivasan, M. J. Conboy, I. M. Conboy, K. Aran, The microfluidic toolbox for analyzing exosome biomarkers of aging, *Molecules* 26 (3) (2021) 535.
- [149] M. He, Y. Zeng, Microfluidic exosome analysis toward liquid biopsy for cancer, *Journal of laboratory automation* 21 (4) (2016) 599–608.
- [150] Q. Zhu, M. Heon, Z. Zhao, M. He, Microfluidic engineering of exosomes: editing cellular messages for precision therapeutics, *Lab on a Chip* 18 (12) (2018) 1690–1703.
- [151] M. Karkhane, H. E. Lashgarian, M. Hormozi, S. Fallahi, K. Cheraghipour, A. Marzban, Oncogenesis and tumor inhibition by micrnas and its potential therapeutic applications: a systematic review, *MicroRNA* 9 (3) (2020) 198–215.
- [152] V. Saini, R. Dawar, S. Suneja, S. Gangopadhyay, C. Kaur, Can microrna become next-generation tools in molecular diagnostics and therapeutics? a systematic review, *Egyptian Journal of Medical Human Genetics* 22 (2021) 1–9.
- [153] Y. Wang, Y. Zhou, Z. Hu, The functions of circulating tumor cells in early diagnosis and surveillance during cancer advancement, *Journal of Translational Internal Medicine* 5 (3) (2017) 135–138.
- [154] F. Tian, C. Liu, J. Deng, J. Sun, Microfluidic separation, detection, and engineering of extracellular vesicles for cancer diagnostics and drug delivery, *Accounts of Materials Research* 3 (5) (2022) 498–510.
- [155] F. Fachin, P. Spuhler, J. M. Martel-Foley, J. F. Edd, T. A. Barber, J. Walsh, M. Karabacak, V. Pai, M. Yu, K. Smith, et al., Monolithic chip for high-throughput blood cell depletion to sort rare circulating tumor cells, *Scientific reports* 7 (1) (2017) 10936.
- [156] Y. Liu, G. Yang, Y. Hui, S. Ranaweera, C.-X. Zhao, Microfluidic nanoparticles for drug delivery, *Small* 18 (36) (2022) 2106580.
- [157] M. B. Kulkarni, S. Goel, Microfluidic devices for synthesizing nanomaterials—a review, *Nano Express* 1 (3) (2020) 032004.
- [158] S. Khizar, N. Zine, A. Errachid, N. Jaffrezic-Renault, A. Elaissari, Microfluidic-based nanoparticle synthesis and their potential applications, *Electrophoresis* 43 (7-8) (2022) 819–838.

- [159] G. Opila, E. Gumieniczek-Chlopek, A. Pietrzyk, N. Rerak, S. Zapotoczny, C. Kapusta, Nanoparticles for space theranostics, in: *Space Resources Conference*, Springer, 2024, pp. 43–55.
- [160] A. Kumar, J. J. Higdon, Origins of the anomalous stress behavior in charged colloidal suspensions under shear, *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics* 82 (5) (2010) 051401.
- [161] D. S. Bolintineanu, G. S. Grest, J. B. Lechman, F. Pierce, S. J. Plimpton, P. R. Schunk, Particle dynamics modeling methods for colloid suspensions, *Computational Particle Mechanics* 1 (3) (2014) 321–356.
- [162] P. R. Schunk, F. Pierce, J. Lechman, A. Grillet, H. Weiss, C. Stoltz, D. Heine, et al., Performance of mesoscale modeling methods for predicting rheological properties of charged polystyrene/water suspensions, *Journal of Rheology* 56 (2) (2012) 353–384.
- [163] A. Malevanets, R. Kapral, Mesoscopic model for solvent dynamics, *The Journal of chemical physics* 110 (17) (1999) 8605–8613.
- [164] C.-C. Huang, A. Varghese, G. Gompper, R. G. Winkler, Thermostat for nonequilibrium multiparticle-collision-dynamics simulations, *Physical Review E* 91 (1) (2015) 013310.
- [165] P. Hoogerbrugge, J. M. Koelman, Simulating microscopic hydrodynamic phenomena with dissipative particle dynamics, *EPL (Europhysics Letters)* 19 (3) (1992) 155–160.
- [166] R. D. Groot, P. B. Warren, Dissipative particle dynamics: Bridging the gap between atomistic and mesoscopic simulation, *The Journal of chemical physics* 107 (11) (1997) 4423–4435.
- [167] E. G. Flekkøy, P. V. Coveney, From molecular dynamics to dissipative particle dynamics, *Physical Review Letters* 83 (9) (1999) 1775.
- [168] P. Espanol, Fluid particle model, *Physical Review E* 57 (3) (1998) 2930.
- [169] W. Dzwinel, D. A. Yuen, K. Boryczko, Mesoscopic dynamics of colloids simulated with dissipative particle dynamics and fluid particle model, *Molecular modeling annual* 8 (1) (2002) 33–43.
- [170] S. A. Bawazeer, S. S. Baakeem, A. Mohamad, A critical review of forcing schemes in lattice boltzmann method: 1993–2019, *Archives of Computational Methods in Engineering* 28 (7) (2021) 4405–4423.
- [171] L.-S. Luo, Lattice-gas automata and lattice Boltzmann equations for two-dimensional hydrodynamics, Georgia Institute of Technology, 1993.
- [172] L.-S. Luo, Analytic solutions of linearized lattice boltzmann equation for simple flows, *Journal of statistical physics* 88 (3) (1997) 913–926.
- [173] X. He, Q. Zou, L.-S. Luo, M. Dembo, Analytic solutions of simple flows and analysis of nonslip boundary conditions for the lattice boltzmann bgk model, *Journal of Statistical Physics* 87 (1) (1997) 115–136.

- [174] X. He, X. Shan, G. D. Doolen, Discrete boltzmann equation model for nonideal gases, *Physical Review E* 57 (1) (1998) R13.
- [175] N. S. Martys, X. Shan, H. Chen, Evaluation of the external force term in the discrete boltzmann equation, *Physical Review E* 58 (5) (1998) 6855.
- [176] L.-S. Luo, Unified theory of lattice boltzmann models for nonideal gases, *Physical review letters* 81 (8) (1998) 1618.
- [177] Z. Guo, C. Zheng, B. Shi, Discrete lattice effects on the forcing term in the lattice boltzmann method, *Physical review E* 65 (4) (2002) 046308.
- [178] A. L. Kupershtokh, D. Medvedev, D. Karpov, On equations of state in a lattice boltzmann method, *Computers & Mathematics with Applications* 58 (5) (2009) 965–974.
- [179] R. Khazaeli, M. R. Nazari, A. Zadehgo, Introducing a modified exact difference method for incorporating unsteady and non-uniform force terms in lattice kinetic models, *Computers & Mathematics with Applications* 78 (3) (2019) 723–740.
- [180] C. S. Peskin, The immersed boundary method, *Acta numerica* 11 (2002) 479–517.
- [181] G. E. Box, M. E. Muller, A note on the generation of random normal deviates, *The annals of mathematical statistics* 29 (2) (1958) 610–611.
- [182] J. Huang, C. Bao, Z. Jiang, X. Zhang, A general approach of unit conversion system in lattice boltzmann method and applications for convective heat transfer in tube banks, *International Journal of Heat and Mass Transfer* 135 (2019) 873–884.
- [183] S. S. Baakeem, S. A. Bawazeer, A. A. Mohamad, A novel approach of unit conversion in the lattice boltzmann method, *Applied Sciences* 11 (14) (2021) 6386.
- [184] B. J. Alder, T. Wainwright, Decay of the velocity autocorrelation function, *Physical review A* 1 (1) (1970) 18.
- [185] Y. Pomeau, J. Piasecki, The langevin equation, *Comptes Rendus. Physique* 18 (9-10) (2017) 570–582.
- [186] R. Everaers, M. Ejtehadi, Interaction potentials for soft and hard ellipsoids, *Physical Review E* 67 (4) (2003) 041710.
- [187] H. C. Hamaker, The london—van der waals attraction between spherical particles, *physica* 4 (10) (1937) 1058–1072.
- [188] M. T. Griffin, Y. Zhu, Z. Liu, C. K. Aidun, D. N. Ku, Inhibition of high shear arterial thrombosis by charged nanoparticles, *Biomechanics* 12 (4) (2018).
- [189] J. Zhou, C. Tu, Y. Liang, B. Huang, Y. Fang, X. Liang, I. Papautsky, X. Ye, Isolation of cells from whole blood using shear-induced diffusion, *Scientific reports* 8 (1) (2018) 9411.
- [190] J. R. Stokes, G. A. Davies, Viscoelasticity of human whole saliva collected after acid and mechanical stimulation, *Biorheology* 44 (3) (2007) 141–160.

- [191] X. Lu, C. Liu, G. Hu, X. Xuan, Particle manipulations in non-newtonian microfluidics: A review, *Journal of colloid and interface science* 500 (2017) 182–201.
- [192] D. Di Carlo, D. Irimia, R. G. Tompkins, M. Toner, Continuous inertial focusing, ordering, and separation of particles in microchannels, *Proceedings of the National Academy of Sciences* 104 (48) (2007) 18892–18897.
- [193] C. Liu, J. Guo, F. Tian, N. Yang, F. Yan, Y. Ding, J. Wei, G. Hu, G. Nie, J. Sun, Field-free isolation of exosomes from extracellular vesicles by microfluidic viscoelastic flows, *ACS nano* 11 (7) (2017) 6968–6976.
- [194] K. Kang, S. S. Lee, K. Hyun, S. J. Lee, J. M. Kim, Dna-based highly tunable particle focuser, *Nature communications* 4 (1) (2013) 2567.
- [195] B. Kim, S. S. Lee, T. H. Yoo, J. M. Kim, Viscoelastic particle focusing in human biofluids, *Electrophoresis* 42 (21-22) (2021) 2238–2245.
- [196] J. Nam, J. Yoon, J. Kim, W. S. Jang, C. S. Lim, Continuous erythrocyte removal and leukocyte separation from whole blood based on viscoelastic cell focusing and the margination phenomenon, *Journal of Chromatography A* 1595 (2019) 230–239.
- [197] S. Yang, J. Y. Kim, S. J. Lee, S. S. Lee, J. M. Kim, Sheathless elasto-inertial particle focusing and continuous separation in a straight rectangular microchannel, *Lab on a Chip* 11 (2) (2011) 266–273.
- [198] J. Zhou, I. Papautsky, Viscoelastic microfluidics: Progress and challenges, *Microsystems & Nanoengineering* 6 (1) (2020) 113.
- [199] A. Van Ombergen, F. Chalupa-Gantner, P. Chansoria, B. M. Colosimo, M. Costantini, M. Domingos, A. Dufour, C. De Maria, J. Groll, T. Jungst, et al., 3d bioprinting in microgravity: opportunities, challenges, and possible applications in space, *Advanced healthcare materials* 12 (23) (2023) 2300443.
- [200] J. G. Oldroyd, On the formulation of rheological equations of state, *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 200 (1063) (1950) 523–541.
- [201] H. R. Warner Jr, Kinetic theory and rheology of dilute suspensions of finitely extendible dumbbells, *Industrial & Engineering Chemistry Fundamentals* 11 (3) (1972) 379–387.
- [202] H. Giesekus, A simple constitutive equation for polymer fluids based on the concept of deformation-dependent tensorial mobility, *Journal of Non-Newtonian Fluid Mechanics* 11 (1-2) (1982) 69–109.
- [203] N. P. Thien, R. I. Tanner, A new constitutive equation derived from network theory, *Journal of Non-Newtonian Fluid Mechanics* 2 (4) (1977) 353–365.
- [204] S. Balasubramanian, P. Kaushik, P. K. Mondal, Dynamics of viscoelastic fluid in a rotating soft microchannel, *Physics of Fluids* 32 (11) (2020).
- [205] Y. Na, J. Yoo, A finite volume technique to simulate the flow of a viscoelastic fluid, *Computational mechanics* 8 (1) (1991) 43–55.

- [206] B. Purnode, M. Crochet, Polymer solution characterization with the fene-p model, *Journal of non-newtonian fluid mechanics* 77 (1-2) (1998) 1–20.
- [207] N. Fiétier, M. O. Deville, Simulations of time-dependent flows of viscoelastic fluids with spectral element methods, *Journal of scientific computing* 17 (1) (2002) 649–657.
- [208] R. B. Bird, P. J. Dotson, N. Johnson, Polymer solution rheology based on a finitely extensible bead—spring chain model, *Journal of Non-Newtonian Fluid Mechanics* 7 (2-3) (1980) 213–235.
- [209] M. Chilcott, J. M. Rallison, Creeping flow of dilute polymer solutions past cylinders and spheres, *Journal of Non-Newtonian Fluid Mechanics* 29 (1988) 381–432.
- [210] J. Marchal, M. Crochet, A new mixed finite element for calculating viscoelastic flow, *Journal of Non-Newtonian Fluid Mechanics* 26 (1) (1987) 77–114.
- [211] T. J. Hughes, M. Mallet, M. Akira, A new finite element formulation for computational fluid dynamics: Ii. beyond supg, *Computer methods in applied mechanics and engineering* 54 (3) (1986) 341–355.
- [212] D. Rajagopalan, R. C. Armstrong, R. A. Brown, Finite element methods for calculation of steady, viscoelastic flow using constitutive equations with a newtonian viscosity, *Journal of Non-Newtonian Fluid Mechanics* 36 (1990) 159–192.
- [213] A. Fortin, R. Guénette, R. Pierre, On the discrete evss method, *Computer methods in applied mechanics and engineering* 189 (1) (2000) 121–139.
- [214] Y. Yu, S. Chen, L. Wang, H.-z. Yuan, S. Shu, An improved lattice boltzmann method with a novel conservative boundary scheme for viscoelastic fluid flows, *arXiv preprint arXiv:2508.16997* (2025).
- [215] M. Sedaghat, A. Bagheri, M. Shahmardan, M. Norouzi, B. Khoo, P. Jayathilake, A hybrid immersed boundary-lattice boltzmann method for simulation of viscoelastic fluid flows interaction with complex boundaries, *Commun Comput Phys* 29 (5) (2021) 1411–1445.
- [216] S. Zou, X. Xu, J. Chen, X. Guo, Q. Wang, Benchmark numerical simulations of viscoelastic fluid flows with an efficient integrated lattice boltzmann and finite volume scheme, *Advances in Mechanical Engineering* 7 (2) (2015) 805484.
- [217] Y.-H. Qian, Y.-F. Deng, A lattice bgk model for viscoelastic media, *Physical review letters* 79 (14) (1997) 2742.
- [218] I. Ispolatov, M. Grant, Lattice boltzmann method for viscoelastic fluids, *Physical Review E* 65 (5) (2002) 056704.
- [219] L. Giraud, D. d’Humières, P. Lallemand, A lattice-boltzmann model for viscoelasticity, *International Journal of Modern Physics C* 8 (04) (1997) 805–815.
- [220] L. Giraud, D. d’Humières, P. Lallemand, A lattice boltzmann model for jeffreys viscoelastic fluid, *Europhysics Letters* 42 (6) (1998) 625.

- [221] P. Lallemand, D. d’Humières, L.-S. Luo, R. Rubinstein, Theory of the lattice boltzmann method: three-dimensional model for linear viscoelastic fluids, *Physical Review E* 67 (2) (2003) 021203.
- [222] J. Onishi, Y. Chen, H. Ohashi, Dynamic simulation of multi-component viscoelastic fluids using the lattice boltzmann method, *Physica A: Statistical Mechanics and its Applications* 362 (1) (2006) 84–92.
- [223] C. Denniston, E. Orlandini, J. Yeomans, Lattice boltzmann simulations of liquid crystal hydrodynamics, *Physical Review E* 63 (5) (2001) 056702.
- [224] D. Marenduzzo, E. Orlandini, M. Cates, J. Yeomans, Steady-state hydrodynamic instabilities of active liquid crystals: Hybrid lattice boltzmann simulations, *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics* 76 (3) (2007) 031921.
- [225] D. Marenduzzo, E. Orlandini, M. Cates, J. Yeomans, Lattice boltzmann simulations of spontaneous flow in active liquid crystals: The role of boundary conditions, *Journal of non-newtonian fluid mechanics* 149 (1-3) (2008) 56–62.
- [226] O. Malaspinas, N. Fiétier, M. Deville, Lattice boltzmann method for the simulation of viscoelastic fluid flows, *Journal of Non-Newtonian Fluid Mechanics* 165 (23-24) (2010) 1637–1653.
- [227] G. Zhao-Li, Z. Chu-Guang, S. Bao-Chang, Non-equilibrium extrapolation method for velocity and pressure boundary conditions in the lattice boltzmann method, *Chinese physics* 11 (4) (2002) 366–374.
- [228] C. Ni, D. Jiang, Three-dimensional numerical simulation of particle focusing and separation in viscoelastic fluids, *Micromachines* 11 (10) (2020) 908.
- [229] P. L. Bhatnagar, E. P. Gross, M. Krook, A model for collision processes in gases. i. small amplitude processes in charged and neutral one-component systems, *Physical review* 94 (3) (1954) 511.
- [230] H. Zhang, C. Shu, L.-P. Wang, Y. Liu, L. Zhu, A lattice boltzmann flux solver for numerical simulation of flows with the viscoelastic fluid, *Computers & Fluids* 292 (2025) 106593.
- [231] E. Carew, P. Townsend, M. Webster, Taylor-galerkin algorithms for viscoelastic flow: application to a model problem, *Numerical Methods for Partial Differential Equations* 10 (2) (1994) 171–190.

Publications

- S. Signorile, D. De Marinis, M.D. de Tullio, *A 3D Lattice Boltzmann method for accurate wetting of ternary fluids with broad rheological variability*, Computer Methods in Applied Mechanics and Engineering 446 (2025) 118246.
- D. De Marinis, S. Signorile, M. Santeramo, M.D. de Tullio, *Three-dimensional Conservative Phase-Field Lattice Boltzmann Simulations of Droplet Generation in a Flow-Focusing Microdevice*, Physics of Fluids 37 (2025) 122027.