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Final Dissertation

A Safety-Optimal Control Method for Virtual Coupling Train Control

by
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A Safety-Optimal Control Method for Virtual Coupling Train Control

Thesis submitted for the degree of Philosophiae Doctor

Italian National Ph.D. Program in Autonomous Systems
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Abstract

Virtual Coupling (VC) is an emerging railway signaling concept aimed at increasing line capacity and reducing train headway through Train-to-Train (T2T) communication. Instead of relying on traditional mechanical couplers, trains under VC operate at similar speeds and close spacing by exchanging real-time information via wireless networks. Although VC promises substantial improvements in efficiency and flexibility, its practical implementation still faces challenges related to safety assurance, control stability, and communication reliability.

This thesis focuses on the longitudinal motion control of VC convoys, addressing the key problem of safety spacing control during operation, under the presence of hazards including communication delays and dynamic uncertainties.

A decentralized control framework based on Control Barrier Functions (CBFs) is developed to ensure safe and efficient spacing among trains. The proposed approach integrates safety constraints and reference control into a series of real-time Quadratic Programs (QPs), guaranteeing collision avoidance while maintaining smooth and stable operation. Simulation studies demonstrate that the controller achieves excellent performance in terms of safety, efficiency, and computational scalability, showing its potential for real-time implementation in real VC applications.

莫听穿林打叶声，
何妨吟啸且徐行。

Acronyms

ABD	Absolute Braking Distance
ATO	Automatic Train Operation
ATS	Automatic Train Supervision
ATP	Automatic Train Protection
CBF	Control Barrier Function
CBTC	Communications-Based Train Control
CLF	Control Lyapunov Function
EB	Emergency Braking
EoA	End of Movement Authority
ETCS	European Train Control System
FB	Fixed Block
HOCBF	High Order Control Barrier Function
MA	Movement Authority
MB	Moving Block
QP	Quadratic Programming
RBC	Radio Block Center
RBD	Relative Braking Distance
RCBF	Relaxed Control Barrier Function
SB	Service Braking
T2G	Train-to-Ground Communication
T2T	Train-to-Train Communication
VC	Virtual Coupling

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Preface

This thesis is submitted in partial fulfillment of the requirements for the degree of *Philosophiae Doctor* in Autonomous Systems

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- [1] **Yike Li**, Yin Tong, Alessandro Giua. *Train Convoy Safe Optimal Control with High Order Control Barrier Functions*. IEEE ITSC 2025: 28th International Conference on Intelligent Transportation Systems. (To appear)
- [2] **Yike Li**, Yin Tong, Alessandro Giua. *Train Formation Safe Optimal Control with Control Barrier Functions*. IEEE CASE 2024: 20th International Conference on Automation Science and Engineering. DOI: [10.1109/CASE59546.2024.10711420](https://doi.org/10.1109/CASE59546.2024.10711420)
- [3] **Yike Li**, Yin Tong, Marco Demuro, Alessandro Giua. *Safety Test Case Generation Using Petri Nets*. PN4TT 2023: International Workshop on Petri Nets for Twin Transitions. [\[Online\]](#)
- [4] **Yike Li**, Yin Tong, Alessandro Giua. *Detection and Prevention of Cyber-Attacks in Networked Control Systems*. WODES 2020: IFAC Workshop on Discrete Event Systems. Published in IFAC-PapersOnLine, DOI: [10.1016/j.ifacol.2021.04.001](https://doi.org/10.1016/j.ifacol.2021.04.001)

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Chapter 1

Introduction

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1.1 Background and Concept of Virtual Coupling

Railways are widely recognized as an eco-friendly and energy-efficient mode of transportation. With many countries prioritizing sustainable mobility and carbon reduction, along with increasing travel demand and environmental awareness among the public, the overall demand for railway **capacity** has risen significantly across all segments, including freight, metro, intercity, and high-speed rail. Passenger transport demand is expected to increase by about 30% by 2050 compared with the year 2000 [1], which is required to be accommodated by the railways in the next 20 years. On the other hand, railway freight demand is forecast to grow by around 50% by 2050 relative to 2010 [2]. In addition, the European Commission has set a strategic planning for the railway sector to reduce greenhouse gas emissions by 60% by 2050 compared with 1990, and to promote a substantial modal shift of both passenger and freight transport from road, air, and water to rail [2]. However, constructing new lines or expanding existing infrastructure requires substantial investments of time, money, and land. As a result, reducing train headways to increase line capacity has become a more cost-effective and environmentally sustainable alternative.

In parallel, there is a growing demand for **flexibility**. Conventional fixed-length train formations cannot accommodate uneven demand, leading

to inefficiencies. For example, empty carriages during off-peak hours and overcrowded ones during peak periods are common on metro and intercity lines [3]. Reducing the number of trains during low-demand periods can mitigate the waste caused by empty carriages, but it also increases passenger waiting times, thereby affecting service quality.

Existing approaches to flexible formation

Traditionally, *fixed formation* is adopted, where trains are coupled as fixed-length sets throughout operation. While easy to manage, it lacks the ability to adjust capacity according to time-varying passenger demand, resulting in inefficiencies during off-peak hours and congestion during peaks.

Flexible formation has therefore been developed and employed as an upgrade solution on existing lines to enhance operational capacity and flexibility without major infrastructure expansion. It allows train length and service capacity to be adjusted in real time, matching fluctuating passenger demand throughout the day. Existing solutions of flexible formation can generally be categorized into two types: *mixed coupling* and *dynamic coupling*.

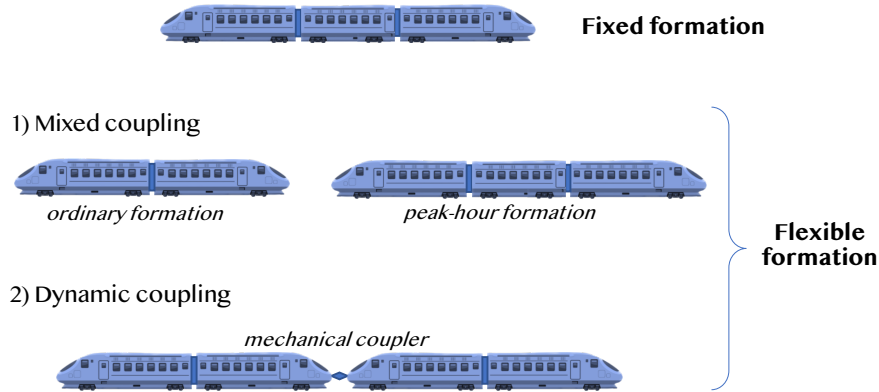


Figure 1.1: Comparison among fixed coupling and two types of flexible formation: mixed coupling and dynamic coupling.

1) Mixed Coupling. In mixed coupling, various train units can be coupled or uncoupled to operate jointly on the same line [4]. Motivated by service-oriented planning, trainsets of different lengths are scheduled to adapt to time-varying passenger flows. For example, the Beijing Metro Line 11 adopts mixed operations of 3-car and 6-car trainsets, while the Amsterdam Metro allows flexible train compositions on Lines 1–4, enabling up to thirteen formation combinations. Similarly, the Vancouver SkyTrain operates with 2-, 4-, or 6-car trainsets [5], providing flexibility in adjusting train length

according to passenger demand. However, such systems require designated locations (e.g., station platforms or shunting yards) for coupling and decoupling, which imposes stringent accuracy and robustness requirements on timetable design. In addition, these systems are prone to delays due to the increased complexity in coordination and scheduling.

2) Dynamic Coupling. Dynamic coupling refers to the process in which two adjacent trains couple on the move without stopping [6]. The system controls the following train to approach and connect with the leading train at low speed, automatically forming a new trainset that continues its journey under a new operational task. Practical implementation has been demonstrated on the Shanghai Metro Line 16, where two trains successfully coupled dynamically within a short time without compromising operational efficiency [7]. This approach significantly enhances operational flexibility, but it also requires extremely high precision in train motion control to ensure safe physical contact between trains. Moreover, the use of mechanical couplers poses challenges in accommodating heterogeneous rolling stock.

To sum up, although existing flexible formation strategies such as mixed coupling and dynamic coupling have been successfully implemented to improve operational efficiency, they still rely on mechanical couplers and designated locations to complete coupling and decoupling. Consequently, despite their flexibility, these systems remain constrained in several aspects, including scalability, control precision, and operational robustness. These limitations motivate the development of *Virtual Coupling* (VC), a non-mechanical, communication-based alternative that enables higher capacity and greater flexibility in railway operations.

One promising solution: Virtual Coupling

Compared with the aforementioned flexible formation solutions, Virtual Coupling overcomes their limitations by replacing physical couplers with continuous communication and cooperative control, allowing trains to form and dissolve convoys dynamically even outside stations. Within a VC convoy, trains with their neighbors through wireless communication, namely *Train-to-Train* (T2T) communication. By exchanging real-time information including speed and position via T2T communication, they can safely follow each other at very small distances. This enables autonomous on-the-run formation of longer convoys virtually at any location along the line. By eliminating mechanical couplers, VC also avoid several drawbacks associated with traditional physical coupling, such as incompatibility between heterogeneous rolling stock and stability issues caused by large in-train buff forces when

passing curves [8]. Furthermore, in emergency situations such as derailments, mechanically coupled trains may experience severe consequences, whereas in a virtually coupled convoy, the following trains can potentially stop in time and avoid collision [9].

The primary concept of VC was first introduced by Bock and Varchmin in 1999 [10], who envisioned a train operation and design concept in which carriages are no longer mechanically connected but instead maintain small safety gaps while each possesses its own traction and control system [11]. However, their work did not provide detailed implementation methods. Stander *et al.* [12] later proposed replacing traditional couplers with wireless communication links and designed procedures for coupling and decoupling. Building on this idea, Henke *et al.* [13], within the framework of the *RailCab* project, developed control strategies for simplified coupling and decoupling operations, though key aspects such as train dynamics, infrastructure, and environmental factors were not fully considered. Due to the technological limitations at that time, particularly in communication, research on VC remained generally inactive until around 2015. With the growing level of railway automation and advances in wireless communication technologies, the implementation of VC has become feasible. Driven by the railway sector's urgent need to improve the utilization of both line and rolling-stock resources, VC has regained wide attention from both industry and academia.

A major turning point came with the launch of the *Shift2Rail* program in 2015 [14], funded by the European Commission. In its technical roadmap, VC was officially identified as a promising solution for increasing line capacity and enabling flexible operation through the formation of multiple train convoys. This recognition stimulated a series of follow-up projects and pilot studies. The *MOVINGRAIL* [15] and *X2Rail* [16] projects, funded under Shift2Rail, investigated migration strategies from the Moving Block system toward Virtual Coupling and analyzed VC's market potential, capacity benefits, and impacts on different railway segments in terms of cost, performance, and feasibility. Meanwhile, the *Safe4RAIL* project [17], initiated in 2016, aimed to develop the next-generation networked Train Control and Monitoring System (TCMS) with Ethernet-based architectures, while the *CONNECTA* project [18] focused on advancing T2T communication technologies, which are essential for reliable VC operation. The outcomes of these projects paved the way for the practical realization of VC. More recently, the *R2DATO* project [19], led by the German Aerospace Center (DLR), developed a novel radio system for VC applications and successfully validated it through field tests [20].

In addition to these initiatives, the potential of VC has been extensively investigated across different railway sectors. Case studies have been

conducted for high-speed lines [21], [22], [23], metro systems [24], [25], freight transportation [26], and main lines [27], as well as for emerging ultra-high-speed systems such as Maglev [28] and Hyperloop [29].

The extensive projects and research activities have demonstrated the enormous potential of VC. It is no longer a purely theoretical concept but a technically feasible and increasingly inevitable direction for the future development of railway systems. At the same time, however, numerous challenges remain before realistic implementation, particularly regarding safety control for VC convoys, which constitute the research focus of this thesis.

Evolution of railway signaling

The evolution of railway signaling can be viewed as a shift from *signaling-based systems* to *train-centered systems* [30]. Representative examples of the two are the *Fixed Block* (FB) signaling and the *Moving Block* (MB) signaling, respectively. Figure 1.2 compares FB, MB, and VC through the simple case of two trains running in the same direction.

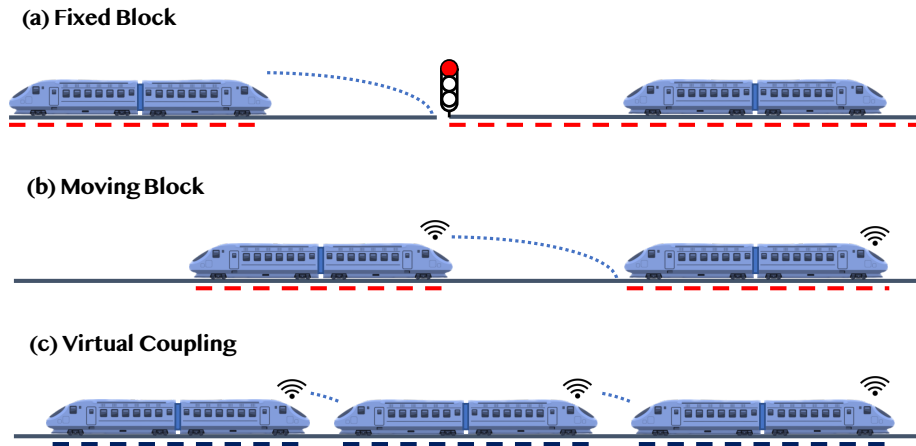


Figure 1.2: Illustrations of train separation schemes in the Fixed Blocking, Moving Block, and Virtual Coupling.

In the traditional Fixed Block system, the track is divided into consecutive sections of fixed length by trackside signaling equipment, and each block can only be occupied by one train at a time. A signal located at the entrance to each block communicates block occupancy to the driver. When determining block length, various factors must be considered, including the safe stopping distance of trains, line conditions, signal visibility, and driver reaction time. This ensures safety but results in large unused sections of track, thereby limiting line capacity.

The Moving Block moves trackside signaling to onboard, eliminating these rigid block sections and permits trains to run closer together [31]. In MB operation, the minimum separation between two consecutive trains is defined as the *absolute braking distance* (ABD) of the following train, plus a safety margin. Enabled by continuous train localization and *Train-to-Ground* (T2G) communication, MB allows more efficient utilization of track capacity [32].

Virtual Coupling extends this idea further. The safe separation is no longer based on ABD but on the *relative braking distance* (RBD), i.e., the distance required for a following train to stop without colliding if both trains perform emergency braking simultaneously. This leads to headway significantly shorter than in MB, enabling higher line capacity and greater operational flexibility [33].

In metro and suburban railway networks, the Moving Block principle has been implemented in the *Communications-Based Train Control* (CBTC) system. Relying on continuous T2G communication, CBTC supports MB operation and significantly enhances both capacity and safety [34]. It adopts a layered architecture comprising *Automatic Train Protection* (ATP), *Automatic Train Operation* (ATO), and *Automatic Train Supervision* (ATS), which together ensure safe, reliable, and energy-efficient operation. ATP and ATO constitute the core of onboard control unit, namely the onboard system. ATS is the centric train management system that supervises and manages the trains within a large geographical area. Cooperating with the trackside *Zone Controller* (ZC) that located distributively in smaller area, the onboard system receives the latest dispatching and routing information from ATS. In addition, ZC also computes and send the *movement authority* (MA) to the train. MA defines the front segment of the track that a train may safely occupy, up to the *end of movement authority* (EoA), where the target speed is zero. Under MB, the EoA is computed based on the rear position of the train ahead.

For mainline and high-speed railways, the *European Train Control System* (ETCS) Level 3 contains the specifications to support MB operation, enabled by continuous localization, train integrity monitoring, and integration with ATO. In ETCS Level 2 and Level 3, MA is dynamically computed by the *Radio Block Center* (RBC) and sent to each train in a centralized manner. The RBC communicates with trains via radio, receiving their real-time positions and issuing MA according the schedule.

Although CBTC and ETCS differ in architecture, both rely on the Moving Block principle and ATO-based operation for capacity enhancement. Virtual Coupling is envisioned as an upgrade to these systems, extending their functionality beyond the absolute braking distance concept.

1.2 Research motivation and scope

VC has gained increasing attention as a next-generation railway signaling solution. Its implementation, however, poses critical challenges especially the V2V communication technologies and a cooperative automatic train operation system.

Among various research topics on VC, one key issue is the longitudinal motion control of VC convoys. The objective is to determine a control law that provides the desired force input for the trains, so that each train can maintain the required spacing from its preceding train. Two main challenges have been identified: one arises from the time-varying maneuvers of the leader train while following the assigned route, and the other stems from interferences, such as communication delays and dynamic uncertainties.

The former challenge is necessary, as the coupled convoy must undergo acceleration and braking phases rather than remaining coupled only during cruising, in order to accomplish operational tasks such as entering stations and stopping precisely at platforms. The latter is inevitable and has already been addressed comprehensively in pre-VC train control studies, where communication delays and dynamic uncertainties have been recognized as crucial issues that require special attention. Moreover, the coexistence of these two challenges further complicates the control design, making safe and efficient control more difficult to achieve.

In order to solve the safety spacing control problem in VC, the overall research scope of this thesis covers three main aspects:

1. **Modeling and formulation:** establishing a unified framework for convoy train control, including system topology, operating scenarios, train dynamics, and the spacing policy;
2. **Controller design:** developing a decentralized safety controller that integrates safety enforcement with robustness against potential hazards, enabling real-time implementation;
3. **Validation and analysis:** evaluating the proposed methods through simulation studies to demonstrate safety preservation, operational efficiency, and computational feasibility.

1.3 Literature Review

This section, we review the existing studies on VC train control and the applied techniques, which nevertheless has a significant impact on our research.

1.3.1 Arising control problems

Existing studies on VC can be broadly categorized into three main research focuses: safety control, stability maintenance, and cooperative coordination. Note that this classification is not mutually exclusive, as stability and coordination strategies often include safety objectives as well. Here, the classification is made according to the primary focus of each study, for the sake of clarity, although many works pursue multiple objectives across these aspects.

1) Cooperative control: distributed coordination and convoy management. VC convoys must dynamically adjust their routes and perform merging or separation operations in a cooperative manner. Such coordination may require higher-level decision-making algorithms, such as the hierarchical control framework proposed in [25]. Beyond pairwise train interactions, cooperative control focuses on network-level convoy management, including dispatching and schedule coordination. These processes mainly involve train-to-infrastructure interactions (for individual trains before coupling) and convoy-to-infrastructure interactions (for coupled trains), particularly in route planning and journey management across both spatial and temporal domains. In this context, operational efficiency is often the primary concern of cooperative control. A typical example is convoy formation, which concerns how two trains can be coordinated to couple efficiently. Wang *et al.* [35] formulated this problem as a cooperative game to enable rapid coupling at junctions. Robust cooperative control strategies against communication topology attacks were investigated in [36], [37], where topology reconfiguration methods were designed to maintain information flow integrity. Wang *et al.* [38] developed a real-time trajectory planning algorithm to coordinate VC train movements through mixed-integer optimization, capable of detecting feasible coupling sections and estimating delay propagation. Dong *et al.* [39] further proposed an event-triggered cooperative control method to handle actuator faults during formation. Overall, the coupling and decoupling commands in VC must be managed analytically and quantitatively, especially under external disturbances such as delays, additional services, or service interruptions, making cooperative control an essential framework for system-level safety and efficiency.

2) Stability control: maintaining consistent spacing and velocity. Another critical aspect of VC control is the stability of the convoy, particularly under uncertainties such as track gradients, parameter variations, and communication noise. The objective is to maintain uniform speed and spacing with minimal deviation. Most stability control studies focus on

the cruise phase of convoy operation, where local and string stability are analyzed [40]. Distributed control laws have also been developed, where each train adjusts its motion in response to local measurements and real-time information exchange with neighboring trains. These approaches aim to suppress disturbance propagation along the convoy, yet maintaining global stability remains challenging when external uncertainties are significant.

3) Safety control: collision avoidance under shortened headway.

One of the foremost challenges in VC implementation is to ensure safety under significantly reduced headway. Unlike conventional systems, where large separations inherently guarantee collision protection, VC requires continuous control to maintain a safe distance between trains in real time. Existing research focuses on two aspects: defining the safety margin and implementing control laws to enforce it. Quaglietta *et al.* [27] proposed a dynamic safety spacing model incorporating multiple risk factors. Some studies achieved the safety by integrating the safety constraint into the optimization problem.

1.3.2 Additional challenges

Dealing with communication delays. The execution of VC heavily relies on continuous and reliable T2T communication. Ideally, each following train periodically receives real-time kinematic information from its preceding train and combines it with its own dynamic model to compute and apply the control input. In practice, however, occasional communication delays are inevitable, especially when trains operate in tunnels or are separated by long distances.

Such delays may render the received data from the preceding train outdated, which can reduce tracking accuracy and, if not properly compensated, compromise safety. Typically, T2T communication is jointly supervised by both trains through a transmission interface module equipped with an embedded safety protocol. A permissible range is defined for the maximum allowable communication delay. If a packet arrives with a delay exceeding this limit, it is discarded. A communication failure is assumed to occur when such excessive delays persist for a certain number of consecutive packets, violating the communication requirements of VC and leading to train degradation [41].

To mitigate performance degradation and improve the system's tolerance to communication delay, the delay effect must be explicitly addressed, with particular attention to its impact on system safety and control performance. Recent studies that considered communication delays in VC train control are summarized in Table 1.1.

In [42], constant communication delays of up to 10 s were introduced between the control center and the leader train in simulation. Although safety was maintained under these delays, the inter-train distances increased significantly, reducing line capacity. Liu *et al.* [43] analyzed string stability under fixed delays. In [44], a two-train VC convoy was investigated through a Particle Swarm Optimization-based online optimization method, where the delay was randomly varied between 0.2 s and 0.5 s. The results showed that both distance and velocity errors were effectively reduced compared to the worst case of a 0.5 s delay, although the distribution of random delays was not specified. Di Meo *et al.* [33] modeled the RBC as a virtual leader under the Predecessor–Leader–Follower (PLF) topology and employed Linear Matrix Inequalities to analyze the effects of stochastic delays along both T2T and T2G links on convoy stability. In [45], the VC convoy was studied under a multi-agent system framework, where a delay model was proposed that decomposes the packet processing time into sending, transmission, and receiving components. These studies typically assumed constant or bounded time-varying delays either during controller design or simulation, with reference to the average end-to-end delay in IEEE 802.11p vehicular networks, which is reported to be no greater than 0.4 ms [46].

Table 1.1: Summary of VC train control studies considering communication delays

Paper	Method	Considered Delay	Main Contribution
[43]	Tube MPC	Constant delay	Stability under delays was analyzed
[42]	Robust MPC	Permanent and punctual delay of constant values	Stability under delays was analyzed
[44]	PSO-based online optimization	Random delay	Reduced the impact of delays
[33]	Feedback control	Multiple delays with uniform distribution	Proved stability under delay
[45]	Observer-based control protocol	Bounded, time-varying communication delays	Guaranteed stability under delay

In this thesis, the proposed controller explicitly accounts for bounded communication delays within the control design framework.

Dealing with uncertainties. Regarding the uncertainty, Table 1.2 lists the representative approaches for VC robust train control in recent studies. Vaquero-Serrano and Felez [47] considered two sources of uncertainty: one added to the acceleration equation of the dynamic model, and the

other related to train positioning. Later, in [42], they further investigated uncertainties in the train dynamics, position and velocity measurements, as well as in the received information from the preceding train. Park et al. [48] proposed a robust gap controller

Zhang et al. [49] proposed a robust control method under nonlinear and time-varying uncertainties caused by external disturbances and parameter variations by solving Udwadia–Kalaba (UK) equations. An optimal design scheme for tunable parameter was proposed. These robust controllers were mostly verified by injecting sinusoidal signals as resources of disturbances. As in [49], [50], time-varying sinusoidal signals with different frequencies and phase shifts are injected into the system as external disturbances to evaluate the robustness of the proposed controllers. In this thesis, similar time-varying disturbances are also constructed and applied to assess the controller performance under dynamic and uncertain conditions.

Table 1.2: Summary of VC train control studies considering uncertainties

Paper	Method	Uncertainty	Main Contribution
[49]	Feedback control	Mass and resistance force	Proved stability under uncertainty
[43]	Tube MPC	Resistance forces	Proved stability with disturbances
[42]	Robust MPC	Resistance force, position error and adhesion loss	Proved feasibility
[48]	SMC	Mass, resistance force and position error	Corrected position error
[50]	Adaptive SMC	Resistance forces and bound disturbances	Proved stability with disturbances

1.3.3 Control approaches

In the following, we summarize the representative control approaches for train control, which had already been well established in pre-VC systems such as MB operations. The categorization follows the two recent comprehensive survey papers by Wu *et al.* [51] and Felez *et al.* [52], where emerging control techniques applied to VC train control were reviewed and classified into five categories: *consensus-based approaches*, *Model Predictive Control* (MPC), *Sliding Mode Control* (SMC), and *Machine Learning* (ML)-based approaches.

These control strategies originate from mature research areas, including multi-agent consensus theory, predictive optimal control, nonlinear robust control, reinforcement learning, and constrained mechanical system theory. Most of these frameworks have been extensively validated in road vehicle

platooning before being adapted to the specific requirements of VC train operations. Their respective advantages and limitations in addressing the safety spacing control problem under VC are discussed in the following.

a. Consensus-based approaches. Consensus-based control has its roots in distributed multi-agent control the networked architectures such as multi-agent systems. By using relative position and speed errors among neighbors, In the context of train control, consensus-based approaches aim to achieve consensus in velocity and spacing, Its main advantage is simplicity and scalability, the controller laws are often linear and can be analyzed via graph theoretic tools. Some studies that adopted consensus-based control were for the MB train operation [53], [54]. As for VC, robust cooperative control strategies against communication topology attacks were investigated in [36], [37], where topology reconfiguration methods were designed to maintain information flow integrity.

Consensus-based approaches are often considered for road traffic control, where vehicles maneuver has directions, merge at intersections, and for unmanned aerial vehicles with extensive cluster activities. Therefore, its extensibility in geometric is less practical in railways. Trains run on tracks with essentially one-dimensional geometry. The operational scenarios are dominated by unidirectional flows, without lateral maneuvers or overtaking. Even when switches and complex station structure are involved, the modeling as binary decisions (mainline or diverging route), without the need for complex graph interactions. Hence, the elaborate networked or multi-agent topologies often studied in CBC could be unnecessary in the railway context.

b. Model Predictive Control. Among the recent studies on VC convoy safety control, MPC is the most widely used as the optimal control method in VC studies [51], [52], [55], due to it naturally incorporates state and input constraints in an optimization framework. Su *et al.* [56] proposed a centralized control approach using MPC to achieve stable spacing under varying track gradients. Work [57] presented a distributed MPC method for maintaining the stable and safety spacing in the VC convoy. Tube-based and min-max formulations were employed, encoded safety constraints under bounded disturbances [42], [43]. The main advantage of MPC is that it provides a great capacity to handle constraints and multi-objective optimizations. Furthermore, the predictive capability of MPC is also an advantage. However, known as receding horizon control, MPC performs the optimization in an iterative way, which brings a high computational cost on both time and hardware resource. Particularly, in the onboard train control implementation, fast computing that satisfies the real-time performance is

necessary. Some studies restrict prediction horizons to only a few steps to maintain real-time feasibility, which in turn sacrifices performance since more information can be considered with a larger prediction horizon [58].

c. Sliding Mode Control. SMC has been explored as an effective approach in terms of uncertainty, due to its robustness against dynamic environmental changes, ensuring the system remains resilient despite external perturbations. As a classical nonlinear robust approach, by enforcing sliding conditions on surfaces defined by spacing and velocity errors. For VC, SMC has shown strong robustness against parametric uncertainties (e.g., mass variations, resistance estimation errors) and external disturbances [50]. Considering the measurement errors in the velocity and position, Park *et al.* [48] applied sliding mode control, and proposed a robust controller that computes a spacing distance reference for a VC convoy of two trains. He *et al.* [59] focused the control for the virtually coupled heavy-haul trains, addressing the input saturation problem by SMC.

SMC has strong robustness and can maintain the normal operation of the system under bounded interference, so it is widely used in train control. However, SMC is inherently discontinuous, which induces chattering. Although smoothing gain functions can alleviate this effect, the theoretical robustness guarantees are weakened, and practical train actuators cannot execute high-frequency discontinuous commands.

d. Learning-based approaches. With the rapid progress of reinforcement learning (RL) and deep neural networks (DNNs), learning-based controllers have recently been explored for VC applications. Their main attraction lies in being *model-free*, which means that controllers can be trained directly from data, thereby bypassing the need for precise longitudinal dynamic models or perfectly data acquired from trackside and other trains.

In [60], a comprehensive analysis was conducted on the potential of RL techniques for VC. Su *et al.* [61] formulated the safe following control of a train as a Markov decision process and solved it using a Deep-Q-Network algorithm. Chai *et al.* [62] combined long short-term memory (LSTM) with MPC to predict the trajectory of the preceding train, achieving a reduction in tracking error of up to 35% in speed and 25% in position compared with conventional MPC.

However, the performance of learning-based methods heavily depends on data quality and coverage. A vast amount of high-quality historical data is required for training, yet such datasets are currently limited for VC. Moreover, even though trained networks can be computationally efficient and adaptive in real time, their lack of interpretability and formal safety

guarantees remains a major concern. As emphasized in data-driven railway research, *if bad data goes in, bad outcomes come out*: without reliable and representative data, AI-based control models may reproduce biases, amplify noise, or generate unsafe control actions. Therefore, while learning-based approaches show promise for long-term VC development, their deployment in safety-critical onboard systems demands rigorous data validation, transparent model behavior, and integration with verifiable control frameworks.

Summary. From above literature review on the techniques applied in VC train control, it can be seen that few previous studies had meet the multiple requirements of VC at same time:

1. ensuring strict safety without sacrificing short intervals, and
2. satisfying various realistic operational constraints including control limits, jerk limits, velocity limits, and
3. being robust to hazards such as communication delays and dynamic uncertainties.

The research gap motivates us to explore Control Barrier Functions (CBFs) as a novel control framework for VC. Recently, safety-critical control has been frequently associated with this notion. This technique is applicable for providing safety guarantees for nonlinear control-affine systems, and thus ideal for applications in robotics and autonomous vehicles. However, as far as we know, there is few studies have applied CBFs in the domain of train control in railways.

CBFs enforce forward invariance of safety sets, thereby ensuring inter-train distance and speed constraints are satisfied. especially when it comes to guaranteeing safety of a nonlinear, large-scale system under complex constraints. Unlike most existing MPC- or SMC-based approaches, which are typically designed for specific operating phases such as cruising or formation, the proposed method unifies the control of the entire convoy operation. It achieves real-time feasibility while fully implementing the relative braking distance principle, thereby ensuring continuous safety protection and stability across all phases, from acceleration to the final stop. The proposed framework also exhibits high computational efficiency, achieving real-time feasibility for onboard implementation. Compared with typical MPC-based methods, it reduces the computation burden by several orders of magnitude, making it suitable for large-scale convoy applications.

1.4 Objectives of the Thesis

VC enables train convoys to operate with flexible formation through Train-to-Train communication, and where the close running is achieved based on the relative braking distance spacing policy. The CBF-based approach has been successfully adopted in various safety-critical applications to achieve real-time collision avoidance, demonstrating its suitability for the VC control framework proposed in this work.

Our research aims to address these gaps by focusing on *safety spacing control* for VC followers over the entire operational cycle. The study adopts the relative braking distance principle as the theoretical foundation for safety definition and develops control algorithms capable of enforcing RBD constraints in real time. Particular emphasis is placed on the robustness of the control law against communication delays and modeling uncertainties, which are intrinsic to practical VC deployment. The core idea is to utilize CBFs to encode safety constraints directly within the control formulation, ensuring forward invariance of the safe set with minimal conservatism and computational burden.

1.5 Organization of the Thesis

This chapter introduces the background and motivation of this thesis. It first outlines the basic concept of VC, its development history, as well as the motivation behind its emergence. A literature review then follows, through which, the research gaps in train movement control and maintaining safe spacing are identified, forming the central focus of this thesis.

After this introduction, Chapter II introduces the fundamental concepts of CBF, including the theoretical origin, formal definition, as well as the related notion of Control Lyapunov Function (CLF). which directly inspired the development of CBF, as it will later be employed to achieve distance convergence in the convoy formation control.

Chapter III presents an overview of the system considered in this thesis. It introduces the adopted topology, spacing policy, VC operation scenarios, and train dynamics, which together form the basis for the subsequent control design.

Chapter IV investigates the optimal train motion control during the VC convoy formation process, where the following train must synchronize its velocity with that of the leading train while adhering to the safety spacing principle that varies dynamically. A CBF-CLF-QP approach is proposed to control the follower.

Chapter V presents the design of the VC controller for the more general

convoy movement process. A reference control force is first developed to achieve spacing convergence and station stopping under comfort and feasibility constraints. Then, safety is formally guaranteed through time-varying High-order Control Barrier Functions (HOCBFs). Additional designs are introduced to handle communication delays and dynamic uncertainties, forming a complete decentralized control framework.

Chapter VI evaluates the proposed controller through numerical simulations. Its performance is assessed in terms of safety, convergence, and operational efficiency under various settings. Extended case studies with heterogeneous convoy, communication delays, dynamic uncertainties as well as emergency braking are also included.

Finally in Chapter VII, the conclusion and future outlook are drawn.

Chapter 2

Preliminaries on Control Barrier Functions

Contents

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Two key requirements for safety-critical controllers are the ability to *provide formal safety guarantees* and the feasibility of *real-time implementation*. To address these challenges, various mathematically rigorous approaches have been proposed. Some methods rely on direct safety verification, such as model predictive control, reference governors, and Hamilton–Jacobi reachability analysis, which explicitly test whether a given task satisfies the safety constraints. These approaches have been applied to safety-critical systems including autonomous vehicles. However, despite advances in computational technology, direct verification methods still face the curse of dimensionality, as their computational complexity grows exponentially with the state dimension [63]. Compared to other safety control schemes, the CBF formulation only requires to solve a small size quadratic program online and thus is more suitable for embedded systems. Extensive researches have shown that CBFs are an idea tool in the practice of safety-critical control. Specifically, CBFs has been successfully demonstrated on many collision avoidance applications, including bipedal robotics [64], [65], unmanned aerial vehicles [66], aircraft [67], and autonomous vehicle [68]. These studies show that CBFs provide a unified framework to express the concept of safety across different application domains, demonstrating the potential of CBFs as an effective tool for safety-critical control design.

In this subsection, we will review historical developments of CBFs, basic notions, and CBF-based optimal controller synthesis that will be applied in following.

2.1 Safety with Barrier Functions

Safety in control theory concerns preventing system states from entering undesirable regions. Let $\dot{\mathbf{x}} \in \mathbb{R}^n$ is the state of a dynamic system. The time evolution of $\dot{\mathbf{x}}$ is given by

$$\dot{\mathbf{x}} = f(\mathbf{x}), \quad (2.1)$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ locally Lipschitz continuous.

Assume that we wish the state of the system always satisfies

$$b(\mathbf{x}) \geq 0,$$

where $b : \mathbb{R}^n \rightarrow \mathbb{R}$ is continuously differentiable. In another word, it should evolve within a given set

$$\mathcal{C} = \{\mathbf{x} \in \mathbb{R}^n : b(\mathbf{x}) \geq 0\},$$

with boundary $\partial\mathcal{C} = \{\mathbf{x} : b(\mathbf{x}) = 0\}$

The set $\mathcal{C}$ is called safe set, or forward invariant set if system (2.1) is initially safe, i.e. the state is initially inside the set $\mathcal{C}$, then it always safe, i.e. the state stay in $\mathcal{C}$, $\forall t$.

Nagumo's theorem [69] states that provided a *necessary and sufficient* condition for set invariance:

$$\dot{b}(\mathbf{x}) \geq 0, \quad \forall \mathbf{x} \in \partial\mathcal{C}. \quad (2.2)$$

It requires the derivative of b on the boundary of $\mathcal{C}$ non-negative.

In the 2000s, the idea was rediscovered in the context of hybrid systems via *barrier certificates* [70], which provide functions $B(\mathbf{x})$ separating safe from unsafe regions. Some literature used the barrier certificate in optimization, where it prevents trajectories from entering undesirable regions [65]. While powerful, classical barrier certificates were often too conservative, since they imposed invariance conditions through inequalities valid on the *boundary*.

Safety vs. Stability: A Dual Perspective

To begin with, we introduce the standard formalization of control affine systems,

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t)) + g(\mathbf{x}(t))\mathbf{u}(t) \quad (2.3)$$

with $t \in T$, functions $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $g : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ locally Lipschitz, $\mathbf{x} \in \mathbb{R}^n$ is the state vector and $\mathbf{u} \in \mathcal{U} \subset \mathbb{R}^m$ is the control input vector. Set $\mathcal{U}$ represents the admissible input set of the system. In the following, unless ambiguity arises, we omit explicitly stating the time dependence of signals $\mathbf{x}$ and $\mathbf{u}$.

Lyapunov functions had long history provided a systematic tool to guarantee *liveness* properties, such as **stability**, through the decrease of a scalar function $V(\mathbf{x})$ along system trajectories. This motivated the introduction of control to close the system, Control Lyapunov Functions to nonlinear systems by requiring

$$V(\mathbf{x}, \mathbf{u}) + \dot{V}(\mathbf{x}, \mathbf{u}) \leq 0,$$

to one seeks analogous conditions for enforcing safety. The idea is not to drive the state to a point, but to *stay within a set*.

This duality inspired the invention of *Control Barrier Functions* (CBFs). Whereas CLFs characterize controllers that stabilize, CBFs characterize controllers that render a set invariant, provides a sufficient condition for the existence of a safe controller. In the subsequent, the formal definitions of CBF and CLF will be given.

Generally, Control Lyapunov Functions (CLFs) are designed for reaching a target state (or set), whereas Control Barrier Functions (CBFs) are designed for avoiding an unsafe set. CBFs are a direct extension of CLFs and are used to enforce stability or state convergence requirements.

Definition 2.1.1

(Lie derivative [71]) Consider a continuous function $y : \mathbb{R}^n \rightarrow \mathbb{R}$, the derivative $\dot{y}$ along the dynamic of (2.3) is given by

$$\dot{y} = \frac{\partial y}{\partial \mathbf{x}} [f(\mathbf{x}) + g(\mathbf{x})\mathbf{u}] \stackrel{\text{def}}{=} L_f y(\mathbf{x}) + L_g y(\mathbf{x})\mathbf{u}, \quad (2.4)$$

where

$$L_f y(\mathbf{x}) = \frac{\partial y}{\partial \mathbf{x}} \cdot f(\mathbf{x}), \quad L_g y(\mathbf{x}) = \frac{\partial y}{\partial \mathbf{x}} \cdot g(\mathbf{x}) \quad (2.5)$$

is called the Lie derivative of y with along to f and g , respectively. $\square$

The Lie derivative provides an abbreviative notation when calculating the derivative.

Definition 2.1.2

(Control Lyapunov Function) Let $V : \mathbb{R}^n \rightarrow \mathbb{R}$ be a continuously differentiable, positive definite, and radially unbounded function. If there exists a positive constant λ such that

$$\inf_{\mathbf{u} \in \mathcal{U}} [L_f V(\mathbf{x}) + L_g V(\mathbf{x})\mathbf{u} + \lambda V(\mathbf{x})] \leq 0, \quad (2.6)$$

then $V(\mathbf{x})$ is an exponentially stabilizing Control Lyapunov Function (ES-CLF) for system (2.3). $\square$

And then, any Lipschitz continuous control law $\mathbf{u}$ that satisfies (2.6) exponentially stabilizes system (2.3). Parameter λ serves as a decay rate of an upper bound of $V(\mathbf{x})$.

2.2 Control Barrier Function

Before giving the definition of CBFs, we review the following definition.

Definition 2.2.1

(Class $\mathcal{K}$ Function) A continuous function $\alpha : [0, a) \rightarrow [0, \infty)$, $a > 0$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. $\square$

In control theory, it is often required to check if a nonautonomous system is stable or not. To cope with this it is necessary to use some special comparison functions. Class $\mathcal{K}$ functions is extensively applied.

Consider a set $\mathcal{C}$ (the safe set) defining by a continuously differentiable function $b : \mathbb{R}^n \rightarrow \mathbb{R}$ as follows:

$$\mathcal{C} := \{\mathbf{x} \in \mathbb{R}^n : b(\mathbf{x}) \geq 0\}. \quad (2.7)$$

Definition 2.2.2

(Control Barrier Function [72]) Given the set $\mathcal{C}$ defined by (2.7), $b(\mathbf{x})$ is a Control Barrier Function (CBF) for system (2.3) if there exists a class $\mathcal{K}$ function α such that

$$\sup_{\mathbf{u} \in \mathcal{U}} [L_f b(\mathbf{x}) + L_g b(\mathbf{x}) \mathbf{u} + \frac{\partial b}{\partial t}(\mathbf{x}) + \alpha(b(\mathbf{x}))] \geq 0, \forall \mathbf{x} \in \mathcal{C}. \quad (2.8)$$

$\square$

It is assumed that $L_g b(\mathbf{x}) \neq 0$ when $b(\mathbf{x}) = 0$. The point-wise set of controllers that satisfy the CBF constraint is defined as

$$K_{\text{CBF}}(\mathbf{x}) = \{\mathbf{u} \in \mathcal{U} \mid L_f b(\mathbf{x}) + L_g b(\mathbf{x}) \mathbf{u} + \alpha(b(\mathbf{x})) \geq 0\}. \quad (2.9)$$

This set characterizes the *permissive set* of control inputs, namely, the set of all u that maintain the safety condition $b(\mathbf{x}) \geq 0$.

Theorem 2.2.1

If b is a CBF and $\nabla h(\mathbf{x}) \neq 0$ for all $x \in \partial \mathcal{C}$, then any Lipschitz controller such that $u(\mathbf{x}) \in K_{\text{CBF}}(\mathbf{x})$ renders $\mathcal{C}$ forward invariant. $\square$

Given a CBF b associated with the set $\mathcal{C}$ from (2.7), any Lipschitz continuous controller $\mathbf{u}$ satisfying (2.8) makes set $\mathcal{C}$ forward invariant for (2.3), which is also called control invariant or safe.

Intuitively, this condition extends Nagumo's theory on boundary 2.2 to the entire safe set, and in a minimally restrictive form. In essence, a barrier

function assigns positive values to the states within a predefined safe set and becomes zero on its boundary. If its time derivative is positive at the boundary, the function value increases, implying that the system remains inside the safe region. This concept has been extended to control design, leading to the development of CBFs as constructive tools to ensure set invariance. Thus, CBFs provide sufficient conditions for safety under mild assumptions. Importantly, they are less conservative than barrier certificates, and unlike Lyapunov-like barrier functions, they do not enforce invariance of all level sets.

In the context of CBFs, class $\mathcal{K}$ functions serve as comparison functions that introduce a *flexible margin* in the dynamic constraints. As the value of the barrier function $b(\mathbf{x})$ approaches zero (i.e., the state gets closer to the boundary of the safe set), the term $-\alpha(b(\mathbf{x}))$ imposes increasingly strict constraints, thereby forcing the controller to avoid boundary violation. Compared to the simpler linear condition ($L_f b(\mathbf{x}) + L_g b(\mathbf{x}) \mathbf{u} \geq 0$), the use of a class $\mathcal{K}$ function $\alpha(b(\mathbf{x}))$ provides a tunable mechanism to regulate both *convergence speed* and *safety margin*.

Intuitive interpretation.

- If $\alpha(s) = \gamma s$ (a linear class $\mathcal{K}$ function), the CBF constraint becomes

$$L_f b(\mathbf{x}) + L_g b(\mathbf{x}) \mathbf{u} \geq -\gamma b(\mathbf{x}).$$

When $b(\mathbf{x})$ is large, the right-hand side allows larger negative values (i.e., the system may temporarily move closer to the boundary). However, as $b(\mathbf{x}) \rightarrow 0$, the right-hand side approaches zero and the constraint becomes increasingly strict, preventing boundary violation.

- If $\alpha(s)$ is nonlinear, e.g., $\alpha(s) = \sqrt{s}$, it enables a more flexible “contraction rate,” forcing the system to slow down more rapidly when it approaches the boundary.

In practice, $\mathcal{K}$ function α usually takes the form of a positive coefficient γ . In this case, γ serves as a maximal decay rate of $b(\mathbf{x}(t))$, i.e. $b(\mathbf{x}(t))$ is not allowed to decay faster than the exponential decay curve $\dot{b}(\mathbf{x}(t)) = -\gamma b(\mathbf{x}(t))$ so that potentially unsafe behaviour smoothes out as it approaches the boundary of set $\mathcal{C}$.

By choosing properly class $\mathcal{K}$ functions, it allow CBFs to enforce forward invariance while avoiding overly conservativeness.

Given a CBF $b(\mathbf{x})$ associated with the set $\mathcal{C}$ from Def. 2.7, any Lipschitz continuous controller $\mathbf{u}(t)$ with $t \geq t_0$ satisfying (2.8) makes set $\mathcal{C}$ forward invariant for (2.3), which is also called control invariant or safe.

2.3 Optimization-based controller synthesis

A key advantage of CBFs lies in the compatibility with optimization-based controllers. This allows a separate design of controllers for achieving control objectives and maintaining safety. Since the CBF constraint is affine in u , it can be enforced through a quadratic program (QP). Two types of optimization-based controller synthesis frameworks employing CBFs are commonly used, which are introduced in the follows.

2.3.1 CBF-QP

Suppose a pre-designed control input u_{ref} achieves desired properties such as optimality or stability, but does not necessarily ensure safety. It may correspond to stabilizing the system to a target point, tracking a reference trajectory, or executing human commands.

$$\begin{aligned} \mathbf{u}(\mathbf{x}) &= \arg \min_{\mathbf{u} \in \mathbb{R}^m} \|\mathbf{u} - \mathbf{u}_{\text{ref}}\|^2 \\ \text{s.t. } & L_f b(\mathbf{x}) + L_g b(\mathbf{x})\mathbf{u} \geq -\alpha(b(\mathbf{x})). \end{aligned} \quad (\text{CBF-QP})$$

Here, the cost function penalizes the deviation from $\mathbf{u}_{\text{ref}}$, while the CBF constraint acts as a filter that modifies an unsafe reference input into a safe one. This yields a minimally invasive correction, which only occurs when necessary (i.e., when $\mathbf{u}_{\text{ref}} \notin K_{\text{CBF}}(\mathbf{x})$). This formulation is also referred to as a *safety filter*, as it filters out potentially unsafe inputs from the nominal input.

Moreover, when both stability and safety are required, the CBF-QP can be extended by incorporating Control Lyapunov Function (CLF) constraints into the same optimization problem.

2.3.2 CBF-CLF-QP

From the above definitions, both CBF and CLF constraints are affine in the control input $\mathbf{u}$. Hence, they can be jointly incorporated into the same quadratic program to form the CBF-CLF-QP framework, defined as:

$$\begin{aligned} \mathbf{u}^*(\mathbf{x}) &= \arg \min_{\mathbf{u}^*=(\mathbf{u}, \delta) \in \mathbb{R}^m \times \mathbb{R}} \|\mathbf{u}\|^2 + p\delta^2, \\ \text{s.t. } & L_f b(\mathbf{x}) + L_g b(\mathbf{x})\mathbf{u} + \alpha(b(\mathbf{x})) \geq 0, \\ & L_f V(\mathbf{x}) + L_g V(\mathbf{x})\mathbf{u} + \lambda V(\mathbf{x}) \leq \delta, \end{aligned} \quad (\text{CBF-CLF-QP})$$

where $\delta \in \mathbb{R}$ is a slack variable that relaxes the CLF constraint to avoid conflict with the CBF constraint, and $p > 0$ is the penalty weight on δ . By introducing δ as an additional variable, the CLF condition becomes a soft constraint, thus ensuring the feasibility of the QP. Moreover, the QP

formulation is computationally efficient, making it suitable for real-time implementation.

Chapter 3

System Overview

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Building on recent research and technological progress, the development pathway of Virtual Coupling has become increasingly clear. However, its technical specifications and system architecture have not yet been standardized. The general consensus is that the realization of VC depends on a new train control system that incorporates an updated separation principle.

To investigate VC from a control perspective, it is essential to clearly define the system structure and control logic adopted in this thesis. Accordingly, this chapter presents the convoy topology, spacing policy, basic operating scenarios, and train movement dynamics that serve as the unified modeling and analytical foundation for the control problems addressed in the subsequent chapters.

3.1 System Architecture

This section provides an overview of the VC convoy system structure. Specifically, we introduce two essential aspects for the subsequent modeling and control: the *convoy topology* and the *spacing policy*. Together, these two aspects constitute the structural basis of the VC convoy considered in this research.

To illustrate their relevance, we first review the corresponding technologies adopted in existing railway systems. By comparing different solutions, we aim to clarify the motivation behind the chosen system configuration and introduce the topology and spacing policy adopted in this thesis.

Reliable communication that ensures the timely exchange of safety-related data between trains is a vital prerequisite for VC operation. If the dynamic information of the leading train is not transmitted or received in time, the efficiency and safety of the convoy may be compromised. For instance, when the leader or an intermediate train initiates braking, a potential safety risk arises if the following train acts on outdated information indicating that the leader has not yet braked.

Currently, most railway networks employ the GSM-R (Global System for Mobile Communications – Railway), used in ERTMS/ETCS Level 2 and Level 3. GSM-R has provided reliable and effective wireless communication between trains and trackside equipment since 2000, and is now deployed in more than 38 countries across Europe, Asia, and North Africa. In recent years, research and standardization efforts have focused on upgrading GSM-R to the *Future Railway Mobile Communication System* (FRMCS) [73], an international 5G-based standard designed to replace GSM-R and support data-intensive applications such as automatic train control and VC. Notably, the world’s first commercial 1900 MHz 5G radio network for FRMCS was tested in September 2025 [74], demonstrating the potential for resilient, high-bandwidth, mission-critical communication. Recent T2T technologies also indicate reliable performance even when trains are separated by up to 10 km [75]. Nonetheless, T2T links remain vulnerable to disruptions or delays in computation or transmission, which may lead to packet loss or outdated information [41]. These factors must be explicitly considered when developing control strategies for VC operation.

3.1.1 Convoy topology

In general, two control architectures are distinguished in VC research: *centralized* and *decentralized* [52]. In a centralized framework, a single controller coordinates all trains in the convoy through a globally optimized strategy [56]. In contrast, decentralized architectures assign one controller to each train, allowing every unit to compute its control input locally. This reduces the computational burden on the control center and leverages the processing capacity of onboard systems. In such configurations, the leading train (the *leader*) executes operational commands that govern the overall convoy behavior, such as entering or leaving a segment or station, while each *follower* adjusts its control force to maintain the desired spacing and velocity relative to its predecessor.

Within decentralized control, typical convoy topologies can be classified into *networked* and *non-networked* architectures [31]. Networked structures usually adopt consensus-based control (CBC) under graph-based topologies, where each agent exchanges information with multiple neighbors. As

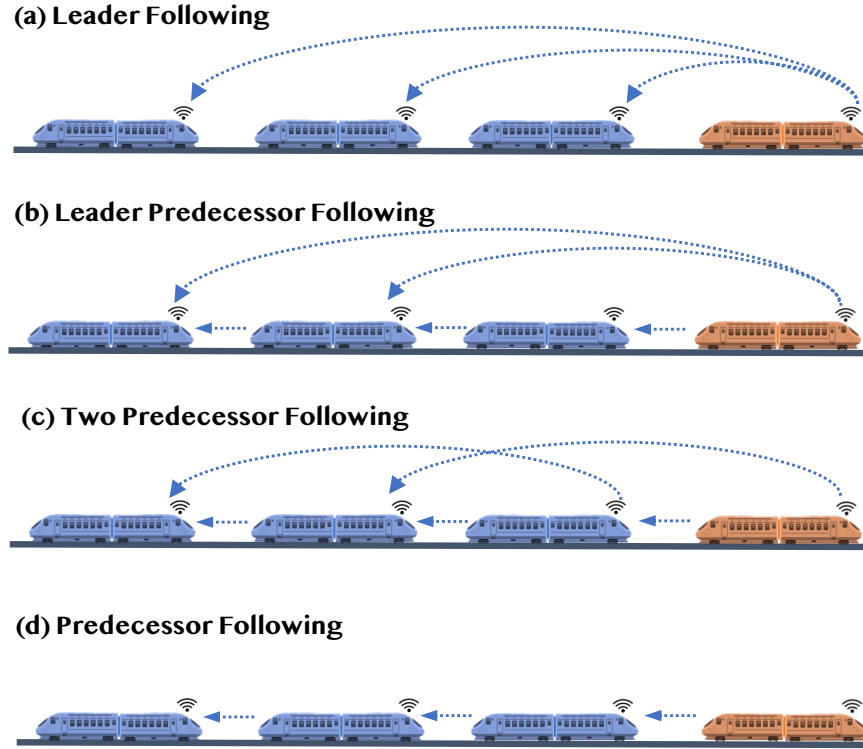


Figure 3.1: Typical VC communication topologies.

discussed in Section 1.3.3, however, non-networked architectures are more suitable for VC train control, as they allow the development of well-adaptive local control laws that are simpler and more practical for railway operations.

Since the controlled entity is the follower train, the main differences among various topologies lie in the communication structure, which determines the state information available to each local controller.

Figure 3.1 illustrates four representative communication topologies that can be used for convoy control:

1. **Leader Following (LF)** — the leader broadcasts its state to all followers.
2. **Leader–Predecessor Following (LPF)** — each follower receives information from both its immediate predecessor and the leader.
3. **Two–Predecessor Following (TPF)** — each follower receives information from the two adjacent trains ahead.
4. **Predecessor–Following (PF)** — each follower uses only the state of its immediate predecessor.

Different topologies offer different trade-offs between global coordination and local simplicity. LF and LPF enable broader system-level coordination

by providing access to leader and network-wide state information, at the expense of increased communication and control complexity. By contrast, PF represents the minimal structure, in which each train ensures a safe headway relative to its predecessor. This configuration aligns naturally with the RBD-based spacing policy and existing railway signaling philosophy, where separation is defined by the braking capability of the preceding train.

In this thesis, the PF topology is adopted as the basis for VC control. While more elaborate information structures such as LPF or TPF may enhance coordination in certain situations, PF already provides sufficient information to achieve the core objective of safety spacing control with high reliability and low complexity. Furthermore, given that VC technology and train-to-train communication are still in their early stages of deployment, adopting a simple and robust communication architecture is both practical and appropriate.

To sum up, we consider the VC convoy as illustrated in Fig. 3.2. The convoy consists of $N+1$ trains: one leader (labeled by 0) and N followers ($1, \dots, N$). The leader plays a commanding role, maintaining constant communication with the train control center to receive operational task orders and enable centralized dispatching management. Each follower determines its driving strategy based on the instantaneous information transmitted from the preceding train via the T2T communication layer, which includes variables such as speed, position, and acceleration. Enabled by high-integrity T2T communication, frequent interventions between the train control center and the followers can be avoided. In ERTMS-based systems, the train control center is the RBC, while in CBTC systems, it corresponds to the ATS and the interlocking. In this thesis, both are collectively referred to as the train control center.

Trains within a VC convoy move synchronously at short inter-train distances and operate cooperatively, behaving as a single train.

The train control center sends the commands of coupling and decoupling, the controller calculates the traction and braking control force in line with the constraint and applies it to the train, so as to maintain the distance between vehicles.

In [76], the key requirements for a new train control system enabling Virtual Coupling were summarized as: (1) management of dynamic coupling and decoupling operations, (2) implementation of a new train separation rule, (3) management of convoys and dynamic route setting/releasing, (4) updated traffic supervision and regulation, and (5) safety assurance and automatic driving through upgraded onboard ATP/ATO systems.

Among these, requirements (1), (3), and (4) are mainly handled by the interlocking or centralized control center, but still require coordination

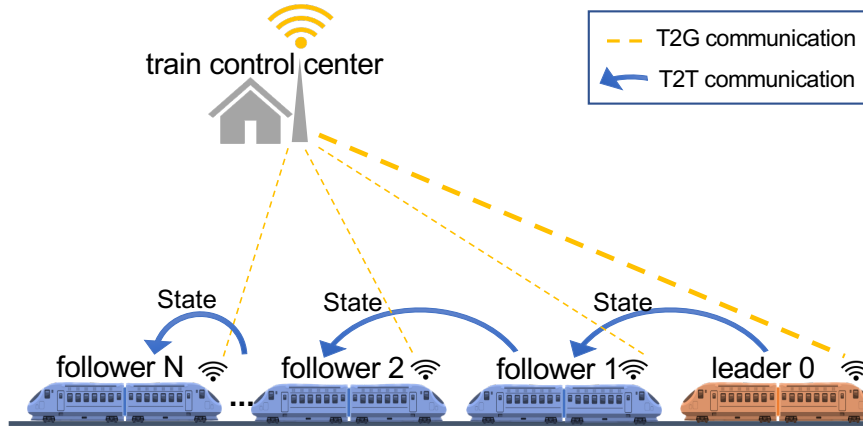


Figure 3.2: A VC convoy of $N + 1$ trains.

with onboard equipment. In contrast, (2) and (5) primarily depend on the onboard controller, as they directly involve train separation logic and real-time control execution.

For the operation control of VC trains, ATO and train-to-train communication are indispensable. Because of the small spacing between trains in the convoy, each single train must constantly consider the driving behavior of the train ahead, which is difficult for drivers to handle in real time while ensuring both safety and comfort. Therefore, we adopt the decentralized architecture for VC convoy, to equip each single train in the convoy with a complete ATO system, so that automatic driving can be realized.

The leader's control requirement is to guide the entire platoon to operate according to the planned trajectory. Its control target is the recommended speed profile. Specifically, the leader's input is the operational plan from the train control center, and measured position and line information, outputting control commands and its own operating state information (e.g., position and speed). This is transmitted to following trains through T2T communication.

The control requirement of each following train is to maintain stable operation in the platoon, keeping consistent spacing and velocity with the leader. After receiving the leader's and predecessor's operating state information, each train calculates its own control commands based on its state and line data, then executes them and transmits updated information to the next train.

Under such topology and control architecture, the VC convoy can be abstracted as multiple subsystems connected in sequence. The system can be designed as a distributed architecture, as shown in Fig. 3.2, where each subsystem independently completes its control calculation and exchanges

information with the predecessor via T2T communication..

For this type of system, the distributed algorithm can be applied: at time t_k , each subsystem sequentially solves its own subproblem, and then the results are transmitted to the next subsystem. Once all subsystems have updated their states, the system enters the next iteration at time t_{k+1} . Repeating this process improves both feasibility and computational efficiency.

3.1.2 Spacing policy

Defining the minimum separation distance between two successive trains is one of the critical aspects of a train control system, i.e., the establishment of a spacing policy. In VC controller design, the spacing policy is enforced through safety constraints, where the following train tracks a control objective while simultaneously satisfying the safety requirement. There often exists a noticeable gap between the control objective and the safety constraint, which represents the distance margin introduced by the nonlinear relative-braking principle [25]. This gap reflects the trade-off between operational efficiency and safety.

Prior to discussing the spacing policies for trains, we first review the well-established policies in car platooning for comparison.

Car platooning has been extensively studied since the 1990s [77], starting from Adaptive Cruise Control (ACC) and evolving into Cooperative Adaptive Cruise Control (CACC) enabled by wireless Vehicle-to-Vehicle communication. In both systems, the spacing policy forms the basis of longitudinal control, defining how each vehicle maintains a safe and efficient distance from its predecessor. With the rise of connected and autonomous vehicles, these policies have become an active topic of research again.

Appropriate spacing policies are crucial to guarantee both safety and stability of the car convoy. Let $d_i^*(t)$ denote the desired spacing between the i -th vehicle and its predecessor $i - 1$, and $v(t)$ the longitudinal velocity of the i -th vehicle at time t . Typical spacing policies include [78], [79]:

1. **Constant Distance (CD):** maintains a fixed spacing independent of speed and acceleration,

$$d_i^*(t) = c,$$

where c is a constant distance representing the desired gap between vehicles.

2. **Constant Time Headway (CTH):** maintains a spacing proportional to vehicle speed,

$$d_i^*(t) = T_h v(t) + d_0,$$

where T_h is the constant headway time, and d_0 is the standstill distance when both vehicles are stationary.

3. **Variable Time Headway (VTH):** generalizes the CTH by allowing the time headway to vary adaptively with system states and operating conditions. In general form,

$$d_i^*(t) = f(v(t), v_{i-1}(t), a_i(t), a_{i-1}(t), \dots) + d_0,$$

where $f(\cdot)$ is a continuously differentiable function that defines the desired spacing as a function of relevant state variables, such as the velocities or accelerations of the ego and preceding vehicles. The function $f(\cdot)$ can be linear or nonlinear, and is typically designed to achieve a trade-off between safety, comfort, and traffic throughput.

In vehicle traffic, experimental results show that the strategy can improve the traffic volume compared with the CTH strategy and the VTH strategy.

Compared with road vehicles, trains operate under fundamentally different physical and operational constraints. They are much heavier, faster, and run on dedicated tracks without lateral maneuvers, which leads to significantly longer braking distances due to limited wheel–rail adhesion.

Moreover, their large passenger capacity imposes stricter safety requirements, and their spacing must be consistent with signaling systems principles, and the definition of movement authorities. Therefore, spacing policies in train convoys must be adapted to account for braking-distance-based safety margins, signaling integration, and communication reliability. Although car-platooning policies provide useful conceptual insights, their direct application to railway systems is infeasible without railway-specific adaptation. In the context of VC, a significant gap that requires addressing lies in the difficulty of effectively handling non-linear safety constraints within real-time control systems.

Relative braking distance principle

Among previous studies on controlling the following train under VC, the general idea is to control the following train's velocity depending on the distance separating it from its predecessor [80]. In the Moving Block systems, each train maintains an *absolute braking distance* (ABD), which equals the distance required for the following train to come to a complete stop after braking initiation [81]. In a VC convoy, T2T communication allows a more efficient spacing principle known as the *relative braking distance* (RBD) [82]. Unlike ABD, RBD accounts for the braking performance of both the leading and following trains, effectively reducing unnecessary gaps. In RBD, train

spacing consists of relative braking distance and a safety margin. The safety margin, or default margin, refers to the minimum distance.

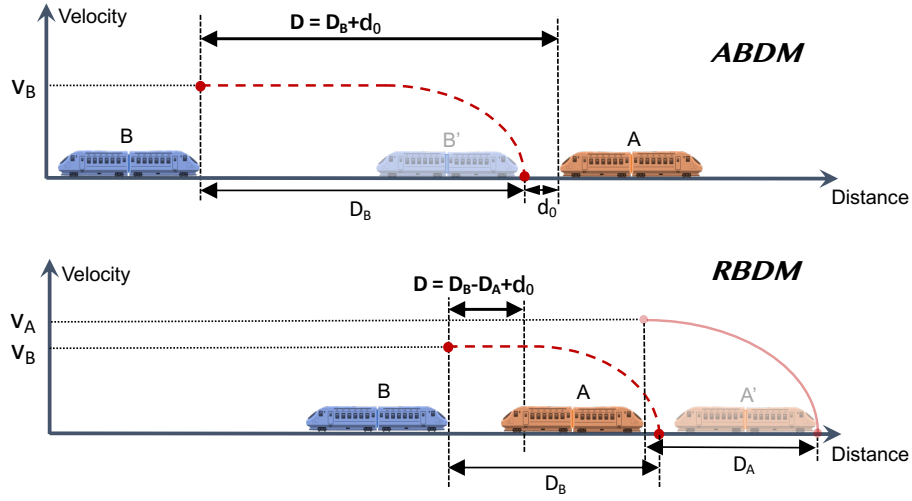


Figure 3.3: Comparison between the absolute braking distance (ABD) and the relative braking distance (RBD).

Figure 3.3 illustrates the conceptual difference between the absolute braking distance and the relative braking distance. In ABD, the safety distance $D = D_B + d_0$ depends solely on the braking distance D_B of the following train B, plus a standstill margin d_0 . In contrast, the RBD adopted in Virtual Coupling defines the spacing as

$$D = D_B - D_A + d_0,$$

where D_A and D_B represent the braking distances of the leading and following trains, respectively. Moreover, in order to make sure that the distance is always equal to or bigger than the margin d_0 ,

Figure 3.4 illustrates the relationship between convoy velocity and the required inter-train spacing defined by the non-linear RBD principle. At standstill ($v = 0$), trains are separated by a default safety margin d_0 . Assuming that ideally the trains within a convoy cruise at nearly the same velocity, as velocity increases, the spacing grows with v^2 . Consequently, maintaining safety while adapting to varying velocities poses a substantial challenge for the control system, especially under acceleration and braking maneuver and operation tasks such as entering the station and stop in the platform precisely.

This formulation assumes that both trains apply brakes simultaneously. When the leading train begins braking earlier or more strongly, the relative term $D_B - D_A$ decreases, allowing a shorter safe distance while still ensuring collision avoidance.

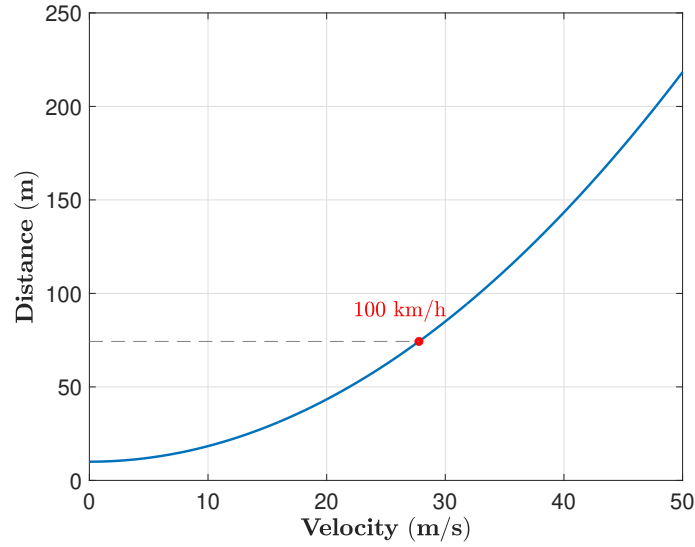


Figure 3.4: Relationship between train velocity and the relative braking distance within a convoy.

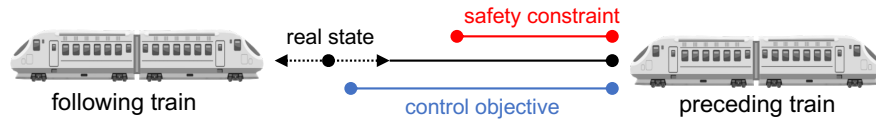


Figure 3.5: Spacing control in the VC convoy.

Figure 3.5 illustrates the essential challenge of spacing control in the VC convoy. The following train must regulate its motion based on the real-time state relative to its predecessor, while ensuring that the actual inter-train distance always satisfies the safety constraint (red). At the same time, the controller seeks to minimize the spacing error and achieve the control objective (blue), which ideally corresponds to the desired distance for efficient operation. Balancing these two requirements (safety preservation as hard constraint and distance convergence as stability goal) is nontrivial, especially under dynamic operating conditions such as acceleration, braking, or even communication delays and dynamic uncertainties.

This RBD principle forms the theoretical basis of Virtual Coupling [82]. By explicitly considering the braking capabilities of both trains, RBD allows consecutive trains to operate with much smaller inter-train spacing, behaving almost as if physically coupled but without mechanical connection. Compared with the conservative ABD-based Moving Block operation, RBD enables significantly shorter headway and improved line throughput, representing the conceptual transition from individual train separation to cooperative convoy movement. While the fundamental idea is based on the relative braking distance, many previous VC studies have employed

alternative spacing methods to simplify control design. Despite the RBD principle being the theoretical basis for VC spacing, many studies adopt a simplified spacing policies rather than the RBD usually in the three different ways: (i) *constant distances*, (ii) *linearized distances*, and (iii) *scenario-based distances*.

Constant spacing is often set empirically, e.g., 130 m in [83], 200–300 m in [31], or 60 m in [84]. Such values are acceptable for high-speed lines. However, they are impractical for metro lines. Metro platforms are shorter, and stations are closer together, where closely coupled operations are crucial to adapt to fluctuating passenger demand and improve service quality.

Some researchers have attempted to linearize the RBD under certain assumptions. For example, [85] and [43] assume that adjacent trains share an identical deceleration rate and impose an upper bound on velocity, leading to a simplified RBD model with $v + v_{i-1} \leq 2v_{lim}$. Although this reduces computational complexity, such an approach neglects realistic variations in braking performance across different trains. Furthermore, simply assuming that the minimum spacing equals the safety margin when the following train achieves the same or a higher velocity than the leading train, these methods may ignore potential safety risks. There are studies that utilized CTH to define the linear spacing principle for train convoys [86], [87]. This criterion of cars lacks practical validation in the railway context, where braking dynamics of train differ significantly from that of road vehicles.

Scenario-based spacing principles [48], [88], [89] adjust separation distances based on multiple operational phases, such as merging, splitting, cruising, acceleration, or deceleration. However, these approaches often rely on discrete state transitions and require an additional decision module or layer, which increase computational costs and system complexity in real-world train control systems. Apart from that, many scenario-based methods are designed under two-train configuration, making their extension to convoys with $N \geq 3$ nontrivial, even though multi-train formations are common in VC applications.

In [40], even the nonlinear RBD spacing policy was employed, the circumstance of the predecessor's braking distance is greater than the follower's was neglected.

Onboard implementation

With the shift from Fixed Block to Moving Block principles, onboard train control has become the core of modern railway signaling, ensuring both safety and efficiency. Virtual Coupling represents the next-generation paradigm in this evolution.

Driven by advances in computing, communication, and automation, train control systems have developed into highly automated architectures that are now widely deployed in both urban and high-speed railways. As illustrated in Chapter 1, the current state-of-the-art train control systems are ATO-based CBTC and ETCS operating under the Moving Block principle. Although these systems have different system architectures, both rely on ATO-assisted control to enhance capacity and automation. Initially, VC was envisioned as a potential future signaling level, sometimes referred to as *ETCS Level 4* [9], [90]. However, more recent studies propose defining VC as an operational mode extending ETCS Level 3. In [91], this concept was introduced as *Full Supervision plus Virtual Coupling (FSVC)*. In FS mode, all train and track data required for supervision are available onboard, allowing fully automatic operation when ATO conditions are satisfied. In [27], ATO becomes the primary subsystem executing VC-related functions while ATP ensures compliance with permitted speeds and the end of movement authority. Thus, VC is regarded as an extension of ETCS Level 3, and an additional operation mode of CBTC [92]. Its implementation will primarily rely on upgrading the existing onboard subsystems and introducing a new Train-to-Train communication layer.

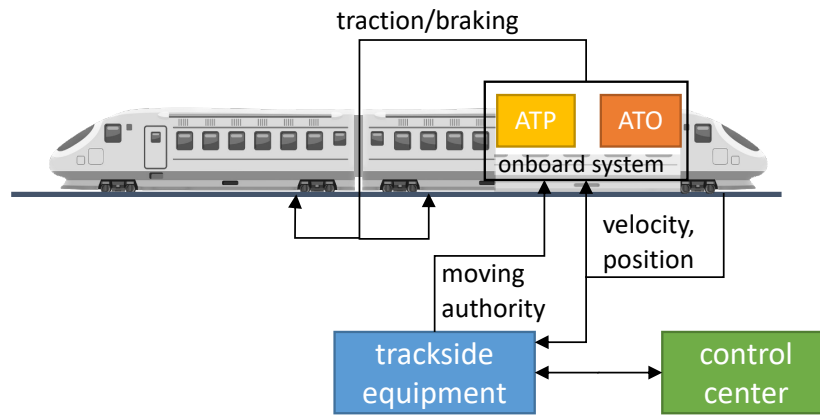


Figure 3.6: Architecture of ATO-based train control systems.

Figure 3.6 depicts the general architecture of modern train control systems integrated with ATO. At the train level, the onboard system consists of the *Automatic Train Protection* (ATP) and *Automatic Train Operation* (ATO) subsystems, which together execute most of the automatic control functions. Through onboard sensors and interface modules, the system acquires real-time measurements of train velocity and position, computes the corresponding control input, and applies it to the traction and braking actuators. This makes the onboard system the central controller of the

train, responsible for enforcing movement commands and ensuring safety compliance.

Externally, the onboard unit communicates with the *trackside equipment* via IP-based or wireless links. The trackside system provides the train with the *moving authority* and receives feedback information such as position and velocity. It also connects to the *control center*, which manages traffic scheduling and operation plans in the centralized manner.

From this perspective, the implementation of VC requires only minimal infrastructural upgrades. Specifically, it involves (i) adding a T2T communication layer for inter-train data exchange, and (ii) upgrading the onboard ATP/ATO software to support decentralized, autonomous control logic. Hence, in future VC deployment, the onboard system—particularly the ATO and ATP modules—will serve as the key functional layer where most VC algorithms are executed.

At the onboard level, ATP and ATO subsystems together constitute the functional core of the onboard system, serving as the foundation for autonomous train operation. ATP guarantees safety by supervising train speed and braking performance, and ATO performs automatic driving tasks such as acceleration, cruising, and precise station stopping. Specifically, the onboard system relies on close coordination between ATO and ATP. ATO calculates the optimal velocity–distance trajectory, while ATP continuously updates the protection curve and triggers braking if limits are exceeded. As VC introduces a new operational mode, its functionalities must be integrated within the ATP/ATO framework. Given that these systems must meet Safety Integrity Level 4 (SIL 4) and strict real-time requirements, the integration of VC must comply with these constraints.

The ATO subsystem plays a crucial role in the transition toward Virtual Coupling. ATO executes traction and braking commands automatically, making it suitable for integration with VC frameworks to realize autonomous following and VC maneuvers.

In summary, realizing VC functionality at the onboard level requires evolving the existing ATO/ATP architecture into a new operating mode. This integration introduces three major requirements: (1) management of mode transitions, i.e., how to enter and exit the VC mode; (2) implementation of a new spacing policy, where the onboard system computes the movement endpoint using updated logic and ensures protection within the VC mode; and (3) management of Train-to-Train communication, including new communication modules and real-time data exchange mechanisms. While dispatching commands remain under the interlocking system, trains operating under VC no longer depend on the movement authority computed by the RBC. Instead, speed supervision and control are executed locally by the

ATO subsystem, marking a shift from centralized to distributed train control.

In this thesis, we won't elaborate the specific duties of ATO and ATP.

These requirements motivate the development of advanced control methods capable of realizing the RBD principle in real time, which forms the core focus of this thesis.

3.2 Scenario Definition and Train Dynamics

3.2.1 Scenario analysis

Table 3.1 presents a general scenario in VC, which consists of four phases in terms of convoy status. In the beginning, the two adjacent trains operate at the Moving Block (i.e. trains are running in separated blocks that are defined by the latter's braking distance), communicating over the T2T channel. The train control center, responsible for dispatching, confirms the coupling need and sends the coupling order to the trains. Both trains must confirm their compliance with the conditions necessary for VC. These conditions cover various operational aspects, including but not limited to the sufficient overlap in their originally planned paths, no ongoing tasks or speed limitations that conflict with the coupling order, and sound communication with the neighboring train and the train control center. If the required conditions are all met, the operation mode of both trains transitions from the Moving Block to "Coupling".

Table 3.1: Virtual Coupling Train Convoy Formation, Running and Separation Processes

PHASE	ACTION	MODE
Preparation	The follower and leader receive the coupling task, and check the conditions for coupling.	Moving Block
Convoy formation	The follower accelerates, then decelerates, while the leader maintains its speed or remains stationary, until the follower matches both the desired distance and the leader's velocity.	VC: Coupling
Convoy running	The follower and the leader run synchronously.	VC: Coupled
Convoy separation	Any VC condition is violated.	VC: Decoupling

Once the follower attains a speed close to that of the leader within a predefined threshold and remains within a distance range from the desired separation, VC is established between them. Afterward, the operation mode of the follower and leader will transition from “Coupling” to “Coupled”. Suppose either the follower or the leader receives a separation order from the train control center, or any violations against conditions for VC such as communication loss or location failures occur. In that case, trains in convoy become separated, known as “Decoupling”. When Decoupling occurs, the control mode of both trains is downgraded to the Moving Block.

The above description provides an overview of the general scenario in which VC is established based on the existing studies’ common sense. Next, we analyze and define ambiguous or undefined key factors in the process of the convoy formation and how to determine its completion.

3.2.2 Train dynamics

The longitudinal dynamics of each follower i can be modeled as a mass-point system and is described by the following continuous dynamic equation [93]:

$$\begin{cases} \dot{p}(t) = v(t) \\ \dot{v}(t) = \frac{u(t) - r(t)}{M} \end{cases} \quad (3.1)$$

where M [kg] denotes the mass of train, $v(t)$ [m/s] is the velocity, and $p(t)$ [m] is the position of the train’s head. External forces include the traction or braking force (T/B force) $u(t)$ [N] and the running resistance force $r(t)$ [N]. The total resistance force r consists of two components: the fundamental resistance and the additional resistance. The fundamental resistance, also known as the running resistance, can be calculated using a second-order empirical equation known as the Davis equation [94]:

$$r(t) = (C_0 + C_1 v(t) + C_2 v^2(t))M, \quad (3.2)$$

where C_1 and C_2 are coefficients associated with mechanical and aerodynamic characteristics, respectively. The parameter C_0 represents the starting resistance, which can be neglected when the train speed exceeds 3 m/s.

The additional resistance, or track resistance, arises from unfavorable track conditions such as gradients, curves, and tunnels. Since this thesis focuses on the general train control problem under VC, the additional resistance is not explicitly modeled in the train dynamics. Nevertheless, its effects are implicitly considered in later analysis by treating it as an external disturbance that may influence the control performance and system robustness.

3.3 Summary

This chapter provides a unified system framework for VC convoy control, including the PF topology, the RBD-based spacing control principle, a general VC operating scenario consisting of three phases, and a second-order train dynamic model. This framework serves as the foundation for the control problems addressed in the subsequent chapters.

Moreover, within the established framework, this chapter identifies three aspects that are crucial for train spacing control in Virtual Coupling:

1. **Real-time computation:** The train-borne implementation of VC primarily relies on upgrading the onboard system, which must satisfy strict safety and real-time computational requirements.
2. **Formalization of the safety spacing policy:** The relative braking distance principle is widely recognized as the most appropriate policy for ensuring operational safety. However, existing studies have yet to achieve control based on this rule without simplifications or restrictions to specific operating phases.
3. **Fail-safe design against potential hazards:** Special attention must be given to possible failures in T2T communication, a newly introduced technology that presents additional challenges to system reliability and safety.

These three aspects collectively form the core concerns of the controller design developed in the following chapters.

Chapter 4

Convoy Formation Control

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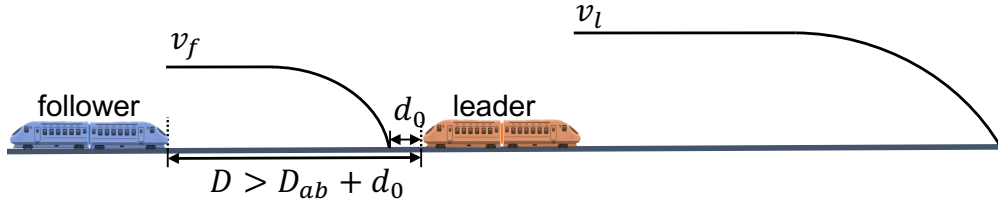
This chapter investigates the optimal train motion control during the Virtual Coupling (VC) convoy formation where the following train (simply called the follower) needs to timely catch up with the velocity of the leading train (simply called the leader) while strictly adhering to the spacing principle that is undergoing change.

First, the dynamic behavior of individual trains and the operating scenarios under VC are analyzed, leading to the definition of spacing control principles used for convoy formation. Building on these foundations, the chapter presents the controller design for the convoy formation, where Control Barrier Functions (CBFs) and Control Lyapunov Functions (CLFs) are employed within a Quadratic Programming (QP) framework to achieve safe and stable formation control.

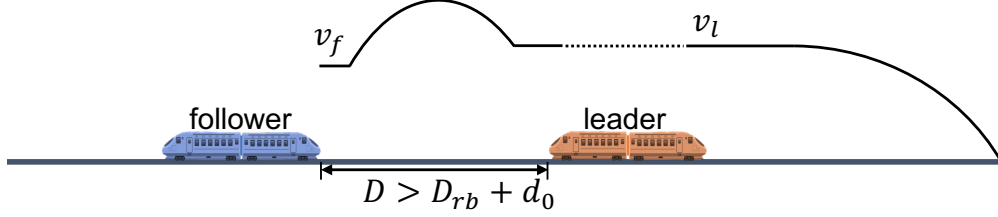
The main contributions of this chapter are summarized as follows:

1. **CBF–CLF–QP-based convoy formation control:** The formation phase is formulated within the Control Barrier Function and Control Lyapunov Function framework, ensuring both safety and convergence. The CBFs enforce the spacing safety constraints, while the CLFs drive the inter-train distances toward the desired formation.
2. **Demonstration of real-time feasibility:** Simulation studies verify that the proposed CBF–CLF–QP formulation can be solved efficiently within the onboard computational limits, confirming its applicability to practical VC scenarios.

0. Moving block



I.VC: Coupling



II.VC: Coupled

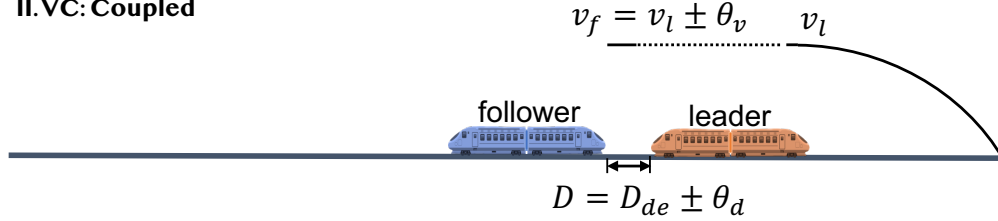


Figure 4.1: Convoy Formation in Virtual Coupling

4.1 Problem Formulation

The scenario analysis in the previous chapter provides an overview of the general operating conditions under which Virtual Coupling is established, based on common assumptions in existing studies. In this section, we focus on the spacing control principle during the convoy formation process and establish the fundamental spacing rule for formation control. We further analyze and clarify the key factors that are ambiguous or undefined in the formation procedure, and define the criteria for determining the completion of convoy formation.

4.1.1 Convoy formation spacing control

The spacing principle determines the minimum safe distance between the trains. This chapter focuses on the follower's motion control, addressing the problem of VC convoy formation. It is assumed that the follower's velocity $v_f(t)$ is time varying while the velocity v_l of the leader is constant. To form the VC convoy, the constraints on the spacing distance are defined as follows.

First, even if two adjacent trains are both in a standstill, they should

be separated by a default margin d_0 [81]. To prioritize safety, there should be sufficient distance for collision avoidance. In the moving block, apart from d_0 , the distance between two successive trains should be longer than the braking distance of the behind train, i.e. the distance it travels from the moment when the brake starts until the train completely stops, which is termed as the absolute braking distance D_{ab} . When calculating the braking distance, we assume the final velocity is zero. Based on this, the absolute braking distance can be calculated using the following equation

$$D_{ab}(t) = \frac{v_f^2(t)}{2a_f}, \quad (4.1)$$

where a_f is the deceleration rate of the following train, which is positive. Therefore, the spatial headway $D(t)$ should be greater than the total of d_0 and $D_{ab}(t)$ as shown in Fig. 4.1.

During the convoy formation phase, the safety distance in the spacing principle for collision avoidance turns to the relative braking distance D_{rb} , which equals to the difference between the braking distances of the two trains. Specifically, we consider the worst case where the leading train uses the emergency braking rate EB , while the following train uses the service braking rate SB and $EB > SB$. Therefore, we define the safety spacing principle for the formation phase as:

$$D(t) > D_{rb}(t) + d_0 = \frac{v_f^2(t)}{2SB} - \frac{v_l^2}{2EB} + d_0. \quad (4.2)$$

Through enforcing (4.2) as a hard constraint, even if the follower's velocity is much larger than that of the leader aiming to catch up with the leader quickly, it is still guaranteed that the follower can avoid rear-end collision in the case of emergency braking by the leader, even when the follower decelerates in a lower service braking rate.

The ultimate purpose of VC is to increase line capacity by minimizing train spacing distance. If (4.2) is the only spacing principle in the VC convoy formation, this may fail for the follower to close up the distance to the leader. Therefore, we introduce a *desired distance* as the target distance that guides the follower to quickly shorten the distance with the leader and form the convoy. This distance must adhere to the safety principle and meets our expectations on the convoy running in the ideal status: the trains run synchronously at a consistent distance. Consequently, the relative braking distance $D_{rb}(v_l = v_f)$ between trains after they are successfully coupled and the convoy has an overall consistent velocity, is chosen as the desired

distance, denoted as $D_{de}(t)$, and equals

$$\begin{aligned} D_{de}(t) &= d_0 + \frac{v_f^2(t)}{2SB} - \frac{v_f^2(t)}{2EB} \\ &= d_0 + \frac{(EB - SB)v_f^2(t)}{2SB \cdot EB}. \end{aligned} \quad (4.3)$$

Let

$$k = \frac{EB - SB}{EBSB},$$

consequently (4.3) can be written as

$$D_{de}(t) = d_0 + \frac{kv_f^2(t)}{2}. \quad (4.4)$$

To determine if the formation of a VC convoy is complete, the following two conditions must be met:

1. The following train attains a desired distance from the leading train, i.e. $|D - D_{de}(t)| \leq \theta_d$.
2. The following train matches the speed of the leading train, i.e. $|v_f(t) - v_l| \leq \theta_v$.

Error thresholds θ_d and θ_v are set to accommodate for interference terms such as location errors, communication delays, sampling noise, etc. Additionally, we define the time of the convoy formation as t_C , i.e., the time from the start of the formation phase to its completion (when the two conditions above are simultaneous fulfilled).

The spacing principle described by (4.2) and (4.3) represents the safe constraint and the control objectives of the follower. In this section, we combine it with the train dynamics to establish the optimal and safe control problem of the follower during the convoy formation by applying the CBF-CLF-QP.

Before applying the CBF-CLF-QP technique to solve the optimal control of the follower for the formation scenario in VC, the control-affine nonlinear model of the following train's dynamic is first formalized.

Remark. To keep terms concise, in this chapter “leader” refers to the reference entity in formation (which may be a single train or a convoy), while “follower” denotes the train or convoy that joins from behind. Even though singular terms are used, the discussion is not restricted to single trains but applies equally to train–convoy or convoy–convoy cases.

4.1.2 Train following model

Let $\mathbf{x} = (x_1, x_2)$ where $x_1(t) = D(t)$ is the distance between the leader and the follower, and $x_2(t) = v_f(t)$ is the follower's velocity. It holds that

$\dot{x}_1 = v_l - x_2$. Taking the T/B force u in (3.1) as the control input, based on the prototype of the control affine system (2.3) and the longitudinal model (3.1), we model the VC train following model as follows:

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -x_2 + v_l \\ -\frac{1}{M}r \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}}_{g(x)} u. \quad (4.5)$$

Note that the leader's velocity v_l is considered a known constant and thus can be regarded as a parameter rather than an input in the model. Besides, although (4.5) is written in a linear form, the model is nonlinear because r depends on the quadratic form of x_2 in (3.2).

As discussed in the previous section, in order to form a VC convoy, the follower needs to reach the distance to the desired distance as close as possible while strictly adhering to the safety distance, meanwhile the two distance depend on the velocities of the follower and the leader. Thus, the spacing principle is described with respect to the model (4.5) as

$$x_1 \rightarrow d_0 + \frac{kx_2^2}{2}, \quad (4.6)$$

$$x_1 \geq d_0 + \frac{x_2^2}{2SB} - \frac{v_l^2}{2EB}. \quad (4.7)$$

Note that (4.6) is a soft constraint that aims to stabilize the system (which can be accomplished by a CLF), whereas (4.7) is a hard constraint that assures the system's safety (can be achieved by a CBF).

Meanwhile, the control input u , i.e. the T/B force, is naturally constrained by the train mechanics defined as

$$-B \cdot M \leq u \leq T \cdot M, \quad (4.8)$$

where $B > 0$ is the maximum braking rate and $T > 0$ is the maximum traction rate.

4.2 Controller Design

We make the following assumptions. First, the parameters of both trains and tracks are obtained after calibration through the corresponding interface modules (e.g., positioning and speed measurement), and the associated errors and uncertainties are assumed to be eliminated. Second, the following train receives real-time information from its preceding train through constant and reliable T2T communication, where the communication delay is considered negligible.

4.2.1 CLF-enforced stability

We employ CLF constraints satisfying (2.6) to achieve the stability objective (4.6), which can be written as $x_1 - D_{de}(t) \rightarrow 0$. The Lyapunov function $V(\mathbf{x}) = (x_1 - D_{de}(t))^2$ is chosen, then

$$\begin{aligned}\dot{V}(\mathbf{x}) &= L_f V(\mathbf{x}) + L_g V(\mathbf{x})u \\ &= \frac{\partial V}{\partial \mathbf{x}} \cdot f(\mathbf{x}) + \frac{\partial V}{\partial \mathbf{x}} \cdot g(\mathbf{x})u\end{aligned}\quad (4.9)$$

where

$$\begin{aligned}\frac{\partial V}{\partial \mathbf{x}} &= \left[\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2} \right] \\ &= \left[2 \left(x_1 - \frac{kx_2^2}{2} - d_0 \right), -2kx_2 \left(x_1 - \frac{kx_2^2}{2} - d_0 \right) \right].\end{aligned}$$

The CLF constraint is

$$\begin{aligned}&L_f V(\mathbf{x}) + L_g V(\mathbf{x})u + \lambda V(\mathbf{x}) \\ &= \left(x_1 - \frac{kx_2^2}{2} - d_0 \right) \cdot \\ &\left[2(-x_2 + v_l) + \frac{2k(r-u)x_2}{M} + \lambda \left(x_1 - \frac{kx_2^2}{2} - d_0 \right) \right] \leq \delta,\end{aligned}\quad (4.10)$$

here δ is the slack variable that serves as the relaxation on the CLF constraint to prevent it from conflicting with the CBF constraint.

4.2.2 CBF-enforced safety

To construct the inequality constraints that enforce the safety control objective (4.7), we consider the CBF derived directly from (4.7), i.e. $b(\mathbf{x}) = x_1 - D_{rb}(t) - d_0 \geq 0$. According to (2.8), the control input should satisfy

$$\begin{aligned}&L_f b(\mathbf{x}) + L_g b(\mathbf{x})u + \gamma b(\mathbf{x}) \\ &= -x_2 + v_l + \frac{rx_2}{MSB} + \frac{-x_2}{MSB} \cdot u \\ &+ \gamma \left(x_1 - \frac{x_2}{2SB} + \frac{v_l^2}{2EB} - d_0 \right) \geq 0\end{aligned}\quad (4.11)$$

Applying the designed CBF in (4.11) and CLF in (4.10), the QP for the control of the follower is constructed by combining the above constraints as

$$\begin{aligned}u^*(x) &= \arg \min \left((u - u_{\text{ref}})^T H (u - u_{\text{ref}}) + p\delta^2 \right) \\ \text{s.t. } &(4.10), (4.11), \text{ and } (4.8),\end{aligned}\quad (4.12)$$

where u_{ref} is the reference value of the control output u , and H is the quadratic objective term, p is the penalty on the slack variable δ .

4.2.3 Feasibility of the QPs

When formulating the CLF constraint, a slack variable is introduced in the right-hand side of the CLF constraints, in order to prevent it from conflicting with the CBF constraint. So that the CLF in (4.12) is treated as a soft constraint. However, the overall QP (4.12) may still become infeasible when the control input is also subject to the bounds in (4.8), which can introduce potential conflicts with the CBF constraint.

Recalling the definition of the set of control inputs that satisfy the CBF constraint (2.9), the feasibility of the QP is equivalent to the non-emptiness of $K_{\text{CBF}}(\mathbf{x})$ under the input limits. If $K_{\text{CBF}}(\mathbf{x})$ and the bounded interval $[U_{\min}, U_{\max}]$ share a non-empty intersection, the QP is feasible; otherwise infeasibility arises. Formally, the condition can be expressed as

$$K_{\text{CBF}}(\mathbf{x}) \cap [U_{\min}, U_{\max}] \neq \emptyset, \quad (4.13)$$

where $U_{\min} = -M T$ and $U_{\max} = M B$ represent the minimum and maximum admissible control inputs, respectively.

Proof. The CBF constraint (4.11) can be rewritten as

$$-\frac{v}{M S B} u \geq v - v_l - \frac{r v}{M S B} - \gamma b(\mathbf{x}).$$

Since $v \geq 0$ (and $M, S B > 0$), it follows that

$$u \leq r + \frac{M S B}{v} (v_l - v + \gamma b(\mathbf{x})).$$

Let

$$U_{\text{ub}}(\mathbf{x}) = r + \frac{M S B}{v} (v_l - v + \gamma b(\mathbf{x}))$$

denote the upper bound of the feasible control input. Hence,

$$K_{\text{CBF}}(\mathbf{x}) = (-\infty, U_{\text{ub}}(\mathbf{x})].$$

Intersecting with the input interval yields a sufficient condition for feasibility:

$$U_{\min} \leq U_{\text{ub}}(\mathbf{x}). \quad (4.14)$$

Assuming the initial condition $b(\mathbf{x}) \geq 0$, and noting $r \geq 0$, the only negative term in U_{ub} is $-M S B$, i.e.,

$$U_{\text{ub}}(\mathbf{x}) = -M S B + r + \frac{M S B}{v} (v_l + \gamma b(\mathbf{x})) \geq -M S B. \quad (4.15)$$

Since the maximum braking force $U_{\min} = -M T$ is smaller than the service braking force (both negative values), we have

$$U_{\min} \leq -M S B. \quad (4.16)$$

Combining (4.15) and (4.16), we can conclude that the feasibility condition (4.14) holds, and consequently, (4.13) is satisfied. Therefore, the constraint (4.11) is always feasible at any state, and $b(\mathbf{x})$ is a valid CBF. $\square$

Table 4.1: Simulation parameters for convoy formation control

Meaning	Symbol	Value
Train dynamic parameters		
Weight of the train	M [kg]	383000
Resistance coefficient	C_0 [N/kg]	0.73
Resistance coefficient	C_1 [Ns/M · kg]	0.008
Resistance coefficient	C_2 [Ns ² /M ² · kg]	0.000148
Emergency braking rate	EB [m/s ²]	1.8
Serving braking rate	SB [m/s ²]	1.5
Maximum braking rate	B [m/s ²]	3
Maximum traction rate	T [m/s ²]	3
Default margin	d_0 [M]	5
Distance threshold	θ_d [m/s]	10
Velocity threshold	θ_v [m/s]	0.2
Control parameters		
CLF constraint coefficient	λ	0.1
CBF constraint coefficient	γ	5
Penalty on deviation from u^*	p	$2e - 2$
Penalty on slack variable	δ	$2e - 2$

Remark. From the above discussion, it can be seen that the bound $U_{ub}(\mathbf{x})$ is affected by the choice of the hyperparameter γ . Specifically, a larger γ enhances the barrier effect when $b(\mathbf{x})$ decreases, thereby improving the responsiveness to reductions in $b(\mathbf{x})$; in other words, $b(\mathbf{x})$ becomes more conservative. However, increasing γ also enlarges the admissible input set, since $U_{ub}(\mathbf{x})$ grows with γ . Conversely, a smaller γ has the opposite effect. In practice, γ is selected to balance conservatism and feasibility.

4.3 Simulation and Results

The effectiveness of the proposed control method is verified by numerical simulations. Parameters used for the simulations are from the Train a Grande Vitesse (TGV) France, as listed in Table 4.1.

We categorize two types of scenarios: 1) Static formation (SF), during which the leader's velocity stays at zero during the formation, and 2) Dynamic formation (DF), which refers that the leader cruises at a constant speed during the formation. For each type, two cases (SF-1, SF-2 for the static formation, and DF-1, DF-2 for the dynamic formation) with varied initial

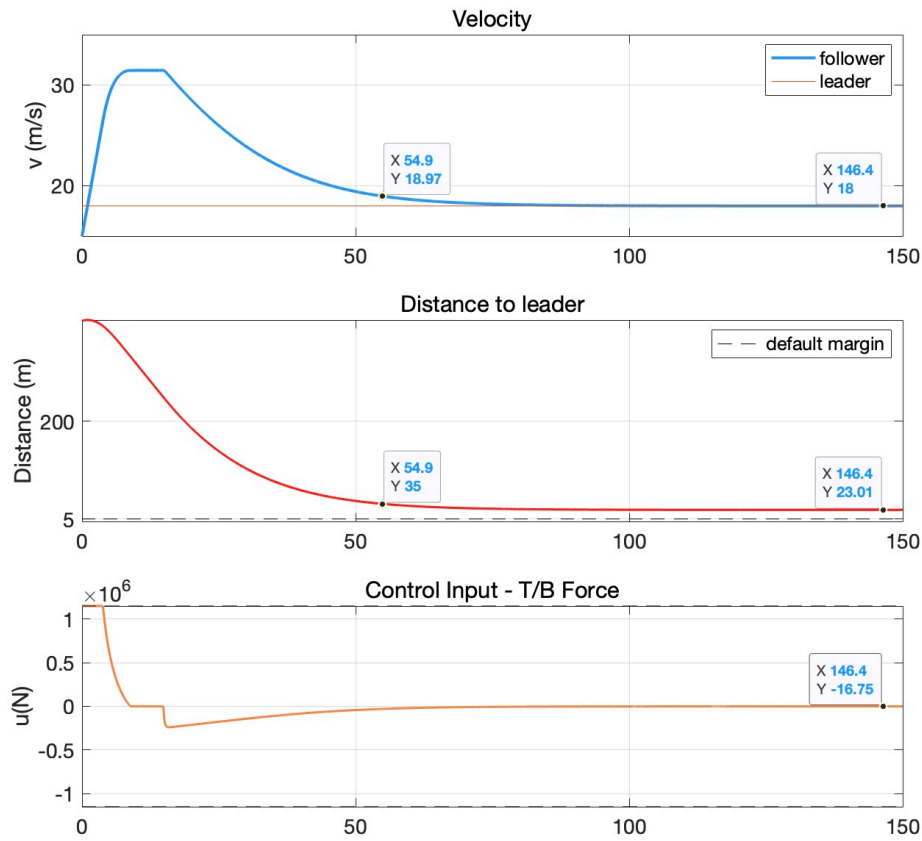
conditions are simulated¹.

Table 4.2 presents the initial conditions and the corresponding formation time t_C . Let $u_{\text{ref}} = r$, and the cost function $H = 2/m^2$. Performing a simulation of $150s$ with the step size $0.02s$, the results of the velocity, distance and control input in the case DF-1 are plotted in Fig. 4.2. The follower undergoes acceleration, idling and deceleration to form a formation with the preceding train at $54.9s$.

After that, the distance continues to narrow down to $23m$, which equals the desired distance by (4.3). And the difference in velocity between the follower and the leader stabilizes to zero.

The total time for running the simulation was around $18.1s$, and the average time for implementing one loop (7500 loops in total) was around $0.0024s$, which meets the real-time requirement of automatic operation module in the train onboard system [95].

Figure 4.2: Result of Case DF-1



¹The QPs are solved using Matlab toolbox `quadprog`. The obtained closed loop system is simulated using function `ode45`.

Table 4.2: Different Initial Conditions and Simulation Results in the Four Cases

Case	v_l	$x_1(0)$	$x_2(0)$	t_C
SF-1	0 m/s	400 m	0 m/s	28.96 s
SF-2	0 m/s	400 m	15 m/s	24.98 s
DF-1	18 m/s	400 m	15 m/s	54.9 s
DF-2	18 m/s	800 m	20 m/s	69.24 s

4.4 Summary

This chapter focuses on the speed control problem of the following train under the Virtual Coupling (VC) concept. It develops the spacing principle for convoy formation and formulates the control objectives on inter-train spacing using the Control Lyapunov Function (CLF) to achieve the desired distance and the Control Barrier Function (CBF) to maintain safety constraints. Quadratic Programs (QPs) are constructed to compute the traction and braking forces that satisfy all control objectives and operational limits, and the feasibility of QPs is analyzed and proved. Simulation results show that the proposed method enables the follower train to converge to the desired spacing, thereby enhancing line capacity while guaranteeing safety.

The proposed CBF–CLF–QP-based control scheme serves as a preliminary step toward the convoy movement control problem investigated in the next chapter. The next chapter extends the proposed control framework to more complex VC scenarios, including coupled convoy operation during station approaches or departures, unexpected braking of the leading train, and realistic line profiles. It also investigates the design of a robust convoy controller that can tolerate communication delays and losses, ensuring safe and stable operation under uncertain and time-varying conditions.

Chapter 5

Convoy Movement Control

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This chapter focuses on the control of the convoy after coupling, i.e., in the coupled status. While the previous chapter addressed the formation phase using the CBF–CLF–QP framework, the present chapter extends the methodology to convoy movement control, where the leading train’s acceleration and deceleration act as external disturbances to the following trains [96]. The objective is to realize the relative braking distance (RBD) principle during dynamic operations and ensure both safety and efficiency under communication delays and model uncertainties.

This chapter is organized as follows. Section 5.1 formulates the control problem and introduces the constraints and objectives. Section 5.2 presents the design of the reference control force and the CBF-based safety enforcement, followed by the delay-aware and disturbance-resilient control designs. Section 5.3 integrates these modules into the final controller synthesis, and a summary is provided in Section 5.4.

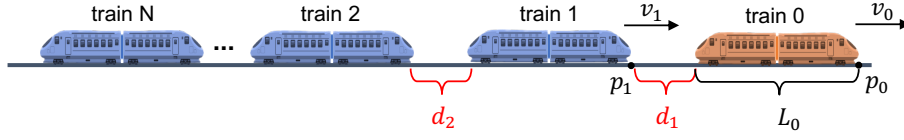


Figure 5.1: Virtual Coupling convoy movement control

5.1 Problem Formulation

During the coupled running phase, as described in Chapter 2, only the first train, i.e. the leading train 0 in the convoy, needs to obtain the operational order from the train control center, and the following trains operate automatically according to the movement authority and the speed, position and other information of adjacent trains.

This thesis won't elaborate on how to control the leader's motion, since it is accomplished by the leader's ATO and there is no VC-related functions involved. Instead, we assume that the leader is tasked with tracking a desired trajectory. In terms of inter space between current convoy and other trains or convoy in front, the absolute braking distance (ABD) is applied.

The convoy movement control falls into the inside control of each follower, is subject to the leader's profile, so we extend the scope from a two-train setting (one follower and one leader), to the general train convoy consisting of $N + 1$ trains (N followers and one leader). We investigate a general VC convoy consisting of $N + 1$ trains as shown in Fig. 3.2. Each following train i ($i = 1, 2, \dots, N$), i.e. the follower, receives real-time kinematic data (e.g., speed, position, acceleration) only from its direct predecessor $i - 1$.

Each train computes its control force locally. The leading train (simply called the leader), indexed by 0, is responsible for processing operational commands that determine the overall convoy behavior, such as entering and departing one segment or stations. Each follower adjust its control input to maintain the desired spacing and velocity relative to its preceding train.

We refer the spatial headway in a VC convoy as the distance between the following train's head and the preceding train's rear. For train i and train $i - 1$, their distance is denoted as d_i , and calculated by $d_i = p_{i-1} - p_i - L_{i-1}$, where L_{i-1} is length of train $i - 1$. To prioritize safety, the distance should always be sufficient for collision avoidance. And this task falls on train i .

Remark. In Chapter IV on convoy formation control, the focus is on the process before a convoy is established between two trains. In that context, the term *follower* specifically refers to the train (or convoy) that is joining from behind. In this chapter, however, *follower* denotes any train within an already formed convoy. The controlled train is referred to as *follower i* , while the term *predecessor* is used for the immediate train ahead of follower i .

5.1.1 Symbols and notations

To describe the problem more clearly, the relevant symbols and notations of convoy movement control are listed below.

Table 5.1: Symbols and notations in Chapter 5

Notation	Description
Train related variables	
a_i	train i 's acceleration
v_i	train i 's velocity
p_i	train i 's position
d_i	actual distance between train i and train $i-1$
$\theta_i(t)$	braking distance difference
d_i^*	desired distance (minimum safety distance)
δ_i^d	deviation from desired distance
a_i^{ref}	reference acceleration
u_i^{ref}	reference control force
u_i	train i 's control input (traction or braking force)
r_i	train i 's resistance force
Train related parameters	
d_0	default margin
ϵ_c	convergence time
$\epsilon_i^{s,1}$	distance-based stopping time
$\epsilon_i^{s,2}$	deceleration-based stopping time
v_{lim}	velocity limit
j_{max}	maximum jerk
T_i	maximum allowable traction rate
B_i	maximum allowable braking rate
SB_i	service braking rate
EB_i	emergency braking rate
L_i	train i 's length
M_i	train i 's mass

5.1.2 Constraints and Objectives

First of all, as the train operating on the line, the system state must be constrained by the train's inherent characteristics as well as the line conditions. To ensure safety and stability of the convoy movement, the following constraints are defined.

Control limits. The control input $u_i(t)$ is constrained by the mechanical limitations:

$$-M_i B_i \leq u_i(t) \leq M_i T_i, \quad (5.1)$$

where $B_i > 0$ and $T_i > 0$ denote the maximum allowable braking and traction rates (in m/s^2), respectively.

Velocity limit. The velocity of each train is subject to a unified velocity limitation from tracks

$$0 \leq v_i(t) \leq v_{\text{lim}}, \quad (5.2)$$

here v_{lim} is the maximum velocity depending on the track condition.

Safety constraint. Collision avoidance is guaranteed by safety constraint on the inter-train distances. Let $d_i(t)$ be the distance between the head of train i and the rear of train $i - 1$, given by

$$d_i(t) = p_{i-1}(t) - p_i(t) - L_{i-1}, \quad (5.3)$$

where $p_{i-1}(t)$ and $p_i(t)$ denote the head positions of the two trains, and L_{i-1} is the length of train $i - 1$.

To ensure safety under all braking scenarios, the distance between train i and train $i - 1$ must always be greater than or equal to a *minimum safe distance* $d_i^*(t)$ determined by the RBD-based spacing principle:

$$d_i(t) \geq d_i^*(t) = \max(\theta_i(t), 0) + d_0, \quad (5.4)$$

where d_0 is a default margin that separates trains even when they are both stationary, ensuring a standstill clearance, and $\theta_i(t)$ represents the difference between the braking distances of train i and train $i - 1$, given by

$$\theta_i(t) = \frac{v_i(t)^2}{2SB_i} - \frac{v_{i-1}(t)^2}{2EB_{i-1}}. \quad (5.5)$$

The operator $\max(\cdot)$ ensures that, for the case $\theta_i(t) < 0$, $d_i^*(t) = d_0$. In (5.5), SB_i denotes the service braking rate of train i , and EB_{i-1} is the emergency braking rate of train $i - 1$, with $SB_i < EB_{i-1}$.

Convergence objective. In traditional optimization control for train convoys, the convergence objective involves minimizing tracking errors in both spacing and velocity. In this work, we focus on spacing convergence, as it directly determines the convoy's compactness and safety. By (5.4) and (5.5), $d_i^*(t)$ increases quadratically with velocity while decreases as the convoy slows down. For example, a 50 m/s convoy requires a minimum safe spacing of 138 m (assuming $SB_i = 1.5 \text{ m/s}^2$, $EB_{i-1} = 1.8 \text{ m/s}^2$), which will reduce to 22 m at the velocity of 20 m/s. Let $\delta_i^d(t) = d_i(t) - d_i^*(t)$ be the spacing deviation. To increase the line capacity, it is necessary to minimize the space

between trains as much as possible. Therefore, the convoy convergence goal is

$$\lim_{t \rightarrow \infty} \delta_i^d(t) = 0. \quad (5.6)$$

Remark. We do not limit the maximum value of $\delta_i^d(t)$, because a large spacing intrinsically offers additional safety. Moreover, excessive distance does not necessarily disrupt VC. Recent T2T communication technologies indicate reliable performance even if the trains are separated up to 10 km [75].

5.1.3 Follower Model

To compute the control force for each following train in the VC convoy, we model the train i as a nonlinear affine control system. Let $\mathbf{x}_i(t) = [p_i(t), v_i(t)]^\top$, based on the longitudinal dynamics (3.1),

$$\underbrace{\begin{bmatrix} \dot{p}_i(t) \\ \dot{v}_i(t) \end{bmatrix}}_{\dot{\mathbf{x}}_i(t)} = \underbrace{\begin{bmatrix} v_i(t) \\ -\frac{r_i(t)}{M_i} \end{bmatrix}}_{f(\mathbf{x}_i(t))} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{M_i} \end{bmatrix}}_{g(\mathbf{x}_i(t))} u_i(t). \quad (5.7)$$

5.2 Controller Design

This section presents the step-by-step development of the proposed convoy controller. First, a reference control force is designed to achieve spacing convergence and timely stopping under comfort constraints. Then, time-varying CBFs and HOCBFs are constructed to ensure collision avoidance, followed by additional designs that address communication delays and modeling uncertainties. Together, these parts constitute the foundation of the optimization-based control framework.

5.2.1 Reference control force

This part establishes the reference control force that guides the nominal motion of each train. The design aims to ensure accurate spacing convergence during convoy formation, timely stopping at stations, and physically feasible acceleration and control force profiles within comfort limits.

Distance convergence. The basis of reference solution is to achieve the convoy convergence goal (5.6), i.e. to minimize the spacing distance. Given $\delta_i^d(t) \geq 0$, we compute a *reference acceleration* $a_i^{\text{ref}}(t)$ for train i such that in the time interval ϵ_c , distance deviation δ_i^d goes to 0 at time $t + \epsilon_c$. Assume that train $i - 1$ maintains the current acceleration $a_{i-1}(t)$, this occurs when

the following formula (5.8) holds

$$\left(v_i(t)\epsilon_c + \frac{a_i^{\text{ref}}(t)\epsilon_c^2}{2}\right) - \left(v_{i-1}(t)\epsilon_c + \frac{a_{i-1}(t)\epsilon_c^2}{2}\right) = \delta_i^d(t) \quad (5.8)$$

where the first element represents the distance traveled by train i over ϵ_c with the recommended acceleration $a_i^{\text{ref}}(t)$. The second represents the distance traveled by train $i - 1$ over the same period ϵ_c with its current acceleration $a_{i-1}(t)$. The relative displacement between train i and train $i - 1$ compensates the spacing deviation δ_i^d .

From (5.8), the reference acceleration for train i is obtained by

$$a_i^{\text{ref}}(t) = \frac{2}{\epsilon_c^2} \delta_i^d(t) + \frac{2}{\epsilon_c} (v_{i-1}(t) - v_i(t)) + a_{i-1}(t). \quad (5.9)$$

In (5.9), ϵ_c is defined as the *convergence time* for all the followers in the convoy, representing the time period over which train i adjusts its state to reduce δ_i^d and achieve the desired spacing relative to train $i - 1$. The effect of the size of ϵ_c on the spacing convergency will be investigated in the next chapter of simulation.

Station-stop logic. When the preceding train comes to a full stop at a station, we have $a_{i-1}(t) = 0$ and $v_{i-1}(t) = 0$. Under this condition, the nominal reference acceleration of the follower derived from (5.9) reduces to

$$a_i^{\text{ref}}(t) = \frac{2}{\epsilon_c^2} \delta_i^d(t) - \frac{2}{\epsilon_c} v_i(t). \quad (5.10)$$

Then $a_i^{\text{ref}}(t) > 0$ when

$$\frac{2}{\epsilon_c^2} \delta_i^d(t) > \frac{2}{\epsilon_c} v_i(t), \quad (5.11)$$

holds. Since the convergence time $\epsilon_c > 0$, this condition is equivalent to

$$v_i(t) < \frac{\delta_i^d(t)}{\epsilon_c}. \quad (5.12)$$

When (5.12) holds, a positive $a_i^{\text{ref}}(t)$ is obtained. In other words, if (5.12) holds when train $i - 1$ has already stopped, the following train will continue to accelerate to shorten the spacing. As $v_i(t)$ increases and the spacing deviation $\delta_i^d(t)$ decreases, the inequality in (5.12) will be reversed to

$$v_i(t) > \frac{\delta_i^d(t)}{\epsilon_c}, \quad (5.13)$$

and according to (5.10), the reference acceleration turns negative, i.e., $a_i^{\text{ref}}(t) < 0$, so that train i begins to decelerate.

However, since the Control Barrier Function (CBF), which will be introduced later, is designed to ensure safety by enforcing the constraint (5.4), i.e.,

$$\delta_i^d(t) \geq 0, \quad (5.14)$$

it prevents $\delta_i^d(t)$ from becoming negative and consequently from decreasing rapidly. As a result, the system may enter a region where $v_i(t)$ and $\frac{\delta_i^d(t)}{\epsilon_c}$ are very close in magnitude. In this region, because the CBF prevents $\delta_i^d(t)$ from decreasing quickly, the reference acceleration computed from (5.10) becomes very small. Consequently, the follower moves with a low speed and a very small deceleration, taking a long time to shorten the remaining distance before the standstill margin. In other words, the train tends to “creep” toward the margin instead of stopping efficiently. This situation can be regarded as a deadlock-like situation, in which the train exhibits both small velocity and small deceleration for an extended period.

It is important to note that a small difference in stopping time between two adjacent trains is desirable, since ideally all trains in a convoy should reach the standstill position in a synchronized manner, which directly affects the overall operational efficiency of the system. To enhance this synchronization, and taking advantage of the guaranteed safety provided by the CBF, a more aggressive reference acceleration is designed for the station-stop phase, so that the follower can decelerate more efficiently while still maintaining safety. This logic can be triggered by the stop signal transmitted by the preceding train via T2T communication.

To determine an appropriate reference deceleration when the leader is at rest, two reference time scales are considered rather than strict physical limits. These time scales provide an estimate of how quickly the follower should decelerate to reach standstill smoothly and efficiently.

The first, denoted $\epsilon_i^{s,1}$, is a distance-based stopping time derived from the remaining distance to the standstill margin. Assuming constant deceleration, the relationship

$$d_i(t) - d_0 = \frac{v_i(t)}{2} \cdot \epsilon_i^{s,1}$$

holds, leading to

$$\epsilon_i^{s,1}(t) = \frac{2(d_i(t) - d_0)}{\max(v_i(t), v_\epsilon)}, \quad (5.15)$$

where v_ϵ is a small threshold introduced only to avoid singularity when v_i approaches zero. This time scale reflects how long the train would need to stop exactly at d_0 under a constant deceleration assumption.

The second, denoted $\epsilon_i^{s,2}$, is a deceleration-based time reference computed from the maximum admissible jerk $j_{\max}$ and the convergence time ϵ_c used in

the nominal design. The corresponding acceleration magnitude is defined as

$$a_i^{\text{ref},\text{stop}} = -j_{\max} \epsilon_c, \quad (5.16)$$

which represents the strongest reference deceleration consistent with the jerk-limited dynamics of the system. The associated stopping time is then expressed as

$$\epsilon_i^{s,2}(t) = \frac{v_i(t)}{j_{\max} \epsilon_c}. \quad (5.17)$$

This value should be interpreted not as a physical bound, but as a smooth reference time derived from the system's rate-of-change constraint.

Finally, the effective stopping time is chosen as

$$\epsilon_i^s(t) = \min(\epsilon_i^{s,1}(t), \epsilon_i^{s,2}(t)), \quad (5.18)$$

so that the train decelerates sufficiently to stop within the available distance without introducing abrupt changes in acceleration. If $\epsilon_i^{s,1}$ is smaller, the resulting deceleration enables the follower to reach standstill approximately at d_0 . If $\epsilon_i^{s,2}$ is smaller, the follower adopts the jerk-consistent reference deceleration in (5.16), ensuring smoothness with respect to the previous control phase.

To unify the reference acceleration under different operating conditions, the overall reference $a_i^{\text{ref}}(t)$ is defined as

$$a_i^{\text{ref}}(t) = \begin{cases} (5.9), & \text{if Train } (i-1) \text{ is moving,} \\ (5.16), & \text{if Train } (i-1) \text{ has stopped.} \end{cases} \quad (5.19)$$

In this way, the proposed law switches from the spacing convergence phase to the station-stop phase, ensuring that the follower train decelerates efficiently while maintaining safety.

Acceleration and force saturation. To ensure riding comfort, a jerk limit is imposed to constrain the rate of change of the reference acceleration:

$$\dot{a}_i^{\text{ref}}(t) \leq j_{\max}, \quad (5.20)$$

where $j_{\max}$ denotes the maximum jerk.

In practice, time is discretized with a sampling interval dt . The discrete reference acceleration is thus saturated according to

$$a_i^{\text{ref}}(t) = a_i^{\text{ref}}(t - dt) + \text{sat}(a_i^{\text{ref}}(t) - a_i^{\text{ref}}(t - dt), -j_{\max}dt, j_{\max}dt), \quad (5.21)$$

where the saturation function is defined as

$$\text{sat}(\cdot, \min, \max) = \begin{cases} \min, & \text{if } \cdot < \min, \\ \cdot, & \text{if } \min \leq \cdot \leq \max, \\ \max, & \text{if } \cdot > \max. \end{cases}$$

To derive the executable reference control force, the reference acceleration $a_i^{\text{ref}}(t)$ is first converted into an equivalent force. According to Newton's second law, the nominal force corresponding to $a_i^{\text{ref}}(t)$ is

$$M_i a_i^{\text{ref}}(t) + r_i(t),$$

where M_i is the train mass and $r_i(t)$ denotes the running resistance. Subsequently, $a_i^{\text{ref}}(t)$ is further saturated by the admissible traction and braking limits (5.1), yielding a feasible reference control force $u_i^{\text{ref}}(t)$ for the subsequent optimization:

$$u_i^{\text{ref}}(t) = \text{sat}(M_i a_i^{\text{ref}}(t) + r_i(t), -M_i B_i, M_i T_i), \quad (5.22)$$

where $\text{sat}(\cdot)$ again denotes the saturation operator.

To summarize, the complete design of the follower controller integrates several functional layers: the spacing convergence law (5.9), the station-stop logic that switches to a_i^{stop} once the predecessor halts, and the jerk filtering that ensures smooth and continuous control actions. These modules are combined with the dynamic model and are subject to the traction and braking saturation limits. Together, they synthesize a unified control law that enables compact convoy operation while guaranteeing safe and smooth standstill behavior.

The resulting $u_i^{\text{ref}}(t)$ serves as the nominal input to the subsequent quadratic programming problem, providing a physically feasible and comfort-aware reference for real-time control implementation.

5.2.2 Collision Avoidance with Time-varying HOCBFs

Our work focuses on maintaining both the flexibility and safety of the original RBD spacing principle, ensuring that VC trains strictly comply with the RBD constraint across various operational scenarios. By imposing (5.4) as a hard constraint, rear-end collisions are always prevented, even in the event of emergency braking by the leading train. This holds true regardless of whether the following train decelerates at a lower service braking rate or is traveling at a higher velocity while attempting to close the gap with its predecessor. Essentially, CBFs can map the constraints from the system states $\mathbf{x}_i(t)$ to the control input $u_i(t)$.

The original collision avoidance constraint (5.4) contains the operator $\max(\cdot, 0)$, making it nondifferentiable at zero. whereas the CBF must be defined as a continuously differentiable function. To overcome this problem, we define two CBF candidates:

$$b_1(\mathbf{x}_i(t), t) = d_i(t) - \theta_i(t) - d_0, \quad (5.23)$$

$$b_2(\mathbf{x}_i(t), t) = d_i(t) - d_0. \quad (5.24)$$

Intuitively, we have

$$\delta_i^d(t) = \min \left(b_1(\mathbf{x}_i(t), t), b_2(\mathbf{x}_i(t), t) \right).$$

Therefore, the constraint $\delta_i^d(t) \geq 0$ holds if and only if both $b_1(\mathbf{x}_i(t), t) \geq 0$ and $b_2(\mathbf{x}_i(t), t) \geq 0$. In other words, enforcing $b_1(\mathbf{x}_i(t), t) \geq 0$ and $b_2(\mathbf{x}_i(t), t) \geq 0$ *simultaneously* is thus equivalent to enforcing the original constraint (5.4).

Substituting (5.23) and (5.24) with $d_i(t) = p_{i-1}(t) - p_i(t) - L_{i-1}$, we notice that the CBF candidates b_1 and b_2 are no longer time-invariant as the CBF defined in (4.11) of Chapter 3. In addition to the follower's state $\mathbf{x}_i(t)$, each barrier function now also depends on the predecessor's position $p_{i-1}(t)$, which is inherently time-varying.

Recalling the convoy formation control discussed in Section 4.3, the system state for (4.5) was defined as $\mathbf{x} = (x_1, x_2)$, where $x_1(t) = D(t)$ represents the inter-vehicle distance and $x_2(t) = v_f(t)$ denotes the follower's velocity. The leader was assumed to move at a constant velocity v_l , which eliminated external time-varying inputs and rendered the follower dynamics autonomous. Consequently, the invariant set defined by the safety constraint (4.7) was also time-invariant, as it involved only the state variables explicitly defined within the system.

In contrast, the present convoy movement control considers a more general case where the predecessor's velocity is neither constant nor incorporated into the follower's system model. In the present setting, the preceding train's motion is time-varying and acts as an external input to the follower's dynamics. As a result, the barrier functions $b_1(\mathbf{x}_i(t), t)$ and $b_2(\mathbf{x}_i(t), t)$ explicitly depend not only on the follower's state $\mathbf{x}_i(t)$ but also on the time-varying position and velocity of the preceding train, $p_{i-1}(t)$ and $v_{i-1}(t)$. For clarity, this dependence is indicated by including t in the argument of b_1 and b_2 . In the following, we omit the explicit time argument for variables other than the HOCBFs, and simply denote $b_1(\mathbf{x}_i(t), t)$ as $b_1(\mathbf{x}_i, t)$.

The original definition of the CBF introduced in (2.8) is time-invariant under the condition $L_g b(\mathbf{x}) \neq 0$, meaning that the constraint depends solely on the system state $\mathbf{x}_i(t)$. However, this property no longer holds in the current setting, where the safety condition explicitly involves the time-varying state of the preceding train.

Moreover, the resulting CBF possessed a relative degree of one: after a single differentiation of b , the control input appeared explicitly as shown in (4.11). Consider (5.24), which enforces that the inter-train distance must remain greater than the constant margin d_0 . According to the standard CBF formulation (2.8), the corresponding constraint on the control input of

train i is expressed as

$$\underbrace{v_{i-1}(t) - v_i(t)}_{L_f b_2(\mathbf{x}_i(t), t)} + \underbrace{0}_{L_g b_2(\mathbf{x}_i(t), t)} \times u_i(t) + \underbrace{\alpha(p_{i-1}(t) - p_i(t) - L_{i-1} - d_0)}_{\alpha(b_2(\mathbf{x}_i(t), t))} \geq 0. \quad (5.25)$$

Here, the control input $u_i(t)$ does not appear explicitly because $L_g b_2(\mathbf{x}_i(t), t) = 0$. As a result, it is impossible to formulate a valid optimization problem as in the CBF-CLF-QP approach presented in Section 4.3.

To overcome this limitation and handle time-varying and higher-order safety constraints arising in the generalized train-following control of VC, the concept of *time-varying High-Order Control Barrier Functions (HOCBFs)* is introduced in the following subsection.

Time-varying HOCBF. As a generalization of CBFs, HOCBFs are capable of handling constraints with arbitrary relative degree [97]. The formal definition of the relative degree is given below.

Definition 5.2.1

(Relative degree) The relative degree of a (sufficiently) differentiable function $b : \mathbb{R}^n \rightarrow \mathbb{R}$ with respect to system (2.3) is the number of times we need to differentiate it along the dynamics of (2.3) until any component of the control $\mathbf{u}$ explicitly appears. $\square$

The relative degree of b also refers to the relative degree of its corresponding constraint $b \geq 0$.

Furthermore, we introduce with the extensional denotations of the Lie derivative:

$$L_g L_f h(\mathbf{x}) = \frac{\partial(L_f h)}{\partial \mathbf{x}} g(\mathbf{x}),$$

$$L_f^k h(\mathbf{x}) = L_f L_f^{k-1} h(\mathbf{x}) = \frac{\partial(L_f^{k-1} h)}{\partial \mathbf{x}} f(\mathbf{x}).$$

For a constraint $b(\mathbf{x}) \geq 0$ with relative degree r , $b : \mathbb{R}^n \times [t_0, \infty) \rightarrow \mathbb{R}$, where t_0 denotes the initial time. A sequence of functions $\psi_i : \mathbb{R}^n \times [t_0, \infty) \rightarrow \mathbb{R}$, for $i \in \{1, \dots, r\}$ are recursively defined as:

$$\psi_i(\mathbf{x}, t) = \dot{\psi}_{i-1}(\mathbf{x}, t) + \alpha_i(\psi_{i-1}(\mathbf{x}, t)), \quad i \in \{1, \dots, r\}, \quad (5.25)$$

where $\alpha_i(\cdot)$, $i \in \{1, \dots, r\}$, denote a $(r - i)^{th}$ order differentiable class $\mathcal{K}$ function, and $\psi_0(\mathbf{x}, t) := b(\mathbf{x}, t)$. The corresponding sequence of *time-varying* forward invariant sets C_i , $i \in \{1, \dots, m\}$, associated with (5.25) are defined in the form:

$$C_i(t) := \{\mathbf{x} \in \mathbb{R}^n : \psi_{i-1}(\mathbf{x}, t) \geq 0\}, \quad i \in \{1, \dots, r\}. \quad (5.26)$$

Definition 5.2.2

(High-Order Control Barrier Function [97]) Let $C_i(t)$, $i \in \{1, \dots, r\}$, be defined by (5.26), and let $\psi_i(\mathbf{x}, t)$, $i \in \{1, \dots, r\}$, be defined by (5.25). A function $b : \mathbb{R}^n \times [t_0, \infty) \rightarrow \mathbb{R}$ is a High-Order Control Barrier Function (HOCBF) of relative degree m for system (2.3) if there exist $(r - i)^{th}$ order differentiable class $\mathcal{K}$ functions α_i , $i \in \{1, \dots, r - 1\}$ and a class $\mathcal{K}$ function α_m such that

$$\sup_{\mathbf{u} \in \mathcal{U}} [L_f^r b(\mathbf{x}, t) + L_g L_f^{r-1} b(\mathbf{x}, t) \mathbf{u} + \frac{\partial^r b(\mathbf{x}, t)}{\partial t^r} + O(b(\mathbf{x}, t)) + \alpha_r(\psi_{r-1}(\mathbf{x}, t))] \geq 0 \quad (5.27)$$

for all $(\mathbf{x}, t) \in C_1(t) \cap C_2(t), \dots, \cap C_r(t) \times [t_0, \infty)$.

The function $O(\cdot)$ denotes the remaining Lie derivatives along f with degree less than or equal to $r - 1$, which is given by

$$O(b(\mathbf{x}, t)) = \sum_{i=1}^{r-1} L_f^i (\alpha_{r-i} \circ \psi_{r-i-1})(\mathbf{x}, t). \quad (5.28)$$

And it is assumed that $L_g L_f^{r-1} b(\mathbf{x}, t) \neq 0$ when $b(\mathbf{x}, t) = 0$. $\square$

With HOCBF, constraints with arbitrary relative degree can be handled. The HOCBF is a general form of the CBF, where an HOCBF with $r = 1$ reduces to a standard CBF. Since (5.27) is also affine in the control input $\mathbf{u}$, the HOCBF can therefore be incorporated as a constraint in the minimum-specification controller, forming a sequence of convex optimization problems, just as the CBF does in CBF-CLF-QP and CBF-QP formulations. In this thesis, we address safety constraints with different relative degrees, and thus adopt the unified term *CBF* to represent both standard and high-order cases.

Constructing constraint for b_1 . The constraint corresponding to $b_1(\mathbf{x}_i, t)$ is

$$\dot{b}_1(\mathbf{x}_i, t) + \alpha(b_1(\mathbf{x}_i, t)) \geq 0, \quad (5.29)$$

where

$$\begin{aligned} \dot{b}_1(\mathbf{x}_i, t) &= \frac{\partial b_1(\mathbf{x}_i, t)}{\partial \mathbf{x}_i(t)} \cdot \frac{d\mathbf{x}_i}{dt} + \frac{\partial b_1(\mathbf{x}_i, t)}{\partial t} \\ &= \begin{bmatrix} -1 & -\frac{v_i}{sB} \end{bmatrix} \dot{\mathbf{x}}_i + v_{i-1} + \frac{v_{i-1} a_{i-1}}{EB_{i-1}}, \end{aligned}$$

the class $\mathcal{K}$ function $\alpha(\cdot)$ is to be chosen later.

Substituting $\dot{\mathbf{x}}_i$ and $\dot{v}_i$ from (5.7), and further substituting in (5.29) (detailed steps omitted), the control constraint is derived as:

$$-\frac{v_i}{M_i SB_i} u_i + \left(\frac{r_i}{M_i SB_i} - 1 \right) v_i + \left(\frac{a_{i-1}}{EB_{i-1}} + 1 \right) v_{i-1} + \alpha(b_1(\mathbf{x}_i, t)) \geq 0, \quad (5.30)$$

which is affine in u_i . All values of u_i satisfying (5.30) ensure that $b_1(\mathbf{x}_i(t), t) \geq 0$ holds.

The feasibility proof of (5.30) is as follows.

Proof. The CBF constraint (5.30), can be written as:

$$\frac{v_i}{M_i SB_i} u_i \leq \left(\frac{r_i}{M_i SB_i} - 1 \right) v_i + \left(\frac{a_{i-1}}{EB_{i-1}} + 1 \right) v_{i-1} + \alpha(b_1(\mathbf{x}_i, t)), \quad (5.31)$$

clearly, $v_i \geq 0$ (and $M_i, SB_i > 0$), multiplying both sides by $\frac{M_i SB_i}{v_i}$ yields

$$u_i \leq U_{b_1}(\mathbf{x}_i, t),$$

where

$$U_{b_1}(\mathbf{x}_i, t) = r_i - M_i SB_i + \frac{M_i SB_i}{v_i} \left[\left(1 + \frac{a_{i-1}}{EB_{i-1}} \right) v_{i-1} + \alpha(b_1(\mathbf{x}_i, t)) \right].$$

Feasibility of the QP reduces to checking

$$u_{\min} \leq U_{b_1}(\mathbf{x}_i, t). \quad (5.32)$$

Assumption 1. The predecessor does not apply a brake that is larger than the emergency braking, throughout the convoy operation, i.e.,

$$a_{i-1} \geq -EB_{i-1}.$$

Here the parameter emergency braking rate EB_{i-1} is set as a positive value.

From this assumption, we can obtain $0 \leq 1 + \frac{a_{i-1}}{EB_{i-1}}$. Thus, $\left(1 + \frac{a_{i-1}}{EB_{i-1}} \right) v_{i-1} \geq 0$ holds. due to the property of class $\mathcal{K}$ function, $\alpha(b_1(\mathbf{x}_i, t)) \geq 0$, as a result, $U_{b_1}(\mathbf{x}_i, t) \geq r_i - M_i SB_i$. Therefore a sufficient condition for feasibility can be reduced as

$$u_{\min} \leq r_i - M_i SB_i. \quad (5.33)$$

To prove 5.33 holds, given that $u_{\min}$ is the largest negative input (maximal braking), we have $u_{\min} \leq -M_i SB_i$. With $r_i \geq 0$, it follows that $-M_i SB_i \leq r_i - M_i SB_i$. Combining the two inequalities, we can deduce that 5.33 holds. Then 5.32 holds, which guarantees feasibility as desired. $\square$

Remark. The discussion above also covers the case where the predecessor train performs emergency braking, i.e., feasibility still holds when $a_{i-1} = -EB_{i-1}$. In both convoy operation and conventional (non-VC) operation, the emergency braking of the leading train is regarded as a hazardous event. On the one hand, such events degrade the performance of train control; on the other hand, if the emergency braking occurs on the follower, the onboard control system must mitigate its consequences in a *fail-safe* manner—this is a fundamental principle in railway signaling, which also applies to VC operation.

In a VC convoy, multiple trains are treated as a single cooperative entity. When the leading train initiates emergency braking, the following trains should also possess the capability to respond safely by applying sufficient braking effort to avoid collision. Although the RBD principle inherently accounts for the leader's emergency braking, the above feasibility analysis of the CBF enforcing the RBD constraint also ensures its effectiveness under such boundary conditions. The corresponding emergency scenario will be illustrated in the simulations.

Constructing constraint for b_2 . For $b_2(\mathbf{x}_i, t)$, the auxiliary functions employing the quadratic class $\mathcal{K}$ function with coefficients ρ_1 and ρ_2 ($\rho_1, \rho_2 > 0$) are defined as

$$\begin{aligned}\psi_0(\mathbf{x}_i, t) &= b_2(\mathbf{x}_i, t), \\ \psi_1(\mathbf{x}_i, t) &= \dot{\psi}_0(\mathbf{x}_i, t) + \rho_1 \psi_0^2(\mathbf{x}_i, t) \\ \psi_2(\mathbf{x}_i, t) &= \dot{\psi}_1(\mathbf{x}_i, t) + \rho_2 \psi_1^2(\mathbf{x}_i, t)\end{aligned}$$

and the HOCBF $b_2(\mathbf{x}_i, t)$ should satisfy

$$\psi_2(\mathbf{x}_i, t) \geq 0. \quad (5.34)$$

With

$$\dot{\psi}_1(\mathbf{x}_i, t) = \ddot{b}_2(\mathbf{x}_i, t) + 2\rho_1 \dot{b}_2(\mathbf{x}_i, t) b_2(\mathbf{x}_i, t),$$

the left hand side of (5.34) is calculated as

$$\psi_2(\mathbf{x}_i, t) = \ddot{b}_2(\mathbf{x}_i, t) + 2\rho_1 \dot{b}_2(\mathbf{x}_i, t) b_2(\mathbf{x}_i, t) + \rho_2 \left(\dot{b}_2(\mathbf{x}_i, t) + \rho_1 b_2^2(\mathbf{x}_i, t) \right), \quad (5.35)$$

where

$$\dot{b}_2(\mathbf{x}_i, t) = v_{i-1} - v_i,$$

and

$$\ddot{b}_2(\mathbf{x}_i, t) = a_{i-1} - \dot{v}_i.$$

Resulting control constraint is given by

$$-\frac{1}{M_i}u_i + \frac{r_i}{M_i} + a_{i-1} + 2\rho_1(v_{i-1} - v_i)b_2(\mathbf{x}_i, t) + \rho_2(v_{i-1} - v_i + \rho_1 b_2^2(\mathbf{x}_i, t))^2 \geq 0. \quad (5.36)$$

The potential conflict between 5.36 and control limits is discussed.

Proof. Rewrite 5.36 to isolate u_i on the one side of the inequality, resulting

$$u_i \leq r_i + M_i a_{i-1} + 2M_i \rho_1(v_{i-1} - v_i)b_2(\mathbf{x}_i, t) + \rho_2(v_{i-1} - v_i + \rho_1 b_2^2(\mathbf{x}_i, t))^2, \quad (5.37)$$

ramping up the RHS into $U_{b_2}(\mathbf{x}_i, t)$, similarly, the feasibility of 5.36 is guaranteed when

$$u_{\min} \leq U_{b_2}(\mathbf{x}_i, t). \quad (5.38)$$

holds.

We observe that $\rho_2(v_{i-1} - v_i + \rho_1 b_2^2(\mathbf{x}_i, t))^2 \geq 0$ always holds, then $U_{b_2}(\mathbf{x}_i, t)$ can be reduced to

$$U'_{b_2}(\mathbf{x}_i, t) = r_i + M_i a_{i-1} + 2M_i \rho_1(v_{i-1} - v_i)b_2(\mathbf{x}_i, t).$$

Correspondingly, 5.38 is equivalent to

$$u_{\min} \leq U'_{b_2}(\mathbf{x}_i, t). \quad (5.39)$$

Assumption 2. The predecessor train is subject to the same or smaller maximal braking rate constraint as train i , i.e., $-B_i \leq a_{i-1}$.

Under this assumption, we have

$$u_{\min} = -M_i B_i \leq M_i a_{i-1}.$$

With $r_i \geq 0$, the feasibility of $b_2(\mathbf{x}_i, t)$ then depends on the term $2M_i \rho_1(v_{i-1} - v_i)b_2(\mathbf{x}_i, t)$. When $v_{i-1} \geq v_i$, inequality 5.39 is guaranteed. When the following train's velocity significantly exceeds that of the predecessor ($v_{i-1} - v_i < 0$ with large velocity difference), infeasibility may occur. With appropriately chosen ρ_1 in the class $\mathcal{K}$ function, feasibility of the QP is guaranteed. $\square$

Remark. Here, ρ_1 is the coefficient of the quadratic class- $\mathcal{K}$ function used in constructing the first-order auxiliary function of the CBF. Choosing a small value of ρ_1 reduces the influence of the negative term when $v_{i-1} - v_i < 0$, but simultaneously leads to a more conservative QP solution when the term is non-negative ($v_{i-1} - v_i \geq 0$). In this case, a smaller ρ_1 yields a smaller U_{b_2} , i.e., a more restrictive permissive set of control inputs u_i that satisfy the CBF constraint $b_2(\mathbf{x}_i, t) \geq 0$. Consequently, the controller may apply unnecessarily small traction forces even when the inter-train distance is large. Therefore, a trade-off arises in the choice of the hyperparameter ρ_1 .

Possible relaxation on assumptions. The feasibility conditions of $b_1(\mathbf{x}_i(t), t)$ and $b_2(\mathbf{x}_i(t), t)$ rely on the assumptions of braking rates of the control subject train i and its predecessor train $i - 1$.

In order to obtain sufficient feasibility guarantees for $b_1(\mathbf{x}_i(t), t)$, Assumption 1 is required. It is reasonable to assume the trains operate in VC mode because once it happens, it will be viewed as “failure”, breaking the basic operation condition for VC. When it occurs in any train in the convoy for any reason, the the convoy will dissemble. Even though, when it happens, the convoy should be capable to handle. the scenario of train $i - 1$ takes an emergency braking maneuver will be viewed as an extreme scenario, simulation in the next chapter will demonstrate the ability of our control methods to cope this case.

For $b_1(\mathbf{x}_i(t), t)$, Assumption 2 is adopted to ensure that the predecessor train is subject to the same or smaller braking rate constraint as Train i . However, this is a strong assumption in realistic convoy operation, heterogeneous braking capabilities are possible, different trainsets may exhibit different braking abilities. Under actual operation conditions, especially the heavy-haul train convoy, is usually heterogeneous rather than homogeneous. Since the cargo are probably of different masses and lengths.

If the predecessor has a weaker braking capability, the required safe spacing increases, and the corresponding permissive set of feasible u_i shrinks. This may lead to infeasibility of the QP even if the follower itself respects its own braking limits. Conversely, if the follower has a weaker service braking capability, then $u_{\min} = -M_i SB_i$ becomes less negative, tightening the feasibility condition $u_{\min} \leq U_{\text{CBF}}(\cdot)$.

Therefore, the feasibility of the CBF-based controller is highly sensitive to the heterogeneity in braking performance. While adjusting hyperparameters (e.g., increasing ρ_1 or γ) can make the CBF more conservative and thereby improve robustness to such heterogeneity, this comes at the cost of reducing the size of the permissive set, i.e., limiting the admissible traction force even when the spacing

The feasibility analysis above highlights that the admissible control region is strongly dependent on the braking performance of both the follower (SB_i) and the predecessor (EB_{i-1}). Taken together, the mismatch between SB_i and EB_{i-1} may shrink this region and potentially render the QP infeasible.

5.2.3 Delay-aware control design

To precisely capture the impact of communication delay, the control and communication processes are assumed to operate synchronously within discrete control cycles of duration dt . Normally, at each cycle t_k , train i obtains the latest available data packet from its preceding train, computes

its control input based on the received information, and applies the control command within the same period $[t_k, t_k+dt)$. Although the system dynamics are modeled in continuous time, a sufficiently small dt is selected to minimize sampling effects. This setting also reflects the digital implementation of communication and control modules in practical onboard systems, where both processes are executed periodically.

The transmitted packet includes the preceding train's position and velocity $\mathbf{x}_{i-1}$, as well as its acceleration a_{i-1} , which can be grouped into the state vector:

$$\Psi_{i-1}(t) = [p_{i-1}(t), v_{i-1}(t), a_{i-1}(t)]^\top.$$

We first discuss the case of a constant communication delay and introduce an adaptive logic to handle delayed information. Subsequently, this analysis is extended to the more general case of time-varying delays, incorporating the modeling of the communication channel and data buffer.

5.2.3.1 Constant delay case.

In our previous work, we investigated the simple case where the communication delay is constant. The T2T channel was modeled as a First-In-First-Out (FIFO) buffer. When a constant delay τ occurs at time t' , no updated packet arrives from train $i-1$ during the interval $[t', t'+\tau)$; that is, train i only possesses the latest available state of train $i-1$ at $t'-dt$. After τ , the system resumes receiving packets, but the information remains delayed until the delay is cleared (by the communication module) at time t'' .

With embedded timestamps, train i can detect the delay τ upon receiving the first outdated packet at $t'+\tau$, and can determine t'' precisely when a packet arrives whose send-time stamp equals t' . Assuming $t'' > t'+\tau$, the received state vector under delay, $\hat{\Psi}_{i-1}(t) = [\hat{p}_{i-1}(t), \hat{v}_{i-1}(t), \hat{a}_{i-1}(t)]^\top$, is defined as

$$\hat{\Psi}_{i-1}(t) = \begin{cases} \Psi_{i-1}(t'), & t \in [t', t'+\tau), \\ \Psi_{i-1}(t-\tau), & t \in [t'+\tau, t''), \\ \Psi_{i-1}(t), & \text{otherwise.} \end{cases} \quad (5.40)$$

The last case covers both the delay-free situation and time intervals outside the delay duration. The delay τ is modeled as a constant bounded by the maximum value $\tau_{\max}$.

Response to constant delay. The reference acceleration under delay is computed based on the availability of the received information, as given in (5.40). Specifically, during the delay period $t \in [t', t'')$, it is determined by the sign of the received (delayed) predecessor acceleration $\hat{a}_{i-1}(t)$ as follows:

1. If $\hat{a}_{i-1}(t) \geq 0$, the following train maintains its velocity (i.e., cruises).
2. If $\hat{a}_{i-1}(t) < 0$, the following train applies service braking.

Once the delay period ends, the computation reverts to the nominal logic defined in (5.9). To summarize, the *delay-aware reference acceleration*, denoted by $\hat{a}_i^*(t)$, is defined as

$$\hat{a}_i^*(t) = \begin{cases} 0, & t \in [t', t''] \text{ and } \hat{a}_{i-1}(t) \geq 0, \\ -SB_i, & t \in [t', t''] \text{ and } \hat{a}_{i-1}(t) < 0, \\ a_i^{\text{ref}}(t), & \text{otherwise.} \end{cases} \quad (5.41)$$

This rule serves as a conservative delay-handling mechanism that does not rely on motion prediction of the preceding train, but guarantees safety without triggering unnecessary emergency braking. By preserving the inter-train spacing within a safe margin under delayed information, it maintains VC functionality during occasional communication delays.

However, this conservative reaction to any detected delay may degrade convoy synchronism, particularly when the delay is short or intermittent. To further assess its effectiveness, this mechanism will later be compared with a compensation-based approach under both constant and time-varying delay conditions in the simulation study.

5.2.3.2 Time-varying delay case

Communication delay in realistic railway systems is influenced by many factors, including queuing, processing, transmission, network congestion, and random packet loss. These factors make the delays intrinsically uncertain and stochastic, often exhibiting significant variability over time.

Modeling. Figure 5.2 illustrates the three communication cases considered in this study: (a) nominal (without delay), (b) constant delay, and (c) time-varying delay. The left diagrams show the information flow between the preceding train $i - 1$ and the following train i , while the plots on the right depict the corresponding delay evolution over discrete time steps.

In the upper part (a), the case without delay is illustrated. Each train transmits its state information periodically at every discrete instant k , and the following train receives the packet within the same step. This does not imply that the transmission time is physically zero, but rather reflects the periodic operation of digital communication and control modules in real ATO/ATP systems, where, at the onboard level, the processes of data reception, computation, and control execution are synchronized within each cycle. It should be noted that this description does not represent a

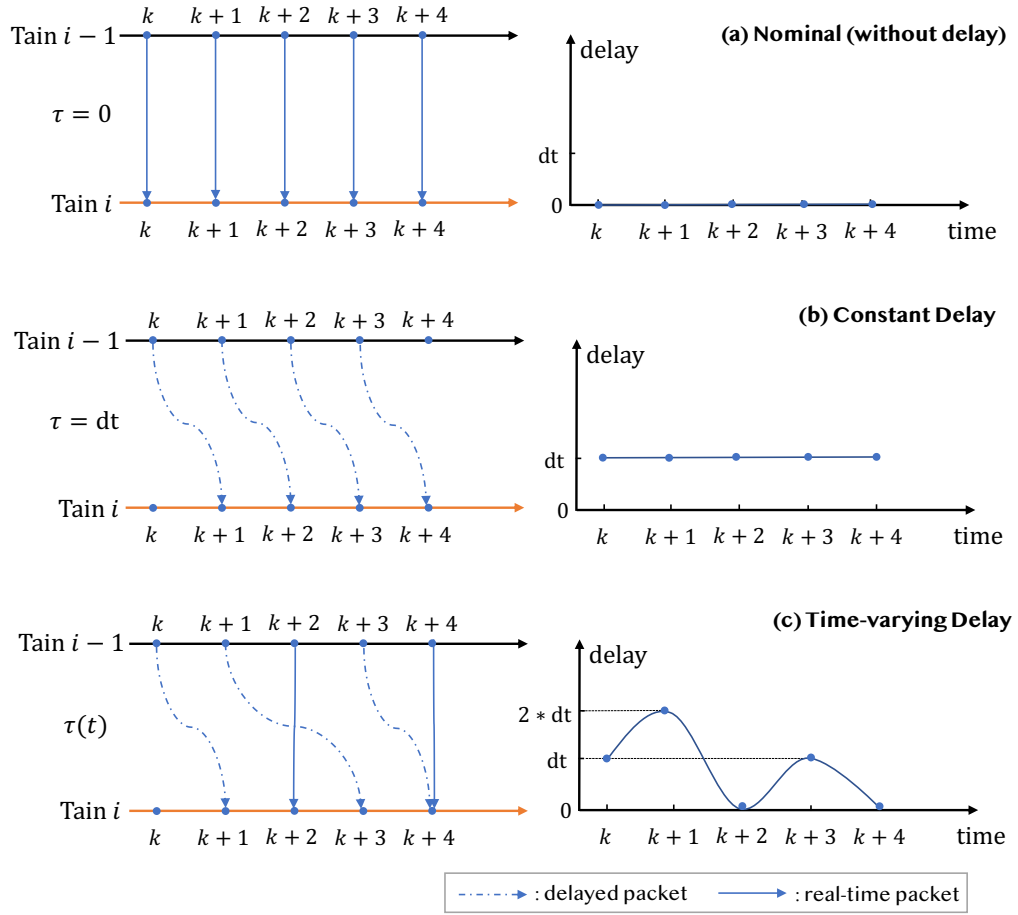


Figure 5.2: Illustrations of constant and time-varying communication delays.

perfectly ideal communication, but a practical simplification that aligns the global timing of transmission and control updates within a unified discrete-time framework [98]. In practice, such synchronization can be achieved through standard mechanisms such as Cyclic Redundancy Check (CRC) and timestamp alignment in the communication protocol. Therefore, in the absence of delay, a packet sent by train $i-1$ at step k is considered to be received by train i at the same step.

In (b), a constant delay $\tau = dt$ is introduced. Each train transmits its state information periodically at every discrete instant k , which corresponds to the control cycle discussed previously. In our simplified representation, a packet transmitted by train $i-1$ at time step k is received by train i one step later, at $k+1$.

The lower part (c) extends this concept to the time-varying delay case. Here, the delay $\tau(t)$ fluctuates dynamically, as illustrated by the wavelike curve on the right. As shown on the right, at k the delay equals dt , meaning that a packet sent at k is received at $k+1$. At $k+1$, the delay increases to $2dt$, so the packet can only be received at $k+3$. Later, at $k+2$, the delay

drops to zero, indicating that the transmission has temporarily returned to real-time; the packet sent at this moment can be immediately received and used by the follower. Such fluctuations may cause packets to arrive out of order or multiple packets to be received simultaneously. For instance, here at $k+4$, when the delay returns to zero, both a newly transmitted real-time packet and a previously delayed one may arrive together. To handle this phenomenon, a data buffer is introduced in the following subsection. Its length corresponds to the maximum admissible delay, ensuring that packets arriving within the delay bound can be processed in sequence, while excessive delay beyond this bound triggers the termination of the VC operation.

To more accurately reflect realistic communication environments, in this thesis, the time-varying delay $\tau(t)$ is modeled as a discrete-time Gauss–Markov process, where τ_k denotes the delay at cycle k :

$$\tau_{k+1} = \tau_k + \alpha(\mu - \tau_k) + \sigma w_k, \quad w_k \sim \mathcal{N}(0, 1), \quad (5.42)$$

where μ is the mean delay, w_k is a Gaussian noise following a standard normal distribution, σ controls the fluctuation intensity, and $\alpha > 0$ determines the rate of mean reversion. The model (5.42) defines a first-order Markov process, since the next delay value τ_{k+1} depends only on the current state τ_k and a white Gaussian noise term w_k , independent of all previous samples. The time-varying delay τ_k is also bounded, i.e. $0 \leq \tau_k \leq \tau_{\max}$.

Motivated by the characteristics of the time-varying delay illustrated above, we now formalize the time-varying delay handling process. As discussed, when the delay fluctuates within a bounded range, packets may arrive in a disordered manner or even simultaneously. To manage such situations, the receiver of each following train maintains a finite buffer to temporarily store the incoming packets, ensuring that delayed information within the admissible bound can still be properly utilized.

Buffer structure. We assume that all trains in the convoy share a global clock, and each transmitted packet carries a timestamp indicating its send time. For simplicity, the discrete index k represents both the control step and the packet timestamp; that is, a packet created by train $i - 1$ at time t_k is labeled by k . Under normal conditions, the packet sent at t_k is received at the same global time,

$$t_{\text{receive}} = t_k.$$

When a time-varying delay $\tau(t) \geq 0$ occurs, bounded by the maximal admissible delay $\tau_{\max}$, the packet becomes available to train i at

$$t_{\text{receive}} = t_k + \tau(t_k) = t_{k + \lceil \tau(t_k)/dt \rceil},$$

where dt is the discrete control period (set to 0.02s in this work). The ceiling operator $\lceil \cdot \rceil$ ensures that packets experiencing partial-cycle delays are processed at the next available control step.

The receiver of train i maintains a bounded buffer $\mathcal{B}(t_k)$ of length $L_{\mathcal{B}} = \lceil \tau_{\max}/dt \rceil$,

$$\mathcal{B}(t_k) = \{\Psi_{i-1}(s_1), \Psi_{i-1}(s_2), \dots, \Psi_{i-1}(s_L)\}, \quad L = \lceil \tau_{\max}/dt \rceil,$$

where $\Psi_{i-1}(s) = [p_{i-1}(s), v_{i-1}(s), a_{i-1}(s)]^\top$ is the state vector contained in the packet sent at time s . The buffer size L guarantees that all packets delayed by no more than $\tau_{\max}$ remain accessible within the storage window. Updates to the buffer follow the following rules.

1) Arrival rule. At each control step t_k , when the buffer receives new packet, it first compares its timestamp with the latest one available in the buffer. If the latest received one has the timestamp greater than the latter, the new packet will be stored in the buffer; otherwise, it is discarded. Concretely, let

$$s_{\max}(t_k) = s_{\max}(\mathcal{B}(t_k)) = \max\{s : (s, \cdot) \in \mathcal{B}(t_k)\}$$

denote the latest (newest) timestamp, corresponding to the latest (newest) packet currently stored in the buffer at t_k . When a new packet $\Psi_{i-1}(s_{\text{new}})$ arrives at t_k ,

$$\text{if } s_{\text{new}} > s_{\max}(t_k) \text{ then insert into } \mathcal{B}; \quad \text{else discard (out-of-order).}$$

If the buffer is full ($|\mathcal{B}| = L$), the oldest element (i.e., with the smallest timestamp) is discarded. After this step, the new packet is added or discarded. Let $\mathcal{B}'(t_k)$ denote the updated buffer after executing the arrival rule. The new latest timestamp of packets in $\mathcal{B}'(t_k)$ is recomputed too, denoted as $s'_{\max}(t_k)$.

2) Selection rule. At same control step t_k , after the buffer update according to above rule, the controller selects the most recent packet currently stored in the buffer:

$$\hat{s}(t_k) = s'_{\max}(t_k), \quad \hat{\Psi}_{i-1}(t_k) = \Psi_{i-1}(\hat{s}(t_k)).$$

Hence, the selection rule always uses the newest timestamp after the arrival update, ensuring that the controller operates on the most up-to-date valid packet available at t_k .

Thus, the selected packet corresponds to the most recent timestamp, which may differ from the most recently received packet when the delay varies. The above mechanism generalizes the constant-delay behavior defined

in (5.40), by (i) rejecting out-of-order packets ($s_{\text{new}} \leq s_{\text{max}}$) and (ii) always using the latest valid timestamp at decision time.

Because $L = \tau_{\text{max}}/dt$, any packet delayed by at most τ_{max} remains available within the buffer window.

Compensation for time-varying delay. In order to reduce the risk brought by time-varying communication delays, we compute a conservative estimate of the predecessor's kinematic states and substitute it with the ones used in the reference control force and the CBF constraints. Let $\hat{\Psi}_{i-1}(t_k) = [\hat{p}_{i-1}(t_k), \hat{v}_{i-1}(t_k), \hat{a}_{i-1}(t_k)]^\top$ be the latest received packet at time t_k with timestamp $\hat{s}(t_k) \leq t_k$. We denote its actual length of delay

$$\tau_{\text{real}} = t_k - \hat{s}(t_k). \quad (5.43)$$

Assume a known jerk bound $j_{\text{max}} > 0$ on train $i - 1$'s acceleration, i.e.

$$|\dot{a}_{i-1}| \leq j_{\text{max}}.$$

Therefore, the unknown status of the preceding train during τ_{real} is estimated based on the latest information, using the maximum possible rate of acceleration change j_{max} . We use $\hat{\cdot}$ to denote the raw received states and $\tilde{\cdot}$ denotes the j_{max} -compensated estimates states. Then the compensated information The j_{max} -compensated velocity is

$$\tilde{v}_{i-1}(t_k) = \max\{\hat{v}_{i-1}(t_k) + \Delta v(t_k), 0\}, \quad (5.44)$$

where the compensation $\Delta v(t_k)$ on $\hat{v}_{i-1}(t_k)$ is derived by

$$\Delta v(t_k) = \hat{a}_{i-1}(t_k)\tau_{\text{real}} - \frac{1}{2}j_{\text{max}}\tau_{\text{real}}^2. \quad (5.45)$$

Corresponding placement during τ_{real} is

$$\tilde{p}_{i-1}(t_k) = \hat{p}_{i-1}(t_k) + \Delta p(t_k), \quad (5.46)$$

with

$$\Delta p(t_k) = \max\{\hat{v}_{i-1}(t_k)\tau_{\text{real}} + \frac{1}{2}\hat{a}_{i-1}(t_k)\tau_{\text{real}}^2 - \frac{1}{6}j_{\text{max}}\tau_{\text{real}}^3, 0\}. \quad (5.47)$$

In turn, the spacing variables in (5.3) and (5.5) are reformed from the compensated states. The compensated relative distance and braking distance difference are

$$\tilde{d}_i(t_k) = \tilde{p}_{i-1}(t_k) - p_i(t_k), \quad (5.48)$$

$$\tilde{\theta}_i(t_k) = \frac{v_i(t_k)^2}{2SB_i} - \frac{\tilde{v}_{i-1}(t_k)^2}{2EB_{i-1}}. \quad (5.49)$$

The corresponding minimum safe distance and spacing deviation become

$$\tilde{d}_i^*(t_k) = \max\{\tilde{\theta}_i(t_k), 0\} + d_0, \quad (5.50)$$

$$\tilde{\delta}_i^d(t_k) = \tilde{d}_i(t_k) - \tilde{d}_i^*(t_k). \quad (5.51)$$

Compensated reference input. The reference acceleration adopts the compensated states to eliminate delay-induced bias:

$$\tilde{a}_i^{\text{ref}}(t_k) = \frac{2}{\epsilon_c^2} \tilde{\delta}_i^d(t_k) + \frac{2}{\epsilon_c} (\tilde{v}_{i-1}(t_k) - v_i(t_k)) + \tilde{a}_{i-1}(t_k), \quad (5.52)$$

where $\epsilon_c > 0$ is the fixed convergence window. This choice improves tracking when delays are present because the predecessor is treated as closer and slower in the conservative sense.

Compensated CBF constraints. The CBF candidates are evaluated with the same compensated states so that feasibility and safety are certified against admissible delays. Define

$$\tilde{b}_1(\mathbf{x}_i, t_k) = \tilde{d}_i(t_k) - \tilde{\theta}_i(t_k) - d_0, \quad (5.53)$$

$$\tilde{b}_2(\mathbf{x}_i, t_k) = \tilde{d}_i(t_k) - d_0. \quad (5.54)$$

Enforcing $\tilde{b}_1(\mathbf{x}_i, t_k) \geq 0$ and $\tilde{b}_2(\mathbf{x}_i, t_k) \geq 0$ guarantees $\tilde{\delta}_i^d(t_k) \geq 0$. The associated HOCBF inequalities retain their affine form in the control input after substituting $\tilde{v}_{i-1}$ and $\tilde{a}_{i-1}$ in place of the received $\hat{v}_{i-1}$ and $\hat{a}_{i-1}$. In this way, both the safety filter and the reference law are aligned to the same worst-case view of the predecessor's motion, which improves robustness to random time-varying delays without altering the QP structure.

5.2.4 Robustness-oriented control design

There does not exist a perfect model. In addition to communication delays, addressing uncertainties beyond the nominal train dynamics is also essential for achieving precise and robust controller design. In VC train control, uncertainties often arise from two main sources:

1. Parametric uncertainties in the train dynamic model, including inaccurate estimates of train mass, traction/braking efficiency, running resistance coefficients, and errors in sensor-based state estimation (e.g., position and velocity).
2. External disturbances acting on the train body, including unmodeled resistance due to track geometry (e.g., slopes, curves, tunnels), environmental factors (e.g., strong winds), and unpredictable weather conditions (e.g., rain or snow causing slippery rails), as well as loss of adhesion during braking.

Note that track data including gradients and curves are typically available to trains via trackside equipment (e.g., balises), which were not considered in the previous section of convoy formation contrl. Here, we include the additional resistances caused by gradient and curve variations within the external disturbance term.

In this thesis, we introduce a perturbation term $\omega(t)$ into the nominal affine dynamics of train i in (5.7), leading to

$$\dot{\mathbf{x}}_i(t) = f(\mathbf{x}_i(t)) + g(\mathbf{x}_i(t)) u_i(t) + \omega(t), \quad (5.55)$$

where $\omega(t)$ collectively introduce variability into train dynamic, and we assume it is bounded. This section discusses how the proposed controller maintains safety in the presence of $\omega(t)$.

Robust CBF overview. The concept of *Robust CBFs* (RCBFs) was introduced to extend CBF-based safety guarantees to uncertain systems. Several RCBF variants have been developed to enhance robustness, including worst-case formulations, observer-based designs, and tunable mechanisms that mitigate conservatism [99]. These methods introduce a compensating term $\sigma(x, u, t)$ into the barrier constraint to bound the effect of model mismatch and external perturbations. While a large σ ensures strict safety at the cost of conservatism, adaptive or observer-based terms alleviate conservatism by tightening the bound online.

In this work, we do not explicitly adopt these formulations. Instead, we analyze whether the nominal CBFs $b_1(\mathbf{x}, t)$ and $b_2(\mathbf{x}, t)$ are already robust to bounded disturbances.

CBF robustness analysis. Following Peng *et al.* [100], the robustness of HOCBF was analyzed for time-varying nonlinear affine systems subject to disturbances. They proved that if $b(\mathbf{x}, t)$ is an HOCBF of relative degree r for the system

$$\dot{\mathbf{x}} = f(\mathbf{x}, t) + g(\mathbf{x}, t) u + \omega(t),$$

then the associated safe set $\mathcal{C} = \{\mathbf{x} : b(\mathbf{x}, t) \geq 0\}$ is *uniformly globally asymptotically stable* (UGAS) with respect to $\mathcal{C}$. This result relies on constructing a Lyapunov function directly from the HOCBF chain and exploiting the relationships among the higher-order derivatives. Hence, the HOCBF not only guarantees forward invariance of $\mathcal{C}$ but also ensures asymptotic convergence to $\mathcal{C}$ under bounded disturbances.

The two CBF constraints defined in Section 5.2.2 can be rewritten to match the HOCBF form used in [100]. For a constraint function $b(\mathbf{x}_i)$ of relative degree r , the r -th derivative satisfies

$$L_f^r b(\mathbf{x}_i) + L_g L_f^{r-1} b(\mathbf{x}_i) u_i + K_j \boldsymbol{\xi}_j(\mathbf{x}_i) \geq -\sigma_j, \quad (5.56)$$

where K_j is a constant gain vector whose entries are selected to render the characteristic polynomial Hurwitz, ξ_j collects the lower-order derivatives of b , and $\sigma_j \geq 0$ is a relaxation variable introduced to preserve feasibility under uncertainties. When $\sigma_j = 0$, the constraint reduces to the nominal CBF form.

By the assumptions of Theorem 1 in [100], if $f(\mathbf{x})$ and $g(\mathbf{x})$ are locally Lipschitz and the control input remains within a compact set $\mathcal{U}$ such that $L_g L_f^{r-1} b_j(\mathbf{x}_i) \neq 0$, the safe set $\mathcal{C} = \bigcap_j \{ \mathbf{x}_i : b_j(\mathbf{x}_i) \geq 0 \}$ is forward invariant and asymptotically stable with respect to $\mathcal{C}$.

This implies that the proposed convoy controller, which enforces the inequalities in (5.56) without additional margins, preserves safety and convergence of the inter-train spacing even when the dynamics are affected by bounded disturbances $\omega_i(t)$. The subsequent simulations further confirm this robustness property: although stochastic and time-varying disturbances are injected into the dynamics, both $b_1(\mathbf{x}_i)$ and $b_2(\mathbf{x}_i)$ remain nonnegative, and the safety constraints are satisfied.

5.3 Controller Synthesis

The proposed VC convoy movement control framework is illustrated in Fig. 5.3. Each follower computes its own control input in a decentralized manner using the real-time information received from its predecessor. The control process begins with the acquisition of the predecessor's kinematic states, which are first processed by the delay compensator to mitigate the effects of delays. The compensated information is then used to calculate the convergence acceleration, which drives the follower toward the desired relative braking distance spacing.

When the preceding train comes to a stop, a station-stop logic is activated to generate a negative reference acceleration, allowing the follower to decelerate timely and stop within the default clearance d_0 . The reference acceleration is subsequently regulated by the jerk constraint and converted into the reference control force, which is further bounded by traction/braking and velocity limits. Meanwhile, the CBF constraints continuously enforce collision avoidance. At each control step, both the reference control and CBF constraints are incorporated into a QP, from which the optimal traction/braking force is obtained and applied to update the train dynamics. This integrated structure enables real-time safe and stable train following across all operational phases, from acceleration to station stop.

Remark. Although the CBF–CLF–QP approach for formation control has already been introduced in the previous chapter, and the simulation results in Section 4.3 show that the controlled follower can smoothly adjust its

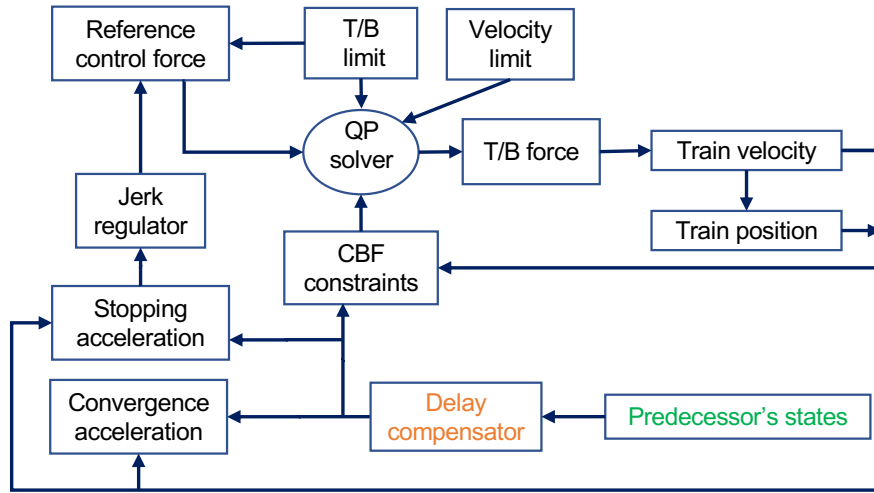


Figure 5.3: Framework of the VC Follower Controller.

velocity to merge with the cruising leader, we observe that the controller developed in this section also has the potential to handle convoy formation. Specifically, it can shorten the inter-train distance to the desired value when the initial spacing is large.

According to (5.9), the reference control force is computed to drive the distance deviation to zero. Since the distance deviation is defined for both coupled and uncoupled trains, the desired distance in either case is the same. The relative braking distance plus a safety margin. In other words, the CBF–CLF-based controller employs the CLF constraint (4.10) to enforce distance convergence, whereas in the CBF-based controller introduced here, this convergence is achieved through the heuristic definition of a_i^{ref} in (5.9).

QP-based algorithm in implementation Our goal is to determine the control input to achieve the distance convergence subject to RBD constraints. To minimizing the deviation from the reference control force in (5.22), the QP of train i is constructed by combining the above CBF constraints as

$$\begin{aligned} u_i(t) &= \arg \min \left(H_i(u_i(t) - u_i^{\text{ref}}(t))^2 \right) \\ \text{s.t. } & \text{Train dynamic (5.7), control limit (5.1),} \\ & \text{velocity limit (5.2), CBF constraints (5.30), (5.36).} \end{aligned} \quad (5.57)$$

where the scalar H_i is the quadratic cost weight. By synthesizing the QP formulation with the defined CBF constraints, a controller for each train i is obtained in a decentralized manner. The detailed procedure of the safe optimal control for the VC followers is summarized in the following Algorithm. In practice, the continuous-time problem is implemented in discrete time with step size dt : at each time step k , a QP with CBF constraints is solved

for each follower. The resulting control input is held constant over the interval and applied to the system until the next update.

Algorithm 5.1 Online Implementation of VC Controller

```

1: Initialize the states  $\mathbf{x}_i$  for all followers  $i = 1, \dots, N$ 
2: for each time step  $k = 1, \dots, K$  do
3:   for each train  $i = 1, \dots, N$  do
4:     Receive predecessor's packet  $\Psi_{i-1}(s_1)$  with timestamp  $s_1 \leq t_k$ 
5:     Update buffer and compute compensated predecessor state  $\tilde{\Psi}_{i-1}(t_k)$ 
6:     Compute reference control force  $u_i^{\text{ref}}(t_k)$  via Eq. (5.22) using  $\tilde{\Psi}_{i-1}(t_k)$ 
7:     Formulate CBF constraints via Eqs. (5.30) and (5.36) using  $\tilde{\Psi}_{i-1}(t_k)$ 
8:     Solve the QP in Eq. (5.57) to obtain the optimal input  $u_i(t_k)$ 
9:   end for
10:  for  $i = 1, \dots, N$  do
11:    Update the state  $\mathbf{x}_i(k+1)$  with  $u_i(k)$  via Eq. (5.7)
12:  end for
13: end for

```

5.4 Summary

This chapter proposed a generalized decentralized control framework for VC followers, integrating reference control design, safety enforcement, delay compensation, and robustness enhancement into a unified optimization-based structure. A reference control force was first designed to achieve spacing convergence and timely stopping based on the instantaneous kinematic states of the preceding train, while ensuring smoothness and feasibility under jerk and force limits. Safety was formally defined by the relative braking distance principle and guaranteed through time-varying High-order Control Barrier Functions, which explicitly incorporate the dynamic and time-dependent spacing constraints. To handle communication imperfections, a delay-aware control design was further introduced, where both constant and time-varying delays were modeled and mitigated through adaptive and compensation-based strategies. To evaluate robustness, unknown bounded disturbances were injected into the follower dynamics without modifying the nominal HOCBF constraints. The results confirmed that the proposed controller preserves safety and spacing convergence under such dynamic uncertainties. All designed components were finally integrated into a sequence of real-time QPs that satisfy the computational constraints of onboard systems. This framework enables safe, efficient, and fully decentralized VC operation without centralized coordination or trajectory prediction of the leading trains.

Main contributions.

1. Realization of the RBD-based spacing policy using time-varying HOCBFs, providing a formal and flexible safety guarantee for convoy operation.
2. Design of a reference control force that achieves distance convergence and station stopping, with comfort ensured through jerk and force saturation.
3. Development of a delay-aware control mechanism capable of coping with both constant and stochastic time-varying communication delays, ensuring safety and reducing unnecessary conservatism.
4. Validation of controller robustness under dynamic uncertainties, demonstrating that the nominal HOCBF structure can maintain safety without explicit robustification terms.
5. Integration of the reference control and HOCBF constraints into a real-time QP formulation, enabling practical onboard implementation within decentralized architecture.

In the next section, simulation results will demonstrate the effectiveness of the proposed controller in various scenarios.

Chapter 6

Simulation and Performance Analysis

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This chapter presents the simulation and case studies to validate the effectiveness of the proposed CBF-based control framework for VC followers in Chapter 5. The chapter are organized into three parts. Section 6.1 introduces the baseline experiments, where nominal and formation scenarios are simulated to verify the controller’s performance under normal operations. The simulation setup and key performance indexes used for this chapter are also defined in this section. Section 6.2 conducts performance analyses by varying design parameters and considering heterogeneous convoys to discuss their influence on the control. Section 6.3 extends the investigation to challenging conditions, including communication delays, model uncertainties, and emergency braking, to further evaluate the robustness of the proposed controller.

For all numerical simulations presented in this thesis, the QPs are solved using the MATLAB toolbox `quadprog`, while the closed-loop system trajectories are simulated with `ode45`. Specifically, MATLAB R2020a was used on a MacBook Pro (1.4 GHz Intel Core i5 CPU, 8 GB RAM).

6.1 Baseline Case Studies

To evaluate the effectiveness of the proposed control method, two representative cases are investigated in this section. The first case, nominal convoy operation, examines the performance of the proposed controller after the convoy is fully coupled and moving under normal conditions. The second, convoy formation scenario, considers the process where initially uncoupled trains form a convoy, allowing a direct comparison with the CLF-based controller introduced in the previous chapter. These cases together provide a comprehensive view of the controller's capability in both steady convoy operations and formation transitions, serving as the basis for the subsequent analyses.

6.1.1 Simulation setup

The focus of the simulations is a typical journey of a VC convoy, which undergoes an acceleration–cruise–deceleration process as it approaches its destination. Two representative railway segments are examined: the high-speed line (HS) and the metro line (MT). For each case, a dedicated movement profile of the leading train is designed.

The leading train's trajectory is pre-defined rather than actively controlled, as its operation is assumed to be governed by an autonomous driving module that is independent of the VC function. In this thesis, the motion of the leading train is manually designed to represent a realistic driving behavior. Under the predecessor–following topology adopted in this work, the follower controllers are responsible for actively responding to their respective predecessors and maintaining safe inter-train spacing throughout the convoy movement.

6.1.1.1 Parameters and configurations.

Firstly, for each railway segment, distinct rolling stocks are considered, and their parameters are listed in Table 6.1. From the table we can find, the HS trains are heavier and longer, exhibiting higher inertia and distinct resistance characteristics compared to the MT trains. In the HS case, a convoy of three coupled trains (Train 0–2) is simulated, whereas in the MT case, a longer convoy consisting of four trains (Train 0–3) is considered.

It is worth noting that the parameters in Table 6.1 are obtained from experimental measurements, and may vary in practice depending on driving conditions. For instance, the vehicle mass changes with passenger load, while the aerodynamic drag coefficient can be affected by meteorological factors. Nevertheless, the case studies in this section are regarded as baseline

Table 6.1: Train dynamic parameters of HS and MT convoy

Parameter	Value	Unit
High-speed train dynamic parameters		
M_0, M_1, M_2	383,000	kg
L_0, L_1, L_2	100	m
$C_{0,0}, C_{0,1}, C_{0,2}$	0.01176	N/kg
$C_{1,0}, C_{1,1}, C_{1,2}$	0.00077616	Ns/m·kg
$C_{2,0}, C_{2,1}, C_{2,2}$	1.6×10^{-5}	Ns ² /m ² ·kg
SB_0, SB_1, SB_2	1.2	m/s ²
EB_0, EB_1, EB_2	1.5	m/s ²
B_0, B_1, B_2	1.8	N/kg
T_0, T_1, T_2	1.8	N/kg
Metro train dynamic parameters		
M_0, M_1, M_2, M_3	115,000	kg
L_0, L_1, L_2, L_3	50	m
$C_{0,0}, C_{0,1}, C_{0,2}, C_{0,3}$	0.01043	N/kg
$C_{1,0}, C_{1,1}, C_{1,2}, C_{1,3}$	0.000113	Ns/m·kg
$C_{2,0}, C_{2,1}, C_{2,2}, C_{2,3}$	2.7×10^{-5}	Ns ² /m ² ·kg
SB_0, SB_1, SB_2, SB_3	1.5	m/s ²
EB_0, EB_1, EB_2, EB_3	1.8	m/s ²
B_0, B_1, B_2, B_3	2	N/kg
T_0, T_1, T_2, T_3	2	N/kg

simulations, and the parameters listed here will be used in the baseline case studies. Further variations in these parameters will later be treated as uncertainties in the subsequent analyses.

Furthermore, the HS and MT convoys adopt different configurations, as summarized in Table 6.2. The default margin d_0 denotes the minimum separation at standstill, while the convergence time ϵ_c specifies the time horizon used in the reference acceleration calculation for distance convergence. The limit velocity v_{lim} represents the maximum permitted line speed, which is set to 75 m/s (270 km/h) for the high-speed convoy and 50 m/s (180 km/h) for the metro convoy. The maximum jerk j_{max} , used for smoothing the acceleration in (5.21), is set to 0.12 m/s³ for the high-speed train and 0.20 m/s³ for the metro train in our simulations.

As for the class- $\mathcal{K}$ functions used in the CBFs and HOCBFs, we choose a linear function with coefficient 10 for $\alpha(b_1(\mathbf{x}_i, t))$ in (5.29), for all following trains in both HS and MT convoys. As introduced earlier, for b_2 , two quadratic class- $\mathcal{K}$ functions with same coefficient 1 ($\rho_1 = \rho_2 = 1$) are

Table 6.2: Operational configurations for HS and MT convoy

Parameter	Value	Unit
High-speed train configurations		
d_0	10	m
ϵ_c	5	s
v_{lim}	75	m/s
j_{max}	0.12	m/s ³
Metro train configurations		
d_0	5	m
ϵ_c	3	s
v_{lim}	50	m/s
j_{max}	0.20	m/s ³

selected. The choice of the class- $\mathcal{K}$ function directly affects the controller performance, particularly in terms of conservativeness and the behavior near boundaries. The adopted parameters were empirically determined through preliminary simulations, which provided a good balance between safety enforcement and control smoothness for both HS and MT scenarios.

6.1.1.2 Leader profile design.

In order to more accurately reflect the operational characteristics of the system under investigation, two leader velocity profiles are designed for integration into the simulation: one for HS convoy operations and the other for MT convoy operations. In each profile, a multiple-phase running scenario that simulates a journey including station departure, interval running, and station arrival, is set up for evaluation [101].

The complete profiles are shown in Fig. 6.1, including the acceleration-time and velocity-position plots for both scenarios. The first column (subfigures (a) and (c)) corresponds to the leader profile of the HS case, while the second column (subfigures (b) and (d)) presents the MT case. The leader's trajectory in each scenario is constructed based on a jerk-limited piecewise acceleration function to simulate the real train operation. In both HS and MT cases, the operation profiles of the leader (train 0) are given and constrained by the same maximum and minimum control force limits as its followers. While ensuring a reasonable jerk (rate of change of acceleration), a cruising state is added between positive and negative acceleration to simulate smooth operation that considers passenger comfort.

The leader's trajectory of HS case follows a three-phase profile: acceleration, cruise, and deceleration.

1. **Acceleration phase:** Starting from rest, the acceleration increases from 0 to 1.5 m/s^2 with a jerk of $+0.1 \text{ m/s}^3$ for 15 s, holds at $a_0 = 1.5 \text{ m/s}^2$ for 35 s, and then decreases to zero with jerk -0.1 m/s^3 for another 15 s. The resulting velocity reaches $v_0 = 75 \text{ m/s}$ at the end of the acceleration phase.
2. **Cruising phase:** The train runs at constant acceleration $a_0 = 0 \text{ m/s}^2$ for 100 s, maintaining the cruising speed $v_0 = 75 \text{ m/s}$.
3. **Deceleration phase:** The train decelerates with jerk -0.064 m/s^3 for 12.5 s, reaching $a_0 = -0.8 \text{ m/s}^2$. It then maintains this deceleration for 75 s, followed by a ramp-up of acceleration from -0.8 m/s^2 to 0 with jerk $+0.032 \text{ m/s}^3$ for 25 s, bringing the train smoothly to a stop.

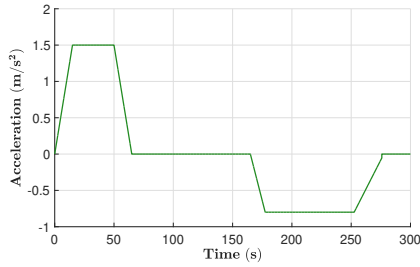
The duration of the cruising phase is set to 150 s to reduce the computational time. By the end of the journey, the leader has traveled a distance of 13,929 m, which is sufficient to demonstrate the effectiveness of the proposed controller, as will be shown in the following experiments. In practice, since high-speed trains often operate with long inter-station distances, the cruising phase can last much longer.

A more sophisticated design is adopted for the leading train in the MT case. Specifically, the trajectory consists of five phases: acceleration, cruising, intermediate deceleration, low-speed cruising, and final deceleration.

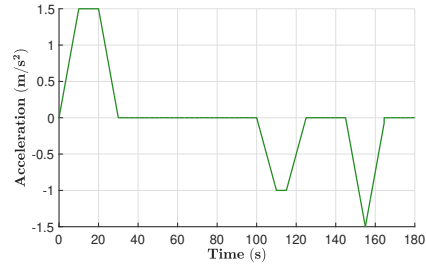
1. **Acceleration phase:** Starting from rest, the acceleration increases from 0 to 1.5 m/s^2 with a jerk of $+0.15 \text{ m/s}^3$ for 10 s, holds at $a_0 = 1.5 \text{ m/s}^2$ for another 10 s, and then decreases to zero with jerk -0.15 m/s^3 for 10 s. The resulting velocity reaches $v_0 = 30 \text{ m/s}$ at the end of the acceleration phase.
2. **Cruising phase:** The train maintains a constant speed of $v_0 = 30 \text{ m/s}$ ($a_0 = 0 \text{ m/s}^2$) for 70 s.
3. **Intermediate deceleration phase:** The train initiates the first braking with jerk -0.1 m/s^3 for 10 s, reaching $a_0 = -1.0 \text{ m/s}^2$. It then maintains this deceleration for 5 s, followed by a smooth reduction of the deceleration rate from -1.0 m/s^2 to 0 with jerk $+0.1 \text{ m/s}^3$ for 10 s, during which the velocity decreases to approximately 15 m/s.
4. **Low-speed cruising:** The train cruises at a constant speed of 15 m/s for 20 s.

5. **Final deceleration phase:** The train performs the final braking with jerk -0.15 m/s^3 for 10 s, reaching $a_0 = -1.5 \text{ m/s}^2$. Afterwards, the deceleration rate is smoothly reduced from -1.5 m/s^2 to 0 with jerk $+0.15 \text{ m/s}^3$ for 10 s, bringing the train to a complete stop.

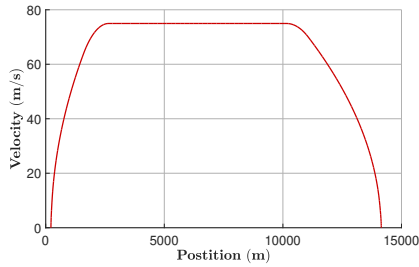
The deceleration process has two stages, with a short cruise with low-speed in between, simulating richer velocity variations and realistic station-approach behaviors of metro. The velocity of the metro train's cruising phase is lower than that in the HS case, and a higher jerk is employed for the metro train during the deceleration.



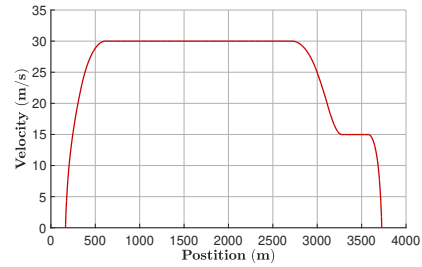
(a) HS scenario: acceleration vs. time



(b) MT scenario: acceleration vs. time



(c) HS scenario: velocity vs. position



(d) MT scenario: velocity vs. position

Figure 6.1: Leader trajectory design for high-speed rail (HS) and metro (MT) scenarios.

6.1.2 Performance indexes

Apart from the safety requirement that must be satisfied first, the synchronized movement among trains in a convoy is essential for evaluating the performance of VC operations. To assess the controller's effectiveness in terms of *capacity enhancement*, *riding comfort*, and *station stopping accuracy*, the following performance indexes are defined.

Let $T_i = [t_0, t_i^s]$ denote the journey time horizon of Train i . This interval is discretized with a sampling step $dt > 0$, such that $t_k = t_0 + k dt$ for $k = 0, \dots, k_i^s$. We set $t_0 = 0$ as the departure instant of the leading train (Train 0) and, by convention, of the whole convoy. t_i^s represents the instant

when Train i comes to a complete stop, and $d_i(t_i^s)$ denotes the distance from its predecessor at that moment.

1) Average spacing deviation: Since the controller is primarily designed to achieve spacing convergence rather than strict velocity matching, it is also important to evaluate how closely the actual spacing follows the desired one throughout the journey. For the i th follower, the *average spacing deviation* is computed in discrete form as

$$\bar{\delta}_i^d = \frac{1}{N_i} \sum_{k=k_0}^{k_i^s} \delta_i^d(k),$$

where $\delta_i^d(k)$ denotes the spacing deviation at sampling step k , and $N_i = k_i^s - k_0 + 1$ is the total number of samples during the journey of Train i . Here, k_0 corresponds to the departure instant of the leader, i.e., the start of the convoy journey. This index represents the mean deviation of the actual spacing from the ideal RBD-based distance. For a three-train convoy, two values $\bar{\delta}_1^d$ and $\bar{\delta}_2^d$ are obtained, which complement the terminal deviation $\delta_i^d(t_i^s)$ by quantifying spacing accuracy throughout the entire journey.

2) Spacing deviation variance: While the average spacing deviation reflects the overall tracking accuracy, it does not capture the fluctuation of the spacing error during the journey. To evaluate the stability of inter-train distance control, the variance of the spacing deviation for the i th follower is defined as

$$\sigma_i^d = \frac{1}{N_i} \sum_{k=k_0}^{k_i^s} (\delta_i^d(k) - \bar{\delta}_i^d)^2.$$

The variance σ_i^d measures this temporal fluctuation of $\delta_i^d(k)$ with respect to its mean. A larger value means that the spacing deviation changes more significantly during the journey.

3) Stopping time difference: Ideally, once coupled, the trains in a convoy should arrive at the station simultaneously. A smaller time interval between adjacent trains at stopping implies a more compact movement, so that passengers may perceive the convoy as a single train. Therefore, the *stopping time difference* between Train i and its predecessor is defined as

$$\Delta t_i^s = |t_i^s - t_{i-1}^s|.$$

In general, $t_i^s > t_{i-1}^s$ holds, meaning that the following train stops slightly later than its predecessor.

4) Stopping spacing deviation: Since platform length is often limited, accurate inter-train spacing at standstill is crucial for applying VC on existing lines. This requires the convoy to stop with the desired clearance between adjacent trains. The ideal spacing at stop is the default safety margin d_0 . Accordingly, the *stopping spacing deviation* of Train i is defined as

$$\delta_i^d(t_i^s) = d_i(t_i^s) - d_i^*(t_i^s) = d_i(t_i^s) - d_0,$$

which indicates whether the spacing between Train i and its predecessor converges to the desired margin at standstill.

5) Relative kinetic energy: Finally, the *relative kinetic energy* $E(t)$ [102] is used to quantify the potential impact of collisions between adjacent trains and thus to reflect the overall convergence performance of the convoy. It is calculated as

$$E(t) = \sum_{i=1}^N M_i \left(v_i(t) - v_{i-1}(t) \right)^2,$$

where M_i denotes the mass of Train i . The mass of the leader is not included in the summation.

The above indexes are consistently used throughout the following analyses, including the baseline experiments, parameter sensitivity evaluations, and extended case studies, to provide a unified and quantitative basis for comparing convoy performance under various operating conditions.

Remark. It is worth noting that the proposed control framework differs fundamentally from conventional stability-oriented methods such as Model Predictive Control (MPC) and Sliding Mode Control (SMC). While MPC-based approaches can provide optimal performance, their computational burden often limits their applicability to real-time operation. In contrast, SMC and other stability-focused controllers are usually designed for specific operating phases, such as cruising or formation, and are implemented in a piecewise manner.

The proposed method, however, is applicable to the entire convoy operation, covering acceleration, cruising, coasting, and braking phases within a unified framework. In particular, it achieves full implementation of the relative braking distance (RBD) principle, thus realizing continuous safety protection based on the dynamic states of trains. This property allows the controller to ensure safety and convergence even in critical stages such as the final stopping process.

No direct performance comparison with MPC or SMC is provided, since the underlying control principles and constraint formulations differ substantially. Specifically, the safety constraints in this work are formulated

through CBFs, which enable continuous-time safety enforcement that cannot be straightforwardly replicated within traditional MPC or SMC frameworks. Therefore, the focus of this study is to validate the effectiveness, real-time feasibility, and full-phase applicability of the proposed CBF-based framework, rather than performing numerical comparisons under artificially aligned conditions.

6.1.3 Convoy movement

We first investigate a nominal case, where the convoys move according to Train 0's operation. The initial states of trains in the two convoys are given in Table 6.3. Each convoy starts from rest, with each pair of adjacent trains initially separated by the default margin d_0 (10 m for HS convoy, and 5 m for MT convoy). For each case, the predefined acceleration trajectory derived in Section 6.1.1.2 is assigned to Train 0 to execute at each same sampling interval $dt = 0.02$ s.

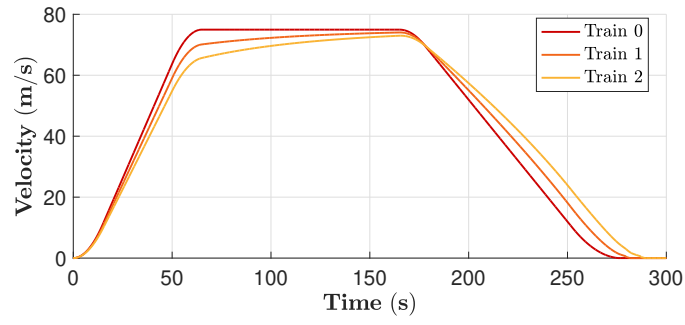
Simulations were performed for 300 s in the HS case and 180 s in the MT case. The results are presented in Fig. 6.2 and Fig. 6.3.

Table 6.3: Initial state configurations for nominal operation of HS/MT convoy

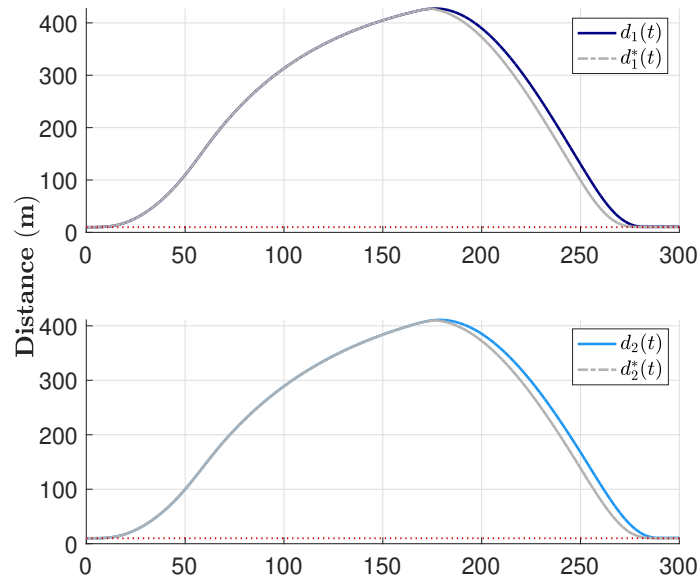
Case	Initial state	Train 0	Train 1	Train 2	Train 3
HS	$p_i(0)$	220 m	110 m	0 m	N/A
	$v_i(0)$	0 m/s	0 m/s	0 m/s	N/A
MT	$p_i(0)$	165 m	110 m	55 m	0 m
	$v_i(0)$	0 m/s	0 m/s	0 m/s	0 m/s

From Fig. 6.2a and 6.3a, it can be seen that the followers can tightly follow the leader throughout the journey. Starting from rest, the convoy gradually accelerates. The followers' velocity increases more slowly than the leader's, since as the speed rises, the desired minimum safety distance also increases. Hence, Train 1 and Train 2 (or Train 3 in the MT case) accelerate successively with slightly smaller rates. When the leader approaches the cruising phase, all trains converge to nearly the same speed, and the inter-train distances become steady. Subsequently, as the leader begins to decelerate, the followers also start to slow down. Because the desired safety distances $d_i^*(t)$ decrease as velocity reduces, the followers' deceleration is slightly milder, which allows the actual distance $d_i(t)$ to converge smoothly to the desired one.

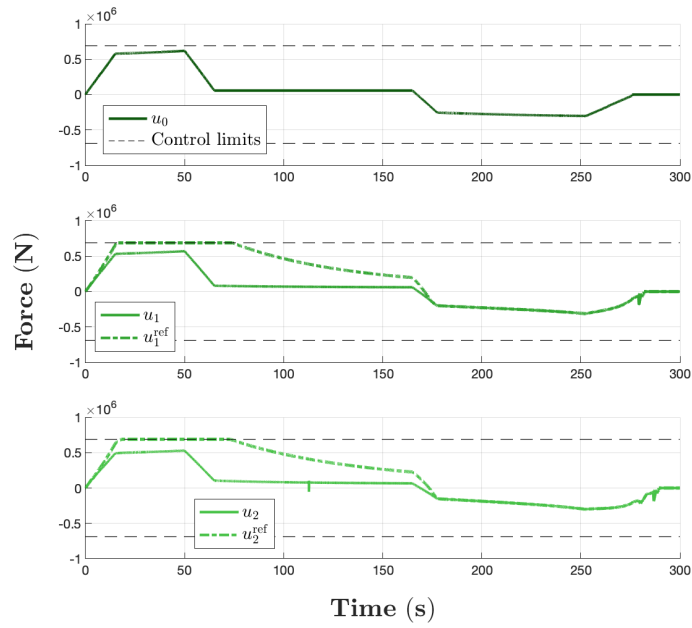
In the distance plots Fig. 6.3b and 6.2b, the dotted red line at the bottom represents the default margin d_0 , i.e., the minimum standstill spacing between two trains. It can be observed that when the trains come to a full stop, the actual inter-train distances effectively converge to this default margin.



(a) Velocity profile.

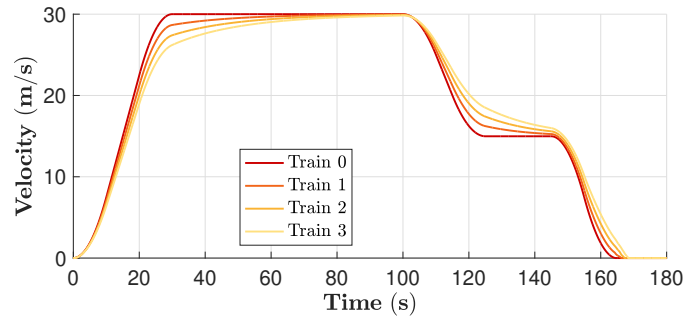


(b) Inter-train distance.

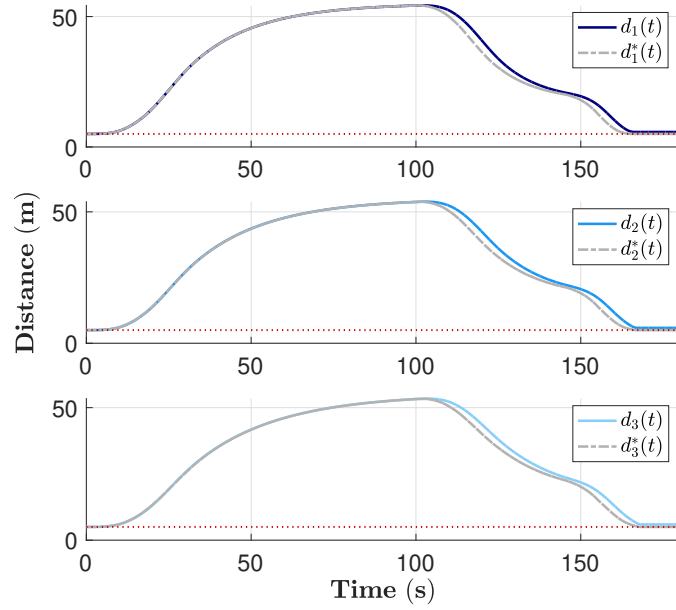


(c) Control input.

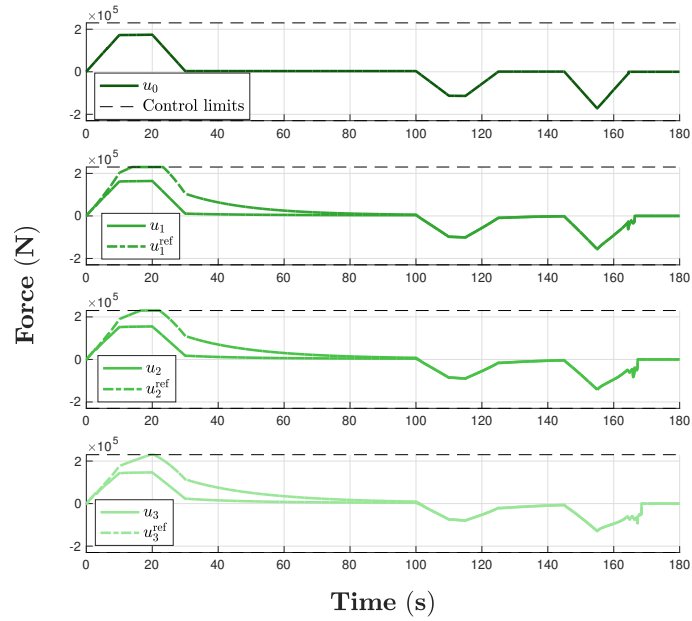
Figure 6.2: Nominal simulation results for the high-speed (HS) case.



(a) Velocity profile.



(b) Inter-train distance.



(c) Control input.

Figure 6.3: Nominal simulation results for the metro (MT) case.

The actual distance (solid line) throughout the entire journey is greater than and close to the desired distance (dash line). The inter-train distances converge smoothly to the desired default spacing d_0 at standstill. A temporary increase in the deviation $d_1(t) - d_1^*(t)$ is observed when Train 0 starts braking at 102 s, as the safety distance $d_1^*(t)$ rapidly increases due to the CBF immediate response to the drop in $v_0(t)$. This braking of Train 0 propagates downstream to Train 2, leading to a similar but milder deviation in $d_2(t) - d_2^*(t)$. Upon stopping, the inter-train distances are effectively reduced to the desired default spacing d_0 . Regarding the control plots in Fig. 6.3c and 6.2c, the regions where the solid and dashed lines diverge indicate the activation of the CBF constraints. during which the following trains closely follow the trajectory of Train 0.

Table 6.4: Simulation results of the nominal convoy movement operation (HS and MT)

Case	Average spacing deviation			Spacing deviation variance		
	$\bar{\delta}_1^d$	$\bar{\delta}_2^d$	$\bar{\delta}_3^d$	σ_1^d	σ_2^d	σ_3^d
HS	7.00 m	6.77 m	N/A	122.36 m	105.58 m	N/A
MT	0.88 m	0.88 m	0.88 m	1.73 m	1.54 m	1.40 m
Case	Stopping time difference			Stopping spacing deviation		
	Δt_1^s	Δt_2^s	Δt_3^s	$\delta_1^d(t_1^s)$	$\delta_2^d(t_2^s)$	$\delta_3^d(t_3^s)$
HS	6.06 s	7.48 s	N/A	0.69 m	0.69 m	N/A
MT	1.84 s	0.92 s	1.18 s	0.75 m	0.84 m	0.94 m

The performance indexes are reported in Table 6.4. All results are rounded to two decimal places. The stopping performance remains preferable: both the stopping time difference and the final spacing deviation stay within small limits, and the latter is kept within 1 m of the default margin.

Note that the larger σ_i^d in the HS case does not imply worse control accuracy. The overall spacing deviation in HS throughout the journey remains small, being almost zero at the early stage and appearing only when the leader begins to decelerate. During the subsequent braking phase, this mild deviation (within about 10 m) persists for a relatively long duration, which causes the squared differences $(\delta_i^d(k) - \bar{\delta}_i^d)^2$ to accumulate and leads to a higher variance compared with the MT case. The higher cruise velocity and longer journey in HS also contribute to the longer duration of deviation, resulting in larger σ_i^d even under stable control behavior.

Remark. Specifically, comparisons of σ_i^d are meaningful when performed among different settings of the HS case, or among different settings of the MT case, rather than directly between HS and MT. In later sections, this index will be used to evaluate the relative stability when various factors such as control parameters are introduced in the HS or MT convoy.

Baseline MPC comparison To provide a baseline indication of the computational burden, a distributed MPC controller is additionally implemented for the HS convoy movement scenario. The MPC formulation reuses the same safety constraint (5.4) as the proposed CBF-QP approach, and the reference control input u_{ref} in (5.22) is incorporated into the cost function. Both controllers are evaluated over a 300 s simulation under identical conditions.

Unlike the proposed method, which relies on **quadprog** to solve a convex quadratic program, the MPC optimization problem is solved using **fmincon**. Given the nominal nature of the baseline scenario and the absence of additional disturbances or delays, a short prediction horizon of 5 steps is adopted, which is sufficient to capture the system behavior in this setting.

The two approaches exhibit comparable control performance in terms of convoy movement and safety maintenance. However, a clear difference is observed in computational efficiency. The mean per-step computation time of the proposed CBF-QP controller is 2.54 ms, whereas the MPC implementation requires 6.87 ms on average. This result already indicates a computational advantage of the proposed approach, even in a simplified baseline scenario.

Since the control performance is similar in this basic case, MPC is not further considered in the subsequent case studies, which focus on robustness-related aspects of the proposed CBF-QP controller.

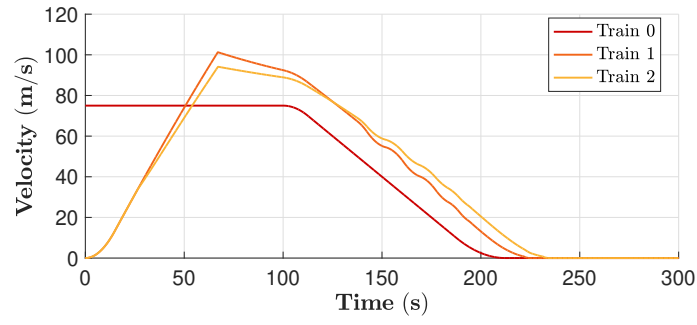
6.1.4 Convoy formation

Different initial states from the nominal scenarios are applied, as listed in Table 6.5, where the adjacent trains are initially separated by larger distances in both HS and MT convoys. For the HS case, the initial spacings are set to $d_1(0) = 110$ m and $d_2(0) = 490$ m. In the MT case, the initial inter-train distances are $d_1(0) = 50$ m, $d_2(0) = 105$ m, and $d_3(0) = 340$ m. No train pairs are initially coupled. Moreover, the leading train is assumed to cruise at a constant velocity, thus skipping the acceleration phase of the predefined trajectory.

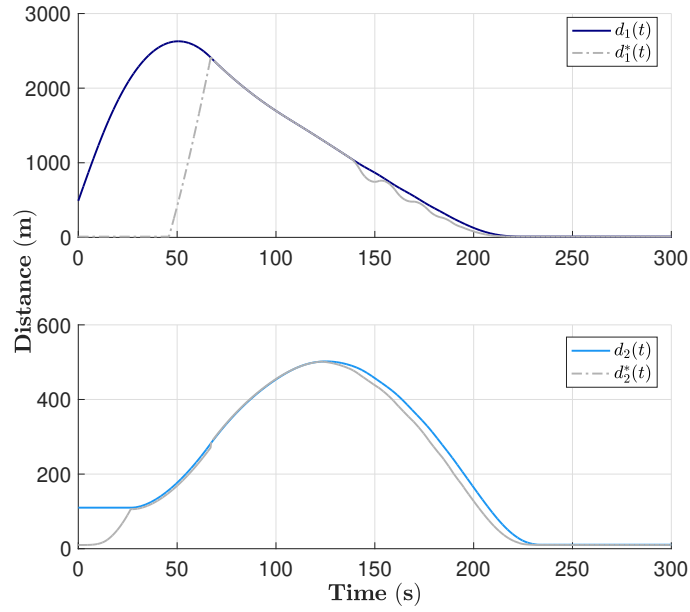
The specific settings for HS and MT scenario are given in Table 6.5 .

Table 6.5: Initial state configurations for formation control of HS/MT convoy

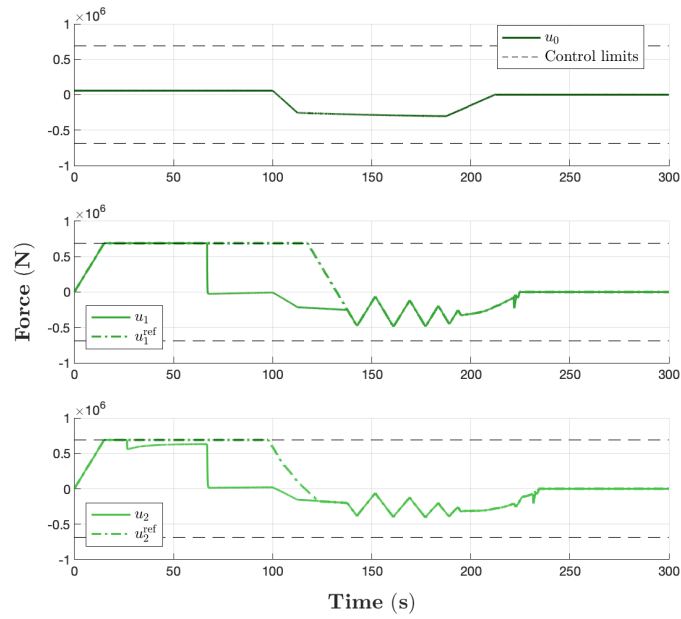
Case	Initial state	Train 0	Train 1	Train 2	Train 3
HS	$p_i(0)$	800 m	210 m	0 m	N/A
	$v_i(0)$	75 m/s	0 m/s	0 m/s	N/A
MT	$p_i(0)$	600 m	210 m	55 m	0 m
	$v_i(0)$	30 m/s	10 m/s	5 m/s	0 m/s



(a) Velocity profile.

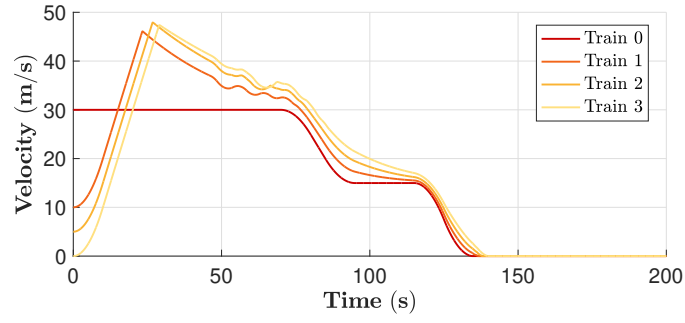


(b) Inter-train distance.

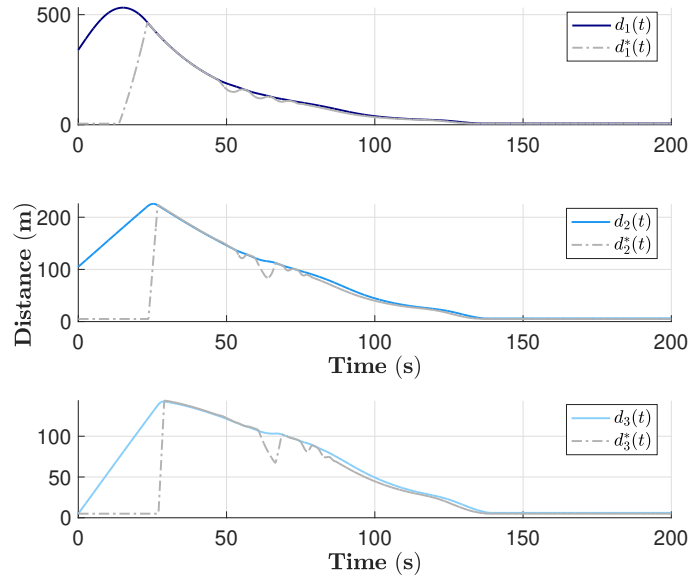


(c) Control input.

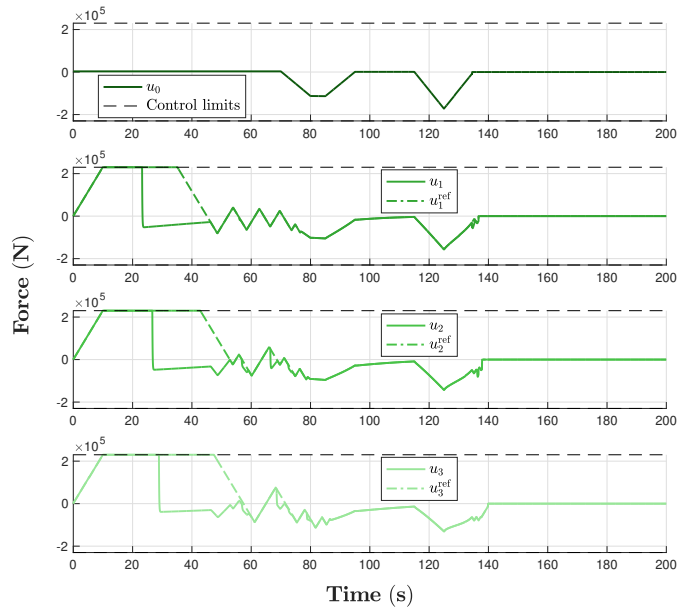
Figure 6.4: Formation simulation results for the high-speed (HS) case.



(a) Velocity profile.



(b) Inter-train distance.



(c) Control input.

Figure 6.5: Formation simulation results for the metro (MT) case.

When the follower approaches coupling, oscillations appear around $t \approx 50$ s in both the velocity profiles and the control inputs. The inter-train distance has already converged to the target earlier ($t \approx 20\text{--}30$ s), yet the velocities synchronize later, during which the oscillations occur.

This phenomenon mainly results from the fact that the reference control force is designed primarily based on distance convergence. When the distance d is close to the desired spacing d_0 but the relative velocity $\Delta v = v_i - v_{i-1}$ remains nonzero, the controller alternates between traction and braking in order to maintain spacing, causing small fluctuations in both the velocity and force profiles. To mitigate this issue, a damping term based on relative velocity could be added to the original reference force formulation.

6.2 Performance Analysis

The control performance of both HS and MT convoys, in their nominal operation and formation scenarios, has been analyzed in the previous section. Building upon these baseline results, this section investigates how key parameters in the convoy control influence the overall performance of the proposed control framework.

The analyses focus on two aspects: i) the convergence time parameter ϵ_c , which determines how rapidly the spacing deviation shortened, and ii) the heterogeneous convoy, where individual trains possess different dynamic characteristics. Specifically, we vary the convergence time ϵ_c in the MT case, as punctuality are one of primary concerns in metro operations. The heterogeneous train characteristics are carried in the HS convoy. including mass and maximum traction/braking rate. This is out of the consideration that high-speed trains are more sensitive to these parameters [103].

By varying these conditions, the study aims to reveal the trade-offs between control responsiveness, riding comfort, and energy efficiency, as well as to assess how the controller maintains spacing convergence under non-identical train configurations. These investigations provide insight into the sensitivity of the proposed method and serve as a preliminary step toward evaluating its robustness under more challenging operational environments.

6.2.1 Effect of convergence time

For MT case, we conduct four simulations adopting different values of parameter ϵ_c (10, 7.5, 5, and 2.5 s) in the reference acceleration calculation for the followers (Train 1-3). Apart from the values of ϵ_c , for each case, the parameters and operation settings of MT convoy trains in Section 6.1.3 are retained. Besides, the convoy is initially established, and trains are

standstill and separated by the default margin. The convoy undergoes a complete journey driven by the identical leader profiles defined previously in Section 6.1.1.2. The results of performance indexes are reported in Table 6.6.

The first two indexes, the average spacing deviation $\bar{\delta}_i^d$ and the invariance σ_i^d , describe the convoy's behavior during the whole journey. For $\epsilon_c = 10$ s, the average deviations of the three followers are 2.58, 2.58, and 2.53 m, respectively, with corresponding standard deviations of 14.89, 13.79, and 12.80 m. As ϵ_c decreases to 7.5, 5, and 2.5 s, both quantities gradually reduce, indicating smaller inter-train spacing errors and less fluctuation along the trajectory. At $\epsilon_c = 2.5$ s, the average spacing deviations fall below 0.8 m, and the corresponding standard deviations drop to around 1 m for all followers.

The stopping performance is reflected by the stopping time difference Δt_i^s and the stopping spacing deviation $\delta_i^d(t_i^s)$. For the largest convergence time $\epsilon_c = 10$ s, the stopping time differences are 12.62, 1.80, and 15.06 s, while the standstill spacing deviations are 0.73, 0.77, and 0.49 m. As ϵ_c decreases, the stopping time differences and spacing deviations gradually become smaller and more uniform across the convoy. At $\epsilon_c = 2.5$ s, the stopping time differences are approximately 1.06, 0.92, and 1.28 s, and the corresponding spacing deviations are 0.78, 0.88, and 1.00 m.

Table 6.6: Simulation results of different convergence time settings (MT convoy)

ϵ_c	Average spacing deviation			Spacing deviation variance		
	$\bar{\delta}_1^d$	$\bar{\delta}_2^d$	$\bar{\delta}_3^d$	σ_1^d	σ_2^d	σ_3^d
10 s	2.58 m	2.58 m	2.53 m	14.89 m	13.79 m	12.80 m
7.5 s	1.96 m	1.96 m	1.91 m	8.84 m	8.08 m	7.48 m
5.0 s	1.35 m	1.35 m	1.36 m	4.14 m	3.73 m	3.36 m
2.5 s	0.75 m	0.76 m	0.78 m	1.10 m	0.98 m	0.89 m
ϵ_c	Stopping time difference			Stopping spacing deviation		
	Δt_1^s	Δt_2^s	Δt_3^s	$\delta_1^d(t_1^s)$	$\delta_2^d(t_2^s)$	$\delta_3^d(t_3^s)$
10 s	12.62 s	1.80 s	15.06 s	0.73 m	0.77 m	0.49 m
7.5 s	8.52 s	0.98 s	7.96 s	0.73 m	0.79 m	0.47 m
5.0 s	4.76 s	0.92 s	1.82 s	0.72 m	0.81 m	0.94 m
2.5 s	1.06 s	0.92 s	1.28 s	0.78 m	0.88 m	1.00 m

The indexes on spacing convergence and stopping synchronization all show clear dependence on the value of ϵ_c . Specifically, they decrease as ϵ_c becomes smaller. In contrast, the standstill deviations $\delta_i^d(t_i^s)$ remain consistently small, around 1 m for all cases. The minor variations between cases are within a few millimeters and are not explicitly reflected, since all numerical results are reported with two decimal places. This suggests

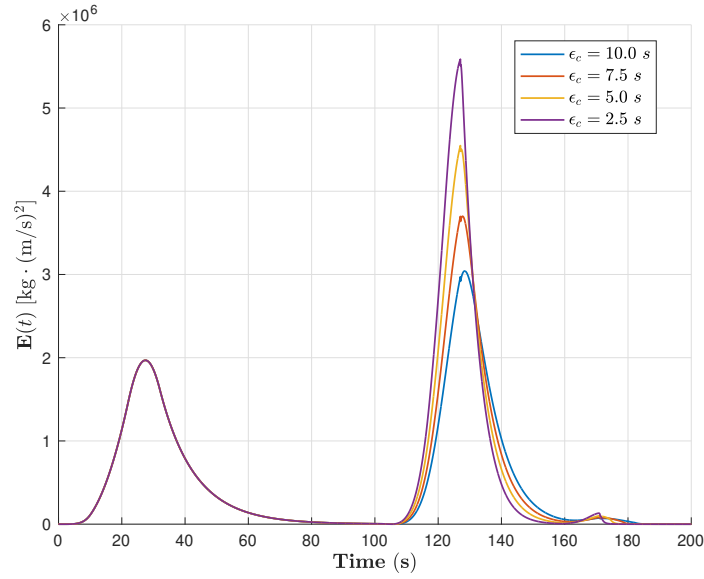


Figure 6.6: Comparison of Relative Kinetic Energy for different ϵ_c .

that the final convergence to the default safety margin d_0 is insensitive to ϵ_c . Overall, these results indicate that different convergence times mainly affect the convergence efficiency during the journey, while the final standstill spacing consistently approaches the default margin d_0 .

We further compare the relative kinetic energy profile each case in Fig. 6.6. It is clear to be seen that a smaller convergence time ϵ_c results in larger $E(t)$. Before 110 s, the lines of each case overlap. Later, they diverge evidently in the interval 110–130 s, when Train 0 starts braking and the distance deviation grows. Driven by smaller ϵ_c , the followers catch up. For example, with $\epsilon_c = 2.5$ s the energy reaches approximately 5.5×10^6 , while with $\epsilon_c = 10$ s it only slightly exceeds 3×10^6 . The reason is that a smaller ϵ_c enforces a more aggressive response: the follower decelerates more slowly in order to quickly reduce the distance deviation. The consequence, however, is a substantially larger kinetic energy.

The results show that a smaller convergence time indeed improves convergence, but it also leads to larger values of $E(t)$ during the convoy deceleration. This suggests that safety impact may be increased, since $E(t)$ reflects the potential collision impact. Therefore, the choice of the convergence time involves a trade-off. Motivated by this, in the future we may relax the assumption of a constant choice of ϵ_c , and explore adaptive or optimal design schemes of a tunable ϵ_c .

6.2.2 Effect of heterogeneous convoy

In the previous simulations, all trains shared identical dynamic parameters, representing a homogeneous configuration. In reality, the dynamic characteristics may vary among trains within a convoy. Here, parameter variations are introduced to assess how such heterogeneity affects the control performance.

This section evaluates the performance of the proposed controller in the heterogeneous VC convoy, where the vehicles differ in mass and traction/braking capabilities. The train length, resistance coefficients, and both service and emergency braking rates are kept identical, while the other parameters are varied according to the conditions defined in Table 6.7. Two cases are considered as shown, Hetero1 represents the case where the leader is heavier and has reduced traction/braking capability, while Hetero2 represents the case where both followers are heavier with weaker traction and braking.

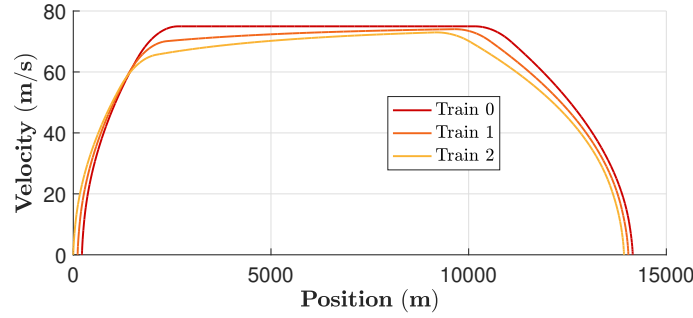
We keep the initial states and journey length identical. In Hetero1, since the leader has a heavier mass and weaker traction and braking performance, Accordingly, its feasible velocity–distance profile is recomputed to satisfy the reduced limits, while the convoy controller in Hetero1 and Hetero2 for the followers, all the other operational settings remain unchanged.

Table 6.7: Train parameters for the heterogeneous HS convoys

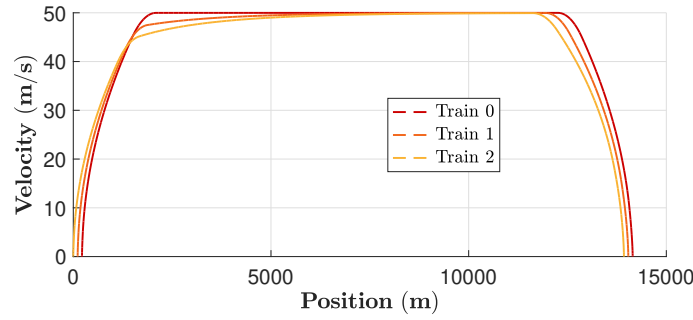
Parameter	Unit	Nominal	Hetero1	Hetero2
M_0	kg	383,000	421,300 (+38,300)	383,000 (0)
M_1			383,000 (0)	393,000 (+10,000)
M_2			363,850 (−19,150)	421,300 (+38,300)
B_0	m/s ²	1.8	1.3 (−0.5)	1.8 (0)
B_1			1.8 (0)	1.5 (−0.3)
B_2			1.9 (+0.1)	1.3 (−0.5)
T_0	m/s ²	1.8	1.2 (−0.6)	1.8 (0)
T_1			1.8 (0)	1.6 (−0.2)
T_2			1.9 (+0.1)	1.3 (−0.5)

Although the total running time is longer in the heterogeneous case, which is a direct result of the leader’s weaker traction and braking, the convoy successfully completes the journey with almost identical terminal spacing. The smaller stopping time differences and spacing deviations suggest that the convoy moves more uniformly when the leader operates at a lower speed with longer acceleration and braking phases. Overall, the results indicate that the nominal controller maintains safe spacing and satisfactory terminal coordination even under capability variation. The extended journey time

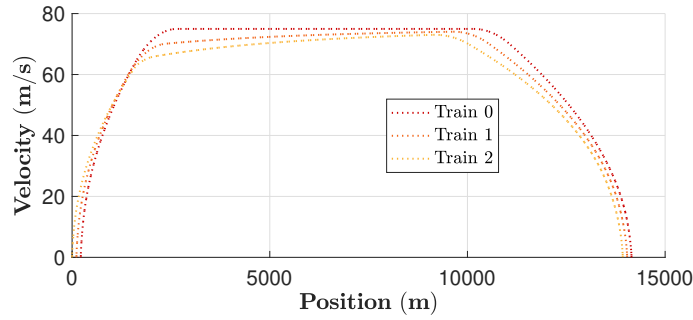
reflects the leader's physical limitations rather than a degradation of control performance.



(a) Nominal convoy



(b) Hetero1 convoy



(c) Hetero2 convoy

Figure 6.7: Velocity–position profiles of the nominal and heterogeneous convoys.

Figure 6.7 compares the velocity–position profiles of the nominal and heterogeneous HS convoys, and the corresponding performance results are summarized in Table 6.8. Under the Hetero1 configuration, the leader possesses a larger mass and weaker traction and braking capabilities, resulting in a reduced cruising velocity and a slower overall journey. The followers, which retain the same traction and braking capacities, converge to the leader's cruising velocity over a longer traveling distance compared to the nominal case. This can also be observed from the notable decreases in $\bar{\delta}_i^d$ and σ_i^d , as well as the improved synchronization of stopping times, as reported in Table 6.8.

In contrast, the Hetero2 configuration increases the mass of the following trains while preserving the leader's parameters. This adjustment leads to only marginal differences in the velocity trajectories compared to the nominal case. Apart from their nearly identical stopping performance in both time and space, the average and variance of the spacing deviation remain close to those of the nominal case, except that the variance σ_2^d becomes larger for Train 2 and Train 1, which results from the weaker traction capability of Train 2. In all cases, the trains reach the terminal with almost consistent inter-train spacing deviations (around 0.70 m) from the ideal margin (10 m). Overall, the proposed controller ensures safe and stable operation even under the considered heterogeneous convoy conditions, indicating that the proposed method exhibits low sensitivity to moderate variations in train mass and traction/braking performance.

Table 6.8: Simulation results of nominal and heterogeneous convoys (HS convoy)

Case	Average spacing deviation		Spacing deviation variance	
	$\bar{\delta}_1^d$	$\bar{\delta}_2^d$	σ_1^d	σ_2^d
Nominal	7.00 m	6.77 m	122.36 m	105.58 m
Hetero1	2.66 m	2.65 m	36.05 m	33.03 m
Hetero2	7.00 m	9.96 m	122.38 m	200.35 m

Case	Stopping time difference		Stopping spacing deviation	
	Δt_1^s	Δt_2^s	$\delta_1^d(t_1^s)$	$\delta_2^d(t_2^s)$
Nominal	6.06 s	7.48 s	0.69 m	0.69 m
Hetero1	4.10 s	5.44 s	0.68 m	0.68 m
Hetero2	6.06 s	7.54 s	0.69 m	0.69 m

6.3 Extended Case Studies

This section extends the analysis to more complex and realistic conditions to further evaluate the robustness of the proposed controller. Three types of disturbances are considered:

1. Communication delays, including both constant and time-varying cases, applied to the MT convoy.
2. Dynamic uncertainties arising from mismatches between the modeled and actual train parameters, applied to the HS convoy.
3. An emergency braking scenario, representing an extreme operational condition for the HS convoy.

In all the following simulations, the CBF settings (including class- $\mathcal{K}$ functions) are kept identical to those in the nominal movement scenario. The design parameter ϵ_c of the reference control force is set to 3 s and 5 s for the MT and HS convoys, respectively.

6.3.1 Constant delay

In this case, a fixed communication delay that equals to $\tau_{\max} = 400$ ms is introduced between Train 0 and Train 1, starting from $t = 100$ s and maintained for the entire convoy journey. The existence of $\tau_{\max}$ is represented by the shaded grey region in the figures. The follower Train 1 receives the leader's state information with a constant delay, and the proposed delay-compensation method is applied. As shown in Fig. 6.8, the overall tracking performance remains stable.

The followers can still synchronize with the leader throughout the acceleration, cruising, and braking phases. And the inter-train distances remain above the desired safe limits with smooth convergence at standstill. In the control input plots, slight oscillations appear immediately after the delay begins, reflecting the sudden activation of the CBF constraints and the adjustment of the reference force caused by the delayed information. However, the control signals gradually recover stability as the delay compensation takes effect. Detailed results of performance indexes will be reported later together with the ones in time-varying-delay case.

6.3.2 Time-varying delay

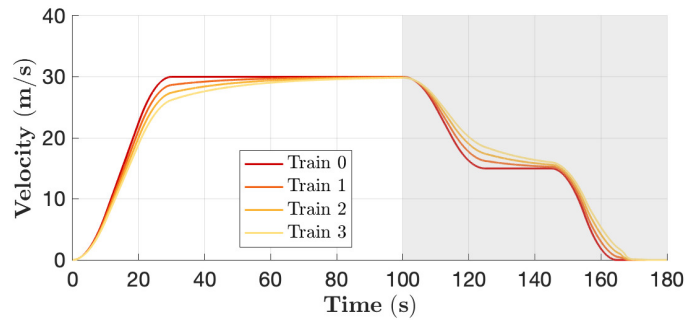
Here, a time-varying delay is adopted. Recalling the model of stochastic communication delay (5.42), the simulation adopts the parameters $\alpha = 0.5$, $\mu = 0.06$, and $\sigma = 0.005$, yielding the discrete update as

$$\tau_{k+1} = \tau_k + 0.05(0.06 - \tau_k) + 0.005 w_k, \quad w_k \sim \mathcal{N}(0, 1). \quad (6.1)$$

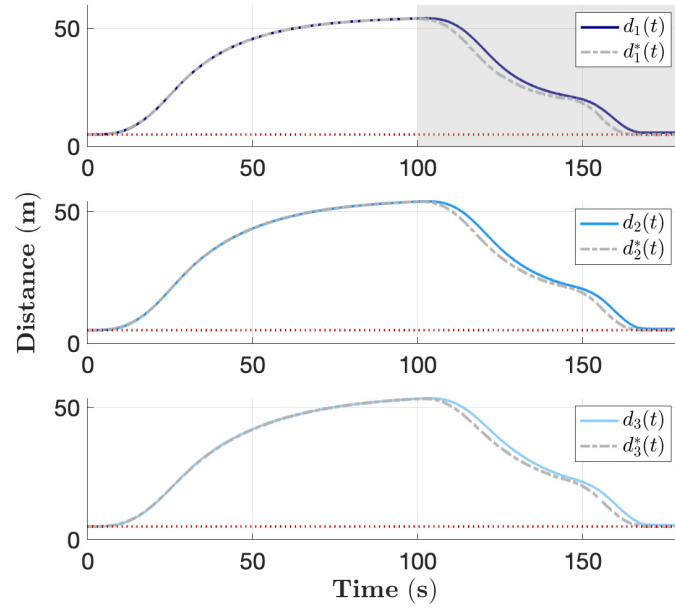
These values produce delays with an average around 60 ms with random fluctuations.

The generated random delay $\tau(t)$ as shown in Fig. 6.9 is introduced, same as the constant-delay-case, starting from $t = 100$ s between Train 0 and Train 1. The corresponding simulation results are displayed in Figs. 6.10, where the period effected by $\tau(t)$ is highlighted in yellow.

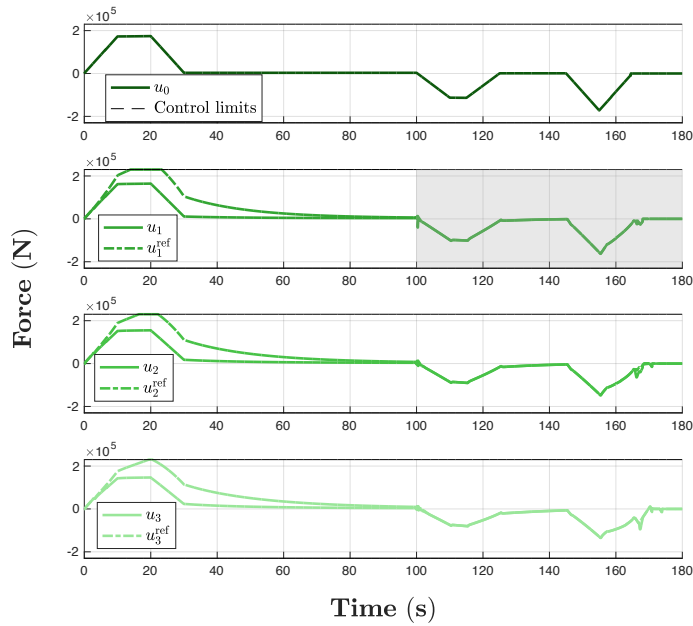
The plots of inter-train distances and follower velocities demonstrate that the proposed controller maintains smooth and safe behavior under random communication delays. In the control input plot, a fluctuation can be observed in Train 1 around $t = 100$ s, which corresponds to the sudden occurrence of the delay. Afterwards, $u_1(t)$ becomes smoother, indicating



(a) Velocity profile (with constant delay).



(b) Inter-train distance (with constant delay).



(c) Control input (with constant delay).

Figure 6.8: MT convoy results under constant delay.

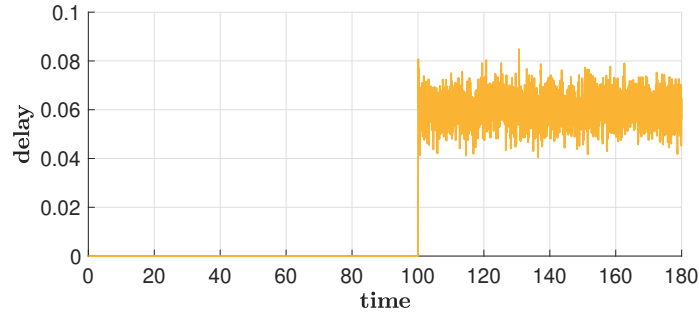
Figure 6.9: Stochastic delay $\tau(t)$

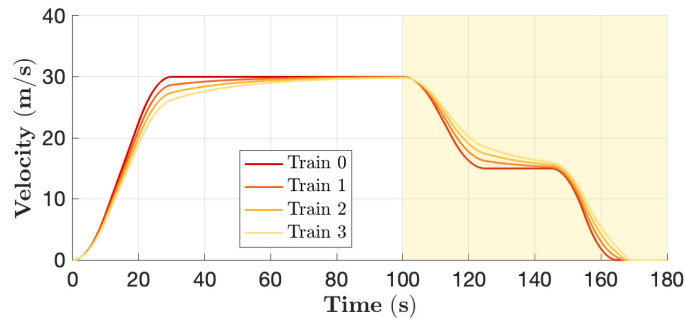
Table 6.9: Simulation results of constant and time-varying delays (MT convoy)

delay	Average spacing deviation			Spacing deviation variance		
	$\bar{\delta}_1^d$	$\bar{\delta}_2^d$	$\bar{\delta}_3^d$	σ_1^d	σ_2^d	σ_3^d
0	0.88 m	0.88 m	0.88 m	1.73 m	1.54 m	1.40 m
$\tau_{\max}$	1.06 m	0.86 m	0.85 m	2.41 m	1.56 m	1.40 m
$\tau(t)$	0.93 m	0.88 m	0.88 m	1.88 m	1.54 m	1.40 m
delay	Stopping time difference			Stopping spacing deviation		
	Δt_1^s	Δt_2^s	Δt_3^s	$\delta_1^d(t_1^s)$	$\delta_2^d(t_2^s)$	$\delta_3^d(t_3^s)$
0	1.84 s	0.92 s	1.18 s	0.75 m	0.84 m	0.94 m
$\tau_{\max}$	3.58 s	2.83 s	2.98 s	0.79 m	0.47 m	0.47 m
$\tau(t)$	2.04 s	0.88 s	1.04 s	0.75 m	0.82 m	0.91 m

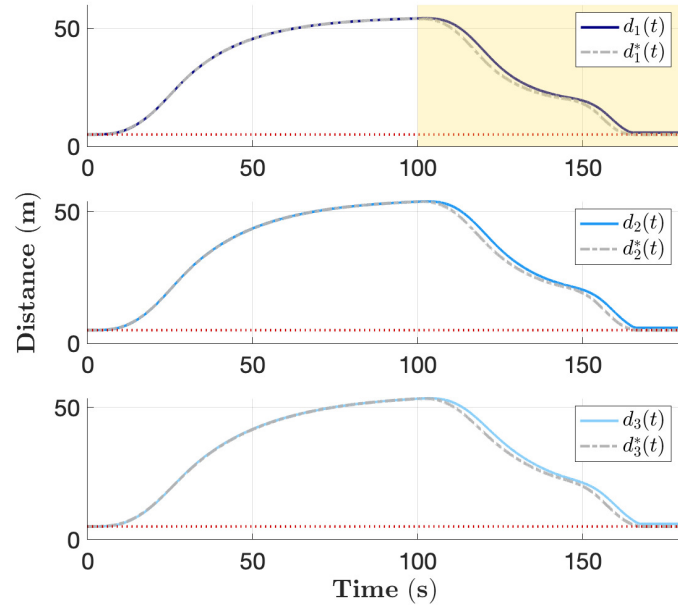
that the delay-compensation strategy effectively stabilizes the control even though the introduced delay exhibits a high degree of randomness, as shown in Fig. 6.9. Figure 6.11 provides a zoomed-in view of $u_1(t)$ during the delayed period.

Table 6.9 compares the performance indexes under three conditions: the nominal case (without delay), the constant-delay case ($\tau_{\max} = 0.4$ s), and the time-varying-delay case ($\tau(t)$ shown in Fig. 6.9, obtained by (6.1)). The results show that the proposed delay-compensation method effectively preserves control accuracy under both delay conditions. The averages and variances of spacing deviations in the constant- and time-varying-delay cases remain close to those of the delay-free case. The increases, compared to the nominal case, are within 0.2 m for $\bar{\delta}_i^d$ and 1 m for σ_i^d , confirming that distance convergence is not compromised.

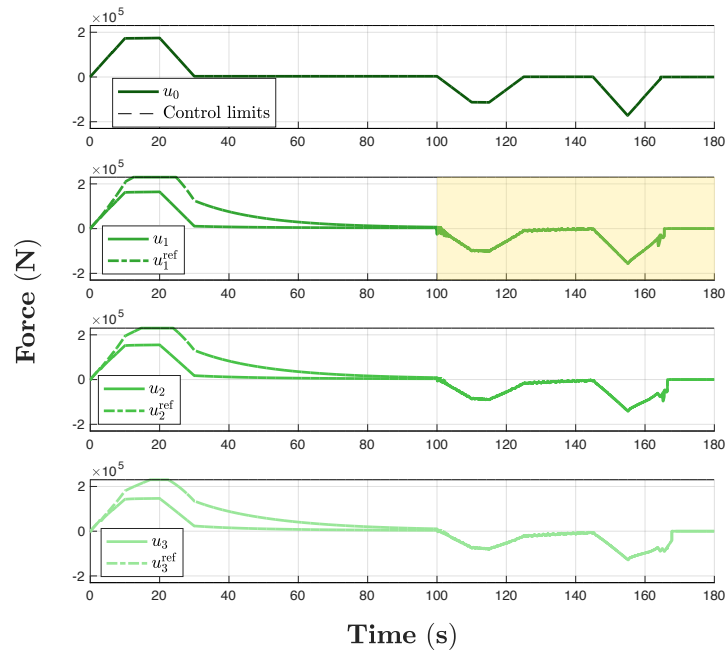
Regarding the stopping behavior, the impact of delay is negligible compared with the delay-free scenario. Interestingly, in the constant delay case with $\tau_{\max}$, the stopping spacing deviations of Train 2 and Train 3 are even slightly smaller. This occurs because, when the leader comes to a full stop, the following trains still receive slightly outdated information and continue adjusting for a short period (especially Train 1) leading to a slightly



(a) Velocity profile (with stochastic delay).



(b) Inter-train distance (with stochastic delay).



(c) Control input (with stochastic delay).

Figure 6.10: MT convoy results under stochastic delay.

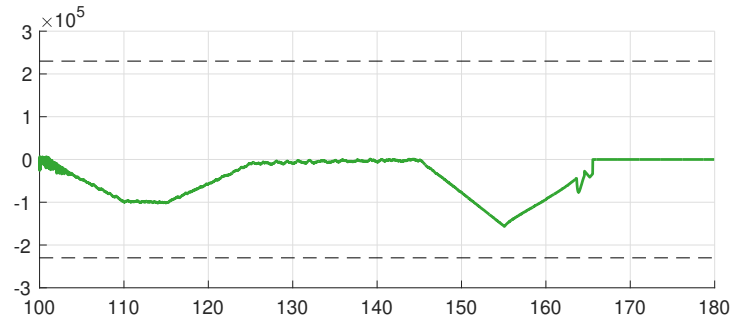


Figure 6.11: Control input of Train 1 during delay.

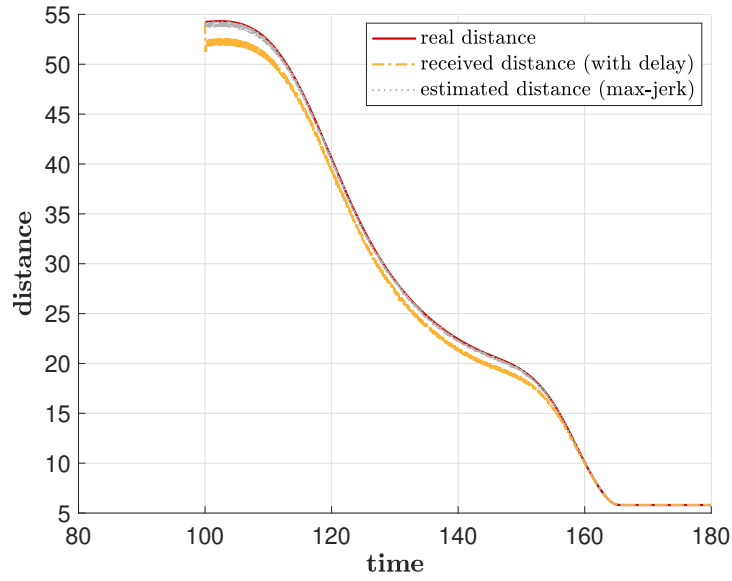


Figure 6.12: Inter-train distance between Train 1 and Train 0 during delay.

longer stopping time and a small final offset in the inter-train distances of the subsequent trains. Overall, the proposed compensation method demonstrates a strong capability to handle both constant and time-varying communication delays.

Figure 6.12 illustrates the distance evolution between Train 0 and Train 1 during the delay, where the solid red line denotes the actual distance $d_1(t)$ and the yellow line represents the delayed measurement $\hat{d}_1(t)$. The grey line shows the estimated distance $\tilde{d}_1(t)$ obtained by the proposed max-jerk-based compensation. As observed, the estimated trajectory closely follows the real distance, effectively compensating for the delay-induced deviation and maintaining accurate spacing information for control execution.

6.3.3 Dynamic uncertainty

Based on the HS convoy under delay-free setting, we assess robustness of the original CBF b_1 and b_2 in (5.23) and (5.24), by injecting bounded disturbances into the train dynamics without adding additional relaxation terms in the CBF constraints (5.30) and (5.36).

Recalling the dynamic of the follower i with uncertain term $\omega_i(t)$ in (5.55), which is unknown to the controller. It represents an additive disturbance, lumping all unknown effects together to the original dynamics.

For the first follower Train 1, the disturbed dynamics are given by

$$\dot{v}_1(t) = \frac{u_1(t) - r_1(t)}{M_1} + \omega_1(t). \quad (6.2)$$

In the following tests, two types of matched disturbances $\omega_1^{(1)}(t)$ and $\omega_1^{(2)}(t)$ are tested separately. Specifically,

$$\omega_1^{(1)}(t) = 0.14 \sin(0.05 t) + 0.05 \tanh(0.04 v_1(t)) + 0.03 \cos(0.005 p_1(t)),$$

and

$$\omega_1^{(2)}(t) = 0.01 \sin(0.0 t) + 0.01 \xi_k, \quad \xi_k \sim \mathcal{N}(0, 1).$$

The disturbance $\omega_1^{(1)}(t)$ emulates the additional resistance caused by slowly varying track geometry such as slopes, curves, or tunnels. Whereas $\omega_1^{(2)}(t)$ models some irregular fluctuations such as adhesion variation, wind, or sensor noise, and is modeled by a sinusoidal component with zero-mean Gaussian noise ξ_k .

Figures 6.13, 6.14 and Table 6.10 show the test results with the two matched disturbances $\omega_1^{(1)}(t)$ and $\omega_1^{(2)}(t)$ injected on Train 1.

Table 6.10: Simulation results of dynamic uncertainties (HS convoy)

disturbance	Average spacing deviation		Spacing deviation variance	
	$\bar{\delta}_1^d$	$\bar{\delta}_2^d$	σ_1^d	σ_2^d
0	7.00 m	6.77 m	122.36 m	105.58 m
$\omega_1^{(1)}(t)$	7.22 m	6.77 m	117.57 m	105.58 m
$\omega_1^{(2)}(t)$	6.92 m	6.76 m	116.66 m	105.62 m
disturbance	Stopping time difference		Stopping spacing deviation	
	Δt_1^s	Δt_2^s	$\delta_1^d(t_1^s)$	$\delta_2^d(t_2^s)$
0	6.06 s	7.48 s	0.69 m	0.69 m
$\omega_1^{(1)}(t)$	6.74 s	7.04 s	0.69 m	0.69 m
$\omega_1^{(2)}(t)$	6.44 s	7.20 s	0.69 m	0.69 m

Under both cases, the follower accelerations in panels (b) remain close to the leader's profile, and the small compensation appears mainly on Train 1.

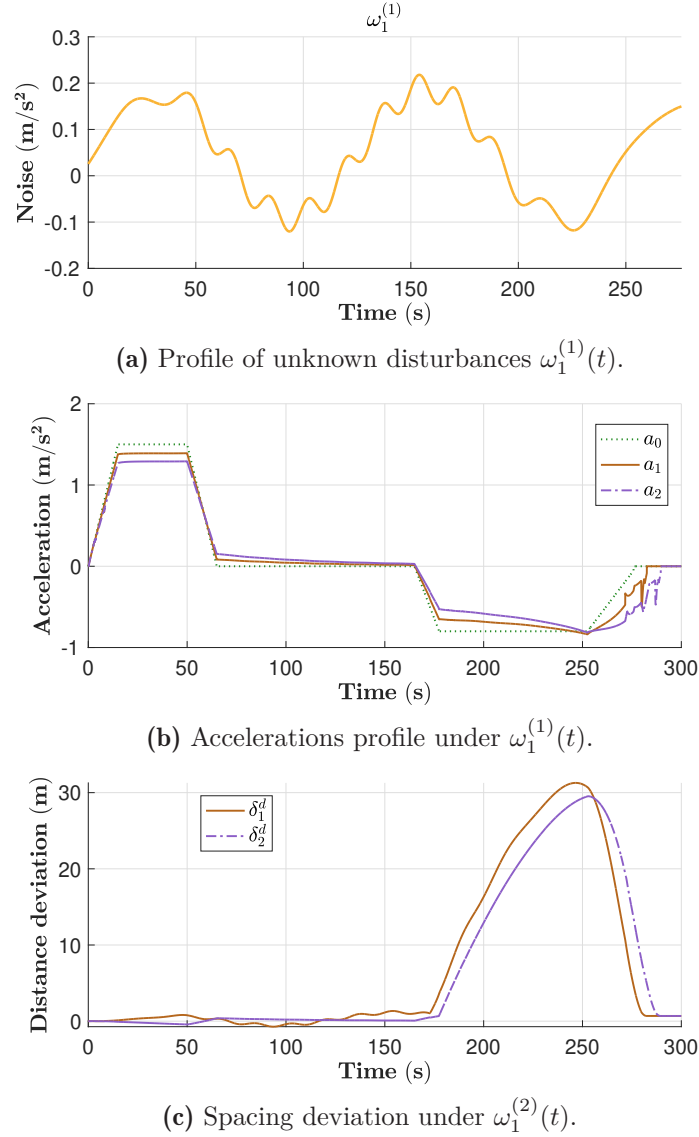


Figure 6.13: Results with dynamic disturbance $\omega_1^{(1)}$.

Panels (c) indicate that the spacing deviations $\delta_1^d(t)$ and $\delta_2^d(t)$ stay small and track the nominal trend during acceleration, cruise, and braking. No amplification along the convoy is observed, which suggests good disturbance containment.

Table 6.10 shows that the average spacing deviations change marginally with respect to the nominal metro case (within about ± 0.3 m). The spacing deviation variances are slightly reduced for Train 1 and essentially unchanged for Train 2. The stopping spacing deviations are identical to the nominal values (0.69 m for both pairs). The stopping time differences vary only moderately. Overall, the controller preserves safety and convoy compactness within numerical tolerance without adding extra margins to the nominal CBFs. These results support the robustness of the CBF-based filter to

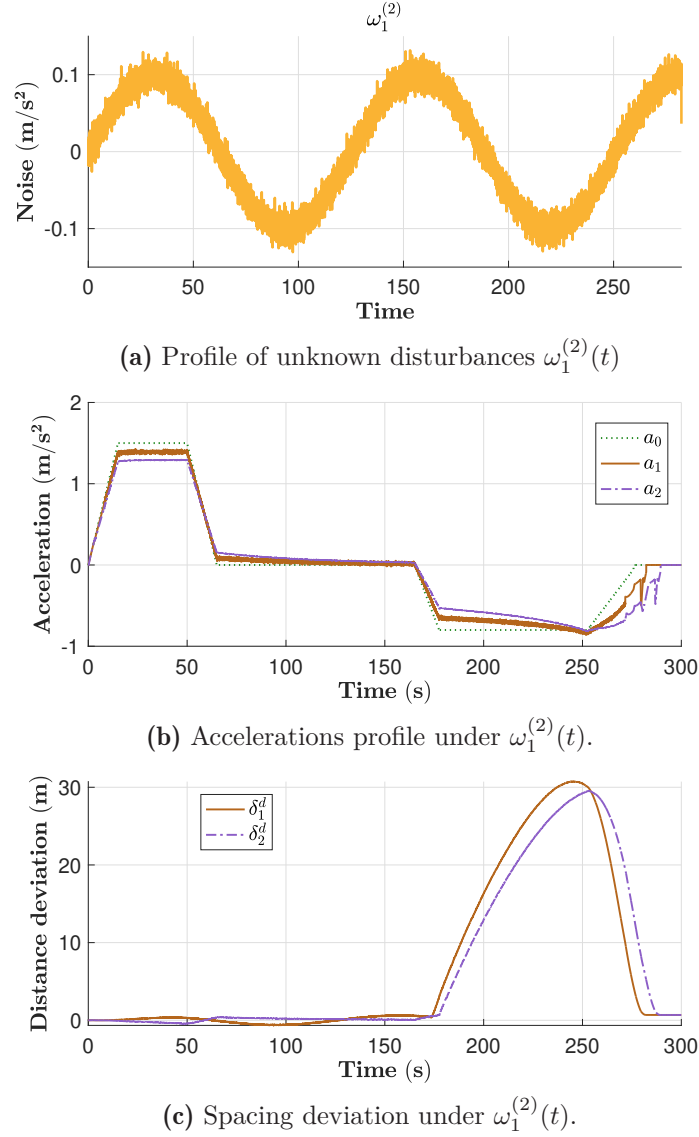


Figure 6.14: Results with dynamic disturbance $\omega_1^{(2)}$.

bounded acceleration disturbances acting on the most safety-critical follower.

Remark. In this section, both the delay and disturbance are injected only into the first follower, since configuration represents the most safety-critical scenario in a VC convoy. Train 1 directly follows the leader and therefore has no preceding follower to buffer its motion. As we see in the test results, the disturbances acting on Train 1 propagate downstream through the convoy. If the proposed method can preserve safety and stability under this condition, the robustness for the remaining followers can be reasonably ensured as well.

6.3.4 Emergency braking

Based on the nominal HS case, we let Train 0 take a sudden emergency braking for 10 seconds at the second cruise phase. The simulation results are reported in Fig. 6.15.

Under the stochastic case $\omega_1^{(2)}(t_k)$, the velocity-spacing profiles also remain smooth. This is expected, the disturbance acts as an additive acceleration (of order $10^{-2} \sim 10^{-1} \text{ m/s}^2$), which is small relative to the available traction/braking authority, and its high-frequency components are attenuated by integration (velocity is the integral of acceleration; spacing is the double integral). Moreover, the QP objective keeps u_1 close to u_1^{ref} while the HOCBF activates only when needed, effectively filtering the injected noise. As a result, the ripple mainly appears in the control input (see the zoomed view), whereas the macroscopic convoy motion remains stable and compact.

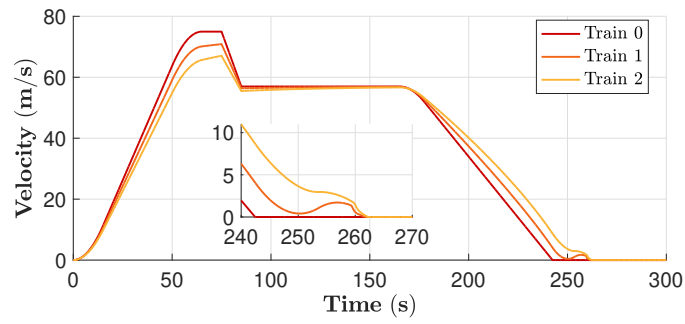
From the velocity profiles, it can be observed that once the leader initiates EB, the followers immediately begin to decelerate and maintain coordinated motion. The delay between the leader's and followers' responses is minimal, demonstrating that the control framework ensures fast and cooperative braking behavior.

The activation of the CBF constraints can be more clearly seen from the control inputs, where the followers' commanded accelerations reach the lower saturation bound at the instant of the leader's emergency braking. This indicates that the safety constraints are actively enforced, and the followers apply the maximum allowable braking to maintain a safe spacing.

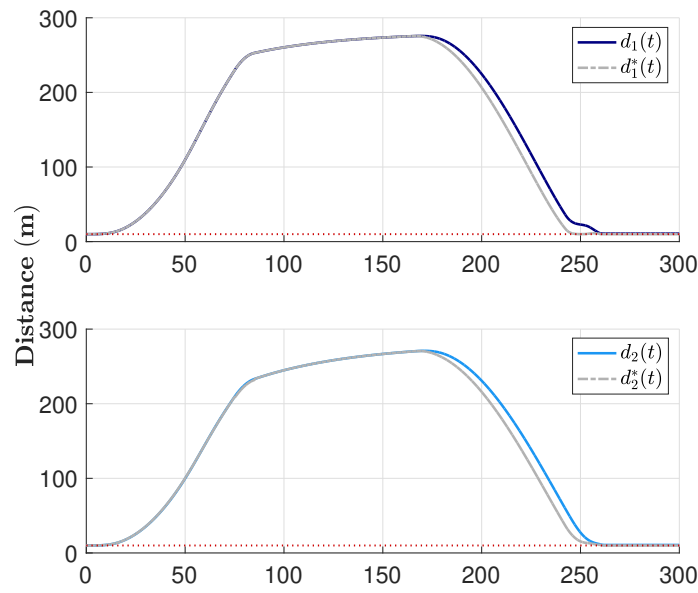
At the final stopping phase, a small "bouncing" behavior can be observed in the control inputs of the followers, leading to minor fluctuations in velocity, as highlighted in the zoomed-in view of Subfig. 6.15a. This behavior results from the followers slightly undershooting the desired spacing, i.e. d_0 , when Train 0 stops abruptly at around $t = 240 \text{ s}$. The controller of Train 1 and Train 0 continue applying a small increasing acceleration (from negative to positive) to reach the default margin. Until around $t = 260 \text{ s}$ the stopping reference acceleration is activated for both. Such behavior indicates that the controller remains responsive and safe even under abrupt braking conditions of the predecessor.

6.4 Summary

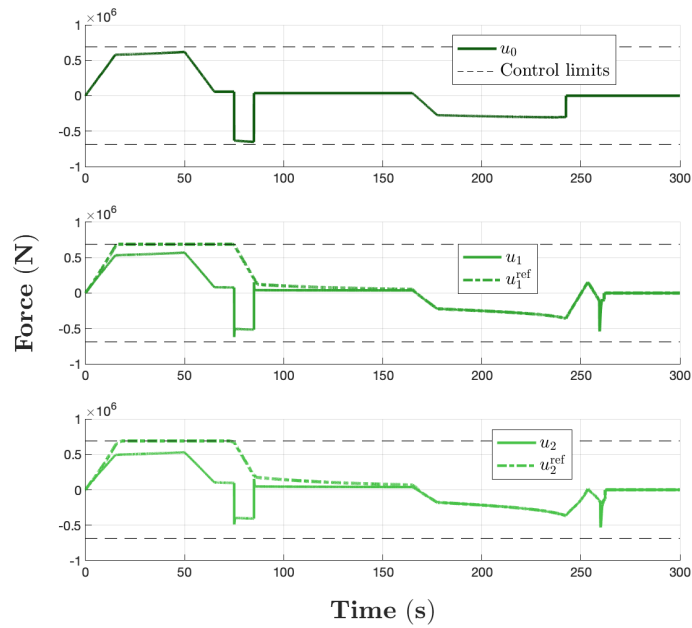
This chapter presented a series of simulation studies to validate the proposed CBF-based controller for VC convoys under both baseline and extended scenarios. The simulations covered different train types (MT and HS), convoy



(a) Train velocity profile.



(b) Inter-train distance.



(c) Control force.

Figure 6.15: HS convoy results with Train 0 emergency braking.

scales, and operating conditions including delay, uncertainty, and emergency braking.

In terms of **computation efficiency**, for both MT and HS convoys, the average computation time per QP (Steps 4–7 in Algorithm 5.1) for each follower is approximately 0.0028 s, which is significantly shorter than a typical train control cycle of 0.2 s. This confirms that the proposed control framework meets the real-time requirements of onboard implementation.

In terms of **control performance**, the results demonstrate that the controller maintains safety and compact convoy motion across all conditions. In the baseline cases, all followers satisfy the RBD-based safety constraints, achieve synchronized stopping, and show small spacing deviations (< 1 m on average for MT operations). The controller effectively adapts to heterogeneous braking capacities and different convergence times without compromising stability. When extended to the scenarios with communication delays or dynamic disturbances, the CBF constraints remain feasible and safety margins are preserved without introducing additional conservatism. Even under injected stochastic noise, the spacing profiles remain smooth and the stopping distances are comparable to the nominal case.

Overall, the simulation results confirm that the proposed method is applicable to convoys with more than two trains and remains effective across all VC-mode scenarios once the convoy formation is established. It features low computational complexity and is easy to implement, making it suitable for real-time onboard deployment. The controller provides a unified and continuously varying spacing function that enables smooth transitions between operating phases, thereby reducing frequent state switching and potential failure points. Moreover, it consistently preserves safety and convoy compactness even in the presence of bounded dynamic uncertainties and communication delays.

These findings collectively verify the practicality and robustness of the proposed controller, paving the way for its application to more complex VC operations in future work.

Chapter 7

Conclusions and Outlook

In this chapter, first a summary of the methods proposed in this thesis is provided. Finally, the thesis conclude by outlining the future work.

7.1 Original Contributions

The main contributions of this work are summarized as follows:

1. The work introduces the use of Control Barrier Functions (CBF) and Control Lyapunov Functions (CLF) in train control to enforce safety constraints directly within the control loop. This approach provides formal safety guarantees without compromising the continuity and optimality of train motion.
2. It achieves a strict implementation of the relative braking distance (RBD) principle through time-varying High-order CBFs, allowing Virtual Coupling to realize its theoretical potential in safety assurance and capacity improvement.
3. A delay compensation method is developed to actively estimate and mitigate communication latency, enhancing robustness under time-varying delay conditions.
4. Through comprehensive simulations, the proposed controller is shown to handle complete convoy operations, covering formation establishment, synchronized cruising, and coordinated station stopping—the key scenarios of real-world train services.
5. The collision-avoidance safety guarantee is always satisfied, and the controller parameters can be flexibly adjusted to accommodate trains with different dynamic characteristics.
6. The controller exhibits high computational efficiency, achieving real-time feasibility and reducing computation burden by several orders of magnitude compared with typical MPC-based approaches. This demonstrates its practical suitability for onboard implementation in the VC system.

Altogether, the results confirm that the proposed CBF-based framework provides a rigorous and practically viable foundation for real-time, decentralized VC train control, bridging the gap between theoretical safety assurance and engineering feasibility.

7.2 Future Outlook

While the proposed control framework successfully achieves real-time, decentralized, and safety-guaranteed VC convoy operation, several open challenges remain for future investigation.

Firstly, although heterogeneous convoys have been examined in the simulations, the current analysis still relies on Assumption 2, which requires the predecessor train to satisfy the same or smaller maximum braking rate constraint as the follower. This assumption guarantees the feasibility of the CBF constraints but may not always hold in diverse operational conditions. Future studies could investigate more generalized formulations that ensure safety and feasibility even when this condition is partially violated or subject to uncertainty.

Secondly, constructing a valid and effective CBF remains a challenging task, especially for the choices of class- $\mathcal{K}$ functions. Future research may explore automated or learning-based methods for generating or tuning CBFs, aiming to improve scalability and reduce conservativeness while retaining formal guarantees.

Third, although sensitivity analyses have been conducted for key design parameters in reference force, the interaction among multiple tunable parameters still requires systematic exploration. More advanced parameter optimization or adaptive tuning strategies could further enhance the balance between safety, comfort, and performance.

Finally, the current study focuses primarily on single-source hazards, such as communication delays or emergency braking. Future work could investigate combined or cascading failure scenarios, where multiple disturbances or uncertainties coexist, to develop a more robust and resilient convoy control architecture.

Overall, addressing these challenges would further advance the practical deployment of CBF-based VC train control and strengthen its applicability to the next generation of intelligent railway signaling systems.

References

- [1] European Environment Agency, *Passenger transport demand – outlook from WBCSD*, <https://www.eea.europa.eu/data-and-maps/indicators/passenger-transport-demand-outlook-from-wbcd/passenger-transport-demand-outlook-from>, 2012.
- [2] European Commission, *Roadmap to a single european transport area: Towards a competitive and resource efficient transport system*, White Paper COM(2011) 144 Final, 2011.
- [3] Zhao, K., Wang, Y., Ding, M., Li, S., Quaglietta, E., and Meng, L., “Train scheduling with flexible coupling and decoupling at stations for an urban rail transit line,” in *2022 IEEE 25th International Conference on Intelligent Transportation Systems (ITSC)*, IEEE, 2022, pp. 1876–1881.
- [4] Wang, P., Xiao, X., and Nießen, N., “Flexible rolling stock composition strategy in urban rail transit lines: The influences of various train units and station capacities,” *Transportation Research Part C: Emerging Technologies*, vol. 162, p. 104609, 2024.
- [5] Yan, Z., Li, X., Zhang, Q., and Han, B., “Seat allocation model for high-speed railway passenger transportation based on flexible train composition,” *Computers & Industrial Engineering*, vol. 142, p. 106383, 2020.
- [6] Nold, M. and Corman, F., “Dynamic train unit coupling and decoupling at cruising speed: Systematic classification, operational potentials, and research agenda,” *Journal of Rail Transport Planning & Management*, vol. 18, p. 100241, 2021.
- [7] J. Cheng and Z. Ji. “The first case of urban rail in China! Shanghai Metro Line 16 will try out the "3+3" online "decoupling and coupling" operation mode from now on.” Accessed: 2025-10-22. [Online]. Available: <http://www.shmetro.com/node49/202111/con115618.htm>.
- [8] Zhang, S., Huang, P., and Yan, W., “A data-driven approach for railway in-train forces monitoring,” *Advanced Engineering Informatics*, vol. 59, p. 102258, 2024.

-
- [9] Mitchell, I. et al., “Ertms level 4, train convoys or virtual coupling,” *IRSE News*, vol. 219, pp. 14–15, 2016.
 - [10] Bock, U. and Varchmin, J., “Enhancement of the occupancy of railroads using virtually coupled train formations,” in *World Congress on Railway Research (WCCR)*, 1999.
 - [11] Bock, U. and Bikker, G., “Design and Development of a Future Freight Train Concept – “Virtually Coupled Train Formations”,” *IFAC Proceedings Volumes*, vol. 33, no. 9, pp. 395–400, 2000.
 - [12] Stander, T., Drewes, J., Braun, I., et al., “Operational and safety concepts for railway operation with virtual train-sets,” *IFAC Proceedings Volumes*, vol. 39, no. 12, pp. 261–266, 2006.
 - [13] Henke, C., Tichy, M., Schneider, T., et al., “Organization and control of autonomous railway convoys,” in *Proceedings of the 9th International Symposium on Advanced Vehicle Control*, Kobe, Japan, 2008, pp. 1–6.
 - [14] Shift2Rail, “Shift2rail strategic master plan, version 1.0,” European Commission, Tech. Rep., Mar. 2015. [Online]. Available: https://rail-research.europa.eu/wp-content/uploads/2021/06/Shift2Rail-Master-Plan_approved-by-S2R-GB.pdf.
 - [15] Aoun, J. et al., “Cost-effectiveness analysis for virtual coupling,” 2020.
 - [16] X2RAIL-4, *Advanced signalling and automation system - Completion of activities for enhanced automation systems, train integrity, traffic management evolution and smart object controllers*, 2019. [Online]. Available: https://projects.shift2rail.org/s2r_ip2_n.aspx?p=X2RAIL-4.
 - [17] Safe4RAIL, *Safe architecture for Robust distributed Application Integration in roLling stock*, 2016. [Online]. Available: <https://safe4rail-1.safe4rail-project.technikon.com/>.
 - [18] CONNECTA, *CONtributing to Shift2Rail’s NExt generation of high Capable and safe TCMS and brAkes*, 2016. [Online]. Available: https://projects.shift2rail.org/s2r_ip1_n.aspx?p=CONNECTA.
 - [19] R2DATO, *Rail to Digital automated up to autonomous train operation*, 2020. [Online]. Available: <https://projects.rail-research.europa.eu/eurail-fp2/>.
 - [20] German Aerospace Center (DLR). “Dlr tests virtual coupling system in the real world.” Accessed: 2025-09-12. [Online]. Available: <https://www.dlr.de/en/latest/news/2025/dlr-tests-virtual-coupling-system-in-the-real-world>.

-
- [21] Schumann, T., “Increase of capacity on the shinkansen high-speed line using virtual coupling,” *International Journal of Transport Development and Integration*, vol. 1, pp. 666–676, Jan. 2017.
 - [22] Liu, Y., Zhou, Y., Su, S., Xun, J., and Tang, T., “An analytical optimal control approach for virtually coupled high-speed trains with local and string stability,” *Transportation Research Part C: Emerging Technologies*, vol. 125, p. 102 886, 2021.
 - [23] Li, S., Yang, L., and Gao, Z., “Distributed optimal control for multiple high-speed train movement: An alternating direction method of multipliers,” en, *Automatica*, vol. 112, p. 108 646, Feb. 2020. Accessed: Jan. 17, 2024.
 - [24] Felez, J., Kim, Y., and Borrelli, F., “A model predictive control approach for virtual coupling in railways,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 20, no. 7, pp. 2728–2739, 2019.
 - [25] Liu, H., Luo, X., Tang, T., Zhang, Y., and Chai, M., “A hierarchical control approach for virtual coupling in metro trains,” *Computer-Aided Civil and Infrastructure Engineering*, vol. 39, no. 9, pp. 1318–1336, 2024.
 - [26] Ma, X., Zhou, M., Wang, H., Song, W., and Dong, H., “Virtual-coupling-based timetable rescheduling for heavy-haul railways under disruptions,” *IEEE Transactions on Computational Social Systems*, vol. 11, no. 5, pp. 7045–7054, 2024.
 - [27] Quaglietta, E., Wang, M., and Goverde, R. M., “A multi-state train-following model for the analysis of virtual coupling railway operations,” *Journal of Rail Transport Planning & Management*, vol. 15, p. 100 195, 2020.
 - [28] Rete Ferroviaria Italiana, “Deliverable d7.2: Technical feasibility study of the maglev,” Rete Ferroviaria Italiana, Tech. Rep., Jan. 2025, Project Deliverable. Accessed: Oct. 12, 2025. [Online]. Available: <https://www.rfi.it/progetti/horizon/deliverables>.
 - [29] Mendes Borges, R. and Quaglietta, E., “Assessing Hyperloop Transport Capacity Under Moving-Block and Virtual Coupling Operations,” en, *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 8, pp. 12 612–12 621, Aug. 2022.
 - [30] Song, H., Gao, S., Li, Y., Liu, L., and Dong, H., “Train-centric communication based autonomous train control system,” *IEEE Transactions on Intelligent Vehicles*, vol. 8, no. 1, pp. 721–731, 2022.

-
- [31] Wu, Q. et al., “Dynamics and control simulation of railway virtual coupling,” *Vehicle System Dynamics*, vol. 61, no. 9, pp. 2292–2316, 2023.
- [32] Versluis, N. D., Quaglietta, E., Goverde, R. M., Pellegrini, P., and Rodriguez, J., “Real-time railway traffic management under moving-block signalling: A literature review and research agenda,” *Transportation research part C: emerging technologies*, vol. 158, p. 104438, 2024.
- [33] Di Meo, C., Di Vaio, M., Flammini, F., Nardone, R., Santini, S., and Vittorini, V., “ERTMS/ETCS Virtual Coupling: Proof of Concept and Numerical Analysis,” en, *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 6, pp. 2545–2556, Jun. 2020.
- [34] Chen, R.-w. and Guo, J., “Development of the new CBTC system simulation and performance analysis,” *WIT Transactions on the Built Environment*, vol. 114, pp. 497–507, 2010.
- [35] Wang, Q., Chai, M., Liu, H., and Tang, T., “Optimized control of virtual coupling at junctions: A cooperative game-based approach,” in *Actuators*, MDPI, vol. 10, 2021, p. 207.
- [36] Xiao, S., Ge, X., and Wu, Q., “Attack-resilient distributed cooperative control of virtually coupled high-speed trains via topology reconfiguration,” *IEEE/CAA Journal of Automatica Sinica*, vol. 11, no. 4, pp. 1066–1068, 2024.
- [37] Xiao, S., Ge, X., and Wu, Q., “Attack-resilient distributed cooperative control of virtually coupled high-speed trains via topology reconfiguration,” *IEEE/CAA Journal of Automatica Sinica*, vol. 11, no. 4, pp. 1066–1068, 2024. DOI: [10.1109/JAS.2023.124011](https://doi.org/10.1109/JAS.2023.124011).
- [38] Wang, P., Zhu, Y., and Zhu, W., “Robust cooperative train trajectory optimization with stochastic delays under virtual coupling,” *IET Intelligent Transport Systems*, vol. 17, no. 7, pp. 1415–1433, 2023.
- [39] Dong, H., Zheng, Y., Song, X., and Gao, S., “Reinforced safe performance cooperative control with event-triggered implementation for train formation,” *IEEE Transactions on Industrial Informatics*, vol. 20, no. 7, pp. 9588–9598, 2024.
- [40] Su, S., She, J., Wang, D., Gong, S., and Zhou, Y., “A stabilized virtual coupling scheme for a train set with heterogeneous braking dynamics capability,” *Transportation Research Part C: Emerging Technologies*, vol. 146, p. 103947, 2023.

- [41] Song, H. and Schnieder, E., “Availability and performance analysis of train-to-train data communication system,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 20, no. 7, pp. 2786–2795, 2019.
- [42] Vaquero-Serrano, M. A. and Felez, J., “A decentralized robust control approach for virtually coupled train sets,” *Computer-Aided Civil and Infrastructure Engineering*, vol. 38, no. 14, pp. 1896–1915, 2023.
- [43] Liu, Y., Zhou, Y., Su, S., Xun, J., and Tang, T., “Control strategy for stable formation of high-speed virtually coupled trains with disturbances and delays,” *Computer-Aided Civil and Infrastructure Engineering*, vol. 38, no. 5, pp. 621–639, 2023.
- [44] Guo, Y., Pei, X., Luo, X., Liu, H., Tang, T., and Hou, T., “A particle swarm optimization-based online optimization approach for virtual coupling trains with communication delay,” *IEEE Intelligent Transportation Systems Magazine*, vol. 15, no. 6, pp. 49–63, 2023.
- [45] Qin, G., Meng, X., Wen, T., and Cai, B., “Virtual coupling trains based on multi-agent system under communication delay,” *Chinese Journal of Electronics*, vol. 33, no. 6, pp. 1545–1554, 2024.
- [46] Alasmay, W. and Zhuang, W., “Mobility impact in IEEE 802.11p infrastructureless vehicular networks,” *Ad Hoc Networks*, vol. 10, no. 2, pp. 222–230, Mar. 2012. Accessed: Oct. 15, 2025.
- [47] Felez, J., Vaquero-Serrano, M. A., and Dios Sanz, J. de, “A robust model predictive control for virtual coupling in train sets,” in *Actuators*, MDPI, vol. 11, 2022, p. 372.
- [48] Park, J., Lee, B.-H., and Eun, Y., “Virtual coupling of railway vehicles: Gap reference for merge and separation, robust control, and position measurement,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 2, pp. 1085–1096, 2020.
- [49] Zhang, X., Yang, D., Zhang, W., and Huang, J., “Optimal robust constraints-following control for rail vehicle virtual coupling,” *Journal of Vibration and Control*, vol. 29, no. 5-6, pp. 1352–1365, 2023.
- [50] Wang, D. and Cao, Y., “Adaptive cruise control of virtual coupled trains based on sliding mode,” in *Journal of Physics: Conference Series*, IOP Publishing, vol. 2224, 2022, p. 012 109.
- [51] Wu, Q., Ge, X., Han, Q.-L., and Liu, Y., “Railway virtual coupling: A survey of emerging control techniques,” *IEEE Transactions on Intelligent Vehicles*, vol. 8, no. 5, pp. 3239–3255, 2023.
- [52] Felez, J. and Vaquero-Serrano, M. A., “Virtual coupling in railways: A comprehensive review,” *Machines*, vol. 11, no. 5, p. 521, 2023.

- [53] Zhao, H., Dai, X., Ding, L., Cui, D., Ding, J., and Chai, T., “Resilient cooperative control for high-speed trains under denial-of-service attacks,” *IEEE Transactions on Vehicular Technology*, vol. 70, no. 12, pp. 12 427–12 436, 2021.
- [54] Wang, X., Zhu, L., Wang, H., Tang, T., and Li, K., “Robust distributed cruise control of multiple high-speed trains based on disturbance observer,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, no. 1, pp. 267–279, 2019.
- [55] Xun, J., Li, Y., Liu, R., Li, Y., and Liu, Y., “A survey on control methods for virtual coupling in railway operation,” *IEEE Open Journal of Intelligent Transportation Systems*, vol. 3, pp. 838–855, 2022.
- [56] Su, S., She, J., Li, K., Wang, X., and Zhou, Y., “A nonlinear safety equilibrium spacing-based model predictive control for virtually coupled train set over gradient terrains,” *IEEE Transactions on transportation electrification*, vol. 8, no. 2, pp. 2810–2824, 2021.
- [57] Liu, Y., Liu, R., Wei, C., Xun, J., and Tang, T., “Distributed Model Predictive Control Strategy for Constrained High-Speed Virtually Coupled Train Set,” en, *IEEE Transactions on Vehicular Technology*, vol. 71, no. 1, pp. 171–183, Jan. 2022. Accessed: Dec. 5, 2023.
- [58] Liu, X., Xun, J., Ning, B., Liu, T., and Xiao, X., “Moving horizon optimization of train speed profile based on sequential quadratic programming,” in *2018 International Conference on Intelligent Rail Transportation (ICIRT)*, IEEE, 2018, pp. 1–5.
- [59] He, J., Long, Y., and Zhang, C., “Anti-Saturation Sliding Mode Control for Virtually Coupled HHTs Under Saturation Constraints,” *IET Intelligent Transport Systems*, vol. 19, no. 1, e70008, 2025. Accessed: Oct. 27, 2025.
- [60] Basile, G., Napoletano, E., Petrillo, A., and Santini, S., “Roadmap and challenges for reinforcement learning control in railway virtual coupling,” *Discover Artificial Intelligence*, vol. 2, no. 1, p. 27, 2022.
- [61] Su, S., Liu, W., Zhu, Q., Li, R., Tang, T., and Lv, J., “A cooperative collision-avoidance control methodology for virtual coupling trains,” *Accident Analysis & Prevention*, vol. 173, p. 106 703, 2022.
- [62] Chai, M., Su, H., and Liu, H., “Long short-term memory-based model predictive control for virtual coupling in railways,” *Wireless communications and mobile computing*, vol. 2022, no. 1, p. 1 859 709, 2022.

-
- [63] Choi, J. J., Lee, D., Sreenath, K., Tomlin, C. J., and Herbert, S. L., “Robust Control Barrier–Value Functions for Safety-Critical Control,” in *2021 60th IEEE Conference on Decision and Control (CDC)*, Austin, TX, USA: IEEE, Dec. 14, 2021, pp. 6814–6821. Accessed: Jan. 11, 2024.
 - [64] Hsu, S.-C., Xu, X., and Ames, A. D., “Control barrier function based quadratic programs with application to bipedal robotic walking,” in *2015 American Control Conference (ACC)*, IEEE, 2015, pp. 4542–4548.
 - [65] Ames, A. D., Coogan, S., Egerstedt, M., Notomista, G., Sreenath, K., and Tabuada, P., “Control Barrier Functions: Theory and Applications,” in *2019 18th European Control Conference (ECC)*, Jun. 2019, pp. 3420–3431.
 - [66] Wen, G., Fu, J., Lu, H., Sun, J., and Shen, H., “Robust collision avoidance and path-following of usvs with reduced conservativeness: A control barrier function-based approach,” *Journal of Field Robotics*, vol. 42, no. 4, pp. 1388–1400, 2025.
 - [67] Molnar, T. G., Kannan, S. K., Cunningham, J., Dunlap, K., Hobbs, K. L., and Ames, A. D., “Collision avoidance and geofencing for fixed-wing aircraft with control barrier functions,” *IEEE Transactions on Control Systems Technology*, 2025.
 - [68] Alan, A., Taylor, A. J., He, C. R., Ames, A. D., and Orosz, G., “Control barrier functions and input-to-state safety with application to automated vehicles,” *IEEE Transactions on Control Systems Technology*, vol. 31, no. 6, pp. 2744–2759, 2023.
 - [69] Nagumo, M., “Über die lage der integralkurven gewöhnlicher differentialgleichungen,” *Proceedings of the physico-mathematical society of Japan. 3rd Series*, vol. 24, pp. 551–559, 1942.
 - [70] Prajna, S. and Jadbabaie, A., “Safety verification of hybrid systems using barrier certificates,” in *International workshop on hybrid systems: Computation and control*, Springer, 2004, pp. 477–492.
 - [71] Khalil, H., *Nonlinear Systems* (Pearson Education). Prentice Hall, 2002.
 - [72] Xiao, W., Cassandras, C. G., and Belta, C., *Safe Autonomy with Control Barrier Functions: Theory and Applications* (Synthesis Lectures on Computer Science). Cham: Springer International Publishing, 2023.

- [73] UIC FRMCS Functional Working Group, “Future Railway Mobile Communication System: User Requirements Specification,” International Union of Railways (UIC), Tech. Rep. FU-7100, version 4.0.0, Jan. 2019, Version 4.0.0, Reference FU-7100, p. 120.
- [74] RailTech. “Nokia and DB deploy first commercial FRMCS network on German test track.” Accessed: 2025-09-12. [Online]. Available: <https://www.nokia.com/newsroom/nokia-deploys-worlds-first-1900-mhz-5g-radio-network-on-deutsche-bahn-test-track-supporting-frmcs/>.
- [75] Lehner, A., Strang, T., and Unterhuber, P., “Direct train-to-train communications at low uhf frequencies,” *IET Microwaves, Antennas & Propagation*, vol. 12, no. 4, pp. 486–491, 2018.
- [76] Goikoetxea, J., “Roadmap towards the wireless virtual coupling of trains,” in *Communication Technologies for Vehicles: 10th International Workshop*, Springer, 2016, pp. 3–9.
- [77] Brackstone, M. and McDonald, M., “Car-following: A historical review,” *Transportation research part F: traffic psychology and behaviour*, vol. 2, no. 4, pp. 181–196, 1999.
- [78] Vogel, K., “A comparison of headway and time to collision as safety indicators,” *Accident analysis & prevention*, vol. 35, no. 3, pp. 427–433, 2003.
- [79] Zhang, Y., Xu, Z., Wang, Z., Yao, X., and Xu, Z., “Impacts of communication delay on vehicle platoon string stability and its compensation strategy: A review,” *Journal of Traffic and Transportation Engineering (English Edition)*, vol. 10, no. 4, pp. 508–529, Aug. 2023. Accessed: Oct. 11, 2025.
- [80] Ketphat, N., Whiteing, A., and Liu, R., “Train Movement Under the Virtual Coupling System,” in *Urban Rail Transit*, ser. Lecture Notes in Mobility, Singapore: Springer, 2021, pp. 257–276.
- [81] Emery, D., “Enhanced ETCS L2/L3 train control system,” en, in *WIT Transactions on State of the Art in Science and Engineering*, 1st ed., vol. 1, Jun. 2010, pp. 113–122. Accessed: Mar. 8, 2024.
- [82] Ning, B., “Absolute braking and relative distance braking-train operation control modes in moving block systems,” *WIT Transactions on The Built Environment*, vol. 37, 1998.
- [83] Liu, Y., Ou, D., Yang, Y., and Dong, D., “A method for maintaining virtually coupled states of train convoys,” *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 237, no. 2, pp. 243–252, 2023.

-
- [84] Liu, C. and Xu, Z., “Multi-agent system based cooperative control for speed convergence of virtually coupled train formation,” *Sensors*, vol. 24, no. 13, p. 4231, 2024.
 - [85] Li, Z., Zhong, L., Yang, H., and Zhou, L., “Distributed cooperative tracking control strategy for virtual coupling trains: An event-triggered model predictive control approach,” *Processes*, vol. 11, no. 12, p. 3293, 2023.
 - [86] Chen, M., Xun, J., and Liu, Y., “A coordinated collision mitigation approach for virtual coupling trains by using model predictive control,” in *2020 IEEE 23rd International Conference on Intelligent Transportation Systems (ITSC)*, IEEE, 2020, pp. 1–6.
 - [87] Luo, X., Liu, H., Wang, J., Chai, M., and Zhang, Y., “Arrival time difference in virtually coupled train set: Cause and solution,” in *2022 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, IEEE, 2022, pp. 474–479.
 - [88] Song, H., ShangGuan, W., Qiu, W., Sheng, Z., and Harrod, S. S., “Two-Stage Optimal Trajectory Planning Based on Resilience Adjustment Model for Virtually Coupled Trains,” en, *IEEE Transactions on Intelligent Transportation Systems*, vol. 24, no. 12, pp. 15 219–15 235, Dec. 2023.
 - [89] Pan, J., Peng, Q., Zhan, S., and Bai, J., “Multiscenario-Based Train Headway Analysis under Virtual Coupling System,” *Journal of Advanced Transportation*, vol. 2021, pp. 1–20, Dec. 21, 2021.
 - [90] Furness, N., Van Houten, H., Arenas, L., and Bartholomeus, M., “ERTMS level 3: The game-changer,” *IRSE news*, vol. 232, pp. 2–9, 2017.
 - [91] Flammini, F., Marrone, S., Nardone, R., Petrillo, A., Santini, S., and Vittorini, V., “Towards railway virtual coupling,” in *2018 IEEE International Conference on Electrical Systems for Aircraft, Railway, Ship Propulsion and Road Vehicles & International Transportation Electrification Conference (ESARS-ITEC)*, IEEE, 2018, pp. 1–6.
 - [92] Zhang, Y., Li, X., Wu, J., Liu, H., and Jia, Q., “Scheme research on train operation control system supporting virtual coupling technology,” *Urban Mass Transit*, vol. 27, no. 6, 2024.
 - [93] Wang, J. and Rakha, H. A., “Longitudinal Train Dynamics Model for A Rail Transit Simulation System,” en, *Transportation Research Part C: Emerging Technologies*, vol. 86, pp. 111–123, Jan. 2018. Accessed: Dec. 4, 2023.

-
- [94] Rochard, B. P. and Schmid, F., “A Review of Methods to Measure and Calculate Train Resistances,” *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 214, no. 4, pp. 185–199, 2000.
 - [95] X2Rail-1, “Deliverable D3.1. User & System Requirements (Telecommunications),” EU-Rail, Tech. Rep., 2016, <https://ec.europa.eu/research/participants/documents/downloadPublic?documentIds=080166e5c0facc5d&appId=PPGMS>.
 - [96] Zheng, Y., Li, S. E., Li, K., Borrelli, F., and Hedrick, J. K., “Distributed model predictive control for heterogeneous vehicle platoons under unidirectional topologies,” *IEEE Transactions on Control Systems Technology*, vol. 25, no. 3, pp. 899–910, 2016.
 - [97] Xiao, W. and Belta, C., “High-Order Control Barrier Functions,” en, *IEEE Transactions on Automatic Control*, vol. 67, no. 7, pp. 3655–3662, Jul. 2022. Accessed: Feb. 1, 2024.
 - [98] Zhao, F., Li, H., Wang, J., and Li, J., “String Stability Based Cloud Predictive Control of Vehicle Platoon With Random Time Delay,” *IEEE Transactions on Vehicular Technology*, vol. 74, no. 6, pp. 8784–8796, Jun. 2025. Accessed: Oct. 11, 2025.
 - [99] Alan, A., Molnar, T. G., Ames, A. D., and Orosz, G., “Generalizing robust control barrier functions from a controller design perspective,” *IEEE Open Journal of Control Systems*, 2025.
 - [100] Peng, J., Wang, H., Ding, S., Liang, J., and Wang, Y., “Robust High-Order Control Barrier Functions-Based Optimal Control for Constrained Nonlinear Systems With Safety-Stability Perspectives,” *IEEE Transactions on Automation Science and Engineering*, pp. 1–11, 2023. Accessed: Nov. 10, 2023.
 - [101] Howlett, P., “The optimal control of a train,” *Annals of Operations Research*, vol. 98, no. 1, pp. 65–87, 2000.
 - [102] Wang, J.-Q., Li, S. E., Zheng, Y., and Lu, X.-Y., “Longitudinal collision mitigation via coordinated braking of multiple vehicles using model predictive control,” *Integrated Computer-Aided Engineering*, vol. 22, no. 2, pp. 171–185, 2015.
 - [103] Liu, X., Peng, Y., Tian, Z., Lu, S., Jiang, L., and Chen, M., “Comparative performance analysis of speed trajectory optimization algorithms for metro and high-speed railways,” *IEEE Transactions on Transportation Electrification*, 2025.

