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Abstract A very recently developed metaheuristic method called Jaya algorithm (JA) is implemented in this study for sizing and layout optimization of truss structures. The main feature of JA is that it does not require setting algorithm-specific parameters. The algorithm has a very simple formulation where the basic idea is to approach the best solution and escape from the worst solution. The original JA formulation is modified in this research in order to improve convergence speed and reduce the number of structural analyses required in the optimization process. The suitability of JA for truss optimization is investigated by solving six classical weight minimization problems of truss structures including sizing, layout and large-scale optimization problems with up to 204 design variables. Discrete sizing/layout variables and simplified topology optimization also are considered. The test problems solved in this study are very common benchmarks in structural optimization and practically describe all scenarios that may be faced by designers. The results demonstrate that JA can obtain better designs than those of the other state-of-the-art metaheuristic and gradient-based optimization methods in terms of optimized weight, standard deviation and number of structural analyses.

Keywords: Metaheuristic optimization methods; Jaya algorithm; Sizing, layout and topology optimization; Truss structures.

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1. Introduction

Metaheuristic algorithms try to achieve a dynamic balance between exploration of search space (i.e. "diversification" phase) and exploitation of accumulated search experience (i.e. "intensification" phase). This balance allows to quickly identify regions of design space containing high quality solutions as well as to bypass regions that either were already explored or are far from global optimum [1-3].

Metaheuristic optimization methods such as, for example, genetic algorithms (GA) [4,5], simulated annealing (SA) [6,7], evolution strategies (ES) [8,9], ant colony optimization (ACO) [10,11], particle swarm optimization (PSO) [12,13], harmony search optimization (HS) [14,15], artificial bee colony algorithm (ABC) [16], big bang-big crunch optimization (BB-BC) [17], charged system search (CSS) [18], firefly algorithm (FFA) [2], teaching-learning-based optimization (TLBO) [19], flower pollination algorithm (FPA) [20], swallow swarm optimization algorithm (SSO) [21] and water evaporation optimization (WEO) [22], have been successfully used in every field of science and engineering.

Truss structures are very often selected as benchmark design problems to test the efficiency of metaheuristic algorithms. Just to cite a few examples from the extensive optimization literature, GA using different search operators and re-analysis strategies [23-30], differential evolution with various search schemes [31-35], SA based on single or multi-level search [36-39], particle swarm optimization with different modelling of social/individual behavior, combination of global/local best and center of mass particles [40-44], harmony search optimization with different search and parameter adaptation strategies [45-49], big bang big crunch optimization with different definition of trial designs and infrequent explosions [48, 50-52], teaching-learning based optimization [53-56], artificial bee colony algorithm with adaptive penalty approach [57], firefly algorithm [58,59], cultural algorithm [60], flower pollination algorithm [61], water evaporation algorithm [62], hybrid methods combining two or more metaheuristic algorithms, as well as metaheuristic algorithms and gradient-based optimization [63-66]. Further information can be found in classical textbooks [2], review papers [67,68] and studies comparing the relative efficiency of several metaheuristic algorithms in static and dynamic truss optimization problems [69,70].

The continuously increasing computational power has favored the blooming of new metaheuristic algorithms that are often claimed by authors to be very competitive with the most popular state-of-the-art optimizers. However, finding the global optimum at a reasonably low computational cost for all problems with a limited sensitivity to the selection (i.e. size and composition) of initial population and the setting/adaptation of internal parameters that drive the search process remains an unresolved issue in metaheuristic optimization.

An interesting metaheuristic algorithm that has a very simple formulation and does not require internal parameters is the JAYA algorithm (JA) developed by Rao in 2016 [71]. The JA algorithm was successfully tested on several benchmark functions. Rao and Waghmare [72] later utilized the JA for solving constrained mechanical design problems such as robot gripper, multiple disc clutch brake, hydrostatic thrust bearing and rolling element bearing. The efficiency of the JA with respect to other metaheuristic algorithms was demonstrated also for these test problems. The JA was used also for sizing optimization of a micro-channel heat sink [73] by taking thermal resistance and pumping power as objective functions and micro-channel width, depth and fin width as design variables. Once again JA resulted very competitive or even better than those obtained for TLBO and multi-objective evolutionary algorithms.

The main objective of this study is to evaluate the suitability of the JA algorithm for weight minimization of truss structures. This test suite is selected because of the tremendously large amount of data available in literature, which allows comprehensive and detailed comparisons to be carried out. Sizing optimization problems of trusses with 200, 942 and 1938 elements, and combined sizing-layout optimization problems of trusses with 25, 45 and 47 elements are solved for that purpose. Sizing optimization problems include up to 204 design variables while combined sizing-layout optimization problems include up to 81 design variables. Discrete sizing/layout variables and simplified topology optimization also are considered. The original JA formulation is modified in order to improve convergence speed thus reducing the number of structural analyses required in the optimization.

The results obtained by the JA are compared with those of other state-of-the-art metaheuristic optimization methods including variants of genetic algorithms, differential evolution, simulated annealing, particle swarm, harmony search, big bang big crunch, artificial bee colony, teaching-learning based optimization, cultural algorithm, firefly algorithm, flower pollination algorithm, water evaporation optimization, hybrid particle and swallow swarm optimization, hybrid particle swarm, ant colony and harmony search optimization. Comparisons with gradient-based optimizers also are presented in the article. The performance of JA is evaluated in terms of minimum weight, standard deviation on optimized weight and number of structural analyses required in the optimization process. In all test problems, JA is compared with the best solutions available in metaheuristic optimization literature as well as with commercial software. A statistical analysis of the best, average and worst optimized weights and corresponding standard deviations obtained over independent optimization runs is performed. Results prove that the proposed algorithm is very competitive with the other metaheuristic methods and its convergence speed is similar to gradient-based optimizers.

The paper is structured as follows. The formulation of the design optimization problem for truss structures is recalled in Section 2. The JAYA algorithm is described in Section 3 along with its implementation for truss structure problems. Section 4 describes the test problems and discusses optimization results. Section 5 summarizes the main findings of this study. Sensitivity of JA convergence behavior to population size is analyzed in detail in the Appendix.

2. Optimization of truss structures

The objective of truss optimization is to minimize the weight of the structure under design constraints such as element stresses and nodal displacements. The sizing optimization problem of a truss structure can be stated as:

Find $A = [A_1, A_2, ..., A_{ng}]$

To minimize $\mathrm{W}(\mathbf{A}) = \sum_{\mathrm{i}=1}^{\mathrm{nm}} \gamma_{\mathrm{i}} \mathrm{A}_{\mathrm{i}} \mathrm{L}_{\mathrm{i}}$

Subjected to (1)

$\sigma_i^c \leq \sigma_i \leq \sigma_i^t$, i=1,2,….,nm

$\delta_{\min} \leq \delta_j \leq \delta_{\max}$, j=1,2,…..,nn

$A_{\min} \leq A_k \leq A_{\max}$, k=1,2,….,ng

where the ***A*** vector contains the sizing variables (i.e. cross-sectional areas of bars), $W(\boldsymbol{A})$ is the weight of the truss structure. γ_i and L_i, respectively, are the material density and the length of member i. A_i is the cross-sectional area of member i with the corresponding lower/upper bounds A_{min} and A_{max}. Each truss design must satisfy design constraints on member stresses σ_i for each element i and displacements δ_j for each node j. σ_i^c and σ_i^t are the allowable compression and tension stresses for member i. $\delta_{\min}$ and $\delta_{\max}$ are the allowable displacements for node j. nm is the number of members in the truss structure, nn is the number of nodes, ng is the number of member groups (i.e. number of design variables).

For optimization problems including also layout variables, the cost function can be rewritten as:

$$\text{Minimize} \quad \mathrm{W}(\boldsymbol{A},\boldsymbol{X}) = \gamma \sum_{\mathrm{i}=1}^{\mathrm{nm}} A_{\mathrm{i}} \sqrt{(x_{\mathrm{i1}} - x_{\mathrm{i2}})^2 + (y_{\mathrm{i1}} - \mathrm{y}_{\mathrm{i2}})^2 + (z_{\mathrm{i1}} - z_{\mathrm{i2}})^2} \qquad (2)$$

where $x_{i1,2}$, $y_{i1,2}$, $z_{i1,2}$ are the coordinates of the nodes limiting the i^{th} element of the structure. The optimization problem hence includes *ndv* design variables where *ndv* is the sum of the *ng* element cross-sectional areas included as sizing variables and *nlay* nodal coordinates included as layout variables. In topology optimization, elements can be removed from the structure if their cross-sectional areas become very small (for example, 10^{-7} in^2).

Stress and displacement constraints to be satisfied are handled by using a penalty function. The penalized objective function F_p is obtained as the product between the truss weight $W(\boldsymbol{A},\boldsymbol{X})$ and the penalty function ψ_p as follows:

$$F_p = W(A, X) \times \psi_{\mathrm{p}} \tag{3}$$

The penalty function is defined as:

$$\psi_p = (1 + \phi)^{\varepsilon} \tag{4}$$

where ϕ is the summation of the stress and displacement penalties, defined as:

$$\phi = \sum_{i=1}^{nm} \phi_\sigma^i + \sum_{j=1}^{nn} \phi_\delta^j \tag{5}$$

The stress constraint penalty ϕ_σ^i for member i and the displacement constraint penalty ϕ_δ^j for node j are respectively defined as:

$$\begin{cases} \phi_\sigma^{\mathrm{i}} = 0 & \text{if} \quad \sigma_{\mathrm{i}}^{\mathrm{c}} \le \sigma_{\mathrm{i}} \le \sigma_{\mathrm{i}}^{\mathrm{t}} \\ \phi_\sigma^{\mathrm{i}} = \dfrac{\left|\sigma_{\mathrm{i}} - \sigma^{\mathrm{t,c}}\right|}{\left|\sigma^{\mathrm{t,c}}\right|} & \text{if} \quad \sigma_{\mathrm{i}} < \sigma_{\mathrm{i}}^{\mathrm{c}} \text{ or } \sigma_{\mathrm{i}} > \sigma_{\mathrm{i}}^{\mathrm{t}} \end{cases} \tag{6}$$

$$\begin{cases} \phi_\delta^{\mathrm{j}} = 0 & \text{if} \quad \delta_{\min} \le \delta_{\mathrm{j}} \le \delta_{\max} \\ \phi_\delta^{\mathrm{j}} = \dfrac{\left|\delta_{\mathrm{j}} - \delta^{\max,\min}\right|}{\left|\delta^{\max,\min}\right|} & \text{if} \quad \delta_{\mathrm{j}} < \delta_{\min} \text{ or } \delta_{\mathrm{j}} > \delta_{\max} \end{cases} \tag{7}$$

The penalty function exponent ε is set equal to 2 in this study. This setting is very common in literature and corresponds to assuming a quadratic variation of penalty terms throughout the optimization process. However, variations can be sharper in the case of non-smooth problems that include different search spaces (i.e. layout and sizing variables) or hundreds of design variables. For this reason, for the combined sizing-layout optimization problems and largest sizing optimization problem solved in this study, the penalty function exponent ε was supposed to increase with the number of iterations as $\varepsilon_{iter}=\varepsilon_o(1+it/it_{max})$ where the initial value ε_o is chosen between 1.001 and 10000, it denotes the current iteration and it_{max} is the limit number of optimization iterations specified by the user. This was done in purpose to amplify the effect of penalty as the search process approaches constraint domain boundaries. Remarkably, JA convergence behavior was found to be insensitive to such a refinement.

3. The JAYA algorithm

The JAYA algorithm (JA) is a new metaheuristic optimization method proposed in 2016 by Rao [71]. The word "Jaya" means "victory" in Sanskrit. This population based algorithm is based on the concept that the search process should always move toward the best design and avoid the worst design. The search engine continuously tries to get closer to success (i.e. to reach the best design) trying at the same time to avoid failure (i.e. by moving away from the worst design) [73]. A definite strength point of JA with respect to other metaheuristic optimization algorithms is that JA does not include any algorithm-specific parameters. In fact, JA only requires two standard control parameters such as population size (i.e. number of truss designs in the population) and maximum number of iterations.

In the optimization process, ndv is the number of design variables (i.e. number of member groups ng in sizing optimization problems; summation of number of member groups and number of layout variables in sizing-layout optimization problems) and *np* is the population size (i.e. number of truss designs). The design corresponding to the lowest penalized objective function (F_p^{best}) is stored as the best design while the design corresponding to the highest penalized objective function (F_p^{worst}) is the worst design stored in the population.

Let $X_{k,l,it}$ denote the value of the *k-th* design variable (cross-sectional areas $\boldsymbol{A}$ and nodal coordinates $\boldsymbol{X}$) for the *l-th* design of the population at the beginning of the *it-th* iteration. The JA algorithm perturbs this design variable using the following equation:

$$X_{\mathrm{k,l,it}}^{\mathrm{new}} = X_{\mathrm{k,l,it}} + r_{\mathrm{1,k,it}}\left(X_{\mathrm{k,best,it}} - \left|X_{\mathrm{k,l,it}}\right|\right) - r_{\mathrm{2,k,it}}\left(X_{\mathrm{k,worst,it}} - \left|X_{\mathrm{k,l,it}}\right|\right) \tag{8}$$

where $X_{\mathrm{k,l,it}}^{\mathrm{new}}$ is the new value assigned to the design variable $X_{\mathrm{k,l,it}}$, $r_{1,k,it}$ and $r_{2,k,it}$ are two randomly generated real numbers in the [0,1] range for the *k-th* design variable in the *it-th* iteration. $X_{\mathrm{k,best,it}}$ is the value of the *k-th* design variable for the best design of the population at the *it-th* iteration while $X_{\mathrm{k,worst,it}}$ is the value of the *k-th* design variable for the worst design stored in the population. The term $r_{\mathrm{1,k,it}}\left(X_{\mathrm{k,best,it}} - \left|X_{\mathrm{k,l,it}}\right|\right)$ indicates the tendency of the solution to move closer to the best solution. The term $-r_{\mathrm{2,k,it}}\left(X_{\mathrm{k,worst,it}} - \left|X_{\mathrm{k,l,it}}\right|\right)$ indicates the tendency of the solution to avoid the worst solution [71].

After all design variables are updated with Eq. (8), the penalized objective function (F_p^{new}) is calculated for the new design. If the F_p^{new} value is better than the previous penalized objective function value (F_p^{pre}) stored in the population (i.e. $F_p^{new} < F_p^{pre}$), the new design replaces the previous one. Otherwise, the previous design remains unchanged. An iteration is completed when the same process is repeated for all designs stored in the population. It is worth pointing out that the random numbers r_1 and r_2 ensure good exploration of the search space and the absolute value of the candidate solution ($|X_{k,l,it}|$) considered in Eq. (8) further enhances the exploration ability of the algorithm [72].

It should be noted that the total computational cost of JA optimization is $np \times it_{max}$ structural analyses if the search process lasts it_{max} iterations. For example, performing 1000 iterations over a population of 100 individuals would entail 100,000 structural analyses, which may become unaffordable for a large-scale truss sizing problem such as, for example, the weight minimization of the spatial 1938-bar tower solved in this study. In order to overcome this limitation, penalized objectives should effectively be compared only if new trial designs defined with Eq. (8) are very likely to improve the designs currently stored in the population (see variant introduced in Section 3.1). Another issue to be considered is whether the best and worst designs should remain unchanged throughout the current iteration or be dynamically updated in the iteration itself each time a new trial design replaces some design in the population. Since preliminary numerical trials confirmed that selecting the most effective updating sequence of best/worst designs may not be an easy task, the classical JA strategy (i.e. best and worst designs remain the same for the whole current iteration) was adopted in this study.

3.1. Implementation of the JA algorithm for truss optimization

In this study, the JA algorithm was applied to design optimization of truss structures. The implementation steps are now summarized.

1. An initial population including *np* truss designs is randomly generated. The optimization problem includes *ng* sizing variables or *ndv* design variables. Each design variable included in a truss design is generated between its lower ($X^{k}_{\min}$) and upper ($X^{k}_{\max}$) bounds using the following equation:

$$X^{o}_{k,l} = X^{k}_{min} + rand(0,1)\,(X^{k}_{max} - X^{k}_{min}) \qquad k=1,2,\dots,ndv;\ l=1,2,\dots,np \tag{9}$$

where *rand*(0,1) is a random number uniformly generated in the [0,1] interval. The generated values $X^{o}_{k,l}$ are stored as $X_{k,l,1}$ to perform the first iteration of the JA algorithm. The maximum number of iterations it_{max} and the penalty function exponent are defined by the user.

2. The penalized objective function values (F_p) are calculated for all truss designs with Eqs. (1) through (7). F_p^{best} and F_p^{worst}, respectively, are the best (i.e. the minimum) and the worst (i.e. maximum) penalized objective functions for the designs stored in the population.
3. Each design included in the population is updated with Eq. (8) and the corresponding penalized objective function value (F_p^{new}) is calculated.
4. If the new design $\boldsymbol{X}^{new}$ is better than previous design $\boldsymbol{X}^{pre}$ stored in the population (i.e. $F_p^{new} < F_p^{pre}$), it replaces the previous one. Otherwise, the previous design remains unchanged. The same process is repeated for all truss designs stored in the population and, hence, an iteration would be completed by performing *np* structural analyses. In order to speed up this step, a novel strategy was introduced in this study. The cost function W^{new} only is preliminary evaluated for the new design. Structural analysis and hence penalty evaluation are then performed only if $W^{new}<W^{pre}$, that is if the new design can potentially improve the previous one. The case $W^{new}>W^{pre}$ entails a new constraint evaluation only if the previous design $\boldsymbol{X}^{pre}$ is infeasible. As is clear, if the previous design is feasible and the new design already yields a larger structural weight regardless of adding penalty terms, it does not make much sense to perturb the design along a non-descent direction.
5. The design corresponding to the lowest value of the penalized objective function is stored as the current best record.
6. If $it < it_{max}$, set $it=it+1$ and repeat steps 2 through 5. Otherwise go to step 7.

7. The optimization process is terminated when the maximum number of iterations it_{max} is completed. The feasible design corresponding to the lowest cost function value stored in the population is taken as optimum design. The maximum number of iterations is the most commonly utilized stopping criterion in metaheuristic optimization. However, some algorithms (see, for example, Ref. [32]) finish the search process also if the absolute value of deviation of the objective function of the best individual and the whole population is smaller than a limit specified by the user. In order to check this issue we analyzed convergence curves recorded for JA in each test problem. The optimization runs carried out in this study indicate that JA converges asymptotically to the optimum design and the standard deviation of design vectors (determined as the maximum standard deviation of each optimization variable assigned to candidate designs) rapidly drops below 10^{-10} as JA performs only 30-50 additional structural analyses after having reached its best solution. Remarkably, the same behavior occurs regardless of problem size and type of design variables (i.e. sizing/layout). In view of this, JA actually behaves as if it included two stopping criteria: the maximum number of iterations and the convergence of populations to the optimum design.

The flow chart of the JA algorithm is shown in Fig. 1.

Multi-stage optimizations can be performed if the truss design problem includes discrete variables. A simplified method for solving mixed continuous-discrete or fully discrete weight minimization problems of truss structures was adopted in this study. This approach was not specifically designed for JA but it can be applied to any metaheuristic algorithm (see, for example, Section 4.7).

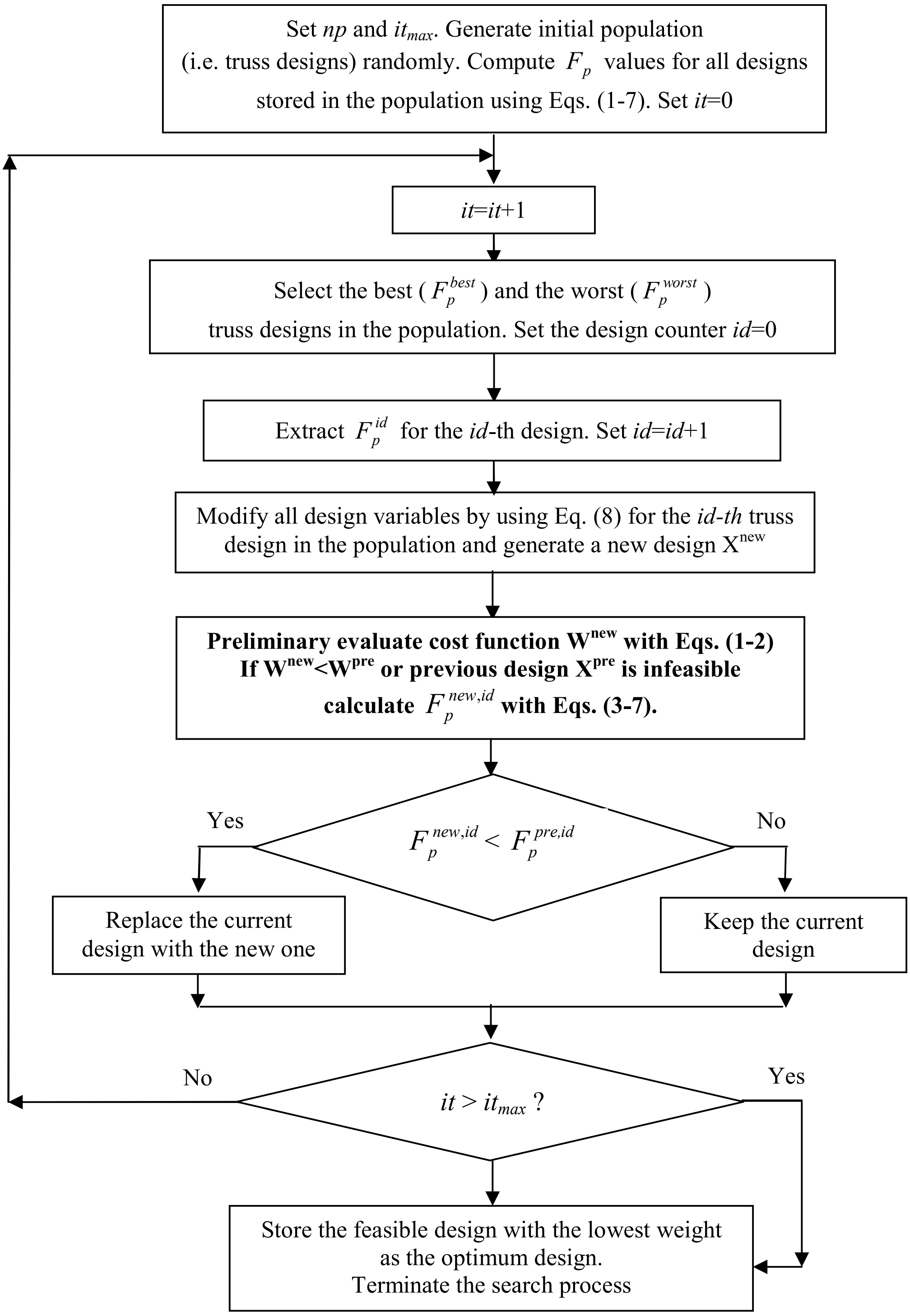


Fig. 1. Flowchart of the JA search process for weight minimization of truss structures. The variant introduced for large-scale and sizing-layout problem is indicated in bold.

4. Test problems, optimization results and discussion

Six trusses previously optimized with various metaheuristic methods were considered in this study in order to demonstrate the efficiency of the JA algorithm in sizing-layout optimization of truss structures. The design examples regarded the sizing optimization of a planar 200-bar truss, a spatial 942-bar tower and a spatial 1938-bar tower; these test cases included up to 204 design variables. Combined sizing-layout optimization was performed for a spatial 25-bar transmission tower, a planar 45-bar truss and a planar 47-bar power line; these test cases included up to 81 design variables. Discrete sizing optimization and simplified topology optimization also were carried out. All test problems solved in this study are very common benchmark examples in structural optimization literature, often selected by researchers in the last thirty years to evaluate the performance of new metaheuristic algorithms.

The best population size *np* for each test problem was determined via sensitivity analysis. The results presented in the Appendix indicate that the best value is *np*=20 for sizing problems while it changes between 30 and 1000 for the sizing-layout problems. Remarkably, the standard deviation of optimized weight never exceeds 0.062% of the corresponding average weight. Furthermore, the standard deviation of the number of structural analyses is always smaller than the 12.1% of the corresponding average number of analyses. Hence, JA performance is practically insensitive to population size.

JA was compared with its standard formulation where constraints are evaluated for all trial designs and other state-of-the-art metaheuristic optimization methods like hybrid big bang big crunch optimization with (HBBBC-LS) or without line search (BBBC), hybrid harmony search with line search (HHS-LS), artificial bee colony algorithm with adaptive penalty approach (ABC-AP), self-adaptive harmony search (SAHS), teaching-learning based optimization (TLBO), multi-stage particle swarm optimization (MSPSO), cultural algorithm (CA), firefly algorithm (FFA), flower pollination algorithm (FPA), water evaporation algorithm (WEA), multi-level and multi-point simulated annealing (CMLPSA), cuckoo search (CS), adaptive evolution strategies (ESs), genetic algorithms (GAs), improved constrained differential evolution (ICDE), hybrid particle swarm and swallow optimization algorithm (HPSSO), hybrid particle swarm ant colony and harmony search optimizer (HPSACO) etc. The above mentioned algorithms were selected according to two criteria: (i) they achieved the best structural weight or required the smallest number of structural analyses indicated in the literature; (ii) they were the most robust optimizers in terms of smallest standard deviation on optimized weight for the test problem at hand. JA was also compared with commercial optimization software in order to gather further information on its convergence behavior and to compare the convergence speed of JA with gradient-based optimizers that should inherently be faster than metaheuristic algorithms.

The JA was executed twenty times for each design example starting from twenty randomly generated initial populations of fixed size. Whenever possible, optimization runs were started from the same initial populations used for the referenced algorithms compared with JA. When the composition of the initial population was not given in literature, the same population size, lower and upper bounds of design variables used in the referenced studies were at least used also in this study in order to perform the most reliable comparison as feasible. Numerical trials with fifty and one-hundred independent optimization runs also were performed for some test problems to cover all cases reported in the literature. Remarkably, the best weight, average optimized weight and standard deviation on optimized weight were less than 0.001% different from the corresponding values obtained for twenty runs.

The best design obtained over the twenty runs and the corresponding number of structural analyses required in the optimization process are reported in tables. Average optimized weight, worst optimized weight and standard deviation on optimized weight recorded in the independent optimization runs also are reported. These quantities are the most commonly adopted parameters in metaheuristic optimization of structures to evaluate the performance of a new algorithm and compare relative merits of metaheuristic search engines.

The JA algorithm was coded in the MATLAB environment and executed on a standard PC equipped with a single 2.2 GHz Intel® Pentium i7-4702MQ CPU. The large-scale 1938-bar problem and the mixed sizing-layout/topology optimization problems were also solved coding the JA-based optimization algorithm in the Fortran 90 programming language. This was done in order to analyze the effect of programming language and software environment. Remarkably, no difference was found between the optimization results obtained for the two implementation modalities. A standard linear elastic finite element solver was implemented by the authors to perform the structural analyses entailed by the optimization process.

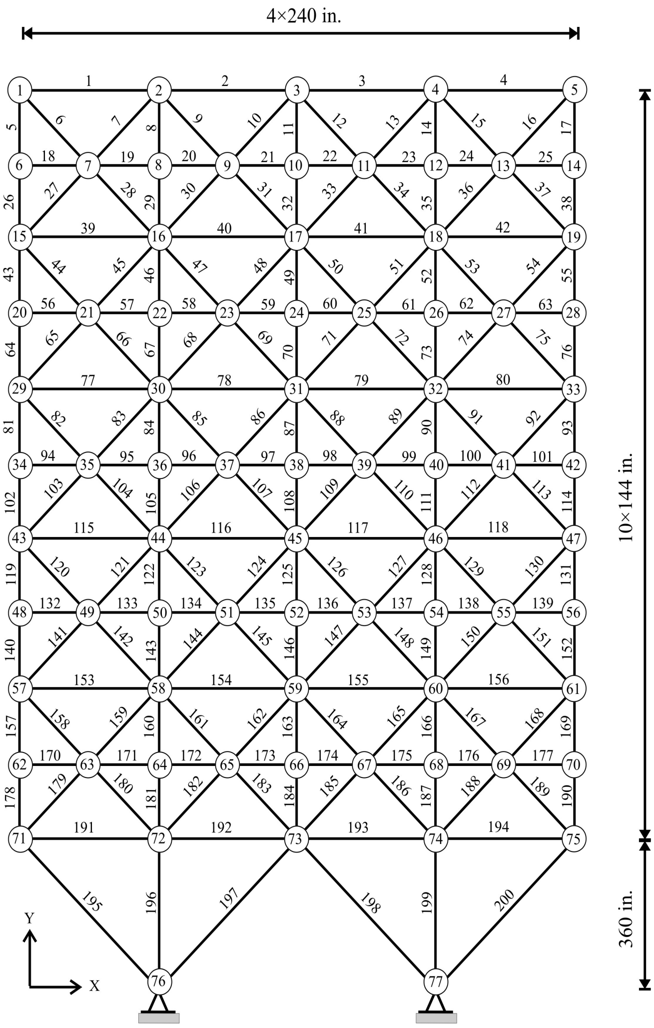
4.1. Sizing optimization of a planar 200-bar truss structure

The planar 200-bar truss schematized in Fig. 2 is a well known average-scale sizing optimization problem. The Young's modulus and mass density are 30 Msi and 0.283 lb/in^3, respectively. The allowable stress in tension/compression is 10000 psi and no displacement constraint is considered. Minimum cross-sectional area of elements is taken as 0.1 in^2. The elements of the structure are divided in 29 groups as illustrated in Fig. 2. The structure must be designed against three independent loading conditions:

(i) 1.0 kip acting in the positive *x*-direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62 and 71;

(ii) 10.0 kips acting in the negative *y*-direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71, 72, 73, 74 and 75;

(iii) loading conditions (i) and (ii) acting together.



Element grouping

(1) 1,2,3,4
(2) 5,8,11,14,17
(3) 19,20,21,22,23,24
(4) 18,25,56,63,94,101,132,139,170,177
(5) 26,29,32,35,38
(6) 6,7,9,10,12,13,15,16,27,28,30,31,33,34,36,37
(7) 39,40,41,42
(8) 43,46,49,52,55
(9) 57,58,59,60,61,62
(10) 64,67,70,73,76
(11) 44,45,47,48,50,51,53,54,65,66,68,69,71,72,74,75
(12) 77,78,79,80
(13) 81,84,87,90,93
(14) 95,96,97,98,99,100
(15) 102,105,108,111,114
(16) 82,83,85,86,88,89,91,92,103,104,106,107,109, 110,112,113
(17) 115,116,117,118
(18) 119,122,125,128,131
(19) 133,134,135,136,137,138
(20) 140,143,146,149,152
(21) 120,121,123,124,126,127,129,130,141,142,144, 145, 147,148,150,151
(22) 153,154,155,156
(23) 157,160,163,166,169
(24) 171,172,173174,175,176
(25) 178,181,184,187,190
(26) 158,159,161,162,164,165,167,168,179,180,182, 183, 185,186,188,189
(27) 191,192,193,194
(28) 195,197,198,200
(29) 196,199

Fig. 2. Schematic of the planar 200-bar truss and element grouping used in optimization.

Table 1 presents the optimization results for JA, CMLPSA [37], ABC-AP [57], SAHS [47], TLBO [53], HPSSO [65], FPA [61] and WEO [62]. It appears that JA obtained the best design overall with a structural weight of 25463.53 lb. This design fully satisfies stress constraints. It should be noted that the best two weights quoted in literature for this problem are 25156.5 lb, achieved by the HPSACO algorithm [63], and 25193.2 lb, achieved by the ray optimization algorithm [74]. However, those designs violate stress constraints, respectively, by 9.97% and 12.7% and hence were not included in the results table. The third best weight quoted in metaheuristic optimization literature is 25445.63 lb, achieved by CMLPSA algorithm, which slightly violates constraints. As the scaled weight of CMLPSA to recover the 0.071% stress constraint violation of its optimized design is 25463.7 lb, the JA algorithm implemented in this study designed a slightly lighter structure than that of CMLPSA.

JA found the optimum design after 31580 structural analyses while TLBO, HPSSO and WEO, respectively, required 28059, 14406 and 19410 analyses. However, JA was faster than these algorithms because it obtained feasible intermediate designs that are better than the optima of TLBO, HPSSO and WEO after only 24510, 8856 and 9432 structural analyses, respectively. Furthermore, JA employed 23663 structural analyses to generate a feasible intermediate design lighter than the 25491.9 lb optimum weight of SAHS: this computational cost is comparable with the 19670 structural analyses required by SAHS. Last, comparison between JA and CMPLSA is not indicative because, unlike the other metaheuristic optimizers of Table 1, the referenced algorithm utilized gradient information that are explicitly available in sizing optimization of truss structures.

Statistical performance of optimizers and constraint margins evaluated at optimum designs are analyzed in Table 2. The JA is very robust because its standard deviation is 24.122 lb, only 0.095% of the average optimized weight; overall, JA ranked 1st, slightly better than TLBO. The optimized weights obtained in the FPA independent runs also are less dispersed than those of JA but the corresponding designs violate stress constraints. Standard deviations of SAHS, WEO and WEO are between 5.9 and 99.6 times as large as JA's standard deviation.

Table 1. Comparison of optimized designs for the 200-bar truss problem

Design variables A_i (in^2)	CMLPSA [37]	ABC-AP [57]	SAHS [47]	TLBO [53]	HPSSO [65]	FPA [61]	WEO [62]	**JA This study**
1	0.1468	0.1039	0.154	0.146	0.1213	0.1425	0.1144	**0.147258**
2	0.9400	0.9463	0.941	0.941	0.9426	0.9637	0.9443	**0.940434**
3	0.1000	0.1037	0.100	0.100	0.1220	0.1005	0.1310	**0.100109**

4	0.1000	0.1126	0.100	0.101	0.1000	0.1000	0.1016	**0.100098**
5	1.9400	1.9520	1.942	1.941	2.0143	1.9514	2.0353	**1.941704**
6	0.2962	0.293	0.301	0.296	0.2800	0.2957	0.3126	**0.296783**
7	0.1000	0.1064	0.100	0.100	0.1589	0.1156	0.1679	**0.100096**
8	3.1042	3.1249	3.108	3.121	3.0666	3.1133	3.1541	**3.106749**
9	0.1000	0.1077	0.100	0.100	0.1002	0.1006	0.1003	**0.100095**
10	4.1042	4.1286	4.106	4.173	4.0418	4.1100	4.1005	**4.108109**
11	0.4034	0.4250	0.409	0.401	0.4142	0.4165	0.4350	**0.403975**
12	0.1912	0.1046	0.191	0.181	0.4852	0.1843	0.1148	**0.193079**
13	5.4284	5.4803	5.428	5.423	5.4196	5.4567	5.3823	**5.434236**
14	0.1000	0.1060	0.100	0.100	0.1000	0.1000	0.1607	**0.100095**
15	6.4284	6.4853	6.427	6.422	6.3749	6.4559	6.4152	**6.434203**
16	0.5734	0.5600	0.581	0.571	0.6813	0.5800	0.5629	**0.575306**
17	0.1327	0.1825	0.151	0.156	0.1576	0.1547	0.4010	**0.135485**
18	7.9717	8.0445	7.973	7.958	8.1447	8.0132	7.9735	**7.980200**
19	0.1000	0.1026	0.100	0.100	0.1000	0.1000	0.1092	**0.100157**
20	8.9717	9.0334	8.974	8.958	9.0920	9.0135	9.0155	**8.980345**
21	0.7049	0.7844	0.719	0.720	0.7462	0.7391	0.8628	**0.709002**
22	0.4196	0.7506	0.422	0.478	0.2114	0.7870	0.2220	**0.437247**
23	10.8636	11.3057	10.892	10.897	10.9587	11.1795	11.0254	**10.89123**
24	0.1000	0.2208	0.100	0.100	0.1000	0.1462	0.1397	**0.100150**
25	11.8606	12.2730	11.887	11.897	11.9832	12.1799	12.0340	**11.89141**
26	1.0339	1.4055	1.040	1.080	0.9241	1.3424	1.0043	**1.049144**
27	6.6818	5.1600	6.646	6.462	6.7676	5.4844	6.5762	**6.610648**
28	10.8113	9.9930	10.804	10.799	10.9639	10.1372	10.7265	**10.77913**
29	13.8404	14.70144	13.870	13.922	13.8186	14.5262	13.9666	**13.87830**
Weight (lb)	25445.63	25533.79	25491.9	25488.15	25698.85	25521.81	25674.83	**25463.53**
NSA	9650	1,450,000	19670	28059	14406	10685	19410	**31580**

The new perturbation strategy introduced in the JA formulation where constraints are evaluated only if trial solutions can effectively improve the current best record (see description of Step 4, Section 3.1) allowed to (i) design a lighter structure than standard JA (25463.53 vs. 25476.48 lb); (ii) reduce the computational cost of the optimization process (31580 vs. 49155 structural analyses); (iii) reduce standard deviation on optimized cost (24.12 vs. 62.59 lb). The original search strategy of JA is inherently efficient because it updates population trying to escape from the worst design and to reach the best design: in fact, the 25476.48 lb weight found by standard JA would rank 2nd overall amongst optimum designs of

Table 1, right after improved JA. The new JA formulation developed in this study further exploits this concept by giving priority to candidate designs that belong to descent directions. The optimized designs found by the new and the standard JAs were very similar. However, the new JA required 36% less structural analyses and was 2.6 times more robust. Rejecting designs that certainly cannot improve current best record allows to detect the most sensitive design variables and always directs search process towards the best regions of design space that are very likely to host the global optimum or nearly global optimum solutions.

Table 2. Comparison of statistical performance and constraint violation for the 200-bar truss problem.

Method	**Best weight** (lb)	**Average weight** (lb)	**Worst weight** (lb)	**SD** (lb)	**CVP** (%)
CMLPSA [37]	25445.63	25446.03	N/A	N/A	0.071
ABC-AP [57]	25533.79	N/A	N/A	N/A	13.136
SAHS [47]	25491.90	25610.20	25799.30	141.85	None
TLBO [53]	25488.15	25533.14	25563.05	27.44	None
HPSSO [65]	25698.85	28386.72	N/A	2403	None
FPA [61]	25521.81	25543.51	N/A	18.13	0.169
WEO [62]	25674.83	26613.45	N/A	702.80	None
JA	**25463.53**	**25477.47**	**25505.32**	**24.12**	**None**

Fig. 3 compares the optimization histories for the best runs of JA, SAHS, TLBO and HPSSO. It appears that JA converges to a nearly optimum design after about 14000 structural analyses and is faster than HPSSO and standard JA. Although SAHS and TLBO initially have better convergence performance, JA practically found the same intermediate designs as SAHS and TLBO after 15000 structural analyses.

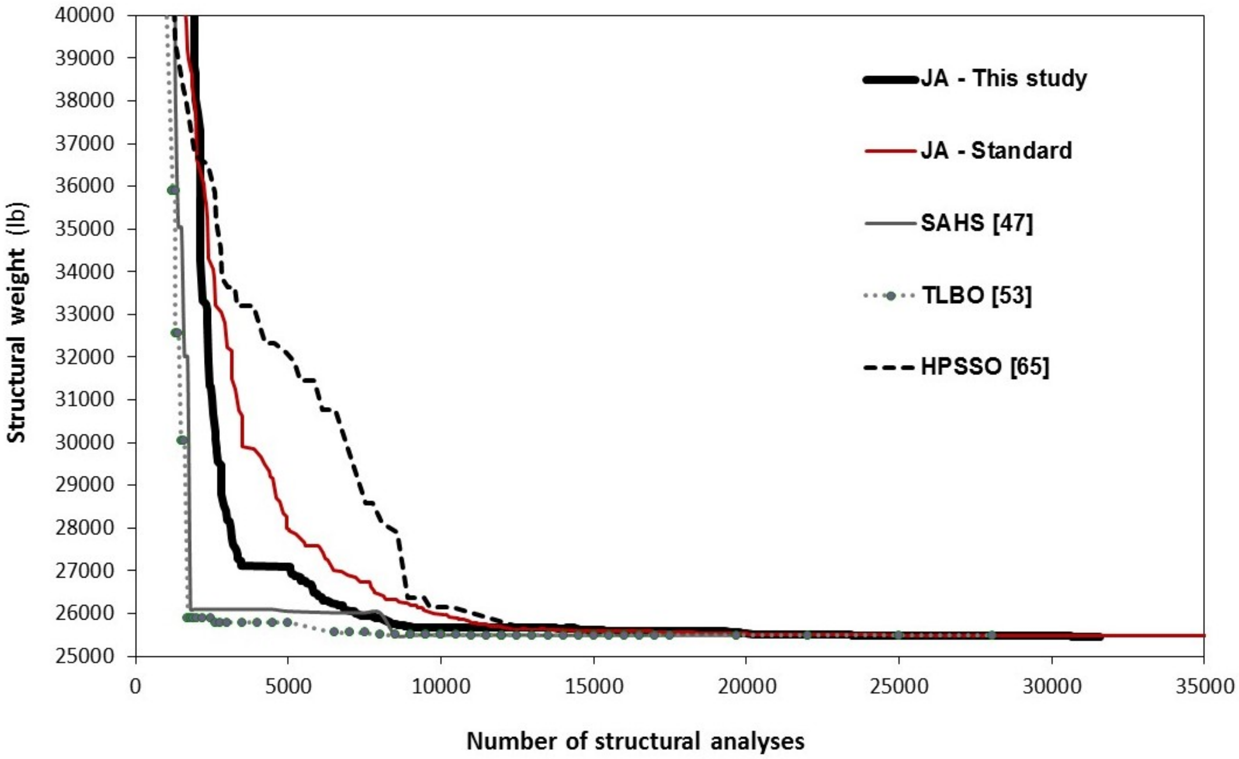


Fig. 3. Comparison of convergence curves for the 200-bar truss problem

4.2. Sizing optimization of a spatial 942-bar tower

The second design example regards a moderately large-scale structure, the spatial 942-bar tower shown in Fig. 4. The structure is symmetrical about *x* and *y*-axes. Elements are divided in 59 groups as indicated in the figure: hence, this test problem includes 59 sizing variables. The Young's modulus and mass density are 10 Msi and 0.1 lb/in^3, respectively. The stress limit for all members (the same in tension and compression) is 25000 psi. The displacements of the four top nodes of the tower in the *x*, *y* and *z* directions must be less than ±15.0 in. Cross-sectional areas of elements can vary between 0.1 and 200 in^2.

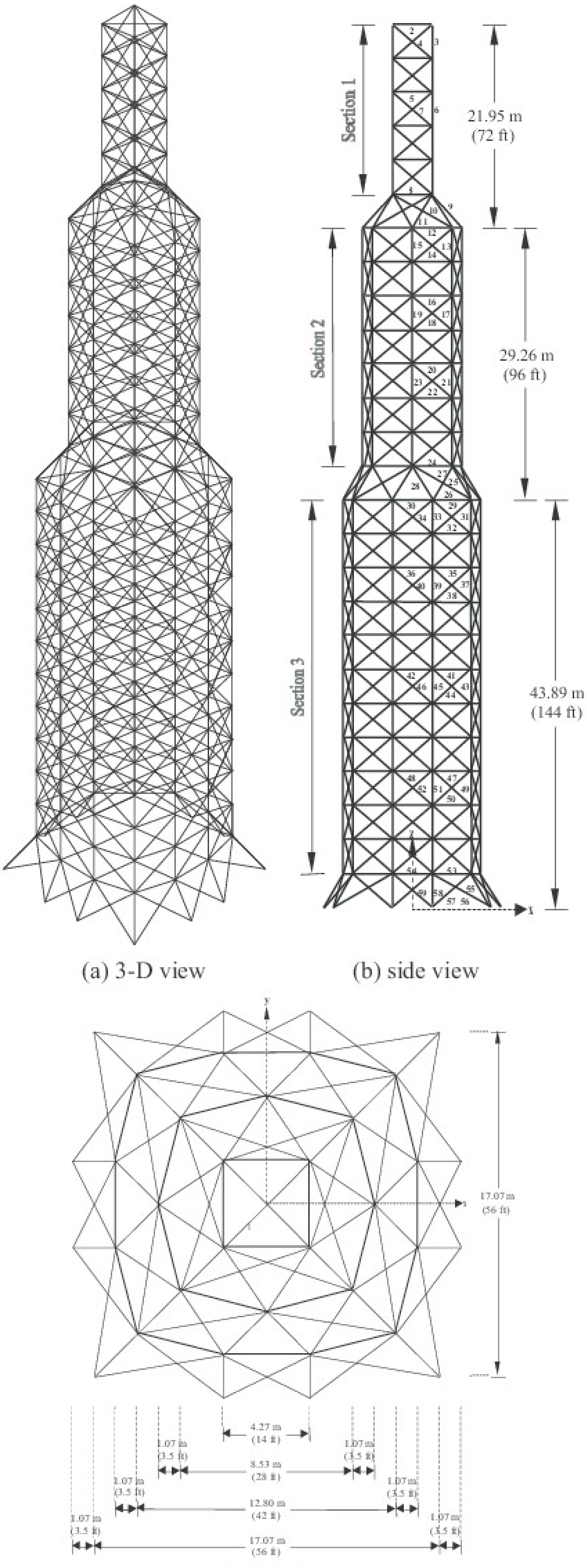


Fig. 4. Schematic of the spatial 942-bar tower.

The structure is subject to a single loading condition comprised of: (i) vertical forces (acting in the negative z-direction) of -3.0, -6.0 and -9.0 kips applied to each node of the first, second and third sections, respectively; (ii) lateral loads (in the y-direction) of 1.0 kips applied to all nodes of the tower; (iii) lateral loads (in the x-direction) of 1.5 and 1.0 kips applied to each node of the left and right sides of the tower, respectively.

Table 3 compares JA results with other metaheuristic algorithms such as GA [23], SA [36], adaptive evolution strategies [75], genetic-Nelder Mead simplex algorithm (GNMS) [27], CS [76] and FFA [59]. This problem has been solved also with common gradient-based optimizers like the Modified Feasible Directions (MFD) and Sequential Linear Programming (SLP) methods implemented in the DOT commercial optimization software [79], and the Sequential Quadratic Programming (SQP) routines implemented in MATLAB [80] and DOT, which were alternatively used in order to maximize convergence speed. Gradient-based optimizations were started from the center of mass of the initial population with np=20. Information on statistical performance and constraint margins evaluated for the optimized designs are given in Table 4; CVP was not evaluated for GA and FFA in this table because optimized cross-sectional areas are not reported in literature.

Table 3. Comparison of optimized designs for the 942-bar tower problem

Design variables A_i (in^2)	GAs [23]	SA [36]	Adaptive ES [75]	GNMS [27]	CS [76]	FFA [59]	**JA This study**	**Combined MFD/SLP DOT**	**SQP MATLAB & DOT**
1	N/A	1	1.02	2.7859	1.00	N/A	**1.045258**	**1.012517**	**1.005961**
2	N/A	1	1.037	1.3572	1.00	N/A	**1.001630**	**1.044112**	**1.011151**
3	N/A	3	2.943	5.0362	3.01	N/A	**3.549999**	**3.285674**	**3.144839**
4	N/A	1	1.92	2.2398	1.75	N/A	**1.924590**	**2.423108**	**2.055534**
5	N/A	1	1.025	1.2226	1.00	N/A	**1.000032**	**1.012567**	**1.048088**
6	N/A	17	14.961	14.9575	14.27	N/A	**15.337079**	**14.966031**	**15.048024**
7	N/A	3	3.074	2.9568	2.93	N/A	**3.108905**	**3.289329**	**3.267696**
8	N/A	7	6.78	10.9038	1.00	N/A	**6.589077**	**6.898785**	**6.564182**
9	N/A	20	18.58	14.4177	1.00	N/A	**16.569661**	**17.443834**	**17.415316**
10	N/A	1	2.415	3.7090	9.38	N/A	**2.553777**	**2.783840**	**3.147816**
11	N/A	8	6.584	5.7076	4.43	N/A	**6.433946**	**6.092343**	**5.894435**

12	N/A	7	6.291	4.9264	4.54	N/A	**5.812166**	**5.695037**	**6.207190**
13	N/A	19	15.383	14.1751	16.41	N/A	**15.836882**	**15.519281**	**15.498274**
14	N/A	2	2.1	1.9043	2.33	N/A	**2.196943**	**2.476988**	**2.427582**
15	N/A	5	6.021	2.8101	7.51	N/A	**4.324553**	**4.362300**	**4.490801**
16	N/A	1	1.022	1.0000	1.00	N/A	**1.000047**	**1.012662**	**1.046294**
17	N/A	22	23.099	18.8070	22.47	N/A	**21.973772**	**21.990211**	**21.864031**
18	N/A	3	2.889	2.6151	2.70	N/A	**2.674909**	**2.955333**	**2.812818**
19	N/A	9	7.96	12.5328	13.58	N/A	**8.722646**	**8.347566**	**8.449910**
20	N/A	1	1.008	1.1314	1.00	N/A	**1.000032**	**1.017148**	**1.103600**
21	N/A	34	28.548	30.5122	28.93	N/A	**29.898613**	**29.342457**	**29.206649**
22	N/A	3	3.349	3.3460	3.23	N/A	**3.249223**	**3.185978**	**3.231316**
23	N/A	19	16.144	17.0450	23.87	N/A	**16.995624**	**16.513879**	**16.527474**
24	N/A	27	24.822	18.0785	41.67	N/A	**25.510407**	**25.126515**	**24.813267**
25	N/A	42	38.401	39.2717	36.02	N/A	**37.634066**	**37.293439**	**37.050391**
26	N/A	1	3.787	2.6062	6.41	N/A	**1.220731**	**1.838838**	**2.464041**
27	N/A	12	12.32	9.8303	23.79	N/A	**11.944077**	**11.691936**	**11.585271**
28	N/A	16	17.036	13.1126	28.39	N/A	**16.515003**	**16.222513**	**15.959013**
29	N/A	19	14.733	13.6897	19.38	N/A	**14.822892**	**14.560158**	**15.062536**
30	N/A	14	15.031	16.9776	20.31	N/A	**15.983565**	**15.683370**	**15.698085**
31	N/A	42	38.597	37.6006	31.41	N/A	**38.514252**	**38.408381**	**38.127149**
32	N/A	4	3.511	3.0602	2.57	N/A	**3.323571**	**3.520728**	**3.398082**
33	N/A	4	2.997	5.5106	4.18	N/A	**3.189674**	**3.041205**	**2.841155**
34	N/A	4	3.06	1.8014	3.33	N/A	**2.822370**	**2.923357**	**2.595424**
35	N/A	1	1.086	1.1568	1.00	N/A	**1.001323**	**1.012537**	**1.021325**
36	N/A	1	1.462	1.2423	1.00	N/A	**1.002606**	**1.012270**	**1.005619**
37	N/A	62	59.433	62.7741	47.11	N/A	**59.530117**	**59.613928**	**59.305347**
38	N/A	3	3.632	3.3276	2.35	N/A	**3.250054**	**3.199754**	**3.124903**
39	N/A	2	1.887	4.2369	3.79	N/A	**2.068093**	**2.121198**	**2.086926**
40	N/A	4	4.072	1.7202	3.30	N/A	**3.084539**	**2.898158**	**3.462332**
41	N/A	1	1.595	1.0148	1.00	N/A	**1.000717**	**1.012534**	**1.032538**
42	N/A	2	3.671	5.4628	1.00	N/A	**1.239938**	**1.217425**	**1.159607**
43	N/A	77	79.511	78.0094	63.33	N/A	**79.891179**	**80.445578**	**80.570210**
44	N/A	3	3.394	3.2206	3.21	N/A	**3.299488**	**3.193447**	**3.202028**
45	N/A	2	1.581	3.5934	4.86	N/A	**1.964128**	**2.011383**	**2.012404**
46	N/A	3	4.204	4.7668	2.22	N/A	**3.489718**	**3.617426**	**3.201326**
47	N/A	2	1.329	1.1531	1.00	N/A	**1.000032**	**1.015906**	**1.082554**
48	N/A	3	2.242	2.1698	1.00	N/A	**1.000032**	**1.012501**	**1.021839**
49	N/A	100	96.886	99.6406	76.93	N/A	**97.181471**	**97.093608**	**97.741588**
50	N/A	4	3.71	4.1469	3.54	N/A	**3.322281**	**3.2531009**	**3.391068**
51	N/A	1	1.055	2.1600	3.91	N/A	**1.002997**	**1.012584**	**1.007845**
52	N/A	4	4.566	4.1499	2.25	N/A	**3.651629**	**3.925643**	**3.732862**
53	N/A	6	9.606	11.2070	11.44	N/A	**7.226228**	**6.877054**	**6.941966**
54	N/A	3	2.984	11.0904	11.64	N/A	**4.544599**	**3.820237**	**3.906502**
55	N/A	49	45.917	35.9499	36.94	N/A	**41.411074**	**40.688316**	**40.215821**
56	N/A	1	1	2.1937	1.00	N/A	**1.002207**	**1.012269**	**1.011849**
57	N/A	62	62.426	66.1705	48.10	N/A	**64.803517**	**65.051505**	**64.863553**
58	N/A	1	2.977	3.3402	5.88	N/A	**2.525618**	**2.418212**	**2.492097**
59	N/A	3	1	4.0525	1.00	N/A	**1.000054**	**1.012579**	**1.038877**

Weight (lb)	170000	143436	141241	142295.75	134120	138878	**137344.356**	**137582.698**	**137549.457**
NSA	N/A	39834	150000	N/A	75000	50000	**58274**	**66482**	**26730**

It appears from Table 3 that JA was the best optimizer also for this test problem. The optimum designs of GA, FFA and JA could not be compared as the referenced solutions are not reported in literature. JA, SA and gradient-based methods converged to feasible optimum designs which are critical with respect to nodal displacements and compressive stresses. The optimum designs of adaptive ES, GNMS and CS instead violate constraints. JA is more robust than CS and FFA: the standard deviation on JA's optimized weights was up to 39 times smaller than for those algorithms (see Table 4). Statistical information were not available for GA, SA, adaptive ES and GNMS.

The new search strategy implemented in JA yield a 0.254% reduction in structural weight with respect to standard JA: from 137694.400 to 137344.356 lb. Similar to the other sizing problems, convergence speed was enhanced using the present JA formulation which required only 58274 structural analyses to complete the optimization process vs. the 72728 analyses of standard JA (i.e. about 20% reduction). The present algorithm was more robust than standard JA: in fact, standard deviation on optimized weight dropped from 320.7 to only 38.346 lb. As mentioned above, selecting candidate designs that lie on descent directions makes it easier for the novel JA algorithm to approach the best regions of design space regardless of the random sequence used for perturbing optimization variables.

JA required less structural analyses than CS and adaptive ES. Furthermore, it found a feasible intermediate design better than the optimized design of SA (i.e. 142893.6 vs. 143436 lb) after only 10421 structural analyses while SA required 39834 analyses to complete the optimization process; a feasible intermediate design weighing only 138491.7 lb (i.e. lighter than the 138878 lb FFA optimum weight) was found after only 21773 structural analyses vs. 50000 analyses required by FFA.

Gradient-based methods obtained slightly heavier (at most 0.174% weight penalty) designs than JA. However, when MFD, SLP and SQP were run as standalone optimizers, optimized weight ranged between 137824 and 138449.1 lb: hence, the corresponding designs were up to 0.804% heavier than the JA's optimum weight. That made it necessary to alternate gradient-based methods by considering the current trend of convergence history. However, this complicated the optimization process with respect to JA runs. It can be seen from Table 3 that combined MFD/SLP was about 14% slower than JA. SQP found the optimum design within only 26730 structural analyses but JA generated a lighter intermediate design than the SQP's optimum design (i.e. 137549.1 vs. 137549.457 lb) after only 40279 analyses.

Table 4. Comparison of statistical performance and constraint violation values for the 942-bar tower problem.

Method	**Best weight** (lb)	**Average weight** (lb)	**Worst weight** (lb)	**SD** (lb)	**CVP** (%)
GA [23]	170000	N/A	N/A	N/A	N/A
SA [36]	143436	N/A	N/A	N/A	None
Adaptive ES [75]	141241	N/A	N/A	N/A	3.05
GNMS [27]	142296	N/A	N/A	N/A	4.35
CS [76]	134120	135244.7	138057.3	1497.06	17.8
FA [59]	138878	139682	142265	1098	N/A
JA	**137344.356**	**137379.616**	**137420.440**	**38.346**	**None**

Fig. 5 compares the convergence curves of JA, FFA, CS and gradient-based methods. The new JA algorithm was definitely faster than the other metaheuristic algorithms (including also standard JA) in finding a nearly optimum solution. The optimization histories of the novel and standard JA became very close after about 25000 structural analyses. SQP soon generated infeasible intermediate designs while JA always searched in feasible regions of design space. Combined MFD/SLP initially had the same convergence rate as JA but, since MFD could not reduce structural weight below 200000 lb, it was necessary to switch the search engine to the SLP method, which exhibited the same oscillatory behavior of SQP. Overall, for this test problem, JA has a more regular convergence behavior than gradient-based optimization methods.

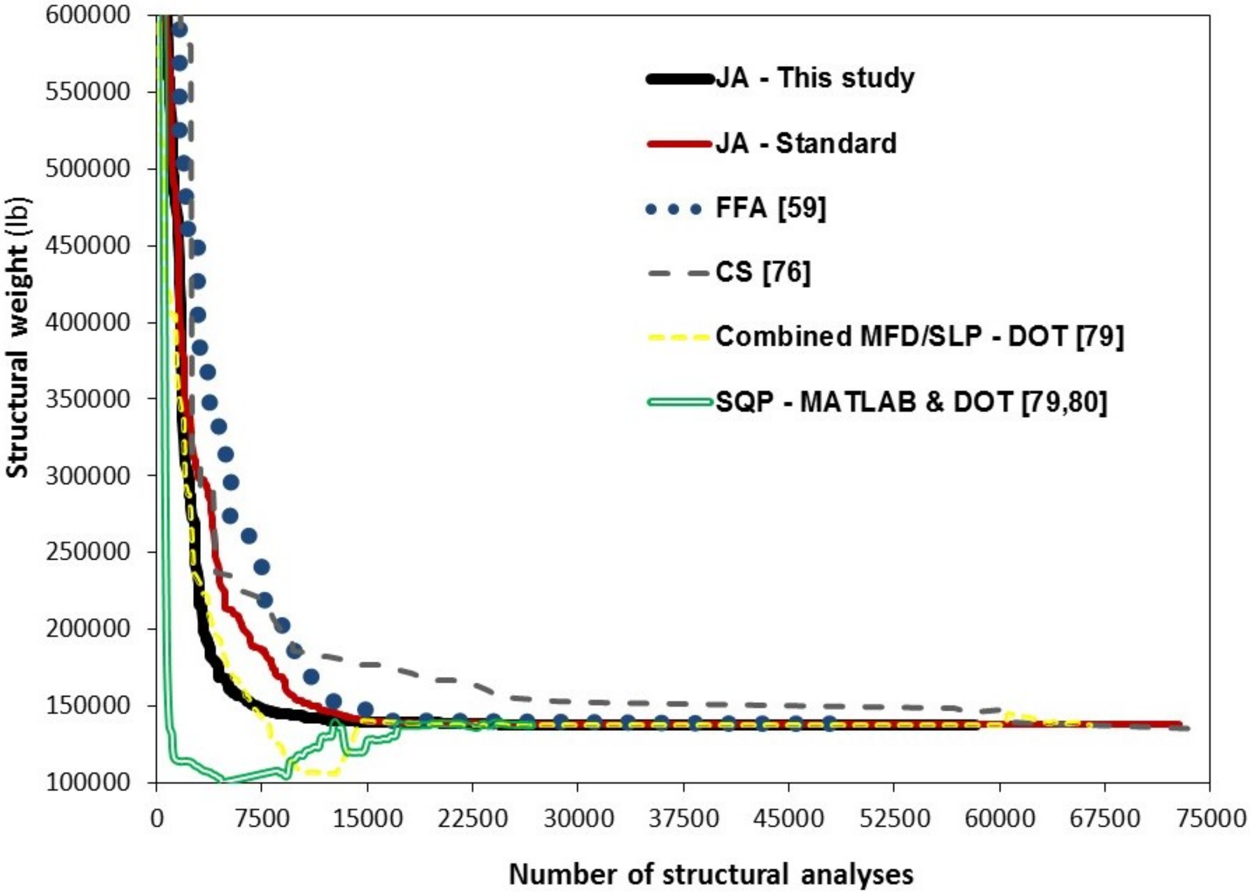


Fig. 5. Comparison of convergence curves for the 942-bar tower problem

4.3. Sizing optimization of a spatial 1938-bar tower

The last sizing optimization problem considered in this study regards the spatial 1938-bar truss tower with 481 nodes shown in Fig. 6. Material properties are the same as for the 942-bar tower. The tower is 285 m tall and is comprised of four modules and two junction modules. From the top to the bottom of the tower there are: (a) a square-based pyramid segment of height 60 m; (b) a square-based prismatic segment of height 40 m and side length 5 m; (c) an octagon-based prismatic segment of height 75 m and radius 5 m; (d) a dodecagon-based prismatic segment of height 100 m and radius 8 m. The two intermediate modules serve to join the adjacent modules that have different profiles. Hence, the layout section of the tower is a regular dodecagon at the ground level and becomes a square at the top segment. Further details of the truss model are given in Fig. S1 of supplementary material.

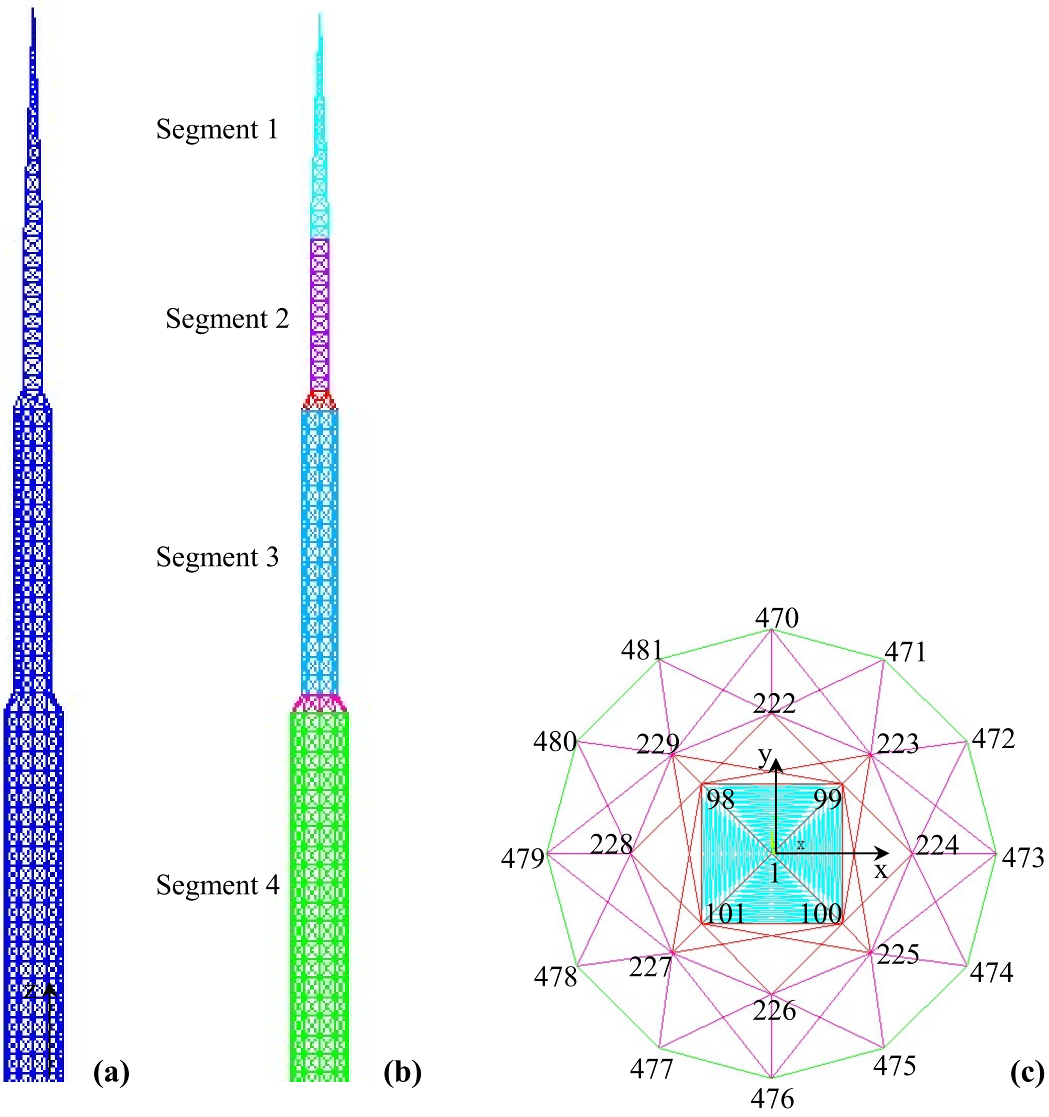


Fig. 6. Schematic of the spatial 1938-bar truss tower: (a) Assembly view of the tower and (b) color representation of the four modules forming the structure and their connecting elements; (c) Layout view of the tower with indication of key-nodes.

Because of the symmetry of the structure, variable linking can be adopted by grouping cross-sectional areas of bars into 204 groups (see Table S1 of supplementary material); nodes, elements and element groups numbering increases from the top to the bottom of the tower. The cross-sectional area of the bars of each group is taken as an optimization variable. Therefore, this large-scale optimization problem includes 204 sizing variables.

The tower must carry three independent loading conditions:

(i) Concentrated forces of 13.5 kN acting downward on nodes 1 through 61; concentrated forces of 27 kN acting downward on nodes 62 through 101; concentrated forces of 40.5 kN acting downward on nodes 102 through 229; concentrated forces of 54 kN acting downward on nodes 230 through 469.

(ii) Concentrated forces of 6.75 kN acting in the positive X-direction on nodes 2, 5, 6, 9, 10, 13, 14, 17, 18, 21, 22, 25, 26, 29, 30, 33, 34, 37, 38, 41, 42, 45, 46, 49, 50, 53, 54, 57, 58, 61, 62, 65, 66, 69 ,70, 81, 82, 85, 86, 89, 90, 93, 94, 97, 98, 101, 108, 116, 124, 132, 140, 148, 156, 164, 172, 180, 188, 196, 204, 220, 228, 239, 251, 263, 275, 287, 299, 311, 323, 335, 347, 359, 371, 383, 395, 407, 419, 431, 443, 455, 467; concentrated forces of 4.448 kN acting in the negative X-direction on nodes 3, 4, 7, 8, 11, 12, 15, 16, 19, 20, 23, 24, 27, 28, 31, 32, 35, 36, 39, 40, 43, 44, 47, 48, 51, 52, 55, 56, 59,60, 63, 64, 67, 68, 71, 72, 75, 76, 79, 80, 83, 84, 87, 88, 91, 92, 95, 96, 99, 100, 104, 112, 120, 128, 136, 144, 152, 160, 168, 176, 184, 192, 200, 208, 216, 224, 233, 245, 257, 269, 281, 293, 305, 317, 329, 341, 353, 365, 377, 389, 401, 413, 425, 437, 449, 461.

(iii) Concentrated forces of 4.45 kN acting in the negative Y-direction on nodes 2, 3, 6, 7, 10, 11, 14, 15, 18, 19, 22, 23, 26, 27, 30, 31, 34, 35, 38, 39, 42, 43, 46, 47, 50, 51, 54, 55, 58, 59, 62, 63, 66, 67, 70, 71, 74, 75, 78, 79, 82, 83, 86, 87, 90, 91, 94, 95, 98, 99, 102, 110, 118, 126, 134, 142, 150, 158, 166, 174, 182, 190, 198, 206, 214, 222, 230, 242, 266, 278, 290, 302, 314, 326, 338, 350, 362, 374, 386, 398, 410, 422, 434, 446, 458; concentrated forces of 4.48 kN acting in the positive Y-direction on nodes 4, 5, 8, 9, 12, 13, 16, 17, 20, 21, 24, 25, 28, 29, 32, 33, 36, 37, 40, 41, 44, 45, 48, 49, 52, 53, 56, 57, 60, 61, 64, 65, 68, 69, 72, 73, 76, 77, 80, 81, 84, 85, 88, 89, 92, 93, 96, 97, 100, 101, 106, 114, 122, 130, 138, 146, 154, 162, 170, 178, 186, 194, 202, 210, 218, 226, 236, 248, 260, 272, 284, 296, 308, 320, 332, 344, 356, 368, 380, 392, 404, 416, 428, 440, 452, 464.

The optimization problem includes 20070 non-linear constraints on nodal displacements, member stresses and critical buckling loads. Displacements of all free nodes in all coordinate directions X, Y and Z must be less than ±40.64 cm (i.e. ±16 in), that is 1/700 of the height of the tower. The allowable tensile stress is 275.9 MPa (i.e. 40000 psi). Structural members are assumed to be tubular and the critical buckling load of the *j-th* member is $-100.01\pi EA_j/8l_j^2$. Cross-sectional areas can vary between 0.1 and 200 in^2.

The weight of the tower was minimized with JA, multilevel/multipoint SA [37], hybrid HS and BBBC with line search (HHS-LS and HBBBC-LS) derived from [48,77], firefly algorithm including line search derived from [78] (FFA-LS), the SLP routine of DOT [79], and the SQP routines of MATLAB [80] and DOT which were alternatively used in order to maximize convergence speed. HHS-LS and HBBBC-LS and FFA-LS were run with *np*=20 since also these algorithms found their best designs for the same population size as JA. Twenty independent runs were carried out for each optimizer in order to statistically compare search engines. Simulated annealing (this metaheuristic algorithm does not use a population in the strict sense) and gradient-based optimizations were started from the center of mass of the initial population that led to the best design. This strategy allowed us to model the average behavior of the population that finally resulted in the best design.

The optimization results found by the different algorithms are compared in Table 5 while convergence curves are shown in Fig. 7. It can be seen that JA found the best design corresponding to a structural weight of 99.255 ton. This design satisfies optimization constraints. The second best feasible design was found by SLP-DOT and weighted 102.789 ton. Because of the computational complexity of this test problem, all other optimizers converged to almost feasible designs that violate constraints between 0.270 and 0.453%: hence, the corresponding scaled weight to recover constraint violation ranges between 101.173 and 102.982 ton.

Table 5. Comparison of the optimization results for the 1938-bar tower problem.

Method	Best weight (ton)	Average weight (ton)	Worst weight (ton)	SD (ton)	NSA	CVP (%)
JA	**99.255**	**99.257**	**99.259**	**0.00175**	**20051**	**None**
HHS-LS Derived from [48,77]	101.247	101.334	101.495	0.140	11399	0.300
HBBBC-LS Derived from [48,77]	100.901	101.083	101.256	0.178	18340	0.270
CMLPSA Derived from [37]	102.518	N/A	N/A	N/A	14057	0.453
FFA-LS Derived from [78]	100.502	100.778	100.959	0.243	15250	0.200
SLP-DOT [79]	102.789	N/A	N/A	N/A	12310	None
SQP [79,80] MATLAB/DOT	101.495	N/A	N/A	N/A	24042	0.258

Table 5 shows also that JA is much more robust than the other population-based metaheuristic algorithms also in this large-scale problem. In particular, the standard deviation on optimized weight determined for JA is about two orders of magnitude smaller than for the other metaheuristic optimizers. The superiority of the present algorithm is demonstrated by the fact that the worst weight obtained by JA in the twenty independent optimization runs (i.e. 99.259 ton) is lower than the corresponding best weights of HHS-LS, HBBBC-LS and FFA-LS (ranging between 100.502 and 101.247 ton).

Since the 1938-bar truss problem included 204 design variables, the different statistical dispersions of optimized weight observed over the independent optimization runs for each algorithm may be due to design space complexity rather than to the search criterion implemented by the algorithm formulation. For this reason, the optimized weights obtained by

JA, HHS-LS, HBBBC-LS and FFA-LS in the twenty independent runs were also analyzed by performing a Wilcoxon test with a level of significance of 0.05. This test gives p-value stating whether a statistical difference is significant or not: the smaller the p-value is, the greater the difference between the considered algorithms will be. Interestingly, p-values determined for each pair of solutions (i.e. JA vs. HHS-LS, JA vs. HBBBC-LS and JA vs. FFA-LS, respectively) always were smaller than 0.05, thus confirming the superiority of JA over the other algorithms. However, performing correlation tests was not strictly necessary because JA proved to be the only metaheuristic algorithm always able to find feasible designs and the worst JA's design was better than the best designs of the other algorithms.

Distribution of optimized cross-sectional areas is practically the same for all algorithms (see Fig. S2 of supplementary material): areas of bars increase from the top to the bottom of each segment and drop down at the transition between adjacent segments (i.e. assembly of top segments 1 & 2, central segment 3, bottom segment 4 in Fig. 6). JA and the gradient-based optimizers size almost the same set of design variables to their lower bound of 0.1 in^2 while the other metaheuristic optimizers tend to increase cross-sectional areas of thinnest elements and reduce cross-sectional areas of largest elements. However, the large number of design variables finally allowed to obtain very similar structural weights.

JA required 20051 structural analyses to complete the optimization process, which is between 1.33 and 2 times as large as the number of analyses required by CMLPSA, FFA-LS, HHS-LS and HBBBC-LS. However, the number of structural analyses for which the present algorithm found some feasible intermediate design lighter than the unfeasible optimized designs found by CMLPSA, FFA-LS, HHS-LS and HBBBC-LS ranges between 15098 and 15222. JA was slower than SLP-DOT but found a 3.44% lighter design than the commercial optimizer. The number of structural analyses for which JA found the same optimum weight of SLP-DOT is 15075, close enough to the 12310 analyses required by DOT.

More information on the relative merits of different algorithms can be gathered from the convergence curves plotted in Fig. 7. JA reduced very quickly the structural weight and found the lightest intermediate designs (which were always feasible) amongst all optimizers in the first 10000 analyses. The moving-toward-best/escaping-from-worst search strategy of JA appears to be more efficient than the line search strategies implemented in HHS-LS, HBBBC-LS, CMLPSA and FFA-LS. Basically, JA tries to perturb each element of the population while the other algorithms are more “global” as they combine simultaneously all optimization variables to form a set of descent directions. Since such a set must not be too large in order to save computation time, HHS-LS, HBBBC-LS, CMLPSA and FFA-LS may miss promising designs. Furthermore, the other metaheuristic optimizers reduce variable step sizes by the ratio between cost function sensitivities and cost function gradient module: this may further slow down the search of very efficient trial designs.

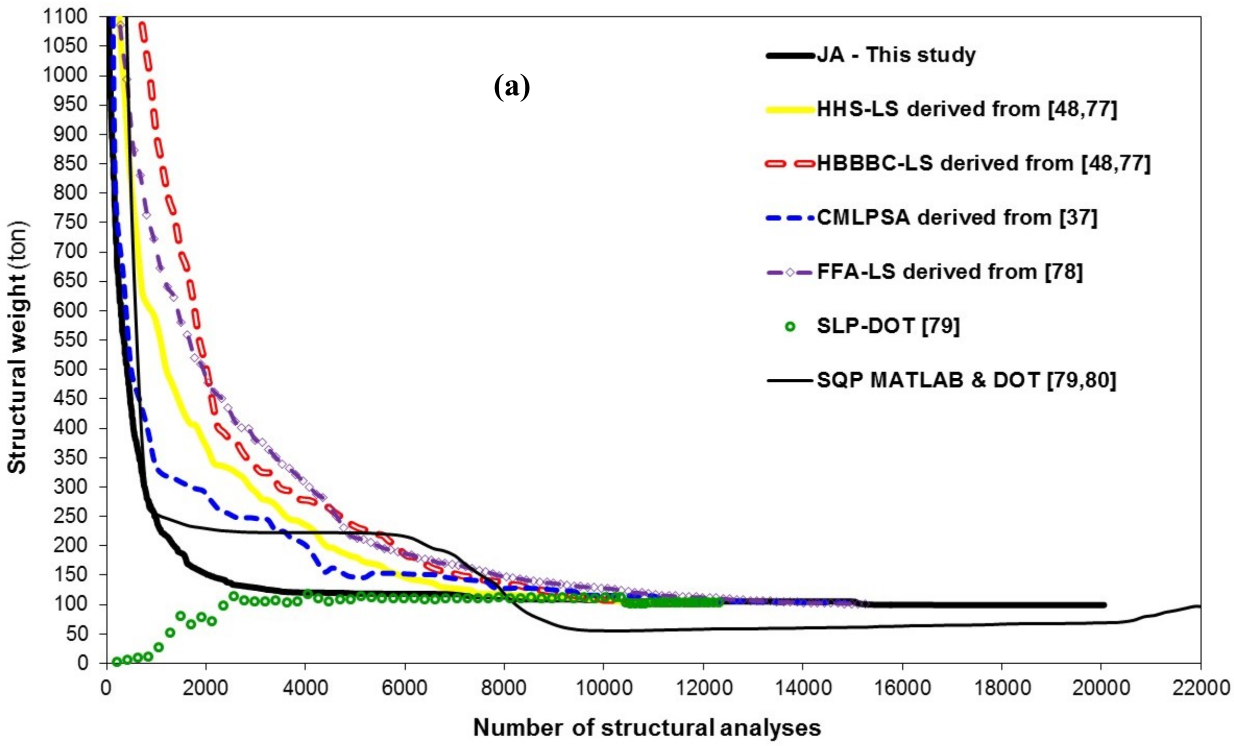
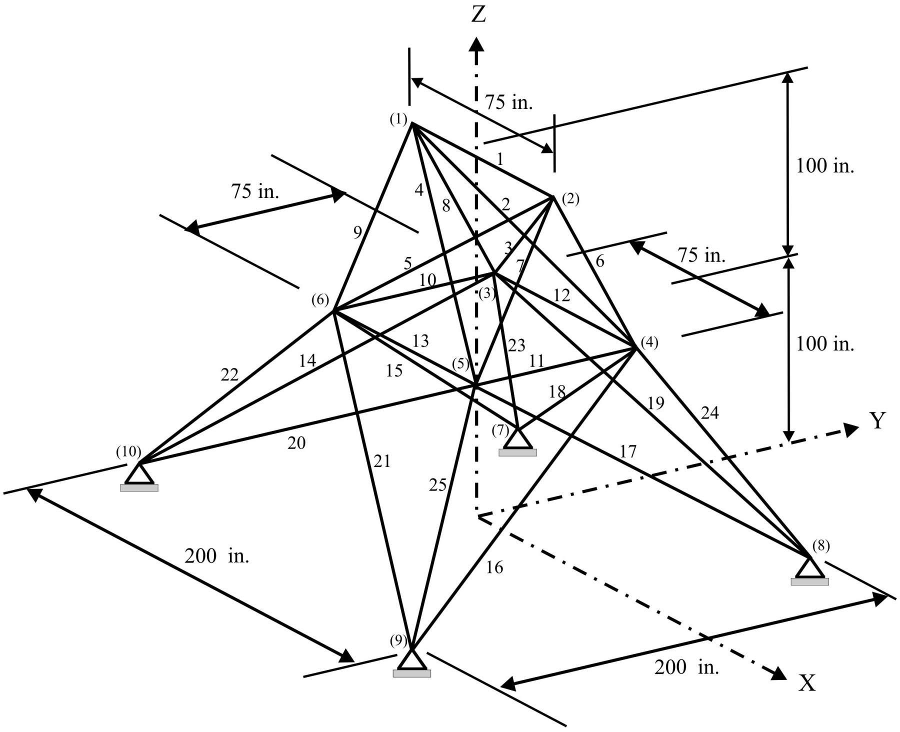


Fig. 7. Convergence curves for the 1938-bar tower problem.

As far as it concerns gradient-based optimizers, Fig. 7 shows that SLP-DOT initially reduced cross-sectional areas of all elements and explored for a while the infeasible space: its convergence curve crossed that of JA after about 7500 structural analyses but SLP-DOT could find feasible designs only after 9000 analyses, that is after about 75% of its optimization history. After 10250 analyses, the commercial optimizer obtained a weight of 110.825 ton vs. only 106.005 ton obtained by JA; the present algorithm obtained a feasible design weighing 110.451 ton after only 8085 analyses. SLP-DOT later converged to a local minimum while JA found the global minimum. SQP got trapped near a local minimum weighing about 220 ton for 2800 analyses (about 14 SQP iterations), then reduced the weight between 6000 and 9000 analyses but generated infeasible designs and could re-enter in the feasible design space only after 22000 structural analyses (hence, similar to SLP-DOT, after about 90% of the optimization history) were completed. The above presented data lead to conclude that JA exhibited a more regular convergence behavior than SLP and SQP. This is due to the fact that sometimes the gradient-based optimizers could not efficiently solve the approximate sub-problems formed in each iteration because of the large number of optimization variables.

4.4. Combined sizing-layout optimization of a spatial 25-bar transmission tower

The first sizing-layout optimization problem solved in this study regards the spatial 25-bar transmission tower schematized in Fig. 8. The Young's modulus and mass density are 10 Msi and 0.1 lb/in^3, respectively. The structure is optimized with 8 sizing variables (the cross-sectional areas of the element groups listed in Fig. 8) and 5 layout variables, the coordinates X_4, Y_4, Z_4, X_8 and Y_8 of nodes 4 and 8. Hence, this test problem includes 13 optimization variables. Cross-sectional areas must be greater than 0.1 in^2. Side constraints for geometry variables are set as follows: $20 \leq X_4 = X_5 = -X_3 = -X_6 \leq 60$ in, $40 \leq Y_3 = Y_4 = -Y_5 = -Y_6 \leq 80$ in, $90 \leq Z_3 = Z_4 = Z_5 = Z_6 \leq 130$ in, $40 \leq X_8 = X_9 = -X_7 = -X_{10} \leq 80$ in, $100 \leq Y_7 = Y_8 = -Y_9 = -Y_{10} \leq 140$ in.



Element grouping
(1) 1
(2) 2,3,4,5
(3) 6,7,8,9
(4) 10,11
(5) 12,13
(6) 14,15,16,17
(7) 18,19,20,21
(8) 22,23,24,25

Fig. 8. Schematic of the spatial 25-bar tower and element grouping used for sizing variables

The tower is loaded by concentrated forces comprised in a single loading condition: 1 kip in the positive x-direction, 10 kips in the negative y and z-directions applied to node 1; 10 kips in the negative y and z-directions applied to node 2; 0.5 kip in the positive x-direction applied to node 3; 0.6 kip in the positive x-direction applied to node 6. The stress limit in tension/compression is 40000 psi for all members while the displacement of nodes of the structure in all coordinate directions must be less than ±0.35 in.

This test problem was solved in literature using many metaheuristic algorithms such as, for example, HS variants [81-83], PSO variants also hybridized with cellular automata [43,44,84,85], ES variants [86,87], FFA [59,83], TLBO [88], GA [25] etc. Those solutions include discrete sizing variables and continuous layout variables. However, the former variables took very few values even if large sets of available cross-sections were used.

JA was compared with multilevel and multipoint SA derived from [37], HHS-LS and HBBBC-LS derived from [48,77], firefly algorithm including line search derived from [78], as well as with the best discrete optimized solutions mentioned above. Optimization results are presented in Table 6. It should be noted that some other discrete solutions available in literature corresponded to even lighter designs than those reported in the table but were penalized as soon as they violated constraints (for example, up to 0.18% on stresses and 0.62% on displacements for PSO). Table 6 proves that JA definitely obtained the best design with continuous variables: in fact, multilevel and multipoint SA, HS and FFA with line search converged to heavier designs (between 2.37% and 3.39%) than JA; BBBC with line search was the least efficient optimizer and violated displacement constraints by 0.2%.

The statistical analysis carried out for this test problem revealed that JA always found in the twenty independent runs carried out for each population size feasible designs weighing between 53.049 and 53.051 kg. Remarkably, the worst design found by JA was lighter than all best designs found by the other metaheuristic algorithms that even converged to infeasible solutions. This confirms with no shadow of doubt the superiority of JA.

The optimized cross-sectional areas of JA are very close to the best discrete solutions available in literature. These designs weigh between 53.186 (best design reported in the table) and 53.9 kg [59], hence at most 1.6% heavier than the optimized weight of 53.049 kg found by JA for the continuous design. It should be noted that the lightest design reported in literature (53.173 kg [85]) violates displacement constraints by 0.62% and hence was not considered for the comparison based on Table 6. However, this design is very similar to those of Table 6 thus confirming that sizing process depends on very few cross-sectional areas.

Table 6. Comparison of the optimized designs for the layout optimization of 25-bar tower

Design Variables	SA [37]	HHS-LS [48,77]	HBBBC-LS [48,77]	FFA-LS [78]	**JA This study**	Modified HS* [82]	FFA* [83]	iPSO* [43,44]	ICDE* [31]	TLBO* [88]	**JA** This study**
A_1 (in^2)	0.1246	0.1041	0.1049	0.1223	**0.1000**	0.1	0.1	0.1	0.1	0.1	**0.1**
A_2	0.1251	0.1189	0.1274	0.1197	**0.1000**	0.1	0.1	0.1	0.1	0.1	**0.1**
A_3	0.9462	0.9156	0.9090	0.8684	**0.9374**	1.0	1.0	1.0	0.9	1.0	**1.0**
A_4	0.1001	0.1028	0.1038	0.1007	**0.1000**	0.1	0.1	0.1	0.1	0.1	**0.1**
A_5	0.1093	0.1424	0.1006	0.1009	**0.1000**	0.1	0.1	0.1	0.1	0.1	**0.1**
A_6	0.1137	0.1192	0.1128	0.1160	**0.1000**	0.1	0.1	0.1	0.1	0.1	**0.1**
A_7	0.1407	0.1405	0.1484	0.2392	**0.1057**	0.1	0.1	0.1	0.1	0.1	**0.1**
A_8	0.9094	0.9254	0.9392	0.8280	**0.9219**	0.9	0.9	0.9	1.0	0.9	**0.9**
X_4 (in)	33.245	34.161	33.556	31.565	**37.801**	37.820	37.401	37.60	36.83	37.657	**37.107**
Y_4	57.016	62.049	61.749	56.007	**55.063**	55.485	55.379	54.46	58.53	54.496	**54.255**
Z_4	125.645	119.690	119.176	129.824	**129.998**	128.730	129.290	130.00	122.67	130.000	**129.998**
X_8	44.745	44.006	42.825	41.620	**51.023**	52.068	51.807	51.89	49.21	51.887	**52.008**
Y_8	136.458	136.921	136.160	139.939	**140.000**	139.590	139.560	139.55	136.74	139.521	**140.000**
Weight (kg)	54.535	54.847	54.820	54.305	**53.049**	53.243	53.229	53.186	53.869	53.187	**53.219**
CVP (%)	None	None	0.2	None	**None**	0.0826	0.0417	None	0.266	0.114	**None**
NSA	3981	3338	3734	4076	**3097**	5000	10000	4870	6000	50007	**3795**

*Cross-sectional areas of bars treated as discrete variables; **Two-stage optimization with discrete bar areas.

The continuous optimum design of JA was re-optimized only with respect to layout variables: sizing variables were hence set equal to the optimum discrete values. After this two-stage optimization process, JA converged to a feasible design weighing 53.219 kg, practically the same as the best weight of 53.186 kg found by iPSO [43,44]. However, JA required only 3795 structural analyses vs. 4870 analyses required by iPSO.

Since JA's discrete solution was obtained by progressively rounding continuous solution and JA always converged to the same continuous optimum, the present algorithm always found the discrete optimum design quoted in Table 6 regardless of population size and number of independent runs. This is a significant improvement with respect to the modified harmony search algorithm of Ref. [82], the firefly optimization algorithms of Refs. [58,83], the particle swarm optimization hybridized with cellular automata of Ref. [85], and the teaching-learning based optimization algorithm of Ref. [88], which obtained standard deviations on optimized weight of 6.970, 3.301, 1.175, 0.656 and 0.587 kg, respectively.

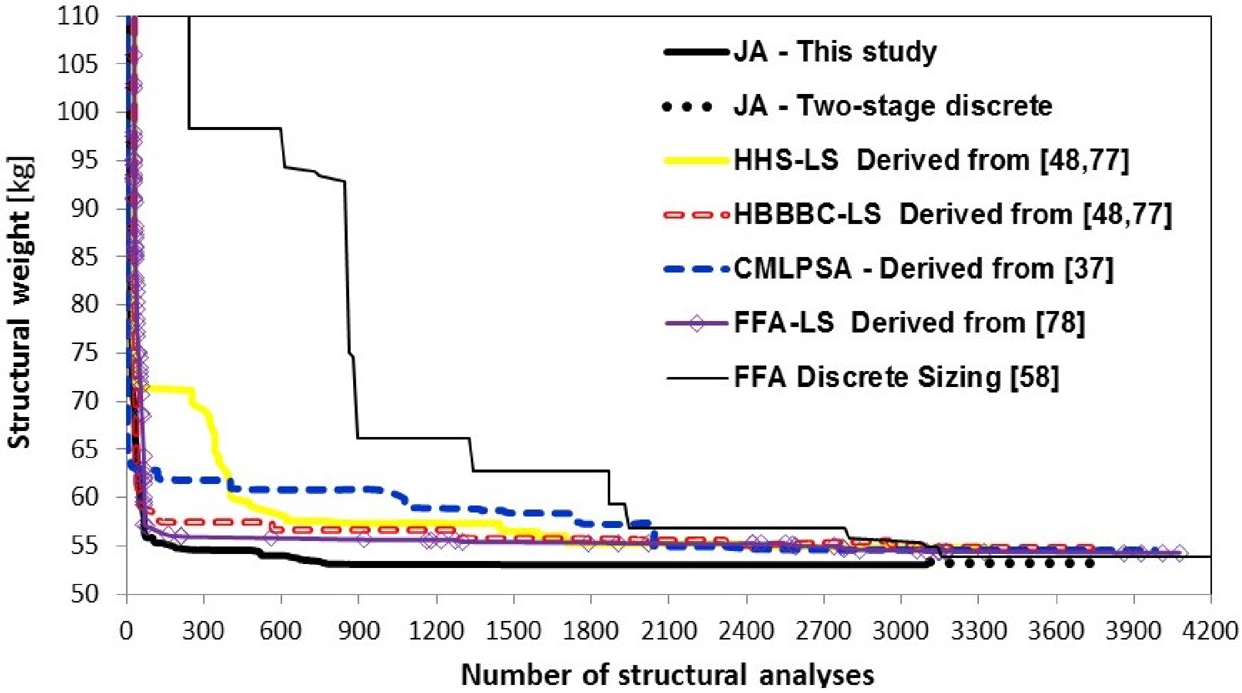


Fig. 9. Convergence curves for the sizing-layout optimization of the 25-bar tower.

Fig. 9 compares the convergence curves of JA with other continuous solutions (multilevel/multipoint SA, HS, BBBC and FFA with line search) and discrete FFA [58], which required over 6000 structural analyses starting from a population whose best design is close to that included in the JA initial population. JA was by a very large extent the fastest optimizer overall. It required about 700 structural analyses to find feasible intermediate designs lighter than the optimized designs of discrete HS, PSO, FFA and TLBO between 4870 and 50007 analyses. The convergence curve for the last part of the JA two-stage optimization process including rounded sizing variables always remains very close to the continuous optimum design. Since JA proved itself to be very robust also in terms of required structural analyses (NSA=3332±204, see Table A3 of appendix), it can be concluded that the present algorithm outperformed the other state-of-the-art optimizers in this test problem.

4.5. Combined sizing-layout optimization of a planar 45-bar truss

The second sizing-layout optimization problem solved in this study regards the planar 45-bar truss schematized in Fig. 10. The Young's modulus and mass density are 10 Msi and 0.1 lb/in^3, respectively. Cross sectional areas of all elements are included as sizing variables. In addition, there are 36 layout variables: the nodal coordinates of all free nodes. Therefore, this test case includes 81 design variables. The structure is loaded by vertical forces: respectively, 150 kips (i.e. 667.461 kN) at nodes 9 and 10 acting downward, and 50 kips (i.e. 222.487 kN) at nodes 11 and 12 acting upward. There are 162 non-linear optimization constraints on nodal displacements, member stresses and critical buckling loads. The displacements of all free nodes in both coordinate directions X and Y must be less than ±2 in (i.e. ±5.08 cm). The allowable tensile stress is 25000 psi. Buckling constraints are the same as for the 1938-bar tower. The lower bound of cross-sectional areas is 0.64516 cm^2 (i.e. 0.1 in^2) while the upper bound is 1290.32 cm^2 (i.e. 200 in^2). More details on this test case can be found in [37].

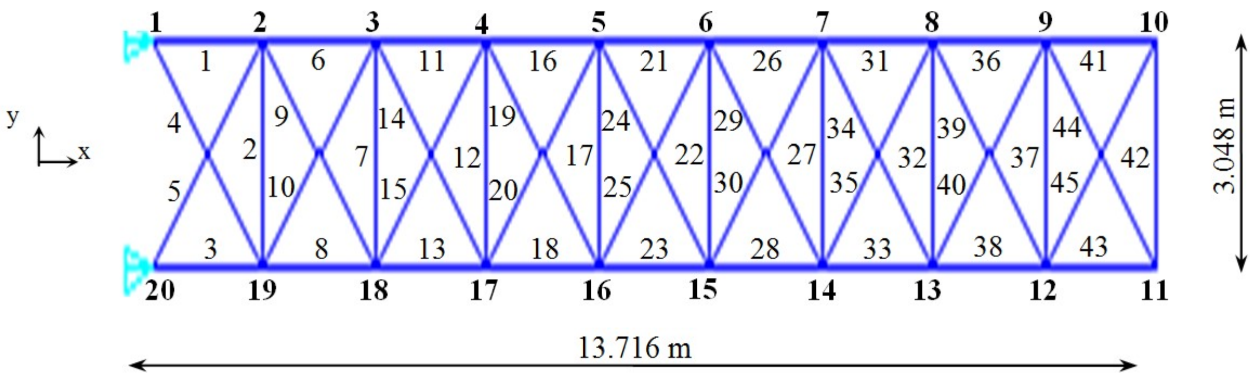


Fig. 10. Schematic of the planar 45-bar truss structure optimized in this study

JA was compared with multilevel and multipoint SA derived from [37], HHS-LS and HBBBC-LS derived from [48,77], FFA-LS derived from [78], and the SQP optimization routine implemented in MATLAB. The optimization runs of HHS-LS and HBBBC-LS were carried out for the population size range (i.e. np=10 to 1000) considered in the sensitivity analysis of JA in order to make a detailed statistical comparison not limited to best designs. The FFA-LS algorithm implemented an adaptive population strategy where population size (always ranging between 10 and 1000) is updated during the optimization process based on the trend of convergence speed: if the design does not improve after 100 consecutive structural analyses, the current population size np_{current} is expanded by a factor λ equal to $\mathrm{Max}\{1+\eta; \eta\times(1000/np_{\mathrm{current}})\}$ where η is a random number in the interval (0,1). The new population size np_{new} is set as the nearest integer to the product ($\lambda\times np_{\mathrm{current}}$). The population size is reset as np_{current}=Nearest integer$\{(np_{\mathrm{new}}+np_{\mathrm{current}})/2\}$ as soon as some trial design has improved the current best record. This approach allowed to maximize performance of FFA-LS for this specific test problem. The initial design selected for SA and MATLAB was the center of mass of the population leading to the best weight of 3193.568 kg found in the JA sensitivity analysis (i.e. np=500, see Table A3).

Optimization results are presented in Table 7 which contains the following information: (i) results of best and worst runs of JA, HS and BBBC with line search; (ii) data relative to optimization runs completed within the largest or smallest number of structural analyses; (iii) CVP (%) for relevant optimization runs and, eventually, the scaled weight obtained by relaxing cross-sectional areas by the amount of constraint violation. The values of standard deviation listed in the table are those obtained by varying population size from 10 to 1000. Dispersions of optimized weight and number of structural analyses were on average one order of magnitude larger than those observed for the independent optimization runs carried out at fixed population sizes. This proves once again the robustness of JA, which is marginally sensitive to the sequence of random numbers utilized to create/update population.

Table 7. Statistical optimization results obtained for the 45-bar truss problem.

Method	**Weight±Std** *Best/Worst/ Fastest/Slowest* (kg)	**NSA±Std** *Best/Worst/ Fastest/Slowest*	**CVP** (%) Scaled weight (kg)
JA - Present	**3193.776±0.169** ***Best*: 3193.568** ***Worst*: 3194.080** ***Fastest*: 3193.677** ***Slowest*: 3193.577**	**5265±633** ***Best*: 5717** ***Worst*: 4768** ***Fastest*: 4323** ***Slowest*: 6061**	**None**
HHS-LS Derived from [48,77]	3194.902±2.740 *Best*: 3191.283 *Worst*: 3199.321 *Fastest*: 3196.255 *Slowest* ≡ *Worst*	4933±351 *Best*: 5130 *Worst*: 5344 *Fastest*: 4252 *Slowest*	0.130% on the best design Scaled weight: 3195.432 kg
HBBBC-LS Derived from [48,77]	3196.176±2.826 *Best*: 3190.699 *Worst*: 3198.855 *Fastest*: 3194.365 *Slowest≡Best*	4077±602 *Best*: 5196 *Worst*: 3829 *Fastest*: 3383 *Slowest≡Best*	0.120% on the best design Scaled weight: 3194.497 kg
CMLPSA Derived from [37]	3193.839	4535	0.0820% on the best design Scaled weight: 3196.458 kg
FFA-LS Derived from [78]	3193.398	5753	0.259% on the best design Scaled weight: 3201.669 kg
SQP-MATLAB	3245.480	11911	None

It can be seen from the table that JA was overall the most efficient optimizer: it converged to the lightest feasible design corresponding to a structural weight of 3193.568 kg. HHS-LS, HBBBC-LS and FFA-LS found optimized designs weighing between 3190.699 and 3193.398 kg. These designs are lighter than the optimized weight of JA, but slightly violate constraints (between 0.12 and 0.259%) and must hence be scaled to become feasible. CMLPSA also converged to an almost feasible design yet 0.271 kg heavier than JA. The scaled optimized weight for HS, BBBC, SA and FFA ranges between 3194.497 and 3201.669 kg, heavier than the worst weight of 3194.080 kg found by JA for the different population sizes or independent optimization runs (for details, see Table 7 of main body and Table A3 of the appendix). MATLAB converged to a feasible design, 1.6% heavier than that found by JA. These results confirm that the convergence behavior of JA does not depend on the sequence of random numbers generated in the search process to form new trial designs.

The complete optimum designs for each optimization algorithm will not be listed in the article in order to save space. Optimized profiles are similar to those shown in Fig. S4-a of supplementary material. Values of cross-sectional areas followed the classical distribution of a tapered beam where lighter members are located near the loaded tip of the structure.

The statistical data of Table 7 indicate that JA was more robust than HHS-LS and HBBBC-LS but required on average more structural analyses than the other metaheuristic algorithms. However, JA found feasible intermediate designs weighing: (i) 3201.42 kg (better than 3201.7 kg, the scaled weight of FFA-LS) after only 4522 structural analyses (vs. 5753 analyses required by FFA-LS to complete the optimization process); (ii) 3196.28 kg (better than 3196.46 kg, the scaled weight of CMLPSA) after 4636 structural analyses (very close to 4535 analyses required by CMLPSA); (iii) 3194.63 kg (better than 3195.43 kg, the scaled weight of HHS-LS) after 4751 structural analyses (vs. 5130 analyses required by HHS-LS); (iv) 3194.19 kg (better than 3194.5 kg, the scaled weight of HBBBC-LS) after 4901 structural analyses (vs. 5196 analyses required by HBBBC-LS). SQP-MATLAB required more than two times the structural analyses of JA.

Another interesting comparison regards the fastest optimization run (see Table 7). JA found better designs than HHS-LS and HBBBC-LS with structural weight 3193.677 kg vs.

3196.255 and 3194.365 kg, respectively. However, the HBBBC-LS optimum design violated stress constraints by 0.065% and was hence scaled to 3196.44 kg. JA obtained a feasible intermediate design weighing 3196.04 kg after 3743 structural analyses, about 20% less than HHS-LS and only 11% more than HBBBC-HS. All of these data prove the very good behavior in terms of converge speed exhibited by JA.

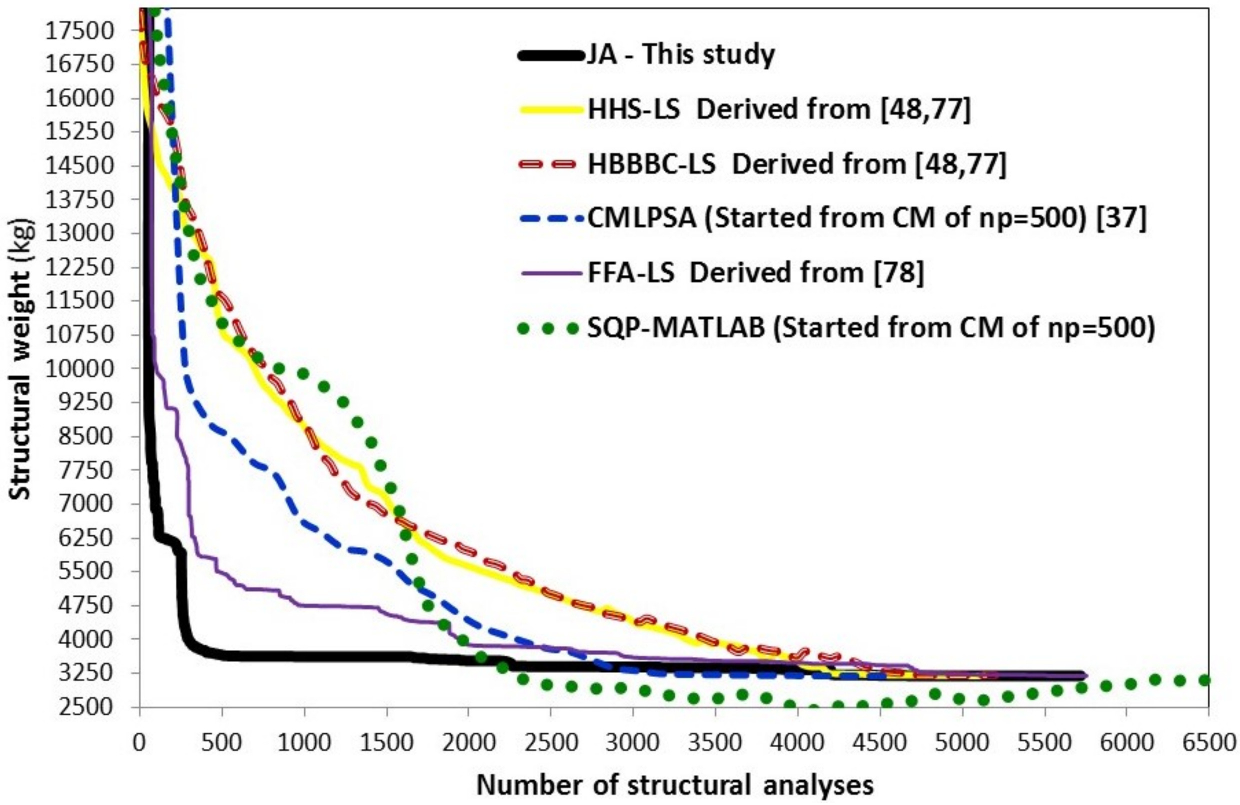


Fig. 11. Comparison of convergence curves for the 45-bar truss problem

The convergence curves for the best optimization runs are compared in Fig. 11, which includes the first 6500 structural analyses for clarity of representation. JA quickly approached the optimum design: in particular, after only 2500 structural analyses, JA had already generated an intermediate feasible solution which is only 6.1% heavier than the final optimum. MATLAB started to generate infeasible designs after 2100 structural analyses and re-entered in the feasible search space only near the end of the optimization process.

The best optimization runs of HHS-LS and HBBBC-LS practically showed the same convergence behavior over the whole optimization process. FFA-LS was the second fastest optimizer in the first half of the optimization history but finally required more structural analyses than the other metaheuristic optimizers. Convergence curves of JA and CMLPSA became very close after about 3000 structural analyses but while the present optimizer always remained in the feasible search space the optimizer derived from [37] had to slightly relax stress and displacement constraints to create efficient trial designs.

4.6. Combined sizing-layout optimization of a 47-bar power line structure

The power line shown in Fig. 12 was schematized as a planar truss comprised of 47 elements and 22 nodes. The Young's modulus and mass density are 30 Msi and 0.3 lb/in^3, respectively. The structure is symmetric about the Y-axis. The bars are grouped into 27 groups (see Fig. 12) thus defining 27 sizing variables. In addition, there are 17 configuration variables: the nodal co-ordinates (X_2, X_4, Y_4, X_6, Y_6, X_8, Y_8, X_{10}, Y_{10}, X_{12}, Y_{12}, X_{14}, Y_{14}, X_{20}, Y_{20}, X_{21} and Y_{21}) to be optimized must preserve symmetry about the Y-axis in each design cycle. Therefore, a total of 44 optimization variables were defined for this combined sizing-layout problem.

The structure must support three independent loading conditions: (i) concentrated forces of 6 kips at nodes 17 and 22 acting in the positive X-direction, and 14 kips at nodes 17 and 22 acting in the negative Y-direction; (ii) concentrated forces of 6 kips at node 17 acting in the positive X-direction, and 14 kips at node 17 acting in the negative Y-direction; (iii) concentrated forces of 6 kips at node 22 acting in the positive X-direction, and 14 kips at node 22 acting in the negative Y-direction.

The optimization problem included 282 constraints: the allowable tensile stress is 20000 psi; the allowable compressive stress is −15000 psi; the Euler buckling compressive stress limit of the *j-th* member is $-3.96 \cdot EA_j/l_j^2$. More details on this problem are given in [89-92].

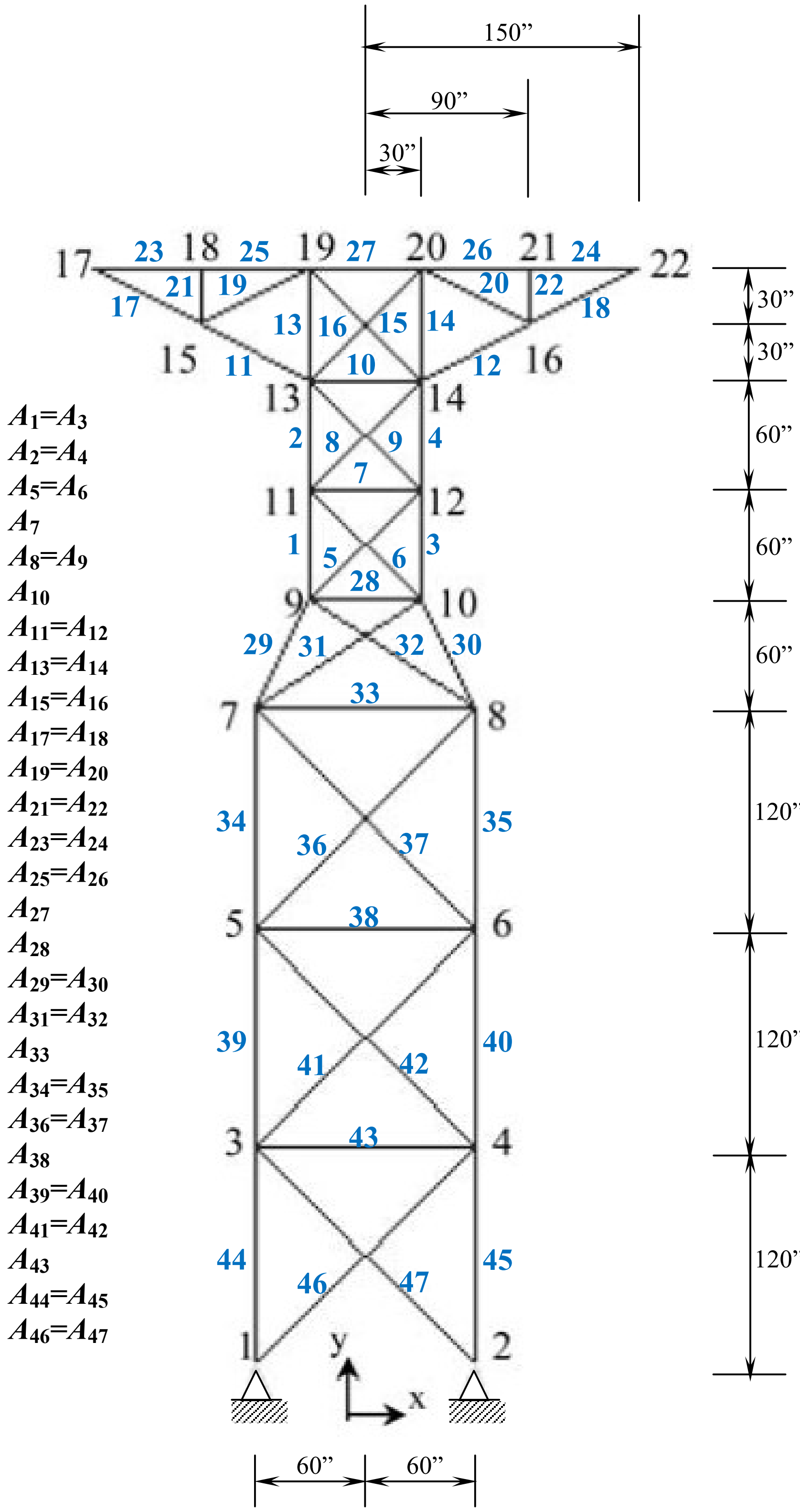


Fig. 12. Schematic of the 47-bar power line and linking of cross-sectional areas used in the optimization process.

This problem was solved in literature considering either continuous [89-92] or discrete sizing variables [28,81,84-87,93]. Table 8 compares the continuous optimization results obtained by JA, multi-level and multi-point SA (derived from [37]), HS, BBBC, FFA with line-search (derived from [48,77,78]) and SQP-MATLAB. Optimization runs of SA and MATLAB were started from the center of mass of the population (*np*=1000) adopted by JA in its best optimization run. Since the optimum design found by MATLAB is between 4 and 6 kg lighter than the optimized weights quoted in [89-92], SQP was selected as the best gradient-based optimizer for this problem. The table also compares the best discrete solutions available in literature − fully stressed design based on evolution strategy (FSD-ES) [87], PSO hybridized with cellular automata (SCPSO) [85] and efficient simulated annealing [36] − with the corresponding solution found by JA using the two-stage optimization strategy where sizing variables optimized in the continuous optimization are rounded up and layout variables are further optimized. The structural weight for the discrete solution quoted in [84] is smaller than the weights reported [28,85,87] (i.e. 816.605 kg vs. 837.567 to 848.988 kg) but this design violates optimization constraints and hence will not be considered here.

It can be seen from Table 8 that JA outperformed the other metaheuristic algorithms in the continuous optimization problem. In fact, the present algorithm designed the lightest structure (833.006 kg) and required the smallest number of structural analyses (4680). All metaheuristic optimizers converged to very similar solutions: structural weight changed by less than 0.1%, sizing variables by less 11.5% (except A_8 and A_{13} for SA, and A_{27} for FFA-LS) and layout variables by less than 12.2% (except X_{20}) with respect to the best design of JA.

Each metaheuristic algorithm was run twenty times for the corresponding population size leading to the best design quoted in the table. JA always converged to feasible designs weighing between 833.006 and 833.101 kg, with a standard deviation on optimized weight of 0.0443 kg. JA’s worst weight was again lower than the best weights obtained by the other algorithms. MATLAB optimizations were also started from 110 designs originally included in the initial population (np=1000) that led to the best JA's design: these initial designs were randomly selected. The commercial optimizer converged to structural weights between 834.201 and 834.531 kg, hence greater than the heaviest weight found by JA.

Table 8. Comparison of the optimized designs for the 47-bar power line

Design variables	SA [37]	HHS-LS [48,77]	HBBBC-LS [48,77]	FFA-LS [78]	SQP Matlab	**JA This study**	FSD-ES* [87]	SCPSO* [85]	SA* [36]	**JA** This study**
A_1 (in^2)	2.6624	2.6174	2.6245	2.6575	2.6140	**2.6642**	2.7	2.5	2.5	**2.7**
A_2	2.5776	2.4457	2.4471	2.4832	2.4459	**2.4680**	2.5	2.5	2.8	**2.5**
A_5	0.6841	0.7370	0.7408	0.7267	0.7468	**0.7330**	0.7	0.8	0.8	**0.7**
A_7	0.1001	0.1003	0.1003	0.1003	0.1000	**0.1005**	0.1	0.1	0.1	**0.1**
A_8	0.7677	0.8957	0.8974	0.8928	0.8976	**0.9350**	0.9	0.7	0.7	**0.9**
A_{10}	1.1619	1.0624	1.0712	1.0545	1.0686	**1.0947**	1.1	1.4	1.3	**1.1**
A_{11}	1.7421	1.7204	1.7267	1.7805	1.7268	**1.7679**	1.8	1.7	1.8	**1.8**
A_{13}	0.7666	0.7089	0.7069	0.6599	0.7107	**0.6698**	0.7	0.8	0.7	**0.7**
A_{15}	0.9338	0.8499	0.8627	0.8625	0.8556	**0.8405**	0.9	0.9	0.9	**0.9**
A_{17}	1.3060	1.2381	1.2372	1.2462	1.2367	**1.3391**	1.3	1.3	1.2	**1.3**
A_{19}	0.3120	0.2991	0.2988	0.3217	0.3016	**0.3143**	0.3	0.3	0.4	**0.3**
A_{21}	1.0750	1.0904	1.0911	1.1758	1.0936	**1.0905**	1.1	0.9	1.3	**1.1**
A_{23}	0.9781	0.9309	0.9300	0.9363	0.9296	**1.0020**	1.0	1.0	0.9	**1.0**
A_{25}	0.9024	0.8978	0.9017	0.8821	0.8988	**0.8982**	0.9	1.1	0.9	**0.9**
A_{27}	0.7585	0.7727	0.7713	0.7190	0.7719	**0.8548**	0.8	5.0	0.7	**0.8**
A_{28}	0.1197	0.1003	0.1000	0.1003	0.1000	**0.1129**	0.1	0.1	0.1	**0.1**
A_{29}	2.6175	2.5962	2.6018	2.6499	2.5924	**2.6879**	2.7	2.5	2.5	**2.7**
A_{31}	0.9059	0.8634	0.8651	0.8481	0.8706	**0.8195**	0.8	1.0	1.0	**0.8**
A_{33}	0.1003	0.1004	0.1001	0.1003	0.1000	**0.1077**	0.1	0.1	0.1	**0.1**
A_{34}	2.9422	2.9195	2.9193	2.9590	2.9158	**2.9578**	3.0	2.8	2.9	**3.0**
A_{36}	0.8553	0.8778	0.8787	0.8771	0.8858	**0.8587**	0.9	0.9	0.8	**0.9**
A_{38}	0.1000	0.1004	0.1002	0.1003	0.1000	**0.1012**	0.1	0.1	0.1	**0.1**
A_{39}	3.1043	3.1138	3.1070	3.1341	3.1108	**3.0970**	3.2	3.0	3.0	**3.2**
A_{41}	1.0538	1.0657	1.0702	1.0584	1.0735	**1.0742**	1.0	1.0	1.2	**1.0**
A_{43}	0.1004	0.1003	0.1003	0.1003	0.1000	**0.1000**	0.1	0.1	0.1	**0.1**
A_{44}	3.2412	3.2473	3.2221	3.2553	3.2428	**3.2108**	3.3	3.2	3.2	**3.3**
A_{46}	1.0764	1.0758	1.0808	1.0486	1.0487	**1.0344**	1.1	1.2	1.1	**1.1**
X_2 (in)	103.435	103.617	105.000	103.055	103.376	**104.136**	100.9724	101.3393	104.0	**100.249**
X_4	85.269	84.776	84.770	85.458	84.455	**88.322**	80.4772	85.9111	87.0	**81.118**
Y_4	128.654	129.483	129.888	124.711	129.402	**121.752**	136.8699	135.9645	128.0	**138.063**
X_6	67.897	67.460	67.186	67.200	67.274	**67.803**	64.3908	74.7969	70.0	**63.520**
Y_6	243.645	246.116	245.967	241.701	245.693	**242.468**	247.0491	237.7447	259.0	**249.861**
X_8	58.529	58.705	58.456	57.733	58.576	**58.009**	55.2589	64.3115	62.0	**54.417**
Y_8	325.032	331.517	331.031	327.019	331.307	**322.627**	338.4534	321.3416	326.0	**338.356**
X_{10}	49.747	50.643	50.444	49.982	50.540	**50.701**	48.7333	53.3345	53.0	**49.238**
Y_{10}	409.643	410.728	410.140	404.820	410.504	**395.426**	409.7380	414.3025	412.0	**404.395**
X_{12}	44.388	44.785	44.585	44.464	44.630	**44.977**	43.4742	46.0277	47.0	**44.082**
Y_{12}	469.679	477.267	476.908	470.819	477.502	**463.933**	472.1479	489.9216	486.0	**467.812**
X_{14}	41.850	45.617	45.398	45.363	45.409	**45.385**	44.8349	41.8353	45.0	**44.534**
Y_{14}	508.000	514.621	514.481	507.943	514.215	**510.969**	512.1901	522.4161	504.0	**511.407**
X_{20}	2.000	0.185	1.002	2.251	0.100	**0.666**	3.8414	1.0005	2.0	**3.851**
Y_{20}	589.782	591.490	591.285	586.223	590.732	**587.372**	591.1449	598.3905	584.0	**590.619**
X_{21}	84.000	89.838	89.993	85.355	89.717	**80.202**	84.5040	97.8696	89.0	**84.215**
Y_{21}	630.009	632.623	632.272	634.216	632.465	**629.338**	630.3472	624.0552	637.0	**630.355**
Weight (kg)	833.816	833.236	833.414	833.410	834.201	**833.006**	837.567	845.540	848.988	**836.975**
CVP (%)	None	None	None	None	None	**None**	None	None	None	**None**

NSA	4819	5274	5112	4946	2854	**4680**	55802	25000	13000♠	**5545**

*Cross-sectional areas of bars treated as discrete variables; **Two-stage optimization with discrete bar areas.
♠Estimated as the product between number of optimization cycles and number of optimized variables.

The number of structural analyses for the different metaheuristic algorithms also changed at most by 12.7%. However, JA found the following intermediate designs (all feasible) weighing: (i) 833.72 kg (better than 833.816 kg, the optimum weight of CMLPSA) after only 3334 structural analyses (vs. 4819 analyses required by CMLPSA to complete the optimization process); (ii) 833.39 kg (better than 833.414 kg, the optimum weight of HBBBC-LS, and 833.410 kg, the optimum weight of FFA-LS) after only 3389 structural analyses (vs. 5112 and 4946 analyses, respectively, required by HBBBC-LS and FFA-LS to complete the optimization process); (iii) 833.18 kg (better than 833.236 kg, the optimum weight of HHS-LS) after only 3732 structural analyses (vs. 5274 analyses required by HHS-LS). This confirms the superiority of JA over multi-level and multipoint SA, advanced HS, BBBC and FFA including line search strategies.

SQP-MATLAB was faster than JA (2854 vs. 4680 structural analyses) but converged to a 0.143% heavier design than the present algorithm. However, JA actually required only 3326 structural analyses to find a feasible intermediate design weighing 834.16 kg, which is lighter than the optimized weight of MATLAB. Remarkably, in the 110 optimizations runs started from randomly selected initial designs, MATLAB could complete the optimization process in less than 3326 analyses (i.e. the corresponding computational cost of JA to find some feasible intermediate design lighter than the optimum design found by MATLAB) only in the 30% of runs. Therefore, convergence speeds of JA and MATLAB may be judged comparable.

The convergence curves recorded for the best optimization run of each metaheuristic algorithm are compared in Fig. 13. JA and SQP-MATLAB rapidly reduced the weight of the power line but while JA's intermediate designs remained feasible the gradient-based optimizer generated infeasible designs and had to increase weight to re-enter in the feasible design space at about 2/3 of the optimization process. Remarkably, JA was the only metaheuristic algorithm comparable with SQP in terms of convergence speed. The other metaheuristic algorithms could never find better intermediate designs than JA. CMLPSA, HBBBC-LS, FFA-LS behaved in a very similar fashion over the first 1500 structural analyses. However, while HBBBC-LS became the closest algorithm to JA in terms of convergence rate, the other algorithms required about 4200 analyses to approach the intermediate designs of JA.

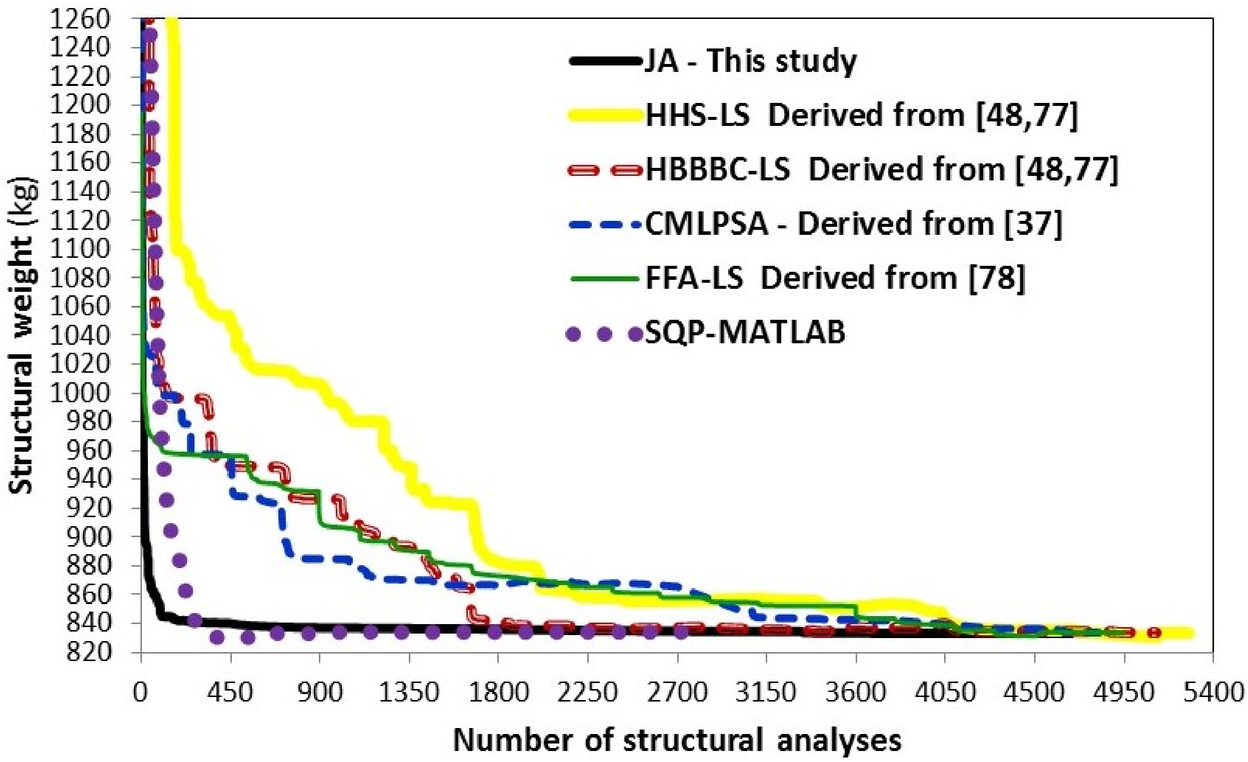


Fig. 13. Convergence curves for the 47-bar power line problem

The optimization results relative to the discrete optimization indicate that JA is again very competitive with state-of-the-art metaheuristic algorithms. Since the continuous solution found by JA was very close to the best discrete solution reported in the literature [87], it was possible to successfully run the two-stage optimization and reduce optimum weight by about 0.6 kg with respect to [87]. This process required only 865 additional structural analyses with respect to the continuous optimization. As shown in Table 8, JA found an optimum design weighing only 836.975 kg which is also considerably lighter than the discrete designs obtained by hybrid PSO with cellular automata [85] and discrete SA [28]. JA was considerably faster than FSD-ES, SCPSO and discrete SA that required at least 13000 analyses to find the discrete solution.

As far as it concerns the robustness of the JA's two-stage discrete optimization process, since the JA's continuous solutions obtained in the twenty independent runs for *np*=1000 vary by less than 0.1 kg in terms of structural weight (i.e. from 833.006 to 833.101 kg), the rounding process always results in the same discrete design quoted in Table 8. The discrete optimization was also conducted starting from the worst continuous solution obtained for *np*=10. Since the twenty independent runs performed for the continuous case with *np*=10 resulted in very similar structural weights (i.e. between 834.446 and 834.467 kg), the cross-sectional areas were always rounded to the same values. At the end of the discrete optimization, JA found a feasible design weighing 837.381 kg, yet lighter than the optimized weights listed in Table 8. The designs obtained for *np*=1000 (Table 8) and *np*=10 are very similar: in fact, the largest difference between corresponding cross-sectional areas is 0.2 in^2 and the average difference between areas is near to zero. The robustness of JA in such a complicated problem represents a significant improvement with respect to the discrete optimization algorithms of Ref. [85] for which it is reported a standard deviation on optimized weight ranging between 13.3 and 15.8 kg (for different computational grid sizes), and Ref. [87] where optimized weight is reported to vary between 837.6 and 852.7 kg.

The above mentioned two-stage optimization process was carried out also for the continuous optimum designs of CMLPSA, HHS-LS, HBBBC-LS and FFA-LS quoted in Table 8. The SA re-optimized weight (i.e. 840.224 kg) is very close to JA and FSD-ES [87] weights (less than 0.39% difference), and 1.03% kg lighter than the best discrete SA design quoted in the literature [36]. The re-optimized weight for HHS-LS is 838.684 kg, much better than the 916.610 kg weight obtained by the discrete HS algorithm developed in [81], and very competitive with the discrete sizing designs quoted in Table 8. The re-optimized weight for HBBC-LS is 837.470 kg, very close to the JA and FSD-ES [87] discrete designs listed in Table 8. Finally, the re-optimized weight for FFA-LS is 839.028 kg, again very competitive with the discrete designs listed in Table 8. These results confirm the efficiency of the two-stage optimization strategy used in this study which works well regardless of the metaheuristic optimization algorithm utilized.

Table 8 shows that optimized cross-sectional areas are very similar while nodal coordinates change more significantly in order to optimally distribute stiffness. Figure 14 compares the layout optimized by JA (i.e. continuous solution and discrete solution found with the two-stage optimization process) with the two best designs quoted in the literature for discrete optimization [85,87]. The overall optimized shape of the structure obviously remained the same but the heights of the segments forming the body of the tower changed by some extent. Interestingly, the profile of the body of the tower designed by SCPSO encloses a larger area than the corresponding profiles optimized by JA and FDS-ES (see Fig. 14). While this profile included the thickest elements, JA could optimally resize the length of heavier elements passing from continuous to discrete solution.

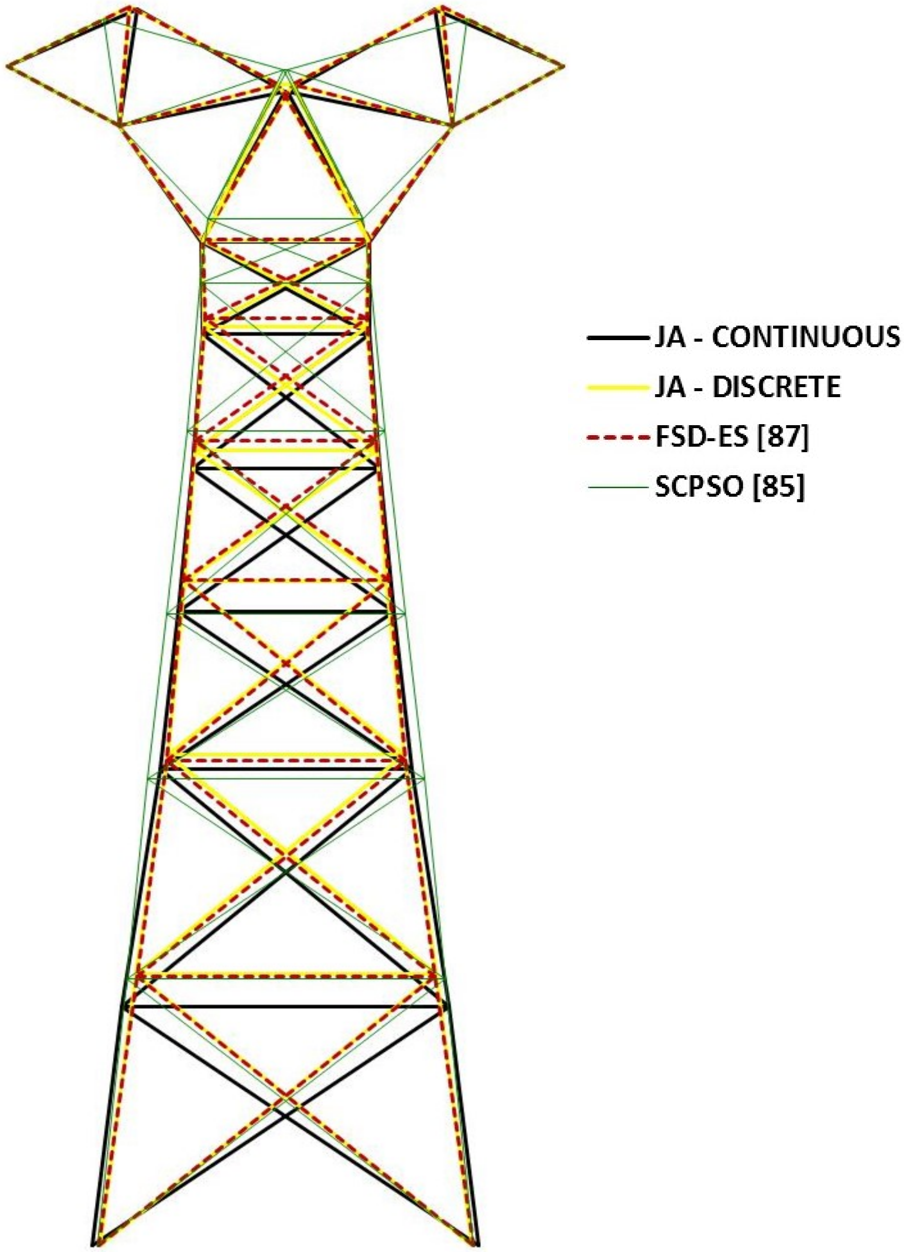


Fig. 14. Comparison of optimized layouts for the 47-bar power line structure

4.7. Simplified topology optimization with discrete sizing variables

Sections 4.4 and 4.6 presented some results for the 25-bar and 47-bar truss problems solved with discrete sizing variables. However, the most difficult scenario is the simultaneous optimization of truss size, layout and topology. For this reason, the 25-bar problem and the 47-bar problem were solved including also topology variables and discrete sizing variables. A simplified topology optimization approach was utilized for that purpose. The optimization process entails the following steps:

(i) Set lower bound of sizing variables as 10^{-7} in^2;

(ii) Perform continuous optimization including all design variables;

(iii) Round sizing variables as they get very close to the available discrete cross-sections;

(iv) Remove sizing variables that get very close to the 10^{-7} in^2 lower bound;

(v) Perform continuous optimization with layout variables and the remaining sizing variables;

(vi) Repeat steps (iii) through (v) until no sizing variables remain to be optimized.

The case of discrete layout variables also was covered by the simplified topology optimization approach. Once step (vi) is completed, layout variables are perturbed to their nearest discrete values and the best design is set the new optimal solution. This solution is then perturbed with respect to sizing variables in order to further improve design. The process ends when no improvement is found passing from one type of variables to another.

The stiffness matrix of the truss structure is not reformulated when the cross-sectional area of some element approaches 10^{-7} in^2. The "weak" elements which are assigned the cross-sectional area of 10^{-7} in^2 practically behave as if they were killed and hence do not contribute by any extent to the global stiffness of the structure. The 10^{-7} in^2 limit was selected as it is 10^{-6} times the lower bound of 0.1 in^2 set for sizing variables. Basically, the stiffness terms (in the case of a truss structure, these terms are proportional to element cross-sectional areas) must be multiplied by a factor 10^{-6} in order to become inactive similarly to what occurs in commercial finite element programs when the "kill element" option is selected.

The continuous optimal solution found by JA for the 25-bar truss structure corresponds to a structural weight of 49.898 kg. Cross-sectional areas of groups 1, 4 and 5 reached the value of 10^{-7} in^2 after 3011 structural analyses. This result was consistent with the optimized topologies reported in [25,58,87,94] and summarized in Table 9. At the end of continuous optimization, cross-sectional areas of element groups 2 and 3 reached the values of 0.1026 and 0.9991 in^2 and were hence rounded to 0.1 and 1 in^2, respectively. The second stage of the optimization process included only cross-sectional area of element group 8 as continuous sizing variable and the five layout variables X_4, Y_4, Z_4, X_8 and Y_8. After 3776 structural analyses, JA reached the weight of 51.348 kg setting the cross-sectional area of element group 8 equal to 0.9034 in^2. This area was rounded to 0.9 in^2 and the optimization process thus included only layout variables. The final weight of 51.388 kg was obtained by JA after 4877 structural analyses.

Table 9. Comparison of the optimized designs for the 25-bar tower topology optimization with discrete sizing variables.

Design Variables	GA [25]	GA [94]	FFA [58]	FSD-ES [87]	**JA This study**
A_1 (in^2)	Removed	Removed	Removed	Removed	**Removed**
A_2	0.1	0.1	0.1	0.1	**0.1**
A_3	0.9	0.9	1.1	0.9	**1.0**
A_4	Removed	Removed	Removed	Removed	**Removed**
A_5	Removed	Removed	Removed	Removed	**Removed**
A_6	0.1	0.1	0.1	0.1	**0.1**
A_7	0.1	0.1	0.1	0.1	**0.1**
A_8	1.0	1.0	0.9	1.0	**0.9**
X_4 (in)	39.91	38.7913	38.50	38.8713	**38.909**
Y_4	61.99	66.1110	64.35	61.5207	**59.087**
Z_4	118.23	112.9787	112.87	119.1785	**123.247**
X_8	53.13	48.7924	49.13	49.4146	**51.227**
Y_8	138.49	138.8910	134.94	137.9423	**140.104**
Weight (kg)	52.045	51.877	52.880	51.899	**51.388**
CVP (%)	None	None	None	None	**None**
NSA	6000	10000	6000	8660	**4877**

The JA's optimum design is listed in Table 9 which compares the performance of the present algorithm with GA variants [25,94], firefly algorithm [58] and FSD-ES (fully stressed design based on evolution strategy) [87]. It can be seen that all optimized designs are very close to JA's solution and satisfy stress and displacement constraints. However, JA designed the lightest structure and required less structural analyses than the other algorithms. Interestingly, JA used a simplified approach to topology optimization with discrete sizing variables while the other formulations were specifically designed for such kind of problems.

The effect of initial population was evaluated by running twenty independent runs for each population size ranging from 10 to 1000. Since all continuous optimizations reduced cross-sectional areas of groups 1, 4 and 5 to 10^{-7} in^2 and converged practically to the same optimum weight of 49.898 kg with only 0.001 kg deviation (this is consistent with the 0.1 in^2 minimum gage case: optimized designs practically coincided and structural weight ranged between 53.049 and 53.051 kg), these members always were removed from the structure to carry out the successive discrete optimization. Furthermore, cross-sectional areas always were rounded to the same values giving the same intermediate designs and finally the optimum design quoted in Table 9. This is a noticeable improvement with respect to the 5.05 kg standard deviation on optimized weight reported for the firefly algorithm of Ref. [58], and the 51.899 to 61.265 kg weight range reported for the FSD-ES algorithm of Ref. [87].

The continuous optimum design found by JA for the 47-bar truss problem weighs 823.805 kg and was obtained after 3520 structural analyses. The cross-sectional areas of element groups 4, 16, 19, 22 and 25 were reduced to the lower bound of 10^{-7} in^2 and the corresponding sizing variables were removed from the optimization process. This was in agreement with the best discrete solutions available in literature (see, for example, [36,93]). SQP-MATLAB converged to a structural weight of 822.403 kg after 2503 analyses but could not reduce below 10^{-5} in^2 the five sizing variables that JA instead reduced to 10^{-7} in^2.

Sizing variables were progressively rounded by JA: (i) cross-sectional areas of element groups 12,13,17,21 immediately after removing element groups 4,16,19,22,25; (ii) groups

5,7,15,26 after 3967 analyses; (iii) groups 10,14,24 after 4370 analyses; (iv) groups 2,9,11,27 after 4751 analyses; (v) groups 6,18,20 after 5289 analyses; (vi) groups 1,3,7,23 after 5842 analyses. From that point, JA optimized only layout variables finding an optimum weight of 829.677 kg after 6855 structural analyses.

Similar to the 25-bar problem, twenty independent optimization runs were carried out for each population size from 10 to 1000. In the continuous optimization process, JA always designed cross-sectional areas of element groups 4, 16, 19, 22 and 25 at their lower bound of 10^{-7} in^2. Hence, these elements were removed from the structure to perform the subsequent discrete optimization. Optimized designs obtained in the continuous optimizations were very close, regardless of population size and independent optimization runs: structural weight ranged from 823.805 to 823.980 kg. The weight dispersion was even smaller than the about 1.45 kg variation observed for the 0.1 in^2 minimum gage. Interestingly, having removed five groups of elements drove the optimizer to distribute cross-sectional areas always in the same way over the remaining bars. Consequently, the rounding process always interested the same cross-sectional areas and JA always converged to the same design quoted in Table 15.

Table 10 compares JA with advanced SA [36] and GA [93]. JA and GA removed the horizontal bars 7, 28, 33, 38 and 43 while SA removed also element 10. However, JA and GA converged to feasible designs while SA violated stress constraints by 6%. JA designed the lightest structure (i.e. 829.677 kg) and required much less structural analyses than literature (i.e. only 6855 analyses for JA vs. 13000 and 100000 analyses estimated for SA and GA, respectively). While these results confirm the efficiency of JA, it should be noted that the referenced GA and SA solutions took also integer nodal coordinates with a resolution of 1 in between available discrete values; discrete values of cross-sectional areas could vary by 0.1 in^2. For this reason, the optimum design of JA was further elaborated by rounding also the layout variables listed in the fourth column of Table 10 (denoted as “JA Discrete sizing variables") to their two lower or upper nearest integer values. Since there are 17 layout variables, this process entailed 68 (i.e. 17x4) new structural analyses: hence, the total number of analyses required by JA raised to 6923. However, this fully discrete design violated buckling constraints by 1.84%. In order to reduce this violation, sizing variables were increased by 0.1 in^2 one at a time. This process led to raise cross-sectional areas of element groups 5 and 17 to 1 in^2 and 2.6 in^2, respectively. The violation on buckling constraints dropped to 1.28% while stress margins remained practically the same. The final structural weight is 834.118 kg. The computational cost of the second rounding turn of JA was 22 new analyses, that is the number of sizing variables not removed from the optimization process. In summary, JA required 6945 analyses, still much less than SA and GA.

Table 10. Comparison of optimized designs for the discrete 47-bar truss topology problem.

Design variables	GA [93]	SA [36]	**JA Discrete sizing variables**	**JA Discrete sizing and layout variables**
A_1 (in^2)	2.6	2.9	**2.6**	**2.6**
A_2	2.4	2.4	**2.5**	**2.5**
A_5	0.8	0.5	**0.8**	**0.8**
A_7	Removed	Removed	**Removed**	**Removed**
A_8	1.1	1.7	**0.9**	**1.0**
A_{10}	1.3	Removed	**1.0**	**1.0**
A_{11}	1.7	1.6	**1.7**	**1.7**
A_{13}	0.6	0.5	**0.8**	**0.8**
A_{15}	1.0	0.9	**0.9**	**0.9**
A_{17}	1.4	1.3	**1.2**	**1.2**
A_{19}	0.5	0.8	**0.3**	**0.3**
A_{21}	1.1	1.1	**1.1**	**1.1**
A_{23}	1.0	1.0	**0.9**	**0.9**
A_{25}	1.0	0.8	**0.9**	**0.9**
A_{27}	0.8	0.5	**0.8**	**0.8**
A_{28}	Removed	Removed	**Removed**	**Removed**
A_{29}	2.7	2.6	**2.5**	**2.6**
A_{31}	1.0	1.1	**0.8**	**0.8**
A_{33}	Removed	Removed	**Removed**	**Removed**
A_{34}	2.9	3.1	**2.8**	**2.8**
A_{36}	0.9	0.5	**0.9**	**0.9**
A_{38}	Removed	Removed	**Removed**	**Removed**
A_{39}	3.1	2.9	**3.0**	**3.0**
A_{41}	1.1	1.4	**1.1**	**1.1**
A_{43}	Removed	Removed	**Removed**	**Removed**
A_{44}	3.2	3.3	**3.3**	**3.3**
A_{46}	1.0	0.3	**1.1**	**1.1**
X_2 (in)	104.0	112.0	**100.048**	**100.0**
X_4	93.0	88.0	**87.011**	**87.0**
Y_4	116.0	140.0	**133.947**	**134.0**
X_6	74.0	68.0	**70.112**	**70.0**
Y_6	223.0	241.0	**260.313**	**261.0**

X_8	64.0	61.0	**60.675**	**61.0**
Y_8	302.0	326.0	**341.972**	**342.0**
X_{10}	54.0	47.0	**53.804**	**54.0**
Y_{10}	391.0	410.0	**411.339**	**411.0**
X_{12}	46.0	44.0	**43.579**	**43.0**
Y_{12}	458.0	450.0	**477.764**	**476.0**
X_{14}	51.0	65.0	**44.497**	**44.0**
Y_{14}	507.0	502.0	**514.208**	**514.0**
X_{20}	19.0	1.0	**2.218**	**2.0**
Y_{20}	595.0	598.0	**594.041**	**594.0**
X_{21}	90.0	58.0	**94.264**	**94.0**
Y_{21}	626.0	635.0	**633.594**	**634.0**
Weight (kg)	855.053	811.603	**829.677**	**834.118**
CVP (%)	None	3.2 (Tens. stress) 6 (Compr. stress)	**None**	**0.89 (Compr. stress) 1.28 (Buckling)**
NSA	100000*	13000+	**6855**	**6945**

* Estimated as the product between number of optimization cycles and population size
+ Estimated as the product between number of optimization cycles and number of optimized variables

The JA solution is much less infeasible than the optimum design of SA. The presence of buckling constraints makes the design process more sensitive to layout variables in spite of the fact that the optimization problem included 27 sizing variables vs. only 17 layout variables. This effect is amplified by topology optimization with discrete variables that further reduces the amount of design freedom. Figure 15 compares the optimized topologies for JA, GA and SA. The most significant differences occur in the upper part of the tower.

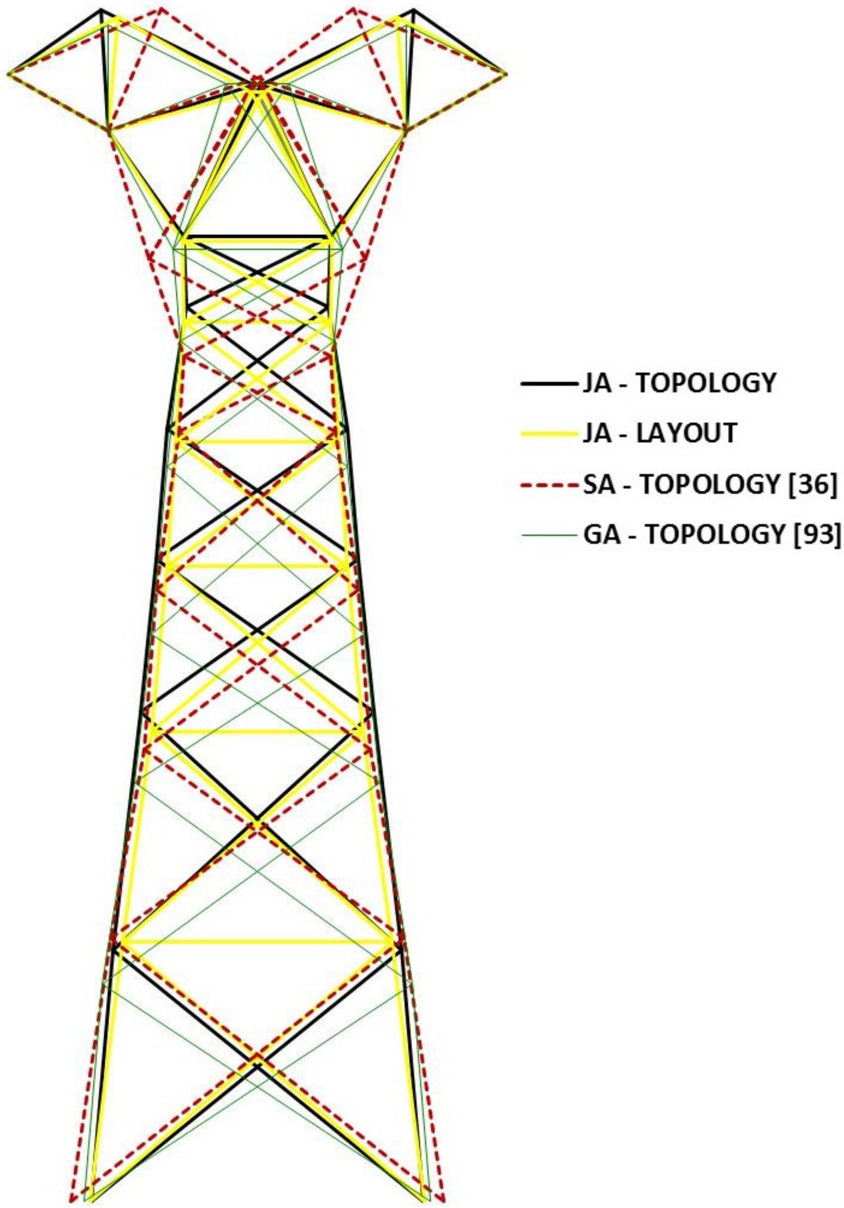


Fig. 15. Comparison of optimized topologies and layouts for the discrete 47-bar problem.

5. Conclusions

A very recently developed metaheuristic optimization method called Jaya algorithm was applied in this study to sizing, layout and topology design of truss structures for the first time ever. Jaya is a non-parametric algorithm that attempts to approach the best design included in the population and simultaneously escape from the worst design. Here, the original formulation of JA was modified in order to improve the convergence speed of the optimization process by reducing the number of required structural analyses.

Truss design problems are the most common benchmark examples used in the structural optimization literature for evaluating performance of new optimization algorithms. JA was tested on three sizing problems including an average-scale structure (with 200 elements and 29 design variables), a fairly large-scale structure (i.e. a 942-bar tower optimized with 59 design variables) and a large-scale structure (i.e a 1938-bar tower optimized with 204 design variables), as well as on three sizing-layout problems (with up to 81 design variables). The case of topology optimization with discrete sizing/layout variables also was developed. For that purpose, a simple multi-stage continuous-discrete process was used. The test problem suite considered here covered all cases that may be encountered by designers.

JA obtained the best design in almost all design examples or was one of the best algorithms paying a very small weight penalty (or even improving) with respect to the global optimum indicated in literature. Statistical performance indicators usually adopted in structural optimization literature such as best, average and worst optimized weights as well as standard deviation on optimized weight obtained over independent optimization runs with different population size and composition proved with no shadow of doubt the efficiency, robustness and reliability of the JA. For all design examples, standard deviation values of the JA are very small, always less than 0.245% of the corresponding average weights. The worst optimized weight of JA is very often lighter than the best weights obtained by the other state-of-the-art metaheuristic algorithms taken for comparison. Remarkably, this always occurred for the very large sizing optimization problem and for the combined sizing-layout-topology optimization problems, thus indicating that JA's superiority becomes more evident as the design space includes a larger number of possible combinations that may lead to the global optimum. This happens when there are hundreds of design variables (i.e. large scale) or optimization variables belong to disjoint spaces (i.e. sizing vs. layout/topology).

JA required less structural analyses than most of the metaheuristic optimization methods compared in this study. The same behavior was observed for large-scale problems as well as for topology optimization with discrete sizing and layout variables. Interestingly, JA’s convergence speed was similar to state-of-the-art gradient based optimizers which usually are one order of magnitude faster than metaheuristic algorithms. Sensitivity analysis demonstrated that JA’s performance is insensitive to population size, composition of initial population and sequence of random numbers used in the search process.

The huge amount of data presented in this article proved that JA is a very powerful algorithm for many truss design problems. Furthermore, JA is a parameter free algorithm that can be easily implemented on computers. A robust JA formulation for discrete sizing optimization which avoids the first stage of continuous optimization is currently under development: preliminary results confirm that JA is very competitive with other state-of-the-art metaheuristic algorithms. Further studies should investigate the suitability of JA for other types of skeletal structures such as 3D frames. The topology optimization approach should be enhanced also considering continuum structures with multi-material designs (see, for example, Ref. [95]) and constraints on natural frequencies (see, for example, Ref. [96]). Comparisons with topology optimization commercial software (e.g. OptiStruct) will have to be carried out as well.

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Appendix

Sensitivity of JA convergence behavior to population size

As mentioned in Section 3, population size (*np*) is the only internal parameter that must be given in input to JA. Sensitivity analysis aimed at finding the most suitable value of *np* was performed for all test problems. The corresponding results are shown in Tables A1 through A3; NSA is the number of structural analyses for which JA finds the optimum weight.

In sizing optimization (Tables A1 and A2), the best performance of JA is achieved for *np*=20. Since the ratio of standard deviation to average optimized weight never exceeded 0.0186%, JA was judged insensitive to population size and all sizing optimization problems were solved with *np*=20. Convergence curves practically coincide after about one half of optimization history (see, for example, Fig. S3-a of supplementary material). Furthermore, optimized designs are very close and stiffness is distributed across the structure always in the same way regardless of initial design/population (e.g. Fig. S3-b of supplementary material).

Table A1. Results of sensitivity analysis on JA convergence behavior with respect to population size for average scale 200-bar and fairly large-scale 942-bar truss sizing problems.

Population size	200-bar truss		942-bar truss	
	Weight (lb)	NSA	Weight (lb)	NSA
10	25463.55	32774	137345.134	59983
20	25463.53	31580	137344.356	58274
30	25475.78	35820	137349.171	59147
40	25467.73	34703	137347.246	60130
50	25465.97	32085	137345.028	59527
60	25463.85	34284	137350.130	61044

Table A2. Results of sensitivity analysis on JA convergence behavior with respect to population size for the large-scale 1938-bar tower sizing optimization problem.

Population size	Weight (ton*)	NSA
10	99.265	21980
20	99.255	20051
50	99.264	20929
100	99.260	21584

* 1 ton=2204.6244 lb

In sizing-layout optimization (Table A3), JA reached its best performance for *np*=30 in the 25-bar tower problem, *np*=500 in the 45-bar truss problem and *np*=1000 in the 47-bar power line problem. JA was very robust also in these problems: in fact, the ratio between weight standard deviation and average weight is, respectively, 0.00123%, 0.0532%, 0.00528% and 0.0612% for the 25-bar, 45-bar and 47-bar truss problems.

Table A3. Results of sensitivity analysis on JA convergence behavior with respect to population size for sizing-layout optimization problems.

Population size	**25-bar tower**		**45-bar truss**		**47-bar power line**	
	Weight (kg)	NSA	Weight (kg)	NSA	Weight (kg)	NSA
10	53.050	3043	3193.768	5072	834.446	3798
20	53.051	3122	3193.677	4323	834.014	3565
30	53.049	3097	3194.080	4768	834.325	3438
40	53.050	3257	3193.998	4475	834.092	4099
50	53.050	3388	3193.708	5663	833.246	4212
60	53.049	3181	3193.664	4505	834.015	3466
70	53.049	3580	3193.981	5497	834.339	4229
80	53.049	3528	3193.587	4748	833.542	3999
90	53.049	3488	3193.874	5920	834.205	4349
100	53.049	3244	3193.810	5783	834.086	3992

250	53.049	3223	3193.577	6061	833.306	3697
500	53.049	3484	3193.568	5717	833.107	4548
1000	53.049	3677	3193.802	5907	833.006	4680

The overall profile optimized by JA is insensitive to population size although some local variations of coordinates in the longest dimension of the truss can be observed especially for the power line (see Fig. S4 of supplementary material). Convergence curves coincide or become very similar between 1/2 and 2/3 of the optimization history. The fastest optimization run always produced better intermediate designs than the slowest run (see Fig. S5 of supplementary material). Using very small or very large populations may slow down the search process. However, the very efficient search strategy of JA and the large amount of design freedom finally allows to find marginally different optimized designs.

The number of structural analyses tends to increase for the very small or the very large populations. However, the ratio of standard deviation to average number of structural analyses never exceeded 12.1%. The corresponding values are: 33541±1652 (200-bar truss sizing problem), 59684±942 (942-bar tower sizing problem), 21136±843 (1938-bar tower sizing problem), 3332±204 (25-bar truss sizing-layout problems), 5265±633 (45-bar truss sizing-layout problem) and 4006±399 (47-bar power line sizing-layout problem). This is a further proof of the robustness of the JA algorithm implemented in this study.