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Original Citation:

Control and learning methods for cognitive spaceborne tracking radars / Sartoni, Matteo. - ELETTRONICO. - (2026).
[10.60576/poliba/iris/sartoni-matteo_phd2026]

Availability:

This version is available at <http://hdl.handle.net/11589/295900> since: 2026-01-19

Published version

DOI:10.60576/poliba/iris/sartoni-matteo_phd2026

Publisher: Politecnico di Bari

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Italian National Ph.D. Program in Autonomous Systems

ACADEMIC DISCIPLINE: SYSTEMS AND CONTROL ENGINEERING (IINF-04/A)

Final Dissertation

Control and Learning Methods for Cognitive Spaceborne Tracking Radars

by

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Control and Learning Methods for Cognitive Spaceborne Tracking Radars

Thesis submitted for the degree of Philosophiae Doctor

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Politecnico di Bari - Università di Bologna

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The doctoral scholarship was funded by the European Union - Next Generation EU, Mission 4 Component [2] CUP [D93D22001390001].

Dissertation submitted for the degree of *Philosophiae Doctor*
Italian National Ph.D. Program in Autonomous Systems

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38th

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Hosting University:
Università di Bologna

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Dedication

This thesis brings my university engineering journey to a close. These have been eight demanding years, but ones I will always happily remember, both for what I learned and for the people I had the pleasure to meet. I dedicate this section to the acknowledgements due for my doctoral work.

First of all, I would like to thank my parents and grandparents for always believing in me over the years and for their help in renovating the apartment I moved into with Karolina and Cleo. Thanks to Karolina, my love and the cherry on top of these past three years. Thank you for the love you feel and show me every day, for waiting for me while I had to go to The Netherlands, and for leaving everything to come and live with me in Montecatini.

A heartfelt thanks to Giuseppe for his academic support and for his patience with me, and thanks to my colleagues at CASY for their company and the great times we shared. Thanks to Andrea and Carlo for hosting me at TAS-I in Rome, it was a great pleasure to spend the six-month research period with you, in a wonderful environment where I met many friendly and down-to-earth people. Thanks also for arranging my activity at ESA; thank you to Max and Gianluca for hosting me and to the other members of TEC-EFP for their support and interest in my research.

A special thought goes to the people at the Shed for Thursday cakes, the 5 p.m. NERF battles, and our Friday afternoon beers. Finally, thanks also to those I have not mentioned here, but whom I had the pleasure of meeting and collaborating with during these three years of my PhD.

Abstract

Making radio frequency remote sensing technologies fully autonomous enables the acquisition of more insightful data during emergency situations. This thesis investigates the application of control techniques and Artificial Intelligence (AI) to cognitive radars. This class of radar systems utilizes information about the surrounding environment, extracted by the receiver, as feedback to the transmitter, thereby rendering the radar a closed-loop system. Within this framework, the aforementioned algorithms can leverage that feedback to adapt the transmission configuration of the radar payload. In this work we mainly focus on cognitive tracking radars. Firstly, we propose an optimal-control algorithm for a spaceborne cognitive tracking radar. The algorithm computes the optimal sequence of transmitted waveforms during tracking, thereby minimizing the estimation error of the trajectory of the target of interest. Simulation results demonstrate that this approach outperforms other methods reported in the literature. Subsequently, the focus shifts to a Tip & Cue configuration for maritime surveillance. This framework involves the cooperation of two satellites to enable higher-resolution data acquisition over an area of interest. Initially, we focused on developing a Gaussian Process Regression method to reconstruct vessel trajectories from sparsely sampled Automatic Identification System (AIS) data. AIS is used by cooperative ships to transmit information such as position, speed, and heading. The algorithm was subsequently implemented and tested on space-qualified Field-Programmable Gate Arrays to evaluate its on-board feasibility. Next, we focused on the development of a target-tracking algorithm based on Synthetic Aperture Radar (SAR) detections. The model employs a combined target-dynamics representation, e.g., partly model-based and partly data-driven (trained via a neural network), to predict the target state, and subsequently refines the estimate using AI-based SAR detections. Subsequently, we extended this problem to a Synthetic Aperture Radar with the objective of detecting and tracking vessels in open sea. In this setting we proposed a Tip & Cue framework, which involves the cooperation between two satellites to enable more accurate imaging of an area of interest. Sensor fusion between Synthetic Aperture Radar and the Automatic Identification System served as an enabling technology for this use case, allowing the association of radar-detected vessels with AIS messages received by the onboard satellite receivers.

Keywords: Cognitive Radar, Synthetic Aperture Radar, Perception-Action Cycle, Target Tracking, Kalman Filter, Optimal Control, Tip & Cue, Automatic Identification System, Waveform Adaptation, Artificial Intelligence.

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Preface

This thesis is submitted in partial fulfillment of the requirements for the degree of *Philosophiae Doctor* in Autonomous Systems.

Acknowledgements

Project co-funded by the European Union-Next Generation Eu under the National Recovery and Resilience Plan (NRRP), Mission 4 Component 2 Investment 3.3 Decree No. 352 (09th April 2022) of Italian Ministry of University and Research Concession Decree No. 2153 (28th December 2022) of the Italian Ministry of University and Research, Project code D93D22001390001, within the Italian National Program PhD Programme in Autonomous Systems (DAuSy).

Introduction

Motivation and Challenges

Spaceborne radar (SBR) and Synthetic Aperture Radar (SAR) systems have gathered great attention in recent years, thanks to their applications across different domains, such as Earth observation and space debris detection and tracking [1], [2]. Traditional SAR systems rely on pre-defined sensing parameters and static processing algorithms, which may limit their adaptability and efficiency in complex and dynamic environments. Furthermore, in the field of target tracking, traditional tracking radar systems employ a feed-forward processing chain in which they first detect the target through low-level processing of the echo signal, and then forward this data on to some higher-level processor that extracts the target kinematic parameters [3]. Paper [4] argues that cognition is mandatory in any useful radar system. However, the cognitive element in current radar payloads is implemented by a human-in-the-loop approach rather than cognition itself. In response to these limitations, the Cognitive Radar (CR) paradigm [5] has emerged as an advanced framework bridging the gap between the radar and neuroscience community. The CR takes inspiration from the visual brain, and it aims at making the radar sensor autonomous and intelligent. A key feature of the visual brain, which can find a meaningful remapping in the radar domain, is its ability to perceive the external environment in one specialized region, and to take actions, based on those perceptions, in another, functionally distinct part of the brain. A cognitive radar is characterized by four fundamental building blocks, with the perception-action cycle, e.g., a feedback loop from the receiver to the transmitter, being the facilitator of intelligence [6]. As such, this paradigm offers much promise for improved performance in target tracking. Indeed, a cognitive tracking radar can exploit the measurements obtained by the receiver as feedback to be provided to the transmitter. Specifically, the receiver computes measurements related to the relative geometry between the radar and the target, which are affected by noise dependent on the transmitted signal parameters [7]. Therefore, control algorithms can be employed to compute the optimal transmission parameters so as to minimize the estimation error [6].

Maritime surveillance is an application of paramount importance [8] in remote sensing. Spaceborne sensor data can be leveraged by governmental entities to monitor both potential illegal activities, such as fishing in protected areas, smuggling, and unauthorized crossings of exclusive economic zones, and possible environmental disasters [9], [10]. It follows that this framework can also be applied to SAR systems to overcome the limitations inherent in traditional SAR payloads. In particular, Cognitive SAR systems may leverage real-time information processing, environmental awareness, and machine learning techniques to dynamically adjust radar parameters such as waveform design, pulse repetition frequency, and imaging modes. The literature on cognitive radar, however, predominantly focuses on single-payload configurations. Therefore, the Tip & Cue configuration, which involves the cooperation of two satellites to obtain higher-resolution data over a given AoI [11], is indeed an interesting framework for the development of a cognitive SAR tracking model. In this setup the first satellite, referred to as the Leader, collects data and identifies the area of interest. It then issues a request (Tip) to the second satellite, referred to as the Follower, which will monitor the AoI at a higher resolution (Cue). Furthermore, the integration of SAR data with Automatic Identification System (AIS) data may be leveraged in the framework of a cognitive SAR system, as it may enable the identification of non-cooperative vessels that may potentially be engaged in illegal activities. This application represents a prime candidate for the development of a use case aimed at the modeling and validation of a cognitive SAR system [12].

Summary of the Contributions

This work presents a contribution to the field of cognitive spaceborne tracking radars. Its primary contribution involves the application of methodologies from control theory, state estimation, and artificial intelligence to improve target tracking performance. This analysis encompasses both real-aperture radars, henceforth simply referred to as radars, and SAR systems.

Within the realm of cognitive tracking radar, the main contribution is the proposal of an optimal control approach to select the parameters of the radar’s transmitted signal. The optimal control framework exploits the system dynamical representation of the state of interest to determine the optimal sequence of control inputs that minimizes a cumulative cost function over a specific time horizon. In this proposed methodology, the objective is to minimize the error in estimating the target’s trajectory. The state estimation algorithm employed is the Extended Kalman Filter (EKF), a prominent extension of the Kalman Filter for non-linear systems. The EKF operates recursively through a two-step process: prediction and update. During the prediction step, the filter uses a dynamic model of the state to project the state estimate forward in time. In the subsequent update step, the filter incorporates new target measurements to refine this prediction. The cost function, defined as the deviation between the true and estimated target states, is formulated as a function of the latter. The principal novelty of this work lies in the formulation of the EKF itself as a dynamical system. This is achieved by employing a cubature rule to approximate the expectation associated with future measurements, which are not yet available from the radar sensor. The proposed approach is benchmarked against other techniques documented in the literature, demonstrating its superior tracking performance.

We subsequently evaluated Gaussian Process Regression (GPR) for reconstructing vessel trajectories using AIS data within a Tip & Cue framework. The Tip & Cue setup involves the collaboration between two satellites to achieve higher-resolution data of an Area of Interest (AoI). Specifically the first satellite, designated as the Leader, selects the AoI under certain requirements, like the presence of targets of interest, and sends the request to the second satellite, denoted as Follower, to obtain data at higher resolution from the AoI (Tip). The Follower subsequently performs a dedicated data acquisition at higher resolution (Cue). Two GPR models were trained, in order to fit the time evolution of the vessel’s longitude and latitude. Therefore, AIS messages were employed. This sensor, installed on cooperative vessels, provides information such as a unique identifier, geodetic coordinates, i.e., longitude and latitude, timestamp, speed, and heading. Furthermore, the proposed algorithm has been ported to two space-qualified Field-Programmable Gate Arrays (FPGAs) for performance evaluation.

Finally, a target-tracking framework that leverages AI-based SAR detections is presented. In the absence of an operational SAR simulator, we first propose a method to simulate target detections as bounding boxes on raw SAR data. The raw data are taken from sub-apertures of the spotlight mode, i.e., data are processed incrementally after successive acquisition intervals rather than as a single batch at the end, thereby producing a time series of detections suitable for tracking. The synthetic bounding-box generator is easily reconfigurable according to the chosen SAR parameters and the number of sub-apertures. The simulator also supports extended-target detections and allows specification of a detection probability and a false-alarm rate (i.e., possible false positives). The centroids of these bounding boxes are georeferenced to obtain Earth-surface coordinates for the detected target, and these coordinates are provided to the filter for real-time state estimation. As a final contribution, the target dynamics are treated with a combined model, partly model-based and partly data-driven. To this end, a neural network was developed that uses the two most recent target positions to estimate the model’s unknown dynamic parameters.

Organization and Chapter Contributions

The organization of this thesis follows the structure of the contributions presented in the previous section. We first provide an overview of radar technologies, SAR systems, and the cognitive radar framework, which serves as a bridge between engineering and neuroscience. Subsequently, we focus on target tracking for cognitive spaceborne radar systems.

In Chapter 1, we first introduce the radar system, focusing on its key measurements, e.g., range, which determines the distance to a target of interest; Doppler shift, which indicates the relative velocity between the radar and the target; and the target elevation and azimuth angles. After presenting the block diagram of a radar system, we discuss the radar equation, which serves as a fundamental tool for radar system design, and finally introduce the concept of resolution. Subsequently, we focus on the SAR system, introducing the concept of synthetic aperture and explaining how it enables a significantly improved azimuth resolution compared to a conventional radar system, a key parameter in spaceborne remote sensing. The main operating modes of this sensor are also introduced. In particular, stripmap mode, a high-swath, low-resolution configuration in which the swath movement over the Earth surface is determined by the platform motion; and spotlight mode, a low-swath, high-resolution configuration in which the payload performs beam steering to continuously illuminate a specific area on the Earth surface for an extended period of time. Finally, the concept of cognitive radar for target tracking is introduced. The four fundamental building blocks of this framework are described: the perception-action cycle, memory, attention, and intelligence. A block diagram is presented to illustrate how these elements enhance a radar system by acting as enablers of intelligent behavior. This is followed by a more detailed description of each building block.

In Chapter 2 we propose an optimal control approach for a basic cognitive spaceborne radar, e.g., that only leverages the perception-action cycle, that has to track a ground-moving target. The target is represented by a linear kinematic model. Motion on the Earth surface is approximated as planar motion in the East-North plane of the East-North-Up Cartesian coordinate frame. The radar performs noisy measurements of both the satellite-to-target range and the range rate. The parameters of the transmitted signal, in this case a linear frequency modulation with Gaussian amplitude modulation, determine the measurement noise statistics of the radar, according to the model introduced in [7]. It is assumed that the radar has full knowledge of the satellite state, which moves on a circular orbit. The spaceborne radar employs an EKF to track the target of interest. During the update step, the filter uses the radar measurements to refine the target estimate. Since these measurements are functions of the transmitted signal parameters, the EKF can be reformulated as a dynamical system, and this formulation is used to cast the waveform-selection problem as an optimal control problem. The controller's objective is to determine the optimal sequence of transmitted waveforms that minimizes the estimation error over the entire simulation horizon. The estimation error — i.e., the discrepancy between the target's true state and the filter estimate — is approximated by the trace of the EKF covariance matrix. Simulations demonstrate that this approach outperforms algorithms commonly used in the literature.

In Chapter 3 we focus on maritime surveillance within a Tip & Cue framework, which envisages cooperation between two satellites to acquire more relevant data over a given area of interest. In the first section, we propose a GPR method to reconstruct vessel trajectories from AIS messages. AIS is commonly employed by cooperative ships to broadcast information such as position, velocity, and heading. Within this framework, two GPR models were trained to learn the temporal evolution of vessel longitude and latitude using only 25% of the available messages. The algorithm was validated in terms of Mean Absolute Error over the entire simulation horizon, which includes a period of observability by the AIS sensors onboard the two satellites and a period of non-observability during which transmissions from the vessels cannot be received by either spacecraft. The algorithm was also ported to space-qualified FPGAs to evaluate computational complexity in terms of training time, inference time, and the number

of vessels that can be tracked within a five-minute interval. In the second section, a target-tracking algorithm was developed that leverages AI-based SAR detections. Lacking an operational SAR simulator, detections were synthetically generated as bounding boxes in the raw SAR data (i.e., range samples - azimuth lines), assuming a specified detection probability and a given rate of false positives. A georeferencing procedure was then presented to map the detected centroid from the raw-SAR domain to coordinates on the Earth surface. Those coordinates were used as measurements in an AI-enhanced EKF for the target of interest. The vessel dynamics, also employed by the EKF to estimate the target state, follow a hybrid model, partly model-based and partly data-driven, whose unknown parameters were learned by a neural network that receives the target's two most recent positions as input.

Notation

The symbols $\mathbb{R}$ and $\mathbb{C}$ denote the set of real and complex numbers, respectively. The i^{th} element of a vector $x \in \mathbb{R}^n$ is denoted as $x[i]$, and its Euclidean norm is denoted as $\|x\|$. Given a vector $x \in \mathbb{R}^n$, its transpose is denoted as $x^\top \in \mathbb{R}^{1 \times n}$. Given a random vector $x \in \mathbb{R}^n$, its expectation is denoted as $\mathbb{E}[x]$. A matrix of zeros of n rows and m columns is referred to as $0_{n \times m}$, while $I_{n \times n}$ denotes the identity matrix of dimension n . Given a matrix $A \in \mathbb{R}^{n \times m}$, its component on the i^{th} row and j^{th} column is denoted as $A_{[i,j]}$. A Gaussian distributed random vector $w \in \mathbb{R}^n$ with mean $\mu \in \mathbb{R}^n$ and covariance matrix $Q \in \mathbb{R}^{n \times n}$ is denoted as $w \sim \mathcal{N}(\mu, Q)$. We denote by $\mathbb{K}$ the sequence $\mathbb{K} = \{0, \dots, K\}$, while $\mathbb{K}_{-1}$ denotes $\mathbb{K}_{-1} = \{0, \dots, K-1\}$. Given the vector $x_k \in \mathbb{R}^n$ at discrete time step k , we use $x_{k:K}$ to denote the sequence $\{x_k, x_{k+1}, x_{k+2}, \dots, x_K\}$. A vector $e \in \mathbb{R}^n$ is the i^{th} canonical vector if $e[i] = 1$ and $e[j] = 0$, for $j = 1, \dots, n, j \neq i$. Given an angle $\theta \in \mathbb{R}$, we use c_θ and s_θ to denote $\cos(\theta)$ and $\sin(\theta)$, respectively.

Part 1

Chapter 1

Foundations of Radar Systems and Cognitive Paradigms

This chapter provides an overview of radar and SAR systems, highlighting their main functionalities and the differences between these two types of sensors. It then introduces the concept of cognitive tracking radar, focusing on the description of each of its fundamental building blocks.

1.1 Introduction to Radar Principles

1.1.1 Basic Principles

Radar is a contraction of the words radio detection and ranging [13]. It follows that the radar is an electromagnetic system used for the detection and location of targets of interest. It operates by transmitting a given waveform and detects the nature of the echo signal.

A radar can be designed to see through those conditions which are impervious to normal human vision, such as darkness, haze, fog, rain, and snow.

The most important attribute of a radar is to be able to measure the distance, or range, to the object.

Key Measurements

A radar system can measure four quantities to localize a given target of interest: range, radial velocity, azimuth angle, and elevation angle.

The range, i.e., the line-of-sight distance between the object and the sensor, is measured from the time delay between the transmission and the reception of a given pulse. Denoting this time delay as T , the range between the radar and the target can be expressed as

$$R = \frac{cT}{2}, \quad (1.1)$$

where c is the speed of light, and the factor 2 is due to the two-way propagation of the radar.

The Doppler shift is a change in the frequency of the received signal caused by the relative motion between the target and the radar. Given the wavelength of the transmitted signal λ , the Doppler shift of the return echo f_d is

$$f_d = \frac{2v \sin \theta}{\lambda}, \quad (1.2)$$

where θ is the off-boresight angle, that is the angle between the target and the line-of-sight direction of the radar, and v is the relative velocity.

The angular position (azimuth and elevation) is determined by the pointing direction of the antenna's main beam. Modern systems employ techniques like monopulse processing to achieve angular accuracy finer than the beamwidth limitation [13, 14]

Radar Resolution

Resolution is one of the key parameters in assessing the quality of a radar system, as it provides an indication of the sensor's capability to distinguish between different targets.

The range resolution of a radar is defined as the minimum separation in range at which two targets can be distinguished as distinct by the radar [14]. It can be computed as

$$\Delta R \simeq \frac{c}{2B}, \quad (1.3)$$

where B is the bandwidth of the transmitted signal.

The Doppler shift resolution Δf_d defines the radar's ability to distinguish between two targets based on their relative radial velocity. It is defined as the inverse of the coherent processing interval T_{cpi} , that is the contiguous time period during which a radar system preserves and processes the phase information of successive received pulses as a single, coherent ensemble

$$\Delta f_d = \frac{1}{T_{\text{cpi}}}, \quad (1.4)$$

therefore, the velocity resolution Δv is

$$\Delta v = \frac{\lambda}{2T_{\text{cpi}}}. \quad (1.5)$$

The azimuth and elevation resolution, denoted as Δx_{az} and Δx_{el} respectively, define the radar's ability to distinguish between two targets based on their angle of arrival. They are fundamentally limited by the antenna beamwidth, which is inversely proportional to the antenna electrical size

$$\begin{aligned} \Delta x_{\text{az}} &\simeq \frac{R\lambda}{L} = R\theta_{\text{az}} \\ \Delta x_{\text{el}} &\simeq \frac{R\lambda}{W} = R\theta_{\text{el}} \end{aligned} \quad (1.6)$$

where L is the antenna length, W is the antenna width, and $\theta_{\text{az}}, \theta_{\text{el}}$ are the antenna beamwidth in azimuth and elevation, respectively.

1.1.2 Block Diagram

A radar system is generally composed by three subsystems: the transmitter, the receiver, and signal processing [15], as shown in Fig. 1.1.

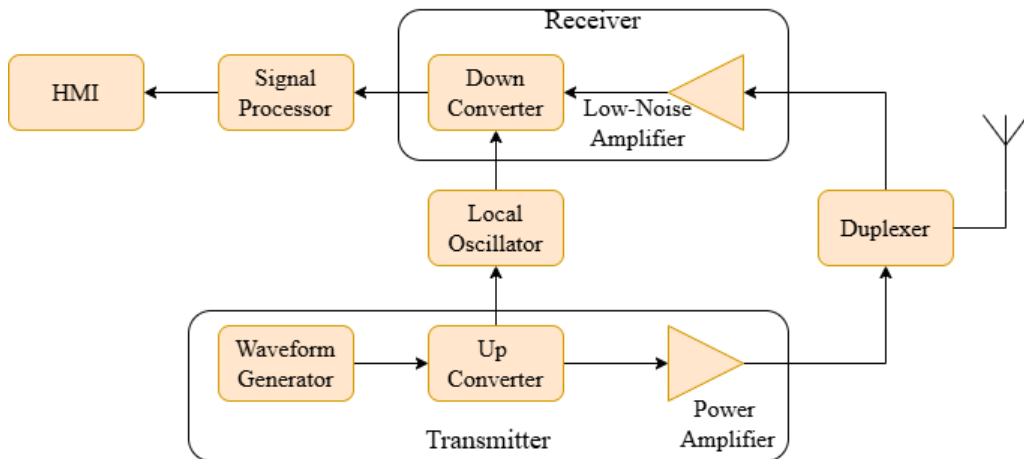


Figure 1.1: Simplified Block diagram of a radar system.

The transmitter subsystem is the source of the signal. The ranging capabilities of the radar are mainly determined by the transmitter design, which makes the power

generated and the associated costs important characteristics to consider. The transmitter is composed of the waveform generator, the up-converter, and the power amplifier.

The receiver subsystem functions as the receiver of the backscattered signal. A portion of the transmitted signal is intercepted by a reflecting object and it is reradiated in all directions. The receiver processes this signal to detect the presence of the target and to extract its location and, if either the radar or the target is non-stationary, the relative velocity. The echo signal, however, is contaminated by clutter and other unwanted signals. Therefore, the receiver should be designed to maximize the Signal-to-Noise Ratio (SNR) of the desired signal by rejecting, or suppressing, unwanted signals. Its main components are the low-noise amplifier and the down-converter.

The signal processing subsystem applies various algorithms to the echo signal to extract relevant information to determine the target position. Powerful signal processing algorithms are required due to the presence of clutter. However constraints such as limited hardware resources and installation space limit the complexity of algorithms that can be implemented.

1.1.3 Radar Equation

The radar equation establishes the relationship between the radar range and the characteristics of the transmitter, receiver, antenna, target, and environment [13]. This equation is employed both to determine the maximum detectable distance between the radar and the target, hence the overall performance of the sensor, and to provide a fundamental basis for radar system design.

If the radar transmits a signal with power P_t through an isotropic antenna, which radiates uniformly in all directions, the non-directional power density S_p at range R from the radar is

$$S_p = \frac{P_t}{4\pi R^2}, \quad (1.7)$$

that is, the ratio between the transmitted power and the surface area of an imaginary sphere of radius R centered at the radar. However, the radar employs a directive antenna to channel the transmitted power P_t into a specific direction in space. The transmit antenna gain $G_t(R, \theta, \phi)$ is function of the spherical coordinate, e.g., range R , azimuth angle θ and elevation angle ϕ , which will be omitted in the following equations for simplicity. It quantifies the increase in radiated power in a given direction with respect to the power that would be radiated by an isotropic antenna. Therefore, the directive power density S_t can be expressed as

$$S_t = G_t \cdot S_p = \frac{P_t G_t}{4\pi R^2} \quad (1.8)$$

The target intercepts the incident power and reflects it back. This property of the radar is referred to as the radar cross section σ . Therefore, the power density reflected by the target S_r , in the vicinity of the radar, is expressed as

$$S_r = \frac{P_t G_t \sigma}{(4\pi R^2)^2} \quad (1.9)$$

The capability of the antenna to capture the received power density is characterized by its effective aperture A_e . Accordingly, the received power P_r is given by

$$P_r = \frac{P_t G_t \sigma A_e}{(4\pi R^2)^2} = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4}, \quad (1.10)$$

where $G_r(R, \theta, \phi)$ is the receive antenna gain.

The maximum range $R_{\max}$ is defined as the distance beyond which a target can no longer be detected. It is obtained when the received power P_r equals the minimum

detectable signal of the sensor $S_{\min}$. Therefore, the maximum range is given by

$$R_{\max} = \left[\frac{P_t G_t A_e \sigma}{(4\pi)^2 S_{\min}} \right]^{1/4} \quad (1.11)$$

Equation 1.11 is the fundamental form of the radar equation.

1.2 Synthetic Aperture Radar Fundamentals

This section provides an overview of SAR systems, introducing both the concept of synthetic aperture, which enables SAR to achieve finer azimuth resolution, and the main operational modes which will be considered in this work.

SAR is an active sensing system mounted on moving platforms, typically aircraft, satellites, or unmanned aerial vehicles. It generates high-resolution imagery by exploiting the motion of the platform along its flight path.

1.2.1 The Resolution Problem and the Synthetic Aperture Concept

As outlined in Section 1.1.1, resolution represents one of the fundamental parameters for assessing the performance of a radar system.

The azimuth resolution, given by Equation (1.6), is strongly dependent on the transmitted wavelength λ and on the antenna length L . Therefore, in order to improve azimuth resolution for a fixed range R and wavelength λ , it is necessary to increase the antenna length L .

The mechanical problems involved in constructing an antenna with a surface precision accurate to within a fraction of a wavelength, and the difficulty in maintaining that level of precision in an operational environment, make it quite difficult to attain values of L/λ greater than a few hundred [16]. However, from a space platform, with $R = 800\text{km}$, even a value of $L/\lambda = 200$ yields only an azimuth resolution $\Delta x_{\text{az}} = 4\text{km}$, which is unacceptable for use in scientific applications that typically require high-resolution imagery.

The key insight that ultimately led to SAR is originally attributed to Wiley [17]. He observed that two point targets, at slightly different angles with respect to the track of the moving radar, have different speeds at any instant relative to the platform. Therefore, the radar pulse when reflected from the two targets will have two distinct Doppler frequency shifts. If the received signal is frequency analyzed, any energy observed in the return at time corresponding to range R and at Doppler frequency f_{d1} will be associated with a target at coordinate

$$x_1 = \frac{\lambda R f_{d1}}{2v}. \quad (1.12)$$

Similarly, energy at a different frequency f_{d2} will be assigned to a corresponding coordinate x_2 . Therefore, even though the targets are at the same range and in the same beam at the same time, they can be discriminated by an analysis of the Doppler frequency spectrum of the return signal.

This insight has been the key element in achieving finer azimuth resolution in a SAR system. Such resolution is computed as

$$\Delta x_{\text{az}} = \frac{L}{2}, \quad (1.13)$$

which is a great improvement with respect to the radar azimuth resolution. We refer the reader to [16] for the complete derivation of Equation (1.13).

1.2.2 SAR Imaging Modes

SAR imaging modes are generally based on a side-looking geometry and take advantage of the motion of the platform to synthesize an aperture.

Several operational modes are available, each characterized by different swath widths, spatial resolutions, and data-processing requirements.

Stripmap

Stripmap SAR imaging is the original mode used in the first systems [18], [19].

As the SAR platform moves along its flight path, the sensor illuminates and images a specific portion of Earth's surface, as shown in Figure 1.2. This area, often referred to as ground swath, can vary between tens and hundreds of kilometers for a spaceborne SAR, and is characterized by the azimuth resolution given in Equation (1.13). For traditional stripmap SAR imaging, the antenna beam is pointed at a fixed off-broadside angle, as the ground swath is illuminated by a continuous sequence of pulses.

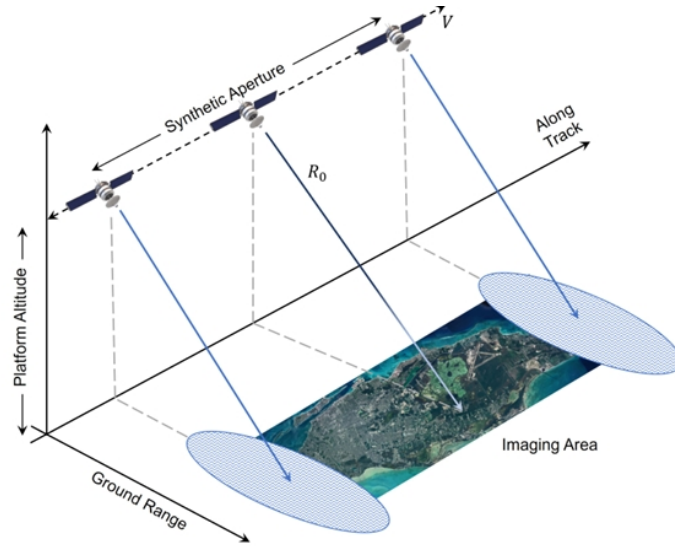


Figure 1.2: Basic geometry of stripmap SAR acquisition. Courtesy of [20].

The area on the ground illuminated by a single pulse is, in general, an ellipse. For a sequence of pulses, the ellipse moves along the ground following the platform's flight path.

Spotlight

Spotlight SAR uses mechanical or electronic beam steering to keep the target area illuminated for a longer period, as depicted in Figure 1.3.

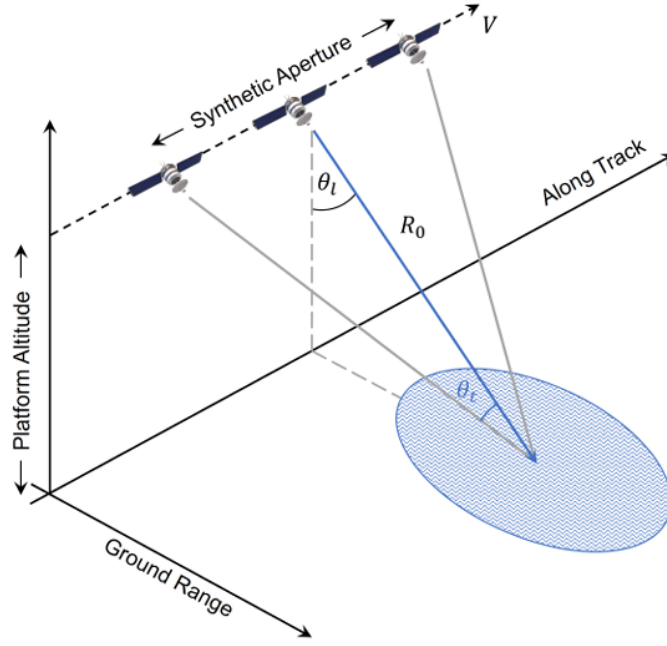


Figure 1.3: Basic geometry of spotlight SAR acquisition. Courtesy of [20].

While this reduces the scene size that is imaged, it leads to higher-resolution images than stripmap mode. Indeed, the azimuth resolution of a SAR operating in spotlight mode is [20]:

$$\Delta x_{az} = \frac{\lambda}{4 \sin(0.5(\theta_{sa}))} = \frac{\lambda}{4 \sin(\theta_t/4)}, \quad (1.14)$$

where $\theta_{sa} = 2\theta_t$ is the angle subtended to the synthetic aperture, and θ_t is the angle from the aperture extremity to the ground swath mid-point.

Whereas a stripmap SAR typically attains azimuth resolutions on the order of few metres, a spotlight acquisition can provide sub-metre resolution [21].

Despite the increased resolution of the spotlight mode, no single mode is universally superior; the choice of mode should be guided by the mission objectives. By its nature, stripmap mode can be exploited to image extensive areas of the Earth surface for initial detection and localization of potential regions of interest, such as zones affected by natural disasters or other relevant targets. Spotlight mode, instead, is particularly suitable for focusing on specific regions of the Earth surface previously identified as areas of interest, enabling the acquisition of more informative data, thanks to the combination of a reduced ground swath and higher spatial resolution.

1.2.3 Raw SAR Data Formation

Transmitted Signal Model

A stripmap SAR system transmits a train of pulses, where each of them is a Linear Frequency Modulated (LFM) signal. The complex baseband representation of a single transmitted pulse is [22]:

$$s(\tau) = \text{rect}\left(\frac{\tau}{T_p}\right) \exp(j2\pi f_c \tau + j\pi \beta \tau^2), \quad (1.15)$$

where

- τ is the fast-time variable, representing time within each Pulse Repetition Interval (PRI),

- T_p is the pulse duration,
- f_c is the carrier frequency,
- β is the chirp rate.

Received Signal Model

The echo signal is a delayed, attenuated, and phase-shifted version of the transmitted signal. For a target at slant range $R(t)$, the received signal is [22]

$$s(t, \tau) = \sigma \cdot \text{rect}\left(\frac{\tau - 2R(t)/c}{T_p}\right) \exp\left(j2\pi f_c\left(\tau - \frac{2R(t)}{c}\right) + j\pi\beta\left(\tau - \frac{2R(t)}{c}\right)^2\right) \quad (1.16)$$

where t is the slow time, which indexes the sequence of transmitted pulses, representing the along-track position of the platform.

Raw SAR Data

The received signal $s(t, \tau)$ is digitized onboard, producing a two-dimensional complex matrix $s[m, n] \in \mathbb{C}^{N_{\text{rg}} \times N_{\text{az}}}$, as depicted in Figure 1.4, where the row index m denotes range samples (fast-time) and the column index n denotes azimuth lines (slow-time).

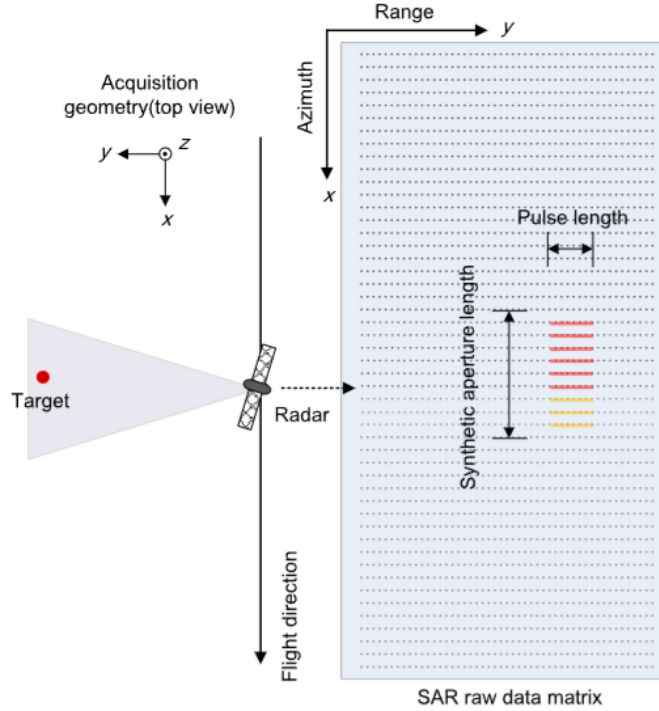


Figure 1.4: Raw SAR matrix acquisition. Courtesy of [23].

The number of columns, i.e. azimuth lines, N_{az} , denotes the number of pulses transmitted during the acquisition, hence

$$N_{\text{az}} = T_{\text{acq}} \cdot \text{PRF} = \frac{T_{\text{acq}}}{\text{PRI}}, \quad (1.17)$$

where PRF is the Pulse Repetition Frequency, and T_{acq} is the time duration of the acquisition.

The number of rows N_{rg} corresponds to the number of data points acquired for each echo signal:

$$N_{\text{rg}} = F_s \cdot \frac{2R_{\text{swath}}}{c}, \quad (1.18)$$

where F_s is the Analog-to-Digital Converter sampling frequency, used to digitize the continuous echo signal, and R_{swath} is the swath width in slant range.

1.3 The Cognitive Tracking Radar Paradigm

Simon Haykin first introduced the concept of Cognitive Radar (CR) in [5]. Drawing on the Oxford English Dictionary's definition of *cognition* as *knowing, perceiving, or conceiving as an act*, he identifies the three fundamental components that constitute such a system:

1. Intelligent signal processing, which relies on learning from the interactions of the radar with its surrounding environment;
2. A feedback mechanism between the receiver and the transmitter;
3. Preservation of the informational content of the radar returns.

Nevertheless, the high-level block diagram of a CR was first presented by Haykin himself in [6], where he focuses on the system architecture for target tracking. The cognitive radar architecture here proposed is based on mimicking the visual brain, since both serve as observers of that environment.

Therefore, there is much to be learned from the visual brain in order to devise a novel radar architecture that outperforms traditional active radar systems.

To contextualize the subsequent description of cognitive radar, the author first delineates the following three classes of radar systems:

- **Traditional Active Radar (TAR):** operates primarily in a feedforward manner. Although TAR lacks a feedback channel between receiver and transmitter, implementations frequently employ adaptive filtering at the receiver for iterative state estimation [24], and adaptive beamforming to better match transmitted waveforms to the propagation environment [25].
- **Fore-Active Radar (FAR):** is characterized by the presence of a feedback link from the receiver to the transmitter. This feedback provides a mechanism for limited intelligence, constituting an initial step toward cognitive capability, and is commonly exploited to inform transmitter waveform selection [7].
- **Cognitive Radar (CR):** is distinguished from TAR and FAR by its capacity to derive rules of behavior in a self-organizing manner through learning from experience, a process that emerges from sustained interactions with the environment [26].

1.3.1 Building Blocks of Cognitive Radar

Much of our understanding of cerebral functions has been based on the following approach: a simplistic mapping of cognitive constructs onto discrete brain regions. This traditional view of human cognition may be regarded as a *modular paradigm*, in which specific brain areas are postulated to operate as independent processors carrying out distinct functions.

However, a new paradigm has emerged in cognitive neuroscience that transcends the modular framework, wherein emphasis is placed on the following postulate [27]:

Conjoint functions of brain areas, which work together as a large-scale network.

Building on this new paradigm, the proposed CR design is based on the four fundamental building blocks of cognition described by Fuster [28]:

1. **Perception-Action Cycle (PAC):** a feedback link about the environment from the receiver to the transmitter.
2. **Memory:** one portion of which resides in the receiver and the other in the transmitter. These two components are coupled together, and the learning process in a cognitive radar takes place predominantly within this memory subsystem.

3. **Attention:** algorithmic in nature, one part operates in the receiver and the other part in the transmitter.
4. **Intelligence:** also algorithmic in nature but distributed throughout the system.

According to Fuster, memory is driven by the PAC; attention is memory driven; and intelligence builds on all three of them.

1.3.2 Perception-Action Cycle

The perception-action cycle serves as the fundamental enabler of intelligence within a CR, establishing a feedback link between the receiver and the transmitter. By analyzing the received signal, the radar can adapt its transmission configuration to improve its performance.

Figure 1.5 illustrates the role of the PAC and memory within the cognitive radar architecture for target tracking [26].

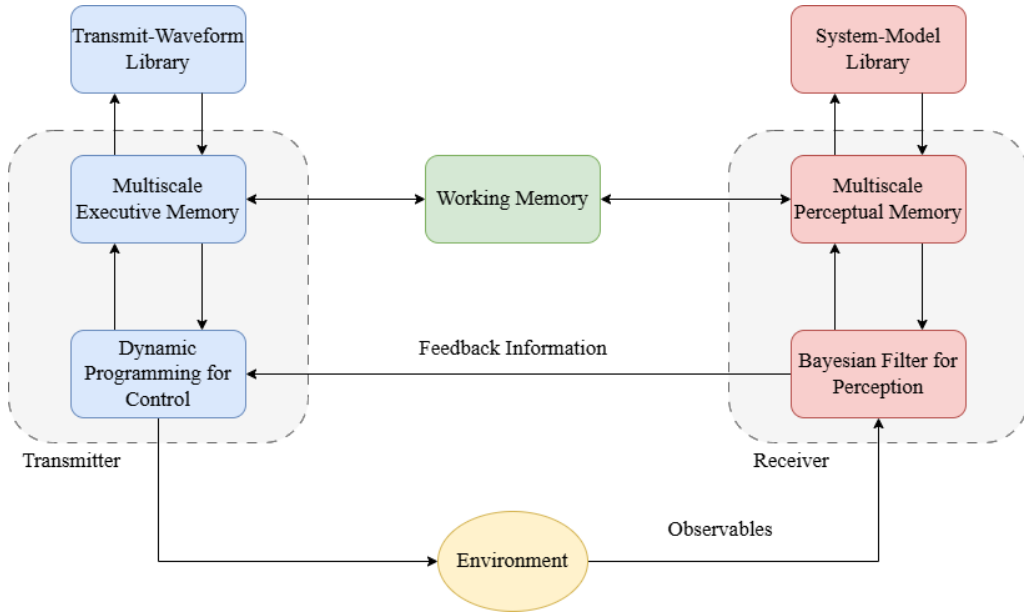


Figure 1.5: Block diagram of a cognitive radar for target tracking.

The cognitive radar gains control over aspects of the sensing process through waveform adaptation, which is performed in the PAC. Therefore, this link ties together estimation (through sensing) and control [6].

A state-space model is required to tackle the target tracking problem. It consists of the following two equations

$$\begin{aligned} x_{k+1} &= f_k(x_k, w_k) \\ z_k &= h_k(x_k) + n_k(u_{k-1}), \end{aligned} \quad (1.19)$$

where

- $x_k \in \mathbb{R}^n$ denotes the target state, which includes variables of interest such as, but not limited to, position and velocity.
- $f_k(\cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the target dynamical model, which describes the target state evolution across time, affected by the noise vector $w_k \in \mathbb{R}^n$.
- $z_k \in \mathbb{R}^m$ is the radar measurement vector.
- $n_k(\cdot) : \mathbb{R}^p \rightarrow \mathbb{R}^m$ denotes the radar measurement error, which is dependent on the transmitted signal parameter vector $u_k \in \mathbb{R}^p$.

- $h_k(\cdot, \cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ represents the radar measurement function.

Perception is executed in the receiver. It processes the radar measurement vector z_k associated with an unknown target, with the dual objectives of reliable target detection and tracking across time.

Target tracking in cognitive radar is treated as a state-estimation problem under a Bayesian framework [5]. The state-space model given by Equation (1.19) may be nonlinear, therefore, we look to an approximate Bayesian filter to perform perception of the environment. The Extended Kalman Filter (EKF), see Appendix A, is a candidate algorithm within this framework.

The receiver is responsible for generating the feedback information about the external environment, and conveying it to the transmitter. Since state estimation falls within the purview of perception, it is therefore logical to express the feedback in terms of the state-estimation error vector $\|x_k - \hat{x}_{k|k}(u_k)\|$, where $\hat{x}_{k|k}$ denotes the input-dependent estimated target state at discrete time k .

The role of the transmitter, in turn, is to exert control over the receiver indirectly by illuminating the surrounding environment and adapting the transmitted waveform, with Bellman's dynamic programming being a candidate approach [29].

To summarize, the PAC framework for target tracking embodies four main steps:

- Bayesian filter for state-estimation in the receiver.
- Feedback information from the receiver to the transmitter.
- Control algorithm for receiver-control performed in the transmitter.
- State-space model of the radar environment.

1.3.3 Memory

A cognitive radar must learn from experience gained through continuous interaction with the environment [5]. This is fulfilled by equipping the radar system with memory.

In a cognitive radar, the memory consists of three subunits: the perceptual memory, the executive memory, and the working memory. The perceptual memory is located in the receiver, the executive memory in the transmitter, while the working memory couples the two, as shown in Figure 1.5.

Perceptual Memory

The perceptual memory resides within the receiver and, ideally, should possess a multiscale structure. In the neural-network literature, this kind of structure is often referred to as *features of features* [30]. Through a learning process, the first layer of perceptual memory extracts the salient features characterizing the measurement vector z_k . These features then serve as input to the second layer of perceptual memory, which goes on to extract the features of features that characterize the measurement z_k , and so on for the third layer.

The *features of features* paradigm corresponds to a learning mechanism that goes on in an associative memory of the heterogeneous kind [31], [32], [33], [34].

In practice, the associative memory functions as a correlator between the measurement space and the system-model space, as shown in Figure 1.6.

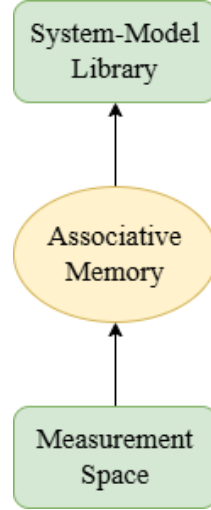


Figure 1.6: Block scheme of perceptual memory as a heterogeneous associative memory.

Its objective is to match each point in the system-model space to a corresponding point in the measurement space.

Executive Memory

The executive memory resides in the transmitter. Its architecture is analogous to that of perceptual memory, with the key distinction that whereas perceptual memory perceives the environment directly via the measurement vector z_k , executive memory perceives it indirectly through the feedback information on the environment provided by the receiver.

It is also reciprocally coupled to another repository, known as the *transmit-waveform library*. Each grid point in this library corresponds to a distinct combination of transmitted-signal parameters u_k . The executive memory processes the feedback information to match a point in the transmit-waveform library.

Working Memory

The working memory serves to couple the perceptual and executive memory, thereby harnessing their combined capabilities. It has a limited capacity and provides the interface between perception and action by linking the perceptual and executive subsystems, as illustrated in Figure 1.5.

Unlike perceptual and executive memory, which are both long-term working memories, it is a short-term store, used for the temporary retention of information [35].

The function of working memory in a CR is to predict the consequences of actions performed by the receiver, the transmitter, or both. In a CR, the working memory consists of a neural network of the heterogenous associative kind [6].

1.3.4 Attention

Memory-based attention manifests itself in the receiver and the transmitter with an inherently algorithmic nature, differently from the PAC and memory, which occupy distinct physical locations within the cognitive radar. Therefore, we have perceptual attention in the receiver and executive attention in the transmitter.

Executive attention in a cognitive tracking radar implements the so-called explore-exploit strategy. The executive memory selects a grid point in the transmit waveform library to closely match the feedback information on the preceding cycle. For the current cycle in the PAC, the radar may therefore look to that previous point as a center point to expand on, e.g., to consider a set of grid points in the transmit-waveform library that lie in the immediate neighborhood of the center point, as illustrated in Figure 1.7.

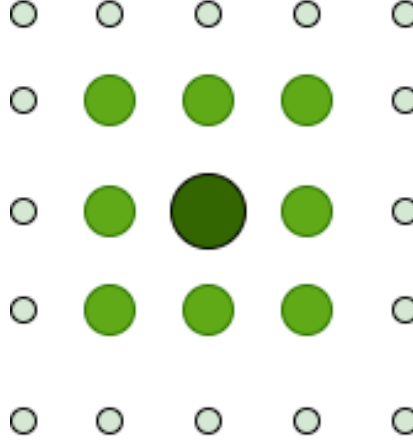


Figure 1.7: Transmit-waveform library grid point. The darkened grid point was selected on the preceedeing cycle; together with the surrounding eight grid points make the cluster to be passed onto the controller.

Hence, the computational effort related to a global search of the entire library is replaced with a local search based on a small cluster of grid points.

1.3.5 Intelligence

Similar to attention, intelligence manifest itself within the cognitive radar through an algorithmic mechanism, driven by the combination of attention, memory, and the perception-action cycle. It follows that intelligence is the most powerful of all the building blocks of cognition under Fuster's paradigm.

Once the executive attention provides the cluster of grid points, as depicted in Figure 1.7, the function of the controller in the transmitter is to find the optimal transmitted waveform parameter vector u_k^* such that

$$u_k^* = \underset{u_k \in U_k}{\operatorname{argmin}} \quad g(x_k, \hat{x}_{k|k}(u_k)), \quad (1.20)$$

where U_k denotes the set of grid points in the transmit-waveform library and $g(\cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ is the cost function.

Two opposing control strategies now arise:

1. **global search** of the grid points in the transmit-waveform library, where the executive attention is not leveraged but global optimization for waveform selection is guaranteed, at the expense of an increased computational complexity;
2. **local search** of the grid points, where executive attention comes into play, resulting in a suboptimal selection of the transmit waveform, leading to a worsened tracking performance. However, this comes with a conservation of computational resources.

This choice is a tradeoff between conservation of computational resources and tracking accuracy, which is an example of the *no-free-lunch* theorem [36].

Part 2

Chapter 2

Optimal Control-Based Target Tracking for Cognitive Spaceborne Radars

In this chapter, we address the problem of cognitive target tracking for a ground-moving target. We consider a framework in which the radar has to select the optimal transmitted waveform to minimize a given objective function depending on (i) the satellite-to-target range and (ii) the range rate. The ground-moving target coordinates are measured via nonlinear mapping affected by noise and, thus, we resort to an Extended Kalman Filter (EKF) for target state estimation. The main contribution presented here is the formulation of the optimal waveform selection problem as an optimal control problem. This is achieved by modeling the EKF as a dynamic system. Such a formulation enables the controller to determine the optimal sequence of transmitted signal inputs. The performance of the optimal control approach is compared with the results obtained through myopic dynamic optimization. Numerical simulations show that the former provides better tracking performance.

2.1 Literature Review

A cognitive tracking radar emulates the visual brain [28] and leverages the received signal to intelligently adapt its transmission configuration [6]. The objective is to minimize the target state estimation error, computed through a Kalman filter, or one of its extensions in the case of a nonlinear state-space model. This filter exploits both the target dynamics and the radar measurements to perform tracking. However, these measurements are affected by noise, whose characteristics depend on the transmitted signal parameters [7]. Consequently, a cognitive tracking radar can exploit this dependency to determine the optimal transmission parameters that minimize the estimation error. Paper [6] is the first to present a description of a cognitive tracking radar that integrates Fuster's four fundamental building blocks [28]: the Perception-Action Cycle (PAC), attention, memory, and intelligence. A basic cognitive radar, or fore-active radar, only employs the PAC to improve its performance [6]. Kershaw's work [7] is a seminal contribution to this domain, providing the first description of a radar system that adaptively modifies the transmitted waveform to optimize the estimation of the measured target, paving the way toward fore-active tracking radars. It also characterizes the dependence of the radar measurement error on transmitted-signal parameters, providing this analysis across several classes of transmitted waveforms. The authors assume linear target dynamics and a ground-based radar. The radar measures are modeled through a linear measurement equation, and it is assumed that the radar can measure the radar-to-target range and range rate. The single-step dynamic optimization approach is used to select the optimal waveform. In [37], the authors extend this model for a single target in clutter. In this updated framework, the transmitted pulses do not yield a single, unambiguously associated measurement. Instead, the radar measurements are modeled to account for potential missed detections and false alarms. The update step of the tracking algorithm employs a probabilistic association mechanism [38] to manage ambiguous measurement-to-track associations. As in the clutter-free scenario, a myopic optimization strategy is utilized for optimal waveform selection. In [39] and [40], the authors tackle the optimal waveform selection problem assuming a nonlinear measurement equation. They also deal with a ground-based radar and the optimal waveform is computed via single-step dynamic optimization. The works [41], [42] and [43] give an emphasis on a longer optimization horizon. In [41], the authors tackle the fore-active tracking radar problem via a moving horizon approach. In [42], instead, the authors propose an infinite horizon optimal control scheme for a ground-based

tracking radar. Furthermore, they present a Monte Carlo Tree Search algorithm with double progressive widening. The work [43] poses the optimal tracking problem in terms of a Markov Decision Process. However, these works deal with ground-based radars.

Spaceborne tracking radar models are proposed in [44], [45] and [46]. The authors of [44] and [45] focus, respectively, on the sensor and on the functional architecture of a Spaceborne Radar (SBR) for space debris detection and tracking. The work [46] provides a first model for the SBR ground-target tracking problem. The authors compare three different nonlinear filters, and the Extended Kalman Filter results as the one with the best performance. However, in these works the radar does not optimize the transmitted waveform in order to increase the tracking performance.

2.2 Problem Formulation

This section provides an overview of the cognitive spaceborne radar ground-target tracking framework.

A spaceborne radar is a radar mounted on a satellite. It transmits a radio-frequency signal and receives the reflected waveform. The radar then processes the backscattered signal to acquire knowledge of the target of interest. The tracking radar employs a filtering algorithm at the receiver to estimate the target state. Differently from traditional settings, we consider the case in which the radar enhances its performance through a feedback loop connecting the receiver to the transmitter. That is, the radar optimizes the transmitted waveform parameters u_k based on the target estimate. The main goal is to improve the estimation accuracy.

The spaceborne tracking radar configuration comprises three reference frames: the Earth-Centered Earth-Fixed (ECEF) frame, the Earth-Centered Inertial (ECI) frame, and the East-North-Up (ENU) frame. Further details on these reference systems are provided in Appendix B. Figure 2.1 depicts the aforementioned coordinate frames.

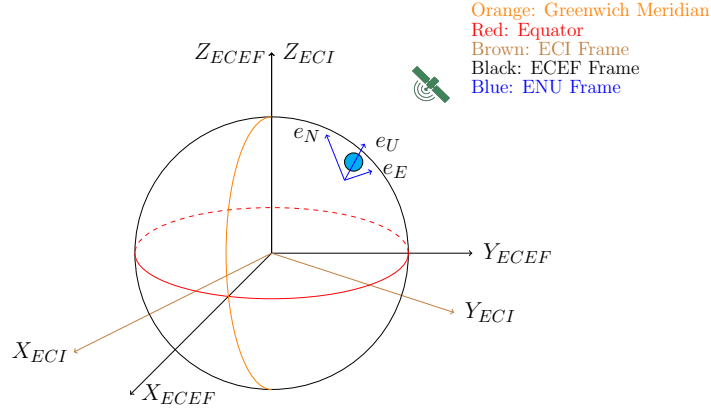


Figure 2.1: ECI, ECEF and ENU Coordinate Frames

In this work, we assume that the Z_{ECEF} and Z_{ECI} axes are aligned, while X_{ECEF} and X_{ECI} are aligned at time step $k = 0$. The target dynamics in the ENU frame is modeled as

$$x_{k+1} = Fx_k + w_k \quad (2.1)$$

where $x_k \in \mathbb{R}^6$ is the target state, $F \in \mathbb{R}^{6 \times 6}$ is the state matrix, and $w_k \in \mathbb{R}^6$ is the system noise. As for the satellite dynamics, we consider a Low-Earth Orbit (LEO) satellite on a circular, noiseless orbit. Under these assumptions, the satellite dynamics in the ECI frame is described as

$$x_{k+1}^e = Ax_k^e, \quad (2.2)$$

where $x_k^e \in \mathbb{R}^6$ is the satellite state in the ECI frame and $A \in \mathbb{R}^{6 \times 6}$ is the state matrix. We refer the reader to Section 2.3 for the definition of the target and satellite state matrix F and A , respectively. In this work, we assume that the radar transmits a Linear Frequency Modulated (LFM) waveform pulse with Gaussian amplitude modulation. The spaceborne radar performs satellite and target-based measurements in the ENU frame

$$z_k = h(x_k, x_k^s) + n_k(u_{k-1}), \quad (2.3)$$

where $x_k^s \in \mathbb{R}^6$ is the satellite state in the ENU frame, $n_k(u_{k-1})$ is the input-dependent measurement noise and $h : \mathbb{R}^6 \times \mathbb{R}^6 \rightarrow \mathbb{R}^2$ is the measurement function. In this work, we adopt an EKF to reconstruct the target state. The target estimate, performed at the receiver, is the feedback information for the transmitter. To select a waveform that reduces the measurement noise, we employ an optimal-control approach.

2.3 Tracking Model

In this section, we provide the considered tracking model. Specifically, we next describe the satellite dynamics, the target dynamics, and the radar measurement function.

2.3.1 Satellite Dynamics

As for the satellite dynamics, we consider a LEO satellite on a circular, noiseless orbit. The satellite state in the ECI frame x_K^e evolves according to (2.2), where the state transition matrix A is

$$A = \begin{bmatrix} I_{3 \times 3} & dt I_{3 \times 3} \\ -\frac{GM}{r_{\text{orb}}^3} I_{3 \times 3} & I_{3 \times 3} \end{bmatrix}. \quad (2.4)$$

that is the Cowell's state transition matrix, assuming a circular, noiseless orbit [47]. Parameter $GM \in \mathbb{R}$ is the geocentric gravitational constant, $r_{\text{orb}} \in \mathbb{R}$ is the orbit radius and $dt \in \mathbb{R}$ is the integration time step. The transformation of the satellite state vector from the ECI frame to the ENU one follows the equations presented in Appendix B. First, the state is transformed from ECI to ECEF by a rotation about the common Z axis. Subsequently, the state is converted from ECEF to ENU according to Equation (B.6), where the reference point is the ECEF coordinates of the ENU-frame origin.

2.3.2 Target Dynamics

We model the ground-target dynamics through the so-called Nearly Constant Velocity (NCV) motion [48] model, according to (2.1). The target state $x_k \in \mathbb{R}^6$ has Cartesian components, namely

$$x_k = \begin{bmatrix} p_k \\ v_k \end{bmatrix} \quad (2.5)$$

where $p_k, v_k \in \mathbb{R}^3$ denote the target position and velocity on the E, N , and U axes of the ENU frame, respectively. As for the state matrix F , it is obtained by discretizing the so-called white noise acceleration matrix $F_c \in \mathbb{R}^{6 \times 6}$ [48] and, thus, it reads as

$$F = \begin{bmatrix} 1 & 0 & 0 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 & dt & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (2.6)$$

We model the additive system noise $w_k \in \mathbb{R}^6$ as a zero-mean multivariate Gaussian distribution $w_k \sim \mathcal{N}(0_{6 \times 1}, Q)$, where the noise covariance matrix $Q \in \mathbb{R}^{6 \times 6}$ is [48]:

$$Q = \begin{bmatrix} q_1 \frac{dt^3}{3} & 0 & 0 & q_1 \frac{dt^2}{2} & 0 & 0 \\ 0 & q_2 \frac{dt^3}{3} & 0 & 0 & q_2 \frac{dt^2}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ q_1 \frac{dt^2}{2} & 0 & 0 & q_1 dt & 0 & 0 \\ 0 & q_2 \frac{dt^2}{2} & 0 & 0 & q_2 dt & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (2.7)$$

where $q_1, q_2 \in \mathbb{R}$ are the power spectral densities along the E and N axes, respectively.

2.3.3 Radar Measurement

We now provide the radar measurement description. To this end, let us introduce the so-called range and range-rate vectors $r_k, \dot{r}_k \in \mathbb{R}^3$ in the ENU frame, defined as

$$\begin{aligned} r_k &= p_k - p_k^s \\ \dot{r}_k &= v_k - v_k^s. \end{aligned} \quad (2.8)$$

Then, we also assume that the radar transmits an LFM waveform pulse with Gaussian amplitude modulation. Hence, the transmitted waveform parameters are $u_k = [\tau_k, b_k] \in \mathbb{R}^2$ [7] where τ_k is the so-called envelope duration and b_k is the so-called chirp-rate parameter of the transmitted signal. With this notation at hand, we can formulate the states-to-measurement function $h(\cdot, \cdot) : \mathbb{R}^6 \times \mathbb{R}^6 \rightarrow \mathbb{R}^2$ of Equation (2.3) as

$$h(x_k, x_k^s) = \begin{bmatrix} \|r_k\| \\ \dot{r}_k^\top \hat{r}_k \end{bmatrix}, \quad (2.9)$$

where $\hat{r}_k = r_k / \|r_k\|$ is a unit vector in the direction of the range vector r_k . The input-dependent measurement noise $n_k(u_{k-1})$ from Equation (2.3) is modelled as a multivariate Gaussian distribution $n_k(u_{k-1}) \sim \mathcal{N}(0_{2 \times 1}, R_k(u_{k-1}))$. The covariance matrix $R_k(u_{k-1}) \in \mathbb{R}^{2 \times 2}$ is, [7],

$$R_k(u_{k-1}) = \begin{bmatrix} \frac{c^2 u_{k-1}[1]^2}{2\eta_k} & -\frac{c^2 u_{k-1}[2] u_{k-1}[1]^2}{2\pi f_c \eta_k} \\ -\frac{c^2 u_k[2] u_k[1]^2}{2\pi f_c \eta_k} & r_{22,k} \end{bmatrix} \quad (2.10)$$

with

$$r_{22,k} = \frac{c^2}{(2\pi f_c)^2 \eta_k} \left(\frac{1}{2u_{k-1}[1]^2} + 2u_{k-1}[2] u_{k-1}[1]^2 \right),$$

where $f_c \in \mathbb{R}$ is the carrier frequency, c is the speed of light, and $\eta_k = (\tilde{r}/r_k)^4 \in \mathbb{R}$ is the signal-to-noise ratio (SNR). Parameter $\tilde{r}$ is the target range at which 0dB SNR is obtained.

Transmitted Waveform

The transmitted signal considered in this work is an LFM signal with Gaussian amplitude modulation. This signal $s_T(t)$ can be described as follows [7]:

$$s_T(t) = \sqrt{2} \text{Re}\{\sqrt{P_t} \tilde{s}(t) e^{j2\pi f_c t}\}, \quad (2.11)$$

where

- P_t denotes the transmitted power,
- f_c is the carrier frequency,

- Re denotes the real part of a complex quantity, whose imaginary unit is denoted by j ,
- $\tilde{s}(t)$ is the complex envelope, given by

$$\tilde{s}(t) = \left(\frac{1}{\pi\tau^2} \right)^{1/4} \exp\left(-\left(\frac{1}{2\tau^2} - jb \right) t^2 \right) \quad (2.12)$$

Figure 2.2 shows the transmitted waveform over 5 seconds, with different values of envelope duration τ and chirp-rate b , and with transmitted power $P_t = 10$ W and carrier frequency $f_c = 9$ GHz.

LFM with Gaussian Amplitude Modulation Waveform

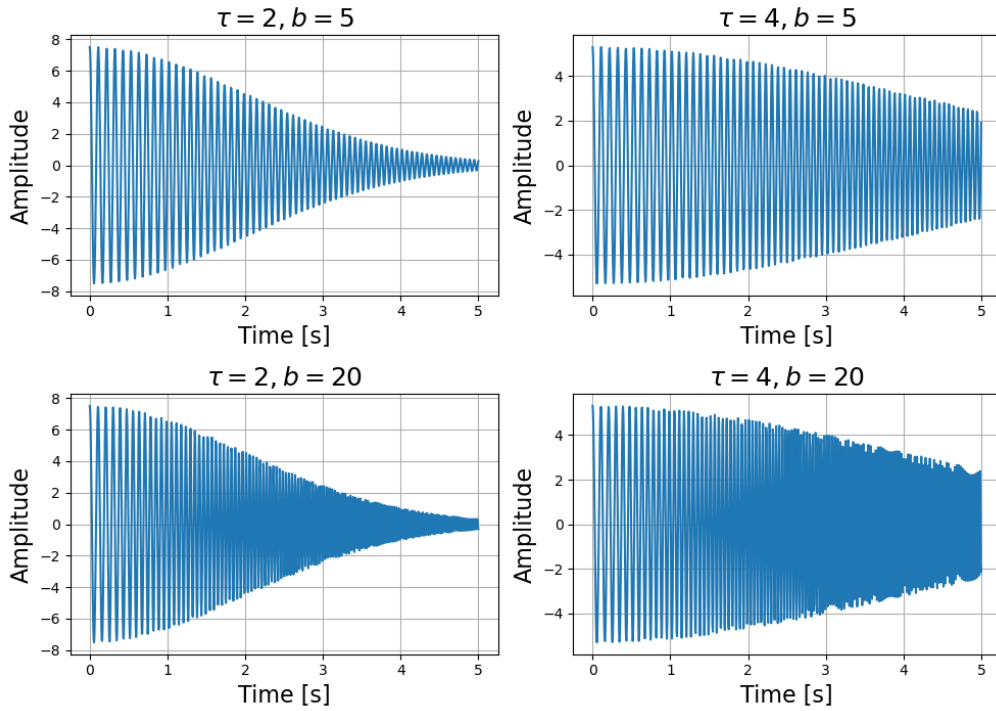


Figure 2.2: Example of transmitted waveforms for different values of chirp rate b and envelope duration τ .

From Figure 2.2, the influence of the chirp rate b and the envelope duration τ on the transmitted signal can be observed. Specifically, increasing the chirp rate b results in a steeper slope of the linear frequency variation over time, whereas increasing the envelope duration, defined as the temporal width of the Gaussian amplitude modulation, leads to a longer pulse duration.

2.4 Extended Kalman Filter

The Extended Kalman Filter [49] is a locally direct approximation to the Bayesian filter under the Gaussian assumption [50]. It is the extension of a Kalman Filter, see Appendix A, for a nonlinear state-space model. Similarly to the Kalman Filter, it incorporates two steps: prediction and update.

2.4.1 Prediction Step

In the prediction step, the EKF employs the state dynamics, along with the estimated state $\hat{x}_{k|k}$ and covariance matrix $P_{k|k}$ at discrete time k , to predict the values that these

quantities will assume at time $k + 1$.

The NCV dynamical model in Equation (2.1) is linear, therefore the prediction step corresponds to the one of the linear Kalman filter, that is,

$$\begin{aligned}\hat{x}_{k+1|k} &= F\hat{x}_{k|k} \\ P_{k+1|k} &= FP_{k|k}F^\top + Q,\end{aligned}\tag{2.13}$$

where we recall that $F \in \mathbb{R}^{n \times n}$ is the NCV state transition matrix and $Q \in \mathbb{R}^{n \times n}$ is the system noise covariance matrix.

2.4.2 Update Step

In the update step, the EKF employs the nonlinear radar measurement equation $h(\cdot, \cdot)$, along with the state-dependent measurement vector z_{k+1} to update the predicted state $\hat{x}_{k+1|k}$ and covariance matrix $P_{k+1|k}$ into the estimated quantities

$$\begin{aligned}S_{k+1} &= H_{k+1}P_{k+1|k}H_{k+1}^\top + R_{k+1}(u_k) \\ K_{k+1} &= P_{k+1|k}H_{k+1}^\top S_{k+1}^{-1} \\ \hat{z}_{k+1|k} &= h(\hat{x}_{k+1|k}, x_{k+1}^s) \\ \hat{x}_{k+1|k+1} &= \hat{x}_{k+1|k} + K_{k+1}(z_{k+1} - \hat{z}_{k+1|k}) \\ P_{k+1|k+1} &= P_{k+1|k} - K_{k+1}S_{k+1}K_{k+1}^\top,\end{aligned}\tag{2.14}$$

where

- $S_{k+1} \in \mathbb{R}^{2 \times 2}$ is the innovation matrix,
- $K_{k+1} \in \mathbb{R}^{6 \times 2}$ is the Kalman gain matrix,
- $\hat{z}_{k+1|k} \in \mathbb{R}^2$ is the measurement residual,
- $H_{k+1} \in \mathbb{R}^{2 \times 6}$ is the Jacobian of the measurement equation computed with respect to the predicted state $\hat{x}_{k+1|k}$.

For the setup under consideration, the Jacobian matrix of the measurement equation can be computed as

$$H_{k+1} = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}_{k+1|k}} = \begin{bmatrix} \hat{r}_{k+1}[0] & \hat{r}_{k+1}[1] & 0 & 0 & 0 & 0 \\ H_{10,k+1} & H_{11,k+1} & 0 & \hat{r}_{k+1}[0] & \hat{r}_{k+1}[1] & 0 \end{bmatrix}, \tag{2.15}$$

where

$$\begin{aligned}H_{10,k+1} &= \frac{\dot{r}_{k+1}[0] - \|\dot{r}_{k+1}\|\hat{r}_{k+1}[0]}{\|r_k\|} \\ H_{11,k+1} &= \frac{\dot{r}_{k+1}[1] - \|\dot{r}_{k+1}\|\hat{r}_{k+1}[1]}{\|r_k\|}.\end{aligned}\tag{2.16}$$

2.5 Optimal Waveform Selection

In the target tracking framework, the considered radar optimizes the transmitted waveform parameters u_k based on the target estimate. For notational convenience, we let

$$\begin{aligned}\hat{\mathbf{x}}_K &:= \{\hat{x}_{0|0}, \dots, \hat{x}_{k|k}\} \\ \mathbf{P}_K &:= \{P_{0|0}, \dots, P_{k|k}\}.\end{aligned}\tag{2.17}$$

Also, as in [41], we introduce the information vector I_k , defined at time step k as

$$I_k := (z_1, \dots, z_k, u_0, \dots, u_{k-1}). \tag{2.18}$$

In this work, we investigate the two optimization approaches presented in [43]

- **Anticipation:** given a time horizon $K \geq 1$, the controller selects the optimal input sequence $\mathbf{u}_K^*$ based on how the system is expected to evolve in the future. Input selection via anticipation of future states and objective values can be considered as a cognitive process.
- **Adaptation:** given a unitary time horizon $K = 1$ the controller selects the optimal input u_k^* based on the system state at current time k . Input selection depending just on the current state can be seen as an adaptive process.

A common objective function in radar target tracking is the trace of the filtered covariance matrix [7]. An anticipative controller for a target tracking cognitive spaceborne radar, anticipates the satellite and the EKF dynamics. This optimization problem can be recast in terms of stochastic optimal control, which can be formulated as

$$\begin{aligned}
 \min_{\hat{\mathbf{x}}_K, \mathbf{P}_K, \mathbf{u}} \quad & \sum_{k=0}^{K-1} \mathbb{E}_{z_k | I_{k-1}, u_{k-1}} \left[\text{Tr}(P_{k+1|k+1}) \right] \\
 \text{s.t.} \quad & \hat{x}_{k+1|k+1} = g_k(\hat{x}_{k|k}, P_{k|k}, u_k, z_{k+1}), \forall k \in \mathbb{K}_{-1} \\
 & P_{k+1|k+1} = d_k(\hat{x}_{k|k}, P_{k|k}, u_k), \forall k \in \mathbb{K}_{-1} \\
 & \hat{x}_{0|0} = \hat{x}_0 \\
 & P_{0|0} = P_0 \\
 & u_k \in U \\
 & z_{k+1} \sim \mathcal{N}(h(F\hat{x}_{k|k}, x_{k+1}^s); S_{k+1}(u_k)),
 \end{aligned} \tag{2.19}$$

where the cost function can be written as [41]

$$\mathbb{E}_{z_k | I_{k-1}, u_{k-1}} [\text{Tr}(P_{k+1|k+1})] = \int_{z_k} \mathcal{N}(\hat{z}_{k|k-1}, S_k) \text{Tr}(P_{k+1|k+1}) d\hat{z}_{k|k-1}. \tag{2.20}$$

The EKF algorithm, presented in (2.13) and (2.14), can be seen as a dynamic system with state $[\hat{x}_{k|k}, P_{k|k}]^\top$.

To this end, the functions g_k and d_k denote the filtered state and covariance matrix equation in (2.14), respectively, with

$$\begin{aligned}
 \hat{z}_{k+1|k} &= h(F\hat{x}_{k|k}, x_{k+1}^s) \\
 S_{k+1} &= H_{k+1}^\top [FP_{k|k}F^\top + Q]H_{k+1}^\top + R_{k+1}(u_k) \\
 K_{k+1} &= [FP_{k|k}F^\top + Q]H_{k+1}^\top S_{k+1}^{-1}.
 \end{aligned} \tag{2.21}$$

Here, we have employed (2.13) to write the predicted measure $\hat{z}_{k+1|k}$, the Kalman K_{k+1} and the innovation covariance matrix S_{k+1} as a function of the filtered state $\hat{x}_{k|k}$ and covariance matrix $P_{k|k}$. Furthermore we remind the reader that H_{k+1} is the Jacobian of (2.9) linearized around the predicted state. The proposed optimization problem is stochastic since, at time step $k+1$, the radar measure z_{k+1} is not known. The cubature rule of the third degree [51] is, therefore, employed to approximate the stochasticity inherent of this framework.

2.5.1 Cubature Rule

The cubature rule approximates a multidimensional weighted integral of the form

$$I(f) = \int_D f(x)w(x)dx, \tag{2.22}$$

using a set of weighted points [51]

$$I(f) \simeq \sum_{i=1}^m \omega_i f(x_i), \tag{2.23}$$

where

- $f(\cdot)$ is some arbitrary function,
- $D \subset \mathbb{R}^n$ is the region of integration, and
- $w(x) \geq 0$ for all $x \in D$ is the known weighting function.

A third-degree cubature rule can be constructed to approximate an n -dimensional Gaussian weighted integral as follows [52]

$$\int_{\mathbb{R}^n} f(x) \mathcal{N}(x; \mu, \Sigma) dx \simeq \frac{1}{2n} \sum_{i=1}^{2n} f(\mu + \sqrt{\Sigma} \zeta_i), \quad (2.24)$$

where $\Sigma = \sqrt{\Sigma} \sqrt{\Sigma}^\top$ and the cubature points are

$$\zeta_i = \begin{cases} \sqrt{n} e_i & i = 1, 2, \dots, n \\ -\sqrt{n} e_{i-n} & i = n+1, n+2, \dots, 2n \end{cases} \quad (2.25)$$

$e_i \in \mathbb{R}^n$ is the i^{th} canonical vector.

2.5.2 Cubature Rule of Third-degree for Optimal Waveform Selection

The cubature rule [51] is used to approximate the expectation on the filtered state $\hat{x}_{k+1|k+1}$ and covariance matrix $P_{k+1|k+1}$. as follows

$$\begin{aligned} \mathbb{E}_{z_{k+1}|I_k, u_k} [\hat{x}_{k+1|k+1}] &= \tilde{x}_{k+1} \simeq \frac{1}{2m} \sum_{i=1}^{2m} g(\tilde{x}_k, \tilde{P}_k, u_k, \tilde{z}_{i,k+1|k}) \\ \mathbb{E}_{z_k|I_{k-1}, u_{k-1}} [P_{k|k}] &= \tilde{P}_{k+1} \simeq d(\tilde{x}_k, \tilde{P}_k, u_k) \\ \tilde{z}_{i,k+1|k} &= h(F\tilde{x}_k, x_{k+1}^s) + \sqrt{S_{k+1}} e_i, \end{aligned} \quad (2.26)$$

where $m \in \mathbb{N}$ is the measurement vector dimension, and $\sqrt{S_{k+1}}$ denotes the Cholesky factor of matrix S_{k+1} . The set $U = \{[u_{min}[1], u_{max}[1]], [u_{min}[2], u_{max}[2]]\} \subseteq \mathbb{R}^2$ contains the upper and lower bounds for the control input u_k . The adaptive controller, at each time step, selects the optimum waveform parameters u_k^* by solving the following myopic dynamic optimization problem

$$u_k^* = \underset{u_k \in U}{\operatorname{argmin}} \quad \operatorname{Tr}(P_{k|k} + d_k(\hat{x}_{k|k}, P_{k|k}, u_k)). \quad (2.27)$$

Myopic dynamic optimization only considers a single-step horizon and, differently from the optimal control technique, it does not leverage the EKF and the satellite dynamics.

The anticipation and adaptation approaches have different computational cost. The former is recasted in terms of an optimal control problem, while the latter is formulated as a single-step dynamic optimization problem.

2.5.3 Simulations

This section provides the values used to model the tracking setup and the waveform optimization algorithms.

In this work, the Earth's semi-major axis a , angular velocity ω and geocentric gravitational constant follow the WGS-84 model [53], see Appendix B. The origin of the ENU frame has geodetic longitude $\lambda_0 = \pi/6$ rad and latitude $\phi_0 = \pi/6$ rad. The integration time is set as $dt = 5s$.

The target NCV dynamics (2.1) is modeled by setting the target initial condition $x_0 \in \mathbb{R}^6$ as

$$x_0 = \begin{bmatrix} 70\text{m} \\ 140\text{m} \\ 0\text{m} \\ 26\text{m/s} \\ 42\text{m/s} \\ 0\text{m/s}, \end{bmatrix} \quad (2.28)$$

and the power spectral densities as $q_1 = q_2 = 0.01\text{m}^2/\text{s}^3$.

The LEO satellite trajectory is a circular orbit, with inclination angle $i = \pi/6$ rad, with respect to the equatorial plane, and radius $r_{\text{orb}} = a + 600 \cdot 10^3$ m from the origin of the ECI frame O_{ECI} . It follows that the magnitude of the satellite velocity is [54]:

$$v_{\text{orb}} = \sqrt{\frac{\text{GM}}{r_{\text{orb}}}}. \quad (2.29)$$

The satellite's initial condition in the ECI frame $x_0^e \in \mathbb{R}^6$ is

$$x_0^e = \begin{bmatrix} r_{\text{orb}} \\ 0 \\ 0 \\ 0 \\ v_{\text{orb}} \cos i \\ v_{\text{orb}} \sin i \end{bmatrix}. \quad (2.30)$$

The radar transmits an electromagnetic signal with fixed carrier frequency $f_c = 10.4\text{GHz}$. The 0dB range is set as $\tilde{r} = 40 \cdot 10^5\text{m}$. The feasible set U contains the lower and upper bounds for the control input u_k , that is $U = \{[10^{-6}, 300 \cdot 10^{-6}], [-300 \cdot 10^8, 300 \cdot 10^8]\}$. The initial state estimate $\hat{x}_{0|0} \in \mathbb{R}^6$ and the covariance matrix $P_{0|0} \in \mathbb{R}^{6 \times 6}$ of the EKF are

$$\hat{x}_{0|0} = \begin{bmatrix} 75\text{m} \\ 150\text{m} \\ 0\text{m} \\ 24\text{m/s} \\ 43.5\text{m/s} \\ 0\text{m/s} \end{bmatrix} \quad (2.31)$$

and

$$P_{0|0} = 0.01 \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (2.32)$$

respectively.

In order to evaluate the quality of the proposed Optimal Control approach, the Root Mean Square Error (RMSE) is chosen as performance metric, as in [6]. The RMSE is computed for the target position and velocity as

$$\begin{aligned} \tilde{p}_k &= \sqrt{\frac{1}{N} \sum_{n=1}^N \|p_{k,n} - \hat{p}_{k,n}\|^2}, \\ \tilde{v}_k &= \sqrt{\frac{1}{N} \sum_{n=1}^N \|v_{k,n} - \hat{v}_{k,n}\|^2}, \end{aligned} \quad (2.33)$$

where $p_{k,n}, v_{k,n} \in \mathbb{R}^3$ denote the true target position and velocity, at time step k , of the n -th simulation, while $\hat{p}_{k,n}, \hat{v}_{k,n} \in \mathbb{R}^3$ are the estimated quantities. The number of simulations performed is N . In this work, the RMSE is normalized over $N = 100$ simulations. The trace of the filtered covariance matrix, normalized over N simulations, is also evaluated

$$\text{Tr}(\tilde{P}_{k|k}) = \frac{1}{N} \sum_{n=1}^N \text{Tr}(P_{k|k,n}), \quad (2.34)$$

where $P_{k|k,n}$ is the EKF covariance matrix at time step k of the n^{th} simulation. This metric is as [41]

$$\text{Tr}(P_{k|k,n}) = \mathbb{E}_{x_{k,n}|I_{k-1,n}, u_{k-1,n}} [(x_{k,n} - \hat{x}_{k|k,n})(x_{k,n} - \hat{x}_{k|k,n})^\top], \quad (2.35)$$

where $I_{k,n}$ and $u_{k,n}$ denote the information vector and control input, at time step k , of the n^{th} simulation.

Figure 2.3 depicts the position RMSE $\tilde{p}_k$ achieved by optimal control, myopic dynamic optimization, and a traditional radar (TRA) that transmits a fixed input. The traditional radar transmits an envelope duration $\tau = 20 \cdot 10^{-6}$ and a negative chirp-rate, as in [41]. Thus, its waveform parameters are set as $u = [2 \cdot 10^{-6}, -5 \cdot 10^8]^\top$.

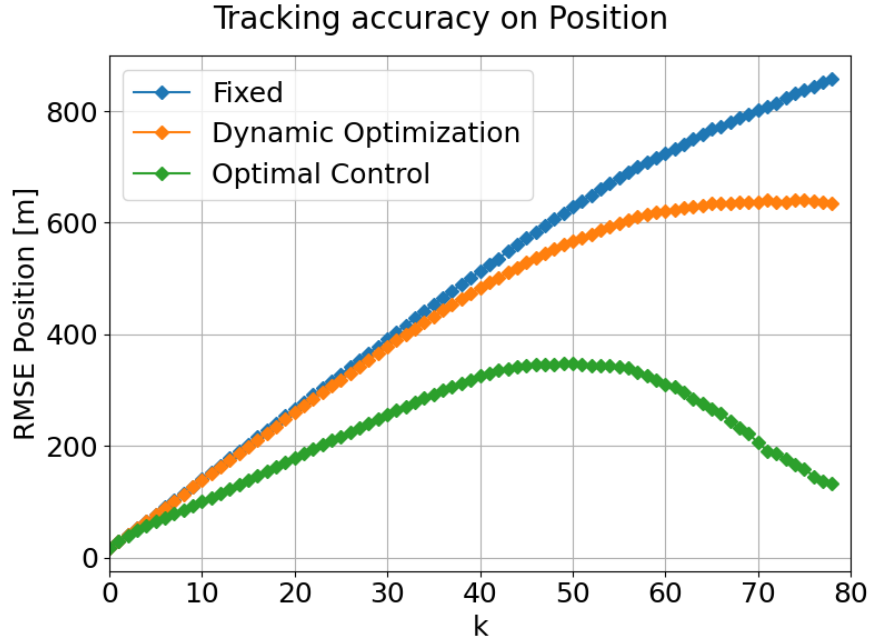


Figure 2.3: RMSE of target position for a traditional (TRA) spaceborne radar (blue), an adaptive spaceborne radar (orange) via Dynamic Optimization (DO), and an anticipative one (green) via Optimal Control (OC).

The optimal control approach achieves a lower position RMSE, thus it provides a smaller estimation error. Figure 2.4 shows the RMSE of the estimated target velocity $\tilde{v}_k$.

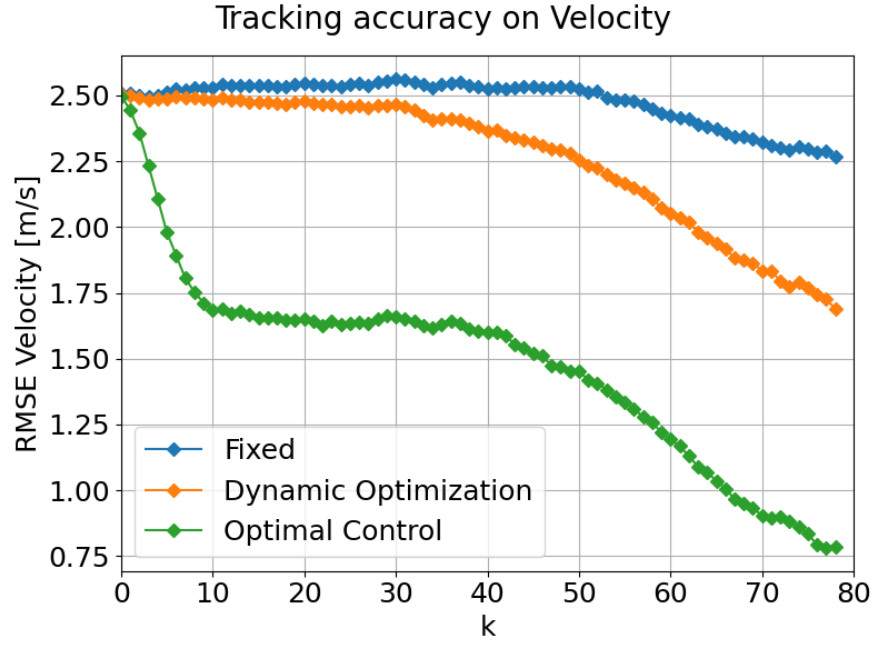


Figure 2.4: RMSE of target velocity for a traditional (TRA) spaceborne radar (blue), an adaptive spaceborne radar (orange) via Dynamic Optimization (DO), and an anticipative one (green) via Optimal Control (OC).

The optimal control technique provides a better performance, hence a better estimate, also for the velocity RMSE. Lastly, the trace of the filtered covariance matrix is shown in Figure 2.5.

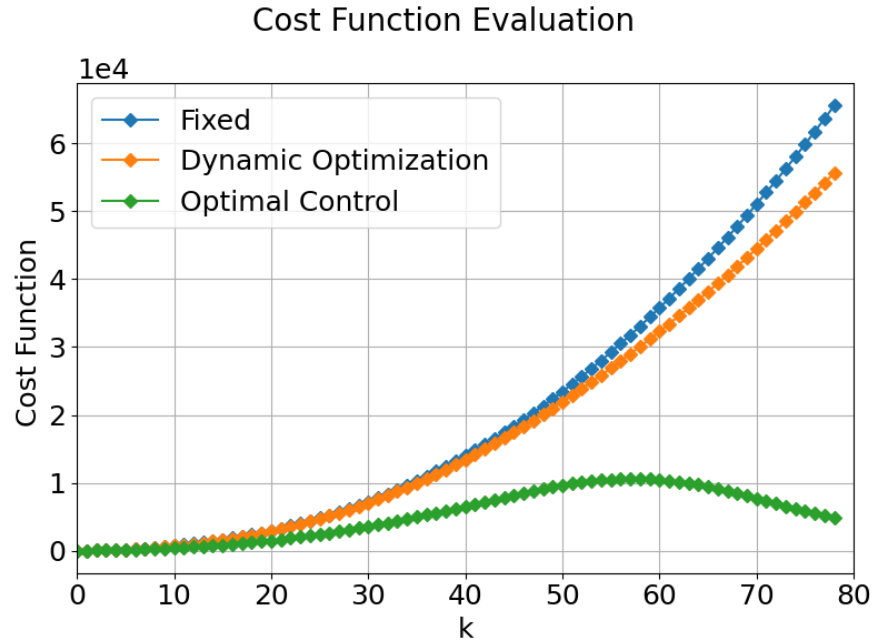


Figure 2.5: Trace of the filtered covariance matrix for a traditional (TRA) spaceborne radar (blue), an adaptive radar (orange) via Dynamic Optimization (DO), and an anticipative one (green) via Optimal Control (OC).

Optimal control achieves the best performance in terms of objective value because it minimizes the cumulative sum of the objective function over the whole simulation

duration L .

Chapter 3

AI-Enhanced Target Tracking for Tip & Cue SAR Systems

In this chapter, we address the problem of maritime surveillance using a Tip & Cue configuration. This approach involves the interaction between two satellites to first detect an area of interest and subsequently acquire more meaningful information about the detected zone.

In the first section, we focus on the use of Gaussian Process Regression to model vessel dynamics using data from an Automatic Identification System (AIS). This sensor is used by cooperative vessels to transmit information such as position, speed, and heading. We assume that both satellites are equipped with AIS receivers, with the second satellite receiving the messages gathered by the first satellite regarding the vessel of interest, in order to model its dynamics. Furthermore, the proposed algorithm is implemented on a space-qualified Field Programmable Gate Array to evaluate its resource consumption.

In the second section, we focus on vessel tracking using Synthetic Aperture Radar. Referring again to the Tip & Cue configuration, we assume that the first satellite operates in a wide-swath, low-resolution mode, e.g., stripmap, to detect a vessel to be tracked. The second satellite, instead, employs a high-resolution, narrow-swath mode, e.g., spotlight, to focus on the vessel of interest and perform tracking. The spotlight data are processed in sub-apertures, rather than as a single batch at the end of acquisition, in order to generate a temporal sequence of detections to be used by the tracking algorithm. Moreover, we assume that these SAR detections are generated from raw (or partially focused) data, thereby avoiding the typical latency associated with downlink and SAR data processing. This enables real-time tracking of the target of interest.

3.1 Literature Review

Maritime surveillance and target tracking are topics of growing importance in the SAR domain. Papers [55] and [56] introduce the cognitive SAR framework for maritime surveillance. In [55], the authors define the imaging and mapping modes, as well as the front-end architecture for a cognitive tracking SAR. Paper [56], on the other hand, formalizes a preliminary algorithmic framework for cognitive tracking SAR mounted on a high-altitude platform. In this scenario the sensor, after detecting a vessel to be tracked, can adapt its beam toward the target by employing a spotlight mode. At the same time, the authors define which transmitted signal parameters could be adjusted to optimize potential performance metrics. However, in this work, the authors do not provide a concrete mathematical formulation of the tracking framework with transmitted signal adaptability, but rather offer high-level guidelines for developing this type of use case. Both industrial and academic interest has emerged around the Cognitive SAR framework, following studies initially conducted on cognitive radar and subsequently on its potential application to SAR sensors [57], [12]. The European Space Agency is directly undertaking studies focused on the application of artificial intelligence to SAR payloads [57], with the goal of enabling real-time autonomous operation in line with the cognitive radar paradigm. These studies have addressed data compression using deep learning, object detection directly from raw SAR echoes, and advancements in signal processing, digital beamforming, and AI-driven decision-making. Paper [12] presents research aimed at formalizing a cognitive SAR framework for various use cases, including the Tip & Cue framework for vessel tracking addressed in this chapter. Performing on-board, real-time target tracking requires the SAR to detect the target in real time, without the need to transmit the signal and metadata to a ground station. Preliminary work in this

direction has been conducted by [23], [58] and [59]. Paper [23] treats the raw SAR data as one-dimensional array to perform target detection. It introduces for the first time the idea of bypassing the standard processing algorithms to enable a real-time on-board target detection. Paper [58] employs a U-Net to detect vessels from real raw SAR data acquired by Sentinel-1 in stripmap mode. However, this framework considers only open and calm sea scenarios, representing a best-case scenario, since rough seas and coastlines, through their backscattered echoes, can affect detection performance. Therefore, a partial processing of the raw SAR data may be required to achieve reliable vessel detection. Lastly, a lightweight deep neural-network for ship detection has been proposed in [59] for vessel detection from real raw SAR data in stripmap and interferometric wide-swath mode. Furthermore, the proposed architectures have been deployed on two space-qualified FPGAs, to prove their onboard feasibility. However, a key challenge is the lack of sufficiently large and diverse labeled raw SAR datasets, particularly with varying sea conditions. The modeling and validation of a cognitive SAR system may involve integrating SAR data with information from an AIS sensor. This field has gained increasing attention in recent years. Paper [60] utilized AIS data as ground truth to assess the effectiveness of SAR-based algorithms for ship detection. Work [61] instead proposes a four-step matching process between AIS and SAR data; Time, Position, Size, and Speed Matching, demonstrating effective correlation and validation of ship information. Another work on the integration between SAR and AIS data is [62], where the authors present a framework to enhance ship detection and recognition, utilizing automatic sea-land segmentation, multi-scale constant false alarm rate detection, and a greedy algorithm for AIS-SAR matchup.

3.2 AIS-Based Gaussian Process Regression for Vessel Trajectory Modeling

The contribution of this section is the evaluation of a Gaussian Process Regression algorithm for vessel-trajectory modeling using AIS messages collected by two satellites operating in a Tip & Cue configuration. The algorithm's performance is assessed in terms of mean absolute error and the proportion of test-set data points that fall within the predictive confidence intervals. Additionally, the algorithm was ported to two space-qualified FPGAs to assess implementation feasibility. The contribution provided in this section has been published as a conference paper [63] and presented at the 2nd European Data Handling & Data Processing Conference 2025.

3.2.1 AIS Data

The authors of [64] present a real-world AIS dataset collected by a ground-based receiver.

This dataset spans a period of 2.5 years, from 9 May 2017 to 26 December 2019, and comprises the positions of anonymized vessels operating within the wider Piraeus port area, one of the busiest in Europe. In total, the dataset contains more than 244 million AIS messages, with an average of over 10.000 records per hour, making it one of the most extensive and densest AIS datasets available.

This dataset is composed by kinematic and static vessel-related data.

Kinematic Part of Dataset

The kinematic component of the dataset comprises variables that evolve over time. The dataset consists of multiple matrices, one for each month under consideration, each of which contains the following columns:

- **Timestamp:** UNIX timestamp of the received message, i.e., the number of seconds elapsed since January 1, 1970, at 00 : 00 : 00 Coordinated Universal time;
- **Vessel ID:** identifier of the vessel, (artificial, for anonymisation purposes);
- **Longitude:** in deg (WGS-84 format);

- **Latitude:** in deg (WGS-84 format);
- **Heading:** relative to true north, in deg $[0, 360)$;
- **Course:** over ground, in deg $[0, 360)$;
- **Speed:** in knots.

Due to its diversity and long spatio-temporal coverage, the dataset can support the training of vessel profiling models, from movement behaviour (e.g. routine vs. unusual movement) to activity profiles (e.g. anchoring vs. cruising vs. fishing, in case of fishery boats).

Fig. 3.1 contains the plot of the longitude transmitted by a vessel thorough AIS data.

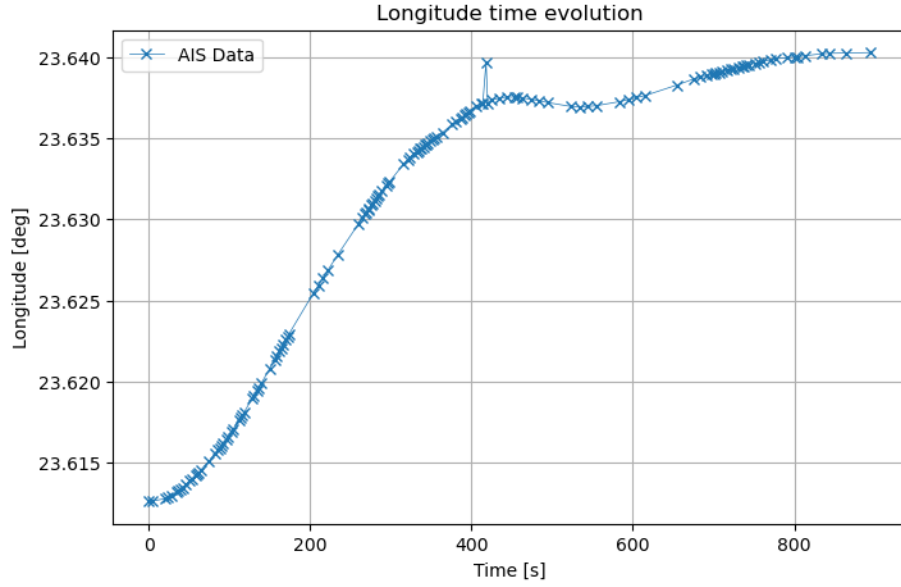


Figure 3.1: Longitude data transmitted via AIS messages in [64]

Fig. 3.2, instead, contains the plot of the latitude transmitted by the same vessel.

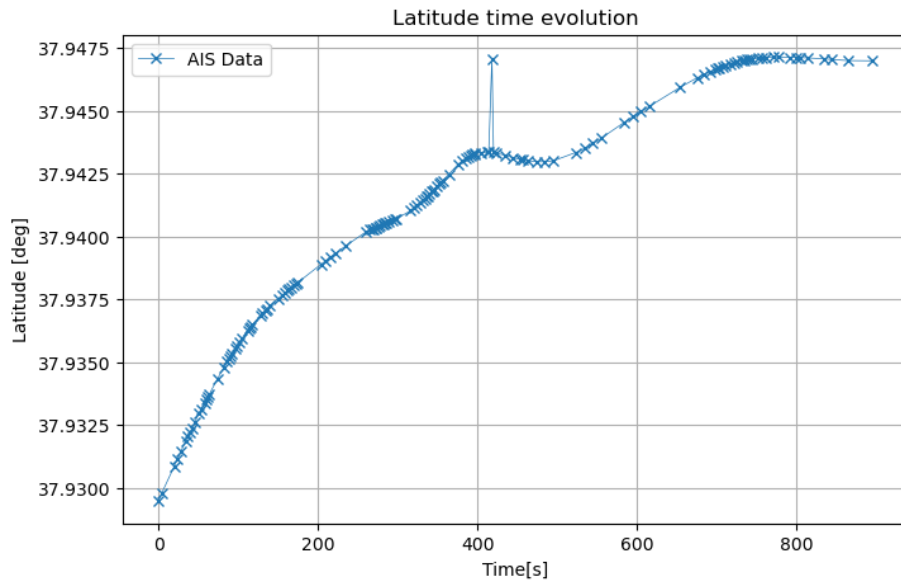


Figure 3.2: Latitude data transmitted via AIS messages in [64]

Static Part of Dataset

The AIS static messages deliver information about vessel characteristics, which basically remain unchanged over the duration of the voyage. It provides, for each vessel ID, the country of origin and the ship type. The ship type is denoted through a positive integer number. The different ship categories are:

- Fishing vessel: 30;
- Sailing vessel: 36;
- Pleasure craft: 37;
- Passenger: 60-69;
- Cargo: 70-79;
- Tanker: 80-89.

The cargo type will be considered in the following simulations.

3.2.2 Gaussian Process Regression

A Gaussian Process is a collection of random variables, any finite number of which have a joint Gaussian distribution [65]. It is completely defined by its mean function $m(x)$ and a parametrized covariance function, called also kernel, $k(x, x', \theta)$, and it is denoted as:

$$f(x) \sim \mathcal{GP}(m(x), k(x, x', \theta)), \quad (3.1)$$

where

$$m(x) = \mathbb{E}[f(x)] \quad (3.2)$$

$$k(x, x', \theta) = \mathbb{E}[(f(x) - m(x))(f(x') - m(x')))]. \quad (3.3)$$

The kernel $k(x, x', \theta)$ encodes the assumptions about the function's smoothness and other properties. It is characterized by the hyperparameter vector $\theta \in \mathbb{R}^p$, omitted hereafter for simplicity.

Gaussian Process Regression with Noisy Data

Consider a training set $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^n$ where $x_i \in \mathbb{R}^D$ is the input and $y_i \in \mathbb{R}$ is the corresponding output. Denote with $X \in \mathbb{R}^{D \times n}$ the input matrix of the training set and with $\mathbf{y} \in \mathbb{R}^n$ the vector containing all the outputs. Therefore we may rewrite the dataset as $\mathcal{D} = \{X, \mathbf{y}\}$

We aim to predict the output $\mathbf{f}^* \in \mathbb{R}^{n^*}$ at new inputs $X^* \in \mathbb{R}^{D \times n^*}$. The joint distribution of the training output $\mathbf{y}$ and the test output $\mathbf{f}^*$ is

$$\begin{pmatrix} \mathbf{y} \\ \mathbf{f}^* \end{pmatrix} \sim \mathcal{N} \left(0_{\bar{n} \times 1}, \begin{pmatrix} K(X, X) + \sigma_n^2 I_{n \times n} & K(X, X^*) \\ K(X^*, X) & K(X^*, X^*) \end{pmatrix} \right), \quad (3.4)$$

where $\bar{n} = n + n^*$ and assuming $m(x) \equiv 0$, and

- $K(X, X) \in \mathbb{R}^{n \times n}$ is the covariance matrix of the training inputs, with $K(X, X)_{[i,j]} = k(x_i, x_j)$.
- $K(X, X^*) \in \mathbb{R}^{n \times n^*}$ is the covariance matrix between the training inputs and the test input, with $K(X, X^*)_{[i,j]} = k(x_i, x_j^*)$.
- $K(X^*, X^*) \in \mathbb{R}^{n^* \times n^*}$ the covariance matrix of the test input, with $K(X^*, X^*)_{[i,j]} = k(x_i^*, x_j^*)$.
- σ_n^2 is the variance of the observation noise, which is assumed to be white Gaussian.

The predictive distribution for $\mathbf{f}^*$ given the training data is Gaussian

$$\mathbf{f}^*|X, \mathbf{y}, X^* \sim \mathcal{N}(\mu^*, \sigma^{*2}), \quad (3.5)$$

with mean and variance

$$\mu^* = K(X^*, X)[K(X, X) + \sigma_n^2 I]^{-1} \mathbf{y} \quad (3.6)$$

$$\sigma^{*2} = K(X^*, X^*) - K(X^*, X)[K(X, X) + \sigma_n^2 I]^{-1} \cdot K(X, X^*). \quad (3.7)$$

The GPR fits the available data by maximizing the log marginal likelihood with respect to the hyperparameter vector θ , given by

$$\log p(\mathbf{y}|X) = -\frac{1}{2} \mathbf{y}^\top (K + \sigma_n^2 I)^{-1} \mathbf{y} - \frac{1}{2} \log |K + \sigma_n^2 I| - \frac{n}{2} \log 2\pi. \quad (3.8)$$

To evaluate the cost function, it is necessary to compute the term $\mathbf{y}^\top (K + \sigma_n^2 I)^{-1} \mathbf{y}$. This term can be obtained by solving a linear system of the form

$$(K + \sigma_n^2 I) \alpha = \mathbf{y} \quad (3.9)$$

for α .

The standard and numerically stable approach to achieve this is via Cholesky decomposition [66];

1. Compute the Cholesky decomposition $K + \sigma_n^2 I = LL^\top$, where L is a lower-triangular matrix.
2. Solve $L\beta = \mathbf{y}$ for β using forward substitution.
3. Solve $L^\top \alpha = \beta$ for α using back substitution.
4. Compute the quadratic form as $\mathbf{y}^\top \alpha$.

Evaluating the cost function involves a matrix inversion, performed through the Cholesky factorization, therefore the cost of a Gaussian Process Regression is $\mathcal{O}(n^3)$.

3.2.3 GPR-Based AIS Data Regression Simulation

In this work we focus on a Tip & Cue framework, depicted in Figure 3.3, for vessel dynamic modeling and maritime surveillance.

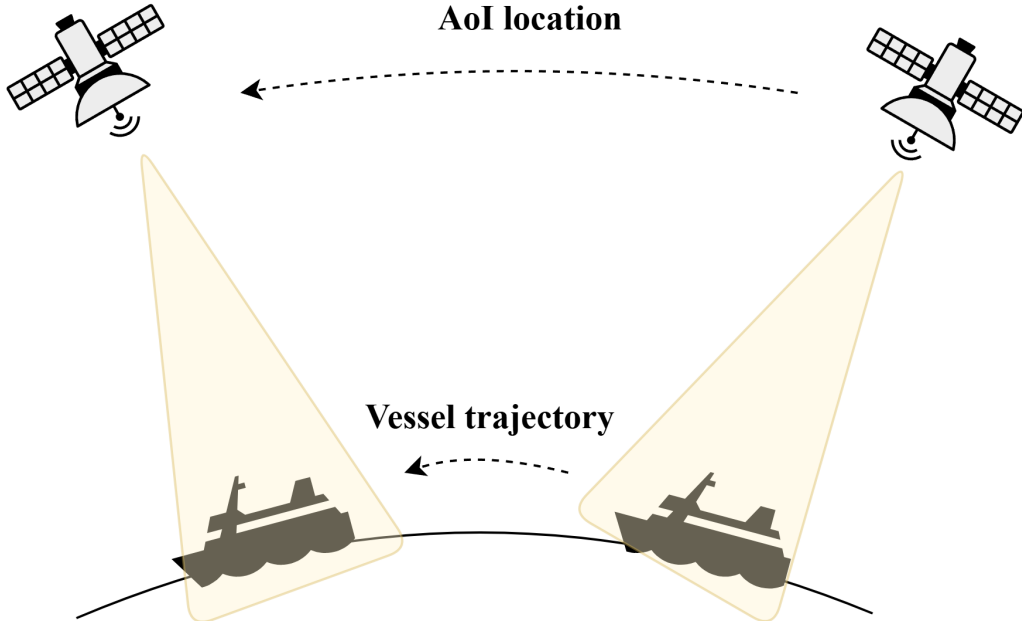


Figure 3.3: Tip & Cue concept with two satellites. The first satellite detects a target and shares the information with the trailing satellite, which can report to the Area of Interest.

This approach leverages the interaction between two spacecraft, referred to as Leader and Follower, to gather more meaningful data from a given Area of Interest (AoI). It consists of the following two steps:

- The Leader satellite performs an initial survey or detection, and identifies a given AoI. It then sends a request to the Follower satellite to gather information on the chosen AoI (Tip).
- The Follower satellite focuses on the AoI to gather more meaningful information (Cue).

Gaussian Process Regression Setup

The choice of kernel is crucial for training the GPR. For both Gaussian Process Regression models, employed respectively to fit the temporal evolution of longitude and latitude, we selected a kernel composed of the sum of two kernels: the Radial Basis Function (RBF) and the Dot Product (DP).

The RBF kernel is defined as

$$k_{rbf}(x, x') = \sigma_f^2 \exp\left(\frac{-\|x - x'\|^2}{2l^2}\right), \quad (3.10)$$

whose hyperparameters are the length scale l and the variance σ_f^2 .

The DP kernel is

$$k_{dp} = \sigma_0^2 x \cdot x', \quad (3.11)$$

where σ_0^2 is the variance, and only hyperparameter, of the DP kernel. This kernel has been used to take into account a general linear trend, with some nonlinear trends incorporated by the RBF kernel. The variance of the observation noise has been set as $\sigma_n^2 = 10^{-5}$, as to denote almost-noise-free data.

Dataset and Preprocessing

The AIS dataset underwent preprocessing prior to the training of the GPR models. A specific class of trajectories was selected according to the following criteria

- a duration of 1100 seconds;
- at least 6 AIS messages within the observation windows of the Leader and Follower satellites;
- an average speed over the trajectory of at least 3 knots.

Data packet loss was modeled by retaining only 25% of the available AIS messages during the visibility periods, both to simulate an AIS acquisition with a spaceborne receiver and to reduce the computational complexity.

To preserve numerical stability, the time instants, which constitute the input features x of our GPR models, were normalized to the interval $([0,1])$.

Performance Metrics and Simulation Results

In this study, we assume that both spacecraft are equipped with onboard AIS receivers, each of which has an observation window of 300 seconds, i.e., 5 minutes. Furthermore, we assume that a 500-second interval elapses between the end of the Leader satellite acquisition and the start of the Follower satellite acquisition; this interval is hereafter denoted the Non-Observability (NO) period.

The performance of the GPR models were evaluated by computing the Mean Absolute Error (MAE) for both latitude and longitude, computed as

$$\text{MAE} = \sum_{n=0}^{N_{\text{traj}}} \frac{\|y^{(n)} - \mu^{(\star, n)}\|}{\bar{n}^{(\star, n)}}, \quad (3.12)$$

where N_{traj} denotes the number of trajectories in the dataset, $\bar{n}^{(\star, n)}$ denotes the number of test data of the n^{th} trajectory, and $\mathbf{y}^{(n)}, \mu^{(\star, n)} \in \mathbb{R}^{n^{\star, n}}$ denote vectors with the real and predicted coordinates, e.g., longitude and latitude, of the n^{th} trajectory.

The results obtained are presented in Table 3.1, for both the whole test set and the test set regarding only the NO period.

Table 3.1: Mean Absolute Error for Longitude and Latitude Data Fitting.

	MAE Longitude [deg]	MAE Latitude [deg]
Test Set	0.0012375725443918627	0.001034888609206028
NO Test Set	0.0016718325938855914	0.0013808886341909938

Such errors can be converted into meters to provide a better understanding of the performance of the algorithm. Given the geodetic latitude ϕ of the targets, the latitudinal error in meters can be computed as follows [67]

$$\text{MAE Latitude}_m = M(\phi) \cdot \text{MAE Latitude}_{\text{rad}}, \quad (3.13)$$

where $M(\phi)$ is the meridian radius of curvature, given by

$$M(\phi) = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{3/2}}, \quad (3.14)$$

where a and e are, respectively, the semi-major axis and the eccentricity of the World Geodetic System 1984 reference, and ϕ was set as $\phi = 37.94\text{deg}$, approximately the geodetic latitude of the port of Piraeus. The longitudinal error, on the other hand, can be converted into meters as

$$\text{MAE Longitude}_m = N(\phi) \cdot \cos \phi \cdot \text{MAE Longitude}_{\text{rad}}, \quad (3.15)$$

where $N(\phi)$ is the prime vertical radius of curvature, given in Equation (B.4). The Total MAE (TMAE) has been computed as

$$\text{TMAE}_m = \sqrt{\text{MAE Longitude}_m^2 + \text{MAE Latitude}_m^2}, \quad (3.16)$$

which is approximately 158.25 m for the full test set and 212.39 m for the NO test set. Consequently, if a SAR sensor were to use this estimate to repoint its beam within the AoI, it would still be able to cover the target of interest, given that the footprint of a Spotlight SAR is on the order of 5-10 km.

Another performance analysis consisted in evaluating the proportion of test points falling within the confidence interval predicted by the GPR. As described in 3.2.2, the GPR prediction follows a normal distribution with mean value $\mu^{\star}$ and standard deviation $\sigma^{\star}$. Given a test set $\mathbf{y}^{\star} \in \mathbb{R}^{n^{\star}}$, it is considered to lie within a specified confidence interval if

$$\mathbf{y}^{\star} \in [\mu^{\star} - z\sigma^{\star}, \mu^{\star} + z\sigma^{\star}]. \quad (3.17)$$

Parameter z corresponds to the quantile of the standard normal distribution $\mathcal{N}(0, 1)$ that captures the desired confidence level

$$z = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \quad (3.18)$$

where Φ^{-1} is the inverse cumulative distribution function (quantile function) of the standard normal, and α is the significance level [68]. The results obtained for the 99, 95 and 90% confidence intervals, e.g. confidence levels of 0.99, 0.95 and 0.9, are reported in Table 3.2.

Table 3.2: Percentage of Data Points Falling within a given Confidence Interval

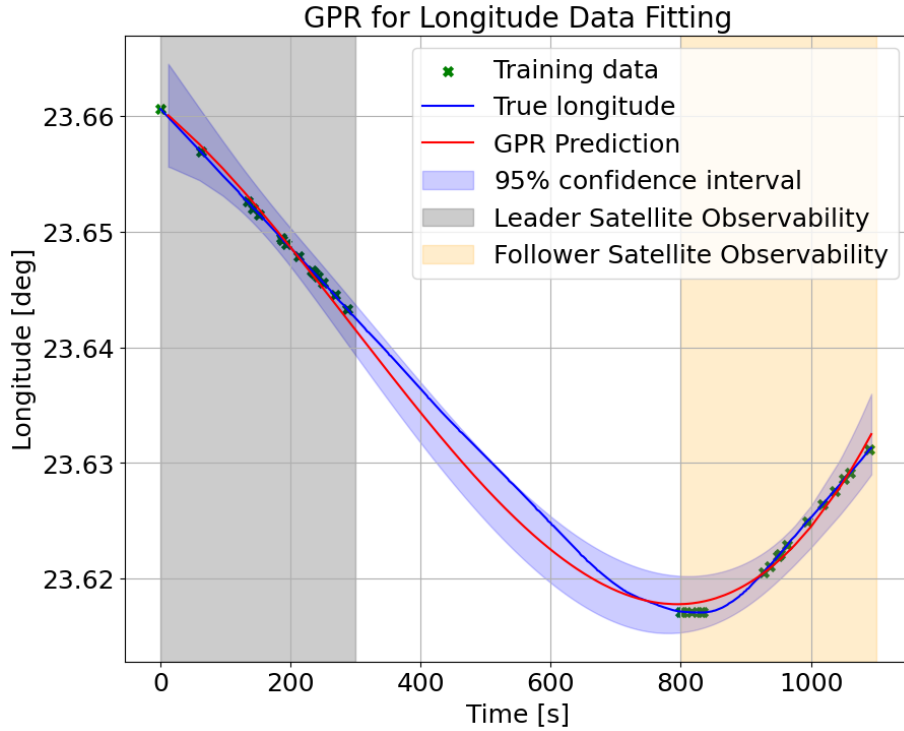
	Longitude Test Data	Latitude Test Data
99%	88.19152670977678%	92.79396609324922%
95%	82.48830704069037%	87.28100474513312%
90%	77.80432837680208%	82.70914259181663%

Despite the period of non-observability and the sparsity of the data used, this performance metric still provides an indication of the potential of the algorithm for reconstructing trajectories of vessels of interest.

Visual Performance

This subsection graphically illustrates the performance of the GPR for a vessel selected from the dataset. The GPR was trained using the same kernels and configuration reported in 3.2.3

Figure 3.4 shows the vessel longitude fitting over time.


Figure 3.4: Longitude data fitting via GPR.

It can be observed that the GPR, using only 1/4 of the available dataset, successfully fits the temporal evolution of the longitude. In this case, the longitudinal trajectory is entirely contained within the 95% confidence interval, despite its parabolic shape. Figure 3.5, instead, shows the vessel latitude fitting over time.

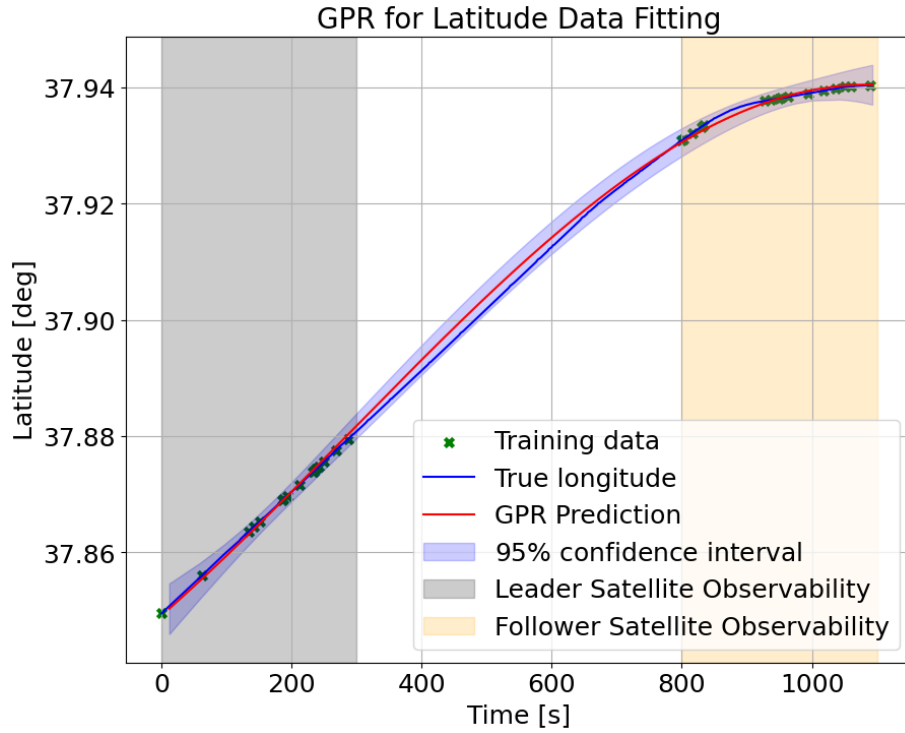


Figure 3.5: Latitude data fitting via GPR.

For the considered vessel, the GPR also adequately captures the temporal evolution of latitude. Figure 3.6 shows the trajectory on the longitude-latitude plane.

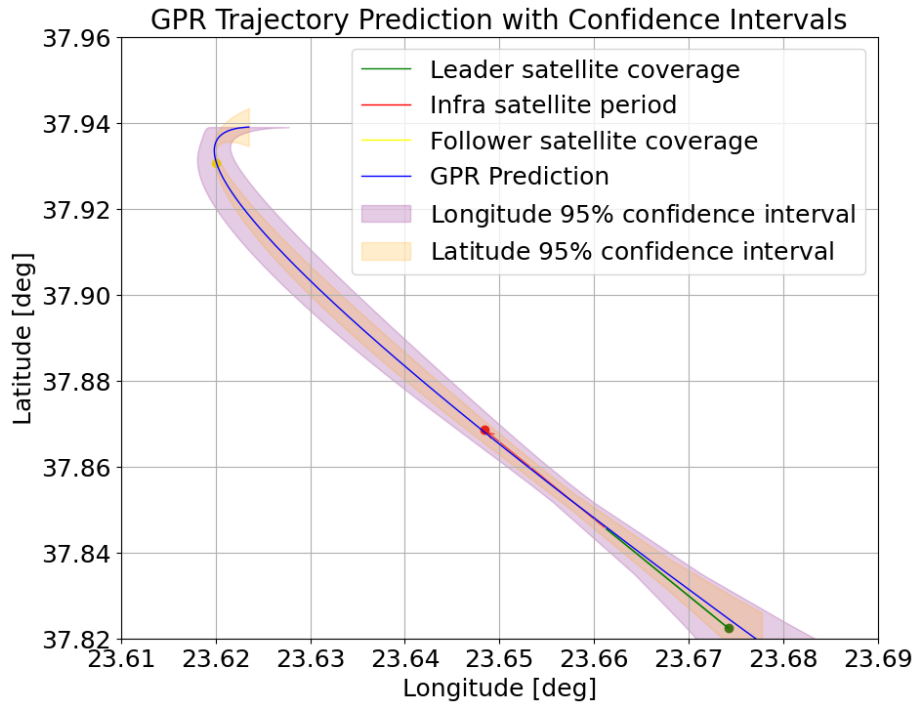


Figure 3.6: Ship track data fitting via GPR.

FPGA Porting and Benchmarking Results

The validation of the proposed algorithm relies on performance evaluation conducted on space-qualified FPGAs. In this work, we evaluate the performance of Gaussian Process Regression on the Zynq UltraScale+ and Versal Edge boards, employing the ZCU102 and VE2302 development kits, respectively. The benchmarking was conducted by running both training and inference of the latitude and longitude GPRs on the space-qualified boards. This was done for each of the 258 selected trajectories.

For the training, only 25% of the available AIS data were used for each course, while the inference was performed by predicting 50 uniformly spaced samples within each ship trajectory. Table 3.3 reports, for both FPGAs, the number of inferences per second and the number of vessel trajectories that can be reconstructed within the 300 seconds (i.e., 5 minutes) of observability of a single AIS. One inference corresponds here to the prediction of the whole 50 test samples for a given ship trajectory.

Table 3.3: Inferences per second and Vessels Tracked every 5 minutes

FPGA	Inference / sec	Vessels Tracked every 5min
VE2302	15.66	56.61
ZCU102	9.91	35.65

Note that the number of vessels tracked every 5 minutes takes into account also the time needed to train each GPR, reported in Figure 3.7 for the trajectories under consideration.

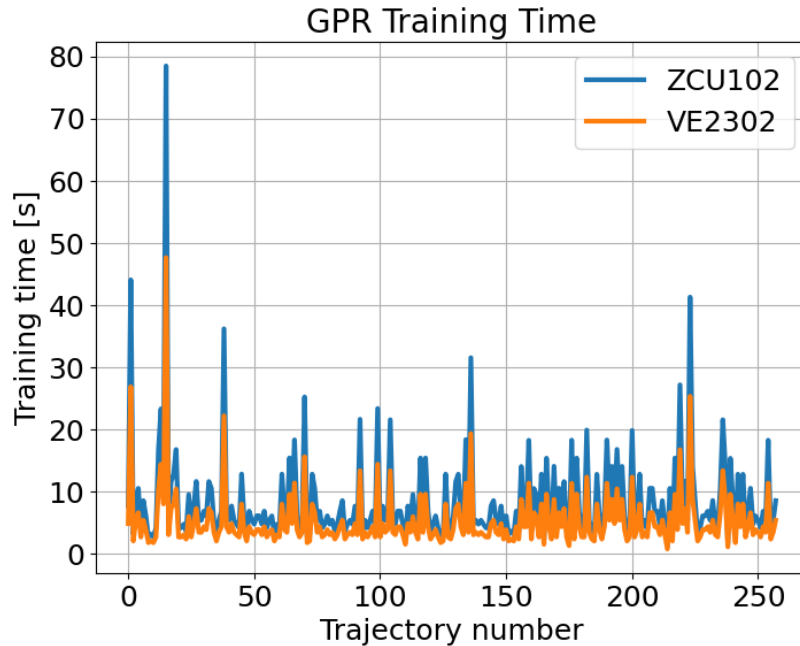


Figure 3.7: Training Time over the Considered Trajectories for the VE2302 and ZCU102 FPGAs.

Training is the most computationally intensive task required by the algorithm and is the reason why the number of trackable vessels is significantly lower than the inference times would suggest. Moreover, it can be seen that the VE2302 enables the fitting of a larger number of vessel trajectories within the same time interval.

Figure 3.8 illustrates the inference time of the two FPGAs under consideration for each of the 258 trajectories analyzed in this study.

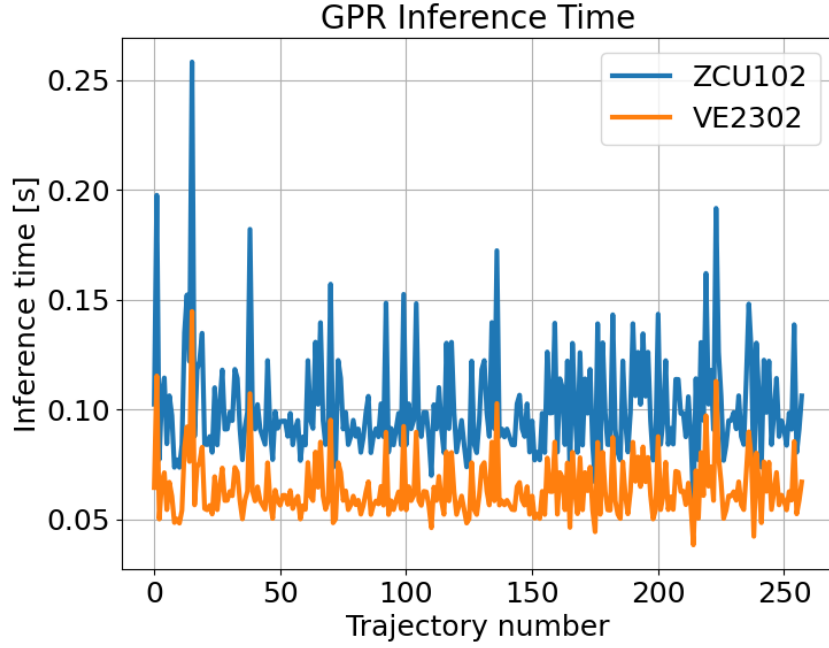


Figure 3.8: Inference Time over the Considered Trajectories for the VE2302 and ZCU102 FPGAs.

As can be observed, the VE2302 outperforms the ZCU102. This is attributable to the fact that the former, being more recently manufactured, is equipped with processors operating at a higher clock frequency. The difference in inference times for different trajectories corresponds to the courses with a higher number of training samples. This is expected, as the processing burden of the inference increases with the size of the covariance matrix $K(X, X)$.

Figure 3.9 shows the percentage by which the VE2302 outperforms the ZCU102 board in terms of inference time.

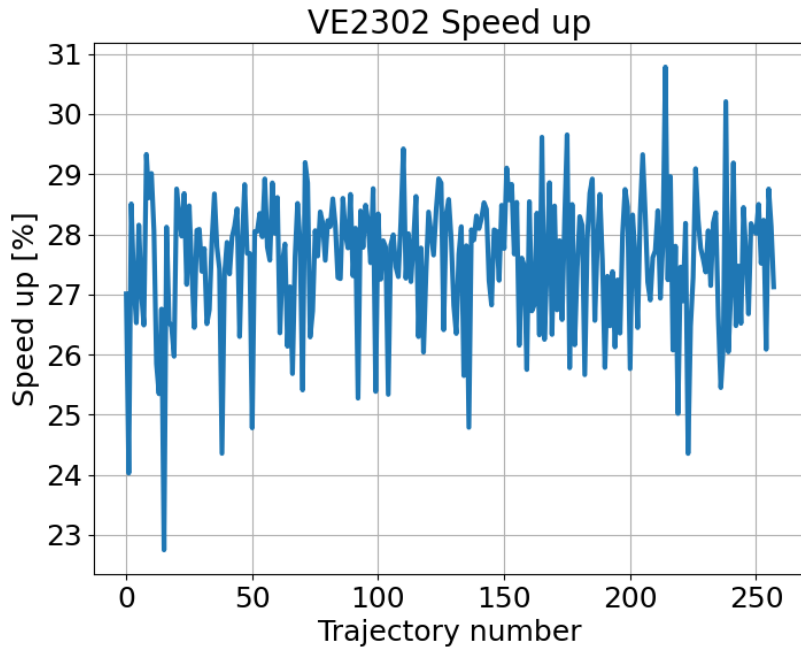


Figure 3.9: Outperformance percentage of VE2302 in Inference Time

For each trajectory considered, the VE2302 outperforms the ZCU102 by at least 23% across all trajectories under analysis.

The results obtained highlight the feasibility of deploying this algorithm on space-qualified FPGAs, with the VE2302 emerging as the most performant among those considered.

3.2.4 Future Work: GPR with Rank-1 Update

If one wishes to use the GPR in an iterative manner, updating the hyperparameters whenever new measurements become available, the approach described in Section 3.2.2 proves to be computationally inefficient, as it incurs a cost of $O(n^3)$, where n is the number of data points.

This computational burden arises from the need to perform a Cholesky factorization of the covariance matrix. Therefore a rank-1 update strategy may be used, in order to iteratively update the GPR weights at a reduced computational cost.

Gill-Murray-Wright Algorithm

Assume we initially have a dataset $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^n$ and its corresponding kernel matrix K_n .

Adding a new point (x_{n+1}, y_{n+1}) amounts to updating the kernel matrix as

$$K_{n+1} = \begin{bmatrix} K_n & k_\star \\ k_\star^\top & k_{\star\star} \end{bmatrix}, \quad (3.19)$$

where

$$k_\star = [k(x_{n+1}, x_i)]_{i=1}^n, \quad k_{\star\star} = k(x_{n+1}, x_{n+1}) + \sigma_n^2. \quad (3.20)$$

Gill, Murray and Wright [69] proposed a numerically stable procedure to update the Cholesky factor L_n without recomputing it from scratch.

Starting from the Cholesky decomposition $K_n = L_n L_n^\top$, compute the vector

$$v = L_n^{-1} k_\star, \quad (3.21)$$

by solving $L_n v = k_\star$ via forward substitution in $\mathcal{O}(n^2)$ time.

Form the residual scalar

$$\gamma = \sqrt{k_{\star\star} - v^\top v}, \quad (3.22)$$

and update the Cholesky factor as

$$L_{n+1} = \begin{bmatrix} L_n & 0 \\ v^\top & \gamma \end{bmatrix}. \quad (3.23)$$

This avoids redoing the $\frac{1}{3}n^3$ operations of a full Cholesky decomposition, since we reuse the existing L_n . Therefore, each update costs only $\mathcal{O}(n^2)$.

An additional strategy to prevent the computational cost from escalating as the number of measurements n increases is the use of a sliding window. In this approach, once the kernel matrix reaches a given dimension $m \times m$, the arrival of a new measurement entails discarding the information corresponding to the oldest one; e.g., by removing the first row and the last column of the kernel matrix.

3.3 DNN-Based Vessel Tracking via Synthetically Generated SAR Detections

The contribution of this section is the formulation of an AI-driven approach for vessel tracking using SAR-based detections. We introduce a method to synthetically generate

vessel detections in raw SAR data. The method is described for extended targets and permits arbitrary specification of the detection probability and the parameters governing the probability density of false detections. The Kalman filter relies on a model-based dynamic model; however, one parameter is unknown and is therefore learned via a neural network that takes as input the vessel's estimated coordinates at the two preceding time steps.

3.3.1 Simulated Vessel Dynamics

The AI-enhanced SAR tracking framework was evaluated using a simulated vessel dynamics. The object under study is described by the state vector $x_k \in \mathbb{R}^4$, at discrete time step k , defined as:

$$x_k = \begin{bmatrix} x_k[0] \\ x_k[1] \\ x_k[2] \\ x_k[3] \end{bmatrix} = \begin{bmatrix} \lambda_k \\ \phi_k \\ \psi_k \\ v_k \end{bmatrix}, \quad (3.24)$$

where λ_k, ϕ_k denote the geodetic longitude and latitude, respectively, ψ_k is the target heading and v_k denotes the target speed.

Dynamical Model

The state equation that may describe the vessel motion is a nonlinear equation of the form

$$x_{k+1} = f(x_k) + w_k, \quad (3.25)$$

where

- $f(\cdot) : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is the nonlinear state transition function,
- $w_k \in \mathbb{R}^4$ is the additive noise, described as $w_k \sim \mathcal{N}(0_{4 \times 1}, Q)$, with covariance matrix $Q \in \mathbb{R}^{4 \times 4}$.

The aim of this work is to track a ship; therefore, the dynamics under consideration must describe the motion of a body moving on the surface of the Earth.

Given the target's absolute speed v and heading ψ , the velocity components along the north and east directions of the ENU frame, see Appendix B, are

$$\begin{aligned} v_N &= v \cos \psi \\ v_E &= v \sin \psi. \end{aligned} \quad (3.26)$$

The small displacements along the east and north directions, using the reference ellipsoid of the WGS-84 model, can be computed as [70]

$$\begin{aligned} ds_N &= (M(\phi) + h)d\phi \\ ds_E &= (N(\phi) + h)\cos \psi d\lambda, \end{aligned} \quad (3.27)$$

where $N(\phi)$ is the prime vertical radius of curvature, given in Equation (B.4), and $M(\phi)$ is the meridian radius, given by

$$M(\phi) = \frac{a(1 - e^2)}{(1 - 2e \sin^2 \phi)^{(3/2)}}, \quad (3.28)$$

where a and e are the semi-major axis and eccentricity of the reference ellipsoid of the WGS-84 model. Computing the time derivative, it follows that

$$\begin{aligned} \frac{ds_N}{dt} &= v_N = v \cos \psi = (M(\phi) + h) \frac{d\phi}{dt} \\ \frac{ds_E}{dt} &= v_E = v \sin \psi = (N(\phi) + h) \cos \phi \frac{d\lambda}{dt}. \end{aligned} \quad (3.29)$$

From these equations, it follows that

$$\begin{aligned}\frac{d\phi}{dt} &= \frac{v \cos \psi}{M(\phi) + h} \\ \frac{d\lambda}{dt} &= \frac{v \sin \psi}{(N(\phi) + h) \cos \phi}.\end{aligned}\quad (3.30)$$

Using a forward-Euler step with integration Δt_k at discrete time-step k , and assuming linear velocity and heading, the dynamical model can be described as

$$x_{k+1} = \begin{bmatrix} x_k[0] + \frac{x_k[3] \sin(x_k[2])}{(N(x_k[1]) + h) \cos x_k[1]} dt_k \\ x_k[1] + \frac{x_k[3] \cos(x_k[2])}{M(x_k[1]) + h} dt_k \\ x_k[2] + \omega_k dt_k \\ x_k[3] + \alpha_k dt_k \end{bmatrix} + w_k, \quad (3.31)$$

where ω_k is the turn rate and α_k is the acceleration.

3.3.2 Synthetic Bounding Box Generation

This subsection describes the synthetic generation of SAR-based detections used in this work.

We assume that detections are produced by an AI algorithm that accepts raw (or partially focused) SAR data as input and outputs a segmentation map for vessel detection. The segmentation map is represented as a two-dimensional matrix in the raw-SAR data domain, where each pixel is assigned a numeric label: 0 denoting absence of the target of interest (corresponding to the black color in the segmentation map) and 1 denoting presence of the target (corresponding to white). Because a sufficiently comprehensive and realistic SAR simulator for producing raw data suitable for training a deep network is not available, this subsection presents a workflow to synthetically generate such detections.

Detection Setup

In this work we focus on generating SAR-based detections in the spotlight mode. As introduced in Section 1.2.2, the spotlight mode employs beam steering to direct the antenna beam toward an AoI. By processing the data acquired during this prolonged illumination, spotlight acquisitions achieve improved azimuth resolution relative to the stripmap acquisition mode.

Therefore, we partition the spotlight acquisition into N_{sub} sub-apertures, as illustrated in Figure 3.10.

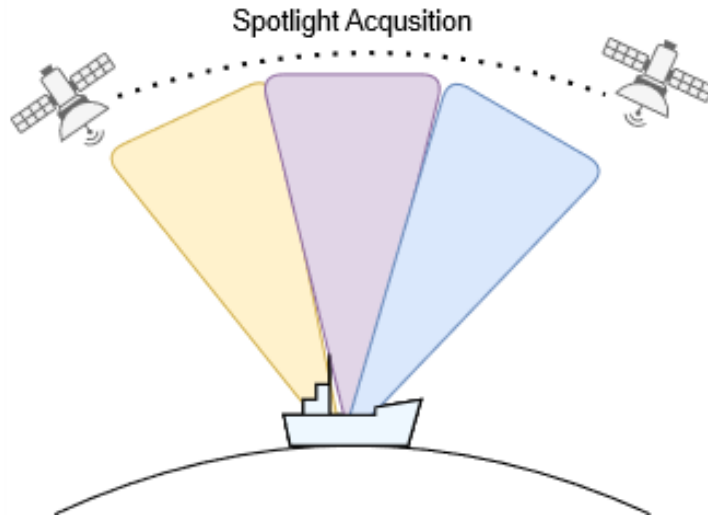


Figure 3.10: Basic geometry of spotlight SAR acquisition with 3 sub-apertures.

In this configuration, the data are not processed only once at the end of the spotlight dwell but are processed incrementally between consecutive sub-apertures. This approach incurs a slight degradation in azimuth resolution compared to the full spotlight acquisition, since the angular extent of the synthetic aperture in Equation (1.14) is reduced to (θ_{sa}/N_{sub}) ; nonetheless, it enables the generation of a time series of detections that can be used as input to sequential tracking algorithms.

Target Detection Generation

For each sub-aperture, the segmentation map is a two-dimensional matrix in the range-samples - azimuth-lines domain, as presented in Section 1.2.3.

Therefore, over the entire spotlight acquisition the segmentation maps can be arranged as a three-dimensional array, formally

$$\mathcal{S} \in \{0, 1\}^{N_{sub} \times N_{rg} \times N_{az}}, \quad (3.32)$$

where N_{rg} and N_{az} denote the number of slant range and azimuth lines, respectively.

Range Sample of Target

Let us denote with $t_{acq,k}$ the acquisition time during the k^{th} sub-aperture, defined as the time elapsed from the beginning of the overall spotlight acquisition. The range line of the target of interest can be computed as

$$l_{rg} = \text{round}\left(F_s \frac{2(R(t_{acq,k}) - R_{near})}{c}\right) \quad (3.33)$$

where we remind that F_s is the sampling rate of the Analog-to-Digital Converter, c is the speed of light, and R_{near} is the slant-range to the near edge of the swath. Parameter $R(t_{acq,k})$ is the instantaneous slant-range, at acquisition time, from the satellite to the target of interest, given by

$$R(t_{acq,k}) = \|p(t_{acq,k}) - p^s(t_{acq,k})\| \quad (3.34)$$

where $p(t_{acq,k}) \in \mathbb{R}^3$ denotes the target position in the ECEF frame at acquisition time $t_{acq,k}$, obtained by converting the geodetic target coordinates in Equation (3.24), and $p^s(t_{acq,k}) \in \mathbb{R}^3$ denotes the satellite coordinates in ECEF frame.

Azimuth Line of Target

The azimuth line at which the target is located corresponds to the pulse whose echo was generated by the reflection from the target itself.

Therefore, given the acquisition time relative to the k^{th} sub-aperture $t_{acq,k}$, and the initial time of the k^{th} sub-aperture $t_{sub-init,k}$ (assuming $t_{sub-init,0} = 0$), the azimuth index l_{az} corresponding to the target location is computed as:

$$l_{az} = \text{round}(f_{prf} \cdot (t_{acq,k} - t_{sub-init,k})). \quad (3.35)$$

where we remind that f_{prf} is the transmitted Pulse Repetition Frequency (PRF).

3.3.3 Extended Target Simulation

Given the location of the target centroid, it is necessary to determine the number of bins occupied by the target along the range and azimuth dimensions.

Azimuth lines extension

The number of azimuth lines occupied indicates how many pulses have been reflected by the target. It can be computed as

$$\Delta N_{az} = f_{prf}(T_{synth} + \Delta T). \quad (3.36)$$

The quantity T_{synth} refers to the synthetic aperture time, e.g., the time during which a point target is within the main radar beam. It is given by

$$T_{synth} = \frac{L_{synth}}{v} = \frac{R_{mid}\theta_{-3dB}}{v} = \frac{0.886R\lambda}{vL_{ant}}, \quad (3.37)$$

where R_{mid} is the slant range to the target at the half time of the sub-aperture, v is the platform velocity, and L_{ant} is the antenna length.

Parameter ΔT , instead, denotes the time taken by the platform to travel a distance equal to the projected length of the target along azimuth

$$\Delta T = \frac{L_{az, proj}}{v} = \frac{|l \cos \theta| + |w \sin \theta|}{v}, \quad (3.38)$$

where we assume that the target has length l , width w , and heading θ .

Range sample extension

In the raw SAR data, a total of N_{rg} range samples are used to represent a swath of width R_{swath} in slant range. Therefore, each pixel has a range dimension equal to

$$n_{sr} = \frac{R_{swath}}{N_{rg}} = \frac{R_{swath}}{\frac{2F_s R_{swath}}{c}} = \frac{c}{2F_s}. \quad (3.39)$$

Given a ship of length l , width w , and heading θ , its projection along the ground range is

$$W_{gr} = |l \sin \theta| + |w \cos \theta|, \quad (3.40)$$

and the projection along slant range is

$$W_{sr} = \frac{W_{gr}}{\sin \theta_i}, \quad (3.41)$$

where θ_i is the incidence angle of the transmitted beam.

Accordingly, the number of range-sample bins occupied by the target can be expressed as

$$\Delta N_{rg} = \frac{W_{sr}}{n_{sr}}. \quad (3.42)$$

3.3.4 Georeferencing Algorithm

Given the bounding boxes generated in Section 3.3.3, the next step consists of performing georeferencing on the centroid of each bounding box, i.e., mapping its location from the range samples-azimuth lines plane into a geographic coordinate system.

The acquisition time $t_{acq,k}$ can be computed from the azimuth line from Eqn. (3.35) as

$$t_{acq,k} = \text{round}\left(\frac{l_{az}}{f_{prf}} + t_{sub-init,k}\right). \quad (3.43)$$

Given the range sample of the centroid l_{rg} , from Eqn (3.33) it follows that the slant range at acquisition time, i.e., the line-of-sight distance between the satellite and the target, is

$$R(t_{acq,k}) = \text{round}\left(\frac{cl_{rg}}{2F_s} + R_{near}\right). \quad (3.44)$$

Starting from these parameters, the centroid of the target is georeferenced with respect to the WGS-84 ECEF reference frame. The georeferenced coordinates of the centroid at acquisition time $(x(t_{acq,k}), y(t_{acq,k}), z(t_{acq,k}))$ must satisfy three constraints.

Slant Range Equation

The georeferenced target coordinates should be at a slant range $R(t_{\text{acq},k})$ given by Equation (3.44), from the satellite

$$\|p(t_{\text{acq},k}) - p^s(t_{\text{acq},k})\|^2 = R(t_{\text{acq},k}), \quad (3.45)$$

where $p(t_{\text{acq},k}) = [x(t_{\text{acq},k}), y(t_{\text{acq},k}), z(t_{\text{acq},k})]^\top$.

Maritime Target Equation

Since ships are maritime moving targets, they must lie on the reference ellipsoid defined by WGS-84, hence

$$\frac{x^2(t_{\text{acq},k}) + y^2(t_{\text{acq},k})}{a^2} + \frac{z^2(t_{\text{acq},k})}{b^2} = 1, \quad (3.46)$$

where a and b are the semi-major and semi-minor axes, respectively, of the reference ellipsoid.

Doppler Equation

The Doppler equation is

$$\frac{2}{\lambda_{\text{tx}}} \frac{(p^s(t_{\text{acq},k}) - p(t_{\text{acq},k})) \cdot (v^s(t_{\text{acq},k}) - v(t_{\text{acq},k}))}{\|p^s(t_{\text{acq},k}) - p(t_{\text{acq},k})\|} = f_{D,\text{sim}}, \quad (3.47)$$

where $v^s(t_{\text{acq},k}) \in \mathbb{R}^3$ denotes the satellite's velocity at acquisition time in the ECEF frame, $v(t_{\text{acq},k}) \in \mathbb{R}^3$ is the target's velocity in the ECEF frame, λ_{tx} is the transmitted wavelength and $f_{D,\text{sim}}$ is the simulated Doppler shift, computed as the left side of Equation (3.47) using the true target and satellite position and velocity.

3.3.5 Turn-rate estimator

This subsection introduces the neural network employed to infer the ship's turn rate, denoted as ω_k . In this work, the tracking algorithm, e.g., an Extended Kalman Filter (EKF), relies on the kinematic model described in Equation (3.31), while the turn rate is assumed to be unknown. Therefore, we developed a neural network that takes the two most recent position estimates of the ship as input to infer the turn-rate parameter.

Problem statement and input representation

At time index k the network receives the two most recent geodetic coordinate estimates and their acquisition times:

$$(\hat{\lambda}_{k-1}, \hat{\phi}_{k-1}, t_{\text{acq},k-1}), \quad (\hat{\lambda}_k, \hat{\phi}_k, t_{\text{acq},k}). \quad (3.48)$$

These are concatenated and flattened to form the input vector

$$\mathbf{x}^{(k)} = \begin{bmatrix} \hat{\lambda}_{k-1} \\ \hat{\phi}_{k-1} \\ t_{\text{acq},k-1} \\ \hat{\lambda}_k \\ \hat{\phi}_k \\ t_{\text{acq},k} \end{bmatrix} \in \mathbb{R}^6. \quad (3.49)$$

The network is therefore a mapping

$$\hat{\omega} = f_\theta(\mathbf{x}), \quad f_\theta : \mathbb{R}^6 \rightarrow \mathbb{R}, \quad (3.50)$$

parametrized by weights θ .

Dataset construction

The dataset is generated as follows. For each generated sequence in the dataset:

1. Draw the per-sequence true turn-rate $\omega \sim \mathcal{N}(1 \times 10^{-3}, (3 \times 10^{-4})^2)$ and clip to $[-2 \times 10^{-3}, 2 \times 10^{-3}]$;
2. Draw a small constant per-sequence acceleration α and clip to a bounded interval (to keep speeds realistic);
3. Set an initial state $(\phi_0, \lambda_0, \psi_0, v_0)$ and propagate the discrete-time dynamics Equation (3.31) for T steps.
4. To train the neural network, we extract the last two observed coordinates and time, and form the vector $\mathbf{x} = [\lambda_{k-1}, \phi_{k-1}, t_{k-1}, \lambda_k, \phi_k, t_k]^\top$.

Note that, while the training is performed using the past two exact target coordinates, the inference is done by using the past two estimated coordinates, together with the acquisition time, as feature vector $\mathbf{x}$.

Neural Network architecture and preprocessing

In this work, we employ a dense MLP, e.g., where every neuron of a given layer is connected to every neuron of the following one. The proposed MLP is structured as follows:

- **Input layer:** receives the feature vector $\mathbf{x} \in \mathbb{R}^6$ as input, therefore it comprises $n = 6$ neurons.
- **Hidden layer 1:** made by 64 neurons with Rectified Linear Unit (ReLU) activation function, see Appendix C.
- **Hidden layer 2:** made by 32 neurons, each with ReLU activation function.
- **Output layer:** scalar prediction.

Prior to training each input feature is standardized using the empirical mean and standard deviation computed over the training set:

$$\begin{aligned}\tilde{\mathbf{x}} &= \frac{\mathbf{x} - \boldsymbol{\mu}}{\boldsymbol{\sigma}}, \\ \mu_j &= \frac{1}{N} \sum_i x_{i,j}, \\ \sigma_j &= \sqrt{\frac{1}{N} \sum_i (x_{i,j} - \mu_j)^2}.\end{aligned}\tag{3.51}$$

The fitted scaler $(\boldsymbol{\mu}, \boldsymbol{\sigma})$ is saved with the model so that the same normalization is applied during deployment.

Training objective and optimization

To train an MLP means to optimize a given cost function $\mathcal{L}(\theta)$, with respect to the parameter vector $(\theta) \in \mathbb{R}^p$, which includes the network weights and biases, see Appendix C.

The training performed in this work minimizes the Mean-Squared Error (MSE) between the predicted and true per-sequence turn-rate:

$$\mathcal{L}(\theta) = \frac{1}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} (\hat{\omega}_\theta(\mathbf{x}^{(i)}) - \omega^{(i)})^2,\tag{3.52}$$

where $\mathcal{B}$ denotes a mini-batch of training examples. Optimization is performed with the Adam algorithm and standard mini-batch training. Per-epoch averages of training and validation MSE are shown in Figure 3.11.

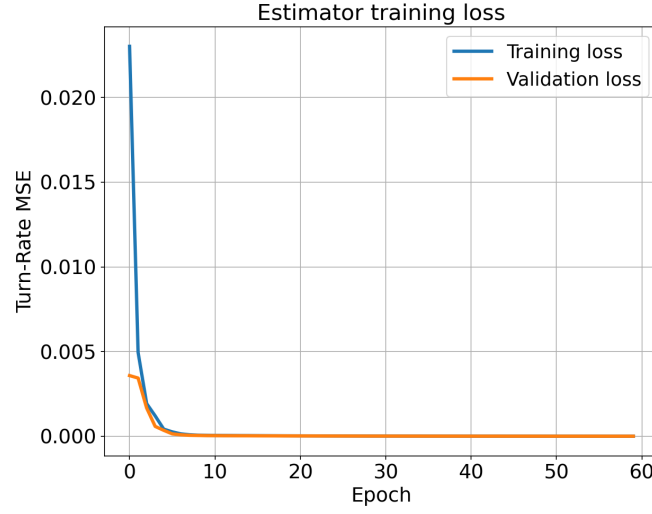


Figure 3.11: Training and validation MSE on the turn-rate prediction.

3.3.6 AI-Enhanced Filtering Algorithm

The AI-enhanced tracking algorithm employed in this work is based on the Extended Kalman Filter to handle the nonlinearity of the dynamic model described in Equation (3.31).

As introduced in Section 2.4, the EKF consists of two steps: prediction and update.

Prediction Step

At time step k , the prediction step employs the dynamic model in Equation (3.31) to propagate the estimated state $\hat{x}_{k|k} \in \mathbb{R}^4$ and covariance matrix $P_{k|k} \in \mathbb{R}^{4 \times 4}$ and compute the one-step-ahead predicted state $\hat{x}_{k+1|k}$ and covariance $P_{k+1|k}$.

Because the tracker does not know the true turn-rate ω_k of the kinematic model, it is estimated by a pre-trained neural network. Hence the prediction step, for $k \geq 1$, is given by

$$\mathbf{x}^{(k)} = \begin{bmatrix} \hat{x}_{k-1|k-1}[0] \\ \hat{x}_{k-1|k-1}[1] \\ t_{\text{acq},k-1} \\ \hat{x}_{k-1|k}[0] \\ \hat{x}_{k-1|k}[1] \\ t_{\text{acq},k} \end{bmatrix} \quad (3.53)$$

$$\hat{\omega}_k = f_{\theta}(\mathbf{x}^{(k)})$$

$$\hat{x}_{k+1|k} = f_{\omega_k}(x_k)$$

$$P_{k+1|k} = F P_{k|k} F^{\top} + Q,$$

where $f_{\omega_k}(\cdot) : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ represents the cargo dynamical model that employs the inferred turn-rate $\hat{\omega}_k$, and $F \in \mathbb{R}^{4 \times 4}$ is the Jacobian of Equation (3.31). For time step $k = 0$, instead, the dynamic model uses a pre-defined turn-rate $\hat{\omega}_0$.

Update Step

The update step leverages the SAR-based detections to refine the predicted state $\hat{x}_{k+1|k}$ and covariance $P_{k+1|k}$ into the estimated quantities $\hat{x}_{k+1|k+1}$, $P_{k+1|k+1}$.

By georeferencing the SAR-detected centroid and converting it to geodetic coordinates, a measurement of the target's geodetic longitude and latitude is obtained, e.g., the first

two components of the target state x_k . Therefore the measurement model can be written as

$$z_k = Hx_k + n_k, \quad (3.54)$$

where the state-to-measurement matrix $H_k \in \mathbb{R}^{2 \times 4}$ is

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad (3.55)$$

and $n_k \sim \mathcal{N}(0_{2 \times 1}, R)$ is the additive measurement noise, with covariance matrix $R \in \mathbb{R}^{2 \times 2}$.

Since the measurement equation is linear, the update step of the tracking algorithm follows the same equations as the classical Kalman Filter, given by Equations (A.12) - (A.15).

3.3.7 Simulation Results

Simulation Setup

The tracking framework exploits SAR detections acquired in spotlight configuration and processes the data in sub-apertures rather than as a single batch at the end of the acquisition. We considered a spotlight acquisition time of $T_{\text{spot}} = 24\text{s}$, which was divided into six sub-apertures of $T_{\text{sub}} = 4\text{s}$ each.

Cargo and Satellite Initialization

In this work, the target is assumed to be a cargo ship. We considered a vessel with length $l = 222\text{m}$ and width $w = 35\text{m}$. The ship dynamics were simulated using Equation (3.31), with the initial state chosen as

$$x_0 = [\lambda_0 \ \phi_0 \ \psi_0 \ v_0]^\top = [23.64\text{deg} \ 37.84\text{deg} \ 45\text{deg} \ 8.4\text{m/s}]^\top, \quad (3.56)$$

and with a constant acceleration of $\alpha = -0.01\text{m/s}^2$ and turn-rate of $\omega = 0.025\text{rad/s}$. Figure 3.12 shows the time evolution of the cargo state components.

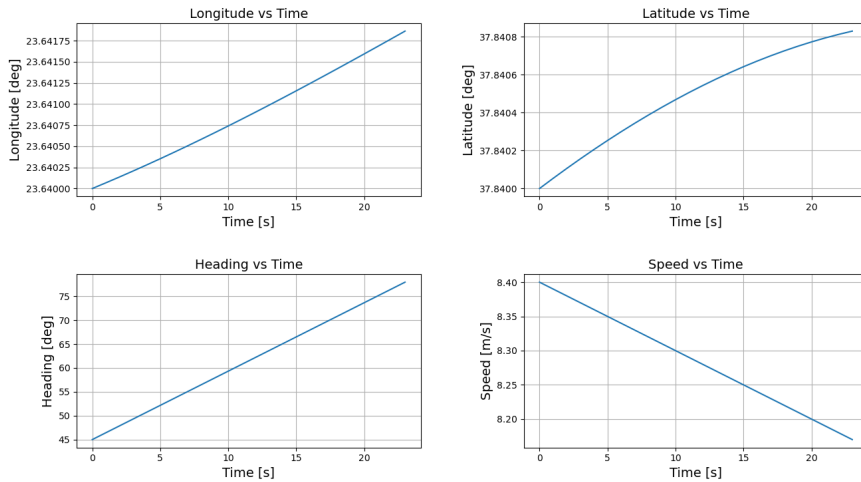


Figure 3.12: Cargo state time evolution.

A Low Earth Orbit (LEO) satellite carrying the SAR sensor was modeled, and its orbital dynamics were simulated using the General Mission Analysis Tool (GMAT) [71].

In order to obtain a trajectory compatible with the target location, we considered a TA1 Modified Julian Epoch of 216300s and, as for the initial state

$$x_0^s = \begin{bmatrix} p_{x,0}^s \\ p_{y,0}^s \\ p_{z,0}^s \\ v_{x,0}^s \\ v_{y,0}^s \\ v_{z,0}^s \end{bmatrix} \simeq \begin{bmatrix} 4802.31\text{km} \\ 2727.36\text{km} \\ 4346.66\text{km} \\ 4.22\text{km/s} \\ 1.99\text{km/s} \\ -5.92\text{km/s} \end{bmatrix} \quad (3.57)$$

where $p_{x,0}^s, p_{y,0}^s, p_{z,0}^s$ denote the coordinates, at discrete time step $k = 0$, of the satellite in an Earth-Centered Inertial frame, while $v_{x,0}^s, v_{y,0}^s, v_{z,0}^s$ are the respective velocities. Figure 3.13 depicts the simulated satellite state in the J2000 Earth-Centered Inertial frame

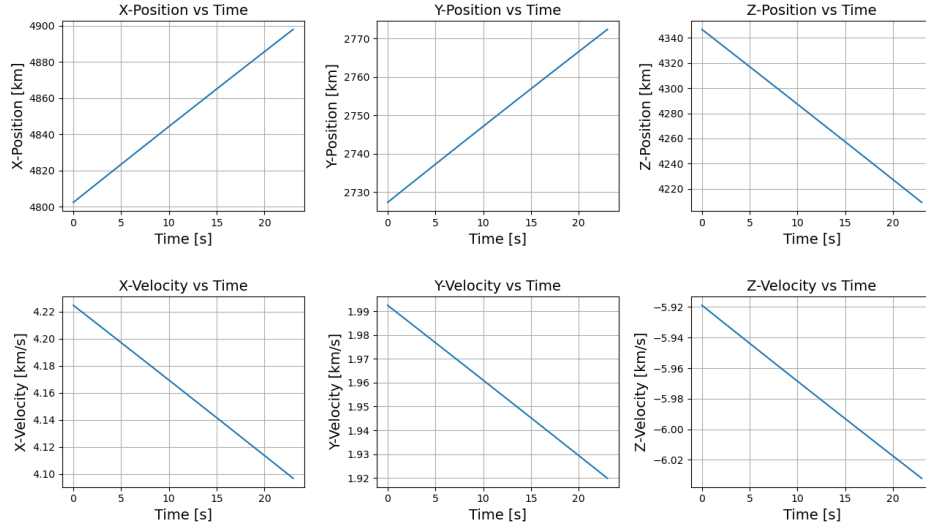


Figure 3.13: Satellite state time evolution in ECI frame.

Bounding Box Generation

As shown in Section 3.3.2, the segmentation map is determined by the SAR sensor parameters. For this work, we configured the following parameters:

- Pulse repetition frequency as $f_{\text{prf}} = 3.7\text{kHz}$,
- Transmitted bandwidth of $B = 150\text{MHz}$, from which the Analog-to-Digital Converter sampling frequency is set as $f_{\text{adc}} = 2.2B$ to avoid aliasing,
- Swath width of $R_{\text{swath}} = 20\text{km}$ in slant range,
- Antenna length of $L_a = 2\text{m}$
- A carrier frequency of $f_c = 5\text{GHz}$,
- Beam center of geodetic longitude $\lambda = 23.64\text{deg}$ and latitude $\phi = 37.84\text{deg}$

Furthermore, we assumed a perfect detectability of the target and the absence of false positives.

An example of a bounding box for a given sub-aperture is shown in Figure 3.14.

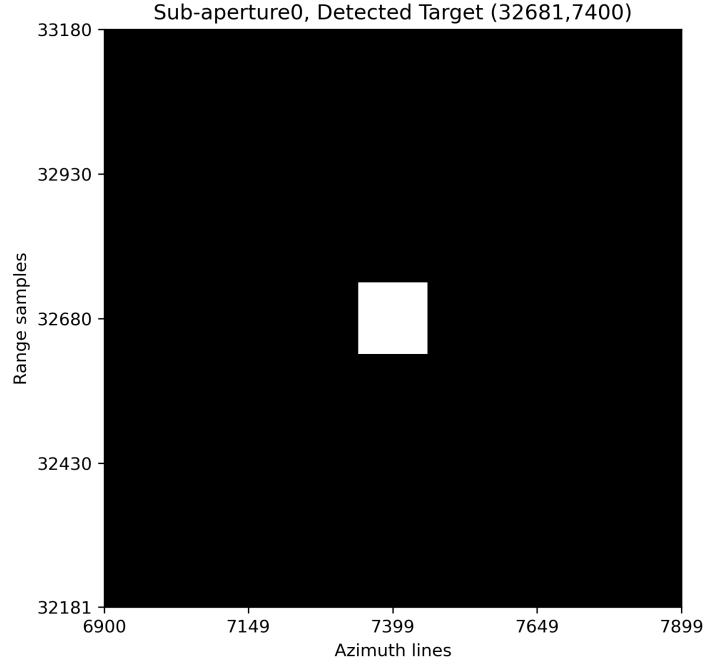


Figure 3.14: Bounding box for SAR-based vessel detection, zoomed around the target centroid.

Georeferencing Results

For each sub-aperture, the detected centroid was georeferenced; specifically, a coordinate transformation was performed mapping the raw SAR coordinate domain to a position on the Earth’s surface.

This coordinate transformation is expressed by the mathematical constraints in Equations (3.45) - (3.47), where in Equation (3.47) we assume complete knowledge of the target’s velocity in order to resolve the constraint. Solving these equations requires knowledge of the satellite state with respect to the adopted ECEF frame. In this case, we assume the satellite is equipped with an onboard Global Positioning System (GPS) [72] sensor that provides measurements of its position $\tilde{p}_k^s \in \mathbb{R}^3$ and velocity $\tilde{v}_k^s \in \mathbb{R}^3$ in the ECEF frame as

$$z_k = \begin{bmatrix} \tilde{p}_k^s \\ \tilde{v}_k^s \end{bmatrix} = \begin{bmatrix} p_k^s \\ v_k^s \end{bmatrix} + r_k, \quad (3.58)$$

where $r_k \in \mathbb{R}^6$ is the GPS measurement, described by $r_k \sim \mathcal{N}(0_{6 \times 1}, I_{6 \times 6})$.

Figure 3.15 shows the georeferencing results for the target longitude.

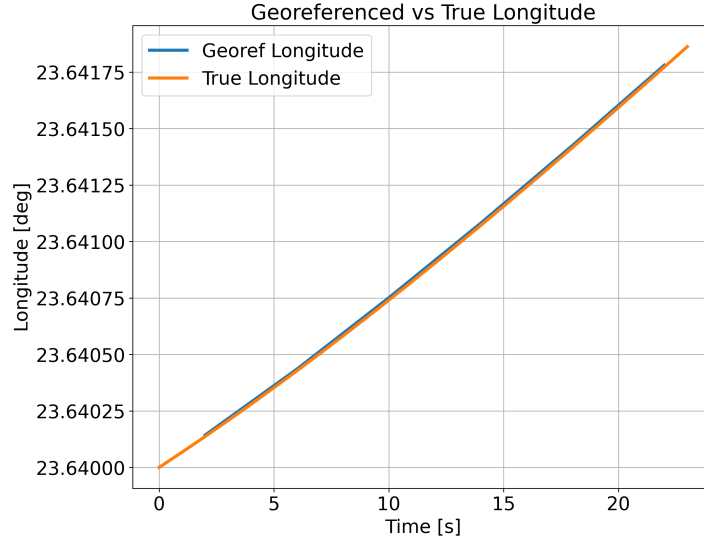


Figure 3.15: True target longitude vs Georeferenced longitude.

It can be easily seen that the georeferencing algorithm correctly computes the geodetic longitude of the simulated cargo.

A similar result is obtained also for the georeferenced cargo latitude, shown in Figure 3.16.

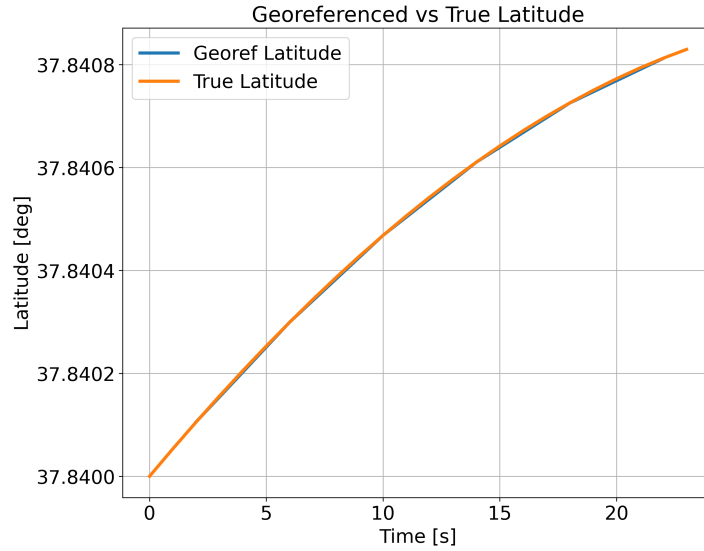


Figure 3.16: True target latitude vs Georeferenced latitude.

Tracking Results

The EKF filtered state $\hat{x}_{k|k}$ and covariance matrix $P_{k|k}$ have been initialized as

$$\hat{x}_{0|0} = \begin{bmatrix} 23.6457\text{deg} \\ 37.8515\text{deg} \\ 45.0172\text{deg} \\ 8.52\text{m/s} \end{bmatrix}, \quad (3.59)$$

$$P_{0|0} = 0.001I_{4 \times 4}. \quad (3.60)$$

The tracking results for the target's longitude are reported in Figure 3.17.

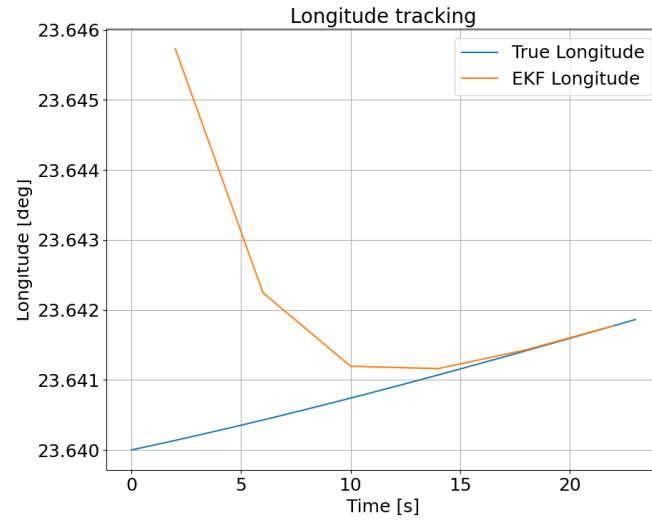


Figure 3.17: True target and AI-enhanced tracked longitude.

The tracked and true target latitude, instead, are reported in Figure 3.18.

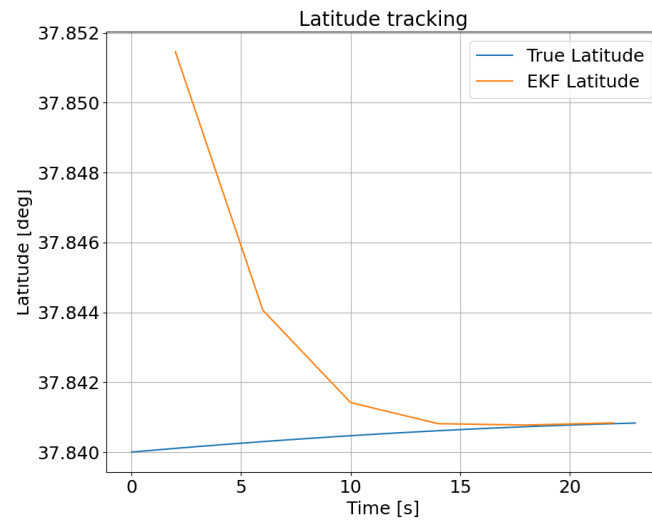


Figure 3.18: True target and AI-enhanced tracked latitude.

From both plots it is worth noting that, as the SAR system detects the cargo vessel, the tracked quantities converge to the true values.

Conclusions

Chapter 4

Conclusions

In this thesis we address the modeling and validation of spaceborne cognitive radars. Making a radar cognitive pursues the ambitious objective of enabling the sensor to autonomously adapt its transmission configuration on the basis of an analysis of the data acquired in reception. This thesis focuses on the application of control algorithms and artificial intelligence techniques for cognitive spaceborne tracking radars. In detail, we first consider cognitive tracking with real aperture radars. By formulating the tracking algorithm equations (e.g., Extended Kalman Filter) as a dynamical system, it was possible to implement an optimal control technique that determines the optimal sequence of transmitted radar signal parameters in order to minimize the estimation error on the target's position and velocity. This approach was then compared with classical methods more commonly employed in the literature, thereby demonstrating its superior performance. Subsequently, our research focused on target tracking with a Tip & Cue setup. This topic represents an emerging research field, with the related literature still in its early stages. The Tip & Cue framework, which entails the interaction between two satellites to achieve a more accurate acquisition of an area of interest, was adopted as the reference setup for maritime target monitoring and tracking. In the first phase, we trained a Gaussian Process Regression model using sparse AIS messages to reconstruct the temporal evolution of the longitude and latitude of the vessel of interest. The algorithm was then ported to two space-qualified Field-Programmable Gate Arrays, and its performance was evaluated in terms of training time, inference latency, and the number of vessels that can be simultaneously tracked. Subsequently, this work presents a target-tracking model that leverages AI-based SAR detections. A method for the artificial generation of such detections is introduced, grounded in radar principles and easily adaptable to different parameter settings. An Extended Kalman Filter then exploits these detections, after georeferencing, as target measurements, combining a model-based and data-driven approach to represent the vessel's dynamics.

Future research activities include the application of sliding-window control techniques to cognitive tracking radars. In this context, methods such as Model Predictive Control may exploit radar measurements as feedback to refine the prediction of the sequence of optimal transmitted-signal parameters. This, however, comes at a higher computational cost compared to the approaches presented in this work. Moreover, further research directions may involve the implementation of a perception-action cycle at the follower payload level within the Tip & Cue framework for target tracking. The presented framework has memory of the last two target coordinates to estimate the unknown parameters of the dynamical model, however a feedback loop could be added on top to optimize the transmitted waveform parameters. In this setting, the SAR payload could adapt the transmitted-signal parameters to optimize a specific performance metric, e.g., Signal-to-Noise Ratio or derived metrics, thereby enhancing high-resolution acquisitions over the area of interest. Furthermore, finding a connection between the waveform adaptation and the performance of the AI-based SAR detection would be of great interest.

Appendices

Appendix A

Bayesian Filtering and Exact Solutions

This appendix provides a description of the Bayesian filtering equations, which are the general equations for computing Bayesian filtering solutions to both linear Gaussian and non-linear non-Gaussian state space models. The Kalman Filter equations are also derived as the closed form solution to the linear Gaussian Bayesian filtering problem.

A.1 Probabilistic State-Space Models

Bayesian filtering is considered with state estimation in general probabilistic state-space models [73].

A probabilistic state-space model consists of a sequence of conditional probability distributions

$$\begin{aligned}x_k &\sim p(x_k|x_{k-1}) \\z_k &\sim p(z_k|x_k),\end{aligned}\tag{A.1}$$

for $k = 1, 2, \dots, K$, where

- $x_k \in \mathbb{R}^n$ is the state of the system at time step k ,
- $z_k \in \mathbb{R}^m$ is the measurements at time step k ,
- $p(x_k|x_{k-1})$ is the dynamical model describing the stochastic dynamics of the system. It can be a probability density, a counting measure, or a combination of them.
- $p(z_k|x_k)$ is the measurement model, which is the distribution.

The model in equation (A.1) is assumed to be Markovian. Therefore, the following two properties hold [73].

Property 1: Markov property of state

The states $x_k : k = 0, 1, 2, \dots, K$ for a Markov sequence, e.g., the state x_k given x_{k-1} is independent of anything that has happened before the time step $k - 1$:

$$p(x_k|x_{1:k-1}, z_{1:k-1}) = p(x_k|x_{k-1}).\tag{A.2}$$

Also the past is independent of the future given the present:

$$p(x_{k-1}|x_{k:K}, z_{k:K}) = p(x_{k-1}|x_k).\tag{A.3}$$

Property 2: Conditional independence of measurements

The current measurement z_k given the current state x_k is conditionally independent of the measurement and state histories:

$$p(z_k|x_{1:k}, z_{1:k-1}) = p(z_k|x_k)\tag{A.4}$$

A.2 Bayesian Filtering Equation

The Bayesian filtering computes the marginal posterior distribution of the state x_k at each time step k given the history of the measurement up to time step k $p(x_k|z_{1:k})$.

Theorem: Bayesian Filtering Equations

The recursive equations for computing the predicted distribution $p(x_k|z_{1:k-1})$ and the filtering distribution $p(x_k|z_{1:k})$ at time step k are given by the following Bayesian filtering equations [73].

- **Initialization:** the recursion starts from the prior distribution $p(x_0)$.
- **Prediction step:** the predictive distribution of the state x_k at time step k , given the dynamical model $p(x_k|x_{k-1})$, can be computed by the Chapman-Kolmogorov equation

$$p(x_k|z_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1})dx_{k-1}. \quad (\text{A.5})$$

- **Update step:** given the measurement z_k at time step k , the posterior distribution of the state x_k can be computed by the Bayes' rule

$$p(x_k|z_{1:k}) = \frac{1}{Z_k}p(z_k|x_k)p(x_k|z_{1:k-1}), \quad (\text{A.6})$$

where the normalization constant Z_k is

$$Z_k = \int p(z_k|x_k)p(x_k|z_{1:k-1})dx_k. \quad (\text{A.7})$$

Reference [73] contains the proof of the previous theorem.

A.3 Kalman Filter

The Kalman Filter (KF) [74] is the closed form solution to the Bayesian filtering problem, where the dynamic and measurement models are linear Gaussian:

$$\begin{aligned} x_{k+1} &= A_k x_k + w_k, \\ z_k &= H_k x_k + n_k, \end{aligned} \quad (\text{A.8})$$

where $x_k \in \mathbb{R}^n$ is the state, $z_k \in \mathbb{R}^m$ is the measurement vectore, $w_k \sim \mathcal{N}(0_{n \times 1}, Q_k)$ is the process noise, with covariance matrix $Q_k \in \mathbb{R}^{n \times n}$, and $n_k \sim \mathcal{N}(0_{m \times 1}, R_k)$ is the measurement noise, with covariance matrix $R_k \in \mathbb{R}^{m \times m}$. The prior distribution is Gaussian $x_0 \sim \mathcal{N}(\hat{x}_{0|0}, P_{0|0})$, with mean $\hat{x}_{0|0} \in \mathbb{R}^n$ and covariance $P_{0|0} \in \mathbb{R}^{n \times n}$. The matrix $A_k \in \mathbb{R}^{n \times n}$ is the state transition matrix and the matrix $H_k \in \mathbb{R}^{m \times n}$ is the state to measurement matrix.

Theorem: Kalman Filter

The Bayesian filtering equations for the linear filtering model in equations (A.8) can be evaluated in closed form and the resulting distributions are Gaussian

$$\begin{aligned} p(x_k|z_{1:k-1}) &= \mathcal{N}(x_k|x_{k|k-1}, P_{k|k-1}) \\ p(x_k|z_{1:k}) &= \mathcal{N}(x_k|\hat{x}_{k|k}, P_{k|k}) \\ p(z_k|z_{1:k-1}) &= \mathcal{N}(z_k|H_k \hat{x}_{k|k-1}, S_k). \end{aligned} \quad (\text{A.9})$$

The parameters of the distributions in equations (A.9) can be computed with the prediction and update steps of the Kalman Filter.

A.3.1 Prediction Step

The prediction step employs the state dynamical model to compute the predicted state $\hat{x}_{k+1|k}$ and covariance matrix $P_{k+1|k}$ given their estimated values at time step k , e.g., $\hat{x}_{k|k}$ and $P_{k|k}$ respectively.

$$\hat{x}_{k+1|k} = A_k \hat{x}_{k|k} \quad (\text{A.10})$$

$$P_{k+1|k} = A_k P_{k|k} A_k^\top + Q_k. \quad (\text{A.11})$$

A.3.2 Update Step

The update step leverages the measurement vector z_{k+1} , and the measurement equation, to refine the predicted state $\hat{x}_{k+1|k}$ and covariance $P_{k+1|k}$ into the estimated state $\hat{x}_{k+1|k+1}$ and covariance matrix $P_{k+1|k+1}$.

$$S_{k+1} = H_{k+1}P_{k+1|k}H_{k+1}^\top + R_{k+1} \quad (\text{A.12})$$

$$K_{k+1} = P_{k+1|k}H_{k+1}^\top S_{k+1}^{-1} \quad (\text{A.13})$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(z_{k+1} - H_{k+1}\hat{x}_{k+1|k}) \quad (\text{A.14})$$

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1}S_{k+1}K_{k+1}^\top. \quad (\text{A.15})$$

Appendix B

Geographical Coordinate System

This appendix provides an overview of the coordinate systems employed throughout the research activities and of the transformations between the different systems presented.

B.1 Coordinate Systems

B.1.1 Earth-Centered Inertial

The Earth-Centered Inertial (ECI) coordinate system is a right-handed Cartesian triad, conventionally denoted as $\{\hat{\mathbf{I}}, \hat{\mathbf{J}}, \hat{\mathbf{K}}\}$, with its origin located at the center of the Earth. The ECI frame is an inertial, e.g., non-rotating, Cartesian frame whose axes are fixed with respect to distant celestial objects.

The fundamental plane is the Earth's equatorial plane, as illustrated in Figure B.1. The $\hat{\mathbf{I}}$ axis points toward the vernal equinox γ , the $\hat{\mathbf{J}}$ axis lies in the equatorial plane 90° eastward of $\hat{\mathbf{I}}$, and the $\hat{\mathbf{K}}$ axis is aligned with the North Pole [47].

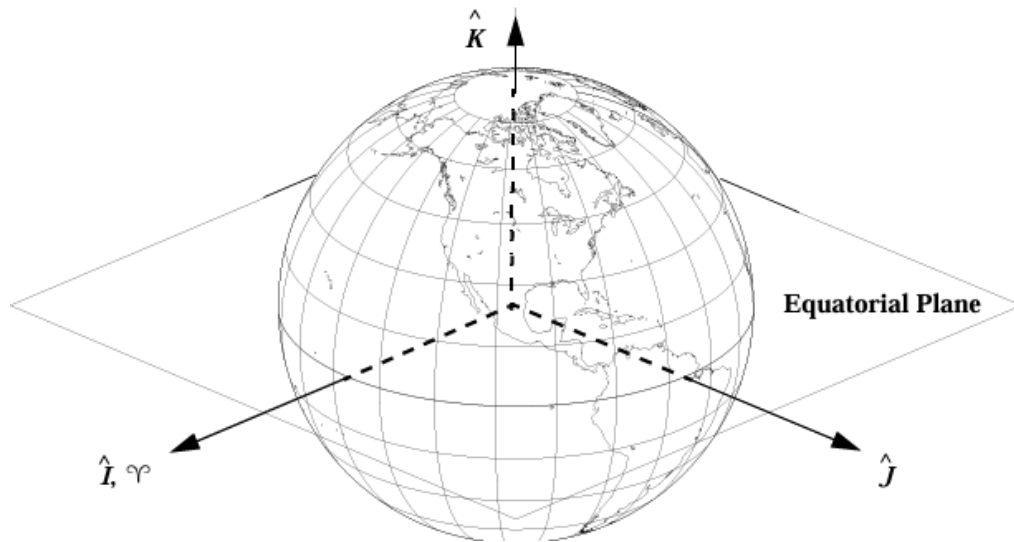


Figure B.1: ECI coordinate system. Courtesy of [47].

The vernal equinox and the Earth's equatorial plane drift slowly with time; therefore an ECI frame must be referenced to a specific epoch [47].

B.1.1.1 J2000

The J2000 frame is a widely used ECI reference whose axes are defined with respect to the mean equator and mean equinox of epoch J2000. The epoch J2000 corresponds to 1 January 2000 at 12:00 Terrestrial Time.

B.1.2 Earth-Centered Earth-Fixed

The Earth-Centered Earth-Fixed (ECEF) reference frame is a right-handed, geocentric Cartesian coordinate system commonly used to represent positions on or near the Earth. It is a non-inertial reference frame, as it rotates together with the Earth.

Generally, coordinates are expressed in the ECEF frame as realized by the World Geodetic System 1984 (WGS-84), which defines the reference ellipsoid parameters and the geocentric terrestrial reference frame used by the Global Positioning System.

B.1.2.1 World Geodetic System 1984

The World Geodetic System 1984 represents the best global geodetic reference system for the Earth for applications of mapping, charting, geopositioning and navigation [75].

The WGS 84 coordinate system is a right-handed, Earth-fixed orthogonal coordinate system, as depicted in Figure B.2.

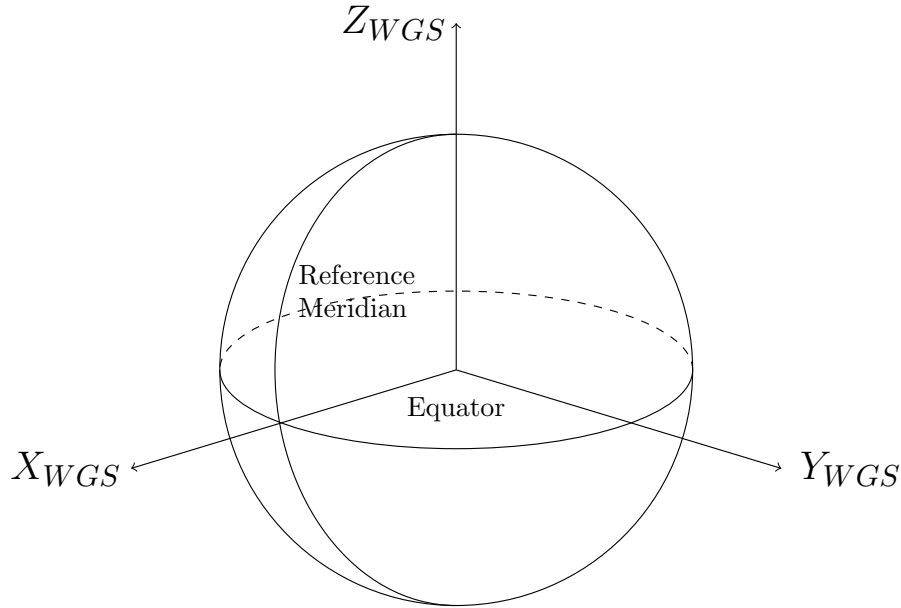


Figure B.2: World Geodetic System 1984 Coordinate System.

The origin and axes are defined as follows:

- Origin: Earth's center of mass.
- Z-axis: the direction of International Earth Rotation and Reference Systems Service (IERS) Reference Pole.
- X-axis: intersection of IERS Reference Meridian and the plane passing through the origin and normal to the Z-axis.
- Y-axis: completes a right-handed coordinate system.

The WGS 84 datum additionally specifies the parameters of the reference ellipsoid:

- Semi-major axis a as $a = 6378137.0\text{m}$.
- Flattening f as $1/f = 298.257223563$.

Other commonly cited parameters of the reference ellipsoid are the semi-minor axis b and the eccentricity e . These may be obtained from the values above, since the flattening satisfies $f = 1 - b/a$ and the eccentricity is given by $e = \sqrt{1 - b^2/a^2}$.

B.1.3 Local-Tangent

Local-tangent reference system is a Cartesian frame whose origin P is located at or near the Earth's surface, and its horizontal plane is approximately coincident with the local level [72].

B.1.3.1 East-North-Up

The East-North-Up (ENU) reference system is a type of local-tangent coordinate system. As shown in Figure B.4, it is formed by the right-handed orthonormal triad $\{\mathbf{e}, \mathbf{n}, \mathbf{u}\}$.

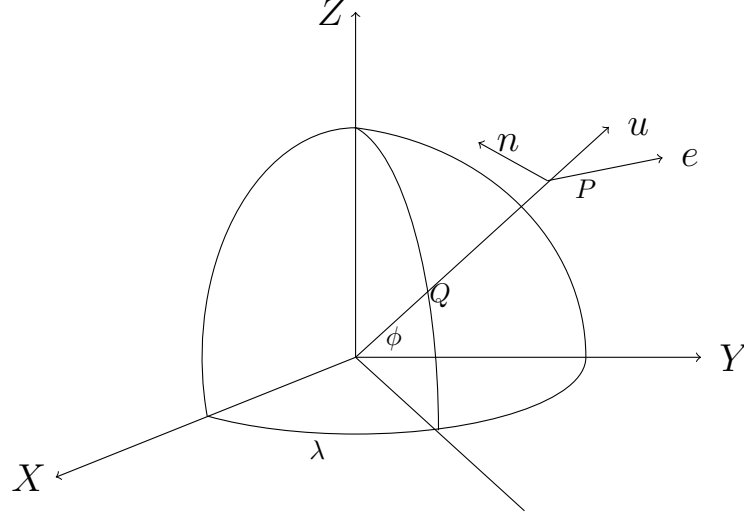


Figure B.3: East-North-Up coordinate system

The Up axis $\mathbf{u}$ is aligned with the outward normal to the reference ellipsoid at the point Q , that is the projection of point P on the ellipsoid's surface, and is positive away from the Earth's surface; the North axis $\mathbf{n}$ is tangent to the geodetic meridian in an Earth-fixed realization and is positive toward the North Pole; the East axis $\mathbf{e}$ completes the right-handed system.

B.1.4 Geodetic System

The geodetic spatial coordinate system is defined by the triplet $\{\lambda, \phi, h\}$, which denote, respectively, the longitude, latitude, and geodetic height [76].

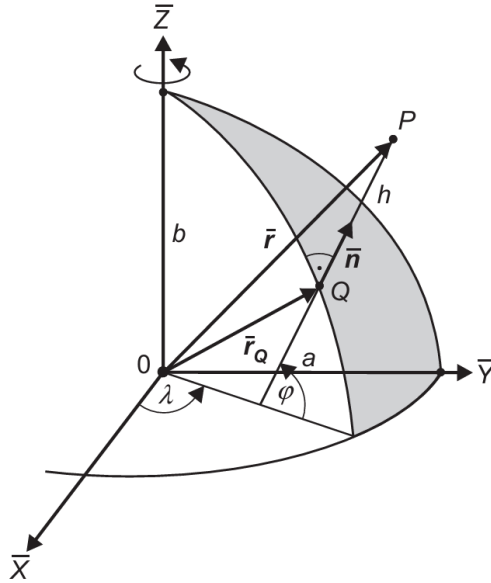


Figure B.4: Visual representation of the geodetic coordinate system. Courtesy of [76].

Considering the section of the reference ellipsoid depicted Figure. B.4, the point of interest P , and its projection on the ellipsoid surface Q , the geodetic longitude λ denotes the angle measured in the equatorial plane ($\bar{X}\bar{Y}$ -plane) between the prime meridian ($\bar{X}$ -axis) and the meridian plane of point P . This quantity is positive when measured eastward.

The geodetic latitude ϕ is the angle measured in the meridian plane between the equatorial plane of the ellipsoid and the normal to the surface at point Q .

Lastly, the geodetic height h is defined as the distance, measured along the normal to the surface at point Q , between point P and the ellipsoid surface.

B.2 Coordinate Conversions

This section describes the transformations between the reference frames introduced above. Additional conversions may be obtained by composing the individual transformations presented.

B.2.1 Earth-Centered Inertial to Earth-Centered Earth-Fixed

Transforming coordinates from the ECI frame to the ECEF one requires the application of a sequence of rotations.

Specifically, given a position vector expressed in ECI coordinates $\mathbf{p}_{\text{ECI}} = [x_{\text{ECI}}, y_{\text{ECI}}, z_{\text{ECI}}]^\top \in \mathbb{R}^3$, its representation in the ECEF frame $\mathbf{p}_{\text{ECEF}} = [x_{\text{ECEF}}, y_{\text{ECEF}}, z_{\text{ECEF}}]^\top \in \mathbb{R}^3$ is obtained as

$$\mathbf{p}_{\text{ECEF}} = \mathbf{W}(x_p, y_p) \mathbf{R}_3(\theta_{\text{GAST}}) \mathbf{N}(t) \mathbf{P}(t) \mathbf{p}_{\text{ECI}}. \quad (\text{B.1})$$

Equation (B.1) accounts for phenomena such as [47]:

- Precession: the Earth's rotation axis is not fixed as it slowly traces out a cone. It shifts the orientation of the equator and the vernal equinox over long timescales. It is denoted with matrix $\mathbf{P}(t)$.
- Nutation: on top of precession, the Earth's axis also makes small periodic oscillations caused mainly by the Moon and Sun's gravitational pull. It is denoted with matrix $\mathbf{N}(t) \in \mathbb{R}^{3 \times 3}$.
- Rotation about Earth's axis: rotation by the Earth's Greenwich Apparent Sidereal Time (GAST), which tells us how far the Earth has turned relative to the stars. It is modelled as a rotation about the z-axis of the GAST angle θ_{GAST} and denoted as $\mathbf{R}_3(\theta_{\text{GAST}})$.
- Polar motion: the Earth's geographic poles are not perfectly stable: the spin axis moves slightly with respect to the Earth's crust. It accounts for the small wobble between the Earth's instantaneous rotation axis and the crust-fixed coordinate system. It is denoted by $\mathbf{W}(x_p, y_p)$, with pole coordinates x_p and y_p .

In our work, for the sake of simplicity, we were not concerned with referencing the transformation to a specific epoch. Consequently, we limited the conversion to the rotation about the Earth's rotation axis, $\mathbf{R}(\theta_{\text{GAST}})$, where θ_{GAST} was computed as the product of the Earth's rotation rate and the elapsed time since 1 January 2000, 12:00 TT.

B.2.2 Earth-Centered Earth-Fixed to Earth-Centered Inertial

Given coordinates expressed in the ECEF frame $\mathbf{p}_{\text{ECEF}} \in \mathbb{R}^3$, the conversion to ECI coordinates $\mathbf{p}_{\text{ECI}} \in \mathbb{R}^3$ is performed through the inverse sequence of rotations of Equation (B.1), namely

$$\mathbf{p}_{\text{ECI}} = \mathbf{P}^{-1}(t) \mathbf{N}^{-1}(t) \mathbf{R}_3^{-1}(\theta_{\text{GAST}}) \mathbf{W}^{-1}(x_p, y_p) \mathbf{p}_{\text{ECEF}}. \quad (\text{B.2})$$

B.2.3 Geodetic to Earth-Centered Earth-Fixed

The geodetic triad $(\lambda, \phi, h) \in \mathbb{R}^3$ can be transformed into ECEF coordinates $\mathbf{p}_{\text{ECEF}} \in \mathbb{R}^3$ using the following equations [76]

$$\begin{aligned} x_{\text{ECEF}} &= (N(\phi) + h) \cos \phi \cos \lambda \\ y_{\text{ECEF}} &= (N(\phi) + h) \cos \phi \sin \lambda \\ z_{\text{ECEF}} &= ((1 - e^2)N(\phi) + h) \sin \phi, \end{aligned} \quad (\text{B.3})$$

where e is the eccentricity of the reference ellipsoid, and N is the prime vertical radius of curvature, given by

$$N(\phi) = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}. \quad (\text{B.4})$$

B.2.4 Earth-Centered Earth-Fixed to Geodetic

Transforming coordinates from ECEF to geodetic entails inverting Equation (B.3). However, this inversion produces a nonlinear equation in $\sin(\phi)$ or $\tan(\phi)$ which, if solved algebraically, gives rise to a high-degree polynomial [77].

Heikkinen [78] derived a quartic equation for computing ECEF coordinates from geodetic coordinates, stated as follows:

$$\begin{aligned} p &= \sqrt{x_{\text{ECEF}}^2 + y_{\text{ECEF}}^2} \\ F &= 54b^2 z_{\text{ECEF}}^2 \\ G &= p^2 + (1 - e^2)z_{\text{ECEF}}^2 - e^2(a^2 - b^2) \\ c &= \frac{e^4 F p^2}{G^3} \\ s &= \sqrt[3]{1 + c + \sqrt{c^2 + 2c}} \\ k &= s + 1 + \frac{1}{s} \\ P &= \frac{F}{3k^2 G^2} \\ Q &= \sqrt{1 + 2e^4 P} \\ r_0 &= -\frac{P e^2 p}{1 + Q} + \sqrt{\frac{1}{2} a^2 \left(1 + \frac{1}{Q}\right) - \frac{P(1 - e^2) z_{\text{ECEF}}^2}{Q(1 + Q)} - \frac{P p^2}{2}} \\ U &= \sqrt{(p - e^2 r_0)^2 + z_{\text{ECEF}}^2} \\ V &= \sqrt{(p - e^2 r_0)^2 + (1 - e^2) z_{\text{ECEF}}^2} \\ z_0 &= \frac{b^2 z_{\text{ECEF}}}{aV} \\ h &= U \left(1 - \frac{b^2}{aV}\right) \\ \phi &= \arctan \left[\frac{z_{\text{ECEF}} + e'^2 z_0}{p} \right] \\ \lambda &= \arctan2[y_{\text{ECEF}}, x_{\text{ECEF}}], \end{aligned} \quad (\text{B.5})$$

where (a, b, e) are the semi-major axis, semi-minor axis and eccentricity of the reference ellipsoid, and $e'^2 = a^2/b^2 - 1$.

B.2.5 Earth-Centered Earth-Fixed to East-North-Up

Transforming ECEF coordinates into ENU coordinates requires the specification of a local reference point. If the reference point is expressed in ECEF frame $\mathbf{p}_{\text{REF}}$, then the vector pointing from the reference point to the target one in the ENU frame $\mathbf{p}_{\text{ENU}} = [x_{\text{ENU}}, y_{\text{ENU}}, z_{\text{ENU}}]^\top \mathbb{R}^3$ can be computed as

$$\mathbf{p}_{\text{ENU}} = \mathbf{T}(\mathbf{p}_{\text{ECEF}} - \mathbf{p}_{\text{REF}}). \quad (\text{B.6})$$

Matrix $\mathbf{T} \in \mathbb{R}^{3 \times 3}$ is a transformation matrix, given by

$$\mathbf{T} = \begin{bmatrix} -s_{\lambda_r} & c_{\lambda_r} & 0 \\ -s_{\phi_r} c_{\lambda_r} & -s_{\phi_r} s_{\lambda_r} & c_{\phi_r} \\ c_{\phi_r} c_{\lambda_r} & c_{\phi_r} s_{\lambda_r} & s_{\phi_r} \end{bmatrix}, \quad (\text{B.7})$$

where (λ_r, ϕ_r) are the geodetic longitude and latitude, respectively, of the reference point.

B.2.6 East-North-Up to Earth-Centered Earth-Fixed

The conversion of coordinates from an ENU reference frame to an ECEF frame corresponds to the inverse transformation of Eqn. (B.6). The ECEF coordinates of the target point $\mathbf{p}_{\text{ECEF}}$ can be computed as

$$\mathbf{p}_{\text{ECEF}} = \mathbf{T} \mathbf{p}_{\text{ENU}} + \mathbf{p}_{\text{REF}}. \quad (\text{B.8})$$

The transformation matrix $\mathbf{T} \in \mathbb{R}^{3 \times 3}$ is

$$\mathbf{T} = \begin{bmatrix} -s_{\lambda_r} & -s_{\phi_r} c_{\lambda_r} & c_{\phi_r} c_{\lambda_r} \\ c_{\lambda_r} & -s_{\phi_r} s_{\lambda_r} & c_{\phi_r} s_{\lambda_r} \\ 0 & c_{\phi_r} & s_{\phi_r} \end{bmatrix}. \quad (\text{B.9})$$

Appendix C

Multi-Layer Perceptron

This appendix provides an introduction to the Multi-Layer Perceptron (MLP), which is used in this work to learn unknown parameters of a kinematic model. It presents the fundamental building block, e.g., the neuron, the activation functions employed, and explains the procedure by which an MLP is trained.

C.1 Neurons and Activation Functions

The neuron, illustrated in Figure C.1, constitutes the fundamental element of an MLP.

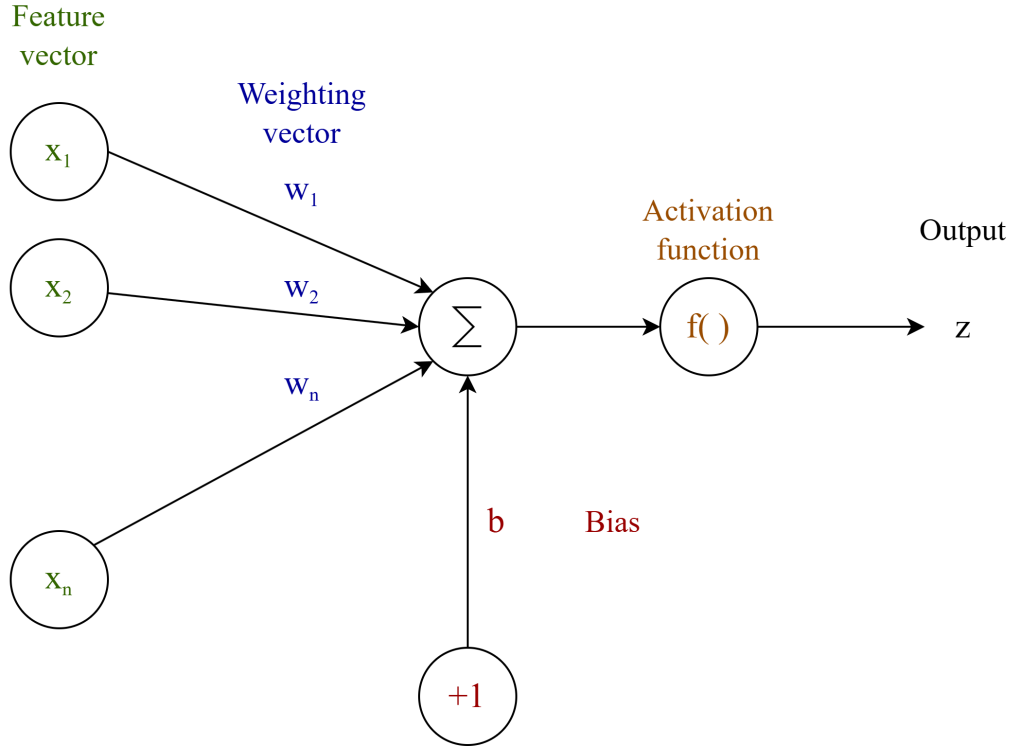


Figure C.1: Block scheme representation of a neuron with unitary bias.

This component accepts an input vector $\mathbf{x} \in \mathbb{R}^n$ and returns a scalar output $z \in \mathbb{R}$, which is commonly expressed as

$$z = f(\mathbf{w}^\top \mathbf{x} + b) \quad (\text{C.1})$$

where

- $\mathbf{w} \in \mathbb{R}^n$ is the weight vector,
- $b \in \mathbb{R}$ is the bias,
- $f(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is the nonlinear activation function.

C.1.1 Activation Function

The activation function produces the neuron's output z from a linear combination of the components of the input vector $\mathbf{x}$. It is parameterized by the weight vector $\mathbf{w}$ and the

bias b , which determines whether and to what extent the input contributes to the inferred output.

Different activation functions are used in the literature, depending on the specific application of the MLP, such as classification or regression. Here we introduce only the Rectified Linear Unit (ReLU), the one employed within this research activity.

Rectified Linear Unit

Rectified linear units use the activation function

$$g(z) = \max\{0, z\} = \max\{0, \mathbf{w}^\top \mathbf{x} + b\}. \quad (\text{C.2})$$

Rectified linear units are easy to optimize because they are similar to linear units [79]. The only difference between a linear unit and a rectified linear unit is that a rectified linear unit outputs zero across half its domain. This makes the derivatives through a rectified linear unit remain large whenever the unit is active. The gradients are not only large but also consistent. The second derivative of the rectifying operation is 0 almost everywhere, and the derivative of the rectifying operation is 1 everywhere that the unit is active. This means that the gradient direction is far more useful for learning than it would be with activation functions that introduce second-order effects.

C.2 Multi-Layer Perceptron

An MLP, depicted in Figure C.2, is a feed-forward neural network, e.g., an acyclical graph made of neurons.

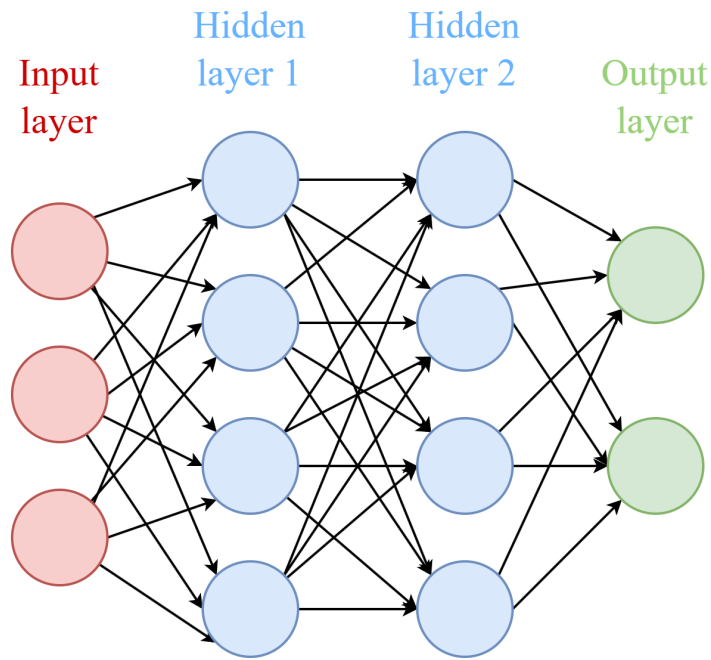


Figure C.2: Block scheme representation of a multi-layer perceptron with two hidden layers.

An MLP consists of several layers, each comprising multiple neurons. The architecture typically includes

- an input layer, which receives the feature $\mathbf{x} \in \mathbb{R}^n$. Therefore, it includes n neurons, one per each argument of the feature vector;
- one or more hidden layers, where each contains l neurons and computes an affine transformation followed by a nonlinear activation, thereby producing an intermediate representation;

- an output layer, producing the inference $\mathbf{y} \in \mathbb{R}^m$.

Since each layer l contains multiple units, we may represent the activations of all its units with an activation vector $\mathbf{h}^{(l)}$. Furthermore, we can represent each layer's weights with a weight matrix $\mathbf{W}^{(l)}$ and each bias with a bias vector $\mathbf{b}^{(l)}$.

Therefore, with respect to the architecture depicted in Figure C.2 that contains two hidden layers, the computation can be written as

$$\begin{aligned}\mathbf{h}^{(1)} &= f^{(1)}(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}), \\ \mathbf{h}^{(2)} &= f^{(2)}(\mathbf{W}^{(2)}\mathbf{h}^{(1)} + \mathbf{b}^{(2)}), \\ \mathbf{y} &= f^{(3)}(\mathbf{W}^{(3)}\mathbf{h}^{(2)} + \mathbf{b}^{(3)}),\end{aligned}\tag{C.3}$$

where $f^{(1)}$, $f^{(2)}$ and $f^{(3)}$ denote the different activation functions employed.

C.2.1 Parameters and Hyperparameters

In machine learning terminology, hyperparameters are design-time choices that remain fixed during training, e.g., they are not optimized by the training algorithm.

For an MLP, typical hyperparameters include the number of hidden layers L , the number of neurons per layer $h^{(l)}$, the activation functions $\phi^{(l)}$, and optimization-related choices such as learning rate and batch size.

By contrast, the parameters of the network, namely the weight matrices $\mathbf{W}^{(l)}$ and bias vectors $\mathbf{b}^{(l)}$, are learned during training via an optimization procedure, e.g., gradient descent, and thus constitute the model's trainable parameters [79].

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