Stochastic resonance of a domain wall in a stripe with two pinning sites

Eduardo Martinez,[1,a)] Giovanni Finocchio,[2] and Mario Carpentieri[3]
[1]*Universidad de Salamanca, Plaza de los Caidos s/n, E-37008 Salamanca, Spain*
[2]*University of Messina, Salita Sperone 31, I-98166 Messina, Italy*
[3]*University of Calabria, Via P. Bucci 42C, I-87036 Arcavacata di Rende, Italy*

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We report on the observation of stochastic resonance phenomenon of a single domain wall in a ferromagnetic stripe with two pinning sites. Under a weak oscillating field, the wall performs irregular transitions between both constrictions in the presence of thermal fluctuations. Ours results indicate that synchronized wall transitions with the driving field can be achieved at the optimal level of noise. The stochastic resonance is quantified by computing the output signal power as a function of temperature. The analysis points out that this system could be used to design well-controlled amplification devices, which could find application as nanodetectors. © *2011 American Institute of Physics*. [doi:10.1063/1.3556314]

The appearance of noise in dynamical systems has been usually considered a nuisance, for example, to design electronic circuits and communication systems, wherein the noise has to be minimized in order to optimize their functionality. However, its presence in certain nonlinear systems can in fact enhance the detection of weak periodic signals, via a mechanism known as stochastic resonance (SR). The phenomenon of SR consists of a nonlinear cooperative effect between periodic and random fluctuation signals, wherein the response of the system modulated by a weak external periodic signal is enhanced at an optimized nonzero noise level. SR was initially introduced by Benzi and co-workers[1] to describe how small periodic perturbations due to earth's wobble could lead to large-scale climatic changes. It took two more years before the first experimental observation of the SR by Fauve and Heslot[2] in a noise-driven electronic circuit known as a Schmitt trigger, and since then SR has become one of the most widespread topics in many different branches of science (see Refs. 3–6 for extended reviews). In the framework of nanomagnetism, the first experimental evidence of the thermally activated switching of a single-domain ferromagnetic particle over a single-energy barrier was addressed by Wernsdorfer *et al.*[7] Although the SR was not analyzed there, their measurements of the telegraph noise suggested that these systems can also be candidates for observing the phenomenon of SR under oscillating fields. More recently, Cheng and co-workers[8] observed the appearance of SR in a spin valve under the combined action between oscillating spin torque and thermal noise.

Although nowadays it is known that SR is even more general than the bistable scenario implies, SR is usually illustrated by considering the overdamped motion of a particle in a bistable potential.[5,6] If the system was noise-free, the particle would relax within the potential well where it was initially launched. However, if the system is coupled to a heat bath, the particle experiences random kicks, so that it can eventually hop over the potential barrier and, therefore, undergoes irregular noise-assisted transitions to the neighboring well with a thermal activation rate which depends on the ratio of barrier height to noise level. If a weak periodic force is applied, the double-well potential is modulated up and down, periodically raising and lowering the potential barrier between both minima. Here, weak means that the modulation is small enough to not excite the particle over the barrier in the absence of noise. However, for some optimal finite amount of noise, the weak force entrains the noise-induced hopping, so the transitions between wells are surprisingly regular. This cooperative effect occurs when the thermal activation rate matches approximately half signal period.[5,6] From a technological point of view, a problem concerns detecting weak signals in a noisy environment. If the system and signal are hidden from view, and information can be only gained by observing the system's output, the phenomenon of SR can be used to enhance the detection output signal.

In this letter, we describe the stochastic resonance in the presence of thermal fluctuations, wherein a single domain wall (DW) is driven by a weak periodic external field along a Permalloy[9] stripe with $L_y \times L_z = 60 \times 3$ nm^2 cross section. A computational region of $L_x = 1.2$ μm in length was discretized by means of a finite-difference scheme using cubic computational cells of $\Delta x = 3$ nm in side. Figures 1(a) and 1(b) depict the two possible equilibrium states of a head-to-head transverse DW at rest in a stripe with two constrictions, each one consisting of two rectangular notches (15 nm long and 6 nm wide) at both edges of the stripe. The spatial dependence of the pinning potential $V_{\mathrm{pin}}(X)$ was computed by

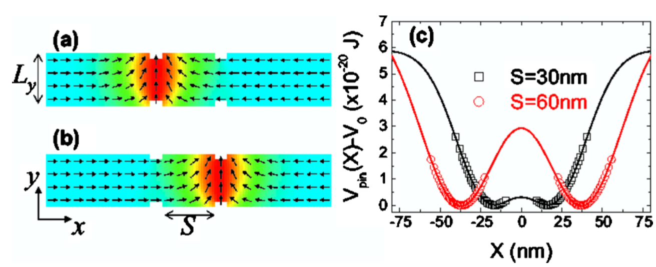

FIG. 1. (Color online) Micromagnetic configurations of a DW pinned at two pinning sites: (a) at the left and (b) at the right. (c) Spatial dependence of the pinning potential $V_{\mathrm{pin}}(X)$ produced by the constrictions for two separations: $S=30$ nm and $S=60$ nm. Dots corresponds to micromagnetic results, and lines are the fits to Eq. (1) (see Ref. 11 for details).

[a)]Author to whom correspondence should be addressed. Electronic mail: edumartinez@usal.es.

means of quasistatic micromagnetic simulations as described elsewhere.[10] It is worth to note that the energy barrier between two pinning sites does not only depend on the individual constriction dimensions, but also depends on the separation S between them. This is evidenced in Fig. 1(c), which shows that the micromagnetically computed pinning potential profiles $V_{\text{pin}}(X)$ [see dots in Fig. 1(c)] are satisfactorily fitted to the following function:

$$V_{\text{pin}}(X) = V_0 - V_d\left\{\exp\left[-\frac{(X+x_L)^2}{L^2}\right] + \exp\left[-\frac{(X+x_R)^2}{L^2}\right]\right\}, \quad (1)$$

where x_L and x_R are the centers of each pinning site, and the constants V_0, V_d, and L determine the curvature of individual pinning sites and the different energy barriers of the system.[11]

Once addressed how the energy barrier between the two wells can be controlled by manipulating the dimensions of the constrictions and the separation between them, we focus our attention on the analysis of the DW dynamics between two constrictions separated by $S=30$ nm driven by a sinusoidal external field along the x-axis $[\vec{B}_{\text{ext}}(t) = B_e \sin(2\pi f_H t)\vec{u}_x]$. In order to do it, a linearized one-dimensional model (1DM) considering the DW as a rigid object is adopted. The DW dynamics is determined by the Langevin equation[10]

$$(1+\alpha^2)m_w\frac{d^2X}{dt^2} = F_{\text{fric}} + F_{\text{pin}}(X) + F_{\text{ext}}(t) + F_{th}(t), \quad (2)$$

where α is the damping parameter and $X(t)$ is the DW position. The DW mass is given by $m_w = 2\mu_0 M_s L_y L_z/\gamma_0^2 H_K \Delta_0$, where γ_0 is the gyromagnetic ratio, μ_0 is the magnetic permeability of the vacuum, H_K is the shape anisotropy demagnetizing field, and Δ_0 is the DW width. The terms at the right hand side are the different forces on the DW. $F_{\text{fric}} = -\Gamma dX/dt$ is the friction viscous force, where $\Gamma = \alpha m_w \omega_d\{1 + (1/\omega_d^2)[\partial V_{\text{pin}}^2(X)/\partial X^2]\}$ and $\omega_d = \gamma_0 H_K$ represents the angular frequency of magnetization oscillations around H_K. The next force $F_{\text{pin}}(X) = -\partial V_{\text{pin}}(X)/\partial X$ is the local pinning force as derived from the pinning potential $V_{\text{pin}}(X)$ of Eq. (1). F_{ext} represents the driving force, which for a time varying magnetic field is given by $F_{\text{ext}}(t) = m_w \gamma_0 \Delta_0[\omega_d H_{\text{ext}}(t) + \alpha(\partial H_{\text{ext}}/\partial t)]$. Finally, thermal fluctuations are included in the formalism by means of a random thermal force $F_{th}(t)$ which is assumed to be a Gaussian-distributed stochastic white-noise process with zero mean value $\langle F_{th}(t)\rangle = 0$ and is uncorrelated in time $\langle F_{th}(t)F_{th}(t')\rangle = 2[\alpha\omega_d m_w K_B T/(1+\alpha^2)]\delta(t-t')$.[12,13]

The inputs $[\Delta_0, H_K$, and $V_{\text{pin}}(X)]$ for the 1DM (Ref. 14) were obtained from full micromagnetic simulations as it was described in detail in Ref. 10, where it was verified that 1DM results are not only qualitatively similar to the ones obtained from full micromagnetic simulations, but also they are in good quantitative agreement. It is worthy to note that DW dynamics in this system cannot be described by an overdamped equation because of the low damping of the system ($\alpha=0.02$). This fact, along with the spatial dependence of the viscous coefficient $\Gamma=\Gamma(X)$, does not allow us to adopt the analytical description of typical overdamped approach.[5,6] Therefore, Eq. (2) is numerically solved by means of a fourth-order Runge–Kutta scheme starting from the initial state depicted in Fig. 1(a). Figure 2 shows typical single-realization under a sinusoidal field with amplitude $B_e = 0.5$ mT and frequency $f_H=5$ MHz, and under three different temperatures T. Note that B_e is significantly smaller than the static value required to promote the DW jump to the right state (≈ 2 mT). It is also one order of magnitude smaller than the Walker breakdown field ($B_W=6$ mT), so that the assumed rigid approximation captures the essential physics of the DW dynamics.

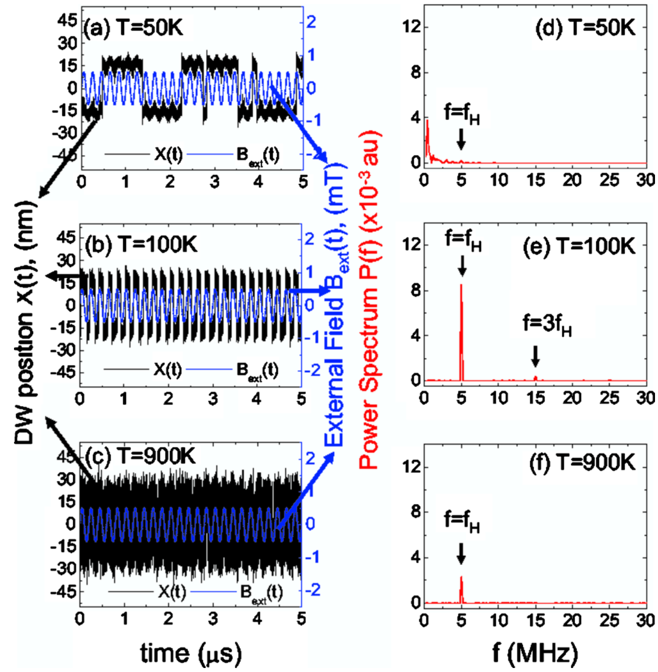

FIG. 2. (Color online) Linearized 1DM results of the DW dynamics driven by a periodic magnetic field $B_{\text{ext}} = B_e \sin(2\pi f_H t)$ with $B_e=0.5$ mT and $f_H=5$ MHz for three different temperatures: from top to bottom $T=50$ K, $T=100$ K, and $T=900$ K, respectively. Graphs at the left side depict the temporal evolution of the DW position $X(t)$, whereas graphs at right side show the power spectrum $P(f)$ as a function of the frequency f as computed from Fourier transform of $X(t)$.

At low temperature [see Fig. 2(a), for $T=50$ K], the average residence time in the two states is much longer than the period of the driving field ($T_H = 1/f_H = 0.2$ μs). Consequently, the DW transitions between both pinning sites occur at unpredictable times. However, if the temperature is increased to $T=100$ K we observe periodic transitions [Fig. 2(b)] where the DW jumps from the left state [x_L, Fig. 1(a)] to the right one [x_R, Fig. 1(b)] and back again once per modulation period. Transitions are most likely at those instants when the corresponding transition rates are maximized and the average residence times in both metastable positions equal half the modulation period. Upon further increasing the noise strength [see Fig. 2(c) for $T=900$ K], too many transitions are thermally activated during 1 cycle of $\vec{B}_{\text{ext}}(t)$, and the cooperative between the DW position $X(t)$ and the noise is lost again. This is the SR effect: The system's response is most regular at a given finite temperature.

For a more quantitative analysis of the synchronization effect visualized in Fig. 2(b), we need a measure of the SR. The most common approach[5,6] is via the power spectrum $P(f)$ of the output variable of the system, the DW position $X(t)$. Right graphs in Fig. 2 display $P(f)$ corresponding to the single-realization depicted at the left side. At low tempera-

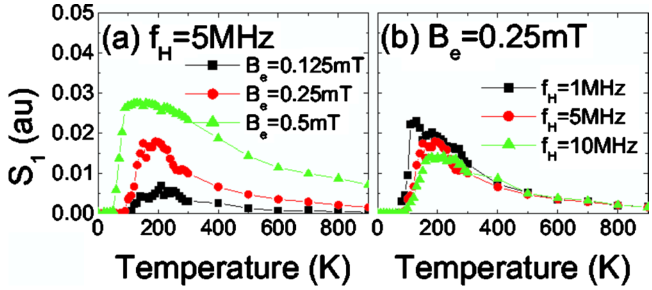

FIG. 3. (Color online) Output signal power (S_1) as a function of temperature T. (a) The frequency of the field remains fixed (f_H=5 MHz), and the amplitude B_e is varied. (b) The amplitude of the driving field remains fixed (B_e=0.25 mT), and the frequency f_H is varied.

tures [see an example in Fig. 2(d) for T=50 K], the main peak of $P(f)$ is observed at a smaller frequency than the driving one (f_H=5 MHz), an evident sign of the absence of synchronization of the DW position $X(t)$ with the driving field $B_{\mathrm{ext}}(t)$. As the synchronization appears [see Fig. 2(e) for T=100 K], the power spectrum depicts a main peak at the input driving frequency ($f=f_H$) and a secondary peak at $f=3f_H$. The generation of only odd higher harmonics of the input frequency in Fig. 2(e) is a typical fingerprint of periodically driven symmetric nonlinear systems.[15] Optimal synchronization is reached when the mean residence time, defined as the interval between two subsequent jumps, matches half the period of the driving frequency.[5] The height of both peaks in the power spectrum starts to decrease as temperature is further increased from the optimal value. The lost of synchronization becomes evident in Fig. 2(f) corresponding to T=900 K, where the height of the main peak has decreased with respect to the optimal case, and the secondary peak has completely disappeared.

Having elucidated the main physical issues of SR in our DW system, we characterize and quantify its temperature (T) dependence. There exist several ways to do it in a bistable system, and the most proper one depends on the experimental accessible observable.[5,6] In the present letter, we adopt the seminal criterion by Benzi *et al.*,[1] where SR was simply quantified by the so-called output signal power S_1, which represents the weight intensity of the peak (S_1) at the frequency of the field, f_H, in the power spectrum. It is defined as the area under the main signal peak at $f=f_H$, and it is independent of the time t_{max}.[6] S_1 reaches a maximum when the DW jumps occur synchronized with $\vec{B}_{\mathrm{ext}}(t)$, and therefore it can be regarded as a quantitative measure of SR. The results are shown in both Figs. 3(a) and 3(b), where the dependence of S_1 on T is depicted for a fixed frequency (f_H =5 MHz) and three different amplitudes, and for a given amplitude (B_e=0.25 mT) and different frequencies, respectively.

Upon decreasing B_e, the position of the maximum of S_1 (optimal synchronization) moves toward smaller temperatures [see Fig. 3(a)] because by increasing B_e the energy barrier between both pinning sites is reduced. For a given temperature, S_1 increases substantially with B_e. On the other hand, if the amplitude remains fixed as in Fig. 3(b) for B_e =0.25 mT, the optimal synchronization (maximum of S_1) occurs at larger temperatures as the frequency of the driving field is increased in the analyzed range (f_H from 1 to 10 MHz).

In summary, we have found that thermal activation can be used to synchronize the DW dynamics between two adjacent pinning sites driving by a weak amplitude driving field and therefore resulting in a SR effect. The system represents a ready candidate to theoretically study and characterize such SR effect in the underdamped limit, because the DW behaves as a rigid object with well-defined mass and position, and more importantly the energy landscape and the energy barrier between pinning sites can be controlled by manipulating both the constriction dimensions and their separation. From a technological point of view, DWs have been proposed to develop memory and logic devices. Our study also predicts the possibility of taking advantage of SR effect to use the DW system as a low frequency detector: for instance, placing two electrodes at both sides of a pinning site in order to measure the anisotropic magnetoresistance (AMR). If the DW is (is not) inside the constriction between electrodes, the AMR signal has a high (low) value. For a given temperature, the DW dynamics between both pinning sites is synchronized with respect to the weak input signal (driving field in this study, but similar results can be obtained by driving current), and therefore a high amplitude periodic AMR signal could be detected. It has to be mentioned that it is also possible to fix the temperature and obtain synchronization if the driving frequency is tuned against the escape rate between both pinning sites. As said, the system presented here can be also used as a frequency nanodetector, which can find application in a wide range of disciplines. We hope that the present work will motivate experimental progress in the direction of characterizing the stochastic resonance effect of a single DW in a ferromagnetic stripe with two pinning sites.

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