Evaluation of the most suitable mode of transport under Uncertainty. Dempster Shafer Theory applied to Analytical Hierarchy Process and Transformable Belief Model

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Valutazione del più adatto modo di trasporto in condizioni di incertezza. La teroria di Dempster Shafer applicata all' Anlaytical Hierarchy Process e al Transformable Belief Model

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Department of Civil, Environmental, Territory, Building Engineering and of Chemistry
RISK AND ENVIRONMENTAL, TERRITORIAL AND BUILDING DEVELOPMENT
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Final Dissertation

EVALUATION OF THE MOST SUITABLE MODE OF TRANSPORT UNDER UNCERTAINTY. DEMPSTER SHAFER THEORY APPLIED TO ANALYTICAL HIERARCHY PROCESS AND TRANSFORMABLE BELIEF MODEL

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Extended abstract (Eng)

The main goal of this work is to formulate a mathematical model of a decision support system for choosing the most suitable mode of transport to implement within a metropolitan context, considering uncertainty and ambiguity embedded in transportation problems. The model should consider not only economic parameters such as transport costs, production costs, and transport demand but, above all, environmental and social parameters, which define the quality of the transport service.

The study applies a hybrid approach based on two different theories: the Analytical Hierarchy Process (AHP) (Saaty, 1980) and the Evidence- or Dempster-Shafer-Theory (DST) (Dempster 1967; 1968; Shafer, 1976). This is the first time that this methodology is used in the field of local public transport. AHP is used to structure the transportation problem and to define the criteria and alternatives. We used the AHP along with the mathematical Theory of Evidence, called also Dempster-Shafer Theory (DST), to evaluate the users' uncertainty in judgments, and to fuse data coming from diverse sources. Finally, we used the Transformable Belief Model (Smets, 1999) to quantify the probability embedded in belief functions.

In the first part of this study, we have carried out a survey to investigate the users' point of view about the quality of transport, expressed through fifteen criteria, representative of the transport quality. The criteria were chosen according to the set proposed by Prioni and Hensher (2000), to the criteria included in the Transportation Research Board Handbook (1999) and to the European Standard EN13816. In particular, the chosen criteria are: Accessibility; Security; Cleanliness; Number of offered seats; Crowding; Frequency; Travel

Time; Punctuality; Regularity of trips; Flexibility; Information; Single ticket(one ticket for all modes of transport, like a transport card); Intermodality; Pollution; Travel fares. In order to obtain more efficient results, we have divided these fifteen criteria into three macro categories. Each categories is composed of five criteria, and that are: Service Criteria, Time Criteria and External Criteria. Within the proposed survey, shared on the web, the users were asked to rank the chosen criteria. To investigate the transport company's point of view, we have extrapolated data, about costs and demand for transport, from financial statements. Among the transport companies that work into the metropolitan city of Bari, we have chosen the Ferrovie Appulo Lucane, which uses both Railway and Roadway transport. Starting from the data about costs and demand of transport, we have obtained the Average Cost curves for Railways and Road Transport. In this way, we have calculated the break-even point between the two curves.

Due to the complexity of the transport problem, we have used the Analytical Hierarchy Process to decompose the problem in different levels. Because of the decomposition, we have obtained priority vectors, both for the pairwise comparison matrix of criteria and for the knowledge matrix, which we have taken as basic probability assignments (bpa) for application of the Dempster-Shafer Theory (DST). The DST is used to fuse different users' opinions, and to fuse users and company points of view, using the Dempster Rule of Combination. Moreover, the DST is used to take into account Uncertainty embedded in human judgment, thanks to the Belief and Plausibility measures, which are respectively the lower and upper bound of likelihood. Finally, in order to obtain the probability measures, we have used the Pigistic Trasnformation by Smets. The results show which alternatives users and transport company consider the best in relation to analyzed criteria.

Keywords: Local Public Transport; Uncertainty; Modal Choice; DS/AHP Method

Extended abstract (ita)

Lo scopo principale del presente lavoro di tesi è quello di formulare un modello matematico di supporto alle decisioni, al fine di individuare il più il più adatto modo di trasporto da implementare all'interno di una città metropolitana. Il modello deve essere strutturato in modo da considerare l'incertezza e l'ambiguità presenti nella pianificazione dei trasporti. Il modello proposto considerare non solo i parametri economici, come i costi di trasporto, i costi di produzione e la domanda di trasporto, ma soprattutto i parametri ambientali e sociali, che definiscono la qualità del servizio di trasporto. A tal fine, lo studio prevede l'applicazione di un modello ibrido, basato su due differenti teorie matematiche: l'Analytical Hierarchy Process (AHP) (Saaty, 1980) e la Dempster Shafer Theory (DST), chiamata anche Teoria dell'Evidenza (Dempster 1967; 1968; Shafer, 1976). Per la prima volta questa metodologia viene applicata nel campo dei trasporti, ed in particolare nel campo della pianificazione del Trasporto pubblico Locale. L'AHP viene utilizzata per strutturare il problema in steps gerarchici e per definire i criteri e le alternative da considerare. Abbiamo utilizzato l'AHP insieme alla teoria matematica dell'evidenza, chiamata Dempster-Shafer Theory (DST), per valutare l'incertezza degli utenti nei giudizi e per fondere i dati provenienti da fonti diverse. Infine, abbiamo usato il Transformable Belief Model (Smets, 1999) per quantificare la probabilità incorporata nelle funzioni di credenza. In primo luogo, abbiamo effettuato un sondaggio, per indagare il punto di vista degli utenti sulla la qualità del trasporto, espressa attraverso quindici criteri. I criteri sono stati scelti in base all'insieme proposto da Prioni e Hensher (2000), ai criteri contenuti nel manuale del Transport Research Board (1999) e alla norma europea EN13816. In particolare, i criteri scelti sono: Accessibilità; Sicurezza; Pulizia; Numero di posti offerti; Affollamento; Frequenza; Tempo di viaggio; Puntualità; Regolarità dei viaggi; Flessibilità; Informazione; Biglietto singolo (un biglietto per tutti i modi di trasporto, come una carta di trasporto); Intermodalità; Inquinamento; Tariffe di viaggio. Al fine di ottenere risultati più efficienti, abbiamo suddiviso questi quindici criteri in tre macro categorie. Ogni categoria è composta da cinque criteri, e sono suddivise in: Criteri di servizio, Criteri di tempo e Criteri esterni.

All'interno dell'indagine proposta, condivisa sul web, gli utenti sono stati invitati a classificare i criteri scelti. Per esaminare il punto di vista della società di trasporto, abbiamo estrapolato i dati, sui costi e sulla domanda di trasporto, dai bilanci economici. Tra le aziende di trasporto che operano nella città metropolitana di Bari, abbiamo scelto la Ferrovie Appulo Lucane, che utilizza sia il trasporto ferroviario che il trasporto stradale. A partire dai dati relativi ai costi e alla domanda di trasporto, abbiamo ottenuto le curve dei costi medi per le ferrovie e il trasporto stradale. In questo modo abbiamo calcolato il punto di rottura tra le due curve. A causa della complessità del problema del trasporto, abbiamo utilizzato il processo di gerarchia analitica per decomporre il problema in diversi livelli. A seguito della decomposizione gerarchica tramite l'applicazione dell'AHP, abbiamo ottenuto vettori prioritari, sia per la matrice di confronto a coppie di quindici criteri che per la matrice di conoscenza, i quali sono stati considerati come assegnazioni di probabilità di base (bpa) per l'applicazione della teoria Dempster-Shafer (DST). La DST è usata per fondere le opinioni dei diversi utenti e per fondere i punti di vista degli utenti e dell'azienda, utilizzando la regola di combinazione di Dempster. Inoltre, la DST è utilizzata per tener conto dell'incertezza presente nel modo di pensare umano, grazie alle misure di credenza e plausibilità, che sono rispettivamente il limite inferiore e superiore della probabilità. Infine, per ottenere le misure di probabilità, abbiamo utilizzato la "Pignistic Transformation" da Smets. I risultati mostrano quali alternative gli utenti e le imprese di trasporto considerano migliori in relazione ai criteri analizzati.

Parole chiave: Trasporto pubblico locale; incertezza; scelta modale; DS/AHP

1. Introduction

There is an extensive discussion in Italy, especially in small metropolitan cities, such as Bari, about the local public transport. In such cities, people prefer almost always, using private cars for most of the trips, thus increasing the congestion and pollution problems and declining the quality of residents' life (Fig. 1).

The Italian National Research Council (CNR) carried out in 2010 a study, which reports the conditions of public regional transport in Italian Regions. From this CNR research, emerges that 61% Italians prefer the use of the car, both as a driver and as a passenger for any kind of movement. The road public transport, such as urban and suburban buses, is ranked second with a percentage of 9%, followed by motorcycle (5%). The rail transport system (train, tram, and metro) barely arrives at 4%, while the percentage of on-foot movements is about 16%.It is important to note that the percentage of people moving by bicycle increased from 2.8% (ISTAT survey 2001) to 3.3% (ISTAT survey 2011).

Moreover, in the last years, the phenomenon of the Urban Sprawl led to an increase of individual trips, in which the vehicles are mostly private cars. Even types and needs of travels in urban and extra-urban mobility changed over time. For example, in the 1980s most of the trips were found as systematic, home- to- work and home-to-school, but, in the last few years, the breakdown between systematic and nonsystematic movement undergone considerable changes. This breakdown is due to the improvement of living conditions (increase in per capita income), the adoption of new lifestyles and the invention of different work and production processes (e.g. "just in time"). In fact, in the 1990s the share of systematic travels in Italy fell to 38%.

Nowadays, the incidence of systematic movements is estimated about 30-35% (source: XIII ° ACI-Censis Report). The prevalence of non-systematic movements determines a lot of changes in transport demand, difficult to predict.

The transport sector affects the life quality and the environmental impacts. In fact, there is a constant pressure from governments, to reduce the impact of transport on climate change. Therefore, it is desirable to consider, among the useful parameters for the planning of transport systems, the environmental and social aspects, as well as the economics ones. Many of the critical issues that arise today in cities derive from the lack of a territorial policy planning, which should deal with the dynamics of mobility.

Nowadays, a transport planning characterized by rational decision is needed, made through the use of quantitative methods for the selection of actions to be realized, as well as greater sharing of choices between all parties involved in the process, both decision-makers and stakeholders.

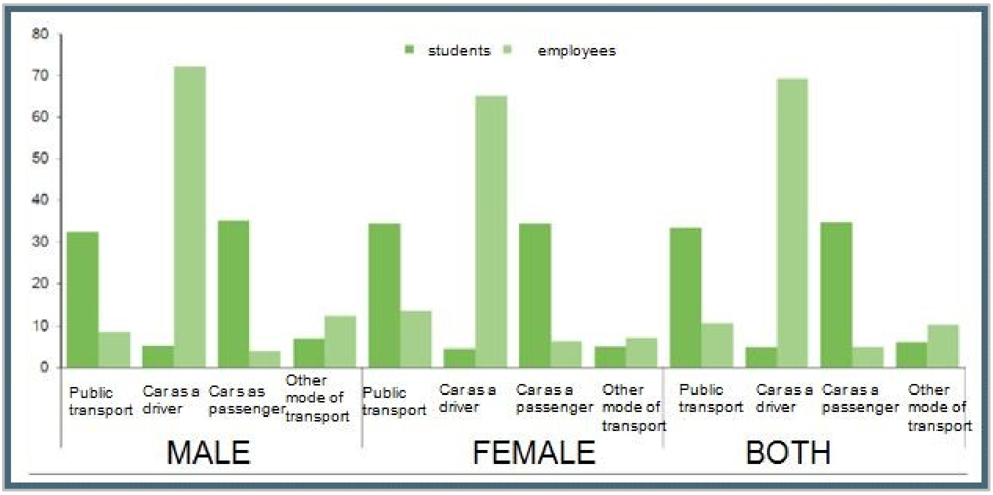

Figure 1 Breakdown between public transport and private transport - ISTAT

1.1 Research scope and objectives

The main goal of this work is to formulate a mathematical model of a decision support system for choosing the most suitable mode of transport to implement within a metropolitan context, considering uncertainty and ambiguity embedded in transportation problems. The model should consider not only economic parameters such as transport costs, production costs, and transport demand but, above all, environmental and social parameters, which define the quality of the transport service.

In this way, transport planning becomes a global activity where industry professionals and technicians make their choices based on real-time data coming from public surveys. In this dissertation, we propose a decision-making method that evaluates transit alternatives in a rational way, accounting for analysts' and users' uncertainty and ambiguity about the system characteristics and performances of alternatives.

The study applies the Analytical Hierarchy Process (AHP) to structure the transportation problem and to define the criteria and alternatives. We used the AHP along with the mathematical theory of Evidence, called also Dempster-Shafer Theory (DST), to evaluate the users' uncertainty in judgments, and to fuse data coming from diverse sources. Finally, we used the Transformable Belief Model to quantify the probability embedded in belief functions.

We apply for the first time the DS/AHP method to transportation problems, being able in this way to represent the uncertainty embedded in the users' judgment related to the quality parameters and the uncertainty embedded in the companies' point of view.

1.2. The Thesis Structure

This work is composed of 6 chapters. The first chapter is the introduction to the research scope and objectives.

The second chapter is focused on the growth of public transport and on uncertainty linked to the choice of the transport mode. In this chapter, we explain also the importance of service quality in transportation planning and the approach to the modal choice in local public transport.

The third chapter reviews the relevant literature. Two main aspects of the research are reviewed: (i) current methods and practices in planning and evaluation of transit systems, and (ii) mathematical theories of uncertainty. In the first section, the characteristics and limitations of different planning and evaluation methods are showed. In the second section, different mathematical treatments of uncertainty, including Probability Theory and Evidence Theory, are presented.

The fourth chapter is about the method we applied, that is the Dempster Shafer Theory combined with the Analytical Hierarchy Process. This chapter describes the main elements and characteristics of the method. It presents also the step-by-step process applied to the proposed methodology, to evaluate transit alternatives in local public transport.

Using the proposed method, the fifth chapter is about the case of study, in which we evaluated the most suitable mode of transport, considering both users' and companies' point of view. In this chapter we present also the data collection, useful in development and validation of the model.

The last chapter summarizes the findings of the model and discuss its benefits and possible applications to other decision problems. This chapter will also present possible recommendation for further studies.

2. Transportation and Uncertainty

The public transportation system is a large-scale system, in which many elements interact with each other. The planning activity of a large-scale system is difficult, because it must satisfy diverse groups of people with a wide range of opinions about benefits, needs, and willingness to pay for its cost (Kronprasert, 2012). The evaluation of a public transport system is characterized by a set of alternatives and objectives, which defines the goal of planning.

Traditional approaches to decision making on transit systems are based on various unrealistic assumption (Ribbons and Timothy, 2007). Examples of these assumptions are: the decision problem is well structured; the evaluation of the objectives is independent; criteria are quantifiable; users belong to a homogeneous group of people. Indeed, these assumptions are not always true; in fact, both Decision Makers and Analysts often do not have complete information about the system and/or alternatives. In traditional decision-making problems, Decision Makers and Analysts uncertainties are studied by the same models; however, the Analyst's uncertainty can be studied efficiently by Probability Theory, while the Decision Maker's uncertainty requires a different approach. In this work, we follow an innovative approach that we will explain in next chapters.

Generally, two major sequential phases compose the transport planning. In the first phase, the identification of project's objectives to address the transportation problem is comprised. In this phase, the collection of data and the travel demand are also studied to develop the transportation model. The second phase, which is the phase that this work deals with, is about the evaluation and decision-making process. At the head of this phase (see fig. 2), there is the definition of the transportation problem, followed by the identification of needs and objectives. After that, a crucial step is the definition of criteria and sets of alternatives. The process to evaluate the transit systems and to reach the best alternative is carried out in three stages: alternatives screening process; alternatives analysis process; project evaluation process for funding recommendations. These processes are labyrinthine, because they deal with both demand and supply characteristics of transit systems and their interactions (Kronprasert, 2012).

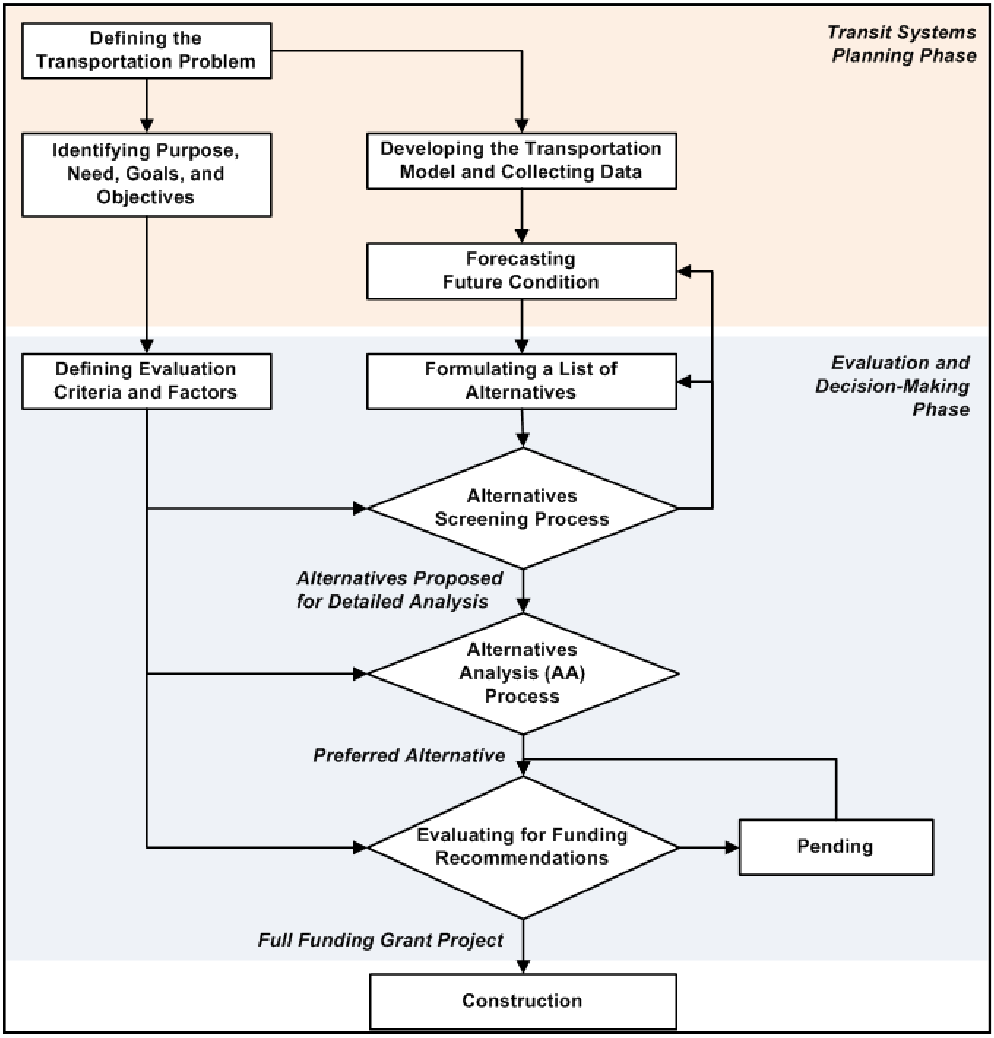

Figure 2 Transit Planning and Project Development Process

(*Adapted from Vuchic, 2005; Sinha and Labi, 2007; FTA 2011,Kronprasert, 2012*).

This study, in which we have analyzed the customer and company's point of view, is focused on evaluation and Decision-Making process, in particular on the choice of the best transport mode. The applied method helps the customers in taking part to the decision problem, expressing their needs and their evaluation on each transport mode. In this way, it is possible to join dissimilar needs and objectives, coming from different sources.

2.1 The public transport

Public transportation systems include a variety of transit options such as buses, light rail, and subways. These systems are available to the public, may require a fare, and run at scheduled times. The purpose of introducing or expanding public transportation is to increase access to and use of public transit while, at the same time, reducing motor vehicle miles driven and traffic congestion.

Public transportation services help ensure that people can reach everyday destinations, such as jobs, schools, healthy food outlets and healthcare facilities, safely and reliably. They play a key role for people who are unable to drive, including those without access to personal vehicles, children, individuals with disabilities, and older adults. Moreover, the use of the public transport reduces the atmospheric pollution and increases the quality of life.

The Public Transport Systems are divided into groups according to specific characteristics, which are:

1. The roadside (Right of way - ROW categories)
   A. Type 1 (ROW category A): the roadside is fully protected, for example the subway (rapid transit)
   B. Type 2 (ROW category B): the roadside is protected, but it can be crossed by pedestrians or cars, for example the tram (semi rapid transit)
   C. Type 3 (ROW category C): the roadside is not separate from the others, for example the buses (street transit)
2. The technologies: it is referred to the mechanical features of the vehicles and travel ways. The most key features are:
   Support, guidance, propulsion, and control
3. The type of service: it includes several classifications as:
   A. by types of routes and trips served;

B. By stopping schedule

C. By time of operation and purpose

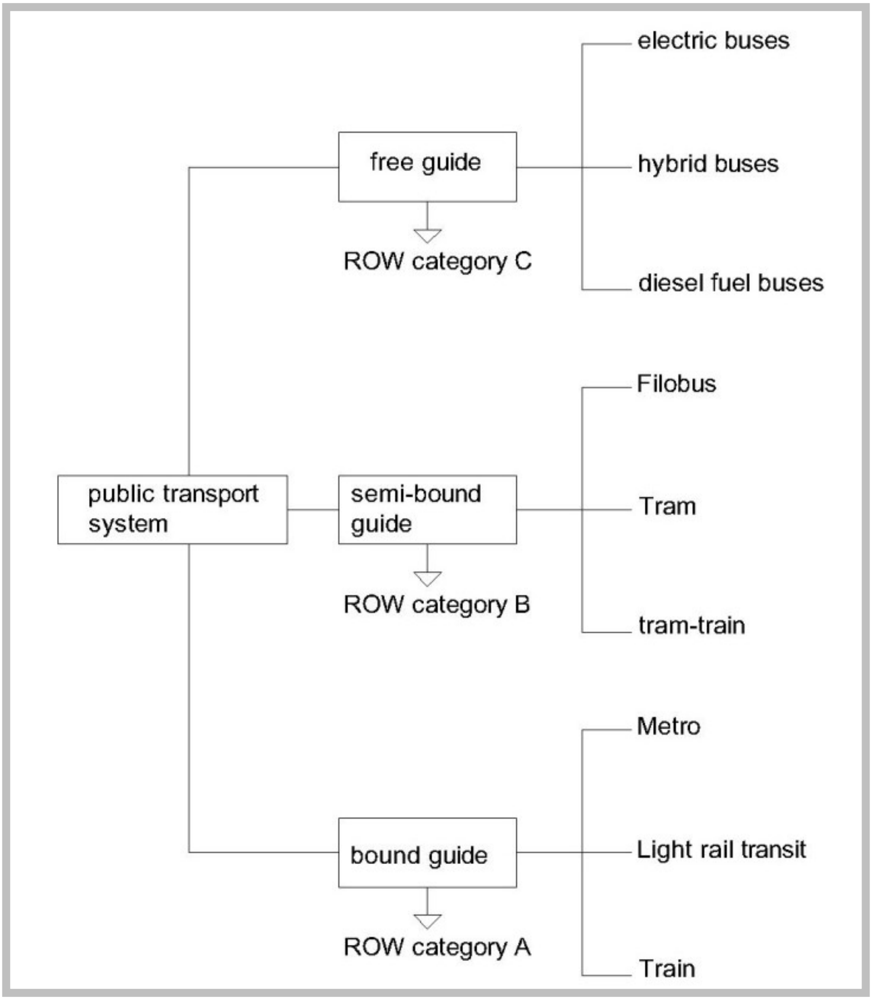

Figure 3 Classification of public transport systems

The most popular used transport in a metropolitan city are buses, tram, and subway.

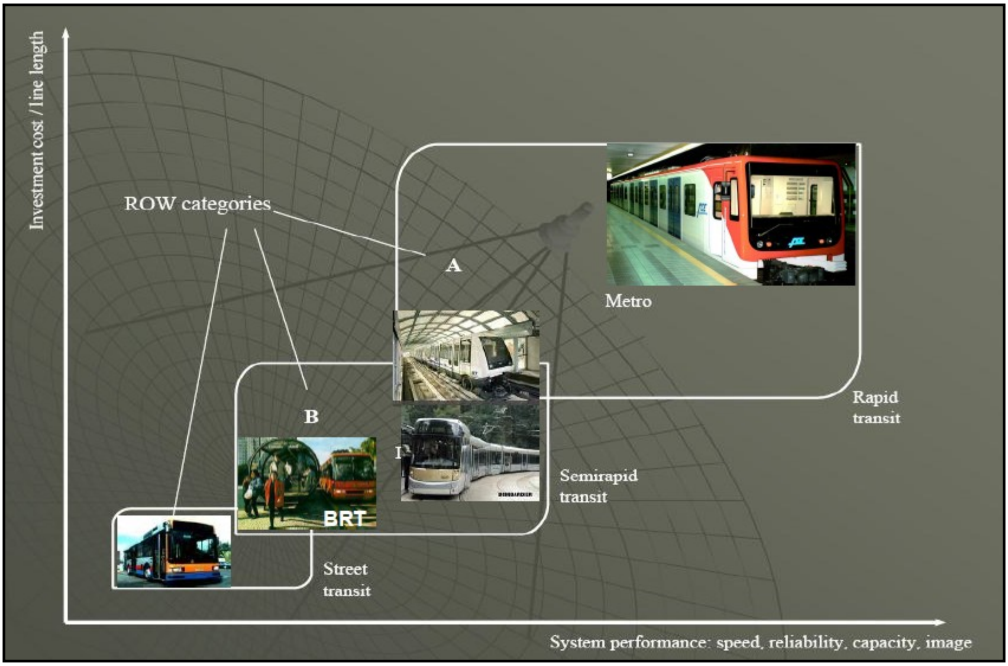

Figure 4 Transit System, Vuchic 2005

The figure 4 underlines that transit services on lightly traveled routes cannot justify large investments. In small cities and suburban areas, buses generally operate the transit services, i.e. the ROW category C. It is safe to switch to ROW category B when the volume of passengers increases. In this case, the logical choice is usually the Light Rail Transit (LRT), because it is more efficient in operations for large passenger volume, it attracts more riders, and its operating costs do not increase linearly with passenger volume (as in the case of buses). (Vuchic, 2005)
In big cities, especially when there is no space for separate road side on the surface and the volume of passengers is more than 20,000 passengers per hour, it is justified the use of ROW category A.
A key parameter, which justifies the use of a transport mode, is the demand, which is the volume of passengers.

The transport system is a complex system, characterized by two interconnected and interacting sub-systems, named as follows:

1. The transport supply, characterized by all physical and organizational elements, interacting among themselves to produce transport opportunities;
2. The transport demand, which is the amount of people and freights that use such opportunities to move from one side to another side of the territory.

The transport demand influences the supply system, because it is the number of people and the amount of freights that intend to move on the territory. In the same way, the supply influences the demand of transport: if it does not fit the needs of users, habitually the demand of public transport decreases, shifting to other modes of transport.

The quality of service perceived by the users is another important aspect that influences the demand and supply dynamics in the transport system. However, the users' set is composed of entities with different socio-economic and behavioral characteristics, characterized by an elevated level of uncertainty in choices. This aspect makes the users' set heterogeneous, thus very difficult to study.

Finally, it results that the choice of the mode of transport depends on a lot of factors, as the transport demand, the road side, the accessibility, the users' needs, and even these factors are related to each other.

2.2Uncertainty in transport planning processes

"There are some things that you know to be true, and others that you know to be false; yet, despite this extensive knowledge that you have, there remain many things whose truth or falsity is not known to you. We say that you are uncertain about them. You are uncertain, to varying degrees, about everything in the future; much of the past is hidden from you; and there is a lot of the present about which you do not have full information. Uncertainty is everywhere and you cannot escape from it."

Dennis Lindley, Understanding Uncertainty (2006)

The analysis of transportation problems is quite difficult because it should deal with uncertainties, related to the human reasoning way. Traditional approaches to Decision Making on public transportation simplify the complexity of the system, for example omitting decision makers' (DM) and users' uncertainty and ambiguity.

There are many facets of uncertainty; usually, Analyst's uncertainty is related to his/her capacity to study the problem and to extrapolate quantitative data, while DM's uncertainty derives from his/her incomplete knowledge of the system and verbal judgments, such as "good" and "bad," "right" and "wrong," and "acceptable" and "not acceptable". Relationships between Analyst and users are often difficult, due to the DMs' verbal expression.

In other words, these uncertainties are associated with the definition of criteria and goals, the lack of knowledge about the system behavior, and the quality of information and data (Kikuchi and Pursula, 1998).

In traditional approaches, the analysts were used to extrapolate from available data the probability of a choice. This kind of process is typical of Probability Measures, which give high-quality results, but represent only <u>empirical uncertainty</u> (Randomness), no other types of uncertainty, like ambiguity and fuzziness (fig. 5).

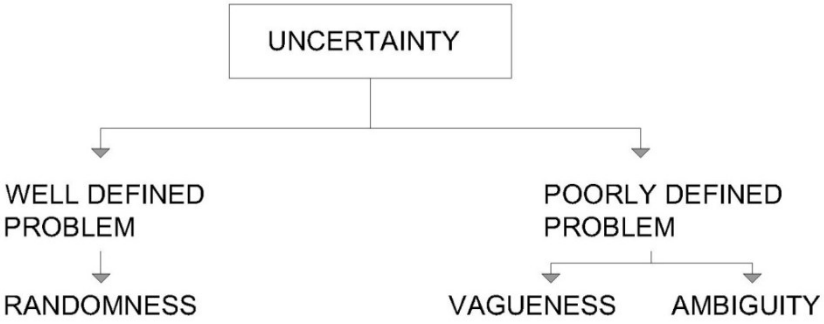

Figure 5 Types of Uncertainty for knowledge representation (Klir and Wierman, 1999)

In the following, we report a brief explanation of the words "Fuzziness" and "Ambiguity" provided by Kikuchi and Pursula (1998).

"Fuzziness" shows an uncertain state in which the transition between the state of concern and its complement is gradual, so it's difficult to make a sharp distinction. This concept was born with the Greek Philosopher Plato, who laid the foundations for the fuzzy logic by proposing a third region between true and false, where the two notions tumbled together. For example, in the proposition "Jane is tall", fuzziness is linked to the adjective tall because its lack of a clear definition. In transportation, this problem is widespread, because sentences like "the bus stop is nearby" or "there is a lot of traffic" or "the quality of service is good," are characterized by no quantitative parameters, dependent on situations, decision makers, location etc...

"Ambiguity" is the uncertainty for which the truth of a proposition, like "the painting was probably painted by Raffaello," is difficult to decide, because the information about the painting is incomplete, due to a lack of evidence, data, and knowledge. Uncertainty related to ambiguity is very popular in evaluation, diagnosis, classification, and judgment. For example, if performances of a transportation plan are known in an approximate way, its evaluation through usual methods can be very difficult because the information about the subject is not clear.

Helton (1997) gave the following dual definition of Uncertainty:

Aleatory Uncertainty- the type of uncertainty that results from the fact that a system can behave in random ways, also known as Stochastic uncertainty, Type A uncertainty, Irreducible uncertainty, Variability, Objective uncertainty.

Probabilistic approaches, such as classical Probability Theory (frequentist) and Bayesian Probability Theory are an effective way to model stochastic uncertainties, like measurement noise, etc. (Ayyub & Klir, 2006).

Epistemic Uncertainty- the type of uncertainty that results from the lack of knowledge about a system and it is a property of the analysts performing the analysis. Also known as Subjective uncertainty, Type B uncertainty, Reducible uncertainty, State of Knowledge uncertainty, Ignorance.

It is well recognized that Aleatory Uncertainty is best dealt with using the frequentist approach associated with traditional probability theory. The Epistemic Uncertainty, instead, needs different mathematical tools to be described. For many years the Bayesian Theory has been used to describe this kind of problems in which an analyst has information on the probability of all events. When this is not available, the uniform distribution function is often used, justified by the Laplace's *Principle of Insufficient Reason* (Savage, 1972).

Afterwards, Possibility theory and Evidence theory emerged as mathematical methods for handling uncertainty, as well as Probability theory. Each one deals with distinct types of evidence; Probability theory deals with mutually exclusive and comprehensive evidence; Possibility theory deals with consonant (or nested) evidence; Evidence theory deals with both consistent and conflicting evidence.

Probability theory is the most established theory that deals with the uncertainty associated with randomness. It presents the degree of support of outcomes through a probability distribution, p where $p: X \rightarrow [0, 1]$ and $\sum p(x) = 1$ $(x \subset X)$. It measures the degree of belief of proposition by probability measure p. Evidence in probability theory must be a singleton (i.e., points to only one outcome.) The probability of the outcome can be defined by three approaches:

(i) uncertainty maximization approach;

(ii) relative frequency-based approach in many trials;

(iii) subjective.

Based on these three approaches, the probability of 0.5 can imply (i) equal (50/100) chance of occurrence (or total ignorance), (ii) observed 500 (out of 1,000) times of occurrence; or (iii) the subjective probability (or belief) of 0.5.

Possibility theory deals with the uncertainty associated with non-specificity. It presents the degree of support of outcomes through the possibility distribution, *r* where $p: X \rightarrow [0, 1]$ and max $\{p(x)\} = 1$ where $x \subset X$. It measures the degree of belief of a proposition by a range between a possibility measure (*Pos*) and necessity measure (*Nec*). *Pos* denotes the degree of belief in an optimistic view and *Nec* denotes the degree of belief in a pessimistic view. Evidence in Possibility theory must be nested. More details of Possibility theory can be found in Zadeh (1978), Dubois and Prade (1988), and Klir (2006).

Evidence theory was pioneered by Dempster in the 1960s and Shafer in the 1970s. It is also known as *Dempster-Shafer theory of evidence* (DST). (Dempster 1967; 1968; Shafer, 1976; Yager, 1994). It has been developed to deal with the uncertainty associated with both nested evidence and singletons.

DST, which is the core mathematical framework in this study, is a generalization of traditional probability theory since it can deal with all patterns of evidence, while the probability theory deals with only singletons. In fact, DST allows assigning degrees of support (basic probability assignments)to one or more sets of outcomes, instead to one of the mutually exclusive outcomes as in probability theory. The Dempster-Shafer theory will be discussed in more details in next chapters.

To face the problems related to uncertainty, the analyst can use three different mathematical frameworks:

1- PROBABILITY → Expected Utility Theory
2- POSSIBILITY → Fuzzy set Theory
3- EVIDENCE → Dempster-Shafer Theory

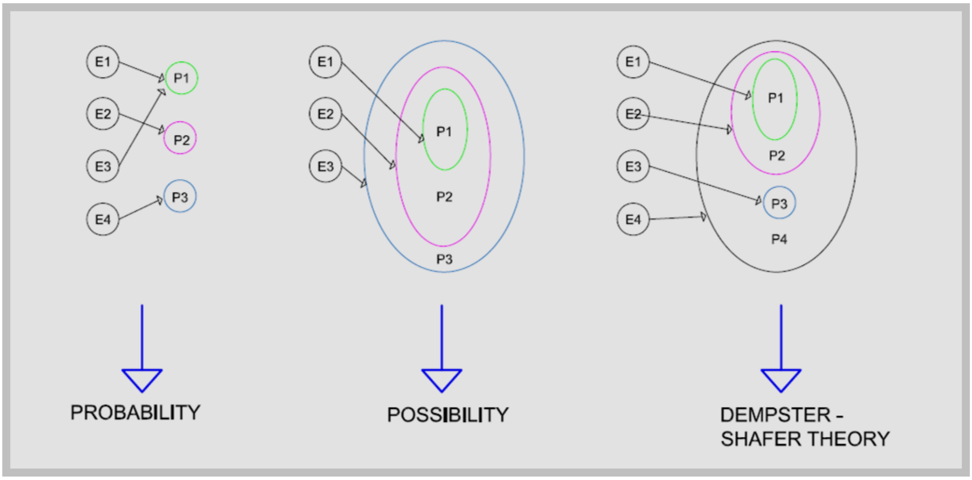

Figure 6 Relationship between Evidence (E) and Proposition (P) –Kikuchi and Pursula (1998)

The difference among the three theories is in the relationship between evidence and proposition. In Probability theory, each piece of evidence points exclusively to a specific clearly defined set. In Possibility theory, each evidence is linked to a nested set of propositions. At last, Evidence Theory, also called Dempster-Shafer Theory, is a summary of these earlier theories. In fact, when the evidence is enough to allow the assignment of probabilities to single events, the Dempster-Shafer model collapses to the traditional probabilistic formulation.

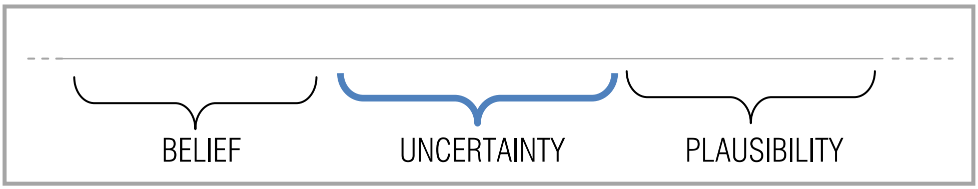

Figure 7 Definition of Uncertainty – author's re-elaboration

2.3 The quality in local public transport

The quality of public transport services is a key issue for both transport companies and analysts. The relevance of this theme is due to the need to offer high-quality transport services to encourage the use of public services and make them more competitive than private cars.

The quality of collective transport services is characterized by various aspects. Some of them, such as travel times, frequency of service, transportation costs are easily measurable; other aspects, such as comfort, information, and security factors are less measurable and more dependent on user attitudes. One of the most important aspects of the quality of service is, in literature, the Reliability of Service, defined by Turnquist e Blume (1980) as "the ability of a collective transport system to respect the timetable programmed and maintain regular intervals and travel times". This is a fundamental aspect of transport systems, because, depending on the degree of reliability, the number of users can increase or decrease (El-Geneidy et al., 2007). The customers' feedback becomes the driver of improvement actions of the service provider.

The overall quality of public transport is made of many criteria.

The criteria represent the customers' point of view about the service provided, and in European Standards have been divided into 8 categories (European Standard EN 13816):

1. Availability
2. Accessibility
3. Information
4. Time
5. Customer care
6. Comfort
7. Security
8. Environmental aspect

In term of service quality, the European Standard defines a scheme in which is explained the relationship between the customer view and the service provider view.

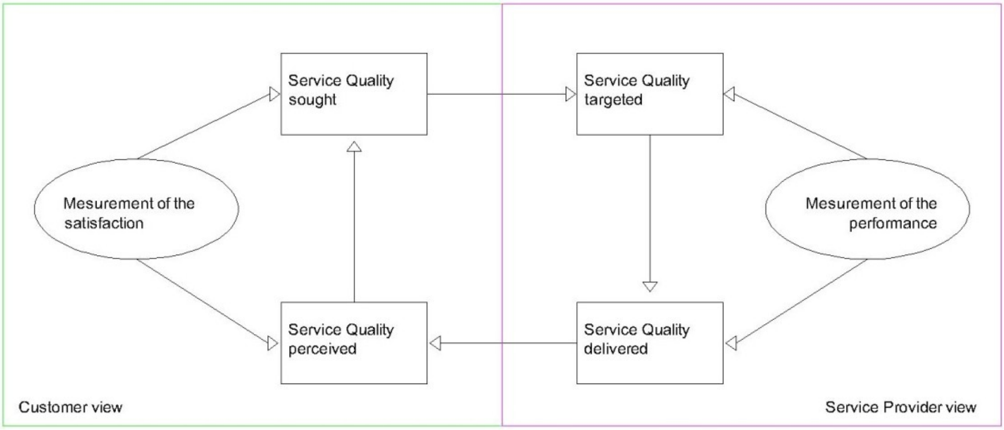

Figure 8 Service Quality Loop – European Standard EN 13816

Passing from the measurement of satisfaction to the measurement of the performance, there are four kinds of service quality, explained above.

The Service Quality Sought is the level of quality, explicitly or implicitly needed by users. Based on the Service Quality Sought, the internal and external pressures, and the technical constraints, the Service Provider defines the Service Quality Targeted.

The Service Quality Delivered is the level of quality achieved on a day-to-day basis. Quality delivered is measured from the customer point of view, and it can be measured using statistical and observation matrices.

The Service Quality Perceived is the customer perception of the quality delivered by the Service Provider, and it depends on their personal experience of the services and on the information they receive about the service.

According to the loop of Service Quality, defined by the European Standard, it is possible to define the degree of Customer Satisfaction as the difference between the Quality Sought and the Quality Perceived.

In this work, we have studied the Quality Sought through users' interviews.

We have chosen the quality parameters explained in the annex of the European Standards 13816, and in the Handbook for Measuring Customer Satisfaction and Service Quality(Transportation Research Board, 1999).

3. The state of the Art

3.1 Quality and uncertainty in public transport

Guirao et al. (2016) stated that, in the field of public transport, the quality is one of the most important criteria for users, when they choose the mode of transport. The public transport is a complex system, whose quality analysis is difficult because it should deal with uncertainty.

Uncertainty has been studied in many fields during the last few centuries. Researchers have tried to identify the different types, dimensions, and evaluation of uncertainty. Among them, Smithson, Smets, Bosc and Prade, Klir and Yuan, Walker and Parsons can be mentioned. Smets (1997) sees uncertainty as the basic part of ignorance. Bosc and Prade (1997) suggest that uncertainty arises from a “lack of information” closely related to the probability theory proponents which assess the probability as lack of knowledge. Klir and Yuan (1995)identify three basic types of uncertainty. These arenonspecificity, strife, and fuzziness. Helton (1996) says that an important question that must be resolved in analyses for many complex systems is exactly what is meant by uncertainty. Analyses for such systems often involve two types of uncertainty: stochastic uncertainty, which results because the system can behave in many different ways and is thus a property of the system itself, and subjective uncertainty, which results from a lack of knowledge about the system and is thus a property of the analysts performing the analysis (Chernoff and Moses 1959; Kaplan and Garrick 1981; Veseley and Rasmuson 1984;Pate-Cornell 1986; Whipple 1986;

Silbergeld 1987; Bogen and Spear1987; Parry 1988; Apostolakis 1989, 1990; Finkel 1990; McKone andBogen 1991; Anderson et *al.,* 1993; Helton 1993b; Kaplan 1993; Hattisand Silver 1994; Hoffman and Hammonds 1994; McKone 1994).Hacking (1975) asserts that the distinction between the two types of uncertainty can be traced back to the beginnings of probability theory.

As for uncertainty treatment, Klir and Wierman (1999)explained the difference between Fuzzy theory, Evidence Theory, Probability and Possibility Theory.

Teodorovic and Kikuchi (1990) apply the Fuzzy Logic Inference to Transport users' behavior simulation on route choice. Kikuchi and Pursula (1998) examine the nature of uncertainty present in transport planning and explored appropriate mathematical treatment, like Fuzzy Set and Evidence Theory.

3.2 Decision Making

Transportation planning is the process of identifying transportation problems and looking for solutions while avoiding future problems. Transportation planners are constantly thinking about the best means of moving people and goods from one place to the next, and doing so in ways that are efficient, safe, cost-effective, and preserve the natural environment (Mid – America Regional Council, 2011).

Many multi-criteria decision-making (MCDM) methods have been proposed to incorporate the needs of different stakeholders involved in decision-making process. MCDM methods use numerical or analytical models to find the alternative that would best meet a wide variety of criteria. They transform both qualitative and quantitative measures into a single objective value (Kronprasert, 2012).

Beynon et al. (1999) develop a methodology in the field of decision making that incorporates Dempster-Shafer Theory with the Analytical Hierarchy Process (DS/AHP) to solve complex problems involving multiple criteria. They utilize knowledge matrix to extract decision makers' preference and use the vectors of knowledge matrix to transform preferences into basic probability assignments.

Dell'Orco and Kikuchi (2003) propose a modeling framework that accounts for the decision maker's uncertainty by Possibility Theory and then calculates the probability of a choice. The Possibility to Probability transformation is performed using the Principle of Uncertainty Invariance.

Kronpasert and Kikuchi (2011) examine the differences between Bayesian and Dempster-Shafer Theory using an example that deals with the choice of the mode of public transportation in a large commercial complex. Moreover, Kronpasert and Kikuchi (2012) propose a new decision-making process, the Belief Reasoning Method, based on the application of Dempster Shafer Theory and Reasoning maps, for evaluating public transportation systems in the planning process.

Dell'Orco and Ottomanelli (2012) propose a model for simulating users' decisional process in a transportation system. In the proposed model, the variables involved are expressed by approximate or linguistic values, like in the humans' reasoning way, to simulate users' mode choice behavior. Utkin et al. (2012), propose an extension of DS/AHP method that considers the multi-criteria decision problem about several levels of criteria. In the method, they reduce the computation procedure for processing and aggregating the incomplete information about criteria and decision alternatives to solving a finite set of linear programming problems.

In recent years, Beynon (2014) reviews in "Reflections on DS/AHP: Lessons to Be Learnt" the impact and the evolution of the DS/AHP method.

Du et al. (2013)use an integrated method for DS/AHP, which gradually integrates pieces of evidence. The integration is carried out over all criteria and members groups by fusing ambidextrous decision information with criteria priority values (CPVs) and member priority values (MPVs).

3.3 Modal choice

The classical transport model is composed of four steps that are: generation of travel, distribution, modal breakdown and assignment. We put our attention on the modal breakdown model, which is very important in planning transport. In fact, the modal choice influences overall efficiency of the transport system and thus the amount of urban space devoted to transport functions as well as the set of alternatives available or not for the traveler. The modal choice models are two different types:

- aggregated models, based on zoning information;
- not aggregated, based on individual data

The modal choice affects the overall efficiency of the transport system and hence the amount of urban space to be devoted to transport functions as well as the range of available alternatives for the traveler. Ortuzar e Willumsen (2006) assert that public transport modes are more efficient in road use.

Choice models are developed from economic theories of random utility, whereas classification models (classifying crash type, for example) are developed by minimizing classification errors with respect to the X's and classification levels Y. Because most of the literature in transportation is focused on choice models and because mathematically choice models and classification models are equivalent, the discussion here is based on choice models.

The model mostly used in recent years for the choice of the mode of transport, is the "modal breakdown model", which provides the fraction of users that, moving between *o* and *d* for the time zone *h,* use the mode of transport *m*. Ortuzar, Willumsen (2006) and Cascetta propose a behavioral model based on the Logit model, in which factors influencing the modal choice are distinct in three categories:

- the characteristics of individuals;
- the characteristics of the trip;
- the characteristics of the mode of transport.

The modal choice affirms that the probability that individuals choose a particular alternative, is a function of their socio-economic characteristics and the relative attractiveness of the alternative. The concept of utility is used to represent the attractiveness of alternatives and, in order to predict the alternative choice, it is necessary to compare the utility value of chosen alternative with the other alternatives, in terms of probability. The main important frameworksare Logit and Probit, characterized by the different distribution of errors.

Starting with the simple binary logit model we have progressed to the multinomial logit model (MNL) and the nested logit (NL) model, the latter becoming the main modeling tool for sophisticated practitioners (Koppelman and Sethi, 2000).

The multinomial logit (MNL) model is the most commonly applied model to explain and forecast discrete choices due to its ease of estimation and foundation in utility theory. The MNL model is a general extension of the binomial choice model to more than two alternatives. The universal choice set is C, which contains j elements, and a subset of C for each individual C_n defines their restricted choice sets. It should be noted that it is not a trivial task to define restricted choice sets for individuals. In most cases J_n for decision maker n is less than or equal to J, the total number of alternatives in the universal choice set, however, it is often assumed that all decision makers face the same set of universal alternatives.

Multinomial probit is an extension of probit models to more than two alternatives. Unfortunately, they are difficult to estimate for more than 4 or 5 alternatives due to the mathematical complexity of the likelihood function as the number of alternatives increases. As computers become faster and/or computational methods become improved, multinomial probit models may be used to estimate models for reasonably sized choice sets.

Another important model is the mixed logit is considered to be the most promising state of the art discrete choice model currently available (Hensher, Greene 2001).

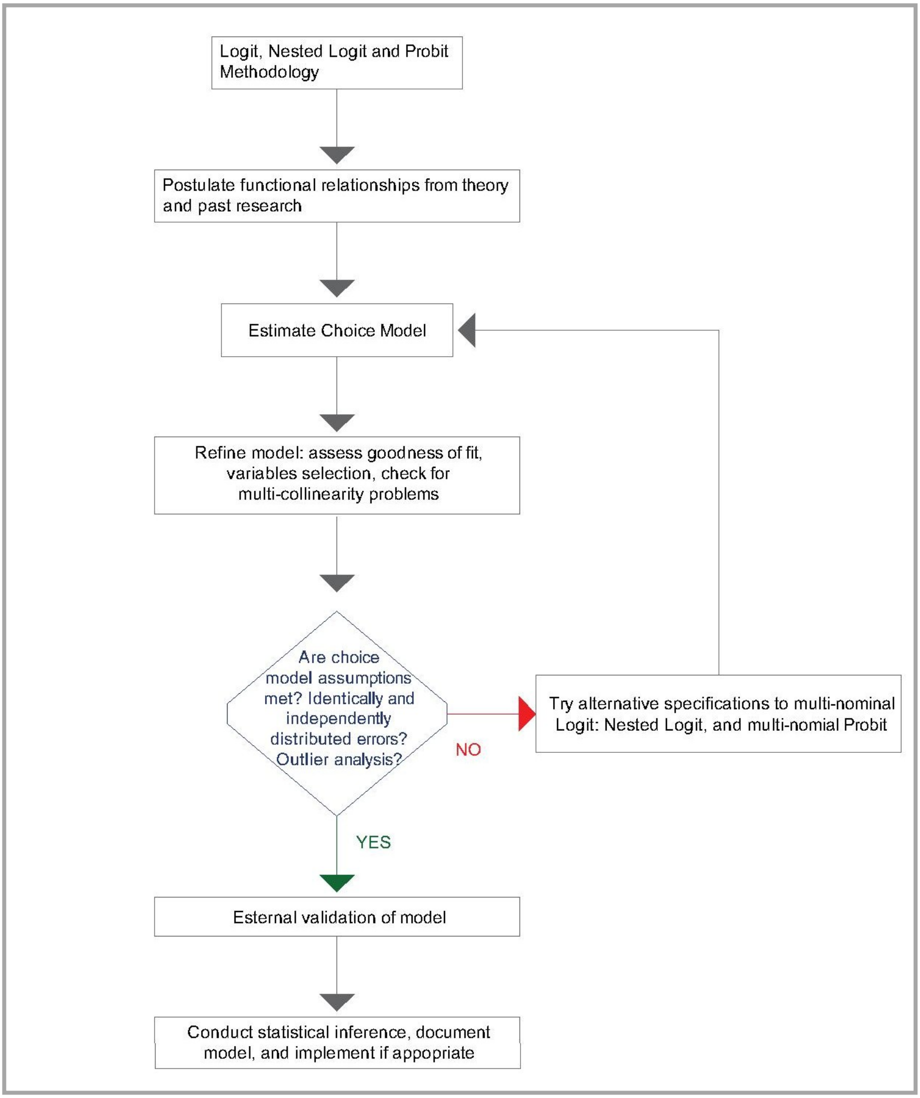

Figure 9 Logit, Nested Logit, Probit Models - Transportation Research Board, chapter 5 - http://onlinepubs.trb.org/Onlinepubs/nchrp/cd-22/v2chapter5.html

4. The Methodology

The complexity of the local public transportation planning process is due to the necessity of considering social, economic, and environmental variables.

For these reasons, the framework of the research considers three main aspects:

1. the organization, in a logical and systematical way, of the transportation problem, through the identification of users' and service's needs;
2. the definition of quality criteria of alternatives mode of transport;
3. the recognition of ambiguity and the uncertainty, embedded both in users' behavior and in analysts' reasoning way.

The need to structure and deepen the research considering these aspects depends on the nature and consequences of the Transit Planning and Evaluation Process.

A lot of stakeholders characterizes the process, namely users, transport company, community, analysts, etc., who are motivated by different purposes and perspectives. Furthermore, the stakeholders consider several, both internal and external, constraints, such as time constraints, financial constraints, political constraints, data and information constraints, and technological constraints. Moreover, the process should deal with the lack of complete and precise information

Thus, it is possible to resume the problem in two statements:

1. Lack of a clear process, which depends on a lack of knowledge, a lack of quantitative measures, high number of alternatives, criteria, and stakeholders;
2. Uncertainty, expressed in different forms, such as analyst's uncertainty, users' uncertainty, uncertainty in data collection, etc.

Our purpose is the evaluation of public transportation, considering the uncertainty in judgments about quality parameters, such as travel time, punctuality, comfort etc. We will analyze the user's point of view, and quantify the quality parameters, often expressed in verbal terms. The proposed method is a hybrid approach based on two different theories: the Analytical Hierarchy Process (AHP) (Saaty, 1980) and the Evidence, or Dempster-Shafer-Theory (DS)(Dempster 1967; 1968; Shafer, 1976).The outcomes of this methods are the values of Belief and Plausibility; afterward, to pass from Belief Function to Probability, we will use the Transferable Belief Model.

4.1 The Analytical Hierarchy Process

The Analytical Hierarchy Process (AHP) is a Multicriteria decision analytical tool that decides the weights of criteria, building a matrix that expresses the relative values of a set of attributes. This method was developed by T. Saaty in 1977. AHP is useful in helping decision-maker facing complex problems with multiple conflicting and subjective criteria (e.g. location or investment choice, projects ranking, etc.) (Ishizaka and Labib, 2011). One of the most important aspects of AHP method is the organization of the problem in a systematic way, like goal, criteria, and alternatives that gives a structured simple solution to decision-making problems (Utkin et al. 2012).

AHP is perhaps the most widely used decision-making approach in the world nowadays. Its validity is based on the many hundreds (even thousands) of current applications, in which AHP results were accepted and used by conscious decision-makers (DMs) (Saaty, 1994b). AHP has three primary functions:

1. Structuring Complexity;
2. Measurement on a ratio scale;
3. Synthesis.

Through the AHP it is possible to break down the problem into hierarchical steps, defining the priorities of elements and comparing their mutual importance with respect to a common attribute.

The basic steps of AHP method are the following:

- Construction of the hierarchical structure, in which elements are arranged in layers, fundamental in the decision problem (see the figure above);
- Collection of data for determination of the preference relation among criteria;
- Estimation of the relative weights, derived from the analysis of pairwise comparisons. To make the pairwise comparison, it is necessary to consider the Saaty's Scale, which is a numerical scale, composed of integers from 1 to 9. A basic, but very reasonable, assumption is that if attribute A is more important than attribute B and is rated 9, then B must be absolutely less important than A and is valued 1/9 (see the above figure);
- Aggregation of relative weights of elements, at each level, to achieve a weighed ordering (ranking)of alternatives.

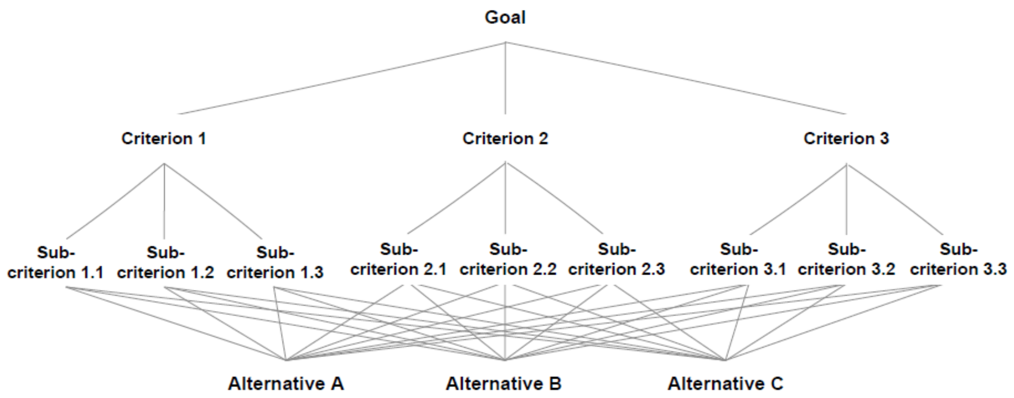

Figure 10 The hierarchical structure of AHP

Intensity of importance	Definition	Explanation
1	Equal importance	Two factors contribute equally to the objective
3	Somewhat more important	Experience and judgement slightly favour one over the other.
5	Much more important	Experience and judgement strongly favour one over the other.
7	Very much more important	Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice.
9	Absolutely more important.	The evidence favouring one over the other is of the highest possible validity.
2,4,6,8	Intermediate values	When compromise is needed

Figure 11 The Saaty's Scale

The mathematical form of AHP is very simple and it is based on the use of matrices and vectors, to set up the weight of criteria and alternatives.

Let us consider n elements to be compared, C_1, C_2, ... C_n, and denote the relative preference of C_i with respect to C_j. In this way, it is possible to obtain a reciprocal square matrix of order n:

$$A = (a_{ij})$$

$$a_{ij} = 1/ a_{ji} \text{ and } a_{ii} = 1$$

It is possible to calculate the vector ω of order n. It is the eigenvector of the matrix A and λ is its eigenvalue.

$$A(\omega) = \lambda\omega$$

For consistent matrix $\lambda = n$

To measure the consistency of expressed judgments, Saaty proposed a measure of the inconsistency, called Consistency Index (CI), that is given by:

$$\mathrm{CI} = \frac{\lambda_{max} - n}{n-1}$$

where λ_{max} is the principal eigenvalue of the judgment matrix and n is its order.
When the reciprocal comparison matrix is consistent, λ_{max}=n and the CI is equal to zero; otherwise, its value is positive. To overcome the order dependency of the CI, Saaty proposed a normalized measure, called the CR, which is given by

$$\mathrm{CR} = \frac{CI}{RI(n)}$$

where RI(n) is the Random (Consistency) Index for matrices of order n. This term is defined as the expected value of the CI corresponding to matrices of order n (RI = E [CI(n)]), when the judgments are simulated in the set {1/9;…; 1; …9) and the Right Eigenvector Method(EVM) is used as the prioritization procedure. The CR gives a measure of where the judgments in the pairwise comparison matrix lie between totally consistent and totally random. When CR = 1, then CI= E [CI(n)] and the judgments are totally random (low precision). High values of CR reflect more inconsistency and thus we are interested in values of CR as low as possible. To accept the consistency of the matrix, Saaty (1980) suggested, as a rule of thumb, a threshold of 10percent or less (CR60:1). More recently, Saaty (1994) suggested thresholds of 5% and 8% for 3by 3 and 4 by 4 matrices, respectively.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Figure 12 The Random Index

In AHP, the Decision Maker (DM) has a significant role, because he/she gives the information about criteria and scenarios, and the AHP converts DM's inputs into numbers. Moreover, AHP reduces the bias in the decision-making process because it checks the consistency of the DM's evaluation. A criticism of AHP is the considerable number of pairwise comparisons to be performed before any rankings that can be considered and also the verification of consistency index for each comparison matrix.

Beynon et al. (2000) highlighted another flaw of AHP model, which does not consider the representation of ignorance.

More details about AHP are reported in Saaty (1980 – 1990).

4.2 The Dempster-Shafer Theory

The main factor of the distinction between Epistemic and Aleatory Uncertainty is the degree of knowledge of the problem. In fact, Epistemic Uncertainty should deal with theill-defined problem, verbal and/or approximate values, lack of information and so on. This needs mathematical tools, different from probability measures, which can consider not only one value of probability measure, but a range of plausible values. This approach has three important consequences:

1. It is not necessary to deduce a precise measure;
2. The principle of insufficient reason is not binding. In case of total ignorance, it is possible to assert the possibility of realizing more events together, without giving a precise probability measure of an individual event.
3. It is not necessary to respect the additive axiom, typical of Probability Theory.

In literature, we found three different approaches, which can represent the uncertainty as a set of values:

- Imprecise Probabilities;
- Possibility Theory;
- Dempster-Shafer Theory of Evidence.

In this work, we applied the Dempster-Shafer Theory, because it allows us to combine information coming from multiple sources, and to quantify the uncertainty embedded in decisions.
The DST subsumes Probability and Possibility theories, as shown in figure 6. From this figure, it appears that DST is useful when many factors influence the choice and individual factors do not exclusively point to one choice. (Kikuchi and Pursula, 1998).

4.2.1 The Mathematical Framework

The Evidence theory, or Dempster-Shafer Theory of Evidence (DST), is a generalized mathematical theory to deal with the all types of uncertainty, particularly uncertainty involving ambiguity (or ignorance). This type of uncertainty is always present in decision-making about large-scale systems like public transit system (Kronprasert, 2012).
The basic idea of Dempster-Shafer Theory is that numerical measures of uncertainty may be assigned to overlapping sets and subsets of hypotheses, events, or propositions as well as the individual hypothesis. In DST, the measures of uncertainty are known as "basic probability assignments (bpa)," and it is possible to refer the bpa not only to singletons but also to sets made up of a number of propositions. DST uses Belief and Plausibility measures instead of Probability measures to represent the degree of support of occurrences. This is a key point of DST.
The basic components of DST are:

- a finite set of hypotheses $\Theta = \{h1, h2, \dots. hn]$,called Frame of Discernment;
- a power set 2Θ is the set of all the subsets of Θ, including itself and a null set, Ø. Each subset is called Focal Element. In the following definition, A and B are Focal Elements.
- the Basic Probability Assignment (bpa), a function m: $2\Theta \rightarrow [0,1]$ that satisfies the following hypotheses:

i) $m(\emptyset) = 0$

ii) $\sum_{B\subseteq A} m(x) = 1$

- the Belief measure, a function Bel: 2Θ→ [0,1], defined by:

Bel (A) $= \sum_{B\subseteq A} m(B)$ for all A⊆Θ

The Belief Measure has to satisfy the following axioms:

1. Axiom 1: Bel(Ø) = 0;
2. Axiom 2: Bel (Θ) = 1;
3. Axiom 3: Assigned a number of subsets n, i.e A1;A2;A3;..;An⊆Θ:

$$Bel\ (\bigcup_{t=1}^{n} A^t \geq \sum_{I\subset 1,2,\ldots,n;I\neq\emptyset} (-1)^{|I|+1} Bel(\bigcap_{t\epsilon I} A^t)$$

If n = 2 and $A_1 \cap A_2 = Ø$, the axiom becomes:

$$Bel\ (A_1 \cup A_2) \geq (-1)^{1+1} (Bel\ (A_1) + Bel\ (A_2)) + \quad |I| = 1$$
$$+(-1)^{2+1} (Bel\ (A_1 \cap A_2)) \quad |I| = 2$$

$$Bel\ (A_1 \cup A_2) \geq Bel\ (A_1) + Bel\ (A_2)$$

- The Plausibility measure, a function Pls: 2Θ→ [0,1], defined by:

Pls (A) $= \sum_{B\cap A\neq\emptyset} m(B)$ for all A⊆Θ

Pls (A) = 1 – Bel ($\overline{A}$)

- The Dempster rule of combination, which allows combining the Basic Probability Assignments to obtain a Belief function that somehow reflects the joint evidence.

The denominator $1 - \sum_{A \cap B = \varnothing} m_{1(A)} m_{2(B)}$ denoted by K, is the "normalization factor," and $\sum_{A \cap B = \varnothing} m_{1(A)}\, m_2\ (B)$ is called also "degree of conflict" between the pieces of evidence. This is very important in the application of DS/AHP because it allows us to verify the goodness of the combination.

$$[m_1 \oplus m_2] = \begin{cases} 0, \quad C = \varnothing \\ \dfrac{\sum_{A \cap B = C} m_{1\ (A)} m_{2\ (B)}}{1 - \sum_{A \cap B = \phi} m_{1\ (A)} m_{2\ (B)}} \end{cases} \qquad (1)$$

The algebraic properties of the Dempster Rule are the Commutative and Associative Property.

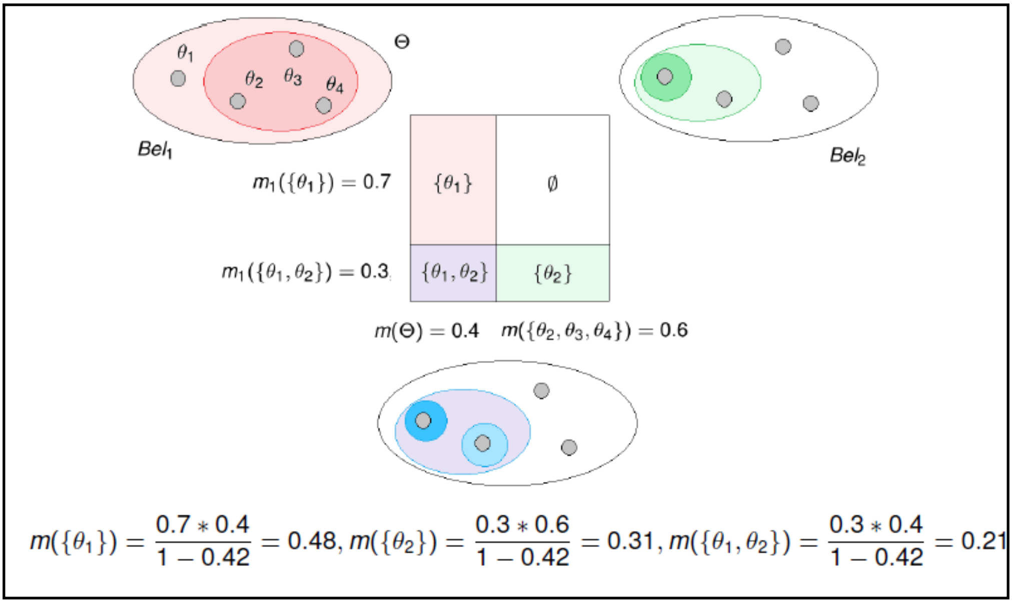

Figure 13 Dempster's Rule example – Adapted by Fabio Cuzzolin "Belief Function: past, present, and future"

In DST, the belief (Bel) and plausibility (Pls) measures are related to one another:

Bel (A) = 1 – Pls (-A) and Pls (A) = 1 – Bel (A) (2)

The Belief is an increasing monotone function, while the Plausibility is a decreasing monotonous function. Both are non-additive measures, in fact, their sum is different than 1.

The Belief measure is the lower bound of the likelihood of A, while the plausibility represents the upper bound of the likelihood of A (see the figure 6).

In conclusion, the probability value of an event falls within the range defined by its lower and upper bounds. If there is an equality between the two values (Belief and Plausibility), the uncertainty does not exist and we can assign a probability value to the single event.

In Dempster Rule, one needs to be cautious in defining the K value, because it contains a SIGNIFICANT INFORMATION to evaluate the consistency of the evidence from the different sources. In fact, the K values give a measure of conflict embedded in the analyzed system. The Conflict Measure (*Con*) can be expressed as follows:

Con (Bel1, Bel2) = log (K)

Con (Bel_1, Bel_2) = $\log \left(\frac{1}{1-\sum_{B \cap C \neq \emptyset} m1\,(B) * m2(C)}\right)$

Con (Bel_1, Bel_2) = $-\log(1-\sum_{B \cap C = \emptyset} m1(B) * m2(C))$

In[3]:= Plot[-Log[1 - x], {x, 0, 1}]

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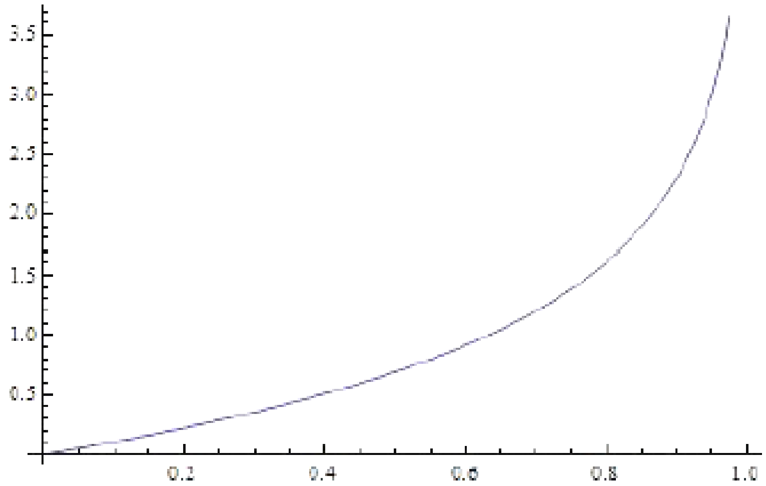


Figure 14 Conflict Measure

As shown in figure 13, the function can assume a value that goes from zero to infinite. In particular, we can observe that:

Con(Bel1, Bel2) $\rightarrow 0$ for $\sum_{B \cap C = \emptyset} m1(B) * m2(C) \rightarrow 0$ no conflict

Con(Bel1, Bel2) $\rightarrow \infty$ for $\sum_{B \cap C = \emptyset} m1(B) * m2(C) \rightarrow \infty$ high conflict

The Belief Function, introduced by Shafer, generalizes the Bayesian probability. It is possible to define the following characteristics:

- classical probability measures are a special class of belief functions(in the finite case) or random sets (in the infinite case)
- Bayes' 'certain' evidence is a special case of Shafer's bodies of evidence(general belief functions)
- Bayes' rule of conditioning is a special case of Dempster's rule of combination

The Belief Function overcomes the limits of Bayesian Probability; in fact, you do not need a prior probability distribution: if you are ignorant, you can use the vacuous BF m_Θ which, when combined with new BFs m' encoding data, won't change the result

$$m_\Theta \oplus m' = m'$$

4.3 The DS/AHP Method

The DS/AHP, introduced in Beyon et al. (2000), is a hybrid method that incorporates two different theories: the Dempster-Shafer Theory, as a mathematical foundation, and the Analytical Hierarchy Process (AHP), as a structure of the method.

The DS/AHP method is structurally similar to AHP with a hierarchy of levels of inherent decision-making. However, its mathematical foundation is based on the Dempster–Shafer theory of Evidence (DST), introduced in the work of Dempster (1968) and Shafer (1976).

The use of DST in DS/AHP allows a DM to make preference judgments on groups of decision alternatives (DAs) rather on individual decision alternative (DA) or through pairwise comparisons of DAs (as in SMART and AHP).

The DS/AHP is a method that should be used in presence of ignorance in a Multi-Criteria Decision Making (MCDM) problem. The ignorance is a set of three categories including incompleteness, imprecision, and uncertainty. Smets (1991) makes a distinction between the three categories of ignorance, based on the objective and subjective components. The uncertainty is the only subjective part, and it is linked to the observer that is not certain about the available information. This information only induces some form of partial knowledge or belief in the observer.

We apply for the first time the DS/AHP method to transportation problems, being able in this way to represent the uncertainty embedded in the users' judgment related to the quality parameters and the uncertainty embedded in the companies' point of view. To consider both at the same time, the proposed model provides the data fusion coming from the two different sources (users and companies). The strength of this model lies in the opportunity to deal, in the same way, with data coming diverse sources.

To describe the DS/AHP method (and results in forthcoming sections), we used an example (hypothetical)problem. The problem considered here is deciding which research proposal(s) are the best from a certain number of proposals. Best may relate to proposals worthy of research funding etc. Here, there are 12 proposals (DAs) labeled A, B, C, D, E,F, G, H, I, J, K, and L, available to be considered over a certain number of different criteria, the four criteria are complexity, expense, originality, and relevance. The DS/AHP hierarchical tree structure for this problem, including example judgments made by a DM, is presented in Fig.14.

A number of groups of DAs have been identified in this figure, over the different criteria based on the opinions of a DM (e.g. A; E;G associated with the complexity criterion). For each criterion, the DM has identified groups of DAs based on their positive preference amongst all the available DAs, to build the frame of discernment {A; B; C;D; E; F ;G;H; I; J;K; L}.In AHP, as even Saaty (1990)highlighted, when the DMs make pairwise comparisons they do keep in mind all the DAs in the evaluation of the level of preference.

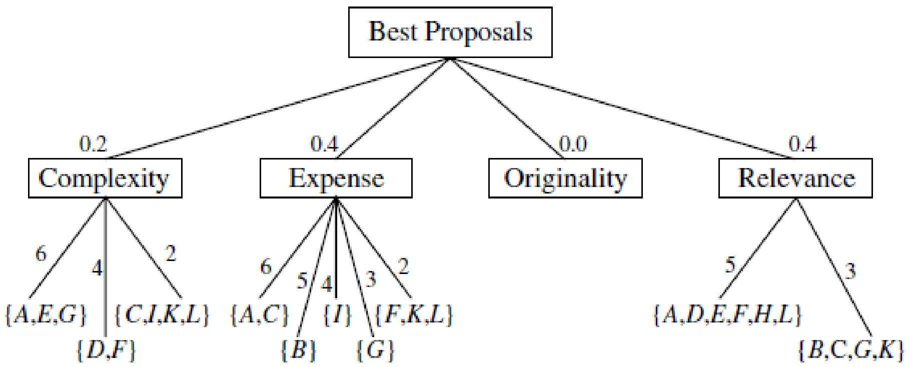


Figure 15 **Hierarchy of DS/AHP, best proposals problem. Adapted by Beynon, 2005**

Within the next DS/AHP analysis, there are intermediate sets of weights to be constructed, starting with the criteria priority values (CPVs) associated with each criterion. The CPVs quantify the levels of importance that a DM has assigned (perceived to have) towards the criteria. As in the traditional AHP, the set of CPVs for the criteria considered sums to unity.

For the hypothetical "best proposals" example, the four criteria complexity, expense, originality, and relevance have the respective CPVs: 0.2, 0.4, 0.0 and 0.4. The CPV for the originality criterion is zero, showing no associated importance; hence no groups of DAs identified on this criterion. For a criterion with a positive CPV it is possible to construct a sequence of weights (bpa values) to the respective groups of DAs identified (focal elements), and a concomitant level of local ignorance.

4.4 The Transferable Belief Model

The Transferable Belief Model (TBM), developed by Philip Smets in 1999, is a model for representing quantified beliefs held by an agent at a given time on a given frame of discernment (Smets, 1990-1994). The TBM is an extension of the Dempster Shafer Theory.

Smets divided Beliefs into two levels:

1. Credal Level, where beliefs are entertained and quantified as belief function;
2. Pignistic Level, where beliefs can be used to make decisions and are quantified by probability functions.

The TBM claims the existence of a Belief functions that describe the credal state on the Frame of Discernment, but when a decision must be made the credal beliefs are transferred to pignistic probability. Considering a credibility space (Ω, R, bel), that describes the belief on R, Smets defines the Pignistc Probability Distribution as:

$$\mathrm{BET\ P}(x) = \sum_{x \subseteq A \in R} \frac{m(A)}{|A|}$$

where:

$m(A)$ is the Basic Belief Assignment;

$|A|$ is the number of atoms of R in A.

BET P is a probability function, but we call it pignistic probability function to stress the fact that it is the probability function in a decision context. The principle underlying this procedure is the Generalized Insufficient Reason Principle since the Insufficient Reason Principle has been used at the level of each focal proposition of the belief function.

It is important to note that the TBM includes two components: one static, the basic belief assignment, and one dynamic, the transfer process. Many authors on Dempster-Shafer model considered only the basic belief assignment and discovered that the basic belief masses are probabilities on the power set of Ω. But usually, they do not study the dynamic part, i.e. how beliefs are updated. Their comparisons are therefore incomplete, if not misleading.

The advantage of the TBM over the classical Bayesian approach resides in its large flexibility, its ability to represent every state of partial beliefs, up to the state of total ignorance. In the TBM, total ignorance is represented by the vacuous belief function, i.e., a belief function such that $m(\Omega) = 1$, $m(A) = 0$ for all A with $A \neq \Omega$. Hence $bel(\Omega) = 1$ and $bel(A) = 0$ for all A strict subset of Ω. It expresses that all you know is that the actual world belongs to Ω. The representation of total ignorance in probability theory is hard to achieve adequately, most proposed solutions being doomed to contradictions. With the TBM, we can, of course, represent every state of belief, full ignorance, partial ignorance, probabilistic beliefs, or even certainty ($m(A) = 1$corresponds to the certainty of A) (Smets, 2006).

5. An application to real case

In transportation field, the uncertainty is a topic discussed mostly in decision making strategy, in which there are a lot of stakeholders that can express their opinion. In recent years, the need to improve public transport to reduce the use of private car led to a change in planning strategies. Current strategies consider the quality of service as a key factor for the growth of public transport demand. In fact, as shown in figure 8, there is a close connection between customer's view and service provider's view. This connection is also present in demand and supply of public transport: the higher the demand, the higher quality levels of a transport supply.

The adoption of a planning strategy that considers the opinion of as many different stakeholders as possible, is important to improve the public transport service and to reduce the use of private cars.

The aim of this research is to formulate a model for Decision Support System, based on the use of DS/AHP/TBM approach, which considers and merges the needs coming from multiple sources, such as users and transport companies. In this way, it is possible to merge subjective criteria, such as the importance of quality parameters for users, to objective ones, such as the transport demand. We applied this model to the choice of the most suitable mode of transport in a metropolitan city. In particular, we considered the case of the metropolitan city of Bari, as shown in next chapter (chapter 5.1).

In the following diagram, there are the reasons that led to the choice of the model. In particular, for each motivation corresponds the adopted theory.

Decompose the complex problem in hierarchical steps;	• Analytical Hierachy Process
Transform qualitative judgments into quantities by assigning them a certain weight;	• Analytical Hierarchy Process
Deal with incomplete and uncertain information;	• Dempster Shafer Theory
Merge information coming from different sources	• Dempster's Rule
Define the range within which to finde the solution	• Belief and Plausibilty mesaures
Assign a probability of choice	• Trasferable Belief Model

Figure 16 The organization of the adopted model

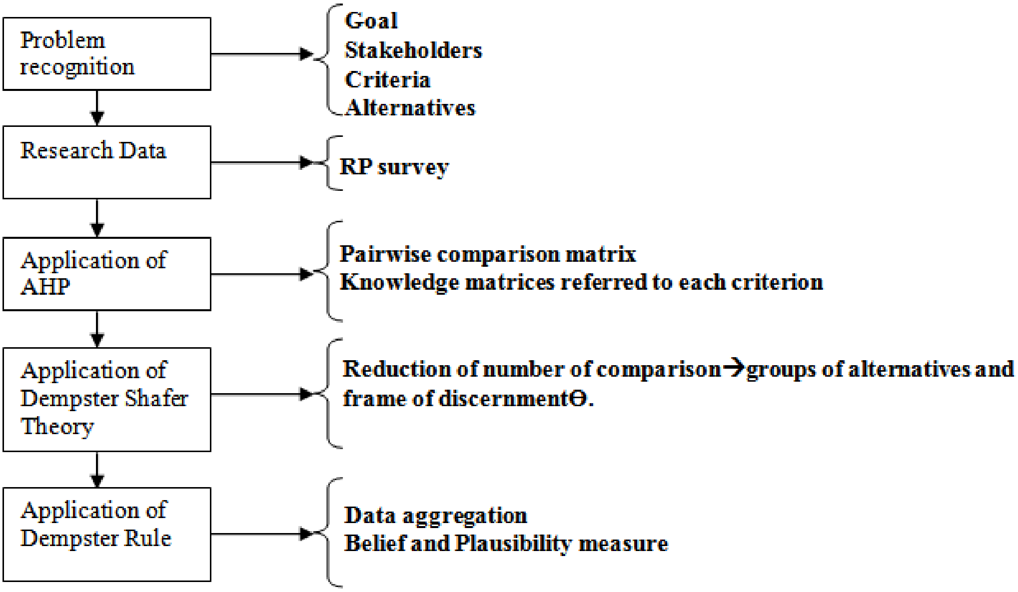
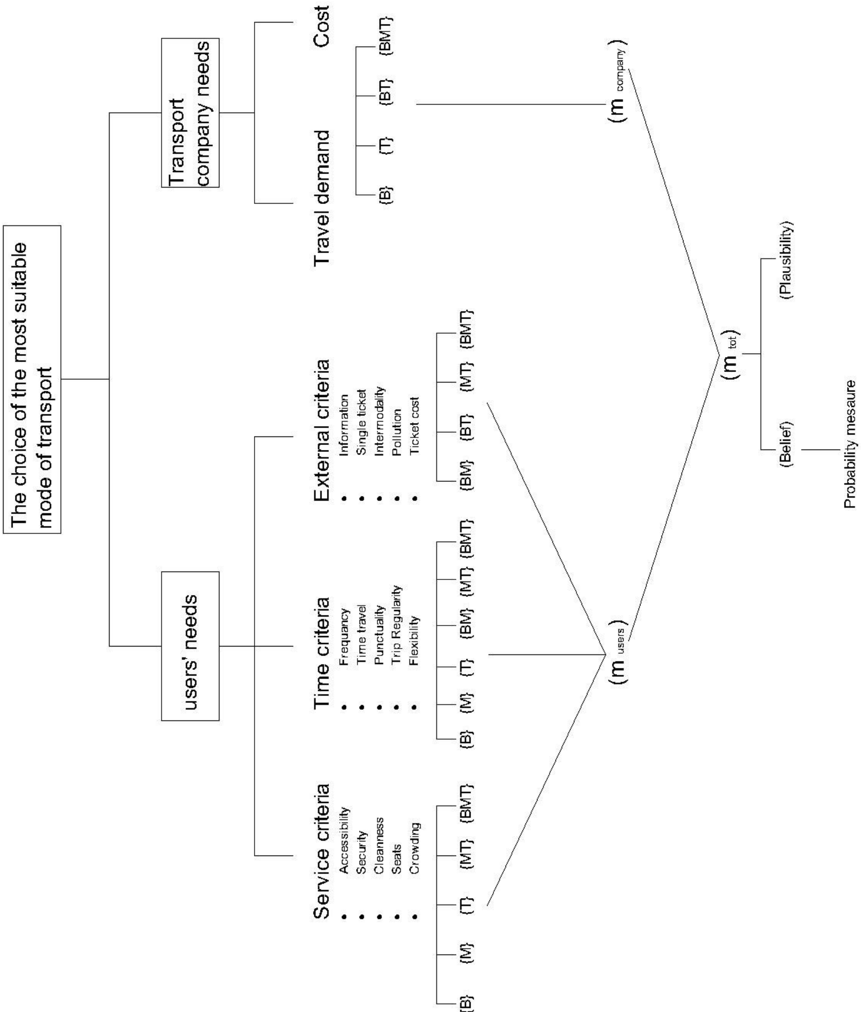


Figure 17 The adopted Model

In this chapter, we applied the proposed method to a real case of study, which regards the choice of the most suitable mode of transport, as is perceived in relation with the ranking of quality parameters, by users and transport companies, for a small metropolitan city in Southern Italy, namely Bari, the capital city of the Apulia Region.


The choice of the most suitable mode of transport
Transport company needs
users' needs
Service criteria
Accessibility
Security
Cleanness
Seats
Crowding
Time criteria
Frequancy
Time travel
Punctuality
Trip Regularity
Flexibility
External criteria
Information
Single ticket
Intermodality
Pollution
Ticket cost
Travel demand
Cost
{B} {M} {T} {MT} {BMT}
{B} {M} {T} {BM} {MT} {BMT}
{BM} {BT} {MT} {BMT}
{B} {T} {BT} {BMT}
(m users)
(m company)
(m tot)
(Belief)
(Plausibility)
Probability mesaure

5.1 The Metropolitan City of Bari

The metropolitan city of Bari is made up of 40 small towns, as well as Bari, and has an extension of 3.825 km^2.According to the ISTAT 2011 Report, the density population has increased by 2.40%, from 1,218,038 inhabitants in 2001 to 1,247,303 in 2011. In the metropolitan city of Bari, 46.2% of municipalities have a population in between 5,000 and 20,000 inhabitants; while 48.80% of municipalities exceed 20,000 inhabitants.

Public transport in the metropolitan city of Bari is carried out by railways and roads. Since the city of Bari had always a leading role, the railways developed in a hub-and-spoke configuration. The metropolitan city of Bari is served by four railway lines, which are:

1. Ferrotramviaria
2. Ferrovie Appulo Lucane
3. Ferrovie del Sud-Est
4. Rete Ferroviaria Italiana

In the following table are shown the main characteristics of the transport company which operates in the metropolitan context of Bari. In particular, in the first column, there is the name of the transport company, followed by the railway route. The Railtrack extension is divided into single track and double track, and into not electrified (NE) and electrified (E). The last column indicates the number of stations for each transport company.

From this table, it is possible to see that most of the railway network is not electrified. The only metro service in town is provided by Ferrotramviaria. It connects the airport with the central station of the National Railways and "S. Paolo" Hospital on the outskirts of Bari. Moreover, the Ferrotramviaria company guarantees some other movements within the city.

Table 1 Railways Extension in Metropolitan City of Bari

Company	Railway	Railtrack Extension				Number of stations
		Single track		Double track		
		NE	E	NE	E	
Ferrotramviaria	Bari central station-Bari “Cecilia”				4.153	5
	Bari-Ruvo		32.32			9
	Ruvo-Barletta				33.44	5
	Airport (metroline)				7.7	2
	Bari central station-Bari “Francesco Crispi”		5.073			5
Ferrovie Appulo Lucane	Bari-Altamura	48.35				14
	Altamura-Gravina	11.71				2
Ferrovie del Sud-Est	Bari FS – Conversano - Putignano	6.277				11
	Putignano-MartinaFranca			34.03		6
	MartinaFranca-Taranto FS			34.85		8
	Mungivacca-Casamassima-Putignano			43.41		11

	MartinaFranca-Novoli-Lecce FS			102.6		15
Rete Ferroviaria Italiana	Bari-Taranto		99.6			11
	Bari-Foggia		122.6			14
	Bari-Lecce		149.3			17
	Rochetta-S.Nicola di Melfi			12.22		2
	S.Nicola di Melfi-Gioia del Colle			127.1		9
	Barletta-Spinazzola			65.68		6

In the metropolitan area of Bari, the railways area crucial transport mode: for example, approximately 25,000 passengers per day arrive in the central station of the National Railways of Bari.

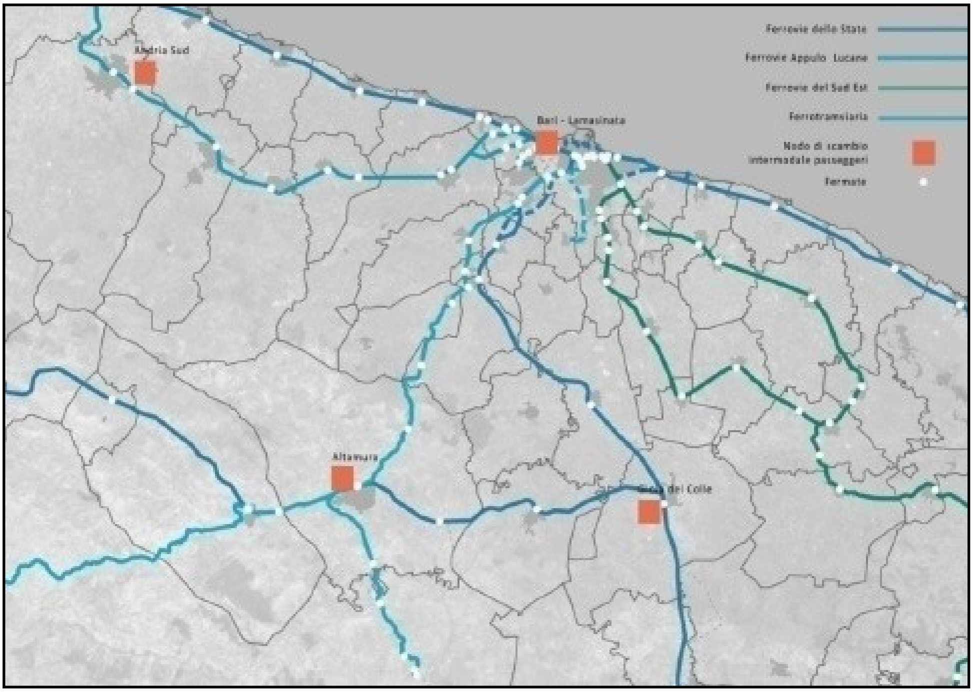


Figure 18 An Extension of Railways in Metropolitan City of Bari

In the Metropolitan City of Bari, the Public Transport is carried out also by buses. In the suburban areas of the metropolitan city, the passengers' traffic was 6,048,395 (2016).

5.2 The Transport Company

In this case of study, we have analyzed one of the Transport Companies, the Ferrovie Appulo Lucane. We chose this company both for the availability of data and for the presence of both road and rail transport.

The following figure shows the analyzed railway, operated by Ferrovie Appulo Lucane, which connects the center of Bari to Gravina in Puglia. This path is characterized by the presence of 8 stations.

Figure 19 The Railway and Bus Service of Ferrovie Appulo Lucane

The whole network (from Bari to Matera) extends 76 km, 61 km in Apulia Region and the remaining part in Basilicata Region, and it is made of a single non-electrified rail track.

As shown in the following figures, the majority of passengers moving on the Bari – Gravina section during a weekday, usually get on the train and get off in Bari station.

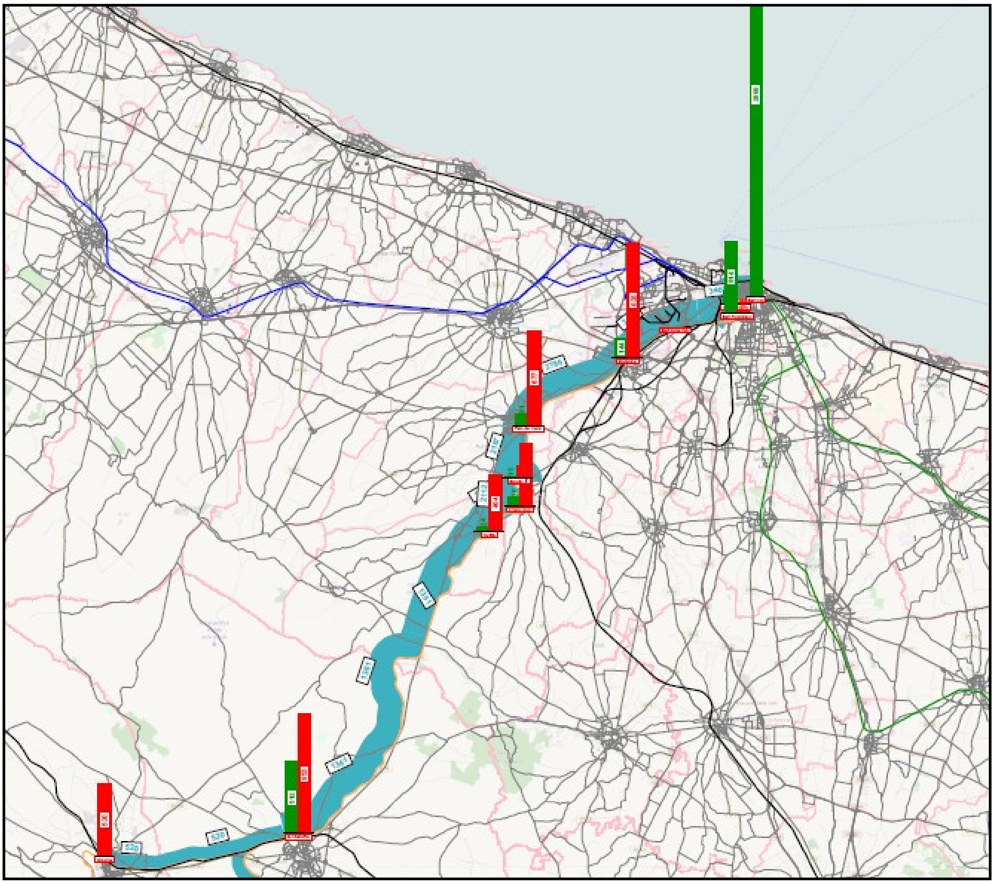


Figure 20 Number of boarding and alighting passengers in the Bari-Gravina direction

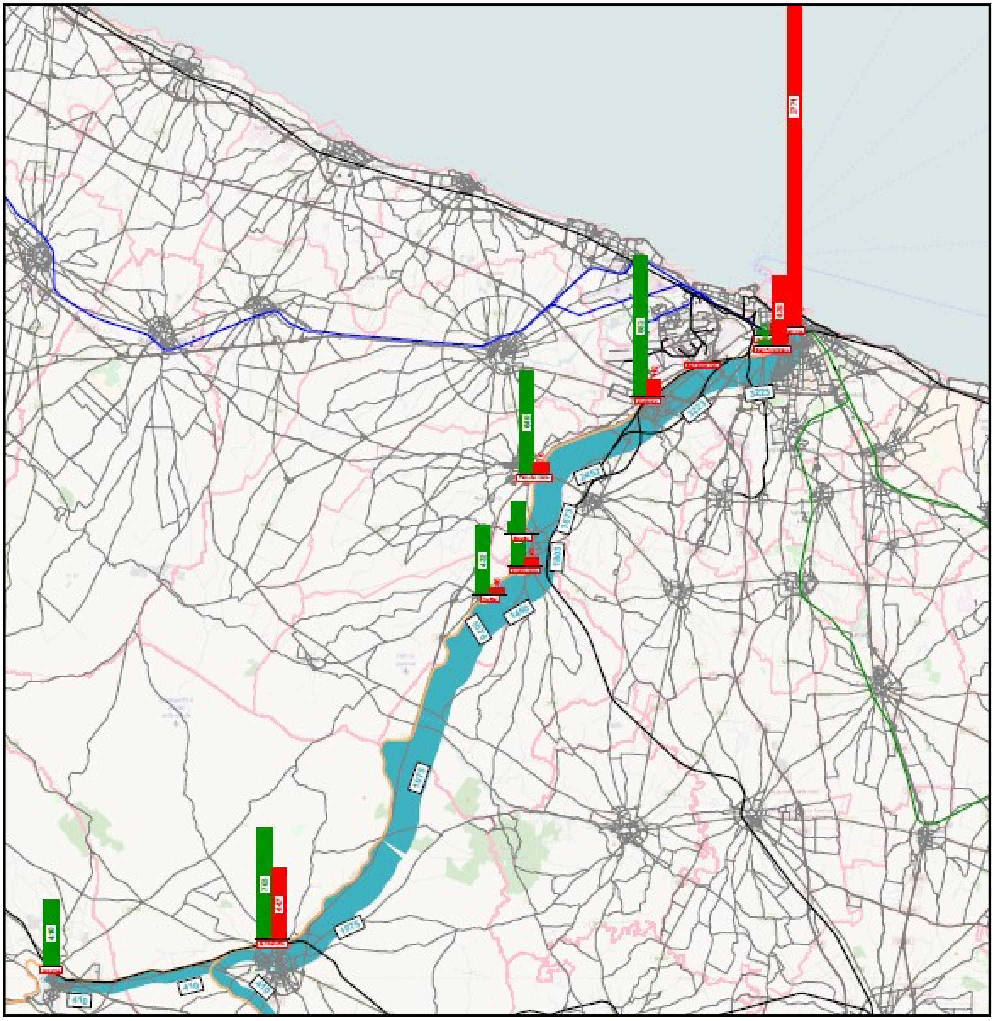


Figure 21 Number of boarding and alighting passengers in the Gravina -Bari direction

5.3 Data Collection

Data search is very complicated in the field of local public transport, as it is a complex and non-homogeneous system.

To validate the proposed model, lots of data, referred to users' point of view and transport company needs, have been analyzed.

5.3.1 Users' data collection

In the field of transport, surveys are a useful tool for the knowledge of the users' opinions. Surveys have different nature, depending on the characteristics and the quality of the information they are seeking. Cascetta (2006) affirms surveys are important tools for calibrating models because, collecting real data, they can adapt the model to the study situation. We can divide surveys into two different classes:

1. Surveys of Revealed Preferences (RP): investigate the actual behavior of users through direct queries;
2. Surveys Stated Preferences (SP): investigate the users' behavior under hypothetical scenarios.

In this research, to analyze the users' point of view in relation to service quality parameters, RP surveys have been carried out using a questionnaire, widely spread among the customers, mostly through the Internet network.

The main goal of the surveys is to obtain a rank, about the chosen criteria, decided by customers and referred to the quality service.

A lot of studies in the literature about the selection of the quality attributes identified a large set of attributes to explain the service quality (see, for example, Prioni and Hensher, 2000). We considered the Transportation Research Board handbook (1999), the European Standard EN13816 and the surveys carried out by local public transport companies for the analysis of service quality to choose the criteria.

In particular, we selected and grouped fifteen criteria into three macro-categories, as shown in the following list:

1) Service criteria
    i) Accessibility;
    ii) Security;
    iii) Cleanliness;
    iv) Number of offered seats;
    v) Crowding.

2) Time criteria:
    i) Frequency;
    ii) Travel Time;
    iii) Punctuality;
    iv) Regularity of trips;
    v) Flexibility.
3) External criteria:
    i) Information;
    ii) Single ticket(one ticket for all modes of transport, like a transport card);
    iii) Intermodality;
    iv) Pollution;
    v) Travel fares.

In the formulation of the questionnaire, we considered the following characteristics:
1) Simple organization of questions
2) Short and targeted questions
3) multiple choice answers

The proposed questionnaire is composed of11 questions; the first four are about the user's characteristics (sex, age, residence, and profession) (table 1), while the others are about the transport mode and the companies mostly used, and their preferences about the quality parameters of the transport service.

To obtain the largest possible number of respondents, the questionnaire link was shared on Facebook and forwarded via email. In addition, some questionnaires were delivered in paper form to students of the Polytechnic University of Bari.

In the following, we report the English translation of the proposed questionnaire.

Figure 22 The figure shows the start page of the questionnaire published online

In the following pages, there is the complete questionnaire that we have proposed to users of the transport company

Survey on the Quality of the Local Public Transportation

Dear user, you're invited to complete this short anonymous questionnaire, useful for collecting data on the perception of the quality of local public transport in our region. The data obtained will be exclusively used for scientific purposes (scientific research related to the transport sector of the Polytechnic University of Bari). We hope that it can reach a great diffusion. Thank you for your participation.

1. Gender
   - Male
   - Female
2. Age
   - <20
   - 21-30
   - 31-40
   - 41-50
   - 51-60
   - 61-70
   - >71
3. Province of Residence
   - Bari
   - Foggia
   - Barletta-Andria-Trani
   - Taranto
   - Brindisi
   - Lecce
4. Employment
   - Student
   - Employee
   - Freelance
   - Housewife
   - Retired
   - Unemployed
5. How often do you use the public transport?
   - Everyday
   - 2-3 times per week
   - 1 time per week

- 1 time per month
- Sometimes in a year
- Never

6. Why do you use the public transport?
   - For studying
   - For working
   - For personal reason
   - For vacation
   - For social reason
7. Which mode of transport do you frequently use?
   - Train
   - Bus
   - Metro
   - Train+bus
   - Metro+bus
   - Train+Metro
8. Which company of Local Public Transport do you usemore frequently?
   - Ferrovie dello Stato
   - Ferrovie Appulo Lucane
   - Ferrovie del sud est
   - Ferrovie del Nord barese
   - Local bus company (AMTAB)
9. In your opinion, on a scale from 1 to 5, which criteria are important in a public transport service?
   - PUNCTUALITY

     ◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5
   - SAFETY

     ◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5
   - COMFORT

     ◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- ACCESSIBILITY

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- FREQUENCY

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- CROWDING

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- TRAVELTIME

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- POLLUTION

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- INTERMODALITY

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- INFORMATION

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- CLEANLINESS

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- NUMBER OF SEATS

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- TRAVEL FARE

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- REGULARITY OF TRIPS

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- FLEXIBILITY

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

10. In your opinion, on a scale from 1 to 5, how do you evaluate the following mode of transport?

- BUS

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- TRAIN

◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5

- METRO

◯1 ◯2 ◯3 ◯4 ◯5

11. How much are you satisfied with public transport service?
    - VERY SATISFIED
    - AVERAGE
    - NOT VERY SATISFIED
    - NOT SATISFIEDAT ALL

Thank you very much for your cooperation

For the data analysis, we have analyzed305 questionnaires.The majority were online questionnaires.

The following figures show, through pie charts, the results obtained in relation to the first part of the questionnaire.

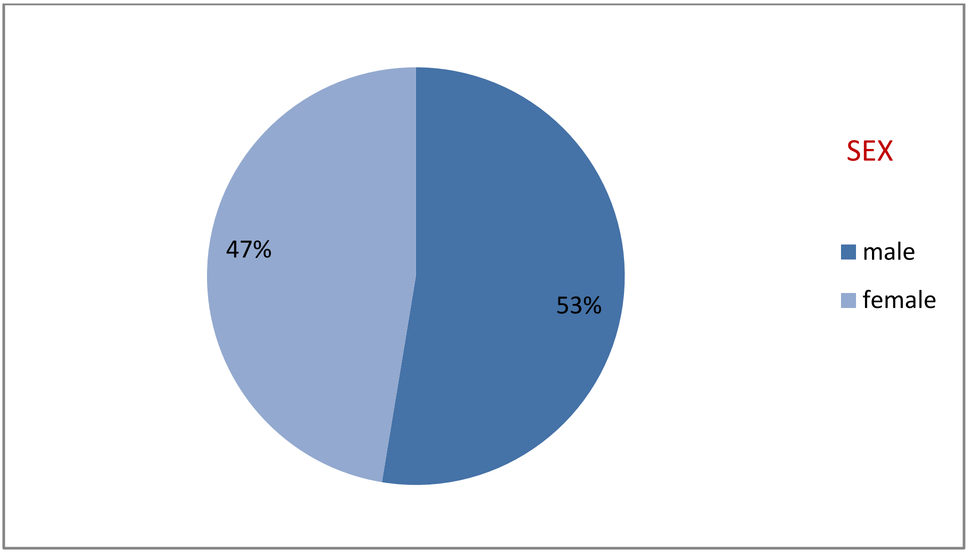


Figure 23 Surveys results about geneder of costumers

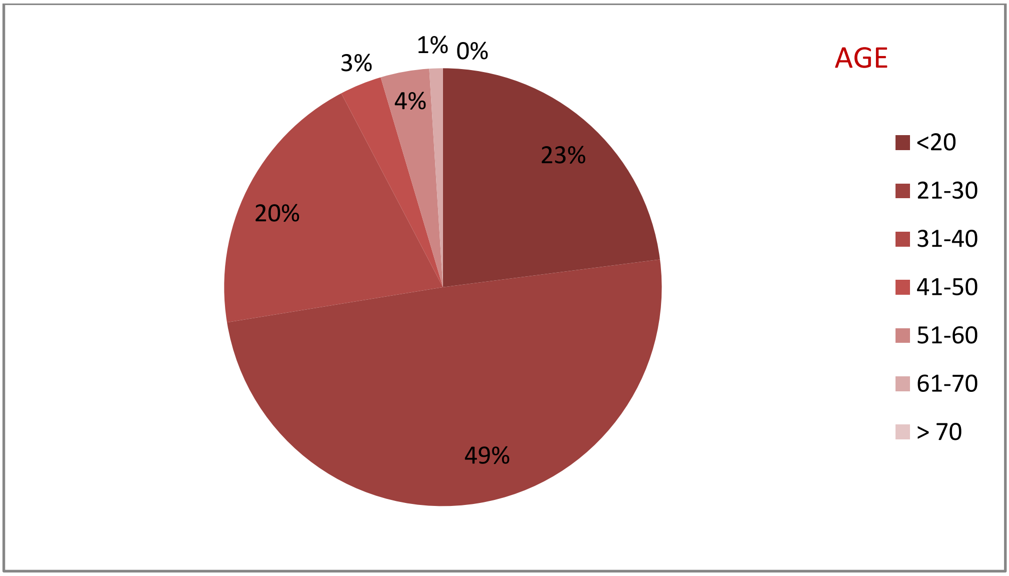
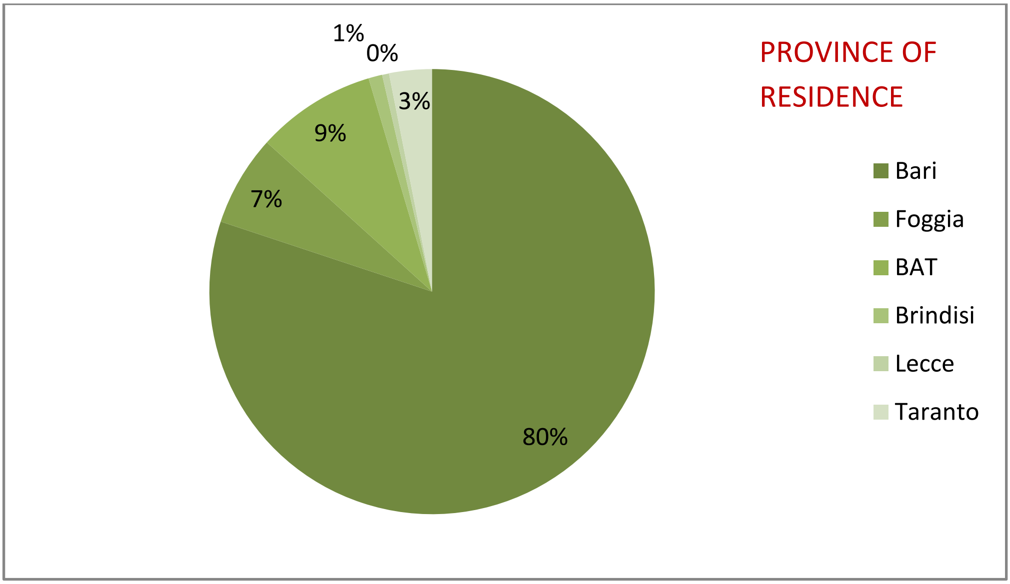


Figure 24 Surveys results about sex and age of customers



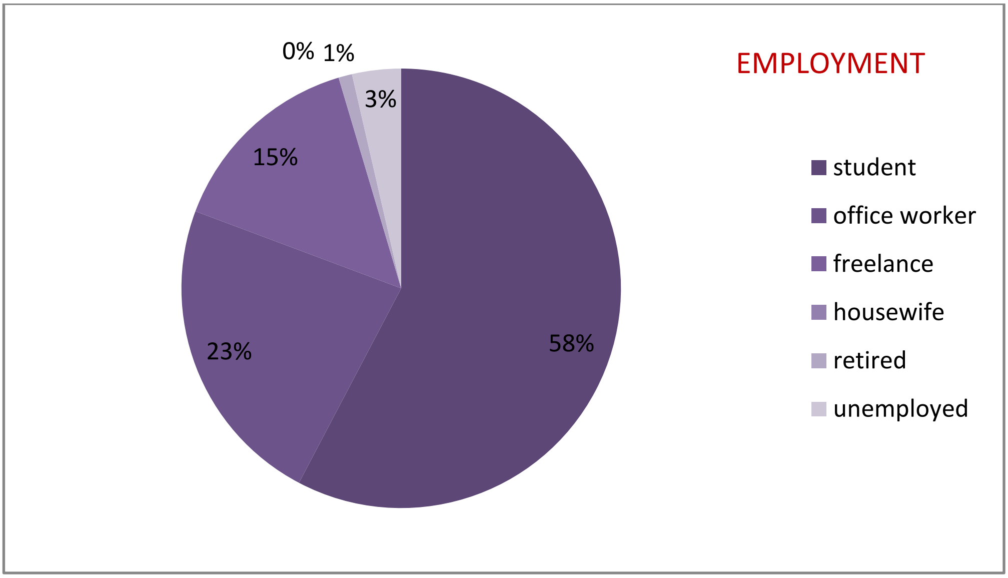


Figure 25 Surveys results about Province of residence and employment

The majority of the customers, according to their employment, uses local public transport for study (51.5%), personal motivation (27%) and work (16.3%).

The means of transport most used is the train (53.1%), followed by the bus (23%) and the mixed transport bus + train (19.4%). A small percentage of 2.6% uses an intermodal transport (train + metro, operated by Ferrotramviaria).

Among the service quality criteria, the customers have assigned the maximum value (5) to punctuality and safety.

The criterion considered less important is the single ticket, which considers the possibility to have an integrated ticket for two or more transportation modes.

The last question of the survey is dedicated to the degree of satisfaction about the used transport mode. The result is quite worrying. In fact, only 1% of customers are very satisfied with transport, while the satisfaction level of the majority, about 40.3%, is below the average.

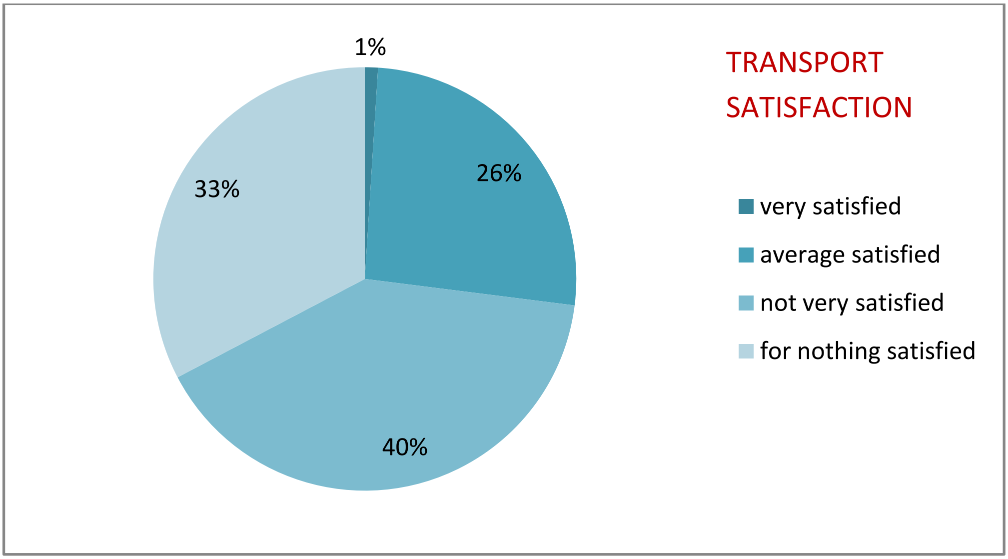


Figure 26 The surveys results about the transport satisfaction

In the next chapter, we explain the way we used these data within the model, to minimize the subjectivity of the analyst.

5.3.2 Company's data collection

The data of the local transport company we analyzed, the Ferrovie Appulo Lucane, were educed mainly from its budget, published on its website.

In particular, we analyzed passengers demand and costs, for both railway and road transport during one year.

The involved costs are the following:

1) Production costs:
   a) Ancillary and consumable raw materials;
   b) Services;
   c) Usage of third parties' goods;
   d) Staff employed;
   e) Amortization and write-downs

f) Provision for risk.

2) Transport costs

a) Buses cost;

b) Trains cost.

Costs and transport demand are related to the time span from 2009 to 2015.

In the following tables are shown data for both trains and buses:

Table 2 Data from Ferrovie Appulo Lucane's budgets - Train

TRAIN	N. PASSENGERS	MILLION PASSENGERS	TRANSPORT COSTS	PRODUCTION COSTS	MEDIUM COSTS
	527584	0,527584	3592000	46328000	94,62
	560000	0,56	3668000	46594000	89,75357
	580000	0,58	4522000	46753000	88,40517
	583956	0,583956	3401000	45592000	83,89844
	1856000	1,856	8620000	46753000	29,83459
	1880000	1,88	7689000	46594000	28,87394
	2099527	2,099527	8335000	45592000	25,68531
	2225373	2,225373	8004000	46328000	24,41478
	2425393	2,425393	7804000	46328000	22,31886
	2985741	2,985741	6994000	46328000	17,85888

Table 3 Data coming from Ferrovie Appulo Lucane's budgets - Bus

BUSES	N. PASSENGERS	MILLION PASSENGERS	TRANSPORT COSTS	PRODUCTION COSTS	MEDIUM COSTS
	944432	0,944432	7967000	46328000	57,48958
	1055000	1,055	8010000	46594000	51,75735
	1075000	1,075	9378000	46753000	52,21488
	1094349	1,094349	8345000	45592000	49,28684
	1250000	1,25	6933000	46753000	42,9488
	1350000	1,35	6014000	46594000	38,96889
	1444607	1,444607	6778000	45592000	36,25207
	1598235	1,598235	6973000	46328000	33,34991
	2037230	2,03723	10173000	46328000	27,73423
	2639290	2,63929	15283000	46328000	23,34378

The diagram, coming from the above data (Table 2,3), shows the relation between transport demand and costs, for both trains and buses. It is worth noting that the break-even point between railway and road transport modes is, in this case, equal to $2.1708e^{+06}$ for the transport demand, and 23.9831 for the medium cost.

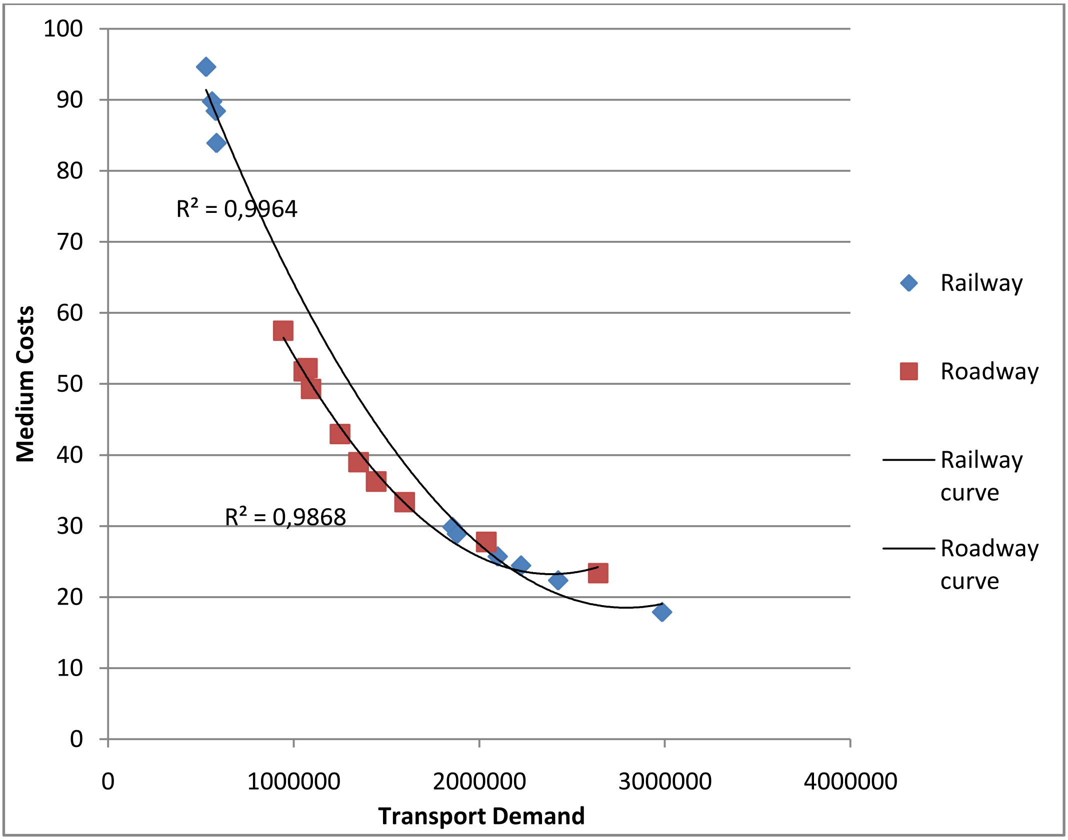


Figure 27 Average Cost curves for Railways and Road Transport.

5.4 The users' optimal mode of transport

To evaluate the users' optimal mode of transport, we have considered the data coming from the survey. These data were used to construct the pairwise comparison of service quality criteria.

As shown in chapter 4, section 4.1, the construction of the pairwise comparison matrix in AHP is made through the Scale of Saaty to make judgments between criteria in pairs. These judgments are typically issued by the decision maker, and we have to check the Consistency of the Matrix. This step is characterized by a high degree of subjectivity in the attribution of judgments, reducing the key rule of surveys in Decision Making Process.

In this model, to avoid subjectivity in attributing judgments of importance among the quality service criteria, we have introduced a system of equation, which allows us to normalize the users' judgments into a scale from 1/9 to 9, as the Saaty Scale required. The users have expressed their preferences on a numerical scale, in which 1 indicates a non-important criterion, 5 indicates a very important criterion.

$$P_{i,j,z} = \begin{cases} 1 & if\ P_{ij} = P_{iz} \\ \frac{P_{ij}}{P_{iz}} * \frac{5}{9} & if\ P_{ij} < P_{iz} \\ \frac{P_{ij}}{P_{iz}} * \frac{9}{5} & if\ P_{ij} > P_{iz} \end{cases}$$

$P_{i,j,z}$ is the preference of i-th user between the j-th criterion and the z- th criterion;

$\frac{P_{ij}}{P_{iz}}$ is the relationship between the user's preference on the j-th criterion and the user's preference on the z-th criterion.

After the normalization of the users' judgments, we have built the service criteria pairwise comparison matrix, which is a 15 x 15 diagonal square matrix.

From this matrix, we have obtained the weights of criteria, called also Criteria Priority Value (*cpv*), which are in the last column of the matrix (see the table n. 4). The *cpvs* quantify the levels of importance (knowledge) a Decision Maker has assigned towards the criteria (Beynon, 2005). To reduce the errors linked to the high number of criteria, we have grouped them into classes, as shown in the section 5.3.1, and then we have calculated the *cpvs*, concerning the classes of criteria. The following table shows the *cpvs* we obtained in relation to criteria and groups of criteria. The main group for users is the group related to the time criteria. In fact, the *cpv* is the highest among the three.

Table 4 Obtained Criteria Priority Values, in relation to criteria and groups of criteria

GROUPS OF CRITERIA	CRITERIA	SINGLE WEIGHT (cpvs)	WEIGHT OF GROUPS (cpvs)
SERVICE CRITERIA	ACCESSIBILITY	0.10	0.30
	SECURITY	0.07	
	CLEANNESS	0.06	
	SEATS	0.03	
	CROWDING	0.03	
TIME CRITERIA	FREQUENCY	0.09	0.47
	TRAVEL TIME	0.06	
	PUNCTUALITY	0.20	
	REGULAR ROUTE	0.06	
	FLEXIBILITY	0.07	
EXTERNAL CRITERIA	INFORMATION	0.05	0.23
	SINGLE TICKET	0.07	
	INTERMODALITY	0.04	
	POLLUTION	0.03	
	TRAVEL FARE	0.03	

After the criteria pairwise comparison, the method involves the formulation of knowledge matrices, which relate each criterion to the chosen alternatives.

In particular, we have considered 3 different alternatives and their combination, as envisaged by the Dempster - Shafer Theory. The chosen alternatives represent the different modes of transport to be included in a metropolitan context. The Decision Alternatives are the following:

{B}(bus), {M}(metro), {T}(train/tram)

To reduce the number of comparisons, in Dempster - Shafer Theory is possible to consider groups of Decision Alternatives, compared with the Frame Of Discernment, which is {B,M,T}. Finally, we have analyzed the following alternatives:

{B}, {M}, {T}, {B,M}, {B,T}, {M,T}, {B,M,T}

In this way, we have obtained 3 knowledge matrix. To construct them, we have used the criteria priority vector, obtained in the criteria pairwise comparison method. We have multiplied these values by the relative importance that the analyst has attributed for each alternative (m_a)and for each criterion:

$$m_a = k * cpv$$

Where:

$m_a =$ is the Basic Probability Assignment for each alternatives;

$k =$ is the *importance value*, from 1 to 9, which the analyst attributes to each alternative in relation to the considered criterion;

$cpv =$ is the Criteria Priority Vector obtained in the criteria pairwise comparison matrix

Table 5 Example of knowledge Matrix –comparison between alternatives for each criterion

criterion	alternatives					*m(bpa)*
alternatives	1	0	0	0	*k*cpv*	*results*
	0	1	0	0	*k*cpv*	*results*
	0	0	1	0	*k*cpv*	*results*
	0	0	0	1	*k*cpv*	*results*
	*1/k*cpv*	*1/k*cpv*	*1/k*cpv*	*1/k*cpv*	1	*results*
TOTAL	results	results	results	results	results	1

The values of the *k* parameter are assigned by the analyst. In this case, we have chosen the *k* value, considering the answers given by the customers about the kind of transport modes they prefer. The obtained Knowledge Matrices are shown below:

Table 6 Knowledge matrix about the Service Criteria

Service Criteria	{B}	{M}	{T}	{MT}	{BMT}	Priority Vectors (*bpa*)
{B}	1.00	0.00	0.00	0.00	1.65	0.16
{M}	0.00	1.00	0.00	0.00	1.98	0.18
{T}	0.00	0.00	1.00	0.00	1.98	0.18
{MT}	0.00	0.00	0.00	1.00	1.98	0.18
{BMT}	0.61	0.51	0.51	0.51	1.00	0.30
TOTAL	1.61	1.51	1.51	1.51	8.57	1.00

Table7 Knowledge Matrix about the Time Criteria

Time Criteria	{B}	{M}	{T}	{BM}	{MT}	{BMT	Priority Vectors *(bpa)*
{B}	1.00	0.00	0.00	0.00	0.00	0.72	0.08
{M}	0.00	1.00	0.00	0.00	0.00	2.51	0.16
{T}	0.00	0.00	1.00	0.00	0.00	1.79	0.14
{BM}	0.00	0.00	0.00	1.00	0.00	1.79	0.14
{MT}	0.00	0.00	0.00	0.00	1.00	2.15	0.15
{BMT}	1.39	0.40	0.56	0.56	0.46	1.00	0.33
TOTAL	2.39	1.40	1.56	1.56	1.46	9.97	1.00

Table 8 Knowledge Matrix about the External Criteria

External Criteria	{BM}	{BT}	{MT}	{BMT}	Priority Vectors (*bpa*)
{BM}	1.00	0.00	0.00	1.25	0.19
{BT}	0.00	1.00	0.00	1.25	0.19
{MT}	0.00	0.00	1.00	2.50	0.28
{BMT}	0.80	0.80	0.40	1.00	0.34
TOTAL	1.80	1.80	1.40	5.99	1.00

We used the Priority Vectors obtained in the knowledge matrices in the Dempster's Rule of the combination. Thus, we have combined the different priority vectors to obtain a single Basic Probability Assignment (*bpa*) for each Decision Alternatives.

Table9 Basic Probability Assignment of transport modes after combing all evidence

bpa_{modes} after combining all evidence						
{B}	{M}	{T}	{BM}	{BT}	{MT}	{BMT}
0.1297	0.3349	0.2423	0.0589	0.0276	0.1581	0.0485

A key parameter while applying the Dempster Rule is the denominator (*1-k*) because it represents the level of conflict between the sources. This measure represents the mass which would be assigned to the empty set if the mass were not normalized. It is crucial to consider this value in the evaluation of the quality of combination: when the value is high (in the case of strong conflict k ≈ 1), the combination may not make sense and may lead to questionable decisions (Beynon, 2000). The k value is also useful because it allows us not to consider the Consistency Index of the AHP.

$$K = 1 - \sum_{A \cap B = \emptyset} m_1(A) m_2(B)$$

$$\mathrm{k} = 0.67$$

In our case the k value is 0.67, showing that the quality of the combination is good.
After the application of Dempster Rule, we have calculated the Belief and Plausibility measures, referred to each mode of transport, as shown in the above table.

Table 10 Belief and Plausibility Measures

MODE OF TRANSPORT	BEL	PLS
{B}	0.1297	0.2647
{M}	0.3349	0.6003
{T}	0.2423	0.4765
{BM}	0.5235	0.7576
{BT}	0.3996	0.6651
{MT}	0.7353	0.8702
{BMT}	1	1

Belief and Plausibility are two key measure in Dempster-Shafer Theory because they allow us to identify the lower and upper bound of the probability of a set of hypotheses. Moreover, using these measures we can get the level of belief in the best mode of transport among a subset of modes within the available alternatives. When the Belief and Plausibility functions are equal, there is no uncertainty in the choice. In this case, the focal elements are singletons, consisting of just one element (Pl (A) = Bel (A)). Yager (2016) underlined that this is a very important special case, the so-called Bayesian belief structure, in which Pl (A) = Bel (A) = Prob (F), where Prob (F) is the Bayesian Probability. In DST, the difference between the Belief and Plausibility measures is defined as the measure of uncertainty.
The table 10 shows that the alternative {Metro and Tram/Train} can be considered the best from the users' point of view. In fact, they have assigned a high value to the punctuality and safety, and both of them respect these characteristics.

5.5 The company's optimal mode of transport

To evaluate the best mode of transport in a metropolitan city from the company's point of view, we have considered the data from the budgets of the Ferrovie Appulo Lucane. We have considered as criteria the demand and the costs of transport, related to the modes of transport. We carried out the evaluation both for railways and road transport modes since the Ferrovie Appulo Lucane operates both modes of transport.

The choice between the two different modes of transport is made on the basis of economic evaluations.

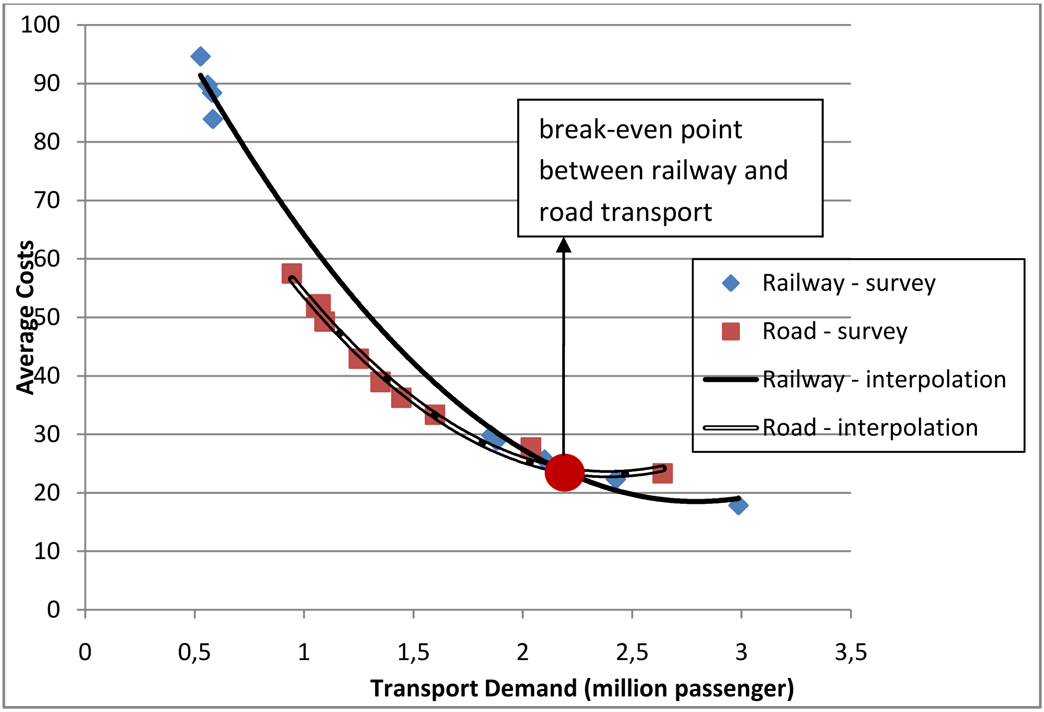


Figure 28 Demand and Average Costs for railways and road transport

In the case of study, we have calculated the break-even point between railway and road transport through the quadratic approximations of the considered data. In the following we have reported the two equations, for railways and road transportation systems:

$$\begin{cases} y_{railway} = 1.42\ E\text{-}11 * x.\hat{}\ 2 - 7.93\ E\text{-}05.*x + 1.29\ E+02; \\ y_{road} = 1.59\ E\text{-}11 * x.\hat{}\ 2 - 7.61E\text{-}05.*x + 1.14E+02; \end{cases}$$

The y parameter represents the average costs, while the x parameter represents the transport demand.
The value of the correlation coefficient $R^2$is 0,996 for the railways, and 0,986 for the road transportation system.
From these equations, we calculated the break-even point that is, in our case, 2.1708 e^{+06} for the demand and 23.98 for the average cost.
The pairwise comparison has been made between two criteria, namely the total cost and the transportation demand. We have assigned the same Criteria Priority Vector, due to the relationship between the two factors.

Table 11 Criteria Pairwise Comparison Matrix, referred to Company's point of view

Criteria	Total Cost	Demand	CPV
Total Cost	1	1	0.5
Demand	1	1	0.5
Total	2	2	1

To formulate the knowledge matrix for the total cost and to apply the AHP method, we made a normalization, starting from the company's data. In our reference system, we assumed that the transportation demand could vary from a minimum value equal to 500,000 passengers to a maximum value equal to 3,000,000 passengers to calculate the preference for a transport mode in relation to the total costs. We have assigned 0 (zero) to the break-even point; in this way, values less than zero show a preference for the road transport, while values greater than zero show a preference for railways. By this method, we can obtain a value of preference between transport modes, knowing the demand for transport or its range.

Table 12 Knowledge Matrix - comparison between alternatives for each criterion

criterion	alternatives					m(bpa)
alternatives	1	0	0	0	k*cpv	results
	0	1	0	0	k*cpv	results
	0	0	1	0	k*cpv	results
	0	0	0	1	k*cpv	results
	1/k*cpv	1/k*cpv	1/k*cpv	1/k*cpv	1	results
sum	results	results	results	results	results	1

We have implemented this model in a Matlab code: entering the available data, we can get the basic probability assignment (*bpa*) and then the Belief and Plausibility measure, referring to the decision problem.

In the considered range of transport demand for both railways and road transport, we have obtained the results shown below:

Table 13 Knowledge Matrix referred to Total Cost Criterion

TOTAL COST	{B}	{T}	{BT}	{BTM}	Priority Vectors (bpa)
{B}	1	0	0	2	0.21
{T}	0	1	0	5	0.32
{BT}	0	0	1	3.0	0.26
{BTM}	0.5	0.2	0.333	1	0.21
Sum	1.5	1.2	1.333	11.00	1

To obtain the Belief and Plausibility measures referred to the Company point of view, we have fused the priority vectors (Table 13) with the Dempster's Rule. For the Metro, the basic probability assignment is equal to zero, because the analyzed company has not the Metro as a mode of transport. Moreover, according to the demand of transport, the use of metro is not justified, due to the low demand.

Table 14 Basic Probability assignment for Transport Company

	{B}	{M}	{T}	{BM}	{BT}	{MT}	{BMT}
$bpa_{company}$	0.2192	0	0.3205	0	0.2451	0	0.2150

The results are shown below:

Table 15 Belief and Plausibility measures

MODE OF TRANSPORT	BEL	PL
{B}	0.219	0.679
{M}	0	0.215
{T}	0.320	0.780
{BM}	0.219	0.679
{BT}	0.784	1
{MT}	0.320	0.780
{BMT}	1	1

These results (Table 15) show that from the company point of view the best mode of transport to implement in the Bari context is given by the intermodal transport. In fact, we have a high value of both Belief and Plausibility with an acceptable difference between them. This result shows also that metro is not suitable in a metropolitan context where transport demand is not so high.

5.6 The joint optimal mode of transport

This research tries to fuse the users' and the company points of view in order to find the most suitable mode of transport in a metropolitan context. In the previous sections, we found the optimal mode for users and the transport company separately. To obtain the joint optimal mode of transport we have fused the two results using the Dempster Rule of combination. In this way, it is possible to make a decision, which takes into account the positions of different stakeholders.

In particular, we have obtained the following total basic probability assignments combining two different sources, namely users and company. In particular, the Matlab code, which we have created for this model, gives us the ability to calculate the basic probability assignments and then the Belief and Plausibility measure, in two different way:

1. If we know the exact demand of transport, we put the real number in order to obtain specific basic probability assignment.
2. If we know the range of transport demand, which is in our case among 500,000 and 3,000,000 of passengers, we calculate several basic probability assignments, increasing the demand on the basis of a fixed step.

The following results have been calculated considering both point of view, the users and the transport company.

Table16 Total Basic Probability assignments

	{B}	{M}	{T}	{BM}	{BT}	{MT}	{BMT}
bpa_{total}	0.3888	0.1293	0.40356	0.01449	0.0392	0.0103	0.0147

In order to represent the degree of belief in a mode of transport, we have calculated the Belief and Plausibility measures as shown in the next table.

Table 17 Belief and Plausibility measures referred to users and company point of view

MODE OF TRANSPORT	BEL	PL
{B}	0.388	0.457
{M}	0.129	0.169
{T}	0.404	0.468
{BM}	0.532	0.596
{BT}	0.831	0.871
{MT}	0.543	0.612
{BMT}	1.000	1.000

The table shows that the most suitable mode of transport for the metropolitan city of Bari considering both the users' and the company's point of view is the intermodal transport bus and tram/train. Furthermore, with the application of this model, it is possible to reduce the uncertainty embedded in the human judgment. In fact, comparing the tables n. 10 – 14 – 16, we can underline that the difference between Belief and Plausibility, as well as uncertainty in the choice of transport mode, decreases to very low values.

In the next figure (Fig. 27) we have reported histograms representing the uncertainty in choosing the most suitable mode of transport for both the transport company and the users. The last column of the histogram represents uncertainty after combining all evidence. In this way, we can see that the joint uncertainty is less than the single uncertainties.

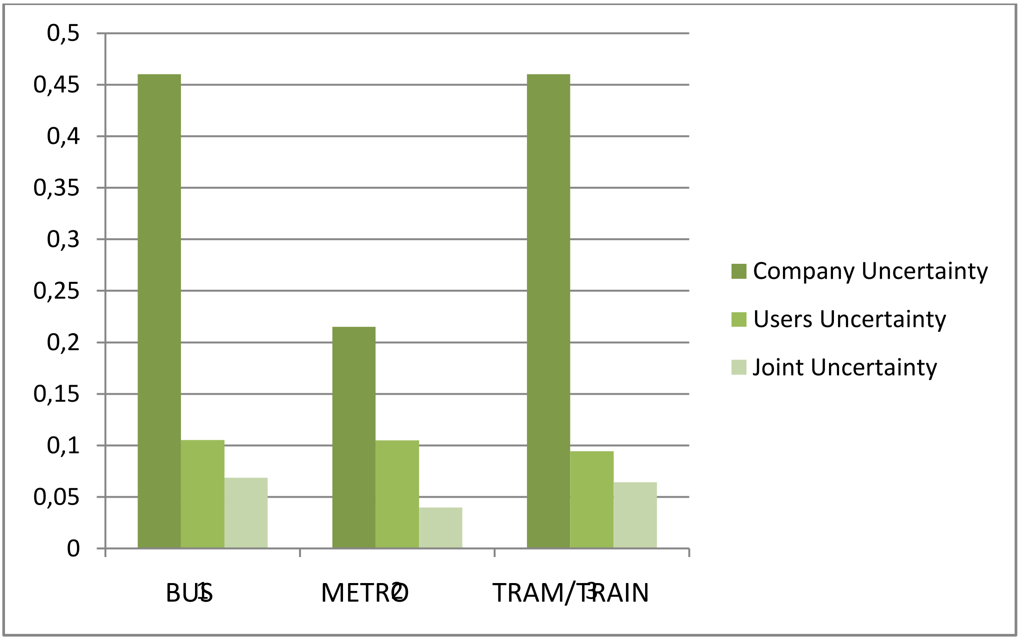


Figure 29 Evaluated Uncertainty for different modes of transport

To obtain the Probability measure, we have applied the Belief Transformable Model (TBM) to the basic probability assignment reported in table 18.

When applying the TBM, the Basic Probability Assignments considered in DS/AHP method become the Basic Belief Assignments (bba). The *bbas* m_n are then transformed into the pignistic probabilities BetP:2Ω→[0.1] by the so-called pignistic transformation:

$$\mathrm{BETP}(x) = \sum_{x \subseteq A \in R} \frac{m(A)}{|A|}$$

In this way, we have obtained the probability value for the three modes of transport, as shown in the following table:

Table18 Pignistic Probabilities referred to the modes of transport

	{B}	{M}	{T}	{BM}	{BT}	{MT}	{BMT}
bpa_{total}	0.3888	0.1293	0.40356	0.01449	0.0392	0.0103	0.0147
BET	0.4205	0.1465	0.4332	//	//	//	//

The results show that the most suitable mode of transport, in probability terms, is the tram/train, with a probability of 0.43 (third column of the previous table). The pignistic probability doesn't allow considering the combination of the focal element in the frame of discernment; however, it is possible to obtain the probabilities associated withalternatives of a particular decision.

In the following figure, we highlight the trend of probability measures related to the three main modes of transport, varying the transport demand. The probability measures are on the ordinates, while the transport demand is on abscises. In this graph, it is possible to see the break-even point between Road and Railway curves, which represents the same usefulness in the use of the bus or the tram/train.

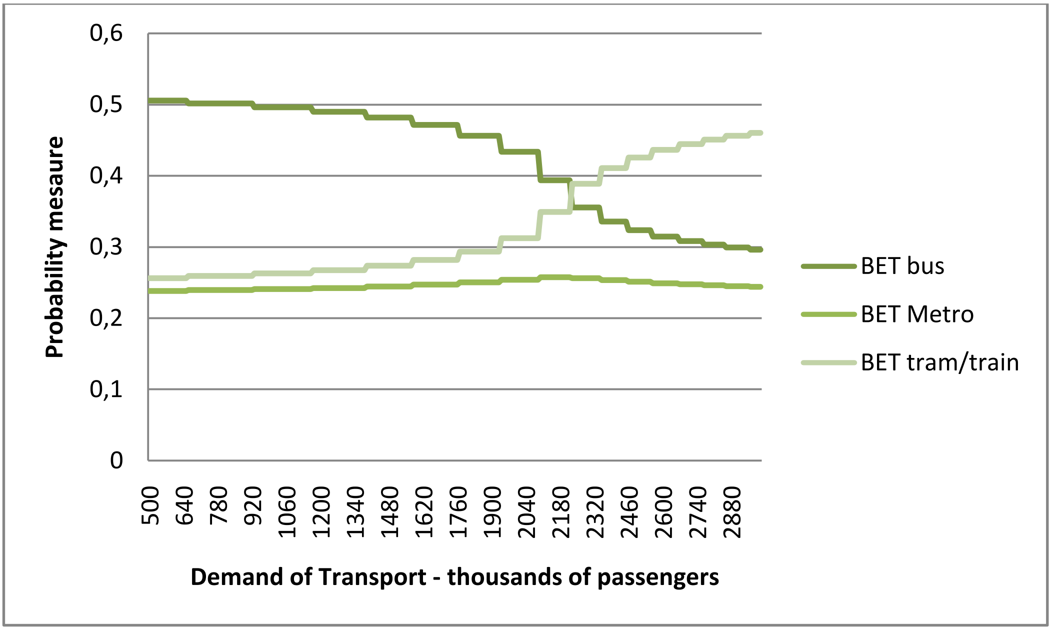


Figure 30 Representation of probability measures as demand of transport varying

As for the probability measures, the trend of Belief Measures, related to the modes of transport and their intermodality, varies according to the transport demand. In this case, on the ordinates there are the Belief Measures. In the graph, it is possible to see the break-even point between the Road and Railway curves. Moreover, there is an intersection between intermodality Bus-Metro and Metro-Tram/Train. Finally, this graph underlines that the most suitable mode of transport is the intermodality Bus-Tram/Train. In fact the {BT} curve has a steady trend in terms of Belief. (Annex A for the total final Belief as varying the demand of transport for each mode of transport)

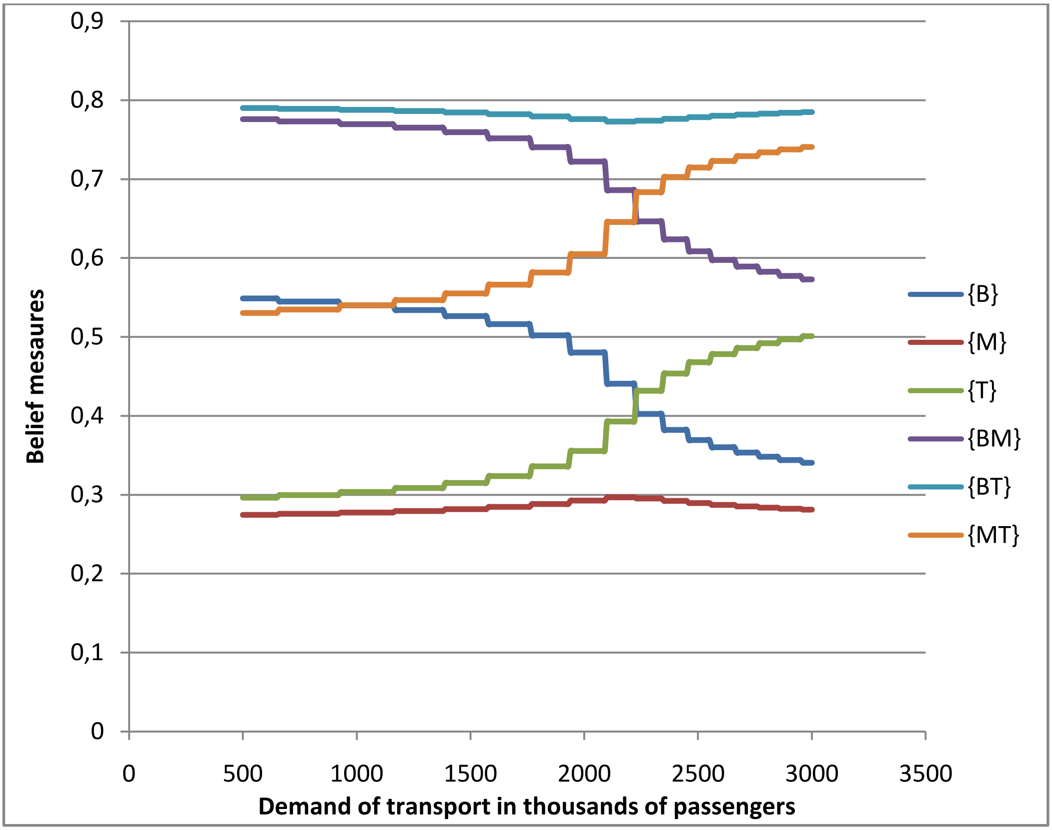


Figure 31 Representation of Belief measures as demand of transport varying

6. Conclusion

The local public transport sector needs a more careful planning, in order to ensure the sustainability of the system. In this work, we have dealt with two main features of the public transport system, which are the quality of the service and the uncertainty linked to the human reasoning way. The main purpose of this work was to combine these two aspects into a single model, to identify the most suitable mode of transport to implement in a metropolitan city. The model takes into account the users point of view about the quality parameters and the company's point of view about the economic aspects (total costs of transport and demand).

In particular, the proposed model takes into account some important characteristics:

1. Understanding the cause and effect relationships among system variables;
2. Modeling the decision-making structure;
3. Considering the uncertainty embedded in human reasoning way
4. Fusing the data coming from different sources
5. Evaluating the goal achievement of the alternatives

Thanks to the fusing of the two different points of view (users and transport company), the analysts have a complete information about the system and also about the needs of different stakeholders. This is a very important aspect in planning transport because the presence of a complete information reduces the analysts' subjectivity.

The methodology applies two theoretical concepts. First, the Analytical Hierarchy Process is developed to describe decision structure to model the transportation decision-making systems. Second, the Dempster-Shafer Theory of Evidence is applied as a mathematical mechanism to analyze the degree of goal achievement of transportation alternatives.

The main advantages of this method are the flexibility in handling different patterns of knowledge and opinions including incomplete, approximate, and conflicting information or even ignorance; also, the capability to measure different types of uncertainty associated with knowledge. In this way, both the analysts and users can focus debates and improve analyses. Another important aspect of this method is its capability to reduce the analyst subjectivity; therefore, it results useful in planning processes, with a lot of factors involved in decision making. In fact, in this way, the rules of the analyst are:

1. collect data coming from users;
2. study the demand of transport and related costs, in order to obtain the break-even point between demand and average costs;
3. apply the model.

In the case study, we confirmed the usefulness of the method in reducing uncertainty, thanks to the fusion of data coming from users and transport company.

In the further studies, the proposed method will be applied to larger scale systems; for example, to all Transport Companies in Apulia Region.

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Appendix A

The total Plausibility Measures as varying the demand of transport, for each mode of Transport and intermodality.

Demand of transport in thousands of passengers	pl{B}	pl {M}	pl {T}	pl {BM}	pl {BT}	pl {MT}	pl {BMT}
500	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
510	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
520	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
530	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
540	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
550	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
560	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
570	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
580	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
590	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
600	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
610	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
620	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
630	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
640	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
650	0,548816	0,274452	0,29639	0,775754	0,789854	0,53008	1
660	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
670	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
680	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
690	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
700	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
710	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
720	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
730	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1

740	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
750	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
760	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
770	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
780	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
790	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
800	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
810	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
820	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
830	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
840	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
850	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
860	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
870	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
880	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
890	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
900	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
910	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
920	0,544852	0,275798	0,299649	0,772902	0,788823	0,534486	1
930	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
940	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
950	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
960	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
970	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
980	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
990	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1000	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1010	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1020	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1030	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1040	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1050	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1060	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1070	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1

1080	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1090	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1100	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1110	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1120	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1130	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1140	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1150	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1160	0,540035	0,27739	0,303631	0,769402	0,787604	0,539824	1
1170	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1180	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1190	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1200	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1210	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1220	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1230	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1240	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1250	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1260	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1270	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1280	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1290	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1300	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1310	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1320	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1330	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1340	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1350	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1360	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1370	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1380	0,534052	0,279299	0,308614	0,764998	0,786142	0,546432	1
1390	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1400	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1410	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1

1420	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1430	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1440	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1450	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1460	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1470	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1480	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1490	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1500	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1510	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1520	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1530	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1540	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1550	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1560	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1570	0,526404	0,281623	0,315043	0,75927	0,784363	0,55484	1
1580	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1590	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1600	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1610	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1620	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1630	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1640	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1650	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1660	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1670	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1680	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1690	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1700	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1710	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1720	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1730	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1740	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1750	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1

1760	0,516246	0,284493	0,323695	0,751486	0,782165	0,565935	1
1770	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1780	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1790	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1800	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1810	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1820	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1830	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1840	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1850	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1860	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1870	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1880	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1890	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1900	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1910	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1920	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1930	0,501998	0,288075	0,336061	0,740199	0,779423	0,581351	1
1940	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
1950	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
1960	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
1970	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
1980	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
1990	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2000	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2010	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2020	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2030	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2040	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2050	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2060	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2070	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2080	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1
2090	0,480201	0,292447	0,355555	0,722017	0,776075	0,604568	1

2100	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2110	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2120	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2130	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2140	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2150	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2160	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2170	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2180	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2190	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2200	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2210	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2220	0,440698	0,296491	0,3929	0,685859	0,772978	0,645356	1
2230	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2240	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2250	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2260	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2270	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2280	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2290	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2300	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2310	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2320	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2330	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2340	0,402363	0,295051	0,43193	0,646333	0,774081	0,683161	1
2350	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2360	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2370	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2380	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2390	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2400	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2410	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2420	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2430	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1

2440	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2450	0,382128	0,292036	0,453704	0,623605	0,77639	0,702367	1
2460	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2470	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2480	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2490	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2500	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2510	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2520	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2530	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2540	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2550	0,369208	0,289299	0,468029	0,608422	0,778485	0,714362	1
2560	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2570	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2580	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2590	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2600	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2610	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2620	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2630	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2640	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2650	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2660	0,360129	0,287009	0,478286	0,597449	0,780239	0,722668	1
2670	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2680	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2690	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2700	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2710	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2720	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2730	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2740	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2750	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2760	0,353359	0,28511	0,486033	0,589109	0,781693	0,728799	1
2770	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1

2780	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1
2790	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1
2800	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1
2810	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1
2820	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1
2830	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1
2840	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1
2850	0,3481	0,283524	0,492109	0,582539	0,782907	0,733524	1
2860	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2870	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2880	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2890	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2900	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2910	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2920	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2930	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2940	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2950	0,343889	0,282187	0,497009	0,577223	0,783931	0,737287	1
2960	0,340437	0,281047	0,50105	0,572828	0,784804	0,740356	1
2970	0,340437	0,281047	0,50105	0,572828	0,784804	0,740356	1
2980	0,340437	0,281047	0,50105	0,572828	0,784804	0,740356	1
2990	0,340437	0,281047	0,50105	0,572828	0,784804	0,740356	1
3000	0,340437	0,281047	0,50105	0,572828	0,784804	0,740356	1