Witnessing quantum steering by means of the Fisher information

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Capturing specific kinds of quantum correlation is of paramount importance for quantum networking. Different routes can be taken to achieve this task, highlighting different aspects of such quantum correlations. Following the recent theoretical results by Yadin, Fadel, and Gessner [Nat. Commun. **12**, 2410 (2021)], we demonstrate experimentally how steering manifests in the metrological abilities of a bipartite state. Our results confirm the relevance of this approach, and compare the outcome with already employed alternatives.

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I. INTRODUCTION

Understanding the potential of processing information by means of quantum systems is bringing about an intense technological effort. This is referred to as the second quantum revolution [1], to distinguish it from the first one that developed the new theory in the early 20th century. The current technology-oriented approach bears conceptual implications as well: Under the new light, quantum properties are considered not only for their physical meaning, but also for their implications in information processing tasks.

This technology-oriented mindset has allowed us to recognize the existence of different categories of quantum correlations, distinguished according to their operational meaning [2]. When observing a bipartite system, one may ask different questions: whether measurement outcomes could be reproduced by means of local realistic models, leading to the concept of quantum nonlocality [3]; whether correlations can be explained only in terms of local quantum states (steering) [4]; and whether there exists a procedure to generate the state locally (entanglement) [5]. These correlations, in turn, provide the means for quantum communications at different levels of security and trust among the nodes [6–8], within the paradigm adopted for this categorization [9].

Steering captures in rigorous terms the effect underlying the Einstein-Podolsky-Rosen (EPR) paradox [10], as originally highlighted by Schrödinger [11], and then formalized by Wiseman *et al.* [12]. Two partners, Alice and Bob, share a bipartite state, however, Bob doubts that Alice can steer his local state better than what she could by local hidden states. There exist criteria for assessing the unsuitability of such models [13–16], many of which have been tested in experiments [17–27]. In particular, Reid's criterion states that steering can be revealed by the inappropriateness of Heisenberg's relations for the variances of Bob's observables, conditioned on his communication with Alice [28]. Based on this approach, Yadin, Fadel, and Gessner (YFG) [29] have established a connection between the presence of steering, as captured by Reid's criterion, and the metrological power of Bob's conditional states. In this work, we present an experimental demonstration of use of the YFG criterion for the measurement of steering on two-photon states produced by nonclassical interference. Our results show that albeit closely related to Reid's criterion, for our class of states the YFG approach delivers a quantitative assessment of the amount of steering at the cost of increased resources being required. These results reinforce the employment of metrological figures beyond problems strictly related to sensing.

II. RESULTS

The concept of steering can be illustrated as follows (Fig. 1). Alice and Bob have access to a bipartite state, and Alice claims that, by performing a measurement of $K = \sum_k k|k\rangle\langle k|$ and collecting the outcome k, she is able to steer the state of Bob's system, and guess the outcome h of a measurement of his observable $H = \sum_h h|h\rangle\langle h|$. Bob, instead, is convinced this is not the case, and Alice merely mimics this process by means of a classical probability distribution for k, and the outcomes are determined by Born's rule applied to a local hidden state $\sigma_k^B(\lambda)$, whose probability $p(\lambda)$ is set by a classical variable λ. The joint statistics of k and h is described by means of the assemblage $\mathcal{A}(k, K) = p(k|K)\rho^B_{k|K}$, which links Alice's result k for the measurement of K, and Bob's conditioned state $\rho^B_{k|K}$; the joint probability is then written as $p(k, h|K, H) = \langle h|\mathcal{A}(k, K)|h\rangle$. The local-hidden-state setting writes $\mathcal{A}(k, K) = \sum_\lambda p(\lambda)p(k|X, \lambda)\sigma_k^B(\lambda)$. Thus, the properties of the allowed assemblages are curtailed, specifically for what concerns the strength of their correlations, in analogy with what occurs with local hidden variables.

Steering is connected with the emergence of the EPR paradox [10], as recognized in Ref. [12], in that the knowledge of Alice's value k allows us to make predictions on Bob's results beyond what would be allowed by Heisenberg's relation—a fact on which also Popper called to attention [30,31]. A way of quantifying these observations in a quantum state considers

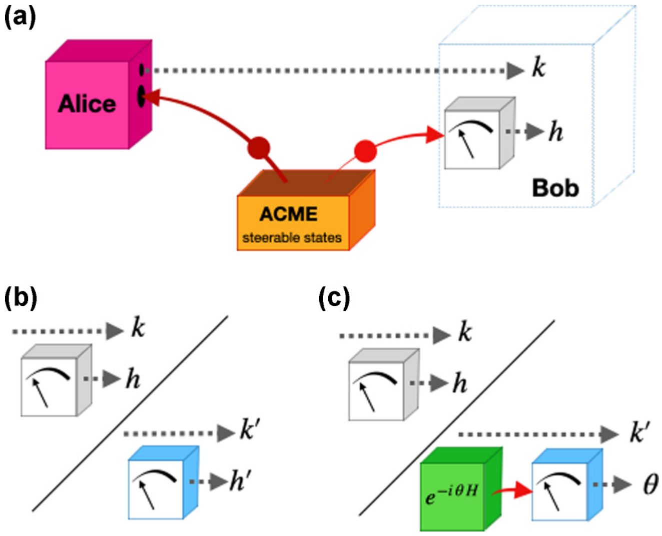

FIG. 1. Conceptual scheme. (a) Alice and Bob share a bipartite state. Alice claims she can steer Bob's state by performing a measurement of K with outcome k. Bob, receiving Alice's outcome and measuring his own, can check whether Alice is indeed steering his state or is pretending to do so by employing local hidden states. Bob can choose two different strategies: either (b) performing the required measurements for implementing Reid's criterion or (c) measuring the Fisher information of his states, conditioned by Alice's outcome.

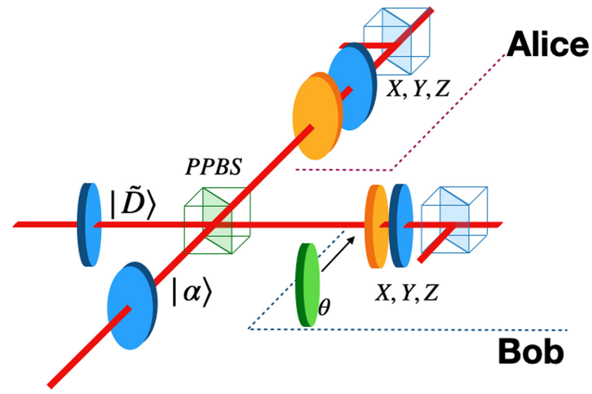

FIG. 2. Experimental setup. Photon pairs are generated via spontaneous parametric downconversion with a continuous wave 405-nm pump through a 3-mm type-I beta barium borate (BBO) crystal. The photons are then spectrally filtered (full width at half maximum of 7.5 nm) and sent through single-mode fibers to the partially polarizing beam splitter (PPBS): The input polarizations are controlled through two half-wave plates (blue), and take into account the necessary prebias to compensate for the different transmission probabilities of the partially polarizing beam splitter. Alice and Bob's measurements of the Pauli operators are then conventionally performed by means of a quarter-wave plate (orange), half-wave plate, and a polarizing beam splitter. For the measurement of the Fisher information, a further rotation is implemented on Bob's side by a half-wave plate (green).

the average deviation of the actual value h with respect to $\check{h}(k)$, the one predicted based on k:

$$\Delta^2 H_{\text{est}} = \sum_{a,h} p(k,h|K,H)(\check{h}(k)-h)^2. \qquad (1)$$

When two incompatible measurements are carried out, a local uncertainty limit holds for nonsteerable states [28,32]

$$\Delta^2 H_{\text{est}} \Delta^2 H'_{\text{est}} \geqslant \tfrac{1}{4}|\langle[H,H']\rangle|^2, \qquad (2)$$

where the average of the commutator is calculated on Bob's unconditioned reduced state ρ^B. Reid's criterion states that the violation of such an inequality may serve as a witness for an EPR paradox.

Inequalities in the form (2) are also found in quantum metrology when considering the estimation of the parameter θ in the transformation $e^{-i\theta H}$. The variance Var(θ) on the value of the parameter is limited by the Fisher information $F(\theta)$ associated with the quantum state, the chosen measurement, and the number n of repetitions of the experiment according to the Cramér-Rao bound $\text{Var}(\theta) \geqslant 1/[nF(\theta)]$. The quantum Fisher information $F_Q(\theta)$, which is a function of the quantum state only, limits all possible Fisher information from above $F_Q(\theta) \geqslant F(\theta)$, which can be saturated for a particular measurement. The resulting inequality $\text{Var}(\theta) \geqslant 1/[nF_Q(\theta)]$ is known as the quantum Cramér-Rao bound [33–39]. For pure states, $F_Q(\theta) = 4\Delta^2 H$, thus we can write the Cramér-Rao bound in a form reminiscent of Heisenberg's relation [40,41]

$$\text{Var}(\theta)\Delta^2 H \geqslant \frac{1}{4n}, \qquad (3)$$

which also applies for generic mixed states [38]. Combining results (2) and (3), YFG have succeeded in establishing a link between the EPR paradox and the metrological power of a state [29]. They consider Bob's conditioned states $\rho^B_{k|K}$ to compute the optimized conditional variance

$$\Delta^2 H_{\text{opt}} = \min_K \sum_k p(k|K)\Delta^2 H_{k|K}, \qquad (4)$$

and the quantum conditional Fisher information

$$F_{\text{opt}} = \max_K \sum_k p(k|K) F_{Q,k|K}, \qquad (5)$$

where $B_{k|K}$ is the value of the quantity B calculated on the state $\rho^B_{k|K}$. For any state admitting a decomposition in assemblages $\mathcal{A}(k,K)$, there holds the limit

$$F_{\text{opt}} \leqslant 4\,\Delta^2 H_{\text{opt}}. \qquad (6)$$

A violation of such an inequality flags the inappropriateness of local-hidden-state models, hence the presence of an EPR paradox. This can be put in direct connection to (2) and (3) by means of the Cramér-Rao bound, and of the property that $\Delta^2 H_{\text{opt}} \leqslant \Delta^2 H_{\text{est}}$.

We tested these ideas in the experiment illustrated in Fig. 2. Bipartite states with a variable degree of steering are produced as follows. Two photons are prepared in the states $|\tilde{D}\rangle = \cos(\pi/3)|H\rangle + \sin(\pi/3)|V\rangle$, and $|\alpha\rangle = \cos(2\alpha)|H\rangle + \sin(2\alpha)|V\rangle$—the kets $|H\rangle$ and $|V\rangle$ denote a photon with horizontal and vertical polarization, respectively. These arrive on a partially polarizing beam splitter (PPBS), with transmittivities $T_H = 1$, $T_V = 1/3$, on which nonclassical interference occurs. By postselecting events when the two photons emerge from different arms, a quantum correlated state is produced [42]. The contributions in the final states are also modulated by the different transmittivities, and these have to be accounted for in the first preparation step: This implies that maximal correlation is expected for $2\alpha = \pi/3$. The two photons are then distributed to two measurement stations, Alice and Bob, for the analysis: A conventional sequence of a quarter-wave plate and a half-wave plate followed by a polarizer implement the measurement of one of the Pauli operators X, Y, or Z. These correspond to discrimination of diagonal (D)/antidiagonal (A) polarizations for X, right-circular (R)/left circular (L) for Y, and H/V for Z.

We detect the presence of steering in Alice and Bob's shared state. The first method we adopt, based on Refs. [12,13], considers a bound on correlations established for a nonsteerable state. For our class of states, this takes the form

$$\langle S\rangle = \frac{1}{\sqrt{3}}|\langle X_A Z_B\rangle + \langle Z_A X_B\rangle + \langle Y_A Y_B\rangle| \leqslant 1. \tag{7}$$

Alice's measurement of Z_A thus prepares Bob's photon in the X_B basis, and similarly for the other two cases. While we have adopted a fully quantum notation for a direct connection to the measured quantities, in Bob's point of view, the expectation values are calculated as average correlations between Alice's classical input and his own quantum observable. The corresponding results are pictured in Fig. 3, where $\langle S\rangle$ is reported as a function of the angle α. For low values of α the existing correlations do not allow us to confidently assess the presence of steering, while for higher values, this is present, despite the nonidealities of the experiment, foremost the reduced two-photon interference visibility.

Consider the observables $H = Y_B$ and $H' = X_B$: Alice should implement a measurement of Y_A and Z_A, respectively, in order to obtain the best predictions of Bob's outcomes. By this choice, the expressions of the variances in (1) for Y_B and X_B can be recast as $\Delta^2 Y_{\text{est}} = 1 - \langle Y_A Y_B\rangle$, and $\Delta^2 X_{\text{est}} = 1 - \langle Z_A X_B\rangle$. According to Reid's criterion (2), their product is bounded in nonsteerable states by $\langle Z_B\rangle$, obtained by tracing out Alice's photon. The corresponding experimental results are reported in Fig. 3(b), and demonstrate a violation of the inequality for all states—except for the first point, for which this is expected. We remark, however, that the amount of violation is not proportional to the level of steering, as it appears evident from the comparison of the two panels in Fig. 3. We consider these results as the set standard for a comparison with the metrological study.

If Bob is convinced of the absence of steering in the distributed state, he would conclude the variance $\Delta^2 Y_{\text{est}}$ sets the achievable precision in an experiment aiming at estimating the parameter θ with the evolution $e^{i\theta Y_B}$. For instance, upon observing $\Delta^2 Y_{\text{est}} = 0$, Bob would infer that the local-hidden-state collection he is receiving is equivalent to an incoherent mixture of eigenstates of Y_B, i.e., $|R\rangle$ and $|L\rangle$. However, Alice has the capability of steering Bob's state to $|H\rangle$ and $|V\rangle$: Contrary to his expectation, Bob could benefit from these coherent superpositions of $|R\rangle$ and $|L\rangle$ for phase estimation.

In our experiment, we have implemented the operation $e^{i\theta Y_B}$ by means of a half-wave plate, set at an angle $\theta/2$, and recorded detection probabilities $p(H|D)$, $p(V|D)$ and $p(H|A)$, $p(V|A)$ in the Z_B basis for Bob, conditioned by a measurement of X_A, i.e., of the D or A polarizations on Alice's side. In order to obtain an expression for the Fisher information, we have fitted the detection probabilities by the function $p(\theta) = [1 + v\cos(\theta + \theta_0)]/2$, with v and θ_0 fit parameters, based on a data set in which θ has been varied from 0 to 48° in steps of 4°. The fit curve allows us to calculate the conditional Fisher information F_D by means of its definition $F_D = [\partial_\theta p(H|D)]^2/p(V|D) + [\partial_\theta p(V|D)]^2/p(V|D)$, and similarly for F_A; typical results are reported in Fig. 4(a). This procedure is compatible with the assumption made upfront that Bob does not question the validity of quantum mechanics for the description of his apparatus—this is implicit in the choice of the fitting function.

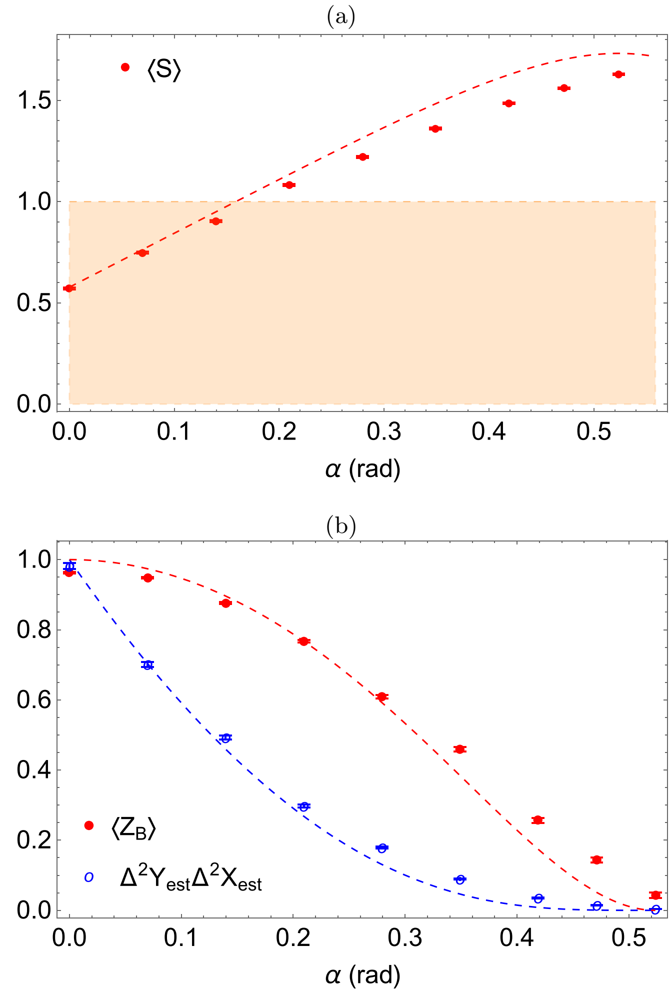

FIG. 3. Steering witness. (a) Assessment of the presence of steering by means of Eq. (7), showing $\langle S\rangle$ as a function of the angle α. The experimental results (red dots) are shown together with the ideal case (dashed red). The orange area indicates values of $\langle S\rangle \leqslant 1$. (b) Test of the steering using Reid's criterion (2). The product of the conditional variances (blue) is well below the nonsteerable measured bound (red), for every α. The points correspond to the measured values, while the dashed lines are predictions for the ideal state. In both panels, the errors are evaluated by propagation of the Poisson statistics of the registered counts.

The maximal values of F_D and F_A are used to compute a lower bound for the quantum Fisher information (5); if the state is not steerable, this is expected to have $4\Delta^2 Y_{\text{est}}$ as the

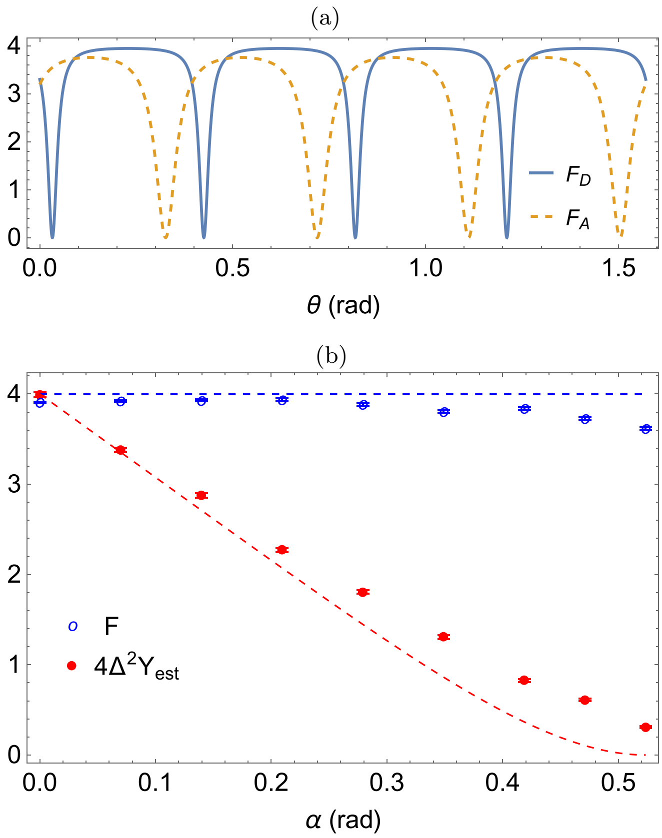

FIG. 4. Fisher information. (a) Conditional Fisher information (F_D solid blue, F_A dashed yellow) as a function of θ for $\alpha = 0.42$ rad. This is derived from the expression for the detection probability $p(\theta)$ obtained by a fit of the data. (b) Measured variance (red) and quantum Fisher information (blue) as a function of α: The points correspond to the measured values, while the dashed lines are predictions for the ideal state. The errors are evaluated through a Monte Carlo routine with 100 runs.

upper limit. The violation of this condition is reported in Fig. 4(b): The solid red points indicate the measured variance, while the open blue points indicate the Fisher information and sit well above the nonsteerable limit in the whole range of α.

III. DISCUSSION

Our results indicate the YFG and Reid's criterion offer consistent information on the presence of steering. Differently from the latter, YFG also provides a quantitative indication on the level of steering in the class of states we have tested; nevertheless it is unclear whether these properties hold in the general case [29]. This information is also captured by the steering witness $\langle S \rangle$, but this is attained only when exceeding a minimum level of steering. Pursuing the YFG route is demanding in terms of resources: Obtaining an estimation of the Fisher information requires performing several measurements, and this compares unfavorably to the more economical method of Fig. 3(a). On the other hand, the closer inspection of Bob's state reveals an EPR paradox for a wider range of values of α. The method preserves its conceptual importance, but appears less appealing for technological purposes, such as verification of steering in networks, especially when aiming at loophole-free arrangements. On the other hand, for many-particle atomic systems, a measurement of the Fisher information represent a convenient alternative [43], and the method may find concrete applications.

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