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# Performance Comparison of Variable-Angle Phase-Shifting Carrier PWM Techniques

Vito G. Monopoli, *Member, IEEE*, Youngjong Ko, *Student Member, IEEE*,  
Giampaolo Buticchi, *Senior Member, IEEE*, Marco Liserre, *Fellow, IEEE*

**Abstract**— This paper analyzes the performances of different Carrier Phase-Shifting PWM techniques to be used with a multilevel cascaded H-bridge converter in case of unbalanced operational conditions. In fact, in many practical applications, the ideal condition of equal DC voltages and equal reference signals for each H-bridge can not be achieved. In such conditions, the conventional Carrier Phase Shifting PWM technique loses its harmonic cancelling capabilities and then the multilevel AC voltage harmonic quality is deeply affected. To overcome this limit of the original technique, different variations have been proposed. All of them still rely on the carrier phase shifting concept and propose to use a different value of the shifting angle for each carrier (unlike the original technique) whenever unbalanced operational conditions occur. In this paper the three main solutions proposed over the last years to extend the capabilities of Carrier Phase-Shifting PWM technique are compared. The analysis is focused on a three-cell cascaded H-bridge converter. Simulation and experimental results are presented.

**Index Terms**— Harmonic analysis, Pulse width modulation, Multilevel converters.

## I. INTRODUCTION

CASCADED multilevel converters have been considered over the last decade a consolidated solution for medium voltage drives and for a profitable interfacing of medium/high power loads and distributed generation sources to the electrical grid by means of a series of single-phase full-bridge power converters. Fig. 1 shows the general schematic of a cascaded multilevel converter. There are some applications of the Cascaded H-Bridge converter (CHB) where limited variations of both the amplitude and the frequency of the output voltage occur. In such cases, PWM can be avoided and equivalent harmonic performance can be achieved through

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V. G. Monopoli is with the Department of Electrical and Information Engineering, Politecnico di Bari, 70126 Bari, Italy (e-mail: vitogiuseppe.monopoli@poliba.it).

Y. Ko and M. Liserre are with the Chair of Power Electronics, Christian-Albrechts Universität zu Kiel, 24143 Kiel, Germany (e-mail: yoko@tf.uni-kiel.de; ml@tf.uni-kiel.de).

G. Buticchi is with University of Nottingham Ningbo China (e-mail: giampaolo.buticchi@nottingham.edu.cn).

fundamental frequency selective harmonic elimination techniques also in presence of unbalanced DC voltages [1]-[2]. Nevertheless, these techniques represent a practical approach only in case of limited output voltage regulation. In fact, whenever the range of variation of the output voltage is wide and rapid changes are required (e.g. motor drives), the pulse width modulation of the CHB becomes again a more viable solution [3]. One of the most used modulation techniques related to such a conversion topology is referred to as Phase-Shifting Carrier Pulse Width Modulation (PSC-PWM).

It allows to reduce the Weighted Total Harmonic Distortion (WTHD) of the overall output voltage waveform by equally shifting the relative phase of the triangular carrier employed in each H-bridge. In fact, in case of  $N$  cascaded bridges, when carrier phase shifts of  $(i-1)\pi/N$  are considered, optimal

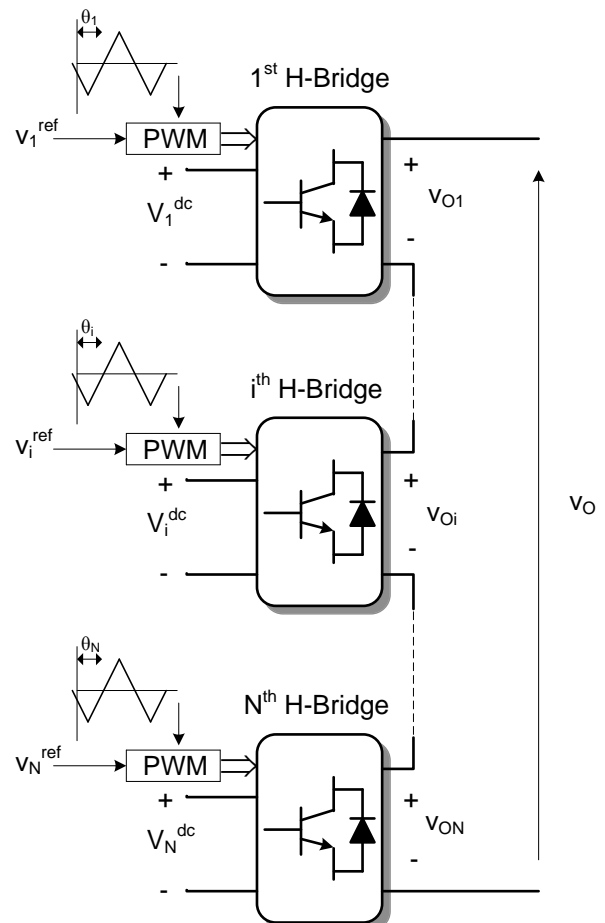


Fig. 1. Cascaded multilevel converter topology

harmonic components cancellation up to the 2Nth carrier multiple occurs [3]. Nevertheless, the main limit with the PSC-PWM is that a perfectly balanced operation is required that means equal DC voltages and equal modulation indexes for all the cascaded bridges. This aspect deeply limits the cases where this technique can provide all its potentiality.

In fact, there are applications where the DC voltage levels are meant to be equal but voltage unbalancing occurs because of unequal power loading [4] or unequal losses among the cascaded bridges [5]. To face such issues some balancing control methods have been proposed in literature. In particular [4] and [6] propose to adjust the modulating signal, while [7] investigates the injection of voltage/current components. Moreover, PSC-PWM itself is inherently unbalancing and for this reason a pulse rotation technique must normally be applied.

In addition to the aforementioned cases, there are some applications that can be considered intentionally unbalanced [8] (asymmetric Cascaded H-Bridges) such as the CHB motor drive capable of regenerating with part of cells [9] and the grid connected multilevel converters that can integrate several kind of DC sources (PV, batteries, supercapacitors, etc.) and can manage different voltage levels [10 – 12]. In all these cases, the conventional PSC-PWM fails since the calculated phase-shifting angles do not allow the cancellation of harmonics up to the 2Nth carrier multiple anymore. To deal with these operational conditions an extension of PSC-PWM was proposed in [13]. It achieves the harmonic cancellation once again, and hence the minimization of the overall output voltage WTHD, through an unequal and dynamically calculated carrier phase shifting that makes null the sum of particular harmonics produced by each bridge. This technique, formerly proposed in [13], has been reconsidered and extended in successive works. In fact, a more comprehensive analysis of the technique proposed in [13] is presented in [14]. It is still focused on the case with unbalanced DC voltages and equal modulating signals for all the H bridges. In particular, details are provided about the harmonic cancellation capability and an upper limit for the equivalent switching frequency with asymmetrical Carrier Phase Shifting PWM is defined. Additionally, a technique that is still a modification of the conventional CPS-PWM is presented in [15] and it is meant for a Cascaded H-Bridges STATCOM. This solution is profitably applied in the case of equal DC voltages which tend to become unbalanced because of different power levels managed by the H-Bridges or different power losses. In this case, different modulating signals have to be used to implement the DC-link voltage balance control method and the conventional CPS-PWM as well as the technique proposed in [13] cannot be applied since a unique modulating signal has to be used for all the H-Bridges. Finally, in [16] is presented a method that uses the Fourier series of the pulses, produced in each PWM period by the H-bridges, to calculate the carrier phase shifting angles. This technique allows the cancellation of the fundamental harmonics (2fPWM) related to the pulses of each H bridge and, in this case, unequal modulating signals and unequal DC voltages can be managed.

In this scenario, this paper aims at comparing the performances of these three main variations of the CPS-PWM

in order to assess which is the one that fits better to the different kind of unbalanced conditions and to provide a means to make a suitable selection among them. To this purpose advantages and drawbacks as well as harmonic cancellation capability and validity limits of each technique are presented in section II. In section III the WTHD's performed by each technique under different unbalanced conditions and the relevant theoretical harmonic spectra are shown. The analysis is focused on a three cell cascaded H-bridge converter. The results of experimental tests are presented in section IV. Finally, conclusions are drawn in section V.

## II. PHASE-SHIFTING CARRIER PWM TECHNIQUES UNDER UNBALANCED CONDITIONS

### A. Phase-shifting carrier PWM technique A [13]

This variation to the original CPS-PWM is suitable mainly for unequal DC voltages and equal modulating signals for each bridge. This technique achieves a WTHD improvement through an unequal carrier phase shifting that allows a cancellation of the carrier harmonics and the sideband harmonics produced by each bridge and belonging to the same harmonic group around multiples of the carrier fundamental.

Under these conditions, a Double Fourier Integral Analysis leads to the following expression for the output AC voltage of the CHB converter:

$$v_0(t) = M \sum_{i=1}^N V_i^{dc} \cos(\omega_o t) + \text{Fundamental} + \frac{4}{\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{2m} J_{2n-1}(m\pi M) \cos([m+n-1]\pi) \cdot \sum_{i=1}^N V_i^{dc} \cos(2m\omega_c t + [2n-1]\omega_o t + 2m\theta_i) \quad \text{Carriers + Sidebands} \quad (1)$$

where  $M$  is the modulation index,  $N$  is the number of cascaded bridges,  $V_i^{dc}$  is the DC voltage of the  $i^{\text{th}}$  converter,  $\omega_o$  is the pulsation of the modulating signal,  $m$  and  $n$  are indexes to account for baseband, carrier and sideband harmonics,  $J_{2n-1}$  is the Bessel function of order  $2n-1$ ,  $\omega_c$  is the pulsation of the carrier signal and  $\theta_i$  is the carrier phase of the  $i^{\text{th}}$  converter.

In order to make null the sum of the harmonics with the same harmonic order produced by each bridge, the following condition must be met:

$$\sum_{i=1}^N V_i^{dc} \cos(2m\omega_c t + [2n-1]\omega_o t + 2m\theta_i) = 0 \quad (2)$$

which can be re-written as:

$$\begin{cases} \sum_{i=1}^N V_i^{dc} \cos(2m\theta_i) = 0 \\ \sum_{i=1}^N V_i^{dc} \sin(2m\theta_i) = 0 \end{cases} \quad (3)$$

where  $2m\theta_i$  represents the phase of all the harmonics (i.e. carriers + sidebands) belonging to the  $m^{\text{th}}$  carrier group produced by the  $i^{\text{th}}$  converter. As the index  $n$  varies, from (1) is evident that the harmonics with the same harmonic order, produced by each bridge, vary their amplitude of the same quantity since the modulation index is equal. For this reason, the condition (3) is valid for all the harmonics belonging to the same carrier group. Therefore, the main advantage of this

technique is that it is possible to cancel out all the harmonics of the same carrier group by fulfilling (3). Nevertheless, to set the problem in a closed form, assuming  $\theta_i=0$ ,  $N-1$  degrees of freedom (i.e.  $\theta_i$ ) can only satisfy  $N-1$  equations in (3) [12]. Consequently, there is a relationship between the number of bridges  $N$  and the number of harmonic carrier groups  $m$  that it is possible to cancel in the converter output voltage. If  $N$  is odd, (3) is determined and  $m_{max}=(N-1)/2$ . While, if  $N$  is even, (3) is undetermined and  $m_{max}=(N-2)/2$ .

Considering the case  $N=3$ , only the harmonics belonging to the first carrier group produced by the three bridges (i.e.  $V_{oi,1n}$ ) can sum up to zero if proper phase shift angles are applied as shown in Fig. 2.a. The multilevel output voltage, the three phase-shifted carrier signals and the modulating signal are shown in Fig. 2.b.

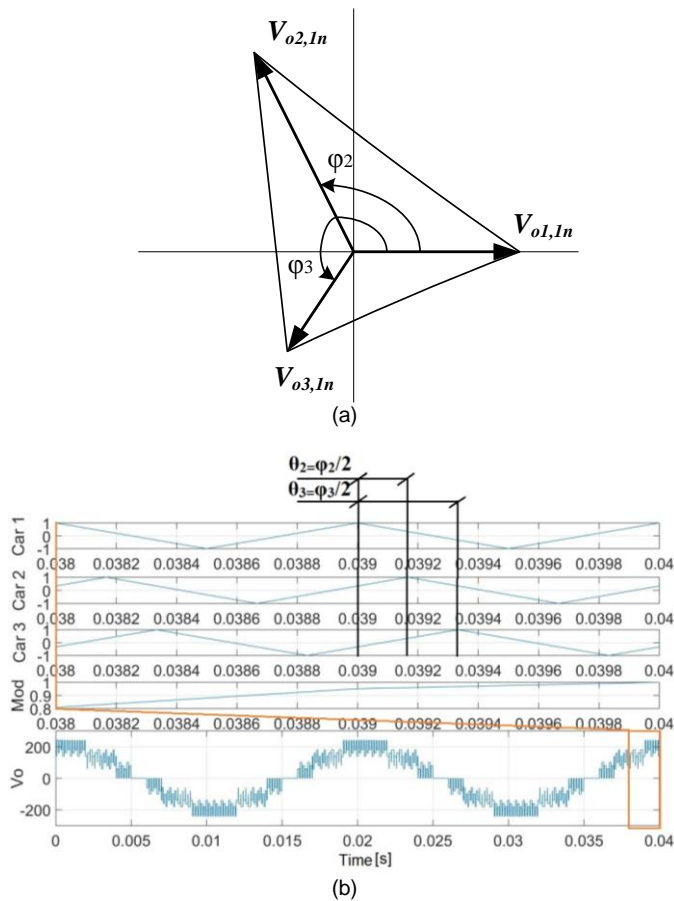


Fig. 2. (a) Harmonics belonging to the first carrier group produced by each bridge. (b) Multilevel output voltage, the three phase-shifted carrier signals and the modulating signal.

In this case (3) can be written as:

$$\begin{cases} V_1^{dc} + V_2^{dc} \cos(\varphi_2) + V_3^{dc} \cos(\varphi_3) = 0 \\ V_2^{dc} \sin(\varphi_2) + V_3^{dc} \sin(\varphi_3) = 0 \end{cases} \quad (4)$$

where  $\varphi_2=2\theta_2$  and  $\varphi_3=2\theta_3$  are respectively the displacement angles between of the harmonics of the second/third bridge and the harmonics of the first bridge belonging to the first carrier group (i.e.  $m=1$ ).

The solution to the equations in (4) is:

$$\begin{aligned} \cos(\varphi_2) &= \frac{1}{2} \left( \frac{-V_1^{dc} - V_2^{dc} + V_3^{dc}}{V_1^{dc} V_2^{dc}} \right) \\ \cos(\varphi_3) &= \frac{1}{2} \left( \frac{-V_1^{dc} + V_2^{dc} - V_3^{dc}}{V_1^{dc} V_3^{dc}} \right) \end{aligned} \quad (5)$$

Valid solutions for the displacement angles  $\varphi_2$  and  $\varphi_3$  are obtained only if the following conditions are met:

$$\begin{cases} |V_1^{dc} - V_2^{dc}| \leq V_3^{dc} \leq (V_1^{dc} + V_2^{dc}) \\ |V_1^{dc} - V_3^{dc}| \leq V_2^{dc} \leq (V_1^{dc} + V_3^{dc}) \end{cases} \quad (6)$$

From (6) is evident that the validity region of this technique is influenced just by the DC voltage values of each bridge.

### B. Phase-shifting carrier PWM technique B [15]

This variation to the original CPS-PWM is suitable for unequal DC voltages and unequal modulating signals for each bridge. This technique achieves a WTHD improvement through an unequal carrier phase shifting that allows a cancellation of selected sideband harmonics produced by each bridge and belonging to the same harmonic group around multiples of the carrier fundamental. Such harmonics are the ones that mostly influence the overall WTHD and are referred to as “main harmonics”.

Under these conditions, a Double Fourier Integral Analysis leads to the following expression for the output AC voltage of the CHB converter:

$$\begin{aligned} v_o(t) &= \sum_{i=1}^N M_i V_i^{dc} \cos(\omega_o t) + \text{Fundamental} \\ &+ \frac{4}{\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{2m} \cos([m+n-1]\pi) \cdot \text{Carriers + Sidebands} \\ &\cdot \sum_{i=1}^N J_{2n-1}(m\pi M_i) V_i^{dc} \cos(2m\omega_c t + [2n-1]\omega_o t + 2m\theta_i) \end{aligned} \quad (7)$$

In this case, the cancellation condition is not the same for all the harmonics belonging to the same carrier group and hence a cancellation of the harmonics of a whole group at the same time cannot be achieved anymore as it was in [13]. In fact, as the index  $n$  varies, from (7) is evident that the harmonics with the same harmonic order, produced by each bridge, do not vary their amplitude of the same quantity since the modulation indexes are unequal. Therefore, this technique makes null just the sum of the “main harmonics” that have  $2f_c/f_o \pm 1$  harmonic order (i.e.  $m=1$  and  $n=[0;1]$ ). The amplitude of these main harmonics depends both on the DC voltage and on the modulation index of each bridge as shown by the following expression:

$$V_i = \frac{2}{\pi} V_i^{dc} J_{2n-1}(\pi M_i), \text{ for } i=1, \dots, N \quad (8)$$

Considering the case  $N=3$ , the sum of the main harmonics produced by each bridge is null if the following condition is met:

$$\begin{cases} V_1 + V_2 \cos(\varphi_2) + V_3 \cos(\varphi_3) = 0 \\ V_2 \sin(\varphi_2) + V_3 \sin(\varphi_3) = 0 \end{cases} \quad (9)$$

Where  $\varphi_2=2\theta_2$  and  $\varphi_3=2\theta_3$  are respectively the displacement angles between the main harmonics of the second/third bridge and the main harmonic of the first bridge.

The solution to the equations in (8) is:

$$\begin{aligned}\cos(\varphi_2) &= \frac{1}{2} \left( \frac{-V_1^2 - V_2^2 + V_3^2}{V_1 V_2} \right) \\ \cos(\varphi_3) &= \frac{1}{2} \left( \frac{-V_1^2 + V_2^2 - V_3^2}{V_1 V_3} \right)\end{aligned}\quad (10)$$

Valid solutions for the displacement angles  $\varphi_2$  and  $\varphi_3$  are obtained only if the following conditions are met:

$$\begin{aligned}|V_1 - V_2| &\leq V_3 \leq (V_1 + V_2) \\ |V_1 - V_3| &\leq V_2 \leq (V_1 + V_3)\end{aligned}\quad (11)$$

From (8) and (11) is evident that both the DC voltage values and the modulation index values of each bridge deeply influence the validity region of this technique.

### C. Phase-shifting carrier PWM technique C [16]

This variation to the original CPS-PWM is suitable for unequal DC voltages and unequal modulating signals for each bridge. This technique achieves a WTHD improvement through an unequal carrier phase shifting that allows a cancellation of the harmonics produced by each bridge in case of unipolar modulation and placed at twice the carrier frequency. Unlike [13] and [15], this technique is not based on the Double Fourier Integral analysis. In this case an analysis based on the Fourier series of the pulses produced by each bridge (with unipolar PWM) every  $T_{\text{PWM}}$  is carried out. Considering that a symmetrical sampled unipolar PWM produces a couple of twin pulses in each carrier period, the analysis framework is just half of carrier period and the calculation of the Fourier coefficients is carried out considering the pulse centered in  $T_{\text{PWM}}/2$  as a square waveform. Therefore, the fundamental frequency of the analyzed signal is  $2f_{\text{pwm}}$ . Consequently, the overall AC multilevel output voltage produced every carrier period can be expressed as:

$$v_0(t) = \sum_{i=1}^N (D_i V_i^{dc} + \sum_{k=1}^{\infty} \frac{2V_i^{dc}}{k\pi} \sin(k\pi D_i) \cos(k\omega t + k\varphi_i)) \quad (12)$$

where  $D_i$  is the ratio between the output voltage and the DC voltage of the  $i^{\text{th}}$  converter.

The proposed technique consists in displacing, every PWM period, the carriers so that the fundamental harmonics ( $2f_{\text{pwm}}$ ) of the pulses of each H bridge sum up to zero. Considering the case  $N=3$ , the sum of the fundamental harmonics of the pulses produced by each bridge is null if the following condition is met:

$$\begin{cases} h_{11} + h_{12} \cos(\varphi_2) + h_{13} \cos(\varphi_3) = 0 \\ h_{12} \sin(\varphi_2) + h_{13} \sin(\varphi_3) = 0 \end{cases} \quad (13)$$

where

$$h_{ki} = \frac{2V_i^{dc}}{k\pi} \sin(k\pi D_i) \quad (14)$$

are coefficients for H-bridge  $i$  and  $k^{\text{th}}$  harmonic order, while  $\varphi_2=2\theta_2$  and  $\varphi_3=2\theta_3$  are respectively the displacement angles between the fundamental harmonics of the pulses of the second/third bridge and the fundamental harmonic of the pulses of the first bridge.

The solution to the equations in (13) is:

$$\begin{aligned}\cos(\varphi_2) &= \frac{1}{2} \left( \frac{-h_{11}^2 - h_{12}^2 + h_{13}^2}{h_{11} h_{12}} \right) \\ \cos(\varphi_3) &= \frac{1}{2} \left( \frac{-h_{11}^2 + h_{12}^2 - h_{13}^2}{h_{11} h_{13}} \right)\end{aligned}\quad (15)$$

Valid solutions for the displacement angles  $\varphi_2$  and  $\varphi_3$  are obtained only if the following conditions are met:

$$\begin{aligned}|h_{11} - h_{12}| &\leq h_{13} \leq (h_{11} + h_{12}) \\ |h_{11} - h_{13}| &\leq h_{12} \leq (h_{11} + h_{13})\end{aligned}\quad (16)$$

From (14) and (16) is evident that both the DC voltage values and the ratio  $D$  of each bridge deeply influence the validity region of this technique.

### III. COMPARISON OF THE TECHNIQUE PERFORMANCES UNDER DIFFERENT UNBALANCED CONDITIONS

Considering the case  $N=3$ , the carrier phase shift angles  $\theta_2$  and  $\theta_3$  have been carried out for the techniques considered in the previous section under different unbalanced conditions. Such calculated values have been used to derive the theoretical harmonic spectra through the expressions presented in [1] for this multilevel converter in case of Technique A and B. In fact, an exact analytical calculation of the harmonic components of a PWM waveform allows a precise determination of the harmonic characteristics and a more effective and rigorous comparison between PWM strategies. While, Fast Fourier Transform (FFT) algorithm, although allowing expediency and reduced mathematical effort, produces results that could be affected by round-off or error due to its practical implementation. As regards technique C, it calculates the value of the carrier phase shift angles  $\theta_2$  and  $\theta_3$  every PWM period. Under unequal modulation indices conditions, the values  $\theta_2$  and  $\theta_3$  vary during a period of the modulating signal and hence a different theoretical harmonic spectrum can be achieved for each PWM period. For this reason it is impossible to carry out a unique spectrum in this case and hence the harmonic performances of technique C will be shown through experimental results

#### A. Unequal DC voltages and equal modulating signals

To compare the performances of the three techniques under these unbalanced conditions it has been considered  $V_1^{dc}=100V$ ,  $V_2^{dc}=80V$ ,  $V_3^{dc}=60V$ ,  $f_o=50\text{Hz}$ ,  $f_c=5000\text{Hz}$ ,  $M=0.8$ .

Under these conditions, the three techniques are equivalent and hence produce the same harmonic spectrum shown in Fig. 3, although technique A should still be preferred because of its lower computational burden. The mathematical demonstration of such an equivalence is included in the Appendix. From Fig. 3 it is possible to notice that the first carrier group is completely cancelled out, although the second carrier group has not disappeared as it would happen in case of equal DC voltages with the conventional CPS-PWM. Fig. 4 shows the WTHD trend as a function of  $V_1^{dc}$  and  $V_2^{dc}$  for all the three considered CPS-PWM techniques. In particular is evident that its value increases as the degree of unbalance among the DC voltages of the three H-bridges increases.

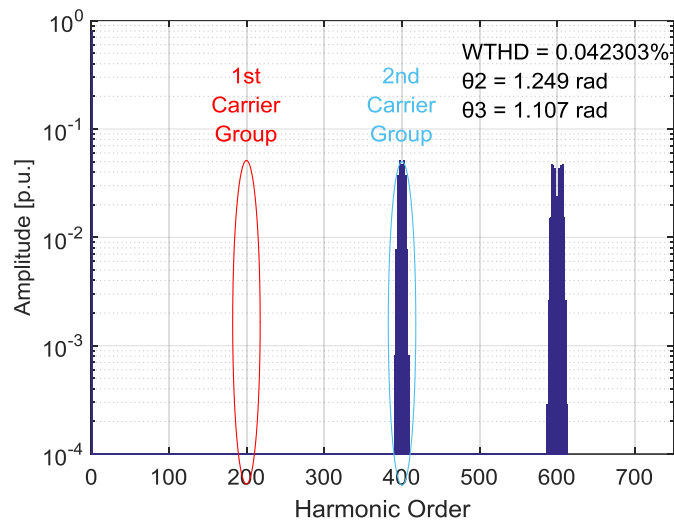


Fig. 3. Normalized theoretical harmonic spectra for all the three considered CPS-PWM techniques in case of unequal DC voltages and equal modulating signals.

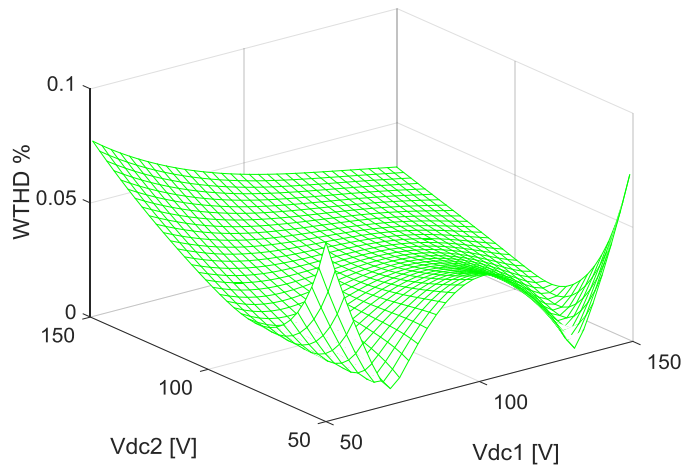


Fig. 4. WTHD trend as a function of  $V_1^{dc}$  and  $V_2^{dc}$  for  $V_3^{dc} = 100$  V e  $M=0,8$  for all the three considered CPS-PWM techniques

### B. Equal DC voltages and unequal modulating signals

To compare the performances of the three techniques under these unbalanced conditions it has been considered  $V^{dc}=100V$ ,  $f_o=50Hz$ ,  $f_c=5000Hz$ ,  $M_1=0.5$ ,  $M_2=0.7$ ,  $M_3=0.9$ . In this case the three techniques are not equivalent and hence different values of the carrier shifting angles are carried out.

Fig. 5 shows that the Technique A does not succeed in cancelling the first carrier group anymore under these unbalanced conditions. Consequently, the WTHD value increases.

The Technique B can cancel out the “main harmonics” in the first carrier group under these unbalanced conditions and, therefore, it can reduce the WTHD value.

Fig. 6 shows the WTHD trend as a function of  $M_2$  and  $M_3$  for Techniques A and B. It is possible to notice that the WTHD values almost the same.

### C. Unequal DC voltages and unequal modulating signals

To compare the performances of the three techniques under these unbalanced conditions it has been considered  $V_1^{dc}=70V$ ,  $V_2^{dc}=50V$ ,  $V_3^{dc}=40V$ ,  $f_o=50Hz$ ,  $f_c=5000Hz$ ,  $M_1=0.95$ ,  $M_2=0.9$ ,  $M_3=0.85$ . Also in this case the three techniques are not equivalent and hence different values of the carrier shifting angles are carried out. Fig. 7 shows that the Technique A again does not succeed in cancelling the first carrier group anymore under these unbalanced conditions. Moreover, it can be noticed that the Technique B again can cancel out the “main harmonics” in the first carrier group under these unbalanced conditions and, therefore, it performs a better WTHD value than the technique A. Fig. 8 shows the WTHD trend as a function of  $M_2$  and  $M_3$  for Techniques A and B. It is possible to notice that the WTHD values almost the same. Fig. 9 shows the WTHD trend as a function of  $V_1^{dc}$  and  $V_2^{dc}$  for Techniques A and B. The two techniques perform almost equivalently unless when  $V_1^{dc}$  is much higher than  $V_2^{dc}$ . In this case technique A performs better.

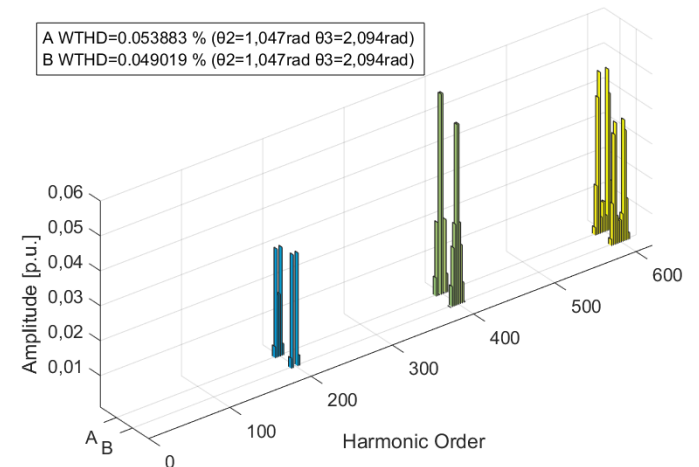


Fig. 5. Normalized theoretical harmonic spectra for techniques A and B in case of equal DC voltages and unequal modulating signals.

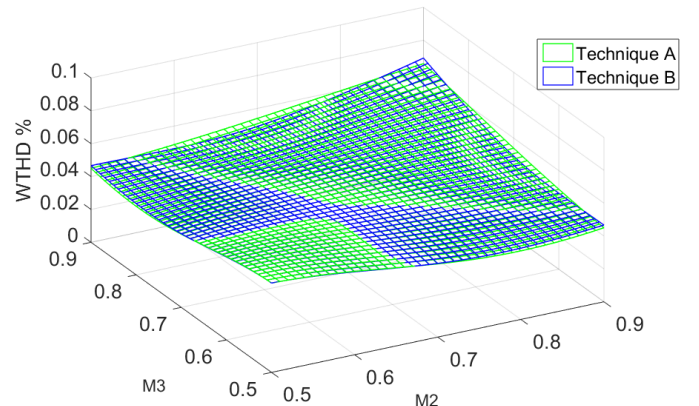


Fig. 6. WTHD trend as a function of  $M_2$  and  $M_3$  for  $V^{dc} = 100$  V e  $M_1=0.5$  for for techniques A and B.

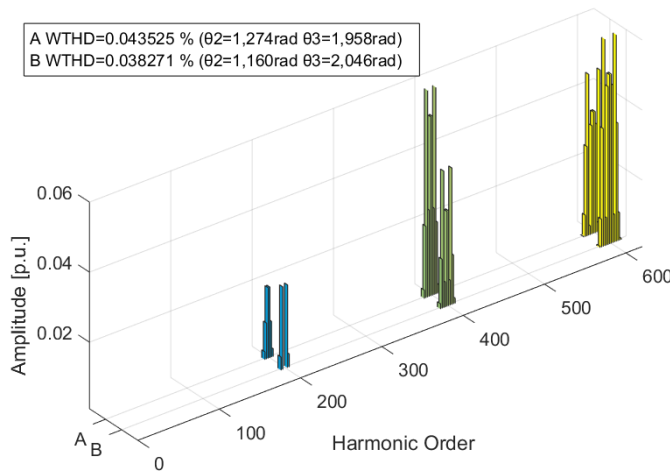


Fig. 7. Normalized theoretical harmonic spectra for techniques A and B in case of unequal DC voltages and unequal modulating signals.

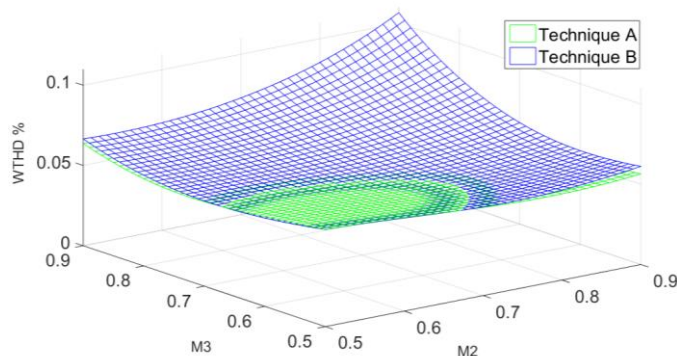


Fig. 8. WTHD trend as a function of  $M_2$  and  $M_3$  for  $V_1^{dc}=100$  V,  $V_2^{dc}=80$  V,  $V_3^{dc}=60$  V and  $M_1=0.5$  for techniques A and B.

#### D. Effects of the DC voltage second harmonic under unbalanced conditions.

The influence on the three considered techniques of a second harmonic (i.e. 100 Hz) in the DC voltage has been

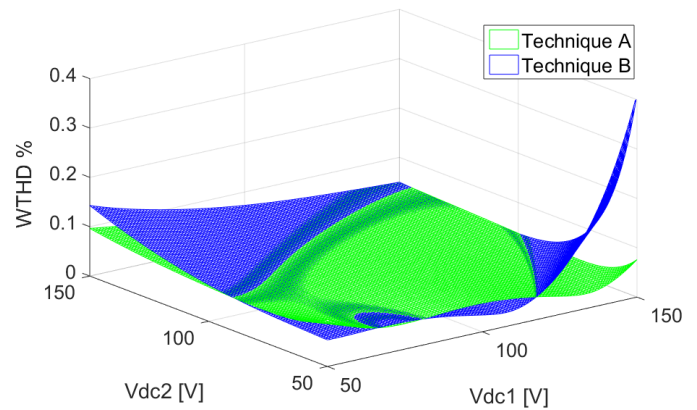


Fig. 9. WTHD trend as a function of  $V_1^{dc}$  and  $V_2^{dc}$  for  $V_3^{dc}=100$  V,  $M_1=0.5$ ,  $M_2=0.7$  and  $M_3=0.9$  for techniques A and B.

investigated by simulations in case of unequal DC voltages and equal modulating signals (case 1), in case of equal DC voltages and unequal modulating signals (case 2) and in case of unequal DC voltages and unequal modulating signals (case 3).

A ripple component of  $5 \cdot \sin(2 \cdot \pi \cdot 100 \cdot t)$  [V] has been considered. As it can be seen in the frequency spectra (Fig. 10 to Fig. 12) in case a 2nd harmonic is present in the DC link, the output voltage has a 3rd harmonic (i.e. 150 Hz). For case 1 (Fig. 10), all techniques still manage to compensate the 1st harmonic group in case of a 100 Hz ripple in the DC voltage but their performance is diminished and the 1st group still appears.

For case 2 (Fig. 11), the technique A already loses the ability with the pure DC voltage. The main effect of the technique B is to cancel out the harmonics  $2m_j \pm 1$  (the components indicated with arrow). However, the components are not completely eliminated when a 2nd harmonic is present in the DC voltage. On the other hand, the technique C shows a similar performance also in presence of a 2nd harmonic.

For case 3 (Fig. 12), the technique A and B show a behavior similar to the previous case 2. The performance of technique C decreases due to the 2nd harmonics in the DC voltage, showing harmonics belonging to the first group.

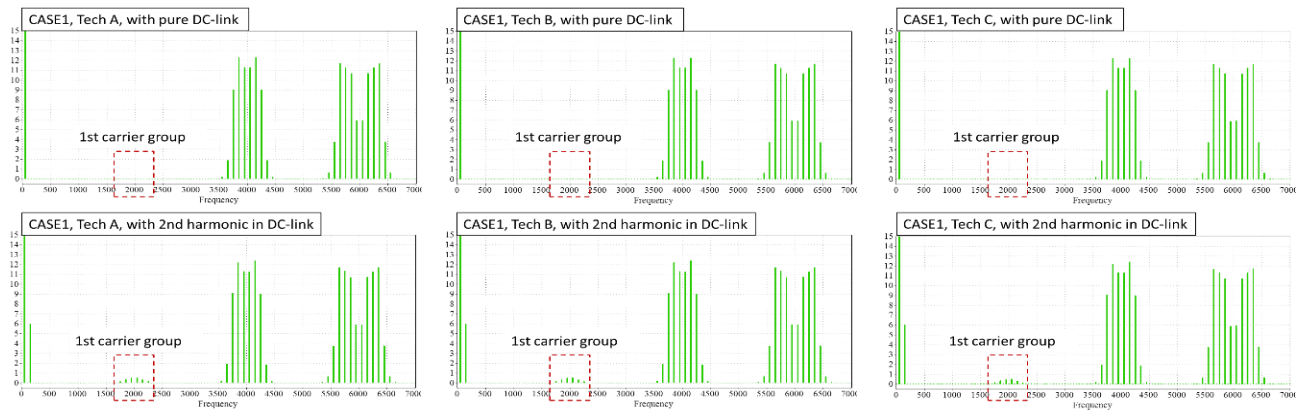


Fig. 10. Frequency spectra of inverter output voltage produced by the three considered techniques in case of unequal DC voltages and equal modulating signals (case 1) without second harmonic in the DC link (up) and with second harmonic in the DC link (down).

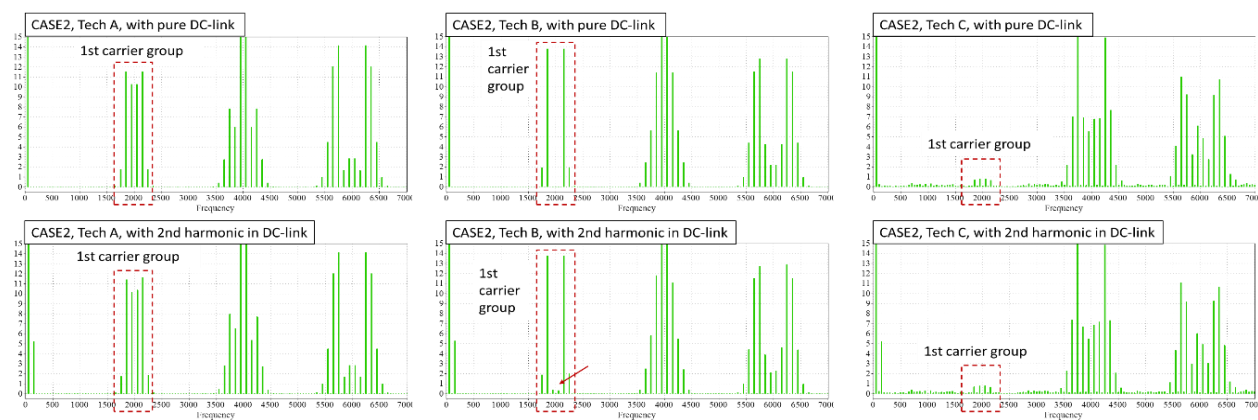


Fig. 11. Frequency spectra of inverter output voltage produced by the three considered techniques in case of equal DC voltages and unequal modulating signals (case 2) without second harmonic in the DC link (up) and with second harmonic in the DC link (down).

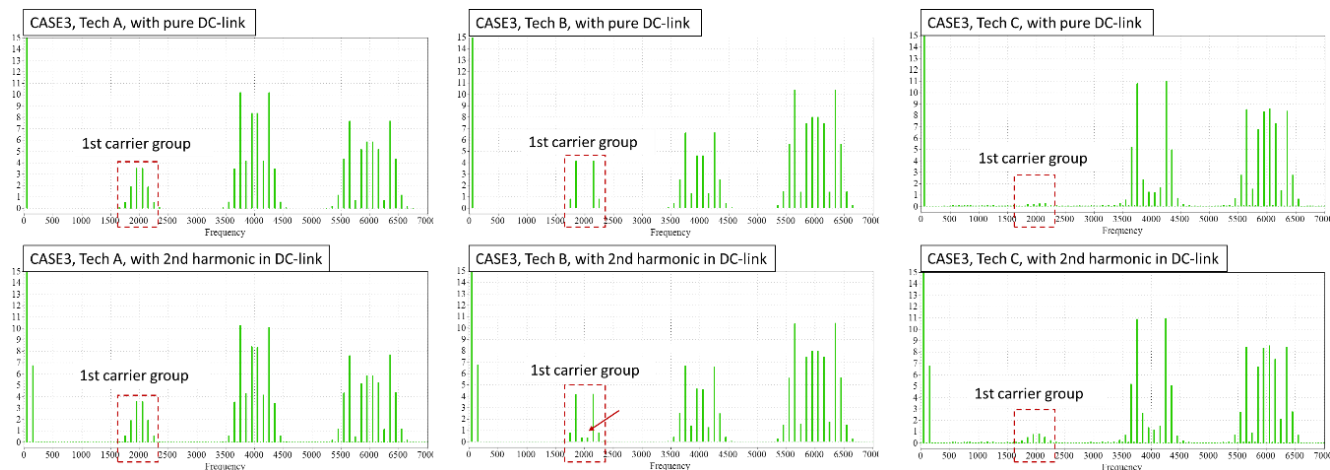


Fig. 12. Frequency spectra of inverter output voltage produced by the three considered techniques in case of unequal DC voltages and unequal modulating signals (case 3) without second harmonic in the DC link (up) and with second harmonic in the DC link (down).

#### IV. EXPERIMENTAL RESULTS

In the following, the three considered PSC-PWM techniques are tested on a developed prototype shown in Fig. 13, considering unbalanced operating conditions. The developed prototype is composed of the seven-level CHB converter with discrete IGBTs (IXYB82N120C3H1), a micro-processor (MPC5643L) to implement the techniques and to generate the gating signals, and three isolated DC sources to emulate the unequal DC voltage cases considered in the analysis section. The experimental test conditions are identical to those considered for simulations.

##### A. Unequal DC voltages and equal modulating signals

Under these operating conditions, the symmetrical PSC-PWM presents an increased WTHD value of 0.0569%. On the other hand, by means of all three PSC-PWM techniques (Fig. 14), the first carrier group is cancelled out while the second carrier group is increasing. Hence, these techniques achieve the reduced WTHD of 0.0447%. All the three techniques allow a WTHD improvement of 21.44% compared to the symmetrical PSC-PWM technique. It should be noted that the carrier shifting angles calculated by each technique are same as  $\theta_2=1.249$  rad and  $\theta_3=1.107$  rad since three techniques are

equivalent under the unequal DC voltages and equal modulating signals as mentioned in section III-A.

##### B. Equal DC voltages and unequal modulating signals

Under these operating conditions, the symmetrical PSC-PWM presents an increased WTHD value of 0.0668%. The effect of each technique is identified and these are shown in Fig. 15 to Fig. 17, respectively.

The technique A does not influence on the carrier angle and the WTHD at all under this condition as shown in Fig. 15. In fact, its WTHD value is identical with that of the symmetrical PSC-PWM.

The technique B makes the main harmonics in the first carrier group to diminish in part. The decreased main harmonics are  $2m_f \pm 1$ , where  $m_f$  is the frequency modulation index (5 kHz/50 Hz=100). Hence, the WTHD decreases to 0.0613% and it is 8.23% lower than that of symmetrical PSC-PWM.

However, the WTHD of the technique C is the best one since the harmonics of  $2m_f \pm 1$  are remarkably decreasing. Therefore, considering the equal DC voltages and the unequal modulating signals conditions, the technique C achieves the best performance since its WTHD is 43.41% lower than that of symmetrical PSC-PWM.



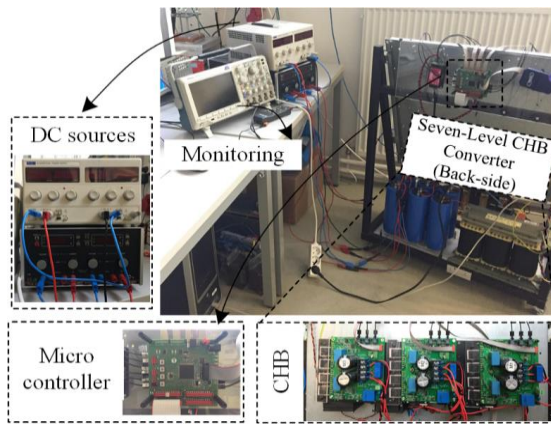


Fig. 13. Experimental setup.

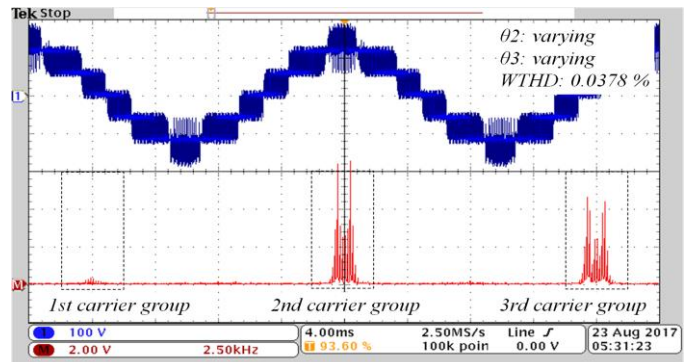


Fig. 17. Experimental converter voltage and its harmonic spectrum with PSC-PWM technique C in case of equal DC voltages and unequal modulating signals.

### C. Unequal DC voltages and unequal modulating signals

Under these operating conditions, the WTHD is 0.0479 % with the symmetrical PSC-PWM. Technique A improves the WTHD value to 0.044 % since harmonics of the first carrier group are attenuated (Fig. 18). Fig. 19 shows the waveforms with the technique B that reduces effectively the harmonics of  $2m_f \pm 1$ , resulting in the improved performance of 0.0406 %. The technique C allows the harmonics of  $2m_f \pm 1$  to diminish without increasing the  $2m_f \pm 3$  like the previous case and the WTHD is 0.0384 % (Fig. 20). All techniques can improve the WTHD performance compared to the symmetrical PSC-PWM technique. For the technique A the WTHD is improved by 8.14 %, while for the technique B and C it is improved by 15.24 % and 19.83 %, respectively.

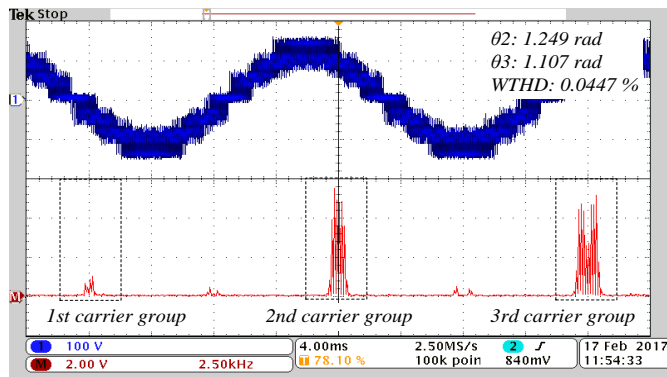


Fig. 14. Experimental converter voltage and its harmonic spectrum with PSC-PWM technique A, B, C in case of unequal DC voltages and equal modulating signals.

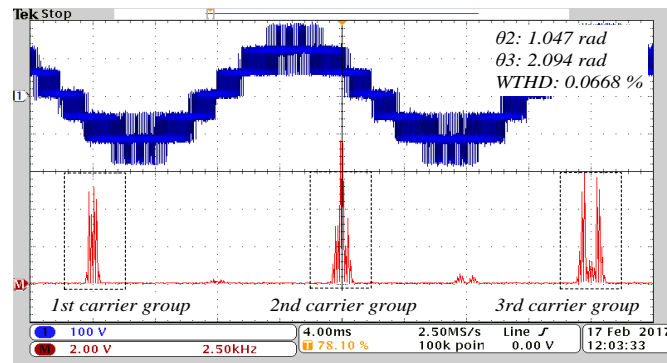


Fig. 15. Experimental converter voltage and its harmonic spectrum with PSC-PWM technique A in case of equal DC voltages and unequal modulating signals.

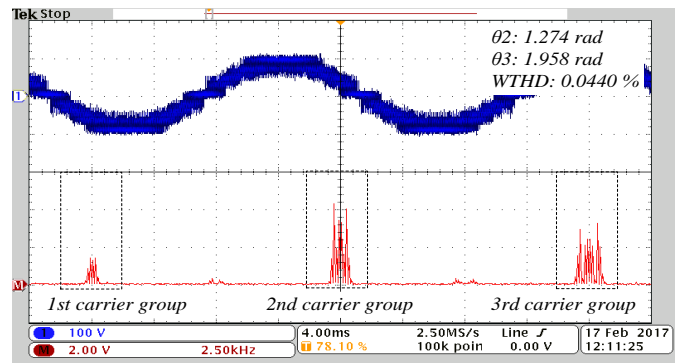


Fig. 18. Experimental converter voltage and its harmonic spectrum with PSC-PWM technique A in case of unequal DC voltages and unequal modulating signals.

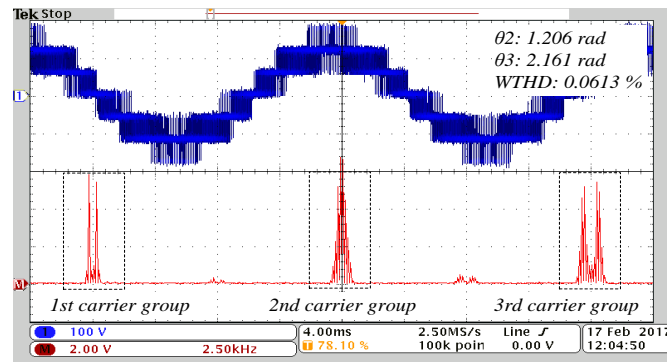


Fig. 16. Experimental converter voltage and its harmonic spectrum with PSC-PWM technique B in case of equal DC voltages and unequal modulating signals.

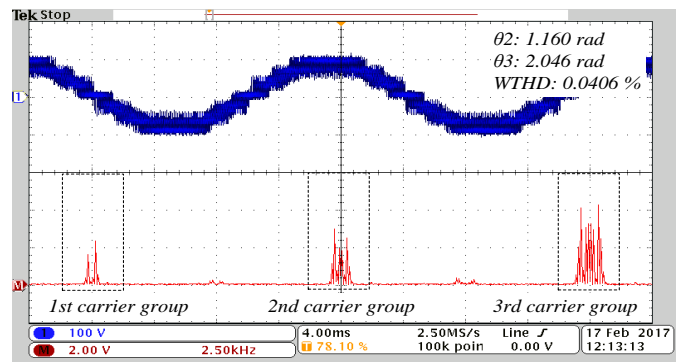


Fig. 19. Experimental converter voltage and its harmonic spectrum with PSC-PWM technique B in case of unequal DC voltages and unequal modulating signals.

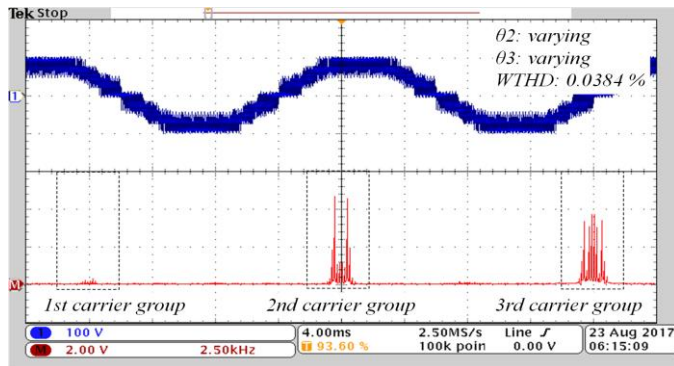


Fig. 20. Experimental converter voltage and its harmonic spectrum with PSC-PWM technique C in case of unequal DC voltages and unequal modulating signals.

## V. CONCLUSION

In this paper three different variations to the conventional symmetrical Carrier Phase-Shifting PWM technique, to be used in case of unbalanced operational conditions, have been compared and their performances have been analyzed. To uniquely assess their performances, the techniques have been tested in the same unbalanced conditions and their spectra as well as the WTHD have been evaluated. Moreover, experimental tests have been carried out on a seven-level CHB converter and the three techniques have been verified in terms of the WTHD for the different unbalanced conditions. Experimental results show a good agreement with the outcomes of the theoretical analysis. In particular, the effect of all the techniques is equal in case of unequal DC voltages and equal modulating signals, whereas the minimum WTHD is achieved by the technique C under equal/unequal DC voltages and unequal modulating signals. In conclusion, this paper provides an effective mean to select the most suitable Carrier Phase-Shifting PWM technique to be profitably used under different unbalanced conditions.

## APPENDIX

Considering unequal DC voltages and equal modulating signals (i.e.  $M_1=M_2=M_3=M$ ), the technique B is equivalent to the technique A since:

$$\begin{aligned} \cos \phi_2 &= \frac{V_3^2 - V_1^2 - V_2^2}{2V_1V_2} = \\ &= \frac{\left(-\frac{2V_{dc,3}}{\pi} J_1(\pi M_3)\right)^2 - \left(-\frac{2V_{dc,1}}{\pi} J_1(\pi M_1)\right)^2 - \left(-\frac{2V_{dc,2}}{\pi} J_1(\pi M_2)\right)^2}{2\left(-\frac{2V_{dc,1}}{\pi} J_1(\pi M_1)\right)\left(-\frac{2V_{dc,2}}{\pi} J_1(\pi M_2)\right)} = \\ &= \frac{\frac{4}{\pi^2} J_1^2(\pi M) (V_{dc,3}^2 - V_{dc,1}^2 - V_{dc,2}^2)}{2\frac{4}{\pi^2} J_1^2(\pi M) V_{dc,1} V_{dc,2}} = \frac{V_{dc,3}^2 - V_{dc,1}^2 - V_{dc,2}^2}{2V_{dc,1} V_{dc,2}} \end{aligned}$$

Similarly:

$$\cos \phi_3 = \frac{V_2^2 - V_1^2 - V_3^2}{2V_1V_3} = \frac{V_{dc,2}^2 - V_{dc,1}^2 - V_{dc,3}^2}{2V_{dc,1} V_{dc,3}}.$$

Considering unequal DC voltages and equal modulating signals (i.e.  $M_1=M_2=M_3=M$  and hence  $D_1=D_2=D_3=D$ ), the technique C is equivalent to the technique A since:

$$\begin{aligned} \cos \phi_2 &= \frac{h_{13}^2 - h_{11}^2 - h_{12}^2}{2h_{11}h_{12}} = \\ &= \frac{\left(\frac{2V_{dc,3}}{\pi} \sin(\pi D_3)\right)^2 - \left(\frac{2V_{dc,1}}{\pi} \sin(\pi D_1)\right)^2 - \left(\frac{2V_{dc,2}}{\pi} \sin(\pi D_2)\right)^2}{2\left(\frac{2V_{dc,1}}{\pi} \sin(\pi D_1)\right)\left(\frac{2V_{dc,2}}{\pi} \sin(\pi D_2)\right)} = \\ &= \frac{\frac{4}{\pi^2} \sin^2(\pi D) (V_{dc,3}^2 - V_{dc,1}^2 - V_{dc,2}^2)}{2\frac{4}{\pi^2} \sin^2(\pi D) V_{dc,1} V_{dc,2}} = \frac{V_{dc,3}^2 - V_{dc,1}^2 - V_{dc,2}^2}{2V_{dc,1} V_{dc,2}} \end{aligned}$$

Similarly:

$$\cos \phi_3 = \frac{h_{12}^2 - h_{11}^2 - h_{13}^2}{2h_{11}h_{13}} = \frac{V_{dc,2}^2 - V_{dc,1}^2 - V_{dc,3}^2}{2V_{dc,1} V_{dc,3}}$$

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Vito Giuseppe Monopoli (S'98–M'05) received the M.Sc. and Ph.D. degrees in electrical engineering from the Bari Polytechnic, in 2000 and 2004, respectively. He is currently an Assistant Professor Bari Polytechnic, Bari, Italy. His research activity concerns multilevel converters and the analysis of harmonic distortion produced by power converters and electrical drives. He is particularly interested in innovative control techniques for power converters. He is member of the IEEE Industry Applications Society, IEEE Industrial Electronics Society and IEEE Power Electronics Society.



Youngjong Ko (S'16) received his B.Sc and M.Sc degrees in Electronic Engineering from the Ajou University, Suwon, South Korea in 2009 and 2012, respectively. Since 2015 he is working toward his Ph.D degree from the chair of power electronics at the University of Kiel, Germany. His research interests include grid-connected power converter and reliability in power electronics.



Giampaolo Buticchi (S'10–M'13–SM'17) received the Masters degree in Electronic Engineering in 2009 and the Ph.D degree in Information Technologies in 2013 from the University of Parma, Italy. In 2012 he was visiting researcher at The University of Nottingham, UK. Between 2014 and 2017 he was a post-doctoral researcher at the University of Kiel, Germany. He is now Associate Professor in Electrical Engineering at The University of Nottingham Ningbo China (UNNC). His research area is focused on power electronics for renewable energy systems, smart transformer fed micro-grids and dc grids for the More Electric Aircraft. He is author/co-author of more than 120 scientific papers.



Marco Liserre (S'00–M'02–SM'07–F'13) received the M.Sc. and Ph.D. degrees in electrical engineering from the Bari Polytechnic, in 1998 and 2002, respectively. He is author of more than 300 technical papers, which received more than 20 000 citations. He is a Full Professor and the Head of the Chair of Power Electronics, University of Kiel, Kiel, Germany. He has published more than 200 technical papers (1/3 in international peer-reviewed journals) and a book at second reprint and also translated in Chinese. These works have received more than 15 000 citations, for this reason he is listed in ISI Thomson report "The world's most influential scientific minds" from 2014. Prof. Liserre has received the European ERC Consolidator Grant, one of the most prestigious in Europe. He is member of the IEEE Industrial Applications Society, IEEE Power Electronics Society, IEEE Power & Energy Society, and IEEE Industrial Electronics Society (IES). He did serve all these societies in various capacities such as a Reviewer, an Associate Editor, an Editor, a Conference Chairman, or a Track Chairman. He has been a Founding Editor-in-Chief of the IEEE INDUSTRIAL ELECTRONICS MAGAZINE, a Founding Chairman of the Technical Committee on Renewable Energy Systems, and IES Vice-President responsible of the publications. He has received several IEEE Awards.