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ON/OFF control for reaching a steady state attractive region in batches Petri nets

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Abstract: This paper addresses an event-driven control problem of controlled generalized batches Petri nets without discrete nodes, i.e., a class of hybrid Petri net formalisms. The goal of the controller is to reach a given steady state from an initial marking qualified as blocking, by controlling the flow of transitions. To this end, we firstly characterize an attractive region of the steady state as an optimal solution of a linear programming problem (LPP). Next, based on the ON/OFF state of transitions, which depends on the enabling conditions and the firing quantities, an ON/OFF event-driven control strategy is proposed for reaching, from a blocking initial marking quantity, the nearest marking quantity in the attractive region. The admissible controlled firing flow vector is computed according to a LPP which maximizes the firing flow of ON-transitions to its maximal firing flow while it imposes a null firing flow to OFF-transitions. The proposed strategy is event-driven and the control actions are updated only when specific events occur. The basic idea of this new control strategy is to fire each transition as fast as possible at any event step until the minimal firing quantity vector is reached.

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1. INTRODUCTION

Petri net is a widely-used formalism for modeling, analysis and control of discrete event systems. To avoid the well-known state explosion problem, the fluidization approach has been developed and has given origin to hybrid Petri nets (David and Alla, 2005), i.e., a new class of formalisms composed of discrete and continuous parts. Generalized batches Petri net (GBPN) (Demongodin, 2001), enriches this class of Petri nets by introducing a new kind of nodes, called batch nodes. Based on the concept of batches, i.e., a group of entities moving through a batch place at a certain speed, GBPNs allow variable delays on continuous flows to be represented. Behavior of such hybrid models is based on a switching continuous-time and event-driven dynamics. When the firing flows associated with continuous and batch transitions and the transfer speeds of batch places are considered as control inputs, the GBPN is called controlled GBPN (cGBPN) (Demongodin and Giua, 2014). In a previous work (Liu et al., 2020a), a controller has been proposed for reaching a given state of a cGBPN by controlling only the firing flows. In particular, the given state is assumed to be a steady state that is a tuple formed by the markings of places and the instantaneous firing flows of transitions, that remain constant for an infinite time. This controller, namely the steady flow controller (SF-controller) is based on an ON/OFF event-driven control strategy, where the control actions are updated only when specific events occur. It assumes that the controlled firing flow of each transition cannot exceed its steady firing flow. To apply this controller, the cGBPN must respect some structural properties: it must be conservative, consistent and without discrete nodes. However, one drawback of this controller is that a blockness issue may occur. More precisely, from an initial marking, qualified as blocking initial marking, the variation of non steady marking quantity may remain null at a certain time and, consequently, the target steady state may not be reached. To avoid this case, inspired by the siphon theory (Liu and Barkaoui, 2016), a marking assumption based on structural conditions has been added on the marked cGBPN.

The goal of this work is to propose a new ON/OFF event-driven control approach that relaxes the previous marking assumption and guarantees the non-blockness issue. To this end, an attractive region of the steady state is firstly defined, knowing that any trajectory starting in this region is guaranteed to converge to the steady state using the SF-controller. Next, given a blocking initial marking, a nearest reachable marking quantity of the region is computed as an optimal solution of a linear programming problem (LPP) that minimizes a distance. Afterwards, a new control law is developed, minimizing the firing quantity needed to reach the computed nearest marking quantity. The admissible controlled firing flow vector is then determined by a LPP which maximizes the firing flow of ON-transitions to its maximal firing flow while it imposes a null firing flow to OFF-transitions. Finally, a new event-driven controller is presented for driving a cGBPN from a blocking initial marking to its nearest one of the steady state attractive region. The content of this paper is structured as follows. Section 2 introduces some basic concepts on cGBPNs,

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recalls the definition of a steady state and presents the SF-controller with its assumptions. Section 3 develops the contribution of this paper. Finally, some conclusions and future works are presented in Section 4.

2. CONTROLLED BATCHES PETRI NETS

In this paper, we first assume that a GBPN does not contain discrete nodes. Thus, all definitions and concepts are presented with this restriction on the net structure. Moreover, due to the space limit, only necessary notions used in Section 3 are given in the following, but the readers can refer to (Demongodin and Giua, 2014) for more details.

2.1 Concepts and definitions

Definition 1. A generalized batches Petri net (GBPN) without discrete nodes, is a 6-tuple, $N = (P, T, \mathrm{Pre}, \mathrm{Post}, \gamma, \lambda)$ where:
- $P = P^C \cup P^B$ is finite set of places partitioned into the two classes of *continuous* and *batch* places.
- $T = T^C \cup T^B$ is finite set of transitions partitioned into the two classes of *continuous* and *batch* transitions.
- $\mathrm{Pre}, \mathrm{Post} : (P \times T \to \mathbb{R}_{0+})$ are respectively the *pre-* and *post-incidence functions* that specify the weight of the arcs from places to transitions and transitions to places.
- $\gamma : P^B \to \mathbb{R}^3_+$ is the *batch place function* that associates with each batch place $p_i \in P^B$, the triple $\gamma(p_i) = (V_i, d_i^{\max}, s_i)$ which represents, respectively, the *maximal transfer speed*, the *maximal density* and the *length* of place p_i.
- $\lambda : T \to \mathbb{R}_{0+}$ assigns to each transition, $t_j \in T$, a nonnegative number $\lambda(t_j) = \Phi_j$ which denotes the *maximal firing flow* associated with the continuous or batch transition t_j. ♢

We denote $\mathbb{R}_{0+}$ (resp., $\mathbb{R}_+$) the set of nonnegative (resp., positive) real numbers. The incidence matrix is defined by $\boldsymbol{C} = \boldsymbol{Post} - \boldsymbol{Pre}$. A nonnegative vector $\boldsymbol{a}$, satisfying $\boldsymbol{a}^T \cdot \boldsymbol{C} = \boldsymbol{0}$, is called a *P-semiflow*, and it is minimal if $||\boldsymbol{a}||$ is not a superset of the support of another one. The *preset* and *postset* of transition t_j are: ${}^\bullet t_j = \{p_i \in P \mid \mathrm{Pre}(p_i, t_j) > 0\}$ and $t_j^\bullet = \{p_i \in P \mid \mathrm{Post}(p_i, t_j) > 0\}$. A non-empty set of places Θ is a siphon iff ${}^\bullet\Theta \subseteq \Theta^\bullet$, and it is minimal if it does not contain another siphon.

Definition 2. Given a batch place p_i, a batch β_i^k of place p_i at time τ, is defined by a triple $\beta_i^k(\tau) = (l_i^k(\tau), d_i^k(\tau), x_i^k(\tau))$, where $l_i^k(\tau) \in \mathbb{R}_{0+}$ is the length, $d_i^k(\tau) \in \mathbb{R}_{0+}$ is the density and, $x_i^k(\tau) \in \mathbb{R}_{0+}$ is the head position. ♢

The marking at time τ is a tuple $\boldsymbol{m}(\tau) = [m_1(\tau)\ m_2(\tau) \dots m_n(\tau)]^T$ that assigns to each continuous place a nonnegative real number and assigns to each batch place p_i, a series of batches ordered by their head positions, $m_i(\tau) = \{\beta_i^1(\tau), \dots, \beta_i^r(\tau)\}$. The initial marking is denoted by $\boldsymbol{m}_0 = \boldsymbol{m}(\tau_0)$. Let $\beta_i^k \in m_i(\tau)$ be a batch in batch place $p_i \in P^B$.
- Batch β_i^k is called an *output batch* if its head position is equal to the length associated with the batch place, i.e., $x_i^k(\tau) = s_i$. A place in a GBPN can have at most one output batch.
- The *output density*, noted by d_i^{out}, of batch place p_i is defined: if at time τ, place p_i has an output batch $\beta_i^k(\tau)$, then $d_i^{\mathrm{out}}(\tau) = d_i^k(\tau)$, else $d_i^{\mathrm{out}}(\tau) = 0$. Note that this density at time τ depends on the marking $\boldsymbol{m}(\tau)$ and can also be denoted by $d_i^{\mathrm{out}}(\boldsymbol{m})$.

Definition 3. The marking quantity vector $\boldsymbol{q} = \mu(\boldsymbol{m}) \in \mathbb{R}^n_{0+}$ associated with marking $\boldsymbol{m}$, is defined as follows:
- if $p_i \in P^C$, $q_i = m_i$,
- if $p_i \in P^B$, $q_i = \sum_{\beta_i^k \in m_i} l_i^k \cdot d_i^k$. ♢

One can remark that $\mu(\boldsymbol{m})$ is an injective mapping which associates a single marking quantity vector $\boldsymbol{q}$ with a given marking $\boldsymbol{m}$. Its inverse is not, since more than one marking $\boldsymbol{m}$ may correspond to a given marking quantity vector $\boldsymbol{q}$. When the marking quantity of a batch place is equal to its maximal one, the place is said to be a *full batch place*, i.e., $q_i = d_i^{max} \cdot s_i$. In the next, we denote by $\boldsymbol{q}^0 = \boldsymbol{q}(\tau_0)$, the initial marking quantity vector.

With a continuous or batch transition is associated an *instantaneous firing flow* (IFF), noted $\varphi_j(\tau) \leq \Phi_j$, that represents the firing quantity of transition t_j by time unit. The IFF vector at time τ is denoted by $\boldsymbol{\varphi}(\tau) \in \mathbb{R}^{|T^C|+|T^B|}$. The behavior of a GBPN is based on a timed discrete event dynamics with linear or constant continuous evolution between timed events. Between two consecutive events, the system is said to be in an *invariant behavior state* (IB-state), which corresponds to a period of time such that the IFF of continuous and batch transitions remains constant and the output density of batch places is also constant. The *input (*resp., *output) flow* of a batch or continuous place p_i at time τ is the sum of all flows entering (resp., leaving) the place and can be written, respectively, as: $\phi_i^{\mathrm{in}}(\tau) = \mathrm{Post}(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$ and $\phi_i^{\mathrm{out}}(\tau) = \mathrm{Pre}(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$. Note that, as $\boldsymbol{\varphi}$ is constant in an IB-state, $\boldsymbol{\phi}^{\mathrm{in}}$ and $\boldsymbol{\phi}^{\mathrm{out}}$ are also constants between timed events. The enabling and firing conditions of continuous and batch transitions are not details in this paper, but could be found in Section 2.3 of (Demongodin and Giua, 2014). At time τ (i.e., date of an event), the set of admissible firing flows forms a convex set described by linear equations (see Proposition 3.2 in (Demongodin and Giua, 2014)). In particular, the firing flow of continuous and batch transitions, and the input/output flow of batch places must satisfy the following constraints: the firing flow of a transition that is not enabled is null; the total flow entering in batch place p_i should not be greater than its maximal flow $V_i \cdot d_i^{\max}$; the total flow exiting batch place p_i should not be greater than the exit flow $V_i \cdot d_i^{out}$ generated by the output batch of the place. Finally, the fundamental equation of a GBPN with respect to its marking quantity vector is expressed as:

$$\boldsymbol{q}(\tau) = \boldsymbol{q}(\tau_0) + \boldsymbol{C} \cdot \boldsymbol{z}(\tau) \tag{1}$$

where $\boldsymbol{z}(\tau) \in \mathbb{R}^n_{0+}$, called characteristic vector or *f*iring quantity vector, denotes the firing count of each transition during time interval $[\tau_0, \tau]$.

Let us denote $R(N, \boldsymbol{m}_0)$ as the set of reachable markings and $RQ(N, \boldsymbol{m}_0) = \{\boldsymbol{q} \mid \exists \boldsymbol{m} \in R(N, \boldsymbol{m}_0) : \boldsymbol{q} = \mu(\boldsymbol{m})\}$ as the reachable marking quantity set. Based on the fundamental equation (1), the potential marking quantity set is defined as $PRQ(N, \boldsymbol{m}_0) = \{\boldsymbol{q} \mid \exists \boldsymbol{z} \in \mathbb{R}^n_{0+} : \boldsymbol{q} = \mu(\boldsymbol{m}_0) + \boldsymbol{C} \cdot \boldsymbol{z} \supseteq RQ(N, \boldsymbol{m}_0)\}$.

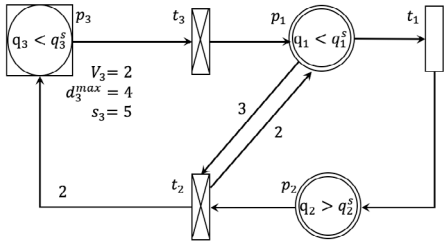

Fig. 1. Batches Petri net.

2.2 Controlled GBPN and steady state

When the instantaneous firing flow of continuous and batch transitions and the transfer speed of batch places are considered as control inputs, this leads to the definition of controlled GBPN (Demongodin and Giua, 2014). However, in this paper, we do not consider the transfer speed as a control input, and define a controlled GBPN as follows.

Definition 4. A controlled generalized batches Petri net (cGBPN) is a GBPN where the instantaneous firing flows of transitions are controlled by a *control input* $\boldsymbol{u}$ with $0 \leq \boldsymbol{u} \leq \boldsymbol{\Phi}$, and $\boldsymbol{\Phi}$ is the maximal firing flow vector. ♢

Note that all transitions are considered controllable. Following (Gaujal and Giua, 2004), the stationary behavior of a continuous Petri net as is said to be strongly stationary, when the control input and the marking of the net are in a constant steady state. This leads to the following definition of a steady state for a cGBPN.

Definition 5. (Steady state) Let $\langle N, \boldsymbol{m}_0 \rangle$ be a cGBPN. The net is in *a steady state* at time τ_s if for any time τ after τ_s, i.e., $\tau \geq \tau_s$, the marking $\boldsymbol{m}^s$ and the IFF vector $\boldsymbol{\varphi}^s$ remain constant. Thus a steady state is defined by a pair $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$. ♢

It should be pointed out that the output density of batch places is constant at the steady state. Demongodin and Giua (2014) characterize the steady state of a cGBPN by solving a linear programming problem that takes into account the net structure and the initial marking. It has also been proved that, at the steady state, the marking has a regular form.

Example 1. Let us consider the cGBPN shown in Fig. 1, where the characteristics of the batch place are given. The maximal firing flow vector is: $\boldsymbol{\Phi} = [2\ 2\ 2]^T$. The initial marking, composed by an output batch in place p_3, is given by $\boldsymbol{m}_0 = [7\ 6\ \{(0.75, 4, 5)\}]^T$, implying an initial quantity vector equals to $\boldsymbol{q}^0 = [7\ 6\ 3]^T$. At this initial state, all transitions are enabled. By solving the LPP of Proposition 4.5 in Demongodin and Giua (2014), an optimal solution $(\boldsymbol{q^s}, \boldsymbol{\varphi^s})$ maximizing the IFF vector is obtained as: $\boldsymbol{q}^s = [8\ 3\ 5]^T$ and $\boldsymbol{\varphi}^s = [1\ 1\ 2]^T$. Moreover, the reachable steady marking is given by the following regular form: $\boldsymbol{m}^s = [8\ 3\ \{(5, 1, 5)\}]^T$. □

2.3 ON/OFF control for reaching a steady state

When a cGBPN is without discrete nodes, the fundamental equation (1) could be expressed as:

$$\boldsymbol{q}(\tau) = \boldsymbol{q}(\tau_0) + \boldsymbol{C} \cdot \int_{\tau_0}^{\tau} \boldsymbol{u}(\rho) d\rho \qquad (2)$$

where $\int_{\tau_0}^{\tau} \boldsymbol{u}(\rho) d\rho$ denotes the sum of firing quantity of continuous or batch transitions during time interval $[\tau_0, \tau]$.

Inspired by (Aguayo-Lara et al., 2012) and (Gaujal and Giua, 2004), the control problem of cGBPNs for reaching a given steady state could be formally defined as follows.

Definition 6. Let $\langle N, \boldsymbol{m}_0 \rangle$ be a cGBPN with $P^D = T^D = \emptyset$ and $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ be a reachable steady state. The *control problem for reaching* $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ deals with the computation of a timed control trajectory $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots, (\boldsymbol{u}^i, \tau_i), \cdots, (\boldsymbol{u}^s, \tau_s)$ feasible from $\boldsymbol{m}_0$ such that $\boldsymbol{m}(\tau_f) = \boldsymbol{m}^s$ and $\boldsymbol{u}(\tau_f) = \boldsymbol{u}^s = \boldsymbol{\varphi}^s, \forall \tau_f \geq \tau_s$. ♢

As a consequence, for the control trajectory $(\boldsymbol{u}^0, \tau_0)$, $(\boldsymbol{u}^1, \tau_1), \cdots, (\boldsymbol{u}^i, \tau_i), \cdots, (\boldsymbol{u}^s, \tau_s)$ such that the controlled firing flow vector $\boldsymbol{u}^i$ is applied at date τ_i and remains constant until τ_{i+1}, corresponding to a timed interval of an IB-state, Equation (2) becomes:

$$\boldsymbol{q}(\tau_s) = \boldsymbol{q}^0 + \boldsymbol{C} \cdot \left(\int_{\tau_0}^{\tau_1} \boldsymbol{u}^0 \cdot d\rho + \cdots + \int_{\tau_{s-1}}^{\tau_s} \boldsymbol{u}^{s-1} \cdot d\rho \right).$$

Based on these concepts, we have proposed in (Liu et al., 2020a), a first ON/OFF control strategy of a cGBPN, called steady flow control law (SF-law), which by controlling the IFF vector according to the state of transitions, allows the system to reach a steady state from an initial one. In the SF-law, the state of transitions depend on the enabling conditions and on the quantities of their input places. At marking $\boldsymbol{m}(\tau)$, transition t_j is said to be an:

- OFF-transition: if t_j is not enabled or, if t_j is enabled and at least one of its input places p_i has a lower marking quantity than its steady marking quantity and, its incidence matrix component is negative, i.e., t_j is enabled and $\exists\ p_i \in {}^\bullet t_j : q_i(\tau) < q_i^s, C(p_i, t_j) < 0$.
- ON-transition: otherwise.

According to the transition states, the SF-law maximizes the firing flow of an ON-transition to its steady firing flow while it imposes a null firing flow to an OFF-transition, i.e., if t_j is ON: maximize $u_j(\tau)$ to φ_j^s and, if t_j is OFF: $u_j(\tau) = 0$. To drive the system from an initial marking to a given reachable steady state, the proposed steady flow controller is based on a pure event-driven dynamics that applies the SF-law at each marking. The controlled firing flow vector is maintained during an IB-state and updated when a particular event (or possibly several events) occurs. Nevertheless, to apply the SF controller, the cGBPN and the steady state must satisfy some assumptions (Liu, 2021) described below. Let $\langle N, \boldsymbol{m}_0 \rangle$ be a cGBPN, $\Pi = \{\Theta_1, \ldots, \Theta_r\}$ be the set of minimal siphons and $\boldsymbol{A} = \{\boldsymbol{a}_1, \ldots, \boldsymbol{a}_l\}$ be the set of minimal P-semiflows.

- H1. No discrete nodes ($P^D = T^D = \emptyset$).
- H2. The net is conservative and consistent.
- H3. The steady firing flow vector is positive, i.e., $\boldsymbol{\varphi}^s > \boldsymbol{0}$.
- H4. Each minimal siphon Θ_i is contained in the support of a minimal P-semiflow.

Assumption (H1) preserves the restricted class of cGBPN for which the steady state analysis has been studied in Demongodin and Giua (2014). Assumption (H2) imposes that each place is contained in the support of a P-semiflow and, each transition is contained in the support of a T-semiflow. Assumption (H3) ensures that the considered steady firing flow of each continuous or batch transition

is not null. Assumption (H4) is a structural property of the net, and allows the set of minimal P-semiflows $\boldsymbol{A}_i$ that contain siphon Θ_i to be determined, i.e., $\boldsymbol{A}_i = \{\boldsymbol{a}_k \in \boldsymbol{A} \mid \Theta_i \subseteq ||\boldsymbol{a}_k||\} \neq \emptyset$.

Another strong structural assumption must be respected to guarantee a non-blocking marking quantity vector.

H5. Each minimal siphon and the marking of the places that constitute it, verify one of the three following conditions: $\forall \Theta_i \in \Pi, \forall \boldsymbol{a}_k \in \boldsymbol{A}_i$, (i) $\Theta_i = ||\boldsymbol{a}_k||$ or (ii) $\forall p_h \in \Theta_i, q_h \geq q_h^s$ or (iii) $\forall p_j \in ||\boldsymbol{a}_k||/\Theta_i, q_j \leq q_j^s$.

Example 2. Consider again the cGBPN without discrete nodes given in Fig. 1. This conservative and consistent net has one minimal siphon, $\Theta_1 = \{p_1, p_3\}$, and one minimal P-semiflow $\boldsymbol{a}_1 = [1\ 1\ 1]$, where $\Theta_1 \subseteq ||\boldsymbol{a}_1||$. Although $\boldsymbol{\varphi}^s = [1\ 1\ 2]^T > 0$, with $\boldsymbol{q}^0 = [7\ 6\ 3]^T$ and $\boldsymbol{q}^s = [8\ 3\ 5]^T$, as we can see, the initial marking quantity does not respect assumption (H5). Thus, if we apply the SF-controller, all transitions are OFF imposing a control input equals to zero (i.e., $\boldsymbol{u}^0 = 0$) and a blocking initial marking. □

3. ON/OFF CONTROL FOR REACHING A STEADY STATE ATTRACTIVE REGION

As we have seen in the previous section, for reaching a steady state by the SF-controller, it is assumed that the initial marking quantity is non-blocking and verifies marking assumption (H5). Assume now that the initial marking does not respect these conditions. In this part, we propose a method to control the net by characterizing an attractive region of the steady state in which any trajectory starting in this region is guaranteed to converge to the steady state using the previous steady flow controller.

3.1 Attractive regions

Let us first introduce the notion of the SF-controllable attractive region of a steady state.

Definition 7. (SF-controllable attractive region). Given a cGBPN $\langle N, \boldsymbol{m}_0 \rangle$ that satisfies assumptions (H1) – (H4), and a reachable steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$. Let $\Pi = \{\Theta_1, \ldots, \Theta_r\}$ be the set of minimal siphons and $\boldsymbol{A} = \{\boldsymbol{a}_1, \ldots, \boldsymbol{a}_l\}$ be the set of minimal P-semiflows. $\boldsymbol{A}_i$ is the set of minimal P-semiflows that contain siphon Θ_i, i.e., $\boldsymbol{A}_i = \{\boldsymbol{a}_k \in \boldsymbol{A} \mid \Theta_i \subseteq ||\boldsymbol{a}_k||\} \neq \emptyset$.

A *SF-controllable attractive region of* $\boldsymbol{m}^s$, denoted by $\boldsymbol{\mathcal{B}^s}$, is the set of non-blocking marking quantities $\boldsymbol{q}$ that satisfies for each siphon Θ_i and P-semiflow $\boldsymbol{a}_k$ in $\boldsymbol{A}_i$, one of following conditions. $\forall \Theta_i \in \Pi, \forall \boldsymbol{a}_k \in \boldsymbol{A}_i$,

$$\begin{array}{ll} \text{(i) } \Theta_i = ||\boldsymbol{a}_k|| & or \\ \text{(ii) } \forall p_h \in \Theta_i, q_h \geq q_h^s & or \\ \text{(iii) } \forall p_j \in ||\boldsymbol{a}_k||/\Theta_i, q_j \leq q_j^s & \end{array} \quad (3) \quad \diamondsuit$$

According to this definition, there exist various SF-controllable attractive regions with respect to the number of minimal siphons and P-semiflows.

Example 3. Let us continue the same example. For this net structure and the steady quantity $\boldsymbol{q}^s = [8\ 3\ 5]^T$, two attractive regions are defined. The first one, denoted $\mathcal{B}_1^s$, satisfies conditions (iii), i.e., the marking of place p_2 should be lower than its steady ones (i.e., $q_2 \leq 3$). The second one, denoted $\mathcal{B}_2^s$, satisfies conditions (ii), i.e., the marking quantity of places p_1 and p_3 should be greater than their steady ones (i.e., $q_1 \geq 8$ and $q_3 \geq 5$). □

In the following, for a blocking initial quantity, we present the computation of the nearest marking quantity in the region $\mathcal{B}^s$. One remark that there exist multiple distances for determining the nearest distance between two vectors, such as Tchebitchev distance, Manhattan distance, Euclidean distance. In this work, we consider the notion of the Tchebitchev distance that defines the greatest differences between two vectors. In details, let $\boldsymbol{q}$ and $\boldsymbol{q}'$ be two marking quantity vectors. The $\mathcal{T}$*-distance* between $\boldsymbol{q}$ and $\boldsymbol{q}'$ is defined as: $d_{\mathcal{T}}(\boldsymbol{q}, \boldsymbol{q}') = \max_{i=1,\cdots,n} |q_i - q_i'|$. This distance can be used to determine the nearest marking quantity vector in the region $\mathcal{B}^s$ from a blocking initial marking quantity vector, as presented in the following definition.

Definition 8. (Computation of nearest marking quantity). Given an initial quantity $\boldsymbol{q}_0$, a nearest reachable marking quantity of region $\mathcal{B}^s$, denoted $\boldsymbol{q}^{\boldsymbol{\mathcal{B}^s}}$, is obtained by solving the following LPP, which minimizes the Tchebitchev distance between the initial quantity and a marking quantity in the region, while ensuring its non-negativeness.

$$\min d_{\mathcal{T}}(\boldsymbol{q}_0, \boldsymbol{q}) s.t. \begin{cases} \text{(a) } |q_i - q_i^0| \leq d_{\mathcal{T}}(\boldsymbol{q}_0, \boldsymbol{q}) & \forall p_i \in P \\ \text{(b) } \boldsymbol{q} \in \mathcal{B}^s & \\ \text{(c) } q_i \geq 0 & \forall p_i \in P \\ \text{(d) } d_{\mathcal{T}}(\boldsymbol{q}_0, \boldsymbol{q}) \geq 0 & \\ \text{(e) } \boldsymbol{q} \in RQ(N, \boldsymbol{m}_0) & \end{cases} \quad (4) \quad \diamondsuit$$

One can remark that constraints (e) in Definition 8 could be linearized by Equation (1) when the net is consistent and there exists no empty siphon (Recalde et al., 1999).

Example 4. The nearest marking quantity vectors in the two attractive regions from the initial quantity, $\boldsymbol{q}^0 = [7\ 6\ 3]^T$, are given as follows. For $\mathcal{B}_1^s$, a nearest reachable marking quantity is a solution of the LPP (4) given by:

$$\begin{cases} \text{(a) } & |q_1 - 7| \leq d; |q_2 - 6| \leq d; |q_3 - 3| \leq d \\ \text{(b) } & q_2 \leq 3 \\ \text{(c) } & q_1 \geq 0; q_2 \geq 0; q_3 \geq 0 \\ \text{(d) } & d \geq 0 \\ \text{(e) } & q_1 + q_2 + q_3 = 16 \end{cases}$$

With an objective function that minimizes the distance from $\boldsymbol{q}_0$, i.e., $\min d = \min d_{\mathcal{T}}(\boldsymbol{q}_0, \boldsymbol{q})$, the optimal solution is obtained with a nearest distance equal to 3, and a nearest marking quantity vector of $\boldsymbol{q}^{\boldsymbol{\mathcal{B}_1^s}} = [7\ 3\ 6]^T$. Let us remark that for the attractive region $\mathcal{B}_2^s$, the optimal solution for the same initial quantity is: d = 3 and $\boldsymbol{q}^{\boldsymbol{\mathcal{B}_2^s}} = [8\ 3\ 5]^T$ that is the steady quantity. □

3.2 Control trajectory for reaching an attractive region

In this part, we first propose a new control strategy for the computation of a control trajectory from an initial marking quantity $\boldsymbol{q}_0$ to its nearest marking quantity $\boldsymbol{q}^{\boldsymbol{\mathcal{B}^s}} \in \mathcal{B}^s$. Inspired by the approach in (Wang et al., 2014), the basic idea of this new control strategy is to fire each transition as fast as possible at any event step until the minimal firing quantity vector $\boldsymbol{z}^{\boldsymbol{q}^{\mathcal{B}^s}}$ is reached.

Always based on the ON/OFF state of transitions which depends on the enabling conditions and the firing quantities, the new strategy, called firing quantity based con-

trol law (ZF-law), maximizes the firing flow of an ON_Z-transition to its maximal firing flow while it imposes a null firing flow to an OFF_Z-transition. More specifically, at marking $\boldsymbol{m}(\tau)$, transition t_j is said to be an:

- OFF_Z-transition: $u_j(\tau) = 0$ if t_j is not enabled, or if t_j is enabled and its firing quantity at $\boldsymbol{m}$ is equal to its minimum firing quantity, i.e., $z_j(\boldsymbol{m}) = z_j^{\boldsymbol{q}^{\mathcal{B}^s}}$.
- ON_Z-transition: maximizing $u_j(\tau)$ to Φ_j, otherwise.

Moreover, to be an admissible ZF control input, the controlled firing flow vector needs to satisfy some constraints given by the definition below.

Definition 9. (Admissible ZF Control Input). A controlled firing flow vector is said to be an admissible ZF control input if it satisfies the following flow constraints that depend on the net structure, its marking and the state of transitions.

- The controlled firing flow of a continuous or batch transition cannot exceed its maximal firing flow.
- The controlled firing flow of a continuous or batch transition which is an OFF_Z-transition is null.
- The input flow of a full batch place cannot exceed its output flow.
- The input flow of an empty continuous place cannot be lower than its output flow.
- The flow arriving in a batch place cannot exceed its maximal input flow.
- The flow leaving a batch place cannot exceed its maximal output flow. ♢

For a marking, the admissible controlled firing flow vector is computed according to the following LPP which maximizes the firing flow vector to the maximal one.

Proposition 1. (ZF-law). Given a cGBPN $\langle N, \boldsymbol{m}_0\rangle$ that satisfies assumptions (H1) – (H4), and a minimal firing quantity vector $\boldsymbol{z}^{\boldsymbol{q}^{\mathcal{B}^s}}$. At marking $\boldsymbol{m}$, the following sets are determined:

- $P_\emptyset(\boldsymbol{m}) = \{p_i \in P^C \mid m_i = 0\}$ be the subset of empty continuous places.
- $P_F(\boldsymbol{m}) = \{p_i \in P^B \mid q_i = Q_i = d_i^{\max} \cdot s_i\}$ be the subset of full batch places.
- $T_{\mathcal{N}}(\boldsymbol{m})$ be the set of transitions that are not enabled.
- $T_{ZC}(\boldsymbol{m}) = \{t_j \in T \mid z_j(\boldsymbol{m}) = z_j^{\boldsymbol{q}^{\mathcal{B}^s}})\}$ be the subset of enabled transitions that have their firing quantities equal to their minimal firing quantities.

At marking $\boldsymbol{m}$, a controlled firing flow vector $\boldsymbol{u}(\boldsymbol{m})$ is an admissible ZF control input, if and only if it is a feasible solution of the following linear constraints set.

$$\begin{cases} \text{(a) } 0 \le u_j \le \Phi_j & \forall t_j \in T \\ \text{(b) } u_j = 0 & \forall t_j \in T_{\mathcal{N}}(\boldsymbol{m}) \cup T_{ZC}(\boldsymbol{m}) \\ \text{(c) } C\,(p_k,\cdot)\cdot \boldsymbol{u} \le 0 & \forall p_k \in P_F(\boldsymbol{m}) \\ \text{(d) } C\,(p_k,\cdot)\cdot \boldsymbol{u} \ge 0 & \forall p_k \in P_\emptyset(\boldsymbol{m}) \\ \text{(e) } \text{Pre}\,(p_k,\cdot)\cdot \boldsymbol{u} \le V_k \cdot d_k^{out}(\boldsymbol{m}) & \forall p_k \in P^B \\ \text{(f) } \text{Post}\,(p_k,\cdot)\cdot \boldsymbol{u} \le V_k \cdot d_k^{\max} & \forall p_k \in P^B \end{cases} \tag{5}$$

A controlled firing flow vector that is an admissible ZF control input is called a ZF controlled firing flow vector if it is a solution of the LPP formed by the previous set of constraints with the objective function $\max \mathbf{1}^T \cdot \boldsymbol{u}$.

Proof. At marking $\boldsymbol{m}$, the objective is to maximize the controlled firing flow vector. According to Definition 9, it is obvious that constraint set (5) satisfies all the conditions to be an admissible ZF control input. ■

Based on an event-driven control strategy, each control action is maintained during an IB-state and updated only when a particular event (or possibly several events) occurs. By considering this new proposed control law, the controller must also consider the following events: (i) the current firing quantity of transition t_j reaches its minimum firing quantity, i.e., $z_j(\boldsymbol{m}) = z_j^{\boldsymbol{q}^{\mathcal{B}^s}}$; (ii) the output density of a batch place changes its value.

Algorithm 1 provides the main procedure that computes a control trajectory with the previous ZF-law.

Algorithm 1: Computation of a control trajectory by the ZF-law for reaching quantity $\boldsymbol{q}^{\mathcal{B}^s}$ from $\boldsymbol{q}_0$

Input: A cGBPN $\langle N, \boldsymbol{m}_0\rangle$, marking quantity $\boldsymbol{q}^{\mathcal{B}^s}$ and, the minimal firing quantity vector $\boldsymbol{z}^{\boldsymbol{q}^{\mathcal{B}^s}}$.
Output: Control trajectory $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots$

1 Initialize : $\boldsymbol{q}^0 = \mu(\boldsymbol{m}_0)$, $\boldsymbol{m}^0 = \boldsymbol{m}_0$, $\boldsymbol{u}^0 = \mathbf{0}$, $\boldsymbol{z}(\boldsymbol{m}^0) = \mathbf{0}$, $\tau_0 = 0$, $i = 0$;
2 **while** $\boldsymbol{z}(\boldsymbol{m}^i) \lneq \boldsymbol{z}^{\boldsymbol{q}^{\mathcal{B}^s}}$ **do**
3 Compute the ZF firing flow vector at time τ_i;
4 Determine all next timed events and select the nearest event(s) in time;
5 Determine the new marking $\boldsymbol{m}^{i+1}$ at time τ_{i+1}, and the marking quantity vector $\boldsymbol{q}^{i+1}$;
6 Determine the firing quantity vector $\boldsymbol{z}(\boldsymbol{m}^{i+1}) = \boldsymbol{z}(\boldsymbol{m}^i) + \boldsymbol{u}^i \cdot (\tau_{i+1} - \tau_i)$;
7 i = i + 1;
8 $\boldsymbol{u}^i = \mathbf{0}$;
9 Return $(\boldsymbol{u}^0, \tau_0), \cdots, (\boldsymbol{u}^i, \tau_i)$;

Step 1 initializes the quantity vector and firing quantity vector as well as the control firing flow vector. The main part (steps 2–7) computes iteratively a new controlled firing flow vector at each event time as long as the firing quantity vector is not equal to the minimum one. Step 3 computes the controlled firing flow according to the ZF-law (see Proposition 1). Steps 4–6 consider the nearest event(s) and update the new marking, the quantity vector and the firing quantity vector. The procedure is repeated until the minimum firing quantity $\boldsymbol{z}^{\boldsymbol{q}^{\mathcal{B}^s}}$ is reached.

Proposition 2. Given a cGBPN $\langle N, \boldsymbol{m}_0\rangle$ that satisfies assumptions (H1) – (H4). Algorithm 1 computes a control trajectory that drives the net from the initial marking quantity to a given marking quantity $\boldsymbol{q}^{\mathcal{B}^s}$ in finite time.

Proof. Constraint set (5) of Proposition 1 are used to compute the admissible ZF control input. Constraints(b) show that once the minimal firing quantity $z_j^{\boldsymbol{q}^{\mathcal{B}^s}}$ is reached, transition t_j is stopped, $u_j = 0$. Since the net is consistent and there exists no empty siphon at $\boldsymbol{q}_0$ by assumptions (H1)-(H4), any solution $\boldsymbol{q}$ of the fundamental equation (see Equation 1) is then reachable (Recalde et al., 1999). ■

The complexity of Algorithm 1 is polynomial since it is based on solving a LPP at each iteration, and the iteration ends within finite steps.

3.3 Example

Let us continue Example 1 with $\boldsymbol{m}_0 = [7\ 6\ \{(0.75, 4, 5)\}]^T$, $\boldsymbol{q}^0 = [7\ 6\ 3]^T$. We remind that the maximal flow vector is given by $\boldsymbol{\Phi} = [2\ 2\ 2]^T$ and the steady state is characterised by $\boldsymbol{q}^s = [8\ 3\ 5]^T$, $\boldsymbol{\varphi}^s = [1\ 1\ 2]^T$ and $\boldsymbol{m}^s = [8\ 3\ \{(5, 1, 5)\}]^T$.

a) Reaching $\boldsymbol{q}^{\mathcal{B}_1^s} = [7\ 3\ 6]^T$ from $\boldsymbol{m}_0$, by ZF controller.

As the firing quantity vector is null at the initial time, i.e., $\boldsymbol{z}(\boldsymbol{m}_0) = [0\ 0\ 0]^T$, the minimum firing quantity vector from $\boldsymbol{q}^0$ to $\boldsymbol{q}^{\mathcal{B}_1^s}$ is equal to $\boldsymbol{z}^{\boldsymbol{q}^{\mathcal{B}_1^s}} = [0\ 3\ 3]^T$. By performing Algorithm 1, a ZF control trajectory that drives the net from its initial blocking quantity to its nearest one is computed as follows. At $\tau_0 = 0$, according to the initial quantity vector, no place is empty or full, i.e., $P_\emptyset(\boldsymbol{m}_0) = \emptyset$ and $P_F(\boldsymbol{m}_0) = \emptyset$. All transitions are enabled, thus, $T_{\mathcal{N}}(\boldsymbol{m}_0) = \emptyset$. Since only transition t_1 has its firing quantity equal to its minimum one, i.e., $z_1(\boldsymbol{m}_0) = z_1^{\mathcal{B}_1^s}$, only t_1 is an OFF$_Z$-transition, i.e., $T_{ZC}(\boldsymbol{m}_0) = \{t_1\}$. Transitions t_2 and t_3 are ON$_Z$-transitions. By applying the ZF-law, the controlled firing flow vector is given by: $\boldsymbol{u}^0 = [0\ 2\ 2]^T$. At $\tau_1 = 1.5$, the nearest event occurs as transitions t_2 and t_3 reach their minimum firing quantities, i.e., $\boldsymbol{z}(\boldsymbol{m}^1) = [0\ 3\ 3]$ The marking is $\boldsymbol{m}^1 = [7\ 3\ \{(3, 2, 3)\}]^T$, implying $\boldsymbol{q}^1 = \boldsymbol{q}^{\mathcal{B}_1^s}$, $\boldsymbol{z}(\boldsymbol{m}^1) = \boldsymbol{z}^{\boldsymbol{q}^{\mathcal{B}_1^s}}$ and the stopping of the algorithm. As a result, at date $\tau_1 = 1.5$, the region $\mathcal{B}_1^s$ is reached and the SF-controller could be applied, as presented in the next.

b) From $\boldsymbol{m}^1 = [7\ 3\ \{(3, 2, 3)\}]^T$ (i.e., $\mu^{-1}(\boldsymbol{q}^{\mathcal{B}_1^s})$) to $\boldsymbol{m}^s$, by SF controller.

At marking $\boldsymbol{m}^1$, t_3 is not enabled and transitions t_1, t_2 are OFF. The next event arrives when batch place p_3 has an output batch. At $\tau_2 = 2.5$, the marking is $\boldsymbol{m}^2 = [7\ 3\ \{(3, 2, 5)\}]^T$ and all transitions are enabled. By applying the SF-law (see Section 2.3), transitions t_1, t_2 are OFF, and the controlled firing flow vector is given by: $\boldsymbol{u}^2 = [0\ 0\ 2]^T$. At $\tau_3 = 3$, places p_1 and p_3 reaches their steady quantities, and the steady quantity vector $\boldsymbol{q}^s$ is reached. At this date, the marking is $\boldsymbol{m}^3 = [8\ 3\ \{(1.5, 2, 4.5), (0.5, 4, 5)\}]$ which does not correspond to the steady marking. Thus, it is still necessary to continue the SF controller which gives $\boldsymbol{u}^2 = [1\ 1\ 2]^T = \boldsymbol{\varphi}^s$ as controlled firing flow vector. Finally, at $\tau_4 = 5.5$, the steady marking is reached.

Figure 2 illustrates the evolution of the marking quantities. For reaching the attractive region $\mathcal{B}_2^s$ where $\boldsymbol{q}^{\mathcal{B}_2^s} = \boldsymbol{q}^s$, the whole control trajectory is same as the one for $\mathcal{B}_1^s$. Thus, Algorithm 1 drives the blocking initial quantity to an attractive region, then the system evolves to the steady state.

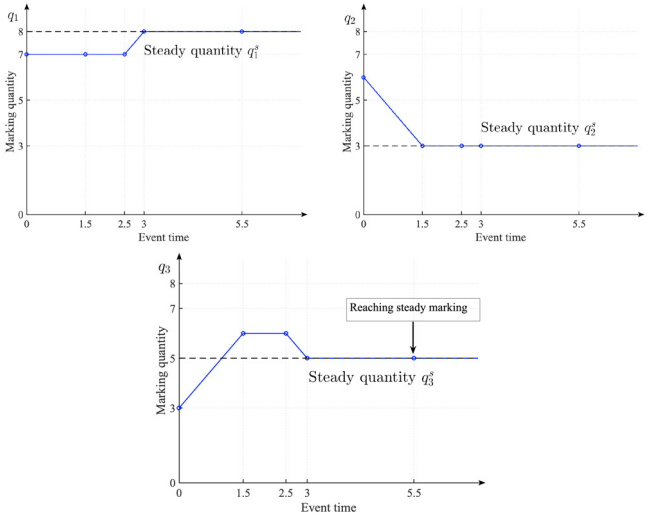

Fig. 2. Evolution of marking quantities.

4. CONCLUSION

In this paper, we propose a new ON/OFF event-driven control strategy for controlled generalised batches Petri net without discrete nodes, that fires transitions as fast as possible at each step until the minimal firing quantity is reached. This controller, inferred by an algorithm, is used to drive the system from a blocking initial marking to an attractive region of a steady state. Future works will focus on the comparison of the complexity and time delay between this new controller and the two previous ones, i.e., steady flow controller and maximal flow controller (Liu et al., 2020b). Another perspective is to investigate the control problem of cGBPNs with discrete nodes.

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