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Probabilistic model for the estimation of *T* year flow duration curves

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[1] A flow duration curve (FDC) is the relationship between any given discharge value and the percentage of time that this discharge is exceeded. It represents the relationship between magnitude and frequency of streamflow discharges. In this paper, a new methodology for the evaluation of the flow duration curve with assigned return period *T* is proposed. The procedure is based on the use of distributions bounded by the interval 0 to 1 and exploits the properties of the beta and complementary beta (CB) distributions. The proposed model, called "EtaBeta," accounts for the interannual variability of the FDC using the distributions of annual minimum daily flow and total annual flow. The intra-annual variability is described by the CB distribution, which (like the beta distribution) has two parameters that characterize the shape of the distribution and in particular of its tails. The model is applied to basins of southern Italy characterized by strong seasonality and highly variable discharge. The results show good performance in terms of the model's ability to represent the analyzed flow duration curves of discharge data even in low-flow conditions. The EtaBeta model also offers interesting possibilities for use in regional analysis.

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1. Introduction

[2] Flow duration curves and related measures are used in many hydrological applications including river ecology, environmental flow management [e.g., *Lane et al.*, 2005], water supply assessment, and hydropower generation design [*Smakhtin*, 2001]. The flow duration curve (FDC) is defined as the relationship between any given discharge value and the percentage of time that this discharge is exceeded. This definition is equivalent to the one provided by *Castellarin et al.* [2004a] where the flow duration curve is introduced as the complement of the cumulative distribution function of discharge *Q*. Depending on the application, FDCs may be constructed using different time resolutions ranging from instantaneous maximum or minimum daily flow to annual, monthly, weekly or, more frequently, daily streamflows.

[3] According to *Mosley and McKerchar* [1993], FDCs cannot be considered as probability distributions, because discharge is correlated between successive time intervals and discharge characteristics are dependent on the season of the year. Nevertheless, the so-called stochastic approach for analysis and modeling of FDCs has been widely applied after the introduction of the "annual flow duration curve" (AFDC) [*Vogel and Fennessey*, 1994]. In fact, according to the probabilistic approach for the FDC evaluation, the percentage of time is considered as a fraction of a reference period (e.g., one hydrologic or calendar year), and the FDC is obtained by associating each discharge to the expected value of the fraction of the reference period during which such discharge is exceeded. This approach allows mean and median FDC to be estimated and also confidence intervals and return periods to be assigned to the FDC. Since the annual FDC stochastic approach was introduced, a number of models were proposed to fit flow duration curves derived from observations. In some cases, they allow for regional analysis and prediction in ungauged basins [*Claps and Fiorentino*, 1997; *Singh et al.*, 2001; *Castellarin et al.*, 2004a]. *Castellarin et al.* [2004b, 2007] provide a review of several studies and different regional methods assessing their effectiveness and reliability. They highlight that difficulties in obtaining accurate FDC estimates for low-flow conditions are commonly observed and well documented in the scientific literature.

[4] In all the mentioned studies, continuous distributions supported on semi-infinite interval, usually $[0, \infty]$, are used to fit annual FDCs. For their evaluation in semiarid basins, *Iacobellis et al.* [2004] introduced a new methodology based on the conjecture that the annual FDCs are distributions supported on a bounded interval [0 1], representing the fraction of time the streamflow exceeds the supplied values during the year, with lower and upper limits respectively identified by the annual minimum and maximum discharge. A two-parameter beta distribution was used to fit the annual FDC while regional distributions of annual minimum and maximum discharges, accounting also for zero flows, were exploited for characterizing the interannual variability. Nevertheless, in work by *Iacobellis et al.* [2004], the estimated values of the beta distribution parameters showed a strong variability not strictly interpretable in terms

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of physical characteristic of the investigated basins. In this paper, a novel bounded FDC model which uses the complementary beta (CB) distribution proposed by *Jones* [2002] is introduced. In particular, the proposed model, hereinafter called EtaBeta, provides a probabilistic framework for the evaluation of the FDC with an assigned return period, and accounts for the variability of the FDC using distributions of annual minimum daily flow and total annual streamflow, integral of the annual FDC. These quantities are much less variable and more predictable than the annual maximum daily discharge. The main advantage of such a model compared with other models available in literature, is provided by its structure which determines the return time of the FDC from parameters related to the annual and minimum flows. This approach avoids using the recorded annual maximum flows. As a consequence, in the case studies presented, the model does not use distributions with more than two parameters, whose selection and estimation can be cumbersome [see, e.g., *Castellarin et al.*, 2004a, 2007; *Hosking*, 1994].

[5] More important, the EtaBeta model explicitly encompasses the analysis and evaluation of the annual minimum daily flow distribution. Thus the estimated annual FDC is particularly reliable for high duration and in low-flow conditions.

[6] As a practical advantage, the EtaBeta model allows to evaluate, within a unique and coherent probabilistic framework, flow duration curves, annual minimum flow distribution, total annual flow distribution and other useful functions for design and management of structural and nonstructural water resources systems, from environmental minimum streamflow requirements to stream diversion and regulation structures.

[7] The paper is organized according to the following outline. In section 2, the EtaBeta model for the representation of the annual FDC is introduced, including definition and derivation of the annual streamflow volume function. In section 3, the derivations of the T year FDC, streamflow volume function, and discharge volume function are presented. In section 4, the model is applied to two series of daily streamflow from the Sinni basin in southern Italy. An analysis of the interannual variability of model parameters is discussed together with an assessment of the model performance. In the final remarks, particular emphasis is given to future research involving the EtaBeta model as a tool amenable for regional analysis and for generation of synthetic FDCs aimed at prediction in ungauged basins.

2. EtaBeta Model for a Single Year

[8] Consider $Q(d)$ as the relationship between the observed discharge values and the total time d in which discharge is less than $Q(d)$ within the reference period D. According to the classical definition of FDC we have

$$\mathrm{FDC}(\delta) = Q(\delta), \tag{1}$$

where the quantity $\delta = (D - d)$ is called "duration" of discharge. Within our probabilistic approach the FDC is considered a probability distribution bounded by the minimum and the maximum observed streamflows. Thus d can be considered as a measure of the probability of nonexceedance of discharge u by means of

$$u = \frac{d}{D}, \tag{2}$$

where $0 \leq u \leq 1$. Hence the duration $\delta = (D - d)$ is a measure of probability of exceedance, $1 - u$, since

$$\frac{\delta}{D} = 1 - \frac{d}{D} = 1 - u. \tag{3}$$

[9] In what follows, we will concentrate our attention on the investigation of the $Q(d)$ function because the FDC is immediately provided by equation (1) once $Q(d)$ is found.

2.1. Definition of the Annual Streamflow Volume Function

[10] If the reference period is selected to be the calendar (or water) year and we are working with daily streamflows, then d is in days, and $D = 365$ d for a nonleap year. In such a case, if the daily streamflows for a generic year are given by Q_j, with $j = 1, 2,., D$, the annual discharge function $Q(d)$ is simply provided by ordering the D flows in ascending order, $Q_1 \leq Q_2 \leq . \leq Q_D$, and assuming $Q(d) = Q_d$ where Q_d, with rank $d = 1, 2, ., D$, is the dth-order statistic of the D random variables Q_j [*Balakrishnan and Rao*, 1998].

[11] The integral function of $Q(d)$ represents a volume of water that ranges from the minimum daily volume $W(1)$ to the total annual volume $W(D)$ if d varies from 1 to 365 d.

$$W(d) = \int_0^d Q(t)dt. \tag{4}$$

For any value of d, $W(d)$ represents the total streamflow volume accumulated over all (even nonconsecutive) days with discharge less than $Q(d)$. Thus it is a measure that is useful for design of hydraulic structures, such as flow diversion structures without storage reservoirs.

[12] For $d = D = 1$ year (a) in equation (2), $W(D)$ equals to the total annual streamflow volume (m^3). A measure associated with $W(d)$ is the corresponding d average discharge $\langle Q(d)\rangle$ simply obtained dividing $W(d)$ by d:

$$\langle Q(d)\rangle = \frac{W(d)}{d} = \frac{1}{d}\int_0^d Q(t)dt. \tag{5}$$

$\langle Q(d)\rangle$ represents the average discharge, in a fixed duration, which is available for flow diversion structures without upstream storage.

2.2. Derivation of the Annual Streamflow Volume Function

[13] Consider the log-transformed variable of W

$$y = \log(W|W > 0), \tag{6}$$

which can be standardized

$$\eta = \left(\frac{y - y_{\min}}{y_{\max} - y_{\min}}\right), \tag{7}$$

where

$$y_{\min} = \log(W(1)), y_{\max} = \log(W(D)). \quad (8)$$

$W(1)$ is the minimum daily flow and $W(D)$ is the total annual flow. The variable η represents a standardized streamflow volume measure that ranges between 0 and 1.

[14] Let us assume η is a random variable that is well described by a beta distribution with parameters $a > 0$ and $b > 0$. Let us denote the beta function as $B(a, b)$, the incomplete beta function as $B(\eta, a, b)$, and the incomplete beta function ratio as $I(\eta, a, b) = B(\eta, a, b)/B(a, b)$. Using this notation, the beta distribution on [0, 1] has density function

$$f_B(\eta) = \frac{1}{B(a,b)} \eta^{a-1} (1-\eta)^{b-1}, \quad (9)$$

cumulative distribution function

$$u = F_B(\eta) = I(\eta, a, b) \quad (10)$$

and quantile function (i.e., inverse of the cumulative distribution function)

$$\eta(u) = F_B^{-1}(u) = I^{-1}(u, a, b), \quad (11)$$

where u is the probability of nonexceedance and $I^{-1}(u, a, b)$ is the inverse of the incomplete beta function ratio.

[15] The incomplete beta function ratio in equation (10) is fitted to the annual function of η that was introduced in equation (7). Hence the stochastic dependence between η and d is analyzed on an annual basis, considering samples of η and corresponding probabilities $u = d/D$, as in equation (2). A presentation and a brief discussion of methodologies commonly used for the estimation of parameters a and b may be found in Appendix A.

[16] Following equations (6) and (7), the annual streamflow volume function $W(d)$ can be expressed as

$$W(d) = \exp\left[I^{-1}\left(\frac{d}{D}, a, b\right)(y_{\max} - y_{\min}) + y_{\min}\right], \quad (12)$$

where η is replaced by $I^{-1}(u, a, b)$ using equation (11), with u evaluated as in equation (2).

[17] Following equation (12), the d average discharge function in equation (5) can be evaluated:

$$\langle Q(d) \rangle = \frac{W(d)}{d} = \frac{1}{d} \exp\left[I^{-1}(u, a, b)(y_{\max} - y_{\min}) + y_{\min}\right]. \quad (13)$$

2.3. Derivation of the Annual Discharge Function $Q(d)$

[18] In addition to the beta distribution, *Jones* [2002] introduced an alternative continuous distribution on [0, 1], the CB distribution, which is obtained from reversing the roles of the distribution and quantile functions of a traditional beta distribution. Within such approach, the quantities u and η introduced above may be considered "stochastic variables" or "probabilities" of each other.

[19] The CB distribution has several properties that are complementary to those of the beta distribution. In particular, the CB cumulative distribution function is the quantile function of the beta distribution and vice versa (likewise for density and quantile density functions). More properties and considerations about the use of beta and complementary beta distributions are reported in Appendix A.

[20] Let us describe this distribution as CB(a, b), where a and b are the parameters. It has cumulative distribution function

$$F_C(u) = I^{-1}(u, a, b) = \eta(u) \quad (14)$$

and density function

$$f_C(u, a, b) = \frac{B(a,b)}{\{I^{-1}(u,a,b)\}^{a-1}\{1 - I^{-1}(u,a,b)\}^{b-1}}, \quad (15)$$

where u is considered a random variable and η is the probability of nonexceedance of u.

[21] Using equation (4), the annual discharge function may be found from the derivative of $W(d)$ as

$$Q(d) = \frac{dW(d)}{dt}. \quad (16)$$

[22] The derivative in equation (16) can be solved using the expression of $W(d)$ from equation (12) with the inverse of the incomplete beta function ratio replaced by the CB cumulative distribution function $F_C(u)$, as in equation (14), to give

$$Q(d) = W(d)(y_{\max} - y_{\min}) \frac{dF_C(u)}{du} \frac{du}{dt}. \quad (17)$$

[23] Finally, replacing the derivative of the cumulative probability function of the CB distribution with its probability density function and using $u = d/D$ from equation (2), one obtains

$$Q(d) = \frac{W(d)}{D}(y_{\max} - y_{\min}) f_C\left(\frac{d}{D}, a, b\right), \quad (18)$$

where functions $W(d)$ and $f_C(u)$ are respectively reported in equations (12) and (15).

[24] It is worth mentioning that the parameters a and b can be estimated from the annual function of η as described in Appendix A. They are representative, considering equations (1) and (18), of the annual FDC shape for values close, respectively, to the lower and upper limits. In particular, according to *Jones* [2004], a and b directly control the relative weights of the left and right-hand tails of the distribution, respectively, while the relative values of a and b actually affect distribution's skewness.

[25] Some graphical examples about the shape of the CB density function are provided in *Jones* [2002]. Moreover, it is known (M. C. Jones, personal communication, 2006) that the way the CB density function depends on a and b is the same way that the tails of the beta density function depend on $1/a$ and $1/b$. Thus, in order to speculate on the shape of

the distribution tails for different values of a and b, the reader, may refer to general knowledge about the beta distribution. More information about CB distribution and the behavior of its tails may be found in work by *Jones* [2007].

3. Annual FDC for Fixed *T* Year Return Period

[26] Consider any stochastic variable which is observed once or globally in a year. The return period of its annual minimum value, which is not exceeded on average once in T years, is determined as a function of its probability of nonexceedance p as $p = 1/T$. On the other hand, the return period of the maximum annual value, which is exceeded on average once in T years, is determined as a function of the same probability of nonexceedance p as $p = 1 - 1/T$.

[27] In any case, following *Vogel and Fennessey* [1994], an annual FDC with assigned return period T is obtained from the pth-quantile annual discharge function $Q_p(d)$. For any fixed d values, for $d = 1, ., D$, $Q_p(d)$ is provided by the pth-quantile of the marginal interannual distribution of the dth-order statistic Q_d for such fixed d. Then, in analogy with relationships (1) and (4) between the FDC, discharge function $Q(d)$, and streamflow volume function $W(d)$, the pth-quantile annual flow duration curve FDC_p is obtained from the corresponding pth-quantile annual discharge function $Q_p(d)$ as

$$\mathrm{FDC}_p(\delta) = Q_p(\delta), \tag{19}$$

with δ "duration" of discharge. The pth-quantile annual streamflow volume function $W_p(d)$ is introduced as

$$W_p(d) = \int_0^d Q_p(t)dt. \tag{20}$$

[28] It is important to highlight that, in principle, an exact estimation of $Q_p(d)$ from stochastic variables a, b, $y_{\min}$, $y_{\max}$ given equation (18), which represents the annual discharge function, involves knowledge of the joint probability distribution $f(Q, W, d, a, b, y_{\min}, y_{\max})$, whose analysis goes beyond the goal of this paper. Nevertheless, we observe that the annual streamflow volume function $W(d)$ can be treated in analogy to the annual flow duration curves in the work by *Vogel and Fennessey* [1994]. Then, by fitting the annual streamflow volume function for all the observed years, one gets a sequence of annual curves, which are considered as independent annual events. This approach allows investigation on the interannual variability of the involved parameters (a, b, $y_{\min}$ and $y_{\max}$).

[29] Dependence of parameters a and b on each other and on $y_{\min}$ and $y_{\max}$ provide an interesting field of analysis that surely deserves careful investigation. Nevertheless, it is already widely recognized that the FDC's tails are shaped by climatic and hydrogeological conditions [*Smakhtin*, 2001]. In particular, a small slope of the low-flow tail indicates persistent discharge, possibly due to significant groundwater/subsurface contribution. In such a case, one would expect a large value of a, indicating a significant volume of water already available for large durations. On the other hand, a steep curve indicates a small and/or variable base flow contribution. This is the case of arid or semiarid basins, including intermittent regimes. Such a shape would be associated with a smaller value of a. Analogous considerations may be made regarding the high flows of the FDC. In fact, one may expect that persistent and frequent precipitation events typical of a humid climate may lead to a flat tail, and a smaller value of b would indicate a less pronounced peak of the FDC. *Vice versa* for arid and semiarid basins where the variability of peak flows is large and may lead to a steep tail, large values of b are expected. Actually, all the mentioned processes are affected by interannual and intra-annual variability, not excluding seasonality effects. Analyzing year by year the annual FDC and the relative a and b values a deeper knowledge about the dependence from climatic and hydrogeological factors, like rainfall, temperature, land cover, permeability and soil behavior, could be reached. Such investigation could then focus on the joint distribution of a and b conditional on $y_{\min}$ and $y_{\max}$, and could be used, for example, in a general framework for generation of synthetic time series of daily discharge. In this paper, which focuses on long- and medium-term predictions of annual FDC, some expedient approximations are introduced in order to constrain the model complexity and allow analytical treatment. In what follows, it is assumed that only geomorphoclimatic variations in a and b are important for long- and medium-term prediction because the interannual variability of a and b does not significantly affect the observed T year FDCs. As a consequence, we assume that the variables $y_{\min}$ and $y_{\max}$ in equation (7) are strongly affected by the interannual variability of the streamflow volume W while the η variable only represents its intra-annual variability.

[30] Then, in order to represent the overall variability of the streamflow volume function, we preserve the structure of equations (12) and let a and b take on the respective mean values μ_a and μ_b, to write

$$W_p(d) = \exp\left[I^{-1}\left(\frac{d}{D}, \mu_a, \mu_b\right)\left(y_{\max,p} - y_{\min,p}\right) + y_{\min,p}\right]. \tag{21}$$

[31] Moreover, following equation (20), and after a derivation analogous to that from equation (16) to equation (18), we obtain

$$Q_p(d) = \frac{W_p(d)}{D}\left(y_{\max,p} - y_{\min,p}\right)f_C\left(\frac{d}{D}, \mu_a, \mu_b\right), \tag{22}$$

where $y_{\min,p}$ and $y_{max,p}$ are found as

$$y_{\min,p} = \log\left(W_p(1)\right), y_{\max,p} = \log\left(W_p(D)\right). \tag{23}$$

[32] In equations (21) and (22), we consider estimates of $W_p(1)$ and $W_p(D)$ from their marginal distributions. In particular, the total annual flow $W_p(D)$ of equation (20) arises as the integral of the pth-quantile annual FDC. In ungauged basins $W_p(1)$ and $W_p(D)$ could be obtained from regional analysis of their data series derived from observations.

[33] This model structure represents a significant alternative to other literature models. In particular the index flood approach used by *Castellarin et al.* [2004a], introduces the

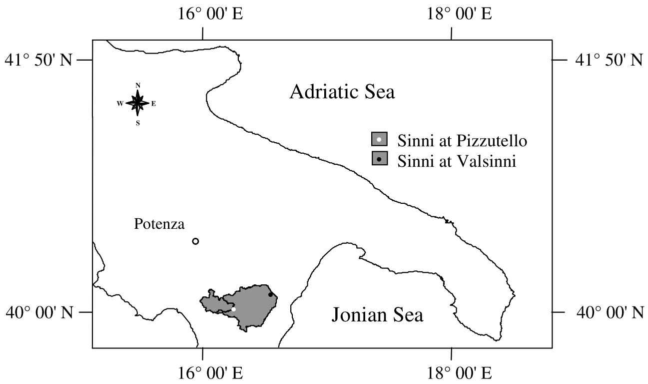


Figure 1. Investigated basins and gauged sites in southern Italy.

dimensionless daily streamflow X' as the ratio between daily streamflow and mean annual flow *AF*. The *AF* describes the long-term climatic regime for a given basin while X' is considered a key signature of the hydrologic behavior of the basin including the influence of hydrological processes, basin size, and permeability. The main difference between the index flood and the EtaBeta model lies in the role of the dimensionless variable. The dimensionless daily streamflow strongly accounts for variability of annual maxima and minima. The η variable used here remains practically unaffected by the hydrological extremes (high and low flows).

[34] The performance of the EtaBeta model will be tested in the next section. Also, the interannual variability of a, b, $y_{\min}$, $y_{\max}$ and their mutual dependence will be analyzed via case studies.

4. Model Application and Discussion

[35] In order to assess the model performance and reliability, two case studies were developed with reference to daily discharge series recorded in southern Italy. The gauged sites, located in two nested subbasins of the Sinni River, are Pizzutello and Valsinni (Figure 1). These discharge series were recorded almost continuously from 1925 to 1980 (see Table 1). The calendar year was used because this particular application is more oriented to the analysis of low flows and volume deficit. In such a case, it is preferable not to split the dry season in separate years. A different choice, like a hydrologic year lasting from September to August, would have been made if floods or high flows was the primary interest.

[36] The main characteristics of the recorded time series and of the investigated basins are shown in Table 1. N, N_c, A, μ_Q, σ_Q and Cv_Q are the number of the available years of observation, maximum number of consecutive years of observation, basin area, mean, standard deviation and coefficient of variation of daily discharge time series, respectively.

[37] The two sites were chosen because they apply to nested basins of the same river, and they show a number of significant hydrological homogeneities as well as some heterogeneities. In fact, for a general climatic characterization, the whole Sinni river basin can be considered typical of a humid Mediterranean climate. Also the lithology and permeability are quite similar over the two basins. Nevertheless, the two nested basins have different sizes (232 km^2 for Pizzutello and 1140 km^2 for Valsinni), and they show some climatic differences because the upstream part of the Sinni basin (average height 932 m a.s.l., distance of outlet from sea 70 km), with gauged station in Pizzutello, is more cold and rainy reflecting a mountain climate (southern Appennine). Its hydrologic behavior is characterized also by significant amounts of spring water, significant seasonal variability, and a land cover dominated by mountain woods. The complementary portion of the Valsinni basin (average height 752 m a.s.l., distance from sea 20 km) is better characterized by pastureland, croplands, low hills, and a sea temperate climate. Such climatic differences are reflected by the climatic index of *Thornthwaite* [1948]:

$$I = \frac{h - E_p}{E_p}, \tag{24}$$

where h is the mean annual rainfall depth and E_p is the mean annual potential evapotranspiration. For the Valsinni basin, I = 0.57, which indicates a humid basin, and for the Pizzutello basin, I = 1.26, which indicates a hyperhumid basin. E_p was calculated according to Turc's formula [*Turc*, 1961], depending on the basin average of mean annual temperature [*Iacobellis and Fiorentino*, 2000, *Fiorentino and Iacobellis*, 2001]. In earlier studies of this region [*Claps et al.*, 1996, 1998], the two nested basins have also shown significant heterogeneities with respect to the coefficient of skewness of annual flood maxima (C_a = 0.75 for Pizzutello, C_a = 2.42 for Valsinni), the mean annual number of flood events (31 for Pizzutello, 19.1 for Valsinni), the average annual rainfall amount (1583 mm for Pizzutello, 1166 mm for Valsinni), the average annual flow amount per unit area (1029 mm for Pizzutello, 592 mm for Valsinni), the variance of annual flow amount per unit area (116017 mm^2 for Pizzutello, 32642 mm^2 for Valsinni).

[38] Following *Castellarin et al.* [2004a], a statistical characterization of daily streamflow regime was performed. In fact, the index flow stochastic model assumes that the dimensionless daily streamflow (X') and the mean annual flow (*AF*) are independent. Then, dependence between *AF* and X' was investigated for data series of Pizzutello and Valsinni. Dependence between the series was assessed by computing the correlation coefficient between the series of *AF* and $X'(r)$ where r indicates a particular day in a nonleap year. They show a nonnegligible scatter, with values ranging from −0.6 to 0.6, and absolute values higher than 0.15

Table 1. Main Features of Gauged Basins and Recorded Time Series

Site	Years of Observation	N	N_c	A, Km^2	μ_Q, m^3/s	σ_Q, m^3/s	Cv_Q
Sinni at Pizzutello	1925–1928, 1930–1942, 1948–1967, 1970–1980	48	20	232	7.3	13.5	1.9
Sinni at Valsinni	1937–1942, 1950–1976	33	27	1140	20.5	32.9	1.6

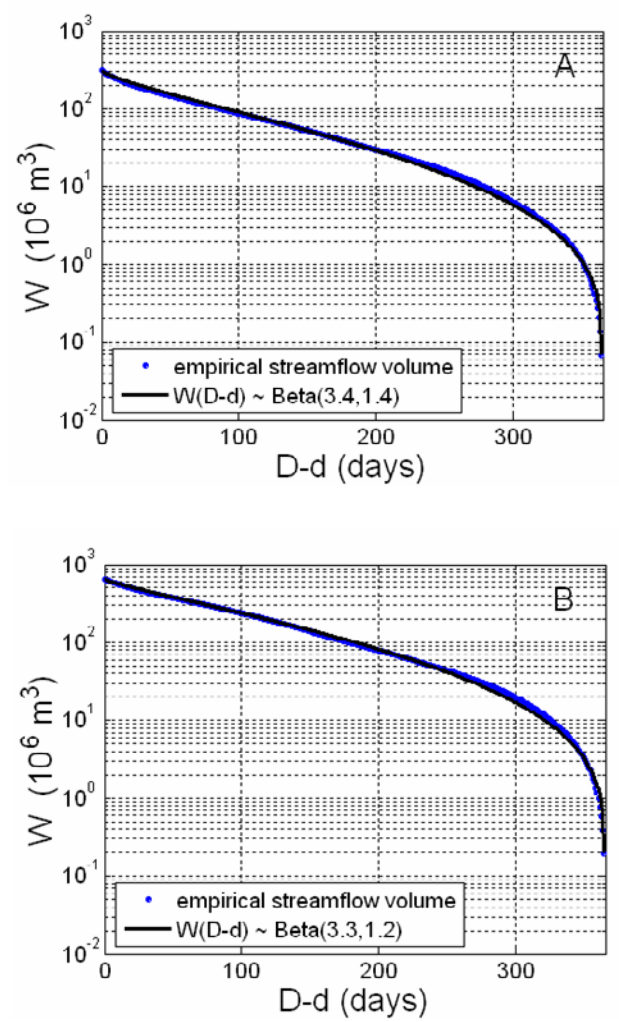


Figure 2. Streamflow volume function $W(D - d)$: empirical values (dots) of year 1937 and model estimates (solid line) from equation (12) for (a) Pizzutello and (b) Valsinni.

in most cases, for both basins. Also the correlation coefficients between the series of *AF* and corresponding values of standard deviation of annual series of X' ($\rho = -0.56$ for Sinni at Valsinni, $\rho = 0.0528$ for Sinni at Pizzutello) and coefficient of skewness of annual series of X' ($\rho = -0.54$ for Sinni at Valsinni, $\rho = -0.30$ for Sinni at Pizzutello) were evaluated.

[39] These results do not suggest the use of the index flow stochastic model which requires data series showing weak correlation between X' and *AF*. This is confirmed, in principle, by *Castellarin et al.* [2007]. In fact, they observe that the model capability to fully represent the interannual variability of daily streamflows generally decreases as duration increases. This is in part ascribed to the "questionable applicability" of the hypothesis of independence between *AF* and X' series for low flows. On the other hand, dependence of η on $y_{\max}$ and/or $y_{\min}$ is irrelevant to the EtaBeta model performance, because the dimensionless variable η is not responsible, for any fixed d, for the FDC interannual variability.

[40] In Figure 2, the empirical streamflow volume functions ($W(d)$ in equation (4)) for 1937 are shown and compared with equation (12) including the fitted beta distribution. In Figures 2, 3, 5, and 6, the traditional descending representation of FDCs and streamflow volume functions is preserved. Thus we report the duration $\delta = D - d$ in the abscissa. The parameter estimates, obtained by means of equations (A1) and (A2) are $a = 3.4$ and $b = 1.4$ for Sinni at Pizzutello and $a = 3.3$ and $b = 1.2$ for Sinni at Valsinni.

[41] It is worth remarking that the parameter values estimated in the same year for the two gauged stations are very close to each other, suggesting catchment similarity with respect to the observed variability of η.

[42] In Figure 3, the empirical annual flow duration curve for the same year is shown and compared with the theoretical equation (18) (solid line), derived using the complementary beta distribution CB(a, b). Parameter values (a = 3.4 and b = 1.4 for Pizzutello, a = 3.3 and b = 1.2 for Valsinni), were already available from the beta curve fitting to streamflow volumes of Figure 2. Also in Figure 3, the d average discharge function $\langle Q(D - d)\rangle$ of equation (13)

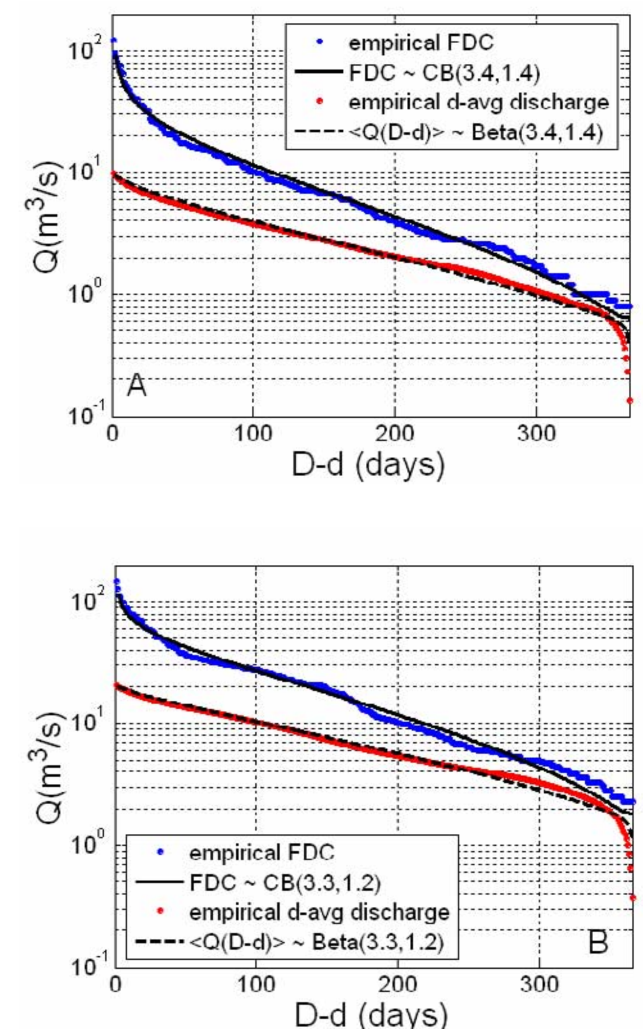


Figure 3. Annual FDC: empirical values (blue dots) of year 1937, model estimates (solid line) from equation (18), and corresponding d average discharge function $\langle Q(D - d)\rangle$ (red dots) for (a) Pizzutello and (b) Valsinni.

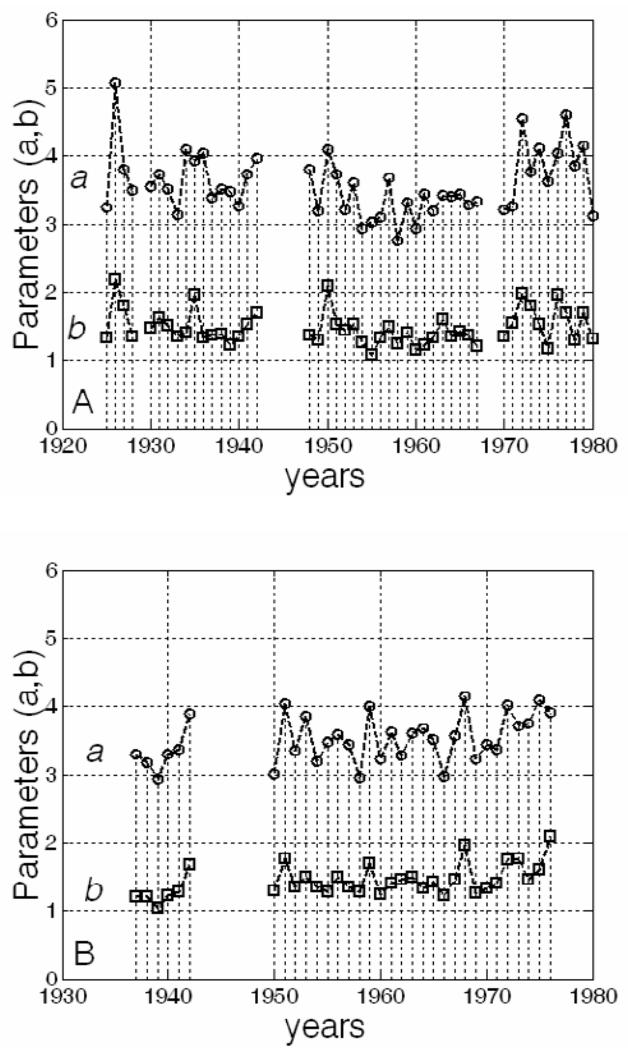


Figure 4. Time series of parameters a (circles) and b (squares) for (a) Pizzutello and (b) Valsinni.

(dashed line) and corresponding observed values are also shown.

4.1. Analysis of Model Performance and Evaluation of Annual FDCs and of *a* and *b* Time Series

[43] For both sites, the estimation of a and b was performed over all the observed calendar years and the χ^2 test was performed on the random variable η and confirmed the null hypothesis that η is beta distributed with significance levels always higher than 20%. In Figure 4, the time series of the yearly estimated values of a and b are displayed, and their main descriptive statistics are reported in Table 2. Only a low variability exists around the mean values as indicated by the low coefficients of variation shown in Table 2.

Table 2. Main Statistics of Parameters a and b Estimated From All Annual Streamflow Volume Functions Obtained From Recorded Series

Site	μ_a	σ_a	CV_a	μ_b	σ_b	CV_b
Sinni at Pizzutello	3.59	0.46	0.13	1.48	0.25	0.17
Sinni at Valsinni	3.51	0.35	0.10	1.44	0.23	0.16

Table 3. Main Statistics of Parameters a and b Estimated From All pth-Quantile Annual Streamflow Volume Functions Obtained From Recorded Series

Site	μ_a	σ_a	CV_a	μ_b	σ_b	CV_b
Sinni at Pizzutello	3.57	0.14	0.04	1.46	0.08	0.05
Sinni at Valsinni	3.51	0.16	0.05	1.42	0.06	0.05

Moreover, as already mentioned for 1937, the time series of a and b at the two gauged stations have similar means and variances. The values of the a parameter, descriptive of the low-flow tail of the FDC, are always higher and also show slightly more variability than the values of b, which characterize the high-flow tail. Such behavior reflects a natural feature of the streamflow volume function, whose right tail is less variable from year to year and less influenced by eventual peak flows. The a values, which have higher mean and variance, also show a low coefficient of variation.

[44] In order to evaluate the simplifications made in section 3 regarding the weak interannual variability of parameters a and b and their primary dependence on basin characteristics (such as climate, geomorphology, land cover, permeability, etc.), we evaluated values of a and b even with respect to all pth-quantile annual streamflow volume functions obtained from recorded series. Table 3 reports mean values, standard deviations, and coefficients of variation of a and b for Valsinni and Pizzutello. Results show that the mean values of a and b are practically coincident with those estimated from annual streamflow volume functions. Moreover, the standard deviation and the coefficients of variation are strongly reduced, thus confirming that the values of a and b are nearly constant and equal to the mean values observed over annual streamflow volume functions when considering the pth-quantile annual streamflow volume functions. These results confirm that fluctuations of the parameters a and b can be considered negligible for long- and medium-term prediction.

[45] Following equation (23), $y_{\min,p}$ and $y_{\max,p}$ were found by evaluating the pth-quantile streamflow volumes $W_p(1)$ and $W_p(D)$. According to the χ^2 test, both variables were found to be lognormal distributed with significance levels higher than 10%. Parameter values are shown in Table 4.

[46] Then, in order to test the model performance, including the effects of all the assumptions which lead to equations (21) and (22), Figures 5 and 6 show the empirical values of $W_p(\delta)$ and FDC_p obtained from discharge series recorded at Pizzutello and Valsinni as well as the model estimates evaluated with equations (21) and (22) for p = 0.25, 0.5, 0.75. It is worth noting that the curves shown in Figure 6 are not fitted to data but theoretically derived as in equation (22) with μ_a and μ_b taken from Table 2.

Table 4. Parameters of the Distribution of $y_{\min,p}$ and $y_{\max,p}$ as in Equation (23)

Station	$\mu[W(1)]$, Mm^3	$\sigma[W(1)]$, Mm^3	$\mu[W(D)]$, Mm^3	$\sigma[W(D)]$, m^3/s
Sinni at Pizzutello	0.053	0.026	230	74
Sinni at Valsinni	0.162	0.076	645	175

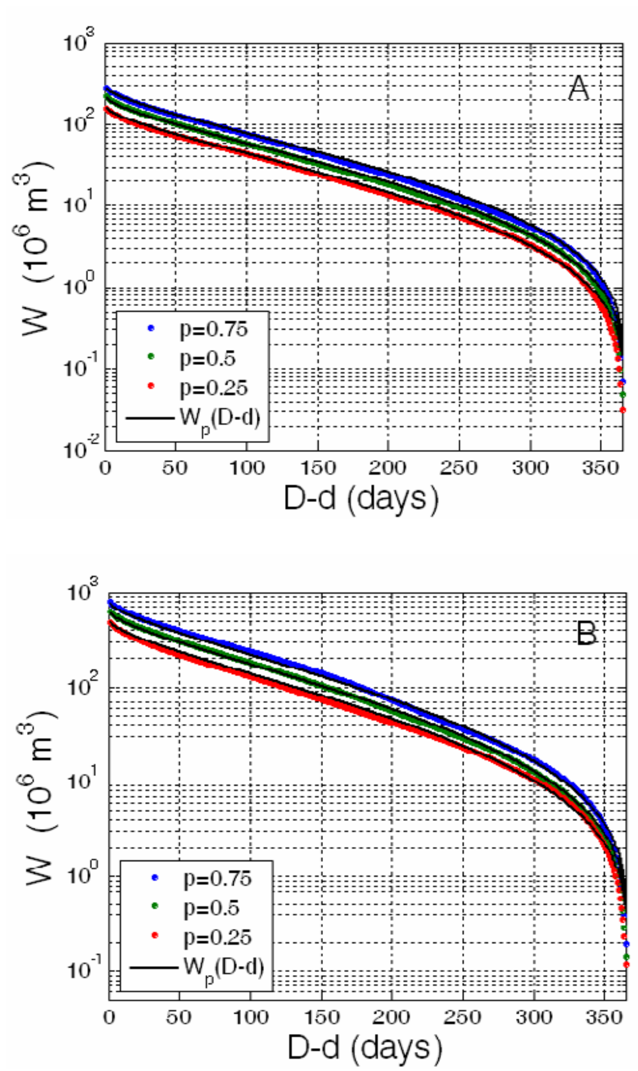


Figure 5. The *p*th-quantile annual streamflow volume function: empirical values (dots) and model estimates (solid lines) from equation (21) for p = 0.25, 0.5, 0.75 for (a) Pizzutello and (b) Valsinni.

[47] Finally, in Table 5 we report the model estimates of some characteristic measures of streamflow volume $W_p(d)$ and discharge $Q_p(d)$, for fixed values of probability of nonexceedance p and, for some significant durations, 7, 182 and 347 [*Singh and Stall*, 1974]. The corresponding empirical values obtained from recorded series are also shown. A graphical comparison is provided in Figures 7 and 8.

[48] It is worth noting that good results are obtained for all flow conditions, including low flows (small p and high duration). Similar results were obtained also from data series recorded in other several basins of Puglia and Basilicata (southern Italy).

4.2. Analysis of the Marginal and Conditional Distributions of *d*

[49] As a further evaluation of model performance, let us exploit the characteristic property of the complementary beta distribution which reverses the roles of the distribution and quantile functions. While considering u (or equivalently d) as the stochastic variable and η its probability distribution, we observe that its variability may be analyzed on an interannual basis with reference to any fixed threshold levels specified in terms of η. In other words, any fixed value of η represents a dimensionless measure of streamflow volume which may correspond to different durations in different years. In Figure 9, we report the empirical distributions of d conditional on different values of η hereafter called $F_{emp}(d|\eta)$. Such distributions are empirically obtained as probability of nonexceedance of d conditional on η from values of d corresponding, in all the observed years, to any fixed values of η = 0.25, 0.50 and 0.75.

[50] From these empirical distributions it is simple to evaluate the expected marginal distribution of d as

$$F_{emp}(d) = \int_0^1 F_{emp}(d|\eta)d\eta. \tag{25}$$

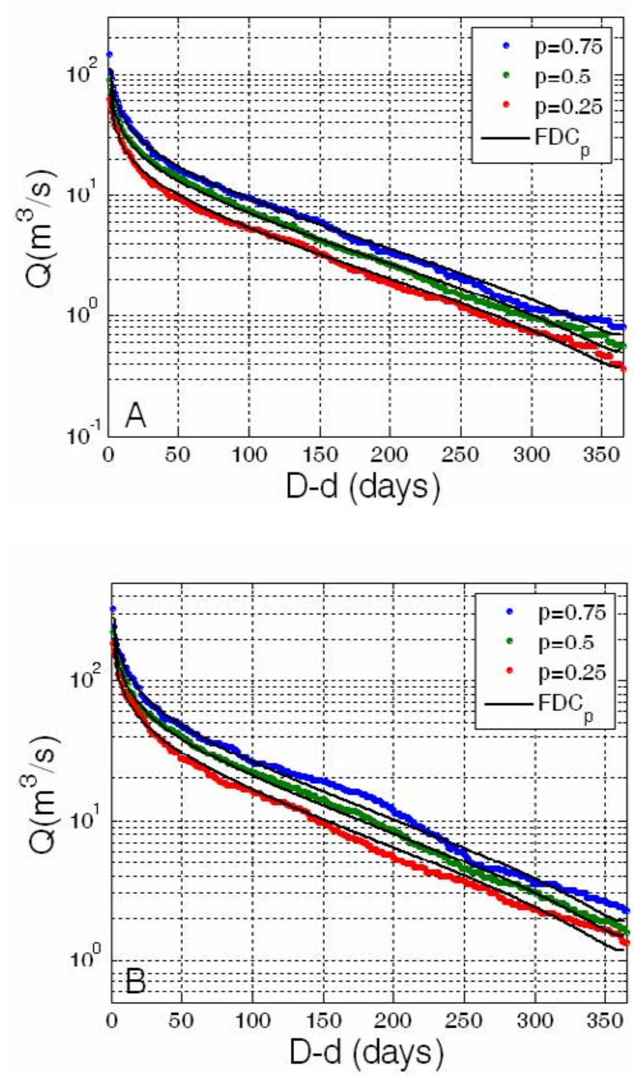


Figure 6. The *p*th-quantile annual flow duration curve: empirical values (dots) and model estimates (solid line) from equation (22) for p = 0.25, 0.5, 0.75 for (a) Pizzutello and (b) Valsinni.

Table 5. Empirical Values and Model Estimates of $W_p(d)$ and $Q_p(d)$ for d = 7, 182, 347 and p = 0.25, 0.5, 0.75

Site	$W_{0.25}(7)$	$W_{0.5}(7)$	$W_{0.75}(7)$	$W_{0.25}(182)$	$W_{0.5}(182)$	$W_{0.75}(182)$	$W_{0.25}(347)$	$W_{0.5}(347)$	$W_{0.75}(347)$
Empirical Values, $W_p(d)$, 10^6 m^3									
Sinni at Pizzutello	0.24	0.34	0.48	16.63	22.45	29.06	107.22	152.48	191.64
Sinni at Valsinni	0.86	1.03	1.42	50.52	68.86	91.33	328.14	445.16	559.73
Model Estimates, $W_p(d)$, 10^6 m^3									
Sinni at Pizzutello	0.35	0.48	0.71	17.62	23.51	31.37	111.44	147.13	194.26
Sinni at Valsinni	1.20	1.53	1.94	55.90	70.44	88.76	344.37	431.69	541.15
Empirical Values, $Q_p(d)$, m^3/s									
Sinni at Pizzutello	0.40	0.57	0.80	2.15	3.11	4.06	18.30	25.90	33.90
Sinni at Valsinni	1.56	1.76	2.40	6.32	10.00	14.6	58.00	68.60	89.50
Model Estimates, $Q_p(d)$, m^3/s									
Sinni at Pizzutello	0.38	0.51	0.69	2.34	3.11	4.12	18.84	24.73	32.47
Sinni at Valsinni	1.19	1.51	1.92	7.42	9.32	11.71	55.90	68.87	87.34

Figure 9 displays also a very good comparison obtained between the quantile function of the theoretical beta distribution obtained as

$$\eta(d) = I_u\left(\frac{d}{D}, \mu_a, \mu_b\right) \tag{26}$$

and the marginal distribution $F_{emp}(d)$ at Pizzutello and Valsinni.

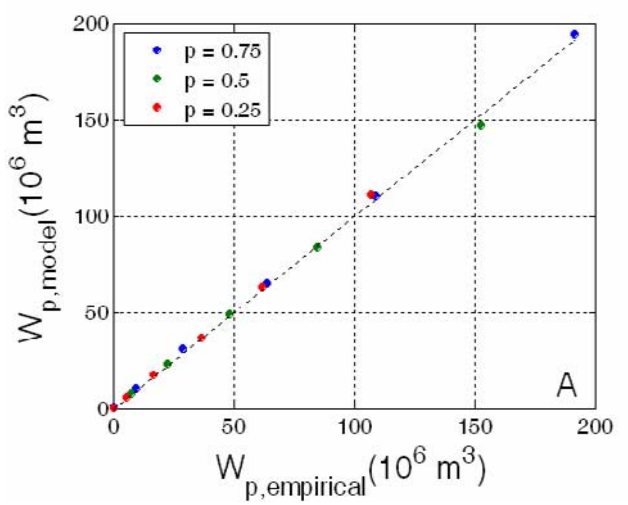
5. Conclusions and Final Remarks

[51] Apart from debates about whether the FDC can be considered as a statistical distribution of discharge, the annual FDC stochastic approach provides an interesting framework for the enhancement of procedures for streamflow analysis and prediction.

[52] In this paper, the annual flow duration curve is considered, in principle, as a distribution function supported on the bounded interval [0, 1] and ranging between the annual minimum and maximum daily discharge. The two-parameter complementary beta distribution is used to model the annual FDC and the incomplete beta function ratio is used in order to fit the streamflow volume annual function. In this way, the overall stochastic variability of the annual FDC is modeled by analyzing the distribution of four variables: the Complementary beta parameters a and b, the annual minimum daily flow and the total annual streamflow volume evaluated as integral of the pth-quantile annual FDC.

[53] The main advantage of the proposed method lies in this simple parameterization which is based on the use of variables dependent on long-term climatic features and well represented by two-parameter distributions. In fact, the EtaBeta model has six parameters that can be easily estimated by using only moments of the first and second order from three different sample series: two from the sample of the annual minima, two from the sample of the annual flows and two from the sample of the standardized streamflow volume η.

[54] As an indirect model comparison, it is worth noting that *Castellarin et al.* [2004a] introduce a dimensionless daily streamflow X' by dividing the daily streamflows by the annual flow AF, for the evaluation of a T year FDC. Such an approach is aimed at describing the long-term climatic regime via the annual flow distribution and the interannual variability of extremes (both minimum and maximum) via the dimensionless daily streamflow. In *Castellarin et al.* [2004a], they use a three-parameter distribution for dimen-



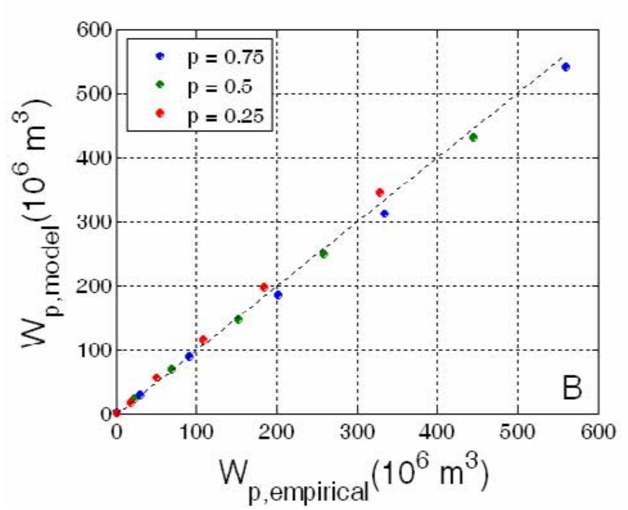


Figure 7. Model estimates versus empirical values of $W_p(d)$ for d = 7, 182, 347 and p = 0.25, 0.5, 0.75 for (a) Pizzutello and (b) Valsinni.

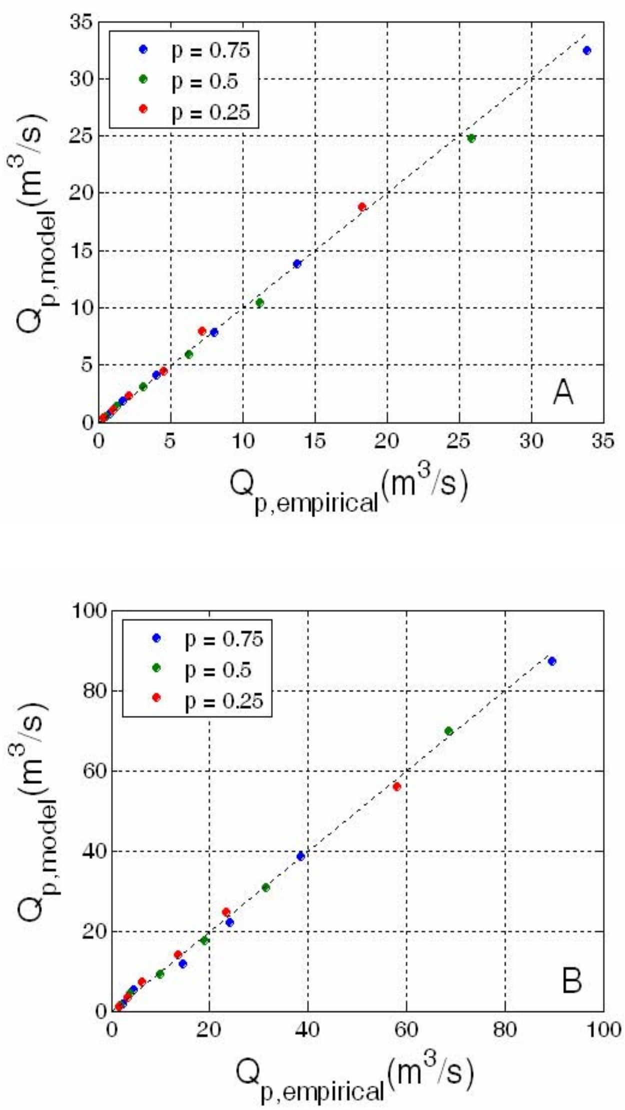


Figure 8. Model estimates versus empirical values of $Q_p(d)$ for d = 7, 182, 347 and p = 0.25, 0.5, 0.75 for (a) Pizzutello and (b) Valsinni.

sionless daily streamflow in order to reduce the number of model parameters. *Castellarin et al.* [2007] use a four-parameter kappa distribution for regional analysis. They show that the four-parameter kappa distribution is not easy to use and requires intensive computation and "repeated constrained optimization" of parameters estimates [see also *Hosking*, 1994]. Moreover, notwithstanding the use of a four-parameter distribution, still performances are not completely satisfying for low-flow conditions. They also discuss the hypothesis of independence between annual flow and dimensionless daily streamflow and its negative effect on model's representation of low flows. We also found this hypothesis not verified for rivers characterized by strong seasonality and very low flows in the dry season. Preliminary investigation performed on several basins in southern Italy show that, in most cases, a significant dependence between AF and X' is present.

[55] Results of application to several basins of Puglia and Basilicata in southern Italy, including case studies here presented, indicates that the EtaBeta model appears as a peculiar model in literature able to improve the FDCs representation in low-flow conditions [see, e.g., *Castellarin et al.*, 2004b; *Castellarin et al.*, 2007; *Cigizoglu and Bayazit*, 2000]. In principle, the EtaBeta model may be advantageous also in arid and semiarid basins that are characterized by an intermittent flow regime, where the annual minimum flow may be assumed equal to zero or to the minimum measurable discharge, thus avoiding the need of evaluating the annual minimum flow distribution. Of course, in such a case, a thorough analysis of the distribution of the annual number of zero flow days would be required to perform the T year FDC evaluation.

[56] Results of the application of the EtaBeta model show that the interannual variability of the annual FDC and hence its return time are completely represented by the annual minimum flow $W_p(1)$ and the total annual streamflow $W_p(D)$. On the other hand, the "standardized streamflow volume" (η) and, in particular, parameters a and b of the

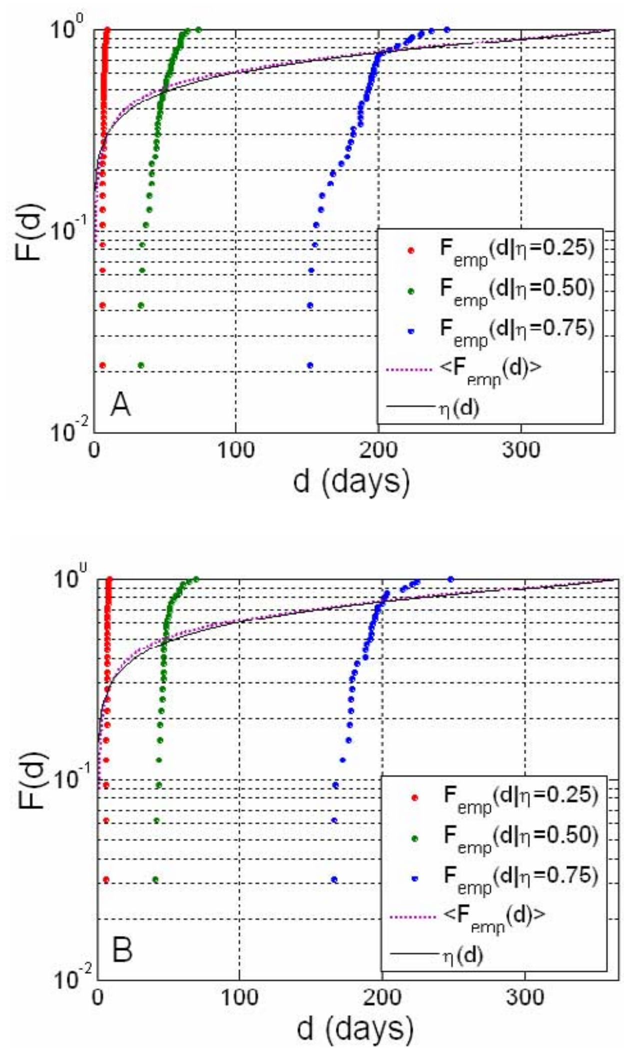


Figure 9. Comparison between theoretical (η_d) and empirical (F_{emp}) distributions of d obtained from discharge time series recorded at (a) Pizzutello and (b) Valsinni.

Table 6. Correlation Coefficients for Parameters *a* and *b*

Parameter	Correlation Coefficient
$\rho_{\text{ap-bp}}$	0.75
$\rho_{\text{av-bv}}$	0.84
$\rho_{\text{pa-W(30)}}$	0.41
$\rho_{\text{pb-med(W)}}$	−0.37
$\rho_{\text{va-med(W)}}$	−0.39
$\rho_{\text{vb-med(W)}}$	−0.54
$\rho_{\text{ap-av}}$	0.51
$\rho_{\text{bp-bv}}$	0.54

complementary beta distribution completely characterize the intra-annual variability of the FDC. They are responsible for the shape of the two tails: *a* characterizes the low-flow part of the FDC, which usually depends on groundwater/subsurface contribution, while *b* characterizes the high-flow part of the FDC, which is more affected by climate and precipitation features. The analysis of time series of *a* and *b* shows a reduced interannual variability. Then, they are replaced by constant values μ_a and μ_b providing accurate evaluation of significant *p*th quantiles $Q_p(d)$ and $W_p(d)$. This result is of particular interest because it opens a field of investigation about the spatial variability of μ_a and μ_b and their dependence on measurable geomorphoclimatic quantities.

[57] With this purpose, a deeper analysis of the time series of *a* and *b* was performed and the results are hereafter reported. In particular, it is worth noting the following.

[58] 1. An interannual dependence between *a* and *b* is visible in Figure 4. In fact, the correlation coefficients $\rho_{\text{pa}-\text{pb}}$ (Pizzutello) and $\rho_{\text{va}-\text{vb}}$ (Valsinni), reported in Table 6, show values of 0.75 and 0.84, respectively.

[59] 2. The correlation coefficients $\rho_{\text{pa}-\text{va}}$, between "*a* series at Pizzutello" and "*a* series at Valsinni," and $\rho_{\text{pb}-\text{vb}}$ between "*b* series at Pizzutello" and "*b* series at Valsinni," also suggest significant dependence.

[60] 3. Other relationships between *a*, *b*, and other characteristic quantities are of interest. We report in Table 6 those showing significant values: the correlation coefficients $\rho_{\text{pa}-\text{W(30)}}$ (between *a* and W(30) at Pizzutello), $\rho_{\text{pb}-\text{med(W)}}$ (between *b* and median(W) at Pizzutello), $\rho_{\text{va}-\text{med(W)}}$ (between *a* and median(W) at Valsinni), and $\rho_{\text{vb}-\text{med(W)}}$ (between median(W) at Valsinni). W(30) is the streamflow volume associated with flow discharges from the annual minimum up to discharge of duration 30 d. Such quantity is representative of the lowest monthly flow in the year. The median(W) is a value representative of the streamflow volume of the dry season (182 d).

[61] The EtaBeta model offers interesting perspectives also in terms of generation of synthetic annual FDCs and prediction in ungauged basins. With this aim the following results are worthy of mention.

[62] 1. In all series of *a* and *b*, only weak traces of nonstationarity are detectable, the autocorrelation functions are shown in Figure 10 in comparison with confidence limits at

$$\rho_l = \pm\frac{2}{\sqrt{N}},$$

and also in this case they allow to detect only weak autocorrelations.

[63] 2. The hypothesis of *a* following a normal distribution is accepted by the χ^2 test for both time series, with significance level higher than 5%. While for *b* at Valsinni it is accepted with 1% significance level, and always rejected for Pizzutello. The hypothesis of *b* lognormal distributed is accepted for both stations with significance levels higher than 5%.

[64] The EtaBeta model allows one to account for flow duration curves, annual minimum flow distribution, and total annual flow distribution. Once these are evaluated other characteristic basin functions can be found. It is worth mentioning the volume W_M of water diverted (in 1 a) by a system without upstream storage and with Q_M maximum flow discharge. W_M can be evaluated as

$$W_M = W(d) + Q_M(d)*(D - d), \tag{27}$$

where *W*(*d*) is the streamflow volume distribution, which can be obtained from equation (12) of the EtaBeta model. The same W_M is a measure of the hydroelectric power which could be generated, downstream of diversion, in 1 a.

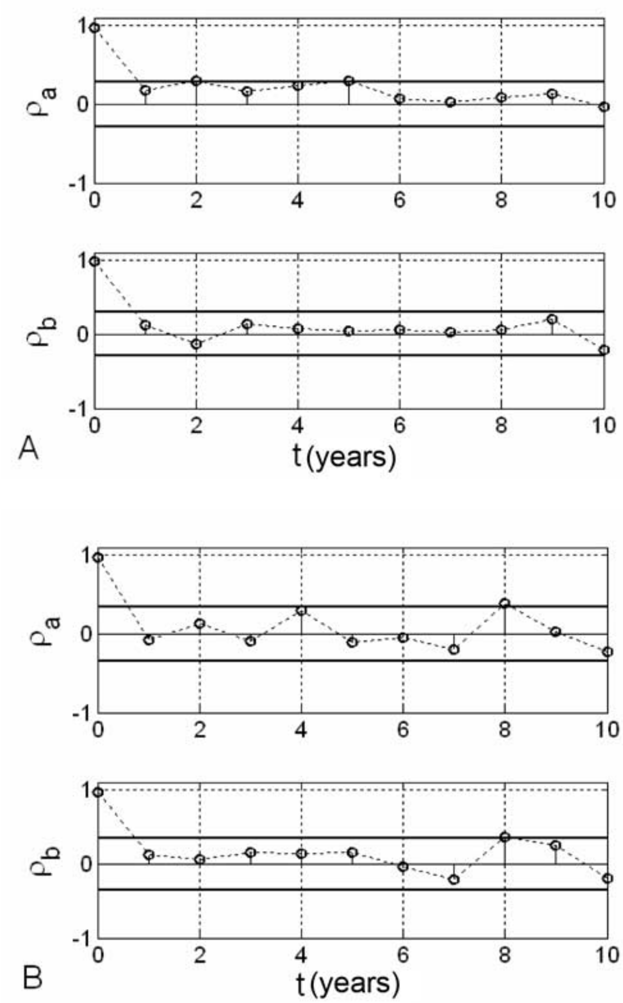


Figure 10. Autocorrelation function of time series of *a* and *b* and confidence limits ρ_l (thick solid lines) for (a) Pizzutello and (b) Valsinni.

[65] Then, within this probabilistic framework, all the following basin features related to water resources management are coherently estimated: (1) annual minimum daily flow $W_p(1)$ in equation (20); (2) total annual flow $W_p(D)$ in equation (20); (3) streamflow volume function $W_p(d)$ in equation (20); (4) flow duration curve $Q_p(d)$ in equation (22); (5) average discharge, in a fixed duration, available for flow diversion structures without upstream storage reservoirs $\langle Q(d)\rangle$ in equation (13); and (6) annual flow available for flow diversion structures without upstream storage reservoir and with Q_M maximum flow discharge W_M in equation (27).

[66] The analysis and prediction of *p*th-quantile $\langle Q(d)\rangle$ and W_M are object of current research activity. Preliminary results are not reported because this paper is conceived with the only purpose of introducing the EtaBeta model.

[67] A comparative analysis of modeling errors, of the proposed and previously available models, accounting for observed natural hydrologic heterogeneities is under study. In fact, a comparative evaluation of model descriptive and predictive ability will be particularly effective in a regional analysis context. At the current state of research, the EtaBeta model is proposed as an "at-site" probabilistic model, which presents characteristics particularly promising for the analysis and detection of spatial variability of its parameters and for the assessment of statistical homogeneity and heterogeneity.

Appendix A

[68] For the estimation of parameters *a* and *b* different methodologies could be used. In this paper *a* and *b* are evaluated by means of their expression in terms of moments of the beta distribution:

$$a = \frac{\mu_\eta\left[\mu_\eta - \mu_\eta^2 - \sigma_\eta^2\right]}{\sigma_\eta^2} \tag{A1}$$

$$b = \frac{a\left[1 - \mu_\eta\right]}{\mu_\eta}, \tag{A2}$$

where μ_η and σ_η are the mean value and standard deviation of the annual series of η obtained from the annual series of daily discharges expediently ordered and transformed as in equation (7).

[69] According to *Jones* [2002, p. 330] "the CB distribution is much more amenable than the beta distribution to exact computations involving expectations of order statistics." In particular, "while the beta distribution is well suited to the explicit computation of moments, ... the complementary beta distribution is very well suited to the explicit computation of expectations of order statistics, ... while the beta distribution is not" [*Jones*, 2002, p. 333]. Moreover, "beta and complementary beta distributions with parameters that are reciprocal of other's parameters are good approximations to one another. ... In particular, this suggests that the complementary beta distribution may be used instead of the beta distribution ... when order statistics are of central interest" [*Jones*, 2002, p. 330].

[70] In such a case, for the estimation of parameters one may use the L moments [*Hosking*, 1990; *Hosking and Wallis*, 1997], i.e., expectations of linear combinations of order statistics. The first L moment, λ_1 is equal to the mean of the complementary beta distribution [*Jones*, 2002], and then,

$$\lambda_1 = \frac{\beta}{\alpha + \beta} \tag{A3}$$

while the second L moment λ_2 has the form

$$\lambda_2 = \frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)} \tag{A4}$$

with

$$\alpha = \frac{1}{a} \tag{A5}$$

and

$$\beta = \frac{1}{b}. \tag{A6}$$

[71] It is worth to mention that according to *Hosking* [2006, p. 198], the CB distribution is an example of a distribution that is uniquely determined by a proper subset of its L moments. In other words, for many distributions including CB, the L moments characterize the distribution with minimum redundancy, since "the information contained in the *r*th L moment is maximally independent of the information given by the first $r - 1$ L moments."

[72] For completeness of analysis, we also performed the evaluation of parameters *a* and *b* assuming η following a CB(α, β) distribution and using equations (A3), (A4), (A5) and (A6). We obtained results practically coinciding with those reported in the paper.

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