

Fast light generation through velocity manipulation in two vertically-stacked ring resonators

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Abstract: Speed manipulation of optical pulses is a very attractive research challenge enabling next-generation high-capacity all-optical communication networks. Pulses can be effectively slowed by using different integrated optical structures such as coupled-resonator waveguiding structures or photonic crystal cavities. Fast light generation by means of integrated photonic devices is currently a quite unexplored research field in spite of its crucial importance for all-optical pulse processing. In this paper, we report on the first theoretical demonstration of fast light generation in an ultra-compact double vertical stacked ring resonator coupled to a bus waveguide. Periodic coupling between the two rings leads to splitting and recombining of symmetric and anti-symmetric resonant modes. Re-established degenerate modes can form when a symmetric and an anti-symmetric mode having different resonance order exhibit the same resonance wavelength. Under degenerate mode conditions, wide wavelength ranges where the group velocity is negative or larger than the speed of light in vacuum are generated. The paper proves how this physical effect can be exploited to design fast light resonant devices. Moreover, conditions are also derived to obtain slow light operation regime.

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1. Introduction

In the last few years, slow and fast light generation and processing is emerging as a very attractive research topic. Group velocity v_g of an optical pulse can be effectively manipulated by a number of techniques which can make v_g less, larger or opposite in sign with respect to the speed of light in vacuum c [1]. One major task in this research field is the theoretical and experimental investigation of novel integrated optical structures enabling the v_g tuning.

In normal dispersion regime the group velocity can be slowed ($v_g < c$) and subluminal pulse propagation can be obtained, whereas in an anomalous dispersion regime a group velocity larger than c or negative leads to a superluminal pulse propagation.

Integrated photonic structures slowing optical pulses have been both theoretically and experimentally largely investigated. These structures, based on either evanescently coupled coplanar ring resonators or photonic crystal cavities, allow very low values of v_g [2–4].

Optical pulses having superluminal velocities have been experimentally demonstrated, without violating Einstein's causality [5], by exploiting stimulated Brillouin scattering in optical fibers [6], coherent population oscillation in amplifying media [7], four wave mixing in semiconductor optical amplifiers [8], vapour atomic medium [9], coupling of modes with different polarization [10], innovative resonant structures formed by zigzag micro-ring resonator chains [11], and structural dispersion in coupled-resonator structures [12]. These pulses exhibit a reduced light matter interaction and so they suffer from a rather low loss. Main applications of fast light are in the gravitational waves, absolute rotation sensing [13, 14] and in the field of information transmission and processing.

Very recently, the relationship between group velocity and information transfer speed in a communication system has been deeply discussed by considering also the effect of the receiver noise [15].

Integrated optical resonators are fundamental building blocks in a large variety of photonic devices such as add-drop multiplexers, physical or chemical sensors, switches and modulators [16–22]. Because of their spectral properties, integrated optical cavities can be effectively exploited as slow or fast light structures. Group velocity of optical pulses propagating in a ring resonator coupled to a straight bus waveguide has been deeply investigated showing that a wide spectrum of v_g values can be obtained by tuning the parameters of the integrated structure [23,24]. Fast and slow light generation in a silicon nitride-based ring resonator has been experimentally demonstrated in [25].

Recently three-dimensional photonic structures formed by vertically-stacked micro-ring resonators have been theoretically proposed and modelled. In [26], a sequence of ring resonators vertically-stacked has been investigated but no application has been envisaged for this structure. The possibility of optically tuning the distance between two vertically-stacked micro-ring resonators by optical attractive and repulsive forces has been theoretically and experimentally explored in [27,28]. All-optical wavelength routing performed by optical gradient force due to radiation pressure in a compact photonic structure including two vertically-stacked ring resonators has been experimentally demonstrated in [29].

In this paper, for the first time, a theoretical investigation of a three-dimensional fast light device formed by two vertically-stacked micro-ring resonators and a straight bus waveguide is presented. Moreover, we discuss the possibility of perturbing the coupling between the stacked rings by either thermo-optic effect or optical forces. We prove that this perturbation enables group velocity manipulation in the proposed structure.

2. Device architecture and modelling

The investigated device, based on two vertically-stacked micro-ring resonators, is shown in Fig. 1. The two micro-ring resonators are on two different planes and are evanescently coupled each other. The resonator on the bottom plane is evanescently coupled also to a straight bus waveguide. The distance between the two rings is d while g is the distance between the ring and the bus waveguide [see Fig. 1 (b)]. The optical power propagates horizontally in the plane of rings and vertically between the adjacent rings. The bus waveguide enables to excite the resonant structure and to observe its spectral response.

This device increases the degrees of freedom in the group velocity manipulation with respect to the case of a single ring resonator discussed in [23]. In fact, in single ring resonator v_g manipulation can be performed only by changing the power transfer between the ring and the straight bus waveguide, whereas in the device of Fig. 1 the group-velocity control can be obtained by shifting the ring-bus power transfer or the distance between the two rings.

The proposed device exhibits the basic features of both an optical resonator and a directional coupler. As in a directional coupler, optical power is periodically exchanged between the rings and supermodes are excited. The device behaves as on optical cavity with a wavelength-dependent transmittivity.

The spectral response of the proposed optical cavity is calculated as the wavelength-dependent ratio between the optical power at the two ends of the straight bus waveguide [input and through ports shown in Fig. 1(a)]. We have assumed that the two rings are identical (their radius has been denoted as R) and only the fundamental quasi-TE and quasi-TM modes are supported by the waveguiding rings. Moreover, we have supposed that only one of these two polarizations is excited at the input port. Finally we have assumed that the coupling between the straight bus waveguide and the top ring is negligible.

For each polarization, the configuration including the two coupled rings supports two supermodes, which are two eigensolutions of the Helmholtz equation solved over the whole cross-section of the structure. According with the coupled mode theory (CMT), the two field

distributions relevant to the supermodes can be obtained as a linear combination of the modes supported by the two uncoupled rings.

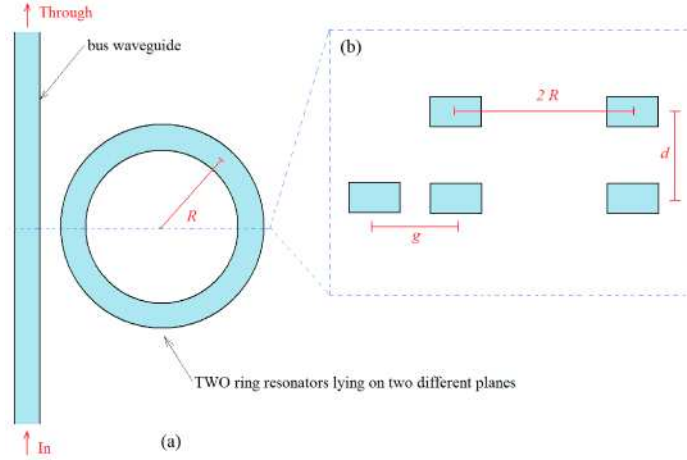


Fig. 1. Two vertically-stacked ring resonators coupled to a straight waveguide. (a) Top-view. (b) Cross-section.

The curvilinear coordinate along the two rings has been denoted with ζ . The amplitudes of the optical signal propagating in the bottom and in the top rings have been denoted with $a_1(\zeta)$ and $a_2(\zeta)$, respectively. The dependence of these two amplitudes on the curvilinear coordinate ζ can be modelled by CMT, which provides these two coupled differential equations [30]:

$$\begin{cases} \frac{da_1}{d\zeta} = i\beta a_1 + i\kappa a_2 \\ \frac{da_2}{d\zeta} = i\kappa a_1 + i\beta a_2 \end{cases} \quad (1)$$

where κ is the coupling coefficient between the rings, β is the propagation constant of the two identical optical modes within the rings and i is the imaginary unit.

Taking into account also the propagation loss within the two rings by the attenuation coefficient α , the general expressions of $a_1(\zeta)$ and $a_2(\zeta)$ can be written as:

$$\begin{cases} a_1(\zeta) = a_1(0) \cos(\kappa\zeta) \exp\left(i\beta\zeta - \frac{\alpha}{2}\zeta\right) + i a_2(0) \sin(\kappa\zeta) \exp\left(i\beta\zeta - \frac{\alpha}{2}\zeta\right) \\ a_2(\zeta) = i a_1(0) \sin(\kappa\zeta) \exp\left(i\beta\zeta - \frac{\alpha}{2}\zeta\right) + a_2(0) \cos(\kappa\zeta) \exp\left(i\beta\zeta - \frac{\alpha}{2}\zeta\right) \end{cases} \quad (2)$$

The two amplitudes $a_1(0)$ and $a_2(0)$ can be determined by imposing the boundary conditions. If $L = 2\pi R$ is the length of each ring, the first boundary condition is quite simple and can be expressed as:

$$a_2(L) = a_2(0) \quad (3)$$

This condition is due to the circular shape of the top ring and to the fact that this ring is coupled only with the bottom ring and not with the bus waveguide.

Coupling between the straight bus waveguide and the bottom ring has been modelled by the following transfer matrix [31]:

$$\begin{bmatrix} E_{thr} \\ a_1(0) \end{bmatrix} = \begin{bmatrix} \tau & ih \\ ih & \tau \end{bmatrix} \begin{bmatrix} E_{in} \\ a_1(L) \end{bmatrix} \quad (4)$$

where E_{thr} is the amplitude of the signal coming out from the through port, E_{in} is the amplitude of signal coming in the bus waveguide, h is the portion of the input optical amplitude that is coupled to the bottom ring, τ is the portion of the input optical amplitude remaining in the bus waveguide.

The second equation of the system in Eq. (4) allows to derive the second boundary condition as:

$$a_1(L) = \frac{a_1(0) - ihE_{in}}{\tau} \quad (5)$$

By imposing the two boundary conditions [Eqs. (3) and (5)] to the system in Eq. (2), we obtain:

$$a_1(L) = - \frac{E_{in} h i e^{i\beta L} \left[e^{i\beta L} - \cos(\kappa L) e^{\frac{L\alpha}{2}} \right]}{e^{L\alpha} + \tau e^{2i\beta L} - (1 + \tau) e^{i\beta L + \frac{L\alpha}{2}} \cos(\kappa L)} \quad (6)$$

From the first equation of the system in Eq. (4) and by using Eq. (6) we derive:

$$E_{thr} = \tau E_{in} + i h a_1(L) \quad (7)$$

Therefore, by using Eq. (7), the ratio between the optical field amplitudes at the two ends of the bus waveguides is given by:

$$\frac{E_{thr}}{E_{in}} = \tau + \frac{h^2 e^{i\beta L} \left[e^{i\beta L} - \cos(\kappa L) e^{\frac{L\alpha}{2}} \right]}{e^{L\alpha} + \tau e^{2i\beta L} - (1 + \tau) e^{i\beta L + \frac{L\alpha}{2}} \cos(\kappa L)} \quad (8)$$

The spectral response $T(\lambda)$ at the through port is given by $|E_{thr}/E_{in}|^2$ and so it is equal to:

$$T(\lambda) = \left| \tau + \frac{h^2 e^{i\beta L} \left[e^{i\beta L} - \cos(\kappa L) e^{\frac{L\alpha}{2}} \right]}{e^{L\alpha} + \tau e^{2i\beta L} - (1 + \tau) e^{i\beta L + \frac{L\alpha}{2}} \cos(\kappa L)} \right|^2 \quad (9)$$

In the expression of the spectral response in Eq. (9) α , h , τ and κ have been assumed to be independent of wavelength. This assumption is valid when a wavelength range narrow enough is considered.

Proposed modelling technique has been validated by the three-dimensional finite-difference time-domain (3D-FDTD) algorithm. Spectral responses of different test structures including two vertically-stacked rings have been calculated by using both Eq. (9) and 3D-FDTD. A very good agreement between results provided by the two numerical methods has been observed. This proves that our CMT-based modelling approach has accuracy comparable with that of the 3D-FDTD method which requires a computation time of several hours.

Simulations performed by 3D-FDTD confirmed all assumptions done in the model development.

3. Resonator spectral response

When the two rings are at a very large distance they can be considered as uncoupled and it results $\kappa = 0$ and $\cos(\kappa L) = 1$. In this case, the spectral response is the same of that of a single ring resonator coupled to a bus waveguide. As it is well known, this response exhibits a periodical set of resonance minima $\lambda_1, \lambda_2, \dots, \lambda_m, \lambda_{m+1}, \dots$ (m is the integer number denoting the resonance order). The distance between two adjacent minima is the free spectral range (FSR = $\lambda_m - \lambda_{m+1}$). The same spectral response as for $\kappa = 0$ can be achieved for $\kappa L = 2 q \pi$ (q is an integer number), when $\cos(\kappa L) = 1$. Therefore, when $\kappa L = 2 q \pi$ the rings are fully decoupled and resonance frequencies correspond to those of the standing alone ring resonator.

When $\cos(\kappa L)$ decreases being slightly less than 1, for each resonance order two resonance minima rather close each other appear in the spectral response. This minimum peak splitting is due to the fact that the two supermodes (symmetric and anti-symmetric) supported by the resonant structure have two different propagation constants and, therefore, they resonate at two different frequencies. Each of them is related to the symmetric or anti-symmetric mode. The magnitude of the splitting between these two minima increases as $\cos(\kappa L)$ decreases. For each resonance order, the synchronous coupling between the two rings is responsible of a power splitting between the two resonant modes in order to lead the system to a minimum energy condition.

If $\kappa L = (2 q + 1) \pi / 2$ and so $\cos(\kappa L) = 0$, the resonator spectral response is the same of that of a single ring having a radius equal to $2 R$. This means that the FSR is halved with respect to the case of uncoupled rings, the rings are fully coupled and complete power transfer between them occurs.

Finally when $\cos(\kappa L) = -1$, $\kappa L = (2 q + 1) \pi$, the spectral response has the same shape as in the case $\kappa = 0$, and it is translated in wavelength of a quantity equal to FSR/2.

The amount of the κ -dependent splitting ($\Delta\lambda$) occurring when $\cos(\kappa L) \neq \pm 1$ can be estimated starting from the coupled differential equations modelling the coupling between the rings. It is given by:

$$\Delta\lambda(\kappa) = \frac{\kappa \lambda_m^2}{\pi n_{eff}} \quad (10)$$

Now, if we consider three adjacent resonant wavelengths λ_{m-1} , λ_m and λ_{m+1} under the condition $\kappa = 0$, we can observe how these resonance wavelengths change as κ increases using the grid of eigenvalues shown in Fig. 2. The lines in the grid show the dependence of resonance wavelengths on κ . For each m value the resonator with two vertically-stacked rings exhibits two resonance wavelengths. Only for specific κ values satisfying the condition $\cos(\kappa L) = \pm 1$ we have that at each resonant order corresponds only one resonance wavelength. In this operating regime, achieved when $\kappa = q \pi / L$, the excited resonant modes are degenerate modes. When a symmetric and an anti-symmetric resonant mode having different resonance order exhibit the same resonance wavelength, the degenerate mode condition is obtained. In the grid of eigenvalues of Fig. 2, degenerate mode condition corresponds to the crossing between two lines corresponding to a symmetric and an anti-symmetric mode and relevant to different resonance orders.

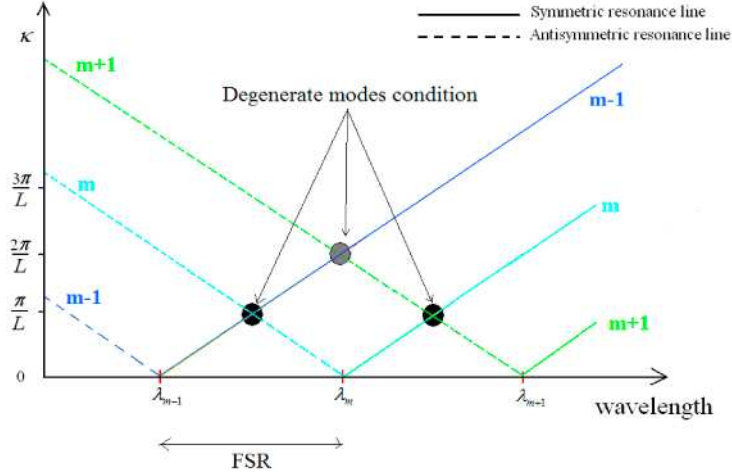


Fig. 2. Spectral properties of the optical cavity including the two vertically-stacked rings (● crossing between lines relevant to adjacent resonant orders, ● crossing between lines relevant to non adjacent resonant orders).

Spectral response of the proposed resonator strongly depends on the power transfer h^2 between the bottom ring and the straight bus waveguide. To show this physical behaviour of the proposed optical cavity we have assumed that the two rings have a radius $R = 5 \mu\text{m}$ and are formed by a squared $300 \text{ nm} \times 300 \text{ nm}$ Si-wire waveguide fully surrounded by silicon dioxide. Under these assumptions, the fundamental quasi-TE mode supported by the Si-wire exhibits an effective index equal to 2.04948. Propagation loss of this mode has been supposed equal to 5 dB/cm. Device operating wavelength is somewhat larger than $1.55 \mu\text{m}$.

We have plotted the cavity spectral response [see Figs. 3(a) and 3(b)] for $h = 0.1$ ($h^2 = 1\%$), i.e. in weak power transfer case. In the contour plot in Fig. 3(a), the two variables are the wavelength and the coupling between the vertically-stacked rings. Wavelength range is from $1.5512 \mu\text{m}$ to $1.5518 \mu\text{m}$, and κ values are close to the degenerate mode condition $\kappa = \kappa^* = \pi / L = 0.1 \mu\text{m}^{-1}$. We can see that resonance curves, corresponding to the minimum transmittance, cross in correspondence of the degenerate mode condition obtained for $\kappa = \kappa^* = 0.1 \mu\text{m}^{-1}$. Degenerate mode resonates at $\lambda = 1.55148 \mu\text{m}$.

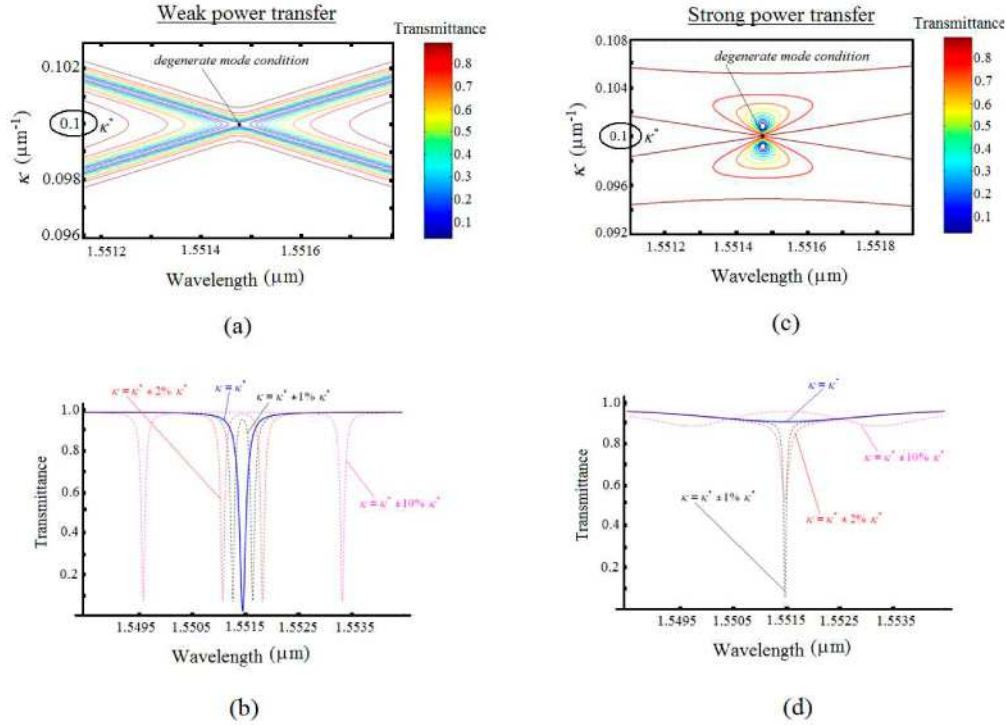


Fig. 3. Spectral response of the proposed optical cavity for both weak and strong power transfer between the bus waveguide and the bottom ring. (a) Contour plot of the spectral response for $h = 0.1$. (b) spectral response for different values of κ and for $h = 0.1$. (c) Contour plot of the spectral response for $h = 0.7$. (d) Spectral response for different values of κ and for $h = 0.7$.

As shown in Fig. 3(b), for $h = 0.1$ a slight shift of κ with respect to κ^* induces the splitting of resonance minimum so that, for each resonant order, two resonance frequency are observed. No significant change of resonance depth is induced by the κ shift.

In case of strong power transfer between the bottom ring and the bus waveguide ($h = 0.7$ and $h^2 = 49\%$) we have newly a degenerate mode condition for $\kappa = \pi / L = 0.1 \text{ } \mu\text{m}^{-1}$ but the contour plot of the spectral response [Fig. 3(c)] is significantly different with respect to that obtained in weak power transfer case [Fig. 3(a)].

In the strong power transfer case, the splitting of the resonance minimum is inhibited in a narrow range of κ values around κ^* . For example, when κ is equal to $\kappa^* \pm 1\%$ κ^* and $\kappa^* \pm 2\%$ κ^* no splitting of resonance wavelength occurs [see Fig. 3(d)]. The slight shift of κ with respect to κ^* induces a clear change in the shape of spectral response and in the depth of resonance minimum but not the resonance splitting. In interval of κ values in which the splitting of resonance minimum is inhibited, there are specific κ values allowing to obtain a resonator spectral response with very deep resonance minima. In particular, for $\kappa^* \pm 1\%$ κ^* , we have that the spectral response of the proposed cavity includes a very deep minimum for each resonant order. Maximum resonance depth, equal to 13 dB, can be achieved for $\kappa = 0.0996 \text{ } \mu\text{m}^{-1}$.

The value of κ maximizing the resonance depth in case of strong power transfer depends on the optical loss α suffered by the resonant mode within the cavity. For $h = 0.7$, κ values maximizing the resonance depth are equal to $0.0996 \text{ } \mu\text{m}^{-1}$ and $0.0992 \text{ } \mu\text{m}^{-1}$ for α equal to 5 dB/cm and 10 dB/cm, respectively. Assuming $\alpha = 5 \text{ dB/cm}$, $\kappa = 0.0996 \text{ } \mu\text{m}^{-1}$ and $\alpha = 10 \text{ dB/cm}$, $\kappa = 0.0992 \text{ } \mu\text{m}^{-1}$ we have plotted the device spectral response (see Fig. 4). We have verified that the quality factor Q (defined as the ratio between the resonance wavelength and

the resonance width) remains practically constant in the two cases being around 1.55×10^5 . Then the Q -factor of the proposed device can be made rather insensitive to propagation loss within the rings properly optimizing the two key parameters h and κ .

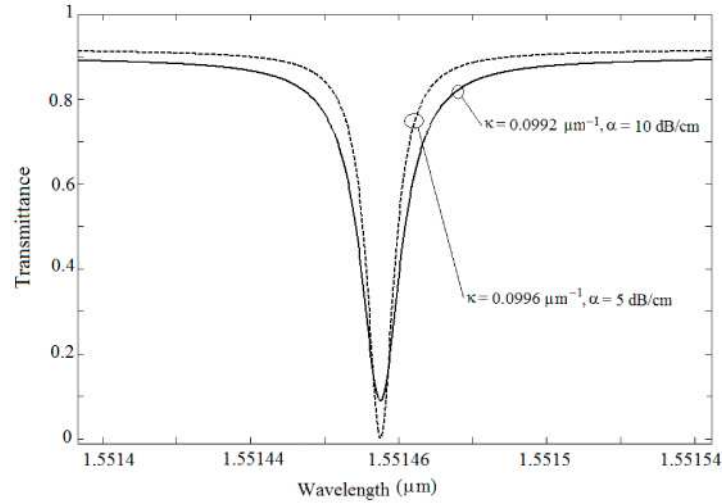


Fig. 4. Spectral response for $h = 0.7$ and α and κ values maximizing the resonance depth.

By analyzing the results obtained for weak ($h = 0.1$) and strong coupling ($h = 0.7$) between the straight waveguide and bottom ring, we can conclude that the interaction between symmetric and anti-symmetric mode, close to the degenerate mode condition, induces the same effect of the coupling between modes having different polarization [10]. In particular, the existence of two dimensional phase singularities with opposite topological charges (π for the positive phase singularity and $-\pi$ for the negative one) allows the orthogonality condition not to be satisfied.

The spikes that appear in the spectrum for $h = 0.7$ are due to the transition from positive to negative phase singularity, with a global phase shift of 2π . This transition does not exist for $h = 0.1$.

In Fig. 5 the phase response in proximity of the degenerate mode condition is plotted. As illustrated in Fig. 5(a), for $h = 0.1$ at the crossing wavelength $\lambda = 1.55148 \mu\text{m}$ the two topological charges, $-\pi$ (green semi-plane in (λ, κ) domain) and π (blue semi-plane), can be well distinguished also near $\kappa = 0.1 \mu\text{m}^{-1}$. On the contrary, for $h = 0.7$ the topological charges are undefined at the crossing wavelength $1.55148 \mu\text{m}$ in the range of κ $[0.099 \mu\text{m}^{-1}, 0.101 \mu\text{m}^{-1}]$, and this effect leads to the creation of two very deep resonance minima, at the positions marked by the black arrows in Fig. 5(b), due to the sharp phase transitions.

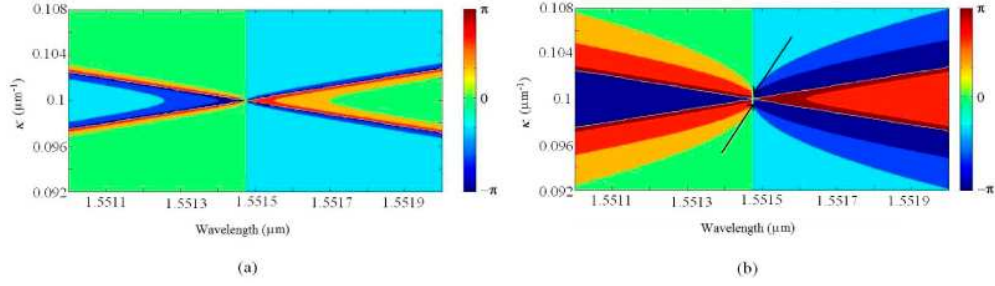


Fig. 5. Resonator phase behavior near the crossing point of two resonance lines having different symmetries. (a) $h = 0.1$. (b) $h = 0.7$.

4. Group velocity manipulation

Launching an optical pulse into the input port of the proposed device, it propagates within the vertically-stacked rings and comes out from the device through port. The envelope of the optical pulse moves at the group velocity v_g , and the pulse distortion depends on the group velocity dispersion (v_g dependence on λ). Group velocity is defined as the ratio between c and the group index n_g ($v_g = c / n_g$). If $n_g < 1$ we have fast pulse propagation being v_g larger than c or negative. On the other hand, if $n_g > 1$ we have $v_g < c$ and subluminal pulses are generated. The derivative of n_g with respect to λ is a measure of pulse distortion and so reduced distortion implies $\partial n_g / \partial \lambda \approx 0$.

Group index of our resonant cavity depends on λ and on the two key parameters h and κ . By properly tuning these last two parameters, v_g can be effectively manipulated, which means that two degree of freedom are available in group velocity tuning.

Previous discussion about spectral properties of the proposed device suggests that cavity behaviour significantly changes whether or not it operates in degenerate mode condition. Therefore we have investigated the resonator group velocity behaviour when $\kappa = \kappa^*$ and $\kappa \neq \kappa^*$.

Discussions about spectral properties of the vertically-stacked rings point out a significant influence of h on device behaviour. Group velocity dependence on h has been explored considering, as in the previous sub-section, the two cases of strong and weak power transfer between the bus waveguide and bottom ring.

To obtain the expression of the group index n_g of the proposed cavity we have derived the resonator phase response Φ . It is given by:

$$\begin{aligned} \Phi &= -\arctan \frac{\text{Im}(E_{thr}/E_{in})}{\text{Re}(E_{thr}/E_{in})} = \\ &= -\arctan \frac{e^{L\alpha} - (\cos \kappa L) e^{\frac{L\alpha}{2}} (1 + \tau)(\cos y + \sin y) + \tau(\cos 2y - \sin 2y)}{e^{L\alpha} - (\cos \kappa L) e^{\frac{L\alpha}{2}} (1 + \tau)(\cos y - \sin y) + \tau(\cos 2y + \sin 2y)} \end{aligned} \quad (11)$$

where $y = \beta L = \omega n_{eff} L / c$, being $\omega = 2\pi c / \lambda$.

Group index can be written as [23]:

$$n_g = \frac{c}{v_g} = \frac{c}{\left(\frac{\partial \beta_{eff}}{\partial \omega} \right)^{-1}} \quad (12)$$

where $\beta_{eff} = \Phi / L$.

The derivative of β_{eff} with respect to ω can be expressed in the form:

$$\frac{\partial \beta_{eff}}{\partial \omega} = \frac{1}{L} \frac{\partial \Phi}{\partial y} \frac{\partial y}{\partial \omega} \quad (13)$$

and so we have that

$$n_g = \frac{c}{\left(\frac{\partial \beta_{eff}}{\partial \omega}\right)^{-1}} = \frac{c}{\left(c/n_{eff}\right)\left(\frac{\partial \Phi}{\partial y}\right)^{-1}} = n_{eff} \frac{\partial \Phi}{\partial y} \quad (14)$$

The derivative of the phase response Φ has been obtained analytically. Also the analytic expression of $\partial n_g / \partial \lambda$ has been derived.

Under degenerate mode condition, i.e. for $\kappa = \kappa^* = 0.1 \mu\text{m}^{-1}$, we have plotted (see Fig. 6) n_g as function of wavelength for $h = 0.1$ and $h = 0.7$. We have assumed the same geometry of the vertically-stacked rings described in previous sub-section.

As shown in Fig. 6, in both cases ($h = 0.1$ and 0.7), it results $n_g < 1$ in the considered wavelength range, which allows to conclude that the achievement of degenerate mode condition induces a superluminal regime. Group velocity is larger than 1 only in a narrow range around the resonance wavelength and so pulse slowing can be predicted in this narrow range around $\lambda = 1.55148 \mu\text{m}$.

Group index dependence on λ is similar for the two considered h values. When $h = 0.1$ larger (in module) negative values of n_g can be achieved with respect to the case of $h = 0.7$.

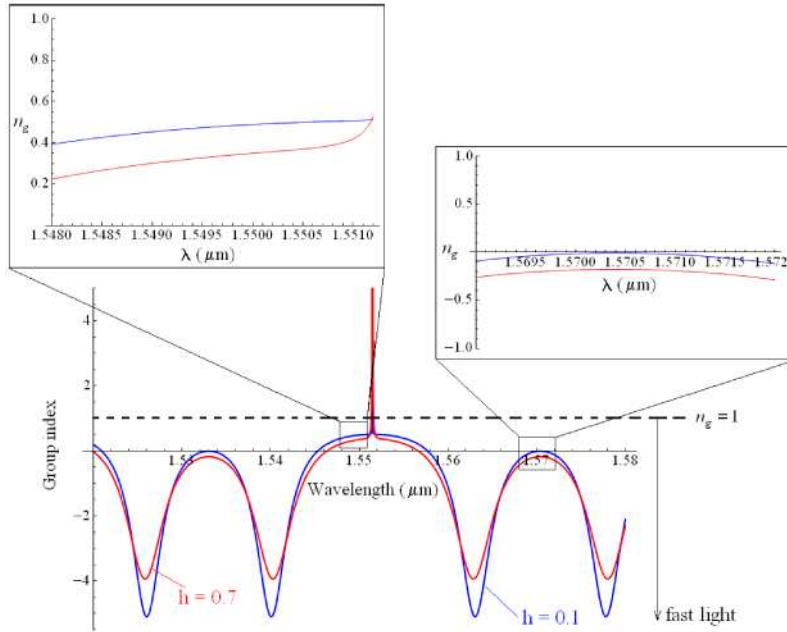


Fig. 6. Group index dependence on λ for $h = 0.1$ and $h = 0.7$ ($\kappa = 0.1 \mu\text{m}^{-1}$).

The dependence of the $\partial n_g / \partial \lambda$ on λ , shown in Fig. 7, exhibits seven λ values in which $\partial n_g / \partial \lambda = 0$ (zero-crossing points). Only two of these zero-crossing points correspond to two quite wide ranges in which n_g dependence on λ is flat. In these two ranges around $1.533 \mu\text{m}$ and $1.57 \mu\text{m}$ we have a very reduced pulse distortion. No other flat section in the plot shown

in Fig. 6 is associated to λ values in which $\partial n_g / \partial \lambda = 0$. In two ranges very close to the resonance wavelength, the derivative $\partial n_g / \partial \lambda$ is about zero. These ranges correspond to two flat sections in the plot showing n_g as a function of λ (Fig. 6). The dependence of $\partial n_g / \partial \lambda$ on λ is similar for the two considered h values. The derivative exhibits larger values for $h = 0.1$. Zero-crossing points are the same in both cases.

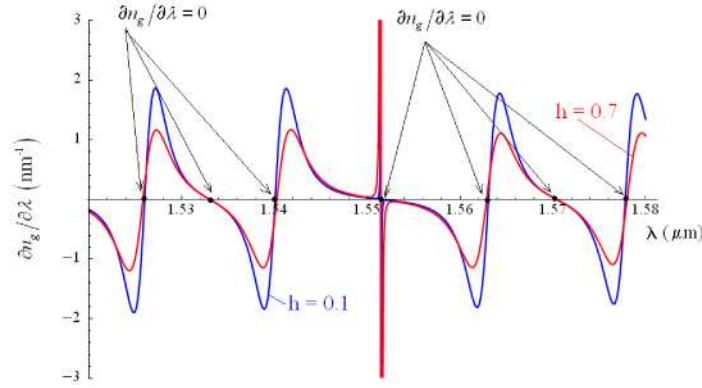


Fig. 7. Dependence on λ of $\partial n_g / \partial \lambda$ for $h = 0.1$ and $h = 0.7$ ($\kappa = 0.1 \mu\text{m}^{-1}$).

In the 3 nm-wide wavelength range between 1.569 μm and 1.572 μm we have observed a very flat n_g dependence on λ . In this range, $\partial n_g / \partial \lambda \approx 0$, and the group velocity is around $-20c$ and $-4.5c$, for $h = 0.1$ and 0.7 , respectively. Moreover, in this range $T(\lambda) = 1$ and the amplitude of pulse spectral components located in this range are not attenuated. Then, we expect that an optical pulse having its spectrum in the range 1.569 μm - 1.572 μm , propagates with a speed (in module) significantly exceeding c , suffering from rather low attenuation and distortion.

In the wavelength range between 1.548 μm and 1.5512 μm we have found that the group velocity is about equal to $2.5c$ and $5c$, for $h = 0.1$ and 0.7 , respectively. A very reduced distortion can be predicted also in this wavelength range, especially for $h = 0.1$.

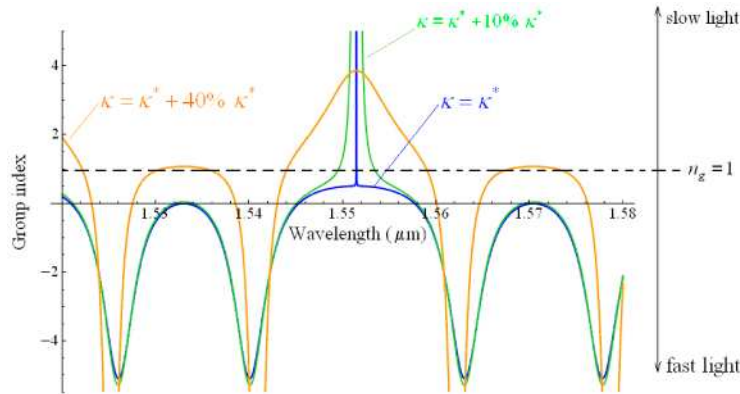


Fig. 8. Group index dependence on λ for three values of κ and for $h = 0.1$.

When the degenerate mode condition ($\kappa = \kappa^*$) is perturbed, the wavelength range around 1.55 μm , in which $n_g > 1$ and $v_g < c$, significantly increases. This means that, when the degenerate mode condition is not satisfied, there are some wavelength ranges in which the cavity acts as slow light device. Fixing a wavelength value, v_g can be tuned in a wide range by changing κ . For $h = 0.1$, we have plotted n_g vs λ for $\kappa = \kappa^*$, $\kappa = \kappa^* + 10\% \kappa^*$ and $\kappa = \kappa^* + 40\% \kappa^*$.

κ^* (see Fig. 8). The wavelength range around 1.55 μm where $n_g > 1$ ($v_g < c$) is 15 nm large for $\kappa = \kappa^* + 40\% \kappa^*$. In this wide range light is slowed. At $\lambda = 1.55 \mu\text{m}$, v_g varies from $2c$ to $0.3c$ when κ passes from $0.1 \mu\text{m}^{-1}$ ($= \kappa^*$) to $0.14 \mu\text{m}^{-1}$ ($= \kappa^* + 40\% \kappa^*$). Moreover, we have verified that, at $\lambda = 1.5516 \mu\text{m}$, n_g can be tuned from 0.5 to 1.5 by shifting κ of $10^{-3} \mu\text{m}^{-1}$. This κ change can be achieved by thermo-optic effect with an uniform increase of the structure temperature of 19° C.

Group index dependence on the coupling coefficient κ between the rings has been investigated for the degenerate mode wavelength ($\lambda = 1.55148 \mu\text{m}$) and three other wavelength values ($\lambda = 1.55 \mu\text{m}$, 1.551 μm , 1.552 μm) close to it. Coupling coefficient has been varied from $\kappa = \kappa^* = 0.1 \mu\text{m}^{-1}$ to $\kappa = 2 \kappa^* = 0.2 \mu\text{m}^{-1}$. Figure 9 shows the possibility to manipulate n_g and the group velocity by finely tuning the coupling coefficient between the vertically-stacked rings.

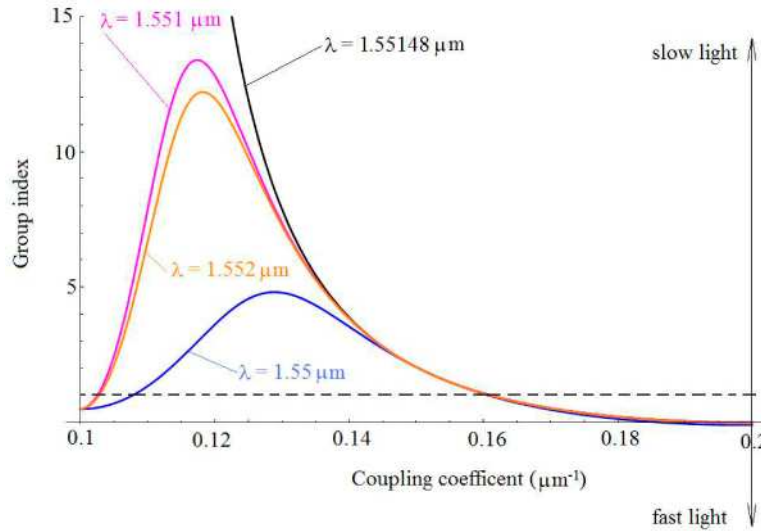


Fig. 9. n_g versus κ for different values of λ ($= 1.55148 \mu\text{m}$, 1.55 μm , 1.551 μm , 1.552 μm).

For $\lambda = 1.55148 \mu\text{m}$, very large values of n_g can be achieved by using κ values very near to $0.1 \mu\text{m}^{-1}$. In the considered κ range, group index monotonically decreases by increasing κ and the condition $n_g = 1$ is achieved for $\kappa = 0.16 \mu\text{m}^{-1}$. For $\kappa > 0.16 \mu\text{m}^{-1}$, fast light condition is fulfilled.

For $\lambda = 1.551 \mu\text{m}$ and 1.552 μm , the slow light condition ($n_g > 1$) is verified for $0.102 \mu\text{m}^{-1} < \kappa < 0.16 \mu\text{m}^{-1}$ and the maximum n_g value (around 13) is achieved for $\kappa = 0.118 \mu\text{m}^{-1}$. The derivative of n_g with respect to κ is about 1 mm when κ is in the range from $0.105 \mu\text{m}^{-1}$ to $0.115 \mu\text{m}^{-1}$. This means that a small change of κ in this range induces a large group index shift. For example, by thermo-optically tuning κ of $0.005 \mu\text{m}^{-1}$, a n_g variation of 5 can be induced.

Finally, for $\lambda = 1.55 \mu\text{m}$ the maximum value of n_g is about 5 and the maximum value of $\partial n_g / \partial \kappa$ is equal to 0.2 mm. In this case, slow light condition is achieved for κ ranging from $0.11 \mu\text{m}^{-1}$ to $0.16 \mu\text{m}^{-1}$.

5. Conclusions

A very compact silicon-on-insulator integrated photonic resonator including two vertically-stacked micro-rings has been theoretically investigated by a very general model suitable for the investigation of resonant devices based on vertically-stacked rings made using any optical material. Cavity spectral response has been in deep discussed showing its dependence on the

two key parameters of the structure, the coupling coefficient κ between the rings and the power transfer h^2 between the bus waveguide and the bottom ring. The developed theoretical model clearly points out the basic features of the investigated structures which simultaneously behaves as directional coupler and ring resonator. This very interesting feature of vertically-stacked rings is valid in general, independently of the specific technology exploited for the device fabrication.

When strong power transfer between the bottom ring and the bus waveguide occurs, it is possible to optimize the κ value so that the resonator quality factor is practically independent of propagation loss. For a resonance depth exceeding 10 dB, optimized Q -factor of the structure is around 155,000. One can expect that, in weak power transfer condition, a loss decrease induces a further Q -factor increase.

The possibility of fast light generation in the proposed structure has been demonstrated. For a specific set of parameters, the cavity exhibits wide wavelength ranges in which the group velocity is larger than c . Around $\lambda = 1.55 \mu\text{m}$ group velocity can be effectively tuned from $0.5 c$ to $1.5 c$, so passing from slow to fast light operating regime, only by a κ variation of 1%. Values of the group velocity derivative with respect to κ of the order of millimetres has been theoretically predicted. The κ shift required for group velocity manipulation can be induced, as an example, by thermo-optic effect.

Although calculations have been made by considering waveguiding rings embedded in silica material system, results we have obtained keep valid also if at least one of the rings can move vertically. Since, as proved in [29], the distance d between the rings can be shifted by an optical control signal, all-optical manipulation of group velocity can be performed by the proposed resonating device. As an example, a κ shift of $0.04 \mu\text{m}^{-1}$ ($= 40\% \kappa^*$) can be induced by a change of 44 nm in the distance d between the rings.

Conclusions about the attractiveness of the proposed structure for group velocity manipulation do not depend on the specific technology considered for device fabrication, i.e. Silicon-on-Insulator, but are related to the potentiality of the resonant photonic structure including vertically-staked rings.