

EWGT 2012

15th meeting of the EURO Working Group on Transportation

## A modular soft computing based method for vehicles repositioning in bike-sharing systems

Leonardo Caggiani<sup>\*</sup>, Michele Ottomanelli

*Politecnico di Bari, via Orabona 4, Bari, 70125 Italy*

---

### Abstract

A crucial issue in bike-sharing systems (BSS) is the unbalanced distribution in space and time of the bikes among the stations. Literature shows several methods, to solve the vehicle reallocation problem and most of them are based on rigid control thresholds and refer to car-sharing systems. In this paper a more flexible fuzzy decision support system for redistribution process in BSS is presented. The aim of the proposed method is to minimize the redistribution costs for bike-sharing companies, determining the optimal bikes repositioning flows, distribution patterns and time intervals between relocation operations, with the objective of a high level for users satisfaction. The proposed method allows to define the best bikes repositioning jointly to the best route for the carrier vehicles. The optimization method has been applied to a simulated BSS that can be considered as a module of a wider real BSS thanks to the scalable architecture of the decision support system. The results of this first tests are interesting even if further investigation are in progress.

© 2012 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of the Program Committee

*Keywords:* bike-sharing; repositioning problem; decision support system; fuzzy inference; sustainable mobility

---

### 1. Introduction

In recent years sustainable transport systems have been gained increasing attention due to the growth of the energy use, noise and air pollution. Different transportation systems with low impact on the environment have been implemented in many industrialized countries such as green vehicles use for public transport, car-pooling, car-sharing or bike-sharing systems. Moreover, there is an improvement of mobility, since a reduction of traffic jam is obtained, due to a decrease in the number of cars moving around and to an increase in the shared use of electric or low pollutant vehicles. In addition to the sustainability of the system, the development of Bike-Sharing

---

<sup>\*</sup> Corresponding author. Tel.: +39-080-5963380; fax: +39-080-5963329.  
*E-mail address:* [l.caggiani@poliba.it](mailto:l.caggiani@poliba.it)

Systems (BSS) is mainly due to its door-to-door feature (DeMaio, 2009). BSS allow to access areas of the city that are forbidden for other kind of vehicles, including electric cars. BSS are also often employed to connect users to public transit networks (for ex. connection between transport hub and final destination). A BSS is generally based on a bike fleet, bike stations and a given (known) number of users. In these systems, customers arrive to rental stations, utilize bicycles for some amount of time, and then return the bicycle to the same or to a different bike station.

The first BSS have been started with small and few expensive systems with low usage rates. The most recent BSS have become high-tech systems with thousands of bikes and major investments requirements. These advanced systems, including electronically-locking racks or bike locks, telecommunication systems, smartcards, mobile phone access, and on-board computers, have made BSS more attractive (Midgley, 2009). The success of a BSS, from a strategic planning point of view, depends on the optimal design of the system in term of number, location and capacity of stations as well as on the consistency of the available bikes fleet.

Nevertheless, a greater diffusion and usage of the BSS is limited by its own drawbacks (Büttner et al., 2011). BSS is used mainly for medium-short distances and for one-way trips. Such a behavior leads to an unbalanced distribution of the bikes over the time and space and consequently to the increase of the probability to find a full (no available slot) or empty station (no available bike). Thus, from the real-time management stand point, to increase the system capacity and users satisfaction, it is necessary to properly relocate the bikes among the stations of the BSS.

As in technical literature, redistribution scheme relevant to systems operating with shared vehicles can be mainly divided into two standard categories: “user based” and “operator based” (Allouche et al., 1999; Barth and Todd, 2001; Kek et al., 2006; Vogel and Mattfeld 2010). In the former the users are stimulated to return to a non-saturated station in order to balance the distribution of the bikes among the stations. In the latter, the relocation is assigned to the service staff. While the indirect customer base redistribution is feasible for mid-term operations, the direct operator based redistribution may be effective in short-term period.

The operator based reallocation problem is defined as Pickup and Delivery Problem (PDP). In literature several methods to solve the vehicle or resources reallocation problem have been presented (Cheung and Powell, 1996; Kochel et al., 2003; Nair and Miller-Hooks, 2011). These models are generally applied to freight transportation (Du and Hall, 1997; Crainic, 2000) or to car-sharing systems (Barth and Todd, 2001; Kek et al., 2006 and 2009).

Recently just a few papers have been dealing with the Bike Sharing Pickup and Delivery Problem (BS-PDP) as a variation of classical PDP. The BSS reallocation can be carried out either during the night when the bikes demand is negligible (static repositioning, as defined in literature), or during the day when the bikes distribution among the stations rapidly changes due to the high demand level (dynamic repositioning). Most of the literature presents static BS-PDP (Benchimol et al., 2011; Shu et al., 2010; Chemla et al., 2011; Forma et al., 2010). Even less authors deal with the dynamic case. In particular Contrado et al. (2012) propose a mathematical formulation of the dynamic problem on a space-time network. In general the dynamic BS-PDP is investigated without focusing at redistribution patterns and time periods (Vogel and Mattfeld, 2010). Some works propose a fixed repositioning time interval (Nair and Miller-Hooks, 2011; Sayarshad et al., 2012) or assume redistribution vehicles moving at random from saturated stations to empty ones (Fricker and Gast, 2012).

Furthermore the relocation rules proposed in literature are usually based on a binary logic with rigid thresholds that could not give a suited demand adaptive system. In addition, when space-time network problem is considered then the optimal route over the road network is not considered. This problem is very important when redistribution have to be made on congested networks and travel time between bikes station is a crucial variable.

In this paper a modular Decision Support System (DSS) for dynamic bike redistribution process is presented. The method is based on a Fuzzy Inference Systems to minimize the vehicles repositioning costs for bike-sharing companies, keeping at a high level users satisfaction, assuming that it increases with the probability to find an available bike or a free docking point in any station at any time. The proposed model considers the dynamic

variation of the demand (for both bikes and free docking slot) and also micro-simulate the BSS in space and in time suggesting optimal repositioning flows, distribution patterns and time intervals between relocation operations by explicitly considering the route choice among the stations. The method has been applied to the case of a single area covered by BSS made up of 5 stations and one carrier vehicle that represents the typical scheme for a part of a wider real BSS. Thanks to the scalability of the approach the DSS can be easily expanded to more complex BSSs.

## 2. Bike sharing system simulator

In order to apply the proposed DSS, we have represented and modeled all the operations of a BSS. We assume that the system runs an advanced control of the number of bicycles in each station as most of the systems currently implemented. The design variables that describe a BSS are the number of stations ( $N_s$ ) and their location in the city network, the number ( $N_{rt}$ ) and the capacity ( $C_{rt}$ ) of the redistribution pick-up or open trailers (proper carrier vehicles used to carry several bikes at the same time), for each station: the number of docking slots ( $N_d$ ) and free ones ( $N_p$ ) and the number of available bikes ( $N_b$ ). The architecture with relevant functions of the BSS operations simulator are depicted by the flow-chart in the Fig. 1.

With respect to the system dynamics, we assume the operating day is divided in  $Z$  discrete time intervals. For each time interval  $z$  and for each station  $j$ , given the pickup bikes demand ( $D_b$ ), the model simulate the destination choice ( $D_p$ ) in order to assess the arrival time for each users. The choice model is based on relative O-D attractiveness and the nature of trip (one-way or round trip). Since in this first stage we have developed the simulation for a low density stations distribution, we assume that each bike request that cannot be satisfied (empty station) turn user away (demand decreasing); as well as each give back not satisfied demand (full station) generate undesired waiting time for user.

At the begin of each interval  $z$  the number of bikes and the available docking points are updated by taking into account in-out users flow (turn-over) and the relocated bikes.

The Decision Support System (Fig. 2) is activated at variable gap time as better explained in the next section. At the end of the day (end of service time period), a static redistribution is carried out in order to restore the starting bikes distribution among the stations.

## 3. The Proposed Decision Support System

### 3.1. Theoretical issues

The problem is to relocate the bicycles from overcrowded station to the stations with a shortage of bikes (i.e. balance the demand-supply system). The first essential data is the prediction of future demand among stations in order to forecast bikes and free docking point requests. The literature proposes different forecasting methods, in particular, for BSS, Borgnat et al. (2009) use a linear regression model; Froehlich et al. (2009) have developed a Bayesian network.

The core of the forecasting demand method in the proposed DSS is based on Artificial Neural Networks (ANN) and Fuzzy Logic. A first advantage of this approach is that, using Neural Networks (NN), there is no need to assume explicit functions for returned or picked up bikes, because a NN learns directly from observed data. Therefore, starting from the time series of bikes requests, the NN is able to relate the hour of the day to entering or exited vehicles number. Furthermore, the high capability and the goodness of NN with respect to forecasting methods proposed for shared use vehicle has been proved by Kek et al. (2005) for the case of car sharing systems.

Another problem faced in the proposed DSS is the decision about when starting the relocation operations. Current approaches are based on binary logic, so their decisional mechanism is quite rigid: as soon as any threshold is exceeded, the relevant rule suggests that relocation should be performed, disregarding how much the

threshold is exceeded or its importance. On the contrary, fuzzy logic allows calculations using variables defined by approximate values, linguistic terms, or both and thus more flexible planning is possible. In fact, differently from traditional logic, fuzzy logic assumes that the truth of a given statement can have more than two values (i.e. true, untrue): every proposition may have a certain degree of truth, defined by the value of its membership function. That is, the degree of truth belongs to the interval  $[0, 1]$ , rather than to the set  $\{0, 1\}$ . Therefore, a fuzzy logic based DSS could lead to better results than a traditional DSS (Dell'Orco et al., 2008 and 2011) in particular when dealing with uncertain, imprecise or ambiguous environment.

### 3.2. Proposed algorithm

In other studies, the relocation process is triggered when the number of available bicycles exceeds minimum or maximum fixed thresholds (Barth and Todd, 2001; Kek et al., 2006). In the model proposed in this paper, the DSS for relocation can start on variable Gap Times (GT) where the value of GT depends on the station status. GT is defined as the maximum time interval in which no station status variation is considered.

The architecture of the proposed DSS is depicted in the Fig. 2 and it starts with the forecasting module that estimates entering and exiting bikes, based on given historical time series. For each station the estimation is performed on GT by two artificial NNs. The first NN is used to estimate the number of entering vehicles; the second one to forecast the exiting vehicles. Both NNs are feed-forward back-propagation networks with a training function that updates weight and bias values according to Levenberg-Marquardt optimization. The NNs have three layers with five neurons for the first and the second layer and one neuron for the third. Each day is divided into  $Z$  steps. The input data of NNs are the vectors of entered and exited bicycles in the station, for each step, during the previous months of the prediction. Actually, such a formulation depends on the available data.

The outputs of the NNs are the entering (PDp) and exiting (PDb) number of bicycles forecasted for each step. Once the predicted request of each user is known, through the simulation of the whole BSS, it is possible to evaluate, in each GT, the availability of a free parking slot or a bicycle. In this paper we aim at describe the relocation algorithm so we will presents any specific test and validation with real data for this forecasting module.

In other words at this stage the outputs are the estimated number of lost users (PLu; i.e. users whose service request cannot be satisfied) and waiting users (NWU; i.e. users that do not find free docking points at the desired station) as well as the estimated waiting time spent to return the bike (UWT).

These parameters and the total predicted waiting time for all stations (STWT) are used as input data for the Fuzzy Inference System (FIS). A FIS simulates the behavior of a human decision-maker by using rules like: IF S is X THEN T is Y. Within the Fuzzy Logic framework, this rule means “the more S is X, the more T is Y”, and the variables X and Y can be represented (or be the representation) by linguistic or approximated values; in other words, fuzzy values. The degree of truth of a given rule depends on fuzzy variables, defined by their respective Membership Functions (MF). For more detail of a FIS structure refer to Zimmermann (1996).

The rules defined in the FIS of this DSS are:

“IF the sum of PLu AND NWU is medium, UWT is high, AND STWT is high, THEN the system is rather out of control”.

Thus, the output of the FIS is the status of the station. If the status is in control then the GT is set as the longest allowed gap time  $GT_0$  for the bicycles relocation. The more the status is out of control the more the GT is set to a shorter established interval.

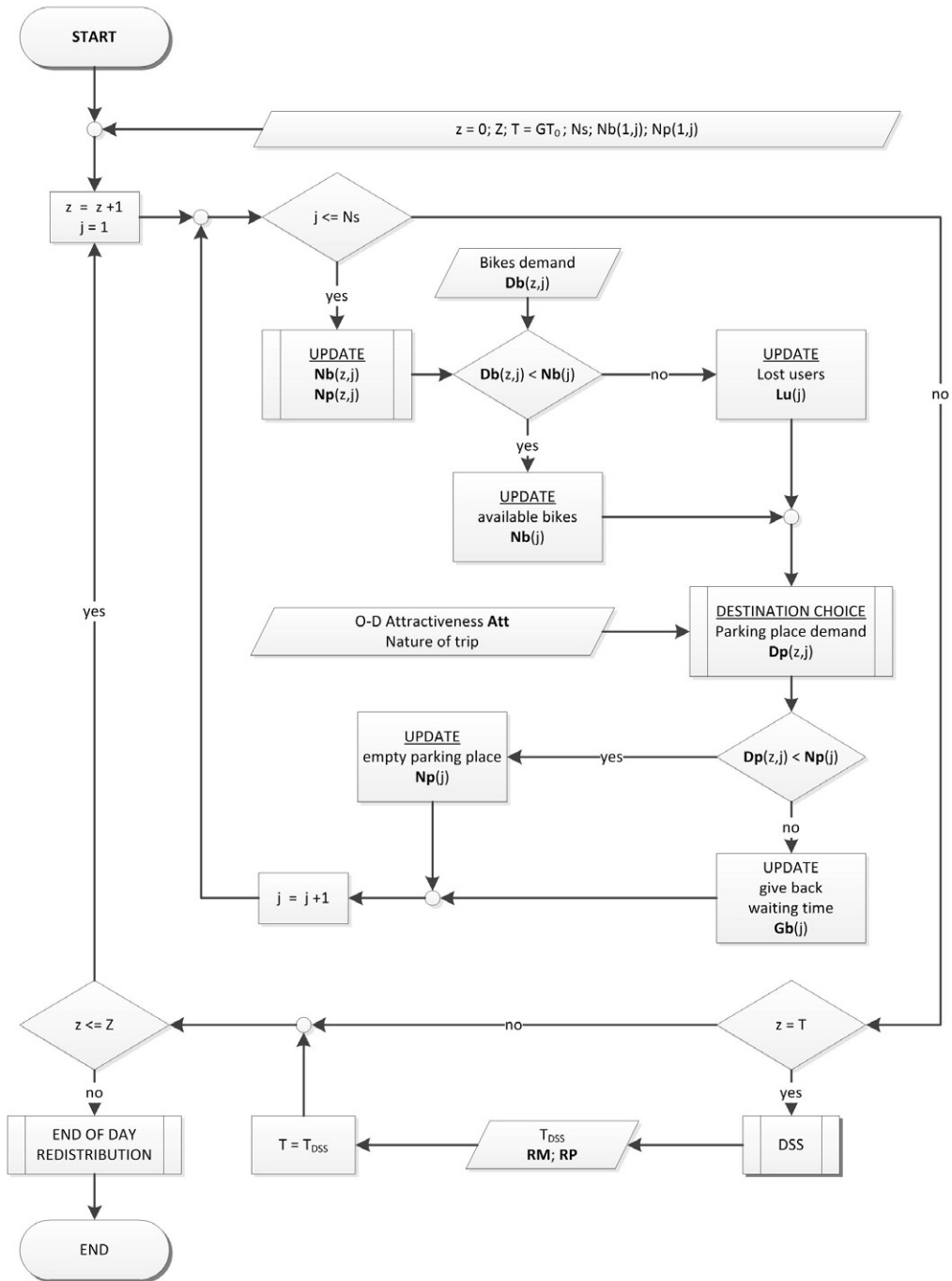


Fig. 1. BSS simulation flow chart

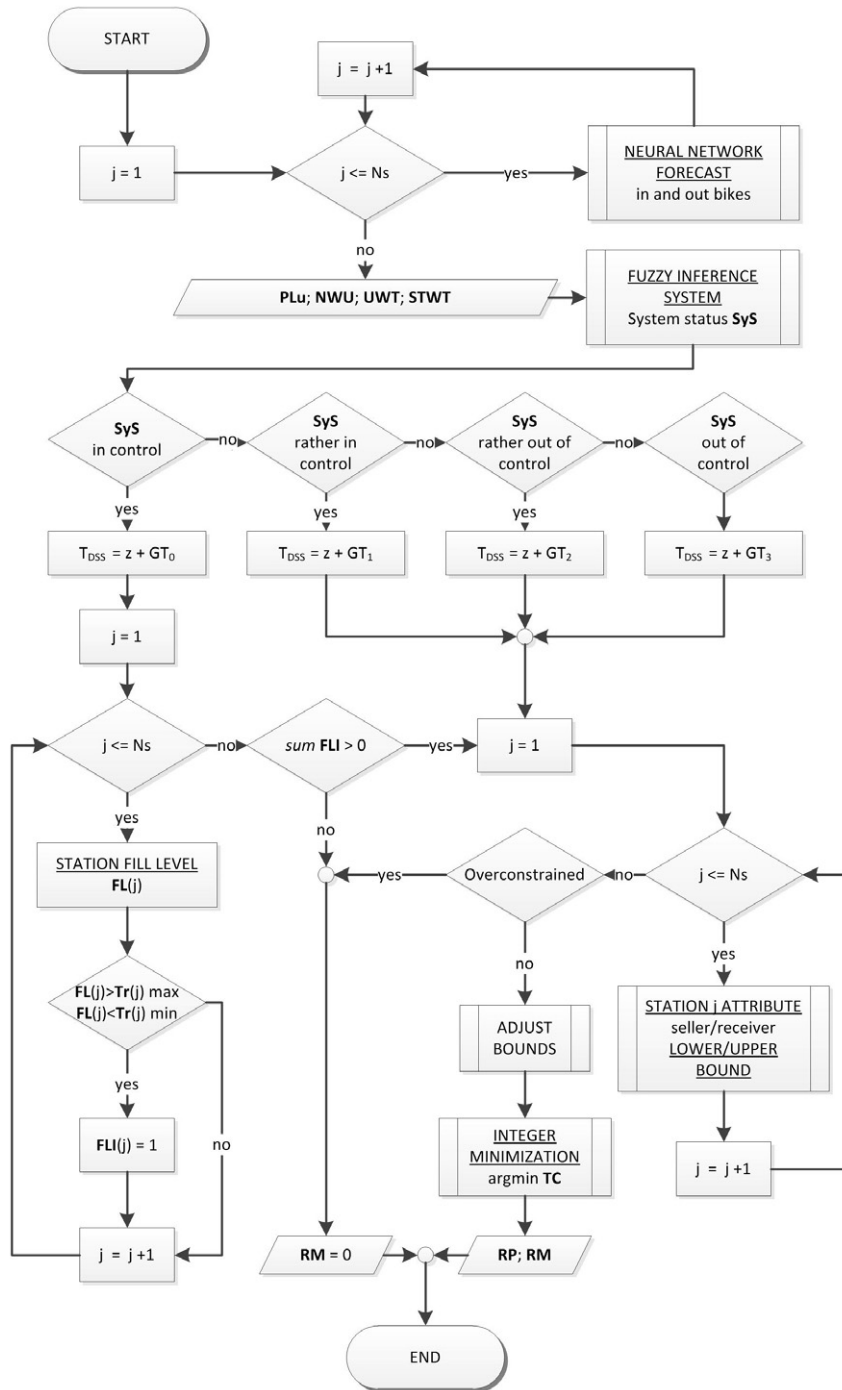


Fig. 2. Flowchart of proposed DSS

If the status identified by the FIS is different from “in control” (i.e. “balanced”), then the algorithm goes on to find whether the conditions exist to perform relocations (i.e. unbalanced system). In particular, since the forecasted entering-exiting bicycles are known, starting from the current number of bicycles in each station, through the simulator it is established the attribute of each station. More simply, the algorithm indicates whether a station is seller or receiver as a function of the forecasted occupation level of the station during the GT. Moreover for each station the DSS evaluates the possible maximum number of bikes that can be moved or docking points that can be made available.

If the problem is over constrained, no relocation is feasible. For example if all station are receivers, no bikes can be delivered.

On the basis of the forecasted in-out bikes within the GT interval, by solving the optimization problem given in eq. 1 we obtain the Relocations Matrix (RM), with  $rm_{ik}$  entries, and the Relocation Path (RelP) of the redistribution carrier vehicles that minimizes the total costs for the provider (sum of relocation costs, RelC, and lost users costs, LuC).

The RM matrix belongs to a set of matrix M such that if a station is seller it must not be receiver. The upper bound (UB with  $ub_k$  entries) and the lower bound (LB with  $lb_i$  entries) of the problem are calculated adjusting the possible maximum number of transferable bikes or free docking points. This correction is due to find the minimum number of relocated bikes that maximize the users satisfaction.

$$(\mathbf{RM}^*, RelP^*) = \arg \min_{\mathbf{RM}} \left( RelC(RelP(\mathbf{RM})) + \sum_{j=1}^{Ns} LuC_j(PDb, PDp, RelP(\mathbf{RM})) \right)$$

s.t.

$$\sum_{i=1}^{Ns} \sum_{k=1}^{Ns} rm_{ik} \leq Crt; \tag{1}$$

$$\sum_{i=1}^{Ns} rm_{i1} \leq ub_1, \dots, \sum_{i=1}^{Ns} rm_{ik} \leq ub_k, \dots, \sum_{i=1}^{Ns} rm_{iNs} \leq ub_{Ns};$$

$$\sum_{k=1}^{Ns} rm_{1k} \geq -lb_1, \dots, \sum_{k=1}^{Ns} rm_{ik} \geq -lb_i, \dots, \sum_{k=1}^{Ns} rm_{Nsk} \geq -lb_{Ns}$$

$$\mathbf{RM} \in \mathbf{M}_{Ns \times Ns}$$

$$\mathbf{M}_{Ns \times Ns} := \left\{ (a_{ik})_{\substack{i=1, \dots, Ns \\ k=1, \dots, Ns}}, a_{ik} \in N \mid \exists k \in \{1, \dots, Ns\}: \forall i \in \{1, \dots, Ns\} \sum_{i=1}^{Ns} a_{ik} \neq 0 \wedge \sum_{k=1}^{Ns} a_{ik} = 0 \right\}$$

This step is necessary to reduce the solution space because the provider costs do not change if the relocation path and lost users do not change.

The relocation paths are carried out by solving a Travelling Salesman Problem in the same level of the optimization problem (fixed point problem). In particular the redistribution truck must previously pass through the seller stations. The lost users costs are calculated with the BSS simulation considering PDP and PDb slots and bikes demands.

Differently from other approaches that use bi-level formulations, the proposed minimization problem is formulated as fixed point Non Linear Integer Optimization. The solution algorithm is based on a Branch and Bound procedure: the outputs are the optimal relocation matrix and the optimal relocation path and bikes distribution among the stations.

If the system status identified by the FIS is “in control”, and if during the gap time  $GT_0$  at least one station is

almost full or almost empty then the DSS provides (or respectively asks for) the number of vehicles previously fixed.

**4. Numerical application**

The proposed algorithm, thanks to the modular properties of the approach, has been applied to a part of a BSS made up of 5 stations that represent the typical scheme of a wider real BSS. The station 1 is located in the city centre, the other four stations are located in the outskirts of the city. The distances in meters between the stations are reported in the Table 1. The DSS can be easily scaled to more complex BSS with a larger number of central or peripheral stations.

The method has been applied considering three working days with 24 hours/day of operations. Each day is divided into 288 steps ( $Z = 288$ ), 5 minutes each long. The starting data are: the number of stations  $N_s = 5$ , the number of carrier vehicles (i.e. redistribution trucks)  $N_{rt} = 1$  with a capacity  $C_{rt} = 20$  bikes, the number of docking points per station  $(Nd_1, \dots, Nd_5) = (40, 30, 30, 20, 20)$ , the number of available bikes  $(Nb_1, \dots, Nb_5) = (10, 10, 10, 10, 10)$ , the Gap Time steps ( $GT_0=11, GT_1=9, GT_2=8$  and  $GT_3=6$ ) and the station occupation parameters ( $Tr_{max} = 95\%$  and  $Tr_{min} = 5\%$ ).

Since all the distances between stations (Tab. 1) are lower than one kilometer we assume that the truck travel and operation time between two stations is lower than five minutes that is a truck movement takes place within one step.

Table 1. Station distances [m]

O/D	1	2	3	4	5
1	0	400	500	300	300
2	400	0	700	700	600
3	500	700	0	400	800
4	300	700	400	0	600
5	300	600	800	600	0

The total redistribution cost per kilometer is assumed to be 0.30 Euros (considering depreciation, taxes, fuel and tires consumption and maintenance costs); the cost for each lost user is fixed to 0.7 Euros (based on subscription and fees of the most of the major BSS). Actually, the costs for lost users should take into account all the impacts given to the consequent use of “no-green” transport means.

The historical data (fifty days input/output demand) are generated by random extractions from a Normal distribution, starting from Fixed Vectors of entering-exiting bikes (FV) (Angeloudis *et. al*, 2012). Each entry of FV is the number of bicycles entered or exited in each step; this number is drawn from the set {0, 1, 2, 3, 4, 5} where each value has different extraction probability as a function of the station position and time of the day. Since we carried out the numerical example for different levels of demand, a different FV has been established. The higher the level of demand is, the higher the probability of a high number extraction.

We assumed triangular and trapezoidal shapes for the MFs in the FIS. For example in Fig. 3 the membership functions of the number of lost and waiting users are shown.

To evaluate the performances of the proposed model, we have tested the algorithm considering two cases and three levels of demand. The two cases are: no relocation and relocation with variable gap time (proposed DSS).



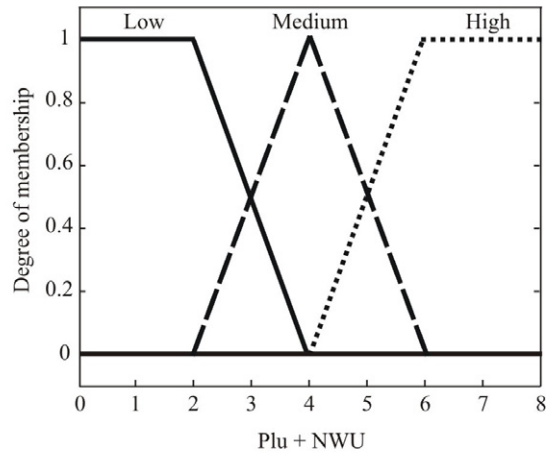


Fig. 3. Membership functions of the number of users not immediately satisfied

Table 2. Excerpt of results

demand level	1			2			3		
number of day	1	2	3	1	2	3	1	2	3
pickup demand	263	302	282	477	505	494	781	720	811
relocated bikes	59	88	76	156	153	172	87	83	70
lost users (no rel.)	52	76	60	161	193	188	361	304	391
lost users (rel.)	29	19	22	88	109	112	324	269	341
lost users reduction (%)	44.2	75.0	63.3	45.3	43.5	40.4	10.2	11.5	12.8
relocation costs	63.9	76.5	59.4	114.6	114.9	134.1	60.9	62.4	55.5

Some results of the mentioned cases are reported in Table 2. In all cases considered cases, the proposed DSS leads to a reduction of the number of lost users. Also case of low demand level, positive results have been reached. Assuming higher level of demand, due to the congestion of the system, the proposed method shows a lower reduced number of lost users.

**5. Conclusions and further developments**

In this paper a DSS for optimal relocation of vehicles in bike-sharing systems has been presented. The method aims at overcome some draw-backs of the existing BSSs that lack in relocation operations or in most of case is made occasionally. In particular, differently from other methods, the proposed one jointly determines the relocation time windows, the optimal carrier vehicles route and the number of bikes to be repositioned. The optimization model has been applied for a single area of a simulated BSS and the early results show that the relocation management leads to the increase of users satisfaction in term of probability of finding available bikes or free slots. The method is modular so that it can be extended to wider and real sized systems.

The proposed DSS reproduce in detailed way the system, thus it can be also used for real-time management

and the strategic design that is to determine the optimal layout of the BSS.

Additional simulation for wider BSS and multiple carrier vehicles are needed and in progress in order to check the robustness of the method. Further research will deal with the implementation of the ANN based model for the real-time estimation of bikes demand based to a real systems with consequent validation using data collected on field data.

## References

- Angeloudis, P., Hu, J. and Bell, M.G.H. (2012) Strategic Repositioning Algorithm for Bicycle-Sharing Schemes, Proc. of the 91th Meeting of the Transportation Research Board, paper 12-3210, Washington D.C., USA
- Barth, M. & Todd, M. (2001), User Behaviour Evaluation of an Intelligent Shared Electric Vehicle System, *Transportation Research Record: Journal of the Transportation Research Board*, 1760, 145-152.
- Benchimol, M., Benchimol, P., Chappert, B., de la Taille, A., Laroche, F., Meunier, F., & Robinet, L. (2011). Balancing the stations of a self service "bike hire" system. *RAIRO-Operations Research*, 45(1), 1-15.
- Borgnat P., Abry P., Flandrin P., Rouquier J.-B. (2009) Studying Lyon's Vélo'V: A Statistical Cyclic Model. Proceedings of the European Conference on Complex Systems (ECCS'09). Warwick, UK, Sept. 2009
- Büttner, J., Mlasowsky, H., Birkholz, T., Groper, D., Fernandez, A. C., Emberger & G., Banfi, M. (2011). Optimising Bike Sharing in European Cities: A Handbook, Intelligent Energy Europe program (IEE). <http://obisproject.com>. Accessed Feb. 13, 2012.
- Chemla, D., Meunier, F. & Wolfler Calvo, R. (2011). Balancing a bike-sharing system with multiple vehicles. 12<sup>e</sup> congrès annuel de la ROADEF, Saint-Etienne, France, 2 au 4 Mars 2011.
- Cheung, R.K. & Powell, W.B. (1996). An algorithm for multistage dynamic networks with random arc capacities, with an application to dynamic fleet management. *Operations Research*, 44, 951-963.
- Contrado, C., Morenci, C. & Rousseau, L.-M. (2012). Balancing a Dynamic Public Bike-Sharing System. <https://www.cirrelt.ca/DocumentsTravail/CIRRELT-2012-09.pdf>. Accessed Mar. 20, 2012.
- Crainic, T.G. (2000). Service network design in freight transportation. *European Journal of Operational Research*, 122, 272-288.
- Dell'Orco, M., Caggiani, L., Sassanelli, D. (2011), Neuro-fuzzy Decision Support System for relocation process in car sharing systems. In: (Crisalli U and Mussone L eds.) *Transport management and land-use effects in presence of unusual demand. Selected papers*. Franco Angeli Pubbl, Milan, Italy
- Dell'Orco, M., Ottomanelli, M., Caggiani, L., Sassanelli, D. (2008) New Decision Support System for Optimization of Rail Track Maintenance Planning Based on Adaptive Neuro fuzzy Inference System. *Transportation Research Record: Journal of the Transportation Research Board*, 2043, 49-54.
- DeMaio, P. (2009). Bike-sharing: History, Impacts, Models of Provision, and Future. *Journal of Public Transportation*, 12(4).
- Du, Y. & Hall, R. (1997). Fleet sizing and empty equipment redistribution for center-terminal transportation networks. *Management Science*, 43, 145-157.
- Forma, I., Raviv, T. & Tzur, M., (2010). The Static Repositioning Problem in a Bike-Sharing System. Proc. of TRISTAN VII, seventh Triennial Symposium on Transportation Analysis, Tromsø, Norway, June 20-25, 2010.
- Fricker, C. & Gast, N. (2012). Incentives and regulations in bike-sharing systems with stations of finite capacity. <http://arxiv.org/pdf/1201.1178v1.pdf>. Accessed Feb. 13, 2012.
- Froehlich, J. Neumann and N. Oliver, (2009) Sensing and predicting the pulse of the city through shared bicycling, Proc. of Twenty-First International Joint Conference on Artificial Intelligence (IJCAI-09), pp. 1420-1426
- Kek, A.G.H., Cheu, R.L. & Chor M.L. (2006). Relocation simulation model for multiple-station shared-use vehicle systems, *Transportation Research Record: Journal of the Transportation Research Board*, 1986, 81-88.
- Kek, A.G.H., Cheu, R.L., Meng, Q. & Fung C.H. (2009). A decision support system for vehicle relocation operations in carsharing systems, *Transportation Research Part E*, 45(1), 149-158.
- Kochel, P., Kunze, S. & Nielander, U. (2003). Optimal control of a distributed service system with moving resources: Application to the fleet sizing and allocation problem. *Int. J. Production Economics*, 81-82, 443-459.
- Midgley, P. (2009). The Role of Smart Bike-sharing Systems. *Urban Mobility Journeys*, May 2009, 23-31.
- Nair, R., & Miller-Hooks, E. (2011). Fleet management for vehicle sharing operations. *Transportation Science*, 45(4), 524-540.
- Sayarshad, H., Tavassoli, S. & Zhao, F. (2012). A multi-periodic optimization formulation for bike planning and bike utilization, *Applied Mathematical Modelling*, Atricle in press, doi:10.1016/j.apm.2011.12.032.
- Shu, J., Chou, M., Liu, Q., Teo, C.-P. & Wang, I-L. (2010) Bicycle-sharing system: deployment, utilization and the value of re-distribution, <http://www.bschooll.nus.edu.sg/Staff/bizteocp/BS2010.pdf>. Accessed Feb. 15, 2012.
- Vogel, P., Mattfeld, D.C. (2010). Modeling of repositioning activities in bike-sharing systems, Proc. of 12th WCTR, July 11-15, 2010 - Lisbon, Portugal.