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# Minimization of proximity losses in electrical machines with tooth-wound coils 

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#### Abstract

Proximity losses are one of the main drawbacks in electrical machines with high rotational speed and electrical frequency. The reduction of proximity losses is usually pursued by adopting thin wires, twisted in the case of parallel connection. Twisting is a very effective solution for reducing proximity losses but also worsens the slot filling factor, heat dissipation, and de resistance. This work proposes an optimal twisting criterion that allows the minimization of the ac Joule losses due to parasitic circulating currents and also reduces the number of twists and the length of the coils. The criterion applies to machines having tooth-wound coils and is validated by finite element analysis (FEA) and experimental results.


## I. Introduction

The demand for weight and volume reduction in electrical machines is becoming more and more important in recent years because of both the requirements of international regulations for the reduction of energy consumption and the growing interest towards aerospace applications. On the one hand, a straightforward solution for size reduction is an increase of rotational speed and fundamental frequency. On the other hand, the related increase of the additional copper losses, due to the slotcrossing leakage flux and skin effect, becomes a critical issue for heat dissipation and efficiency.

The subdivision of the winding conductor into multiple insulated strands, reducing the wire area presented to the slot transverse flux lines, leads to the reduction of the copper losses. However, in [1] and [2] it has been experimentally demonstrated that this strategy is not sufficient to minimize the copper losses. The imbalance of the strand flux linkages causes the circulation of parasitic currents among the strands. In high-frequency machines, where conductors ${ }^{1}$ of large cross sections and low number of turns are used, these circulating currents can be very high despite the thinness of the bundled strands. Losses can be reduced by performing

[^0]the transposition of the strands either continuously through the length of the conductor (Litz type or Roebel bars) or only in the winding end connections. The first solution is the most straightforward, since it drastically reduces the circulating currents, but results in a lower slot-fill factor. Aiming at loss minimization, the second solution requires a design effort in order to choose the optimal transposition positions along the winding length, resulting in a better exploitation of the slot area.

The first, most general work on additional copper losses dates back to 1922 [3]. The analytical model developed by Lyon predicts the leakage impedance of a polyphase distributed winding, made of multi-stranded rectangular conductors, possibly transposed outside of the slot. The work in [4] applies the same method to determine the ac copper losses of transformer windings. Only in recent years, a renewed interest on this topic has led to the development of bi-dimensional analytical models, which allow investigating the effect of the machine design variables, with much lower computational cost than numerical models, [5] and [6]. The work in [5] accounts for the winding topology, the slot geometry, and the magneto motive force spatial harmonics, showing high accuracy in predicting the effects of eddy currents due to open circuit and armature reaction fields, in the absence of ferromagnetic saturation. In recent works [7] and [8] the issue of uneven current sharing of parallel strands is approached by FE-based methods, exploiting simplified numerical models to determine the impedance matrix of the inter-strands electromagnetic coupling. Despite the relevance of losses due to parasitic circulating currents, in particular for high-speed machines, only a few papers have recently addressed this problem. Moreover, none of them provided a closed-form analytical model capable of predicting the effect of the transposition of the strands on the ac Joule losses.

The present work applies the method in [3] to toothwound coils, validating it by means of both FEA and experiments, and demonstrates that a single transposition can cancel the bundle-level parasitic currents, which is another name by which circulating-currents are known in the literature. Despite the limitations of the analytical model, which applies to rectangular conductors embedded in open
rectangular slots, in the absence of the rotor field, the derived criterion for the optimal transposition is valid also for round wire stranded conductors and for semi-closed slots.

## II. Nomenclature

$\mathrm{n}_{\mathrm{w}}$ number of half turns disposed along the slot width
$n$ number of half turns disposed along the slot height
$\mathrm{n}_{\text {tr }}$ position along the slot height of the turn preceding the transposition point
m number of strands
d net height of a conductor
a increase of the strand height due to insulation
s slot width
w net conductor width
Is length of the stator stack
le sum of the lengths of the inactive parts of a half-turn
$\mu_{0}$ vacuum permeability
$f$ frequency
$\sigma$ electrical conductivity
$\delta=\sqrt{1 /\left(\pi f \mu_{0} \sigma\right)}$ skin depth

## III. DESCRIPTION OF COILS AND THEIR TRANSPOSITION

Consider a couple of series-connected solid conductors placed in a slot of a ferromagnetic body as shown in Fig. 1a. The time variation of the magnetic flux that crosses the slot produces eddy currents, of the strand-level type, circulating within the slot-bound portion of the conductors, thus leading to a non uniform distribution of the current over the conductor cross section (skin effect). The bottom conductor links more flux lines than the top one, thus its induced voltage is greater. Now, consider the same conductors connected in parallel as in Fig. 1b. The induced voltage difference among the strands before the connection is now compensated by an imbalance of their net currents, which can be seen as the effect of a current circulating among the strands through the terminals at which they are joined together. The transposition of the strands tends to make equal the flux linkages of each strand thus minimizing this parasitic current, which is of the bundle-level type. The strand transposition can be performed outside the slot by twisting either an individual conductor or the entire coil. The latter is a technique widely adopted during the manufacturing process of preformed coils of distributed windings. This paper focuses on the first type of transposition, performed once per each tooth-wound coil at the end connection between two consecutive turns.

Consider a tooth-coil made of n turns of a m-strand rectangular conductor, as reported in Fig. 2. The half-turns are counted from the input to the output terminal. The strands are insulated throughout the coil and the conductor is wound in an edgewise manner such that the strands lie one above the other along the slot height. Let the strands be numbered counting from bottom to top of the slot at the first
half-turn. The transposition of the strands is obtained by a twist of the conductor performed along the end connection between the $n_{t-t}$-th and the ( $n_{t_{t}}+1$ )-th turn (see Fig. 3) so as the strands of the next half-turns result numbered from top to bottom of the slot. If the tooth coil is made of more than a concentric layer $n_{w}>1$ (see Fig. 4) the transposition must be performed once on each layer at the same slot height.

Tooth-wound coils could be more easily manufactured by bending the conductor flatwise on the tooth/mandrel. In this case, the long edge of the strand cross section is oriented along the slot height. However such a flatwise-wound coil is generally not used, nor here represented, because it is exposed to higher losses, as shown in Fig.10, because of an enhanced skin effect.


Fig. 1. (a) Eddy currents due to parasitic currents in series connected turns (b) Circulating currents in parallel connected turns.


Fig. 2. Representation of the reciprocal position of the strands of a n-turn, 3strand coil with transposition after the $\mathrm{n}_{\mathrm{tr}}$-th turn.


Fig. 3. Conductor twisting performed along the end connection among the $\mathrm{n}_{\mathrm{tr}^{-}}$-th and the $\left(\mathrm{n}_{\mathrm{tr}}+1\right)$-th turn.


Fig. 4. Positions of the half-turns in the slot and representation of the net cross sections of the strands of a half turn separated by the insulation (grey).

## IV. LEAKAGE IMPEDANCE IN CONCENTRATED WINDINGS

The analytical formula for the leakage impedance at the terminals of a tooth-wound coil is presented here as function of the position of a single strand transposition. The leakage impedance at the coil terminals is that part of the total impedance associated with the magnetic energy stored in the conductors and with the Joule losses.

The formulae presented here, derived from the 1D analysis presented in [3], apply to single-layer concentrated windings made of multi-stranded rectangular conductors embedded in open rectangular slots. The model is based on the following assumptions: 1) the flux lines within an individual strand are perpendicular to the lateral edges of the slot, 2) the resistivity of the conductor is uniform, and 3) no flux lines penetrate the conductors along the end connections.

Being that the field lines are horizontal by assumption, the lamination of a conductor along the width of the slot has no influence on losses; therefore is not taken in consideration here.

## A. Multistrand conductor

By applying the analysis developed in [3], one obtains in (1) the expression of the leakage impedance per half-turn of the described coil.

$$
\begin{align*}
\dot{\mathrm{Z}} & =\mathrm{R}\left\{\mathrm{M}+\left[\left(\frac{\mathrm{n}_{\mathrm{tr}}^{2}}{\mathrm{n}}-\frac{\mathrm{n}}{2}\right)^{2}-\frac{1}{4}\right] \mathrm{N}+\right. \\
& \left.-\left[\left(\frac{\mathrm{n}_{\mathrm{tr}}^{2}}{\mathrm{n}}-\frac{\mathrm{n}}{2}\right)^{2}-\frac{4 \mathrm{n}^{2}-1}{12}\right] \mathrm{T}+\frac{\mathrm{n}^{2}+\mathrm{n}_{\mathrm{tr}}^{2}}{2 \mathrm{n}} \mathrm{~S}\right\} \tag{1}
\end{align*}
$$

where

$$
\begin{gather*}
R=\frac{l_{s}+l_{e}}{\sigma w d}  \tag{2}\\
M=\frac{1_{s}}{1_{s}+l_{e}} \alpha d \frac{\frac{\alpha a}{2}+\tanh \frac{\alpha d}{2 m}}{\tanh \left(\frac{\beta d}{2 m}\right)} \operatorname{coth}(\beta d)  \tag{3}\\
N=\frac{1_{s}}{1_{s}+l_{e}} \alpha d \frac{\frac{\alpha a}{2}+\tanh \frac{\alpha d}{2 m}}{\tanh \left(\frac{\beta d}{2 m}\right)} 2 \tanh \left(\frac{\beta d}{2}\right) \tag{4}
\end{gather*}
$$

$$
\begin{gather*}
\mathrm{T}=\frac{l_{\mathrm{s}}}{1_{\mathrm{s}}+l_{\mathrm{e}}} \alpha \mathrm{~d} 2 \mathrm{~m}\left(\frac{\alpha \mathrm{a}}{2}+\tanh \frac{\alpha \mathrm{d}}{2 \mathrm{~m}}\right)  \tag{5}\\
\mathrm{S}=\frac{1_{\mathrm{s}}}{l_{\mathrm{s}}+\mathrm{l}_{\mathrm{e}}} \alpha \mathrm{~d} \alpha \mathrm{a}  \tag{6}\\
\alpha^{2}=\mathrm{j} 2 \pi \mathrm{f} \sigma \frac{\mu_{0} \mathrm{wn}_{\mathrm{w}}}{\mathrm{~s}}  \tag{7}\\
\sinh ^{2} \frac{\beta \mathrm{~d}}{2 \mathrm{~m}}=\frac{\frac{\alpha \mathrm{d}}{2 \mathrm{~m}}\left(\frac{\alpha \mathrm{a}}{2}+\tanh \frac{\alpha \mathrm{d}}{2 \mathrm{~m}}\right)}{\left(\frac{\frac{\alpha d}{m}}{\sinh \frac{\alpha d}{m}}+\frac{l_{\mathrm{e}}}{l_{\mathrm{s}}}\right)} \tag{8}
\end{gather*}
$$

With $n_{t r}=n$, equation (1) gives the impedance per turn of a non-transposed coil. ${ }^{2}$

## B. Solid slot-bound conductors connected in series

The leakage impedance per half-turn of a coil made with a solid conductor is obtained in (9) by letting $\mathrm{m}=1, \mathrm{n}_{\mathrm{tr}}=$ n , and $\mathrm{a}=0$ in (1),

$$
\begin{equation*}
\dot{\mathrm{Z}}=\mathrm{R}\left(\mathrm{M}+\frac{\mathrm{n}^{2}-1}{3} \mathrm{~N}\right) \tag{9}
\end{equation*}
$$

The analysis in [4] gives an analogous expression for the leakage impedance of a winding portion of $\mathrm{n}_{\mathrm{w}} \mathrm{n}$ half-turns. However, it does not account for the turn-end length. Then letting $\mathrm{l}_{\mathrm{e}}=0$ in (2)-(8) and multiplying (9) by $\mathrm{n}_{\mathrm{w}} \mathrm{n}$, one obtains the same expression given by Dowell.
As will be demonstrated in the next subsection, for multistranded coil with an optimal transposition, the height and number of the strands can be determined as if the strands were connected in series, thus allowing the designer to determine the ac Joule losses on the basis of (9).


Fig. 5. Normalized resistance of the slot conductors connected in series against the normalized conductor height. Isofrequency curves (dashed blue).

[^1]The ratio $\frac{\Re e(\mathrm{z})}{\mathrm{R}}$ refers to only the active parts of a coil embedded in slots of the same width $\left(l_{e}=0\right.$ and $\left.\mathrm{wn}_{\mathrm{w}}=\mathrm{s}\right)$ and is plotted in Fig. 5 as function of the conductor height normalized to the skin depth, for several values of $n$. Isofrequency curves are represented with dashed blue lines, thus helping to visualize the effect of different stranding solutions. Note that the maximum of the blue curves occurs at $\mathrm{d} / \delta \approx 2$. Therefore increasing the number of strands is always beneficial if the normalised strand height is less than 2.

## C. Optimal transposition

An optimal strand transposition should cancel the circulating currents between the strands, so as each strand carries the same net current, the losses reduce to those obtained with the strands connected in series, keeping constant the coil ampere-turns.
If the strands of a n-turn, m -strand coil are connected in series, the leakage impedance per half-turn, $\dot{\mathrm{Z}}_{s}$, can be obtained by applying the following substitutions in (9):

$$
\mathrm{n} \rightarrow \mathrm{~nm} ; \mathrm{m} \rightarrow 1 ; \mathrm{d} \rightarrow \mathrm{~d} / \mathrm{m}
$$

The derivative of the real part of $(1), \Re e(\dot{\mathrm{Z}})$, with respect to $n_{\text {tr }}$ is equal to zero ${ }^{3}$ for $n_{\text {tr }}=n / \sqrt{2}$. By substituting this value in (1), it is possible to demonstrate that the following equation holds:

$$
\begin{equation*}
\mathfrak{R e}(\dot{\mathrm{Z}})=\frac{1}{m} \Re e\left(\dot{\mathrm{Z}}_{s}\right) . \tag{10}
\end{equation*}
$$

In other terms, a single twist placed along the conductor at about $70 \%$ of the coil height (from bottom to top of the slot), is able to cancel the bundle-level parasitic currents. The cancellation of the bundle-level losses corresponds to a minimization of losses due to parasitic circulating currents. With the proposed twisting procedure this result can be obtained with a single transposition outside the slot, achieving a minimization of the copper quantity (each transposition increases the length of the coil). Then, also dc Joule losses are indirectly minimized. The above stated condition applies accurately in the case of coils made with flat solid strands in which the position of strands at each turn is exactly known. In the case of a bundle of parallel round strands, it is more complicated to precisely exchange the position of two or more conductors. Even so, the experiments shown later demonstrate that the optimal transposition condition gives reasonably good results also in the case of a bundle with numerous thin circular wires.

## V. EXPERIMENTAL VALIDATION

## A. Test bench

Experiments have been carried out on two slightly different setups. The first one, constituted by an E-shaped ferrite core and a single tooth-wound coil embedded in open slots in Fig. 6a (hereinafter E-core), was used to validate the analytical model. This set-up guarantees a good

[^2]approximation that the flux lines are perpendicular to the lateral edges of the slot. The second one, constituted by a Ushaped core and two identical tooth-wound coils connected in series and embedded in a semi-closed slot, Fig. 6b, (hereinafter U-core) was used to validate the transposition criterion in a more general condition. In both cases the slot depth $\left(l_{\mathrm{s}}\right)$ is 75 mm , while the slot width (s) is 11.7 mm for the E-core and 20 mm for the U-core. The ferromagnetic core is made of ferrite with relative permeability almost stable at 2000 with a flux density value below 400 mT , a value never approached during the experiments. The coils were placed to align the coil top with the top of the slot.

The test bench circuit is represented in Fig. 7. A programmable power supply (Pacific 360AMX) supplies the coils under test with a sinusoidal voltage of adjustable frequency and amplitude, within the ranges [50-5000 Hz] and $[0-300.0 \mathrm{~V}]$, respectively. The readouts taken on the power analyzer (Yokogawa WT130) are the coil voltage and current, the active power, and the electrical frequency. The choice of the ferrite core guarantees very low core losses in the whole considered frequency range, then the measured active power has been considered as the measure of the conductor losses. The coil temperature has been monitored.

Several coils were manufactured using flat rectangular wires having cross section equal to $6 \times 1.12 \mathrm{~mm}^{2}$, round wires having 0.5 mm diameter and round wires having 0.2 mm diameter (Litz wire). With the exception of Litz wire that was an off-the-shelf product, the coils were realized for this purpose. Details about the coils are reported in Table I. The considered coils have in common the overall external dimension. They all almost completely fill the slot and have height approximately equal to $46.5 \pm 1 \mathrm{~mm}$. They differ for the number of turns and strands and for the copper filling factor. The loss results will be compared by maintaining constant the ampere-turns for all the considered coils. The ampere-turns value was chosen equal to 242 to limit coil voltage and currents to well below the limits of the power supply and also reduce coil warming during tests.

(a)

(b)

Fig. 6. Coil under test and ferromagnetic cores: (a) E-core and (b) U-core.


Fig. 7. Representation of the test bench.


Fig. 8. Coil under test with exchange of the reciprocal position of strands 1 and 3 (a). Coil made of a circular wires bundle, with a $180^{\circ}$ twist, indicated by the white ellipse (b).

Fig. 8a shows how the parallel strands were transposed by exchanging their relative position. Fig. 8 b shows how the strand transposition of the bundle of circular wires is performed by twisting the bundle by $180^{\circ}$. In this case the bundle is randomly wound and it tends to assume a rectangular flatwise-wound shape. The position of each conductor is not precisely controlled, but after twisting they will tend to exchange their position with respect to the center of the bundle.

## B. FEA

The first experimental setup described above was modelled by a couple of slots placed at distance $l_{e}$ in a ferromagnetic body of constant relative permeability equal to 2000 . The strands were modelled as rectangles of size equal to the actual one, placed in the slot at uniform distance (a) so that the coil height was equal to the actual one. The conductivity of copper was set to $5.5 \cdot 10^{7} \mathrm{~S} / \mathrm{m}$. The slot width was set equal to the value reported in Table I. A time-harmonic problem was set and the electrical variables of each strand
were constrained by a circuit topology of the type represented in Fig. 9, where the reference for each voltage is taken on each strand as represented in the legend, on turn referred to the strands depicted in Fig. 2. The current I was imposed to the FEA model by means of a generator placed between the external circuit and the joint of the strands. The resistances $R_{s 1}, \ldots, R_{s m}$ account for all the end connections of each strand.

$$
\begin{equation*}
\mathrm{R}_{\mathrm{s} 1}=\mathrm{R}_{\mathrm{sm}}=2 \mathrm{n} \frac{\mathrm{ml}_{\mathrm{e}}}{\sigma \mathrm{wd}} \tag{11}
\end{equation*}
$$

The tooth-coil ac resistance $R_{a c}$ is obtained by integration of the Joule power over the copper regions according to the following expression:

$$
\begin{equation*}
\mathrm{R}_{\mathrm{ac}} \mathrm{I}^{2}=\mathrm{l}_{\mathrm{s}} \frac{\oiint_{\mathrm{Cu}} \mathrm{~J}_{\mathrm{z}}^{2} \mathrm{~d} \Omega}{\sigma}+2 \mathrm{n} \frac{\mathrm{ml}_{\mathrm{e}}}{\sigma \mathrm{wd}} \sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\oiint_{\mathrm{Cu}} \mathrm{~J}_{\mathrm{z}} \mathrm{~d} \Omega\right)^{2} \tag{12}
\end{equation*}
$$

where $\mathrm{J}_{\mathrm{z}}$ is the z -component of the current density phasor.

## VI. FEA AND EXPERIMENTAL RESULTS

With the aim of validating the proposed test bench, the losses obtained using coils "T36S1_36 flatwise wound' and "T36S1_36 edgewise wound", as named in Table I, are firstly compared using the E-core. The code "TnSm_ $n_{t r}$ " at the beginning of the coil name indicates the number of turns ( n ), strands ( m ), and turns preceding the transposition point $\left(n_{t r}\right)$. Losses given by equation (1) agree quite well with the experimental results reported in Fig. 10. The losses of the flatwise-wound coil clearly evidence the presence of a corner frequency where the slope of the losses decreases. The ratio between the losses of flatwise- and edgewisewound coil halves ranges from 1000 to 3000 Hz . The results shown next have been limited below 1500 Hz because this is a frequency range of practical interest for high-speed electrical machines.

TABLE I COILS USED FOR THE TESTS

| Coil name | Elementary wire | Strands (m) | Bundle position in the slot | Turns <br> (n) | Twisting/ Transposition ( $\mathrm{n}_{\mathrm{tr}}$ ) | Net bundle copper area |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T7S5_7 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 5 | Edgewise | 7 | None | $33.60 \mathrm{~mm}^{2}$ |
| T7S5-5 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 5 | Edgewise | 7 | Transposed between turns 5 and 6 | $33.60 \mathrm{~mm}^{2}$ |
| T7S5_2 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 5 | Edgewise | 7 | Transposed between turns 2 and 3 | $33.60 \mathrm{~mm}^{2}$ |
| T12S3_12 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 3 | Edgewise | 12 | None | $20.16 \mathrm{~mm}^{2}$ |
| T12S3_4 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 3 | Edgewise | 12 | Transposed between turns 4 and 5 | $20.16 \mathrm{~mm}^{2}$ |
| T12S3_8 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 3 | Edgewise | 12 | Transposed between turns 8 and 9 | $20.16 \mathrm{~mm}^{2}$ |
| T36S1_36 flatwise wound | $6 \times 1.12 \mathrm{~mm}^{2}$ | 1 | Flatwise | 36 | None | $6.72 \mathrm{~mm}^{2}$ |
| T36S1_36 edgewise wound | $6 \times 1.12 \mathrm{~mm}^{2}$ | 1 | Edgewise | 36 | None | $6.72 \mathrm{~mm}^{2}$ |
| T18S2_18 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 2 | Edgewise | 18 | None | $13.44 \mathrm{~mm}^{2}$ |
| T18S2_6 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 2 | Edgewise | 18 | Transposed between turns 6 and 7 | $13.44 \mathrm{~mm}^{2}$ |
| T18S2_12 | $6 \times 1.12 \mathrm{~mm}^{2}$ | 2 | Edgewise | 18 | Transposed between turns 12 and 13 | $13.44 \mathrm{~mm}^{2}$ |
| Non-twisted bundle | Diameter 0.5 mm | 31 | Flatwise | 24 | None | $6.09 \mathrm{~mm}^{2}$ |
| Bundle twisted @ each turn | Diameter 0.5 mm | 31 | Flatwise | 24 | Twisted at each turn | $6.09 \mathrm{~mm}^{2}$ |
| Bundle twisted @ 2/3 | Diameter 0.5 mm | 31 | Flatwise | 24 | Twisted at $2 / 3$ of the coil height | $6.09 \mathrm{~mm}^{2}$ |
| Litz wire | Diameter 0.2 mm | 350 | Flatwise | 12 | Continuously twisted and formed into a rectangular profile | $11 \mathrm{~mm}^{2}$ |



Fig. 9. Equivalent electric circuit of parallel strands used for FEA.
Fig. 11 shows the trend of losses for the coil "T12S3_ntr", determined by (1) and also by FEA, against the position of the conductor twist through the coil. There are also plotted the losses of the same coil having strands connected in series, for the same ampere-turns. As expected from the analysis reported in the previous sections, the bundle-level parasitic currents cancel out when the conductor twist is placed among the eighth and the ninth turn $\left(\mathrm{n}_{\mathrm{tr}}=8\right)$, since eight is the nearer value to $12 / \sqrt{2}$, resulting in a minimization of the losses. In particular, they reduce to $29 \%$ of their value for the non-transposed coil. A good accuracy of the analytical model can also be observed. FEA results reported in Fig. 12 for several slot openings show that it has a slight influence on the position of the minimum.

The benefit of the optimal transposition has been confirmed by analytical and experimental results obtained using the coils named as "T12S3_ntr" and "T36S1_36 edgewise wound', and shown in Figs. 13a and 13b. In fact, the curves of the coils with strands in series and with strands transposed at the end of the eighth turn are almost overlapped. The reduction of losses due to transposition agrees quite well between FEA and experiments, as confirmed by Fig. 13c. At 1 kHz , from the non-transposed to the optimal transposition condition, the losses reduce at $29 \%$ and $31 \%$ according to FEA and experiments, respectively. This result is also highlighted in Fig. 11. Similar analyses were also performed considering coils with five strands and seven turns and the obtained analytical, FEA, and experimental results are reported in Fig. 14. With the number of parallel strands also grows the loss reduction from nontransposed to the optimal transposition condition. In this case at 1 kHz the losses reduce at about $11 \%$ and $19 \%$ according to FEA and experiments, respectively. The increased difference between experimental and FEA results is justified by the poor control on the shape of the hand-made coils that degrades with the number of parallel strands. Even if the results are good enough to demonstrate the analytical approach, the loss reduction would benefit from a machinecontrolled coil-manufacturing process.

Fig. 15 shows the losses obtained using flat wires and the U-core with series- and parallel-connected wires. As said, this experiment utilizes a more realistic semi-closed slot shape. Also in this case, when the transposition of parallel strands is realized at $2 / 3$ of the coil height, losses are very close to the case of series connection. It is also evident that a
single transposition far from the optimal condition (e.g. 1/3 of the coil height) gives a small reduction of bundle-level losses with respect to the not-transposed coil.

Fig. 16 shows the losses obtained with a bundle of circular wires. As mentioned before, such coils tend to assume a flatwise shape unless forced to stay edgewise with some complicated winding tools. The drawback is evidenced by the losses obtained without twisting, which tend to increase quickly with frequency.


Fig. 10. E-core. Experimental determination of the T36S1_36 flatwise(higher) and edgewise- (lower) wound coil losses as function of frequency.


Fig. 11. E-core. Losses of the coil "T12S3_n $\mathrm{n}_{\mathrm{tr}}$ " against the position of the point of conductor twist.


Fig. 12. E-core. Losses of the coil "T12S3_n ${ }^{t r}$ " against the position of the point of conductor twist, for several different slot-opening values.


Fig. 13. E-core. Losses of the coils "T12S3_n $\mathrm{n}_{\mathrm{tr}}$ " and "T36S1_36 edgewise wound" versus frequency: (a) analytical (red) and FEA (black), (b) experiments, and (c) percentage losses referred to non-transposed condition.

Losses obtained twisting the coil at each turn or only at $2 / 3$ of the coil height are comparable. There is an advantage of the "Bundle twisted @ 2/3" at lower frequencies due to the reduced length of the coil. The proposed twisting technique could be then applied to any number of parallel connected strands allowing the selection of the most appropriate number of turns for the specific application.

(a)

(b)

(c)

Fig. 14. E-core. Losses of the coils "T7S5_n $\mathrm{n}_{\mathrm{tr}}$ " and "T36S1_35 edgewise wound" versus frequency: (a) analytical (red) and FEA (black), (b) experiments, and (c) percentage losses referred to non-transposed condition.

Finally, the losses obtained using the proposed optimal transposition technique are compared with those of a state-of-the-art Litz wire in Fig. 17. It is worth stressing that the bundles of parallel conductors were made using a manual winding machine and could benefit from an automatic production procedure in terms of compactness and accuracy of bundle positioning inside the slot.


Fig. 15. U-core. Experimental determination of the copper losses versus frequency of the coils T18S2_n ${ }_{t r}$ and for the coils T36S1_36 with the strands in series.


Fig. 16. U-core. Losses obtained using flat rectangular conductors (T36S1_36) and a bundle of round conductors.


Fig. 17. U-core. Comparison of the losses obtained using the proposed optimal twisting strategy with those obtained with state-of-the-art commercial Litz wire.

Nevertheless, owing to the proposed technique, the considered Litz wire is a competitive solution only above 1
kHz . Note that a Litz wire with thinner and more twisted wires would produce a poorer filling factor and increased low-frequency losses, whereas a Litz wire with ticker and less twisted wires would be more sensitive to frequency changes, thus approaching the performances of the proposed bundle twisted at $2 / 3$.

## VII. Conclusions

This paper presented an original methodology for reducing losses due to parasitic circulating currents in electrical machines with tooth-wound coils. The method is based on a single transposition of the parallel connected strands realized in the coil passive side. Among the benefits of the proposed method, there are the reduction of dc copper losses, the increased filling factor, and the simplicity of the coil-manufacturing process.

An analytical expression describing the circulating current losses for a coil with a single transposition has been introduced and verified using extensive FEA and experimental results. Although the method was introduced for flat rectangular wires, it has been demonstrated with experiments that it is possible to extend the results also to bundles of circular wires.

The proposed method appears to be an effective and lowcost alternative to Litz wire in high-frequency electrical machines.

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## References

[1] Mellor P., Wrobel R., McNeill N., "Investigation of proximity losses in a high speed brushless permanent magnet motor" 41st IAS Annual Industry Applications Conference, Vol. 3, 2006, pp.1514-1518.
[2] Mellor P.,Wrobel R., Salt D., Griffo A., "Experimental and analytical determination of proximity losses in a high-speed PM machine", IEEE Energy Conversion Congress and Exposition (ECCE), 2013, pp. 3504-3511.
[3] Lyon, W. V. "Heat Losses in Stranded Armature Conductors". Journal of the American Institute of Electrical Engineers, Vol. 41, n. 1, 1922, pp. 37-49.
[4] Dowell, P. L. "Effects of eddy currents in transformer windings." Electrical Engineers, Proceedings of the Institution of, Vol. 113, n.8, 1966, pp. 1387-1394.
[5] Wu L.J., Zhu Z.Q., Staton D.A., Popescu M., Hawkins D., "Analytical model of eddy current loss in windings of permanentmagnet machines accounting for load" Magnetics, IEEE Transactions on (IEEE) 48, n. 7 (2012): 2138-2151.
[6] Reddy P.B., Jahns T.M., Bohn T.P., "Modeling and analysis of proximity losses in high-speed surface permanent magnet machines with concentrated windings", IEEE Energy Conversion Congress and Exposition (ECCE), 2010, pp. 996-1003.
[7] Fang Jiancheng, Liu Xiquan, Bangcheng Han, Kun Wang "Analysis of Circulating Current Loss for High-Speed Permanent Magnet Motor", Magnetics, IEEE Transactions on (IEEE) 51, n. 1 (2015): 113.
[8] van der Gest M., Polinder H., Ferreira J.A., Zeilstra D. "Current sharing analysis of parallel strands in low-voltage high-speed machines", Industrial Electronics, IEEE Transaction on (IEEE) 61, n. 6, 2014, pp. 3064-3070.


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[^0]:    ${ }^{1}$ Any bundle of wires, of circular or rectangular cross section, which constitutes one of the directions of the phase winding is referred to as "conductor." A solid wire, of circular or rectangular cross section, is hereafter referred to as "strand." A conductor can be either a bundle of strands or a single, solid wire.

[^1]:    ${ }^{2}$ Letting $9=0$ and $n$ be substituted by $n / 2$ in the final expression of $Z$ for the case 4 in [3], one obtains the same expression given by (1) for $n_{t r}=n$.

[^2]:    ${ }^{3}$ Observe that the term in $S$ of (1) is imaginary.

