Nonlinear Dynamics of Topological Ferromagnetic Textures for Frequency Multiplication

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We propose that the nonlinear radio-frequency dynamics and nanoscale size of topological magnetic structures associated with their well-defined internal modes advocate their use as *in materio* scalable frequency multipliers for spintronic systems. Frequency multipliers allow for frequency conversion between input and output frequencies, and thereby significantly increase the range of controllably accessible frequencies. In particular, we explore the excitation of eigenmodes of topological magnetic textures by fractions of the corresponding eigenfrequencies. We show via micromagnetic simulations that low-frequency perturbations to the system can efficiently excite bound modes with a higher amplitude. For example, we excite the eigenmodes of isolated ferromagnetic skyrmions by applying half, a third, and a quarter of the corresponding eigenfrequency. We predict that frequency multiplication via magnetic structures is a general phenomenon that is independent of the particular properties of the magnetic texture and works for magnetic vortices, droplets, and other topological textures.

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I. INTRODUCTION

Frequency multiplication is an important phenomenon of nonlinear oscillators, from which one obtains high-frequency outputs given low-frequency inputs. It has broad application in communication circuits [1–6], optical experiments [7–11], and spintronic devices [12–17]. In spintronic experiments, different methods going beyond pure spintronics techniques are often employed to controllably create and manipulate magnons at different frequencies. We predict that the nonlinear dynamics of topological magnetic structures provides an *in materio* scalable pure spintronics-based alternative for frequency multipliers, see Fig. 1.

The excitation modes of magnetic vortices [18–20], skyrmions [21–24], and droplets [25–28] have relevant applications in magnetic tunnel junctions [20,29], racetrack memories [30,31], microwave generators [20,32], and nonconventional computing [33–36]. Excitation modes of ferromagnetic topological textures are bound magnon modes [24,26,37–41]. They are typically classified by an integer number, n, associated with a quantized angular momentum: (i) breathing modes ($n = 0$) [25,42–44], (ii) gyration modes ($|n| = 1$) [19,45,46], and (iii) higher-order modes ($|n| > 1$) [37,39,46,47]. The sign of n determines the rotation direction: clockwise or counterclockwise. So far, most applications of skyrmions rely on linearly approximating the dynamics. These excitation modes, however, are fundamentally nonlinear. This implies, for example, an amplitude dependence of the excitation-mode frequencies, as well as the existence of harmonic generation, i.e., the excitation of integer multiples of applied frequencies.

In general, the nonlinearity of magnetization dynamics is associated with many-magnon scattering [48–54]. An example of the use of nonlinear phenomena to create multiple magnons is parallel pumping [55–61], which is demonstrated both theoretically and experimentally. Parallel pumping is used to generate magnon Bose-Einstein condensates, for example [54,60,62,63]. In this nonlinear process, associated with parametric excitation, one can excite a certain magnon mode by applying an ac field with twice the corresponding eigenfrequency [64–67]. The high frequency of the required driving field, as well as resonant magnon excitation at the driving field's frequency, are caveats of the experimental implementation of parallel pumping. Moreover, the resonance at the frequency of the driving field is very sensitive to defects [55,68]. Interactions of magnons with topological magnetic textures provide alternative

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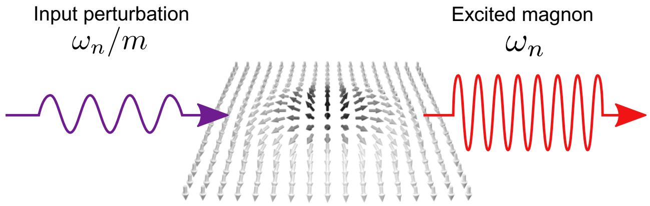

FIG. 1. Sketch of frequency multiplication using a magnetic skyrmion. Perturbations with a fraction of eigenfrequency ω_n can excite the corresponding eigenmode of the skyrmion. Moreover, when input perturbation is created by an ac magnetic field, the amplitude of the excited eigenmode can be larger than the amplitude of the input magnon.

possibilities for controlled and robust many-magnon scattering manipulation [24,38,46,69–74].

Here, we propose the excitation of topological magnetic textures via frequency multiplication. Applying a perturbation with a fraction of an eigenfrequency of a bound mode leads to a resonance at the corresponding eigenfrequency, see Fig. 1. In particular, we achieve excitation of the breathing $n = 0$ mode of isolated skyrmions by applying ac magnetic fields, both in-plane and out-of-plane, with half and a third of the corresponding eigenfrequencies. We also analyze the amplitude dependence of the excited eigenmode in terms of the amplitude of the applied ac field and the damping parameter. The resonance of the bound eigenmodes with perturbations at fractions of the eigenfrequencies presents several advantages for applications: (i) above a certain threshold amplitude for the applied field, excitation with a fraction of the eigenfrequency is more efficient than that with the same frequency, i.e., the amplitude of the eigenmode is bigger when applying a fraction of the eigenfrequency, rather than applying the eigenfrequency itself; (ii) perturbations with fractional frequency are not eigenstates of the system, and thus, decay quickly away from the topological texture that does not contribute significantly to instabilities; and (iii) the eigenfrequencies can be tuned by changing the material's parameters, for example, with temperature changes or by applying constant magnetic fields. This means that, for given a frequency, it is possible to design a topological texture that allows for resonant-frequency multiplication.

This manuscript is organized as follows. In Sec. II, we discuss the general model, which we use to predict the possibility of exciting eigenmodes of magnetic topological objects by fractions of the eigenfrequency. We describe the general case of frequency multiplication and the underlying physical behavior and assumptions. The theory is valid for topological objects, in general, and is expected to occur independently of the particular local topological magnetic configuration. In Sec. III, we present frequency multiplication for the excitation of skyrmions, as an example. We show that it is possible to excite the skyrmion modes, including higher-order modes that have not yet been experimentally studied. We show that the method of excitation by a fraction of the eigenmode can be more efficient than the usual linear excitation. Moreover, we study, via micromagnetic simulations, the efficiency of the frequency-multiplication dependence on the amplitude of the driving force and damping. We conclude in Sec. IV by emphasizing the advantages of frequency multiplication and discussing its difference from parametric excitation. Furthermore, we finalize by proposing applications for the method described here.

II. GENERAL MODEL

Frequency multiplication is a well-known phenomenon in nonlinear optics, where it is also named higher-harmonic generation [75–77]. It relies on the nonlinear properties of systems and can be explained by a general picture. Considering a field ϕ of a nonlinear system, the dynamics of small perturbations, $\tilde{\phi}(t) \ll 1$, of the static background, ϕ_0, can be expressed as the expansion

$$\frac{d\tilde{\phi}}{dt} \approx \mathcal{L}_1(\phi_0)\tilde{\phi} + \mathcal{L}_2(\phi_0)\tilde{\phi}^2 + \cdots + \mathcal{L}_p(\phi_0)\tilde{\phi}^p + \cdots . \tag{1}$$

The first term on the right, $\mathcal{L}_1\tilde{\phi}$, corresponds to a linear approximation for the nonlinear system and provides the eigenstates of the system with frequencies ω_n. Terms $\mathcal{L}_k$ with $k > 1$ correspond to interactions between perturbations. They renormalize the value of the eigenfrequencies ω_n and lead to amplitude-dependent frequencies. If we consider a perturbation with a fraction of an eigenfrequency, such as $\tilde{\phi}(t) \approx \phi_{\omega_n/2}\cos(\omega_n t/2)$, the quadratic term generates a contribution $\tilde{\phi}(t)^2 \approx \tilde{\phi}^2_{\omega_n/2}\cos(\omega_n t)$, which corresponds to a solution to the linear term. This principle can be extended to $m > 2$ and reveals why fractions of the eigenfrequency may lead to excitation of the corresponding eigenmode. In general, the amplitude of eigenmode ω_n grows with the mth power of the driving amplitude with frequency ω_n/m. Moreover, Eq. (1) is independent of the amplitude of perturbation φ.

In particular, the dynamics of unitary magnetization, $\mathbf{m} = \mathbf{M}/M_s$, where M_s is the saturation magnetization, is well described by the Landau-Lifshitz-Gilbert (LLG) equation [78]:

$$\frac{d\mathbf{m}}{dt} = -\frac{\gamma}{M_s}\,\mathbf{m} \times \mathbf{B}_{\text{eff}} + \alpha\,\mathbf{m} \times \dot{\mathbf{m}}. \tag{2}$$

Here, γ is the gyromagnetic ratio, and $\mathbf{B}_{\text{eff}} = -\delta\mathbf{E}[\mathbf{m}]/\delta\mathbf{m}$ is the effective magnetic field in a system with total magnetic energy density $E[\mathbf{m}]$. The nonlinearity of Eq. (2) allows for the existence of solitons corresponding to localized stable noncollinear magnetic configurations to be

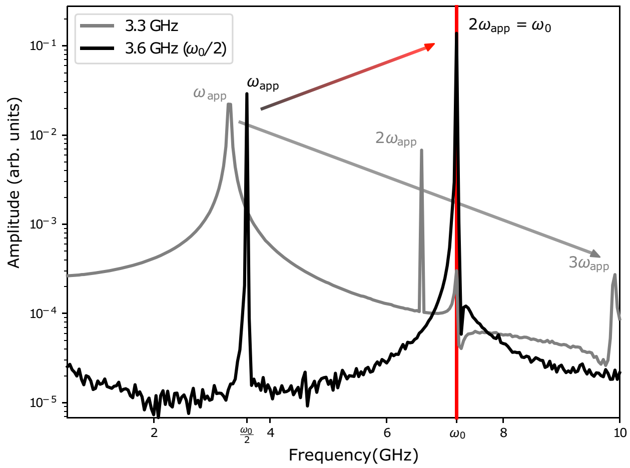

FIG. 2. Illustrative example of general principles of excitation by frequency multiplication. We show excitation of a skyrmion by magnetic fields with amplitude $B = 5$ mT at two different driving frequencies, ω_{app}. Due to localized nonlinear potential of the skyrmion configuration, both driven excitations generate harmonics corresponding to integer multiples of frequency. For the case in which the harmonic coincides with an eigenstate, there is resonance and the corresponding multiple has a higher amplitude.

characterized by topological properties. Moreover, low-frequency perturbations to these topological configurations are bound to the vicinity of the noncollinear structure and behave as localized traveling waves [24,26,37–39,41]. The nonlinear potential associated with these topological objects gives rise to frequency multiplication of driven perturbations. If we consider perturbations $\delta\mathbf{m}$ of the topological configuration, given by $\mathbf{m_0}$, we can expand Eq. (2) to Eq. (1), including higher orders of $\delta\mathbf{m}$.

The exact expansion depends on the configuration $\mathbf{m_0}$ and its symmetries. From an intuitive picture, however, one can derive some general principles, taking into account traveling perturbations in the noncollinear region of the topological object, see Fig. 2. First, we consider that any low-frequency perturbation $\delta\mathbf{m}$ around the noncollinear region of configuration $\mathbf{m_0}$ behaves as an underdamped traveling wave given by

$$\delta\mathbf{m} = \sum_n \phi_1(r,\psi,t)(\mathbf{m}_0 \times \hat{\mathbf{n}}) + \phi_2(r,\psi,t)[(\mathbf{m}_0 \times \hat{\mathbf{n}}) \times \mathbf{m}_0)], \tag{3}$$

where $\phi_1, \phi_2 \ll 1$ are dynamically conjugated and can be described in terms of collective coordinates [79–81]. Here, r, ψ are polar coordinates, and $\hat{\mathbf{n}}$ corresponds to the direction of the field-polarized background. If the polar and time dependence of $\phi_{1\,n}, \phi_{2\,n}$ are given by the combination of $\omega_n(t - t_0) - n\psi$, with n as an integer, the perturbation corresponds to the eigenmode of topological excitation with eigenfrequency ω_n. Perturbations propagate in the localized nonlinear potential of the topological structure as an underdamped traveling wave. The frequency of the amplitude depends on the amplitude of the perturbation and spatial landscape of the potential. Since damping has a viscous nature, faster perturbations experience greater damping.

Traveling waves propagating in nonlinear potentials naturally generate harmonics corresponding to excitations with integer multiples of the original frequency [75–77]. When a harmonic of the driven perturbation coincides with the natural eigenfrequencies of the system, there is resonance and the corresponding eigenmode is excited, see Fig. 2. Since excitations with lower frequency are less damped, one expects that, at a certain regime of the amplitude of the driving field and range of damping values, excitation by frequency multiplication will be more efficient than the linear resonance of the eigenmode.

The basic assumption for frequency multiplication is the existence of a localized nonlinear potential associated with the noncollinear topological structure. Frequency multiplication happens for any driven perturbation. For this reason, there is no threshold for the driving field. Increasing the amplitude of the driving perturbation, however, leads to a higher efficiency of the nonlinear process. In addition, selection rules may be derived from the spatial distribution of the driven perturbation. We consider a Fourier expansion of the perturbation in terms of the eigenmodes of the topological object, such as

$$\phi_{i\,n}(r,\psi,t) = \sum_n \phi_{i\,n}(r)\phi_n(r, n\psi - \omega_n t), \tag{4}$$

where ϕ_n are the eigenmodes. We claim that a perturbation can excite, via frequency multiplication, any mode with nonvanishing $\phi_{i\,n}(r)$, see Fig. 3, for example. Notably, increasing the driving force may lead to an increase of amplitudes $\phi_{i\,n}(r)$, which almost vanish for smaller driving forces.

A quantitative analysis depends on the specific magnetization dynamics and the underlying topological object. In the next section, we consider the skyrmion as an example to demonstrate frequency multiplication in magnetic topological objects.

III. EXAMPLE: MAGNETIC SKYRMIONS

While the results above are quite general, here, we focus on the excitation modes of isolated skyrmions stabilized by perpendicular anisotropy [47,82] without loss of generality. The excitation modes of skyrmions are studied by analytical and numerical methods, which consider both linearized equations of motion and full micromagnetic simulations [38,39,46,47,70,83]. The experimental excitation and detection of these internal modes, in particular, modes with $|n| > 1$, are still a challenge [24,38,44,84,85].

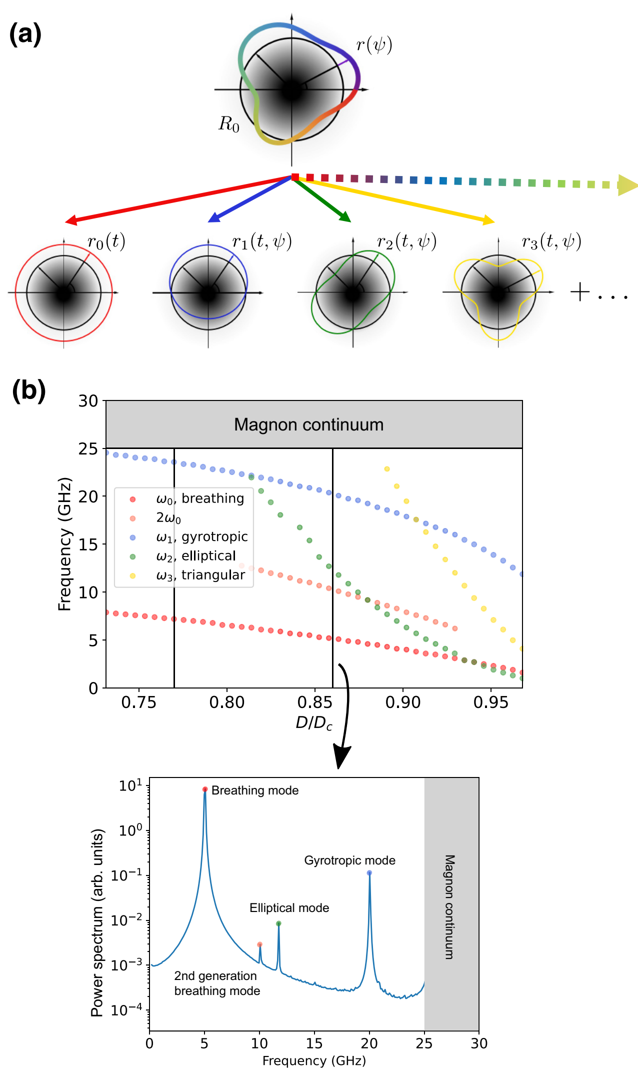

FIG. 3. Excitation modes of skyrmions. (a) Sketch of the decomposition of smooth deformation into internal skyrmion modes. Magnetization inside and outside the boundary has opposite directions, going from black to white. R_0 is the ground-state radius of the skyrmion, and r_n characterizes bound modes. Mode $n = 0$ represents the breathing mode. Modes with $|n| > 0$ rotate (counter-)clockwise with an amplitude-dependent frequency. (b) Frequency spectrum of Néel skyrmion as a function of rescaled DM interaction strength, D/D_c (top panel), and detailed analysis for a fixed DM interaction strength $D/D_c = 0.86$ (lower panel). Power spectrum in the lower panel is obtained by performing a Fourier transform of magnetization dynamics [88].

This is because some modes only couple at the linear level with fields that obey certain spatial distributions and symmetries [38,86]. We demonstrate that bound modes can be excited by homogeneous alternating fields with fractions of the corresponding eigenfrequencies. Since fractional excitation is more efficient for certain field amplitudes, this provides a path for the experimental excitation and detection of all skyrmion modes.

To obtain explicit results for an anisotropy-stabilized isolated skyrmion, we consider the following model:

$$E[\mathbf{m}] = \int dV[A(\nabla\mathbf{m})^2 - D\mathbf{m}\cdot[(\hat{\mathbf{z}}\times\nabla)\times\mathbf{m}] - K\,(m_z^2 - 1)], \tag{5}$$

where A is the magnetic stiffness; D characterizes the strength of the interfacial Dzyaloshinskii-Moriya (DM) interaction; and K is the strength of the effective uniaxial anisotropy, which incorporates a correction due to a local approximation of the magnetostatic interactions [87]. We consider $D < D_c = 4\sqrt{AK}/\pi$, such that the ferromagnetic state is the ground state of the system [43,47,82]. The energy contribution due to an external applied magnetic field, B_{ext}, is given by the Zeeman term,

$$E_{B_{\text{ext}}}[\mathbf{m}] = \mu_0 B_{\text{ext}}(t)\int dV\hat{\mathbf{n}}\cdot\mathbf{m}(\mathbf{x},t), \tag{6}$$

where μ_0 is the vacuum permeability.

For a skyrmion modeled by Eq. (5), the spin-wave eigenstates are given by the bound modes at the skyrmion with frequencies ω_n, as well as a continuous distribution of magnon modes above the anisotropy gap with frequencies $\omega = (2\gamma/M_s)(K + Ak^2)$, where k is the wave number [47]. In Fig. 3(b), we show the magnon spectrum obtained from micromagnetic simulations. Beyond the eigenmodes obtained from linearization calculations [47], we are able to identify the second harmonic of the breathing mode and a gyrotropic mode. It is important to note that, for anisotropy-stabilized skyrmions (at zero magnetic field), most modes exist only below the magnon gap for D close to D_c [39,47]. The micromagnetic simulations of this manuscript are performed with MuMax3 [89] using the following parameters: $M_s = 1.1\times10^6$ A/m, $A = 1.6\times10^{-11}$ J/m, $K = 5.1\times10^5$ J/m^3.

To reveal the resonance with a fraction of the eigenfrequency, we compute the amplitude of the eigenmodes with frequency ω_n as a function of applied frequency ω for an in-plane magnetic field excitation. Perturbations of an isolated circular skyrmion driven by out-of-plane magnetic fields are radially symmetric and couple only to the breathing mode. In the case of an out-of-plane field applied to a skyrmion with no radial symmetry or an in-plane driving field, perturbations are generated that can excite any eigenmode. In Fig. 4, we show the results of exciting (a) the breathing $n = 0$ mode and (b) the elliptical $n = 2$ modes at different applied frequencies but with the same amplitude of the in-plane applied field. We notice that the eigenmodes are excited by applying fractions of the corresponding frequency, i.e., $\omega = \omega_n/m$, where $m \in \mathbb{Z}$. The same behavior is obtained for the triangular $n = 3$ mode. The observation of resonance peaks at integer fractions of the corresponding eigenmodes is noteworthy.

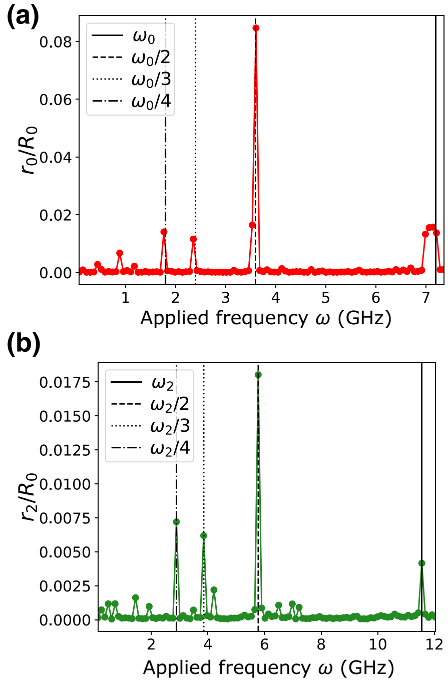

FIG. 4. Amplitude of (a) breathing (ω_0) and (b) elliptical (ω_2) modes as a function of the frequency of in-plane applied field $B_{\text{ext}} = 0.05$ T with $\alpha = 10^{-3}$. Fractions of the corresponding frequency can still strongly excite the desired mode.

We focus on excitation of the breathing mode by an out-of-plane field to analyze the amplitude of the eigenmode r_n, the dependence on the applied-field amplitude B_{ext}, and material damping α [90]. We apply out-of-plane ac magnetic fields with half and one third of the breathing-mode frequency ω_0. In Fig. 5(a), as a function of the applied ac magnetic field strengths B_{ext}, we show the amplitudes, which are extracted from the frequency spectrum (see right panel), of the breathing mode r_0 and the forced perturbation $\tilde{m}$ at the ac field frequency as a $\log_{10}$-$\log_{10}$ plot. The top (bottom) panel corresponds to $\omega_0/2$ ($\omega_0/3$) for a fixed damping constant of $\alpha = 10^{-3}$. While $\tilde{m}$ grows linearly with B_{ext}, the amplitude of the breathing mode grows with a power of two for $\omega_0/2$ and with a power of three for $\omega_0/3$, over a certain range of applied field, as expected for second- and third-harmonic generation. As the amplitude of the breathing mode grows beyond the linear approximation, the power-law coefficient reduces due to further scattering of the magnons. Furthermore, as another key observation, we find that, above a certain amplitude of the applied field, the breathing mode is more excited than $\tilde{m}$.

In Fig. 5(b), we show r_0 and $\tilde{m}$ as a function of the damping parameter α for $B_{\text{ext}} = 0.003$ T. Analogous to Fig. 5(a), the left (right) panel corresponds to an applied ac magnetic field with frequency $\omega_0/2$ ($\omega_0/3$). We see that the forced perturbation amplitude $\tilde{m}$ is independent of damping, as expected. The breathing mode r_0, however, decreases as

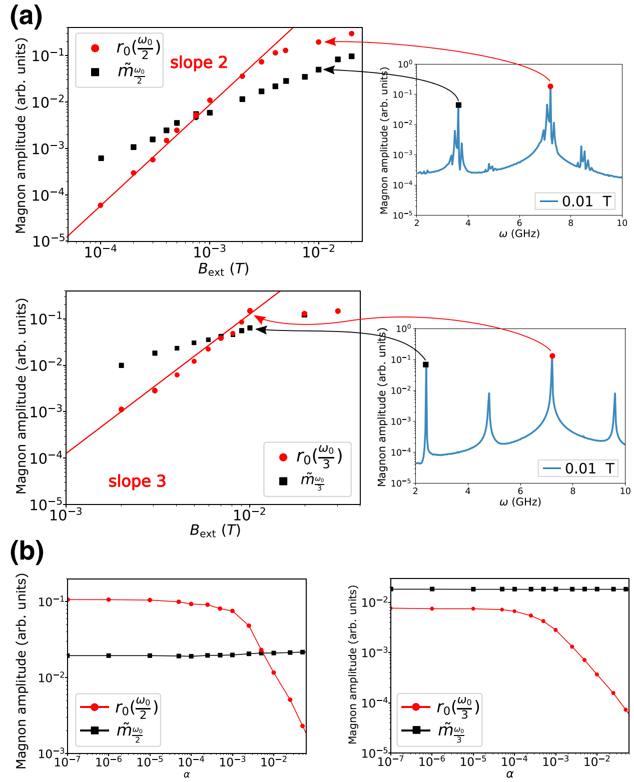

FIG. 5. Plots of amplitudes of excited modes in terms of (a) amplitude of the applied field with $\alpha = 10^{-3}$ and (b) damping parameter for an out-of-plane ac magnetic field $B_{\text{ext}} = 0.003$. In (a), straight lines with slope = 2 (3) indicate growth with a power of 2 (3) for the second- (third-) harmonic generation. We notice that, above a certain amplitude of applied field, the breathing mode is more excited than the mode with the same frequency as that of the perturbation. Other two peaks in the inset of third-harmonic generation correspond to second- and fourth-harmonic generations. They do not resonate with any eigenmodes of the skyrmion.

a function of α. The Gilbert dissipation damps the energy flow from the forced perturbation to the resonant mode and reduces the amplitude from the resonant mode. Thus, frequency multiplication is more efficient for systems with small damping.

IV. DISCUSSION

From the results above, we notice that the phenomenon of frequency multiplication might resemble that of well-known parametric excitation [64–67]. The main difference, however, is that the resultant frequency is a multiple and not a fraction of the applied frequency. This crucial difference presents considerable advantages of frequency multiplication for two main reasons: (i) lower frequencies are easier to produce and guide experimentally; and (ii) there are no magnons that are resonantly excited at the pumping frequency, since the applied frequency is well below the magnon gap. The latter presents an experimental obstacle for parametric pumping, since directly excited magnons at the pumping frequency are hard to avoid in a real experimental situation. These magnons can interact with the parametrically excited ones and, thus, hinder the ability to fully control the excited modes. Moreover, the excitation and scattering of magnons with higher frequency during parametric excitation are very sensitive to sample inhomogeneities.

To summarize, we propose excitation of the eigenmodes of topological magnetic textures by frequency multiplication based on the general nonlinear properties of magnetization dynamics. While we explicitly demonstrate this effect by means of micromagnetic simulations on the excitation modes of isolated magnetic skyrmions, our theoretical analysis reveals its universal behavior, i.e., being independent of the microscopic details of the magnetic structures and the source of the perturbation. Not only does this provide an alternative method to excite the eigenmodes by applying lower-frequency amplitudes, this frequency multiplication mechanism propounds applications in spintronics devices, such as an *in materio* frequency multiplier for magnonic applications [20,71,91–93]. In particular, bound modes not excited via linear resonance and that are weakly coupled (e.g., by dipolar stray fields or spin-wave excitations) serve as a building block for so-called parametrons, a computing scheme that has seen a strong revival in recent discussions of oscillator-based computing [94]. Furthermore, the tunability of the magnetic textures by altering the material properties, temperature, and applied fields make them very versatile.

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