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Abstract

We present a novel theory of the adhesive contact of linear viscoelastic materials against rigid substrates moving at constant velocity. Despite the non-conservative behavior of the system, the closure equation of the contact problem can be rigorously formulated in the form of a local energy balance. In the case of adhesiveless contacts, this is equivalent to enforce the stationarity of the total energy stored into the viscoelastic material. However, in the presence of interfacial adhesion, the appearance of non-conservative terms leads to different values of the energy release rates G1 and G2 at the contact trailing and leading edges, respectively. Specifically, the present theory predicts a non-monotonic trend of G1 and G2 as function of the indenter velocity, as well as a very significant enhancement of hysteretic friction due to the coupling between adhesion and viscoelasticity, compared to the adhesiveless case. Both predictions are in very good agreement with existing experimental data.

Keywords: viscoelasticity, adhesion, friction, crack propagation, hysteresis

I. INTRODUCTION

Understanding the origin of friction in adhesive sliding or rolling contact of rubber-like materials is a long-standing problem. Recent theories [21–23, 26] have shown that part of the friction is ascribable to dissipative phenomena occurring in the bulk of the material, associated with the intrinsic viscoelastic behavior of natural rubbers and elastomers. However, experimental observations made by Grosch in his seminal paper [14], have demonstrated the key role that interfacial adhesion plays in determining the overall frictional response in sliding contacts [12]. Nonetheless, although a comprehensive theory of adhesive viscoelastic friction is still lacking, the adhesive contribution to friction is commonly postulated in describing the frictional behavior of both micro- and macro-scale systems [17, 30, 38]. Adhesive layers adsorption [4, 18, 35, 36], and the dynamics of adhesive links debonding/rebinding [10, 11, 34] are some of the mechanisms responsible for adhesive hysteresis even in the case of rigid and purely elastic interfaces, which eventually lead to a friction force opposing the interface sliding. However, regardless of the detailed description of the adhesive links, specific scale-dependent mechanisms of interaction between adhesion and viscoelasticity can be identified. Indeed, at the large-scale, the effect of adhesion is to increase the real contact area and, in turn, the portion of bulk material undergoing cyclic deformation, which eventually leads to the aforementioned bulk energy dissipation and to viscoelastic friction [29]. On the other hand, at the asperity level, as observed in laboratory tests [3, 9, 16, 28, 37], local adhesion hysteresis modifies the energy release rates at the leading and trailing contact edges thus producing an additional contribution to friction, usually referred to as adhesive friction [12, 15, 31]. This mechanism, usually ascribed to the so-called small-scale viscoelasticity [1, 25, 27], is triggered by adhesion (i.e., it vanishes for adhesiveless contacts), and leads to an increase of the contact area and an asymmetric distribution of contact stresses [5]. In this letter we present a novel theory, based on energy balance, to study adhesive viscoelastic contacts in steady sliding motion. The proposed approach allows to investigate the interplay between adhesion and viscoelasticity across the scales for speed values spanning the entire viscoelastic spectra of the material. In agreement with experimental evidences [14, 28, 37], we predict, for the first time, a specific velocity-dependent frictional behavior, which proves that the adhesive contribution to viscoelastic friction cannot be neglected.

II. FORMULATION

We consider the case of a rigid periodic indenter, under displacement controlled conditions, in adhesive contact with a linear viscoelastic half-space. The indenter moves at a constant velocity $\mathbf{v}$. According to [8, 22], the surface normal displacement $u(\mathbf{x})$ and stress $\sigma(\mathbf{x})$ fields (in a reference frame co-moving with the indenter) are time-independent and related to each other by a spatial convolution product

$$u(\mathbf{x}) = \int d^2x_1 \mathcal{G}_{\mathbf{v}}(\mathbf{x} - \mathbf{x}_1)\, \sigma(\mathbf{x}_1) \tag{1}$$

where, $\mathbf{x}$ is the in-plane position vector, and $\mathcal{G}_{\mathbf{v}}$ is the viscoelastic Green's function, which parametrically depends on $\mathbf{v}$. Notably, $\mathcal{G}_{\mathbf{v}}(\mathbf{x})$ is non-symmetric, i.e. $\mathcal{G}_{\mathbf{v}}(-\mathbf{x}) \neq \mathcal{G}_{\mathbf{v}}(\mathbf{x})$ and, therefore, can be decomposed in a symmetric (even) part $\mathcal{G}^E_{\mathbf{v}}(\mathbf{x}) = \mathcal{G}^E_{\mathbf{v}}(-\mathbf{x})$ and antisymmetric (odd) one $\mathcal{G}^O_{\mathbf{v}}(\mathbf{x}) = -\mathcal{G}^O_{\mathbf{v}}(-\mathbf{x})$, i.e.

$$\mathcal{G}_{\mathbf{v}}(\mathbf{x}) = \mathcal{G}^E_{\mathbf{v}}(\mathbf{x}) + \mathcal{G}^O_{\mathbf{v}}(\mathbf{x}) \tag{2}$$

We recall that the contact area is an unknown of the problem and must be determined as a part of the solution. To this aim we need an additional closure equation, which can be phrased in terms of energy balance between the change of surface energy $\Delta\gamma\delta A$, caused by an infinitesimally small change δA of the contact area A, and the corresponding work of the internal stress δL,

$$\delta L = \Delta\gamma\delta A \tag{3}$$

where $\Delta\gamma$ is the surface energy per unit area also referred to as the Duprè work of adhesion. Note that in the case of elastic materials δL equates the change of the elastic energy δU stored into the material. However for non-conservative systems, as in our case, $\delta L = \delta U \ + \delta L_P$ where δL_P is a non conservative (positive or negative) term. Aiming at expressing δU and δL_P in terms of $\sigma(\mathbf{x})$ and $u(\mathbf{x})$, we consider a quasi-static (or infinitely slow) change $\delta u(\mathbf{x})$ of the displacement field $u(\mathbf{x})$. The work δL made by internal stresses is $\delta L = \int d^2x\sigma(\mathbf{x})\,\delta u(\mathbf{x})$. Using (1) and (2) yields

$$\delta L_P = \int d^2x d^2x_1 \mathcal{G}^O_{\mathbf{v}}(\mathbf{x}-\mathbf{x}_1)\,\sigma(\mathbf{x})\,\delta\sigma(\mathbf{x}_1) \tag{4}$$

$$\delta U = \int d^2x d^2x_1 \mathcal{G}^E_{\mathbf{v}}(\mathbf{x}-\mathbf{x}_1)\,\sigma(\mathbf{x})\,\delta\sigma(\mathbf{x}_1) \tag{5}$$

Moreover, since $\mathcal{G}^E_{\mathbf{v}}(\mathbf{x})$ is symmetric, we have that $U = \frac{1}{2}\int d^2x\mathcal{G}^E_{\mathbf{v}}(\mathbf{x}-\mathbf{x}_1)\,\sigma(\mathbf{x})\,\sigma(\mathbf{x}_1)$; similarly, with $\mathcal{G}^O_{\mathbf{v}}(\mathbf{x})$ being antisymmetric, we have $\int d^2x\mathcal{G}^O_{\mathbf{v}}(\mathbf{x}-\mathbf{x}_1)\,\sigma(\mathbf{x})\,\sigma(\mathbf{x}_1) = 0$. Hence, from Eq. (5), the elastic energy U can be rephrased in a more convenient form as

$$U = \frac{1}{2}\int d^2x\sigma(\mathbf{x})\,u(\mathbf{x}) \tag{6}$$

which is the standard form already known for purely elastic materials. Notably, since $\mathcal{G}^O_{\mathbf{v}}(\mathbf{x})$ is an odd function, the term δL_P vanishes when the shape of the displacement field during the loading process does not change, i.e. for $\delta u(\mathbf{x}) = u_0(\mathbf{x})\ \delta\eta$, where η is a dimensionless control parameter, and $u_0(\mathbf{x})$ represents the fixed shape of the displacement field. Linearity (1) yield $\sigma(\mathbf{x}) = \sigma_0(\mathbf{x})\ \eta$ and $\delta\sigma(\mathbf{x}) = \sigma_0(\mathbf{x})\ \delta\eta$ with the condition $u_0(\mathbf{x}) = \int d^2x_1\mathcal{G}_{\mathbf{v}}(\mathbf{x}-\mathbf{x}_1)\,\sigma_0(\mathbf{x}_1)$. A particular case leading to $\delta L_P = 0$ occurs for concentrated loads, e.g. when $\sigma(\mathbf{x}) = \delta_D(\mathbf{x}-\mathbf{x}_0)\ \eta$, i.e. when the stress distribution is represented by a concentrated force at point $\mathbf{x}_0$, the quantity $\delta_D(\mathbf{x})$ being, in fact, the Dirac delta function.

Aiming at calculating the term δL of Eq. (3), we consider an infinitesimally small (and extremely slow) change δA of the contact area A, about an equilibrium condition, due to a small perturbation of its boundary ∂A. In the presence of extremely short-range adhesive forces, the contact area boundary ∂A can be regarded as the tip of a crack, and the contact area change δA as a small propagation (advancing or receding) of the crack itself. In such a case, the displacement field $u(\mathbf{x})$ discontinuously jumps of a quantity $\Delta u(\mathbf{x})$ as the contact domain is subjected to the small change δA. It follows that $\Delta u(\mathbf{x}) = 0$ for $\mathbf{x}\in A$ and $\Delta u(\mathbf{x}) = u_+(\mathbf{x}) - u_-(\mathbf{x}) \neq 0$ for $\mathbf{x}\in\delta A$, where $u_+(\mathbf{x}) = u(\mathbf{x}, A+\delta A)$ and $u_-(\mathbf{x}) = u(\mathbf{x}, A)$. Recalling that for $\mathbf{x}\notin A$ but close to the boundary ∂A the displacement field takes the form $u(\mathbf{x})\propto\sqrt{d}$, where d is the distance from the boundary, one concludes that the 'quasi-static movement' of the crack occurs through a succession of single point loadings as in the case of zipper opening or closing [2, 32]. Then, the infinitely small contact propagation over the area δA can be regarded as an opening (closing) displacement field governed by a

single parameter η slowly increasing from zero to one, i.e. $u(\mathbf{x},\eta) = \eta\,\Delta u(\mathbf{x}) + u_-(\mathbf{x})$, which yields $\delta u(\mathbf{x},\eta) = (\partial u/\partial\eta)\,\delta\eta = \Delta u(\mathbf{x})\,\delta\eta$. Linearity (1) between stresses and displacement then yields $\sigma(\mathbf{x},\eta) = \eta\ \sigma_+(\mathbf{x})$ with $\mathbf{x} \in \delta A$ and $\sigma_+(\mathbf{x}) = \sigma(\mathbf{x}, A+\delta A)$. Note that for $\mathbf{x} \in \delta A$, the stress $\sigma_-(\mathbf{x}) = \sigma(\mathbf{x}, A) = 0$. Then, the work $\delta L = \delta U + \delta L_P$ done by the internal stresses during the quasi-static crack opening (closing) process can be simply calculated as

$$\begin{aligned}\delta L &= \int_{\delta A} d^2x \int_0^1 \sigma(\mathbf{x},\eta)\,\Delta u(\mathbf{x})\,\delta\eta \\ &= \frac{1}{2}\int_{\delta\Omega} d^2x \sigma_+(\mathbf{x})\,\Delta u(\mathbf{x})\end{aligned} \tag{7}$$

Noting that $\int_{\delta A} d^2x\sigma_+(\mathbf{x})\,\Delta u(\mathbf{x}) = \int d^2x\,[\sigma_+(\mathbf{x}) + \sigma_-(\mathbf{x})]\,\Delta u(\mathbf{x})$, and recalling that $\Delta u(\mathbf{x}) = u_+(\mathbf{x}) - u_-(\mathbf{x})$, Eqs. (3, 6) then yield

$$\delta U = \Delta\gamma\delta A - \delta L_P \tag{8}$$

where

$$\delta L_P = -\frac{1}{2}\int d^2x\,[\sigma_+(\mathbf{x})\,u_-(\mathbf{x}) - \sigma_-(\mathbf{x})\,u_+(\mathbf{x})] \tag{9}$$

is the non-conservative contribution to the work of internal stresses related to the hysteretic behavior of the material. In fact, using Eq. (1) and recalling that $\mathcal{G}^O_{\mathbf{v}}(\mathbf{x}) = \frac{1}{2}[\mathcal{G}_{\mathbf{v}}(\mathbf{x}) - \mathcal{G}_{\mathbf{v}}(-\mathbf{x})]$, we write $\delta L_P = -\int d^2x d^2x_1 \mathcal{G}^O_{\mathbf{v}}(\mathbf{x}-\mathbf{x}_1)\,\sigma_+(\mathbf{x})\,\sigma_-(\mathbf{x}_1)$, thus showing that this contribution is associated with the antisymmetric component of the Green function. It follows that in the case of purely elastic materials, i.e. for symmetric Green's functions, $\delta L_P = 0$.

Recalling the definition of the energy release rate $G = \delta U/\delta A$ we rewrite (8) in the form

$$G(\mathbf{v}) = G_0 - \frac{\delta L_P}{\delta A} \tag{10}$$

where we set $G_0 = \Delta\gamma$. Note that standard theories of viscoelastic crack propagation [6, 7, 27] usually assume a wide non linear process zone at the crack tip leading to values of G_0 greater than $\Delta\gamma$. The present theory do not include such non linear processess as cavitation and bond breaking. However, at least for adhesive cracks, where the tensile stress is not so high as to generate cavitation or other ruptures, the effect of crack process zone may not be important unless the rubber compound is very weakly crosslinked.

Interestingly, Eq. (10) shows that the effect of the viscoelasticity can be either to increase or decrease the energy release rate, depending on the sign of the δL_P. This leads, therefore, to a different behavior at the leading and trailing edges of the contact. We also observe that in the case of adhesiveless contact, i.e. $\Delta\gamma = 0$, the stress intensity factor at the edge of the contact vanishes so that the displacement field does not present any discontinuous jump as the contact area changes of the infinitesimally quantity δA, leading to $\delta L = \delta U = \delta L_P = 0$. Thus, for adhesiveless steady-state viscoelastic contacts (under displacement controlled conditions), the equilibrium solution can be found by simply enforcing $\delta U/\delta A = 0$, as in the purely elastic case.

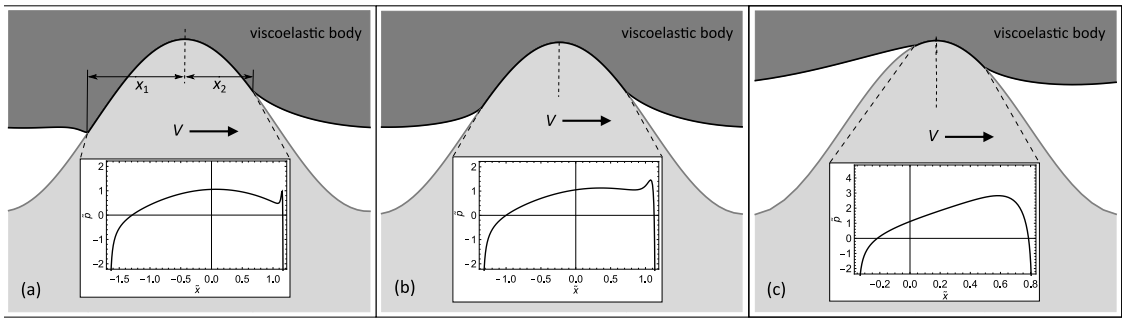

FIG. 1: The sliding contact of a viscoelastic material against a moving rigid indenter, in the presence of adhesion. The inset shows the asymmetric pressure distribution at the contact interface. The contact half-width is $a = (x_2 + x_1)/2$, and contact eccentricity is $e = (x_2 - x_1)/2$. Small-scale viscoelasticy (also referred to as adhesion hysteresis) occurs at small velocities (regime I), (a); coupled small and large scale viscoelasticy (regime II) occurs at intermediat velocities (b); large scale viscoelasticity (regime III) occurs at higher velocities, (c).

III. RESULTS AND DISCUSSION

In what follows, we comment on the viscoelastic adhesive behavior predicted for a 1D+1D contact of a viscoelastic half-plane in partial contact with a wavy rigid profile of wavelength λ, wavevector $k = 2\pi/\lambda$ and amplitude h, (see Fig. 1), sliding at constant velocity v. The material obeys the standard linear viscoelastic model with one single relaxation time τ, and creep function $J(t) = 1/E_0 - (1/E_0 - 1/E_\infty)\exp(-t/\tau)$, where E_0 and E_∞ are respectively the low and high frequency real values of the viscoelastic modulus of the material. The contact is modelled by relying on Green's function approach described in Refs [19–21]. The closing conditions to calculate the contact area half-width $a = (x_2 + x_1)/2$, and the eccentricity $e = (x_2 - x_1)/2$, are given by Eq. (10). Specifically, we consider two independent contact area variations at the two edges of the contact. Due to material viscoelasticity the contact interface will experience an asymmetric distribution of normal pressure, depending on the specific contact conditions. This leads to viscoelastic friction, so that, according to Ref. [19–21, 23, 26], the global friction coefficient μ can be calculated as

$$\mu = -\frac{1}{\lambda p_\infty}\int p(x)\,u'(x)\,dx \tag{11}$$

where p_∞ is the remote pressure acting on the solid and $u'(x)$ the spatial derivative of the normal displacements. Notably, the only source of contact pressure asymmetry is ascribable to the viscoelastic energy dissipation occurring during the cyclic deformation of viscoelastic half-space. The viscoelastic material is indeed excited at different frequencies. The bulk of the material is excited at frequency $\approx v/\lambda$; therefore, depending on the indenter sliding velocity v, the bulk of the material may be in the rubbery, transition or glassy regions. Close to the edges of the contact the material is excited at frequency v/ρ, where ρ is the velocity dependent radius of curvature of the blunted edges of the contact (in general, ρ may differ at the trailing and leading edges). This allows us to roughly identify three regimes, depending on the indenter sliding velocity v: (I) the small-scale viscoelastic regime [Fig. 1(a)], for $v \leq \rho/\tau < \lambda/\tau$, in which viscoelastic hysteresis is mostly localized close to the edges of the contact (the so-called adhesion hysteresis) and the bulk of the material

behaves elastically with modulus E_0; (II) the coupled small/large scale viscoelastic regime [Fig. 1(b)], for $\rho/\tau < v < \lambda/\tau$, in which the whole solid experiences viscoelastic hysteresis; (III) the bulk viscoelastic regime [Fig. 1(c)], for $\rho/\tau < \lambda/\tau \leq v$, in which viscoelastic hysteresis mostly affects the bulk material, whereas the response of the material close to the edges of the contact is elastic, with modulus E_∞. Notice that for $v \ll \rho/\tau$ and $\lambda/\tau \ll v$ the whole half-plane behaves elastically, with Young moduli E_0 and E_∞, respectively. Regarding the frictional response, in the small-scale viscoelastic regime (i.e. regime I), the friction coefficient is dominated by adhesion hysteresis and we get $\mu \approx \mu_a = (G_1 - G_2)/\lambda p_\infty$. According to [5] we refer to μ_a as the adhesive friction coefficient. In the bulk viscoelastic regime (i.e. regime III), we expect a vanishing adhesive contribution to friction (i.e., $G_1 \approx G_2 \approx \Delta\gamma$), as most of the energy dissipation occurs into the bulk of the solid, so that $\mu \approx \mu_0$, with μ_0 being the adhesiveless friction coefficient. Interestingly, regime II is characterized by a strong coupling of small and large scale viscoelastic hysteresis, which leads to a very strong enhancement of friction due to the combined effect of adhesion hysteresis and bulk viscoelasticity.

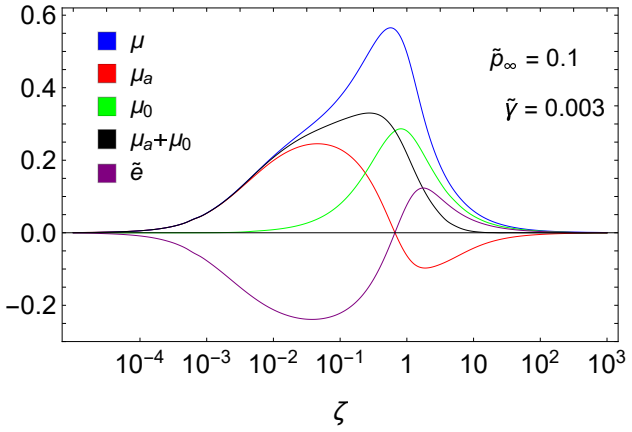

FIG. 2: The frictional response under load controlled conditions. μ is the overal friction coefficient for the viscoelastic adhesive contact here considered, μ_a is the adhesive friction coefficient, and μ_0 the corresponding adhesiveless viscoelastic friction coefficient. μ is always larger than $\mu_a + \mu_0$, thus showing that friction calculations based on two-scale approaches, i.e. made by summing up the adhesive friction (μ_a) and the adhesiveless viscoelastic friction (μ_0), does not predict correctly the overall frictional behavior of adhesive vicoelastic interfaces.The quantity $\tilde{e}$ is the dimensionless eccentricity of the contact area.

Fig. 2 reports the friction coefficients μ, μ_a, μ_0, $\mu_a+\mu_0$, and the dimensionless eccentricity $\tilde{e} = ke$ as functions of the dimensionless sliding speed $\zeta = kv\tau$, at given dimensionless pressure $\tilde{p}_\infty = 2p_\infty/(E_0^* kh) = 0.1$, with $E_0^* = E_0/(1-\nu^2)$, Poisson ratio $\nu = 0.5$, $E_\infty/E_0 = 10$, and dimensionless adhesion energy $\tilde{\gamma} = 2\gamma/(E^*\lambda) = 0.003$. Fig. 2 clearly confirms that at small values of the dimensionless speed ζ, the dominant contribution to friction arises from adhesion hysteresis, i.e., $\mu \approx \mu_a$ (small-scale viscoelasticity, regime I). In the intermediate range of speeds (regime II), the real frictional behavior is strongly enhanced by the combined presence of adhesion hysteresis and bulk viscoelasticity, which leads to an overall friction coefficient μ significantly greater than the sum of the adhesive friction coefficient μ_a and the adhesiveless friction coefficient μ_0. In such conditions, the value of the friction coefficient μ may rise up to almost two times the values $\mu_a + \mu_0$. The latter

result confirms the fundamental role of adhesion in determining the frictional performance of viscoelastic materials, as for example in the case of grip and handling performance of racing tires [30]. We stress that, in regime II, there is no possibility to separate the small and the large scales viscoelastic losses. Models based on such an assumption fall short in predicting the frictional and adhesive behavior of polymeric materials in the intermediate range of sliding velocities, as observed by Grosch in Ref. [14] (see Figs. 7-10 therein), and also pointed out by Roberts [28]. In agreement with Grosch's observations, the trend of the overall friction coefficient μ (Fig. 2), presents a hump localized where the maximum adhesive friction μ_a occurs (low velocity side). As the velocity is increased, a maximum of μ is then observed at dimensionless speed ζ close to 1, at which the adhesiveless friction coefficient μ_0 also takes its maximum. We note that the contribution of adhesion to friction can also occur from processes related to molecules binding-stretching-detaching cycles [33], which are not included in the present treatment.

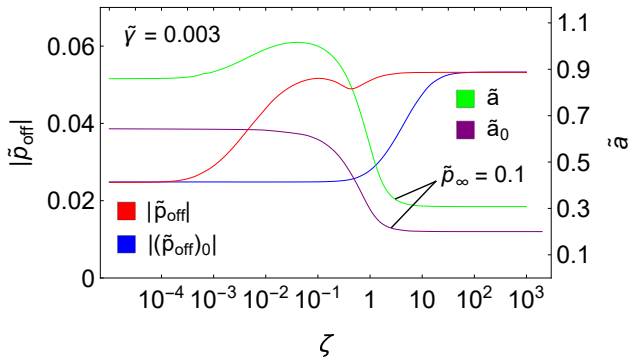

FIG. 3: The pull-off load $|\tilde{p}_{off}|$, the pull-off load $\left|(\tilde{p}_{off})_0\right|$ (see text for a detailed description), the contact area $\tilde{a}$ and the adhesiveless $\tilde{a}_0$ contact area (at given contact pressure) as functions of the dimensionless speed ζ. At intermediate speeds, the pull-off load and the contact area are strongly enhanced due to a strong increase of the effective adhesion caused by viscoelastic dissipation.

Fig. 3 reports the dimensionless half width $\tilde{a} = ka$ of the contact area, the dimensionless width $\tilde{a}_0 = ka_0$ of the corresponding adhesiveless viscoelastic contact, the critical remote pull-off load $|\tilde{p}_{off}|$ and the pull-off load $\left|(\tilde{p}_{off})_0\right|$. Note that the latter has been estimated assuming a perfect elastic material with elastic modulus $|E(\omega = \zeta/\tau)|$. All these quantities are plotted versus the dimensionless speed ζ. By comparing the trend of the quantity $\tilde{a}$ with $\tilde{a}_0$, we note that, in contrast with the monotonically decreasing trend of $\tilde{a}_0$, the half-width $\tilde{a}$ presents a significant non-monotonic behavior characterized by a very strong enhancement of the contact area. Specifically, such a feature occurs in the same range of intermediate speeds (regime II) corresponding to the aforementioned increase of μ well above the the values of $\mu_a + \mu_0$. This considerable increase of the contact area takes place as a consequence of the combined effect of adhesion hysteresis and bulk viscoelasticity. Coming to the pull-off remote pressure, we observe that $\left|(\tilde{p}_{off})_0\right|$ monotonically increase because of the monotonic material stiffening (3) occurring as ζ is increased. However, the effect of adhesion hysteresis and viscoelasticity causes the real pull-off load $|\tilde{p}_{off}|$ to rise up much beyond $\left|(\tilde{p}_{off})_0\right|$, already at very low velocities (regime I). A further increase of ζ leads to a slightly non-monotonic trend of $|\tilde{p}_{off}|$, likely owed to the combined effect of adhesion hysteresis and bulk viscoelasticity (regime II). Experimental observations on rolling cylinders in contact with

viscoelastic soft substrates strongly support this picture [9].

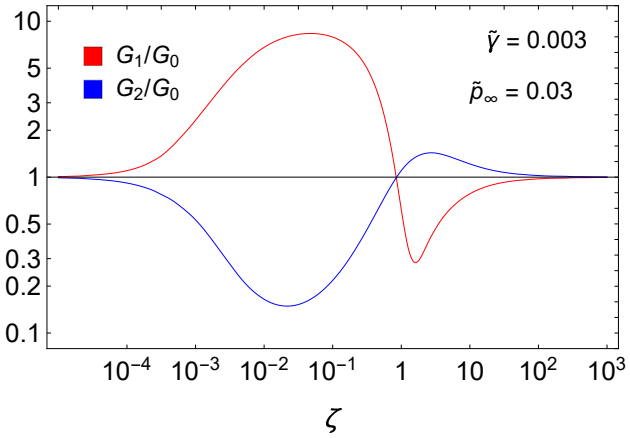

FIG. 4: The energy release rates G_1/G_0 and G_2/G_0 at the trailing and leading edges respectively, as a functin of the dimensionless sliding velocity ζ, in a log-log diagram.

To close the discussion we present in Fig. 4. the trend of the energy release rates G_1, at the contact trailing edge, and G_2, at the contact leading edge, as a function of the dimensionless sliding speed ζ in a log-log diagram. Note that, G_1 and G_2 follow a non-monotonic trend. More specifically G_1/G_0 (red curve), at very low speed takes the unit value as the material behaves elastically (with modulus E_0) everywhere and the energy release rate must equate the adhesion energy for unit area $G_0 = \Delta\gamma$. As the velocity is increased, G_1/G_0 increases up to a maximum value $\approx E_\infty/E_0$ [6, 7, 13, 27], and then decreases to values even smaller that those attained by G_2/G_0, followed by a further increase which asymptotically leads to $G_1/G_0 \to 1$ at very high velocity. Indeed, in this case, the material is again elastic everywhere (with modulus E_∞). Regarding G_2/G_0 (blue curve), it takes the unit value at very low velocity, and with ζ increasing, it firstly decreases to a minimum value and then increases again to a local maximum. At very high speed, G_2/G_0 asymptotically decrease toward the unit value. This non-monotonic behavior is not observed in infinite systems [6, 7, 13, 27] and is a characteristic of the finiteness of contact area as inferred in [24, 25], where a bell shaped behavior of G_1 is reported.

IV. CONCLUSIONS

We present a comprehensive theory of the adhesive contact mechanics of linearly viscoelastic materials in steady contact with rigid substrates moving at constant velocity. The theory allows to investigate the interplay between viscoelastic hysteresis and interfacial adhesion over a wide range of velocity, thus encompassing several characteristic length scales and viscoelastic regimes. The present theory clearly indicates that adhesion plays a fundamental role in determining the overall frictional response of the contact over the entire range of sliding speeds. Specifically, we show that, at intermediate speeds, the concurrent presence of small scale viscoelasticity (i.e. adhesion hysteresis) and large-scale viscoelasticity (bulk hysteresis) leads to an enhancement of the overall friction coefficient μ which has been only observed experimentally and never explained before. Our predicted friction versus sliding velocity presents a strong agreement with experimental observations reported by Grosch in his seminal work [14].

Acknowledgments

This work was partly supported by the Italian Ministry of Education, University and Research under the Programme "Progetti di Rilevante Interesse Nazionale (PRIN)", Grant Protocol 2017948, Title: Foam Airless Spoked Tire – FASTire (G.C.) The authors acknowledge support from the Italian Ministry of Education, University and Research (MIUR) under the program "Departments of Excellence" (L.232/2016)

[1] J. M. Baney and C.-Y. Hui. Viscoelastic crack healing and adhesion. *Journal of Applied Physics*, 86(8):4232–4241, October 1999.

[2] M. Barquins and D. Maugis. Tackiness of elastomers. *The Journal of Adhesion*, 13(1):53–65, December 1981.

[3] M Barquins, D Maugis, J Blouet, and R Courtel. Contact area of a ball rolling on an adhesive viscoelastic material. *Wear*, 51(2):375–384, 1978.

[4] Bharat Bhushan, Jacob N. Israelachvili, and Uzi Landman. Nanotribology: friction, wear and lubrication at the atomic scale. *Nature*, 374(6523):607–616, April 1995.

[5] G Carbone. Adhesion and friction of an elastic half-space in contact with a slightly wavy rigid surface. *Journal of the Mechanics and Physics of Solids*, 52(6):1267–1287, June 2004.

[6] G. Carbone and B. N. J. Persson. Crack motion in viscoelastic solids: The role of the flash temperature. *The European Physical Journal E*, 17(3):261–281, July 2005.

[7] G. Carbone and B. N. J. Persson. Hot cracks in rubber: Origin of the giant toughness of rubberlike materials. *Physical Review Letters*, 95(11), September 2005.

[8] Giuseppe Carbone and Carmine Putignano. A novel methodology to predict sliding and rolling friction of viscoelastic materials: Theory and experiments. *Journal of the Mechanics and Physics of Solids*, 61(8):1822–1834, August 2013.

[9] J.-C. Charmet and M. Barquins. Adhesive contact and rolling of a rigid cylinder under the pull of gravity on the underside of a smooth-surfaced sheet of rubber. *International Journal of Adhesion and Adhesives*, 16(4):249–254, January 1996.

[10] Manoj K. Chaudhury. Interfacial interaction between low-energy surfaces. *Materials Science and Engineering: R: Reports*, 16(3):97–159, March 1996.

[11] Yu.B. Chernyak and A.I. Leonov. On the theory of the adhesive friction of elastomers. *Wear*, 108(2):105–138, March 1986.

[12] André Le Gal, Xin Yang, and Manfred Klüppel. Evaluation of sliding friction and contact mechanics of elastomers based on dynamic-mechanical analysis. *The Journal of Chemical Physics*, 123(1):014704, July 2005.

[13] J A Greenwood. The theory of viscoelastic crack propagation and healing. *Journal of Physics D: Applied Physics*, 37(18):2557–2569, September 2004.

[14] K A Grosch. 274(1356):21–39, June 1963.

[15] S. Hao and L. M. Keer. Rolling contact between rigid cylinder and semi-infinite elastic body with sliding and adhesion. *Journal of Tribology*, 129(3):481–494, January 2007.

[16] Brodie K. Hoyer, Rong Long, and Mark E. Rentschler. A tribometric device for the rolling contact of soft elastomers. *Tribology Letters*, 70(2), March 2022.

[17] Seong H. Kim, David B. Asay, and Michael T. Dugger. Nanotribology and MEMS. *Nano Today*, 2(5):22–29, October 2007.

[18] Nobuo Maeda, Nianhuan Chen, Matthew Tirrell, and Jacob N. Israelachvili. Adhesion and friction mechanisms of polymer-on-polymer surfaces. *Science*, 297(5580):379–382, July 2002.

[19] N. Menga, L. Afferrante, and G. Carbone. Effect of thickness and boundary conditions on the behavior of viscoelastic layers in sliding contact with wavy profiles. *Journal of the Mechanics and Physics of Solids*, 95:517–529, October 2016.

[20] N. Menga, L. Afferrante, G.P. Demelio, and G. Carbone. Rough contact of sliding viscoelastic layers: numerical calculations and theoretical predictions. *Tribology International*, 122:67–75, June 2018.

[21] N. Menga, G. Carbone, and D. Dini. Exploring the effect of geometric coupling on friction and energy dissipation in rough contacts of elastic and viscoelastic coatings. *Journal of the Mechanics and Physics of Solids*, 148:104273, March 2021.

[22] N. Menga, C. Putignano, G. Carbone, and G. P. Demelio. The sliding contact of a rigid wavy surface with a viscoelastic half-space. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 470(2169):20140392, September 2014.

[23] B. N. J. Persson. Theory of rubber friction and contact mechanics. *The Journal of Chemical Physics*, 115(8):3840–3861, August 2001.

[24] B. N. J. Persson. Crack propagation in finite-sized viscoelastic solids with application to adhesion. *EPL (Europhysics Letters)*, 119(1):18002, July 2017.

[25] B. N. J. Persson. On opening crack propagation in viscoelastic solids. *Tribology Letters*, 69(3), August 2021.

[26] B N J Persson, O Albohr, U Tartaglino, A I Volokitin, and E Tosatti. On the nature of surface roughness with application to contact mechanics, sealing, rubber friction and adhesion. *Journal of Physics: Condensed Matter*, 17(1):R1–R62, December 2004.

[27] B. N. J. Persson and E. A. Brener. Crack propagation in viscoelastic solids. *Physical Review E*, 71(3), March 2005.

[28] AD Roberts. Looking at rubber adhesion. *Rubber Chemistry and Technology*, 52(1):23–42, 1979.

[29] M Scaraggi and B N J Persson. Friction and universal contact area law for randomly rough viscoelastic contacts. *Journal of Physics: Condensed Matter*, 27(10):105102, February 2015.

[30] R. S. Sharp, P. Gruber, and E. Fina. Circuit racing, track texture, temperature and rubber friction. *Vehicle System Dynamics*, 54(4):510–525, January 2016.

[31] Hongquan She, David Malotky, and Manoj K. Chaudhury. Estimation of adhesion hysteresis at polymer/oxide interfaces using rolling contact mechanics. *Langmuir*, 14(11):3090–3100, May 1998.

[32] Kenneth R. Shull. Contact mechanics and the adhesion of soft solids. *Materials Science and Engineering: R: Reports*, 36(1):1–45, January 2002.

[33] A. Tiwari, L. Dorogin, M. Tahir, K. W. Stöckelhuber, G. Heinrich, N. Espallargas, and B. N. J. Persson. Rubber contact mechanics: adhesion, friction and leakage of seals. *Soft Matter*, 13(48):9103–9121, 2017.

[34] Katherine Vorvolakos and Manoj K. Chaudhury. The effects of molecular weight and temperature on the kinetic friction of silicone rubbers. *Langmuir*, 19(17):6778–6787, July 2003.

[35] Hisae Yoshizawa, You Lung Chen, and Jacob Israelachvili. Fundamental mechanisms of interfacial friction. 1. relation between adhesion and friction. *The Journal of Physical Chemistry*, 97(16):4128–4140, April 1993.

[36] Hisae Yoshizawa and Jacob Israelachvili. Fundamental mechanisms of interfacial friction. 2. stick-slip friction of spherical and chain molecules. *The Journal of Physical Chemistry*,

97(43):11300–11313, October 1993.

[37] Yuyan Zhang, Xiaoli Wang, Hanqing Li, and Weixu Yang. A numerical study of the rolling friction between a microsphere and a substrate considering the adhesive effect. *Journal of Physics D: Applied Physics*, 49(2):025501, November 2015.

[38] Ming Zhou, Noshir Pesika, Hongbo Zeng, Yu Tian, and Jacob Israelachvili. Recent advances in gecko adhesion and friction mechanisms and development of gecko-inspired dry adhesive surfaces. *Friction*, 1(2):114–129, June 2013.