IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received February 11, 2021, accepted February 24, 2021, date of publication March 8, 2021, date of current version March 18, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3064427

Min-Max Approach for High-Speed Train Scheduling and Rescheduling Models

CHIARA BERSANI, MARÍA A. DEL CACHO ESTIL-LES,
AND ROBERTO SACILE, (Member, IEEE)
Department of Informatics, Bioengineering, Robotics and Systems Engineering, University of Genova, 16145 Genova, Italy
Corresponding author: María A. Del Cacho Estil-Les (mariadelcachoestil@gmail.com)

ABSTRACT The definition of train scheduling is going to shift its current static approach, as generally resulting in a timetable, towards a dynamic approach where the timetable can be modified in order to enhance the satisfaction of passengers' demand. In this paper, demand-oriented scheduling and rescheduling models are formalized in order to propose a dynamic timetable that can be modified to satisfy groups of passengers. More specifically, the assessment is based on a min max method. The proposed approach allows the definition of the train timetables for scheduling rail services along a multiline rail network according to operational constraints related to train capacity, train speed limits, passenger's train transfers, possible conflict in the track section use, with the main objective to minimize the travel time of a set of passengers' groups. The timetable resulted by the scheduling problem have been used as the input to a rescheduling model where predefined disturbances have been assumed to some nodes of the network. The proposed models are applied to the real case study of the rail network for high-speed trains in the Northern Italy.

INDEX TERMS High-speed train, min-max approach, train rescheduling, train scheduling.

I. INTRODUCTION

Train-scheduling (TS) allows defining the planning of the arrival and departure times of trains at the rail stations. The TS general objective is to minimize the completion time or/and the delays of the planned train services [1]. So, the main issue is to determine the best feasible timetable for a set of trains in order to satisfy restrictive operational constraints related to the track capacity, maximum travel speed, and to avoid possible conflicts on the rail network in using each single track.

In literature, the TS problem has been classified into three main categories [2]: classic, real-time and robust TS. The classic TS problem finds the timetable for high or medium speed trains on a single or double track set of railway sections minimizing the makespan of the overall train activities [3], the railway throughput [4] or train fuel consumption [5]. The real-time TS problem is based on the current position of trains proposing dynamic solutions in case of disturbances on the railway infrastructures and solving, in real time, forthcoming conflicts [6] minimizing, for example, eventual delays as in [7] or [8]. The research [9] proposes a real-time scheduling model, avoiding propagating delays without degrading the quality of service to passengers. In [6], a branch and bound model introduces an approach to the conflict between two trains using alternative graphs. The work in [10] describes four methods to solve the problem of a conflict in a single, double and multi tracks railway lines. The study in [11] defines a TS problem taking also into account the crew scheduling and possible changes to the network. Finally, works on the robust TS show robust optimization techniques on delay management problems [12] and on the control of the distance between two successive trains [13].

From a performance-based viewpoint, TS problems can be investigated on three different levels, which depend on the increased data required for the analysis: microscopic, macroscopic and mesoscopic levels [14]. The first one computes the TS according to a deep analysis of each infrastructure component such as safety signals and dynamic information, as in [15]. The macroscopic approach can optimize operations at network level [16]. In the mesoscopic one, the optimization is addressed to a fine-tuning train routing among the main corridors on the network, likely to optimize energy efficiency [17].

In a modern view, the rail managers have also to adapt the train schedule to customer requirements, ensuring an adequate number of trains to meet the passenger needs in term of demand for each destination, assuring speed and

The associate editor coordinating the review of this manuscript and approving it for publication was Bilal Alatas.

punctuality [12]. In this context, the definition of the robustness level of the system network may be relevant to offer the best service. In addition to the timetable efficiency, a robust TS must have the ability to avoid delay propagation in case of disturbances on the network usually caused by operational problems, design errors or changes in functional parameters, minimizing the primary delays in respect to the nominal scheduling [18].

Different approaches in literature focus on the study of the TS robustness [19]. A robustness index may represent the expected probabilistic increase in passenger discomfort when a delay occurs [20] or the measure in which a nominal timetable may absorb the upcoming disturbances [2].

In [21] the authors solved an TS problem integrating the microscopic and macroscopic framework. In this case, at the microscopic level, a stable timetable has been generated by considering train running and headway times at local level. The macroscopic model, applied to the overall network, defines the timetable including robust methods to estimating delay propagation.

Another approach [22] proposed the performance measure of timetable robustness as the percentage of the process completed within the scheduled time or as a percentile of the process time. As a general approach to improve the timetable robustness, the TS planner extend the arrival instant of the train with a buffer time to deal with the possible deviation in time with respect to the scheduled departure in order to prevent delay propagation to the following trains, so called secondary delay [23]. One of the methods to carry out robust optimization is the min-max approach, in which a feasible solution is calculated by optimizing the worst case given a set of scenarios. In [24], a min-max method is used to optimize the organized and balance use of train station tracks while in [25] the authors optimized the passenger robustness, which aims to minimize the total travel time of the passengers when possible delays affect the network.

This paper can be positioned in this context proposing two approaches for the optimal scheduling and rescheduling problem evaluating both robustness and resilience, using a min-max approach.

Firstly, a train scheduling problem is solved to find the optimal TS to minimize passengers' discomfort in terms of the traveling time. Besides, the proposed approach implements the ability to solve possible conflicts that appear when two or more trains must cover the same rail block section during their journeys. A min-max model is presented to face this problem and it refers to the so called Conflict Resolution Problem (CRP) [26], [27]. It is based on the concept to add, for each couple of nodes, a pair of additional arcs, which may be activated when a conflict appears, and to define the precedence TS to enter a track in order to minimize the possible delay. The second part of this paper deals to present a rescheduling problem, which proposed a min max approach to minimize the maximum delay in the scheduling tasks in case of occurrence of disturbances on the network according to safety and operation restrictions.

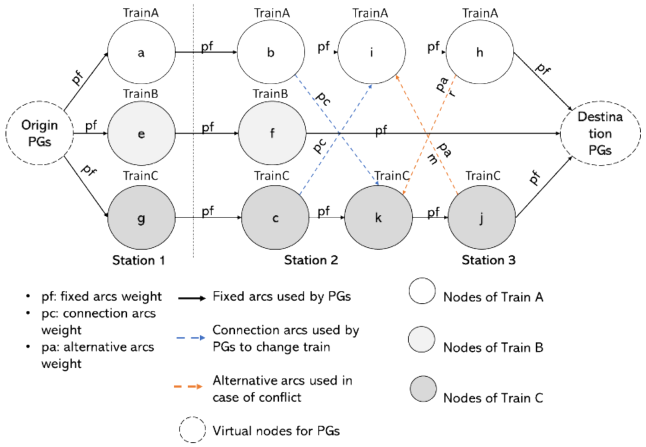

FIGURE 1. **Network scheme with its main components.**

The paper is organized as follows. Section 2 presents the microscopic min-max criteria train scheduling model (MTSM) and train rescheduling model (MTRM). In Section 3 the proposed approaches are applied to a real case study which considers the high-speed line infrastructure in the Norther Italy and in the section 4 the results are discussed. Section 4 also contains the performance evaluation of the proposed approaches against a rescheduling model formalized as a mathematical programming problem. Finally, conclusions and future developments appear in Section 5.

II. MODEL DESCRIPTION

A. NETWORK DESCRIPTION

The scheduling and rescheduling problems are implemented through a network modelled as a graph, which consists of a set of nodes and a set of arcs. In this approach, the nodes represent the events associated to each train (arrivals, departures) and the arcs the different operational tasks among nodes whose weights define the execution time.

In the graph, each row in horizontal direction represents the path of each train while, the vertical columns are associated to the track sections of the railway network. Another characteristic of the model is related to the passengers' groups with cover the network according to their corresponding information about the trips. Figure 1 shows an example of a graph with its main components.

- The Passengers Group (PG), $W = \{1 \ldots W\}$ represents an indivisible group of passengers that travels from one station to another, with the aim of reaching its destination. Each group PG is characterized by data about its journey: the origin station (α_w), destination station (β_w), the number of passengers of the group (π_w) and the arrival time at the origin station (dt_w).
- Set of nodes, $N = \{1 \ldots N\}$. Each node represents a logistic operation in a network section. Each train is represented by a specific group of nodes that in the graph appear horizontally. Two consecutive nodes in each row of the graph may represent a station in which the train stops and departs or the movement of the train

from one station to next one. Each node is characterized by a parameter (l_i) which represents the starting time of a logistic operation related to the associated train. In order to implement the model, for each PG, two Virtual Nodes (VNs) have been added. The VNs do not represent a true station but just the virtual origin and destination stations of the PGs. The PG departs and arrives, respectively, at the virtual origin node (α_w) and to the virtual destination node (β_w), which are connected to the nodes of the network associated to the available trains. The VNs connect passengers to the stations and not directly to the trains. VNs provide the PG with the possibility to choose the train and the route that best fits the needs of the group in term of departure time.

- Set of arcs, $S = \{1 \ldots S\}$. Each directed arc joins two nodes and, depending on the type, allows the train to travel through the network. Three types of arcs exist: fixed arcs, connection arcs and alternative arcs.
- Fixed Arcs (FF), $F = \{1 \ldots F\}$. These horizonal arcs link the nodes from the departure station to the last planned station designing the path of each train. Each arc represents, with the respective nodes, a stop area or a siding area. The travel time to cover the arc $s, s \in$ F, is indicated by the parameter pf_s, which also regulates the speed of the train. PG can travel on these arcs. The origin and destination virtual nodes are also connected to network through fixed arcs, but in this case, the weight of the arc is zero.
- Connection Arcs (CC), $C = \{1 \ldots C\}$. These arcs connect nodes that represent train's stop/departure at railway stations in order to allow PG changing the train if necessary (blue dashed line in the Figure 1). These arcs are included in track sections. The parameters of the connection arcs (pc_s) are related to the time spent by PG to change train at the related station.
- Couples of Alternative Arcs (AA), $\boldsymbol{A} = \{1 \ldots \boldsymbol{A}\}$. They join nodes that belong to different trains and to two consecutive block sections (red dashed line in the Figure 1). They are used to solve possible conflicts when two or more trains need to enter the same block section at the same time. In case of CRP, one of the two alternative arcs may be activated to allow one train to enter the block section preceding the other one. The weight $\boldsymbol{pa}_s$ indicates the time that the trains need waiting the permission to proceed on the arc $\boldsymbol{s}, \boldsymbol{s} \in$ C, in case of conflict. PGs cannot travel through these arcs.

B. MIN-MAX APPROACH TRAIN SCHEDULING MODEL (MTSM)

The proposed train scheduling model (MTSM) computes an optimal train timetable based on passengers' demand. It defines the TS according to PG's requirements, considering a min-max approach. The problem minimizes the completing time for PGs paths and it skips the economic cost of the journeys. In the following, the notation, parameters and variables of the model are listed.

1) NOTATION

$W = \{1 \ldots W\}$	Set of PG
$N = \{1 \ldots N\}$	Set of Nodes
$S = \{1 \ldots S\}$	Set of Arcs
$F = \{1 \ldots F\}$	Set of Fixed Arcs
$C = \{1 \ldots C\}$	Set of Connection Arcs
$A = \{1 \ldots A\}$	Set of Alternative Arcs

2) PARAMETERS

α_w	Origin node for the path PG, $w \in W$
β_w	End node for PG, $w \in W$
pf_s	Weight for FA, $s \in S$
pc_s	Weight for CA, $s \in S$
pa_s	Weight for AA, $s \in S$
$orig_{i,s}$	Binary variable, if i is the source node of arc s, the parameter assumes value $1, 0$ otherwise.$i \in N, s \in S$
$dest_{i,s}$	Binary variable, if i is the destination node of arc s, the parameter assumes value 1, 0 otherwise. $i \in N, s \in S$
π_w	Number of passengers for PG, $w \in W$
dt_w	Departure time for PG, $w \in W$
$o_{m,r}$	Binary parameter which assumes value 1 if m and r are a couple of alternative arcs,$m, r \in A$
cap_i	Indicates the maximum passenger capacity of the train at the node $i \in N$
M	Big value in respect to the value of the variables of the problem

3) DECISIONAL VARIABLES

l_i	Starting time of a logistic operation at the node $i \in N$
at_w	Arrival time at destination station for the PG, $w \in W$
$y_{m,s}$	Binary variable that indicates the presence of a CRP between two nodes associated to arcs m and s. It assumes value 1 if arc$s$ is selected to solve the CRP and it assumes value 0 if the arc m is selected. $m, s \in A$
q_f^w	Binary variable that assumes value 1 if arc f belongs to the path of the PG $w.f \in F \cup C, w \in W$
K_{SM}	PG's paths completion time index in the scheduling model
K_{RM}	PG's paths completion time index in the rescheduling model

4) MODEL DEFINITION

The MTSM is formalized by a min-max criteria associated to a minimization problem with a linear objective. The min–max version consists of finding a solution having the best-worst case value in the traveling time of the considered set of PGs.

$$\min\ K_{SM} + \gamma_s \sum_{i=1}^{N} l_i \tag{1}$$

The objective function (1) has two terms. The first one minimizes K_{SM} which represents the maximum value of the completion time of the PGs' paths to reach the destination weighted by the number of passengers of each PG and implemented by the constraint (2). The second term is the sum of the overall time to compute the logistic operations in the rail network. The parameter γ_s weights the second term of the objective function.

$$\pi_w \left(at_w - dt_w\right) < K_{SM} \quad w \in W \tag{2}$$

Constraints (2) allow the implementation of the min max approach.

$$l_j \geq l_i + \left(orig_{i,f} dest_{jf} pf_f\right) - M\left(1 - \left(orig_{i,f} dest_{j,f}\right)\right) \quad \begin{matrix} i, j \in N \\ f \in F \end{matrix} \tag{3}$$

Constraints (3) ensure that, for two consecutive nodes, the starting time of a logistic operation at the node j $\in N$ has to start after the completion of the preceding operation. So l_j has to be greater than the starting time at node *i-th* and the traversing time from node *i-th* to *j-th*, if the nodes *i-th* and *j-th* represent, respectively, the origin and the destination of the arc $f \in F$. This means that the arrival time of a train to the destination node of an arc has to be greater than the sum of the starting time at the origin node and the travelling time on the same arc in respect to the speed limits.

$$\begin{aligned} l_i \geq\ & l_j + \left(orig_{j,m} dest_{i,m} pa_m\right) \\ & - M\left(1 - \left(orig_{j,m} dest_{i,m} o_{r,m}\right)\right) \\ & - M\left(1 - \left(orig_{j,m} dest_{i,m} y_{r,m} o_{r,m}\right)\right) \\ & i, j \in Nm, \quad r \in A,\ m \neq r \end{aligned} \tag{4}$$

$$\begin{aligned} l_k \geq\ & l_h + \left(orig_{h,r} dest_{k,r} pa_r\right) \\ & - M\left(1 - \left(orig_{h,r} dest_{k,r} o_{r,m}\right)\right) \\ & - M\left(orig_{h,r} dest_{k,r} y_{r,m} o_{r,m}\right) \\ & k, h \in Nm, \quad r \in A,\ m \neq r \end{aligned} \tag{5}$$

According to the alternative graph construction as in [26] by equation (4) and (5), the model decides to activate one of the alternative arcs r or m in case of CRP ($o_{r,m} = 1$) among a couple of conflict operations at nodes *i-th* and *k-th* for one which belong to two different trains (see figure 1). Assuming that the alternative arc m connects nodes *j-th* and *i-th* and arc r connect nodes *h-th* and *k-th*, if $o_{r,m} = 0$, the starting times of the logistic operations at the nodes are not constrained. By the big M approach and the binary variables associated to the matrices node-arc related to the alternative arcs, constraints (4) formalize the possibility to activate the arc m in order to allow the train, whose the node *j-th* belongs to, accessing firstly the disputed rail section. If $y_{r,m} = 1$ and contemporary $o_{r,m} = 1$, arc m is activated, so the operation at node *j-th* is scheduled before the operation at *i-th* whose starting time l_i has been postponed and it can start only after the starting time of the previous operation l_j added to the alternative arc cost pa_m, which represents a delay in the execution.

Complementary to constraints (4) are constrains (5) which allow to activate arc r in case of CRP ($y_{r,m} = 0$) giving precedence to l_h.

$$\sum_{i=1}^{N} \sum_{u=1}^{F \cup C} orig_{\alpha_w,u} dest_{i,u} q_u^w = 1 \quad w \in W \tag{6}$$

Constraints (6) ensure that the PGw starts its travel at the predefined station α_w.

$$\sum_{i=1}^{N} \sum_{u=1}^{F \cup C} orig_{i,u} dest_{\beta_w,u} q_u^w = 1 \quad w \in W \tag{7}$$

Constraints (7) control that the PGw finishes the travel at predefined destination station β_w.

$$\sum_{u=1}^{F \cup C} dest_{i,u} q_u^w = \sum_{m=1}^{F \cup C} orig_{i,m} q_m^w \quad \begin{matrix} i \in N, w \in W, \\ i \neq \alpha_w, \\ i \neq \beta_w \end{matrix} \tag{8}$$

Equations (8) guarantee PGs flow conservation at the node *i-th*.

$$\begin{aligned} & l_i \geq dt_w - M\left(\left(1 - orig_{\alpha_w,c}\right) dest_{i,c} q_c^w\right) \\ & \quad i \in N \quad w \in W\ c \in C \end{aligned} \tag{9}$$

Constraints (9) ensure that the train on which PG can just depart after the arrival of the selected PG at the origin station.

$$\begin{aligned} l_j \geq\ & l_i + \left(orig_{i,c} dest_{j,c} pc_c\right) - M\left(1 - \left(orig_{i,c} dest_{j,c}\right)\right) \\ & - M\left(1 - \left(orig_{i,c} dest_{j,c} q_c^w\right)\right) \quad i, j \in N,\ c \in C,\ w \in W \end{aligned} \tag{10}$$

By equation (10), the model guarantees that PG may change train in case of transfer. A train cannot leave until the PG has reached the station at node *j-th*.

$$\begin{aligned} & at_w \geq l_i - M\left(1 - \left(orig_{i,u} dest_{\beta_w,u} q_u^w\right)\right) \\ & \quad i \in N, \quad u \in F \cup C,\ w \in W \end{aligned} \tag{11}$$

Constraints (11) compute the PGs arrival time at_w at the respective destination stations.

$$\sum_{w=1}^{W} orig_{i,u} dest_{j,u} q_u^w \pi_w \leq cap_i \quad \begin{matrix} l_i \geq 0 \\ i, j \in N \\ u \in F \cup C \end{matrix} \tag{12}$$

Constraints (12) assure that the number of passengers traveling on the train does not exceed the maximum allowed capacity at the node *i-th*.

$$y_{r,m}, o_{m,r}, q_r^w \in \{0, 1\} \quad \begin{matrix} m, r \in F \cup C, \\ m \neq r \\ w \in W \end{matrix}$$

$$orig_{i,c} dest_{i,c} \in \{0, 1\} \quad \begin{matrix} l_i \geq 0 \\ i \in N \\ c \in \mathrm{S} \end{matrix} \tag{13}$$

Finally, (13) define that the optimization variables as positive and binary.

C. MIN-MAX APPROACH TRAIN RESCHEDULING MODEL (MTRM)

Based on the MTSM results, the MTRM provides the new timetable adapted to the presence of possible disturbances on the railway network. The MTRM has been formalized assuming the same notation and using the same parameters of the MTSM. However, TRM takes, as input variables, the optimal value associated to the decision variables in output to the MTSM related to the starting time of the logistic operations $l_i, i \in N$.

In the proposed model, the disturbances are represented by delays that occur on one or more nodes of the network. In this case, for the nodes affected by perturbations, a new value of the starting time is assumed to be worsened in respect to the optimal value coming from TSM application.

In order to apply the MTRM, the following new decision variables have to be added.

- pfR_f : the new weight of fixed arcs concerning the traversing time;
- lR_i : new logistic operational time at node *i-th*;
- d_i : delay time of the logistic operational at node *i-th* with respect the optimal timetable obtained by applying MTSM;
- a_i : the advanced time of the logistic operational at node *i-th* with respect the optimal timetable obtained by applying MTSM.

Besides, a new parameter θ_i has been introduced in the model to represent the priority of the train at node *i-th*. The priority level decides which train has more importance to access firstly the track block when a CRP appear. Higher priority is given, more resilience is associated to the train.

The MTRM consists of the new objective (1’) and a new set of constraints ((2’) and (14) – (19)) but also it is subject to the constraints (3) – (13) coming from MTSM.

The new objective and the new constraints for the MTRM are introduced below.

$$\min\ Obj1 + \gamma_{r2} Obj2 + \gamma_{r3} Obj3 + \gamma_{r4} Obj4 \qquad (1')$$

where

$$Obj1 = K_{RM}$$

$$Obj2 = \sum_{w=1}^{W} \pi_w \left(at_w - dt_w\right)$$

$$Obj3 = \sum_{i=1}^{N} \theta_i . d_i$$

$$Obj4 = \sum_{i=1}^{N} a_i$$

In the objective (1’), the first component, *Obj*1, implements the min max approach minimizing the maximum delay for each PG as computed in the constraint (2’). The *Obj*2 minimizes the overall makespan of the PGs’ travels compensating the drawbacks of the PGs’ performances in terms of travel time. The *Obj*3 minimizes the delay time for the trains with higher priority. Finally, also the advance time for the overall logistic operations are minimized in *Obj*4. The parameters γ_{r2}, γ_{r3}, γ_{r4} weighting the different components in the objective function.

$$\pi_w \left(at_w - dt_w\right) \leq K_{RM} \quad w \in W \qquad (2')$$

Constraints (2’) implement the min max decision making which implies to minimize the worst scenario with respect to the set of PGs’ paths in which higher is the difference between the arrival and the destination time. Those values are weighted anyway also by the number of persons for PG.

$$d_i \geq lR_i - l_i \quad i \in N \qquad (14)$$

Constraints (14) compute the delay time of the logistic operations at each node of the network. It must be greater than or equal to the difference between the starting time of the logistic operation decided in the MTRM solution and the optimal one coming from the previous scheduling phase MTSM. The presence of the third addendum *Obj*3 in the objective function guarantees that d_i will be as small as possible.

$$a_i \geq l_i - lR_i \quad i \in N \qquad (15)$$

By equations (15), the model guarantees that the advance time of the logistic operation at node i has to be greater than or equal to the difference between the starting time generated in the MTSM and the new one generated by the MTRM.

$$pfR_f \geq pf_f \quad f \in F \qquad (16)$$

Constraints (16) define the new weight of fixed arcs as greater than or equal to the predefined one in the MTSM.

$$lR_i \geq l_i \quad i \in N \qquad (17)$$

By equations (17), the new time of the logistic operation must to be greater than or equal to the optimal logistic operational time in the MTSM. If $d_i = 0$, constraint (17) is equal to constraint (14).

$$lR_i \geq 0 \quad i \in N \qquad (18)$$

Finally, equations (18) defines the variables domain.

III. CASE STUDY

This section presents the case study based on a real application located on the railway network in Italy. Trenitalia is the main important rail company in Italy. The selected network consists of the railway infrastructures which connect the 6 main cities in the Northern Italy by 10 high-speed trains services. Two types of train have been considered: “freccia” and “intercity”. Freccia is the high-speed train of Trenitalia, it can travel on all European high-speed networks and it can reach a max speed of 400km/h. It has a maximum passenger’s capacity lower than intercity trains. Intercity trains connect major and minor cities in Italy to meet the different mobility clients’ requirements of medium to long distances.

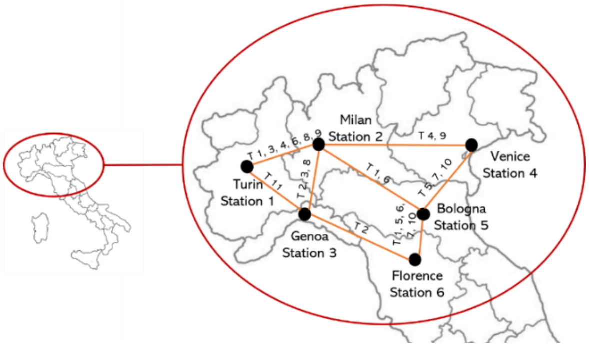

FIGURE 2. Northern Italy Trenitalia network. Tx on the links represent the considered train lines.

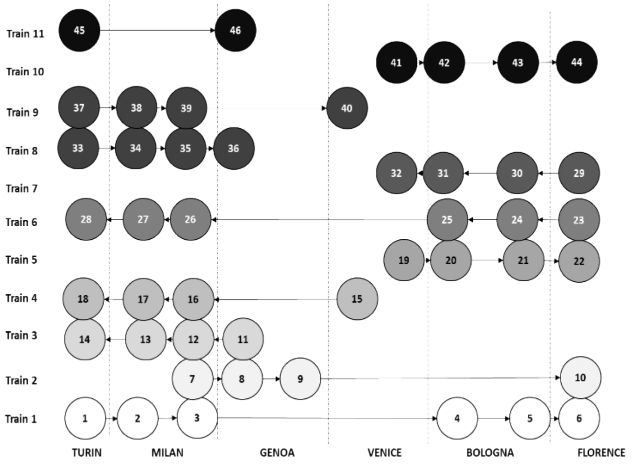

FIGURE 3. Network scheme.

Figure 2 represents the map of the network in North Italy, with the main stations in the leading cities.

As Figure 2 shows, Station 1 (Turin) is linked to Station 2 (Milan) through trains 1, 3, 4, 6, 8, and 9; Station 1 is also liked to Station 3 (Genoa) by train 11. Station 2 (Milan) is linked to Station 3 (Genoa) through trains 2, 3 and 8, to Station 4 (Venice) through trains 4 and 9, and to Station 5 (Bologna) through trains 1 and 6. Station 3 (Genoa) is also linked to Station 6 (Florence) through train 2. Finally, Station 5 (Bologna) is also linked to Station 4 (Venice) through trains 5, 7, 10 and to Station 6 (Florence) through trains 1, 5, 6, 7 and 10.

Figure 3 shows the railway network scheme with each train stops and siding areas. In the graph, each line represents a different train; the nodes represent the logistic operation related to the selected train while the vertical sections represent the track blocks associated to the stations of the network. Turin and Venice stations may be only origin or destination for the PGs' paths while the other station also include origin, stop or siding area.

For better readability of the network scheme in Figure 3, some examples area presented. The nodes 1 to 6 belong to Train 1 which departs from Turin and stops at Milan and Bologna, with Florence as destination. Node 2 represents the train arrival in Milan station while node 3 its departure to the Bologna station. It is a freccia train with a capacity to 492 passengers. Nodes 23 to 28 belong to Train 6, a freccia train, traveling from Florence to Turin, with stops in Bologna and Milan. Train 8 departs from Turin with node 33, stops at Milan station (nodes 33 and 34) and finishes the path at Genoa station at node 36. It is an intercity train with a maximum capacity of 600 passengers.

TABLE 1. Train data.

	Origin Station	*Intermediate Stations*	*Destination Station*	*Type of Train*	*Maximum passenger's capacity*
T1	Station1	Station2 Station5	Station6	*Freccia*	492
T2	Station2	Station3	Station6	*Freccia*	492
T3	Station3	Station2	Station1	Intercity	600
T4	Station4	Station2	Station1	*Freccia*	492
T5	Station4	Station5	Station6	*Freccia*	492
T6	Station6	Station5 Station2	Station1	*Freccia*	492
T7	Station6	Station5	Station4	*Freccia*	492
T8	Station1	Station2	Station3	Intercity	600
T9	Station1	Station2	Station4	*Freccia*	492
T10	Station4	Station5	Station6	Intercity	600
T11	Station1	-	Station3	Intercity	600

Table 1 summarizes train's path features with origin, destination and intermediate stations, type of train and maximum passenger's capacity.

The proposed case study has to manage the paths of 9 PGs with a different starting time, origin and destination stations for the paths. The PGs data, a priori known, appear in the Table 2.

IV. RESULTS

This section presents the results obtained with the application of the MTSM and MTRM. The models have been implemented in Cplex. Once the optimal train schedule results from the MTSM, a set disturbances (delays) which affect the network has been applied. Finally, the MTRM model generates the optimal timetabling adapted to manage the applied perturbations in order to minimize the secondary delays and the failure propagation.

A. MTSM MODEL APPLICATION

Each PG intended to reach the destination station in the shortest possible time according to the operational constraints related to the network. The following Tables 3 and 4 present, respectively, the optimal routes for trains and PGs.

Four PGs used a direct route to reach their destination while other PGs needed to change, at least one time, the train. In the Table 4 it is possible to observe the travel time of each PG (time of the trip) and the related completion time which starts at the PG's arrival at the virtual starting node and the arrival time at destination.

TABLE 2. PGs data.

	Origin station	Destination station	π_w	dt_w
PG 1	Station3 Genoa	Station6 Florence	230	8:00
PG 2	Station1 Turin	Station6 Florence	200	8:00
PG 3	Station1 Turin	Station3 Genoa	200	8:00
PG 4	Station4 Venice	Station3 Genoa	100	7:00
PG 5	Station5 Bologna	Station3 Genoa	200	8:00
PG 6	Station4 Venice	Station1 Turin	90	7:00
PG 7	Station2 Milan	Station6 Florence	100	8:00
PG 8	Station6 Florence	Station4 Venice	100	8:00
PG 9	Station4 Venice	Station6 Florence	250	7:00

TABLE 3. Train timetable by MTSM.

	Departure time	Time at intermediate stations	Arrival time
T1	9:46	10:49-11:00 12:10-12:12	12:43
T2	8:00	9:46-10:00	11:25
T3	12:00	13:42-13:44	15:30
T4	8:00	10:00-10:50	11:50
T5	8:00	9:50-9:52	10:25
T6	8:30	9:03-9:05 10:15-11:51	12:51
T7	8:00	8:30-9:51	11:41
T8	8:00	9:46-10:17	11:59
T9	8:00	9:00-10:00	12:00
T10	11:42	13:31-13:33	14:30
T11	8:00	-	9:58

TABLE 4. PG's optimal paths.

	Train departure station	Train arrival station	Departure time	Arrival time (at_w)	Travel time	Completion time ($at_w - dt_w$)
PG1	T2	T2	10:00	11:25	1:25	3:25
PG2	T11	T2	8:00	11:25	3:25	3:25
PG3	T11	T11	8:00	9:58	1:58	2:58
PG4	T4	T8	8:00	11:59	3:59	4:59
PG5	T6	T8	9:05	11:59	2:54	3:59
PG6	T4	T3	8:00	15:30	7:30	8:30
PG7	T1	T10	11:00	14:30	3:30	6:30
PG8	T7	T7	8:00	11:41	3:41	3:41
PG9	T5	T5	8:00	10:25	2:25	3:25

In the proposed solution, the trains 2, 11 and 8 are mostly used by PGs while train 9 travels empty, as we can see in Figure 4.

Figure 5 shows the route of each PG through the different available trains and the stations where the train changing takes place. As an example, PG6 departs from Venice with Train 4 and, at Station 2 (Milan), it changes from Train 4 to Train 3 to reach Turin at 15:30. The PG9 remains on Train 5, from the beginning (Venice) to the end of its travel (Florence). By this approach, PG's paths completion time index is $K_{SM} = 854, 2$.

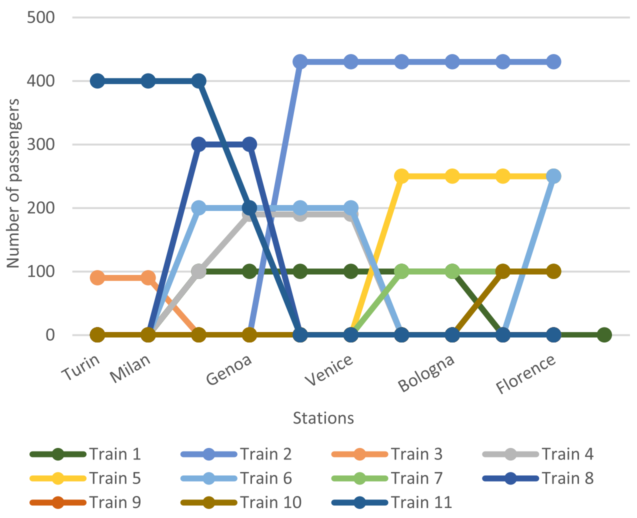

FIGURE 4. Train occupancy.

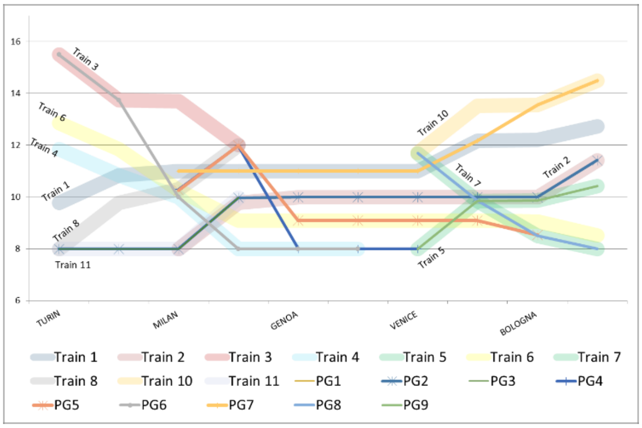

FIGURE 5. MTSM results and PGS' allocation to trains.

According to the presented MTSM results, the MTRM is then applied to recover the delays generated to the operational disturbances on some nodes.

B. MTRM MODEL APPLICATION

In order to evaluate the MTRM performances, the disturbances have been added to the nodes which belong to the paths of the mainly used trains. A delay of half an hour on the departure time has been forced for train 2 and train 8 in particular at nodes 7 and node 33 of the network in figure 3, so let's $l_7 = 8.30$ and $l_{33} = 8.30$.

The MTRM results appear in Table 5.

In Table 5 we can see that the input disturbances on the path of the Train 2 and 8 also affect timetable of Train 1, 4 and 6, generating delays in some or all the stations where the trains stop. In case of Train 1, the disturbance causes the delay already in the beginning of the trip. Instead, in Train 4 and 6, the delay only occurs in the last two stations.

Table 6 shows how the new timetable affects the passengers' journeys.

The input disturbances at the node l_7 affect the paths of PG1 and PG2, which reach the destination with a delay

TABLE 5. Train timetable by MTRM after disturbances application.

	Departure time	*Time at intermediate stations*	*Arrival time*
T1	**10.16**	**11:19-11:30** **12:40-12:42**	**13:13**
T2	**8:30**	**10:16-10:19**	**11:45**
T3	12:00	13:42-13:45	15:30
T4	8:00	10:00-**11:20**	**12:20**
T5	8:00	9:50-9:52	10:25
T6	8:30	9:03-9:05 10:15-**12:21**	**13:21**
T7	8:00	8:30-9:51	11:41
T8	**8:30**	**10:16-10:18**	**12:00**
T9	8:00	9:00-10:00	12:00
T10	11:42	13:31-13:33	14:30
T11	8:00	-	9:58

TABLE 6. PGs' paths in MTRM after disturbances (in bold the difference in time in respect to the MTSM in Table 4).

	Train departure station	*Train arrival station*	*Departure time*	*Arrival time* (at_w)	*Travel time*	*Completion time* $(at_w - dt_w)$
PG1	T2	T2	**10:19**	**11:45**	**1:26 (+0:01)**	**3:45 (+0:25)**
PG2	T11	T2	8:00	**11:45**	**3:45 (+0:20)**	**3:45 (+0:25)**
PG3	T11	T11	8:00	9:58	1:58	2:58
PG4	T4	T8	8:00	**12:00**	**4:00 (+0:01)**	**5:00 (+0:01)**
PG5	T6	T8	9:05	**12:00**	**2:55 (+0:01)**	**4:00 (+0:01)**
PG6	T4	**T4**	8:00	**12:20**	**4:20 (-3:10)**	**5:20 (-3:10)**
PG7	T1	**T1**	**11:30**	**13:13**	**1:47 (-1:43)**	**5:13 (-0:43)**
PG8	T7	T7	8:00	11:41	3:41	3:41
PG9	T5	T5	8:00	10:25	2:25	3:25

of 25 minutes with respect to the MTSM. PG3, PG8 and PG9 do not perceive any consequences on their trips. On the other hand, PG 7 obtains the advantage to compute its path with one hour and 43 minutes in advance. Mainly benefits are produced for PG6. The reason is related to the priority variables associated to the trains. The PG6, in fact, starts its travel on the Train 4, which is a freccia train, whose nodes are classified as higher priority. In the MTSM, the PG6 changes train at intermediate station, while in the MTRM, it continues its trip through the Train 4 with any changing. This provides the PG6 with arriving at the destination 3 hours and 10 minutes in advance. The reason may be reconducted to the objective function: in scheduling model, MTSM, the objectives minimizes the maximum completion time of the PGs weighted for the number of PG's passengers. Unfortunately, PG counts the smaller number of passengers, $\pi_6 = 90$, so it may leave Train 4 and to complete its path by Train 3. On the contrary, in the rescheduling model MTRM, the Train 4 holds the nodes at higher priority, so this path has precedence on the Train 3 which is classified as intercity, with lower priority. This means that PG6 can proceed on the same train and reach its destination in advance. It's verified by a CRP detection between Train 3 and 4 to enter the block section at Station 1 (Turin) coming from Station 2 (Milan). Between the pair of alternative arcs, AA1, which connects node 14 of Train3 to node 17 of the Train4, and AA2, which connects node 18 of Train4 and node 13 of Train3, only the arc AA2 is activated.

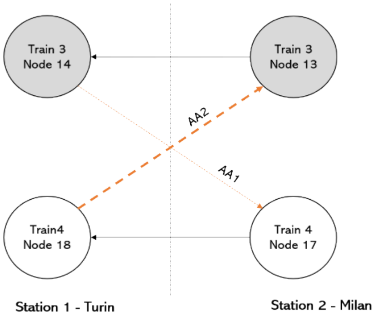

FIGURE 6. CRP between Train 3 and Train 4 in the case study.

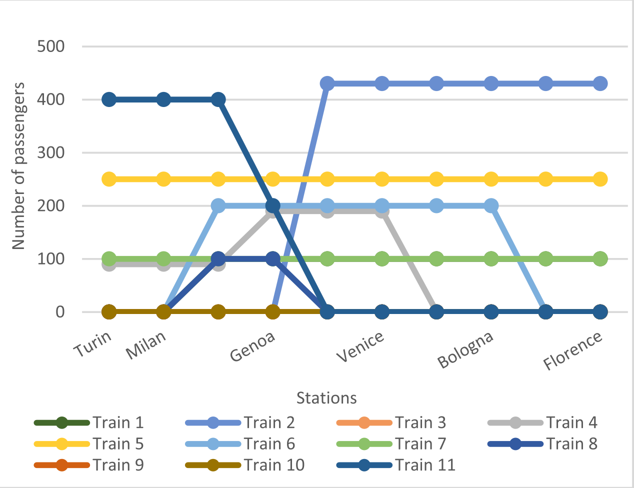

FIGURE 7. Occupancy f each train.

This means that priority is given to Train 4 while the access of Train 3 is delayed, as shown in Figure 6. The starting time of operation at node 13, l_{13} may start only after the completion of l_{18} plus the alternative arc AA2 cost. The MTRM approach produces the PG's paths completion time index $K_{RM} = 862, 5$.

Figure 7 shows train capacities. Troughs Figure 7 and Figure 8, which represents PGs allocation on the train paths, it is highlighted also the PGs 6 and 7 do not change trains with respect to the MTSM.

C. MTRM VS RESCHEDULING MODEL (RM)

In order to evaluate the MTRM performances, a rescheduling model based on a traditional minimization problem has been implemented.

The objective function in rescheduling model (RM) is replaced and formalized considering only the last three objectives in the eq. (1') omitting the min max criteria. The new objective in the RM (1'') is the following, where $\vartheta_1, \vartheta_2, \vartheta_3$ are weighting parameters.

$$\min\ \vartheta_1 \sum_{w=1}^{W} \pi_w \left(at_w - dt_w\right) + \vartheta_2 \sum_{i=1}^{N} \theta_i . d_i + \vartheta_3 \sum_{i=1}^{N} a_i \tag{1''}$$

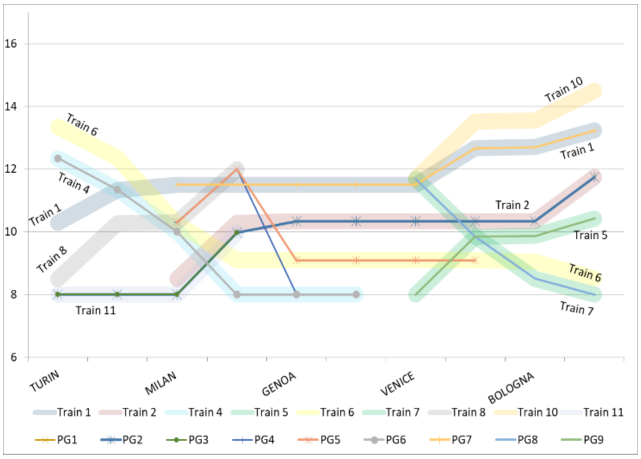

FIGURE 8. MTRM results and PGs' allocation to trains.

TABLE 7. Train timetable by RM after disturbances application.

	Departure time	*Time at intermediate stations*	*Arrival time*
T1	**10.16**	**11:19-11:30 12:40-12:42**	**13:13**
T2	**8:30**	**10:16-10:19**	**11:45**
T3	12:00	13:42-13:45	15:30
T4	8:00	10:00-**11:20**	**12:20**
T5	8:00	9:50-9:52	10:25
T6	8:30	9:03-9:05 10:15-**12:21**	**13:21**
T7	8:00	8:30-9:51	11:41
T8	**8:30**	**10:16-10:18**	**12:00**
T9	8:00	9:00-10:00	12:00
T10	11:42	13:31-13:33	14:30
T11	8:00	-	9:58

TABLE 8. Optimal PGs' paths in RM after disturbances (in bold the difference in respect to the MTSM in Table 4).

	Train departure station	*Train arrival station*	*Departure time*	*Arrival time* (at_w)	*Travel time*	*Completion time* $(at_w - dt_w)$
PG1	T2	T2	**10:19**	**11:45**	**1:26 (+0:01)**	**3:45 (+0:25)**
PG2	**T1**	**T1**	**10:16**	**13:13**	**2:57 (-0:28)**	**3:45 (+0:25)**
PG3	T11	T11	8:00	9:58	1:58	2:58
PG4	T4	T8	8:00	12:00	4:00 (+0:01)	**5:00 (+0:01)**
PG5	T6	T8	9:05	12:00	2:55 (+0:01)	**4:00 (+0:01)**
PG6	T4	T4	8:00	12:20	4:20 (-2:50)	**5:20 (-3:10)**
PG7	T1	T10	**11:30**	14:30	**3:00 (-0:30)**	6:30
PG8	T7	T7	8:00	11:41	3:41	3:41
PG9	**T10**	**T10**	**11:42**	**14:30**	**2:48 (+0:23)**	**7:30 (+4:05)**

The RM is subject to constraints (3) – (13) by MTSM and to (14) – (19) by MTRM. The constraint (2) and (2') which implement the min max approaches are skipped.

The results of the RM application appear in Table 7 and Table 8.

In Table 7 shows that the timetable in the RM approach, applying the same disturbances, reflects the MTRM results.

On the other hand, Table 8 shows that the new optimal PGs' paths present variances in respect to the MTRM application.

TABLE 9. PGs' paths completion time indices (in bold the maximum value for each approach).

	$\pi_w(at_w - dt_w), w \in W$		
	MTSM	**RM**	**MTRM**
PG1	785,8	862,5	**862,5 (K_{RM})**
PG2	683,3	750,0	750,0
PG3	393,3	593,3	593,3
PG4	498,3	500,0	500,0
PG5	796,7	800,0	800,0
PG6	765,0	480,0	480,0
PG7	450,0	650,0	521,7
PG8	368,3	368,3	368,3
PG9	**854,2 (K_{SM})**	**1875,0**	854,2

TABLE 10. Performance comparison.

	Trave time of the PGs		*Completion time of the PGs* $\pi_w(at_w - dt_w)$	
	Average value	**Maximum value**	**Average value**	**Maximum value**
MTSM	03:14	07:30	04:12	08:30
RM	03:00	04:20	04:42	07:30
MTRM	02:54	04:20	04:07	05:20

Now, PG2 and PG9 use different trains to carry out their tours. While the completion time of PG2 is just the same, PG9 suffers a delay both in travel and in completion time. This latter presents a delay of four hours and five minutes in respect to the MTRM approach. This result demonstrates as the min max approach fits the maximum disturbance dejection. In the RM results, in fact, the larger time completion is generated for the PG9 which, due to the great number of passengers, generates the worse objective function value associate to the term $\pi_w\,(at_w - dt_w)$ among the set of PGs (see Table 9).

The concept of min-max criteria should provide the best solution in the worst case minimizing the main impact of perturbations on the system.

In the proposed case study, the MTRM minimizes the maximum values of delay in the time schedule, associated to the PG9, reducing to more than the halved the objective function and providing feasible solutions for the other PGs. However, from Table 10, it is evident that RTRM approach dominated the solutions of the other models in term of average and maximum value both for travel time and completion time for the selected set of PGs.

V. CONCLUSION

In this paper, high-speed TS and train rescheduling models are presented in order to evaluate how the network reacts to possible disturbances on the network. The main aim is to adapt the train time schedule to passengers' demand. Furthermore, the models can resolve possible conflicts between two or more trains which want to enter the same block section at the same time instant. This conflict is solved by adding alternative arcs, which are activated if necessary, giving importance to the train with higher priority.

The TS model provides the optimal timetable for trains, with the objective to minimize passengers' travel times.

After that, introducing some disturbances, the min-max approach for the rescheduling model highlights that perturbation may be reduced in respect to the simple minimization of the objective function. The introduction of the concept of priority of trains at hight speed profile guarantees more resilience for the secondary delay propagation favouring the passengers' requirement to reach their destination with a minimum travel time. The comparison between the min-max algorithm and the minimization model proves the efficiency of the proposed approaches. The PG's paths completion time index appears significantly inferior in the min-max approach which means that the maximum value of the secondary delay on the network has been reduced improving the real time traffic management.

Future research should anyway consider the integration in the model the speed profile of trains and dwell time for the logistic operations or other strategic goals of the decision makers as the ticket costs for passengers or crew management and economic profits for railway managers.

REFERENCES

[1] C. Scheffer, "Train scheduling: Hardness and algorithms," in *Proc. Int. Workshop Algorithms Comput.*, Cham, Switzerland: Springer, 2020, pp. 342–347.

[2] M. A. Shafia, S. J. Sadjadi, A. Jamili, R. Tavakkoli-Moghaddam, and M. Pourseyed-Aghaee, "The periodicity and robustness in a single-track train scheduling problem," *Appl. Soft Comput.*, vol. 12, no. 1, pp. 440–452, Jan. 2012.

[3] W. Tian and E. Demeulemeester, "Railway scheduling reduces the expected project makespan over roadrunner scheduling in a multi-mode project scheduling environment," *Ann. Oper. Res.*, vol. 213, no. 1, pp. 271–291, Feb. 2014.

[4] A. Caprara, M. Monaci, P. Toth, and P. L. Guida, "A lagrangian heuristic algorithm for a real-world train timetabling problem," *Discrete Appl. Math.*, vol. 154, no. 5, pp. 738–753, Apr. 2006.

[5] K. Ghoseiri, F. Szidarovszky, and M. J. Asgharpour, "A multi-objective train scheduling model and solution," *Transp. Res. B, Methodol.*, vol. 38, no. 10, pp. 927–952, Dec. 2004.

[6] A. D'Ariano, D. Pacciarelli, and M. Pranzo, "A branch and bound algorithm for scheduling trains in a railway network," *Eur. J. Oper. Res.*, vol. 183, no. 2, pp. 643–657, Dec. 2007.

[7] E. König, "A review on railway delay management," *Public Transp.*, vol. 12, no. 2, pp. 335–361, Jun. 2020.

[8] S. Yu, X. Han, B. Lin, Y. Gao, X. Dai, and D. Cui, "Delay propagation mechanism model for high-speed train operation under arrival/departure time delay," in *Proc. IEEE 23rd Int. Conf. Intell. Transp. Syst. (ITSC)*, Sep. 2020, pp. 1–6.

[9] H. Belhomme, S. Dauzere-Peres, M. Gagnon, and F. Ramond, "Using a new multi-objective concept for real time rescheduling in dense railway systems," in *Proc. IEEE 23rd Int. Conf. Intell. Transp. Syst. (ITSC)*, Sep. 2020, pp. 1–6.

[10] J. Törnquist and J. A. Persson, "N-tracked railway traffic re-scheduling during disturbances," *Transp. Res. B, Methodol.*, vol. 41, no. 3, pp. 342–362, Mar. 2007.

[11] C. G. Walker, J. N. Snowdon, and D. M. Ryan, "Simultaneous disruption recovery of a train timetable and crew roster in real time," *Comput. Oper. Res.*, vol. 32, no. 8, pp. 2077–2094, Aug. 2005.

[12] V. Cacchiani, J. Qi, and L. Yang, "Robust optimization models for integrated train stop planning and timetabling with passenger demand uncertainty," *Transp. Res. B, Methodol.*, vol. 136, pp. 1–29, Jun. 2020.

[13] C. Bersani, S. Qiu, R. Sacile, M. Sallak, and W. Schön, "Rapid, robust, distributed evaluation and control of train scheduling on a single line track," *Control Eng. Pract.*, vol. 35, pp. 12–21, Feb. 2015.

[14] R. M. P. Goverde, N. Bešinović, A. Binder, V. Cacchiani, E. Quaglietta, R. Roberti, and P. Toth, "A three-level framework for performance-based railway timetabling," *Transp. Res. C, Emerg. Technol.*, vol. 67, pp. 62–83, Jun. 2016.

[15] B. He, P. Chen, A. D'Ariano, L. Chen, H. Zhang, and G. Lu, "A microscopic agent-based simulation for real-time dispatching problem of a busy railway passenger station," in *Proc. IEEE 23rd Int. Conf. Intell. Transp. Syst. (ITSC)*, Sep. 2020, pp. 1–7.

[16] F. Corman, A. D'Ariano, A. D. Marra, D. Pacciarelli, and M. Samà, "Integrating train scheduling and delay management in real-time railway traffic control," *Transp. Res. E, Logistics Transp. Rev.*, vol. 105, pp. 213–239, Sep. 2017.

[17] V. Cacchiani, "Models and algorithms for combinatorial optimizaion problems arising in railway applications," Doctoral Dissertation, Springer, DEIS, Bologna Univ., Bologna, Italy, 2009.

[18] V. Cacchiani and P. Toth, "Nominal and robust train timetabling problems," *Eur. J. Oper. Res.*, vol. 219, no. 3, pp. 727–737, Jun. 2012.

[19] F. Corman, A. D'Ariano, and I. A. Hansen, "Evaluating disturbance robustness of railway schedules," *J. Intell. Transp. Syst.*, vol. 18, no. 1, pp. 106–120, Jan. 2014.

[20] Y. Takeuchi, N. Tomii, and C. Hirai, "Evaluation method of robustness for train schedules," *Quart. Rep. RTRI*, vol. 48, no. 4, pp. 197–201, 2007.

[21] N. Bešinović, R. M. P. Goverde, E. Quaglietta, and R. Roberti, "An integrated micro–macro approach to robust railway timetabling," *Transp. Res. B, Methodol.*, vol. 87, pp. 14–32, May 2016.

[22] R. M. P. Goverde and I. A. Hansen, "Performance indicators for railway timetables," in *Proc. IEEE Int. Conf. Intell. Rail Transp.*, Aug. 2013, pp. 301–306.

[23] P. Jovanović, P. Kecman, N. Bojović, and D. Mandić, "Optimal allocation of buffer times to increase train schedule robustness," *Eur. J. Oper. Res.*, vol. 256, no. 1, pp. 44–54, Jan. 2017.

[24] H. Wang and Y. Yue, "Optimizing train-platforming plans at arrival-departure tracks of high-speed railway stations," in *Proc. IEEE 5th Int. Conf. Intell. Transp. Eng. (ICITE)*, Sep. 2020, pp. 461–466.

[25] S. Burggraeve and P. Vansteenwegen, "Robust routing and timetabling in complex railway stations," *Transp. Res. B, Methodol.*, vol. 101, pp. 228–244, Jul. 2017.

[26] A. D'Ariano, M. Pranzo, and I. A. Hansen, "Conflict resolution and train speed coordination for solving real-time timetable perturbations," *IEEE Trans. Intell. Transp. Syst.*, vol. 8, no. 2, pp. 208–222, Jun. 2007.

[27] F. Donzella, M. D. C. Estil-les, C. Bersani, R. Sacile, and L. Zero, "Train scheduling and rescheduling model based oncustomer satisfaction. Application to genoa railway network," in *Proc. 13th Annu. Conf. Syst. Syst. Eng. (SoSE)*, Jun. 2018, pp. 593–600.

CHIARA BERSANI received the Ph.D. degree in service science applied to transport and logistics systems from the University of Genoa, Italy.

Since 2007, she has been a Research Assistant with the University of Genova. Her main research interests include transport and logistics systems, with special reference to hazardous material management and inventory routing problems. She has been co-editor of the recent books *Hydrogen Infrastructure for Energy Applications: Production, Storage, Distribution and Safety* and *Advanced Technologies and Methodologies for Risk Management in the Global Transport of Dangerous Goods*.

MARÍA A. DEL CACHO ESTIL-LES received the degree in telematics engineering from the Rovira i Virgili University, Spain, in 2016. She is currently pursuing the Ph.D. degree in computer science and systems engineering with the University of Genoa, Italy. Her main research interest includes logistics problems.

ROBERTO SACILE (Member, IEEE) received the Laurea degree in electronic engineering from the University of Genova, Italy, in 1990, and the Ph.D. degree, in 1994. Since 2000, he has been with the University of Genova, where he is currently a Professor of automation and systems engineering. His research interests include systems engineering and its application to the transportation, logistics, and energy sectors. More recently, he has been working in human machine interfaces to enhance safety in the automotive sector.

• • •