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Modeling horizontal and vertical equity in the public transport design problem: A case study

Rosalia Camporeale, Leonardo Caggiani, Michele Ottomanelli*

Polytechnic University of Bari, Dept. of Civil, Environmental, Land, Building Engineering & Chemistry, Transportation@Poliba Research Group, Via Orabona, 4, 70126 Bari, Italy

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ABSTRACT

In the transportation literature, equity has been and is still used with a variety of meanings and purposes. Traditionally, equity has been considered in strategic transport planning but very few works have been addressing it in a quantitative way, detailing how to explicitly consider it at a transportation design level (tactical and/or operational) focusing on the consequent social role of transportation.

This paper deal with how quantitatively incorporate spatial and social equity principles in the Transit Network Design Problem. With respect our previous preliminary study, this paper goes a step further in the definition of the solution to the problem, proposing a starting candidate route set generation procedure as preliminary step to solve before the main optimization. The objective function considers at the same time the cost of users, operators and unsatisfied demand, and a comprehensive horizontal and vertical equity indicator is also specified among the constraints of the problem. An extensive sensitivity analysis investigates how the costs of the system vary with respect to the achieved level of equity. Then, an application to a real case of study is presented to validate of the proposed methodology and highlight its usefulness and performances.

1. Introduction

The role of public transport is crucial in any society, especially to promote a more sustainable and equitable urban development. It can bridge the mobility gap between captive and choice riders (Welch, 2013) since it can provide people with mobility and access to employment, education, health and any other kind of social, recreational and community facilities. To properly perform this task, public transport systems must guarantee a high-quality transit service, in order to be attractive to non-captive users and at the same time be affordable for those groups that lack private transportation. It is often difficult achieving these goals, being still financially viable for subsidizing local and central governments (Ibarra-Rojas et al., 2015).

The equitable distribution of transit services is a major concern of transportation planners and policymakers worldwide. As stated by Krumholz and Forester (1990), promoting a wider variety of choices for people who have fewer ones is the first step towards an equitable planning. With a suitable level of access and geographic coverage for everybody, it is possible to achieve a spatial equity (Murray and Wu, 2003), i.e. a spatial distribution of stops able to ensure a good balance of travel speeds and short access distances, according to the urban structure and the related pattern of transit lines.

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* Corresponding author.
Email addresses: rosalia.camporeale@poliba.it (R. Camporeale); leonardo.caggiani@poliba.it (L. Caggiani); michele.ottomanelli@poliba.it (M. Ottomanelli)

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There are a number of reasons – such as the aging of population, the rising fuel prices, the renewed health and environmental concerns, and so on – that are contributing to changing consumer preferences. Transport policies are changing accordingly, contributing to reduce automobile travel and increase demand for alternative modes, provided they are comfortable, convenient and affordable (Litman, 2006).

In particular, the last decades are witnessing a gradual shift from a 'mass transit' planning (whose philosophy suggests that all people living within a community deserve equal access to public transportation), to the idea that social inclusion needs to become an integral part of the transit planning process (Kaplan et al., 2014). Some social groups are more likely to require public transport services: above all, low-income and socially disadvantaged individuals that are the most transit-dependent (Denmark, 1998; Pucher and Renne, 2003; Sanchez et al., 2004; Dodson et al., 2007), as they cannot afford a car and therefore often are troubled by having access to their desired destinations (Lucas, 2012). To ensure the members of these vulnerable groups an equal range of opportunities, societies should strive for guaranteeing an equitable provision of public transport services. We could cite some good examples of social equity objectives pursued in several urban transportation plans (Manaugh et al., 2015); however, too often these aims are not adequately translated into specific objectives, and it seems to be a lack of quantitative indicators able to assess the related achievements.

In this framework, this paper contributes with a step forward to our previous research (Camporeale et al., 2016). As first part of the main solution framework, an alternative candidate route set generation procedure has been introduced: it has the advantage to be easily applicable to real-sized case studies. Moreover, we provided two sensitivity analysis and an application to a real case study, that were missing in our preliminary formulation, in order to prove the validity of the model. The final goal of this paper is to propose a method able to embrace the needs of vulnerable categories since the planning stage of a public transportation system, trying to avoid them experiencing an exacerbate social exclusion and pointing at a more equitable community. In other words, we assert that a satisfactory level of equity in transportation systems can be reached only if equity is already included in the Transit Network Design Problem (TNDP), determining a set of bus routes (and associated frequencies) convenient for both users of the system and operators.

In the following sections, a literature review on both the issues covered (i.e. TNDP and Equity) aims at explaining what a Transit Network Design Problem is, providing also a detailed explanation of the equity concept. Then, we introduce the proposed two-steps solution methodology, specifying how it embeds equity considerations in the formulation of TND Problem. By means of an extensive sensitivity analysis and by an application to a real case of study a test of the effectiveness of the model and the opening of a discussion about the achieved results are carried out.

2. Literature review

In this section, we present some basic theoretical background of the problem and the relevant main literature. Since the proposed methodology involves different topic, for sake of clarity, we have divided this section into three subsections. The first one deals with the Transit Network Design Problem; the second one focuses on the problem of equity in transportation and, finally, we point out the problem of quantifying the equity in TNDP.

2.1. The transit network design problem

The transit planning process requires the solution of the Transit Network Design problem (TNDP), as it encloses every decision taken before the system comes into operation. The TNDP problem is commonly divided into several sub-problems that embrace strategical, tactical, and operational decisions (Desaulniers and Hickman, 2007; Ceder, 2007), including the design of routes, frequencies, time schedules, fleet size, and number of employees.

In particular, the objective of TNDP consists in the design of the bus lines layout, together with the determination of their associated operational characteristics. It involves:

- the definition/arrangement of lines/routes and bus stop locations;
- the selection of the frequencies, to the time of the day and the synchronicity of transfers, fleet size, and resources, which should be assigned to each transit line. TNDP is crucial because the overall cost of the system largely depends on it.

Several objective functions, constraints, solution approaches and methodologies, as well as search algorithms, are proposed in the literature (Pattnaik et al., 1998; Fan and Machemehl, 2006; Cipriani et al., 2012; Cancela et al., 2015). Solution methodologies for TNDP can be roughly categorized into three categories, namely: mathematical optimization searching for exact solutions, heuristics, and meta-heuristics. In this paper, we are dealing with a meta-heuristic approach.

Meta-heuristics are approximate methods that efficiently implement iterative mechanisms to explore a large part of the solution space aiming to find the global optimal solution or at least a local one (D'Acierno et al., 2014). Some examples are constituted by Genetic Algorithm (GA) (Chakroborty and Dwivedi, 2002; Fan and Machemehl, 2011; Szeto and Wu, 2011; Cipriani et al., 2012; Chew et al., 2013; Amiripour et al., 2014; Nayeem et al., 2014), Simulated Annealing (Fan and Machemehl, 2006; Yan et al., 2013) and Ant Colony Optimization (ACO) (Yang et al., 2007; Yu et al., 2012).

Once having defined the transit network to operate on, the aim of a TNDP is the minimization of an objective function, usually meeting multiple and conflicting objectives. The purposes of several stakeholders can be embedded: the users/passengers of the transit system; the authorities with responsibility for the system regulations; and the service operator (Ceder and Israeli, 1998; Deb et al., 2002). Each of them tends to have different goals, and consequently, it becomes important to reach a trade-off between their interests.

A recent comprehensive literature review concerning the TNDP is reported in Ibarra-Rojas et al. (2015). Reviewing the works from 1975 to 2014, we can see that the equity aspects are neglected in most cases, except for Chen and Yang (2004) and Fan and Machemehl (2011). In particular, these last propose a bi-level optimization model to solve the public transportation network redesign problem, in which the spatial equity issue is explicitly considered for the first time. Among more recent works, we can mention Li et al. (2016), that addressed the development of an optimal routing design for feeder buses in suburbs, able to provide social benefits; and Ruiz, Segui-Pons and Mateu-LLadó (2017), that proposed an integrated methodology of bus frequency modeling that could enhance at the same time the public transport service level and the social equity.

2.2. The equity issue

The concept of equity has been mainly referred to a fair distribution of benefits and costs. However, entailing in its definition a moral judgment, what constitutes an equitable distribution has been always difficult to define with certainty (Wee and Geurs, 2011). Individuals, groups of people and regions inevitably have an unequal access to different destinations (activities/opportunities). This lack of equity (although not necessarily problematic), might become less acceptable when dealing with disadvantaged categories of people.

In this section, the two main perspectives of equity that may be considered in transportation planning are briefly described: horizontal and vertical equity. The former emphasizes the importance of treating equally people in equal circumstances. It is essentially based on egalitarian theories, and consequently, opposes unjustifiable preferences of one individual or group over another. As an example, a planner assuming this point of view would try to distribute burdens and benefits deriving from a public transportation project evenly throughout the community.

However, one of the major problems with horizontal equity approach is that it fails to consider appropriately the existing social inequalities. Therefore, an alternative is represented by a vertical equity perspective, that concerns the distribution of benefits between groups with different needs. In this case, a distribution is considered fair if it provides larger/better resources to the most disadvantaged individuals or groups (Krumholz and Forester, 1990): in other words, according to the purpose of this paper, if these vulnerable categories receive priority consideration for public transportation projects.

It is clear that often these two types of equity can overlap or conflict. A decision might seem fair according to one criterion but inequitable according to the other. If disadvantaged groups are being prioritized, then everyone is not being treated equally.

2.3. Quantifying spatial and social equity

The attempt of incorporating equity principles in the development of transportation systems has been increasingly concerning transportation planners and decision makers, especially in the last decades (Bertolaccini, 2013).

Delbosc and Currie (2011) have recently suggested a single system-wide measure able to quantify the horizontal (i.e. spatial) equity of transit service distribution throughout a metropolitan region. Basically, they modified the traditional Gini coefficient (Gini, 1912) that compares a population's distribution income to a line representing perfect equality. Instead, the Delbosc and Currie's modified Gini coefficient (from now on, we denote it as D&C_Gini), can be properly specified to measure how well transit supply meets transit demand. A perfectly even distribution of supply would result in a Gini coefficient of 0, while a perfectly unequal distribution would result in a coefficient of 1. As an example, two overall coefficients of the D&C_Gini index calculated by Delbosc and Currie (2011) are: for Melbourne, Australia was 0.68, i.e. around 70% of population shares only 19% of transit service; while Baltimore City has a slightly lower equity of transit services with a D&C_Gini index of 0.7083.

In Currie (2010), it is suggested to calculate the level of transit service supply in a given area as

$$\mathrm{SI_D} = \sum_b \left(\frac{\mathrm{Area}_{\mathrm{B}b}}{\mathrm{Area_D}} \cdot \mathrm{SL}_{\mathrm{B}b} \right) \tag{1}$$

where $\mathrm{SI_D}$ is the supply index for the district (traffic zone) D under analysis, b is the number of walk access buffers to stops/stations in each district, B_b is the buffer b for each stop/station in each district, Area is the square kilometer spatial area, $\mathrm{SL}_{\mathrm{B}b}$ is a service level measures (number of public vehicle arrivals within a given time period).

The transit supply index accounts for both the spatial coverage of a district by walk catchments to public transport and for the quality of the service itself. Despite it has some limitations, the ease of calculation makes it a practical choice for practitioners to usefully characterize the level of supply in a certain reality.

The most important function of public transit lies in providing access to all members of a society, particularly to those with limited mobility choices (Manaugh and El-Geneidy, 2010). Since socially disadvantaged groups should receive some priority in public transportation planning, it is important to define correctly these groups. A common way to do it is by means of a social indicator

(Foth et al., 2013), that are instruments capable of identifying underprivileged groups lacking access to goods and resources, comparing them to the rest of society (Townsend et al., 1988).

A number of factors contribute to transport disadvantage: age, disability, income, ethnicity, just to mention some of them (have a look at Murray and Davis, 2001 for a more extensive list of groups potentially involved). Choosing properly the variables to be included is the most important aspect that leads to the generation of social indicators.

We could say that, in practical applications, these indicators can be built from socio-demographic and economic information. We can find an example in Ruiz et al. (2014), who obtained a social indicator of Public Transport Need (PTN) for each using the following equation:

$$\mathrm{PTN_D} = \sum_{y} \mathrm{w_y x_y} \tag{2}$$

where D is the district under analysis, *y* is the considered variable (i.e. adults without cars, persons aged over 65 years, persons with a disability pension, low-income households, students, etc.); $\mathrm{w_y}$ is the weight assigned to each variable, representing its relative importance within the social framework of the study area/city; $\mathrm{x_y}$ is the value of the variable *y*. $\mathrm{PTN_D}$ has to be less or equal to $\mathrm{Pop_D}$, i.e. the total population residing in the district D. Each area/city or region under investigation has its own social features to be studied and examined, and expertise and data are strictly required to identify the patterns of transport disadvantage that need to be included in the construction of the final index.

A considerable number of works in literature deals with the measure of transit equity. Only looking at the most recent among them, we can mention Ricciardi et al., 2015, that focused on the equity public transport provision among disadvantaged cohorts; Griffin and Sener, 2016, that demonstrates a method for income-based transit equity analysis providing results for nine cities in the US; Mortazavi and Akbarzadeh, 2017, with their calculation of public transit service quality for each traffic analysis zone, that indicates the amount of benefit that each zone is receiving from the transit system; or Jang et al., 2017, that developed a methodology for calculating the index of the spatial equity for the public transportation services for the city of Seul, using a Gini coefficient based on the accessibility to the services. In a recent work, Gallo (2017), proposes to use as equity indicator the satisfaction variable computed within the framework of a random utility-based travel demand systems model.

However, we can notice that what these and most other works in literature have in common is that they evaluate equity ex-post: none of them deals with the application of the concept in the planning stage of a new (or re-designed) transit network, but they study the situation of existing cases or compare different areas and/or vulnerable groups. On the contrary, the best way to be confronted with societies with fairer transit systems is to plan them differently, trying to incorporate a renewed set of principles in designing new realities. In the following, we want to fill this apparent gap in the literature by suggesting a two-step methodology which adds an equity constraint to the transport network design problem, accounting for both spatial and social equity aspects.

3. Mathematical notation

Every road/transit network can be modeled by means of a directed graph G = {N, A}, establishing a finite number of nodes $n \in \mathrm{N}$ to be connected by arcs $a \in A$. We define 'route' a sequence of adjacent nodes (and then arcs) in G and 'transfer path' a cumulative path using more than one route. Each ar*c* has an associated cost $\mathrm{c_a}$ that represents the in-vehicle (or onboard) travel time, i.e. the time spent by vehicles to travel on it. The demand corresponding to a given zonal partition (traffic zone) is considered concentrated in centroid nodes and it is represented by an origin–destination matrix OD = $\{d_{ij},\ i,j \in [1 \ldots \mathrm{n}]\}$, where d_{ij} denotes the demand from node *i* to node *j*, expresses in trips per time unit in a given time period.

Along the paper the following notation will be used:

	Sets and indices
$n \in \mathrm{N}$	nodes
$\mathrm{a} \in \mathrm{A}$	arcs
$t_k \in T$	terminal pairs
$\bar{t}$	number of terminal pairs
$m \in M$	length intervals
$\bar{m}$	number of length intervals
$\mathrm{r_k}$	generic route
$\mathrm{f_k}$	frequency of the generic route $\mathrm{r_k}$
$\mathfrak{R}$	candidate set of routes
$\mathfrak{R}'$	generic set of routes that can be selected, $\mathfrak{R}' \subset \mathfrak{R}$,
$\mathfrak{R}^*$	optimal set of routes, $\mathfrak{R}^* \subset \mathfrak{R}$
F'	generic set of frequencies that can be selected, associated to the routes $\mathrm{r_k} \in \mathfrak{R}'$
F^*	optimal set of frequencies associated with the optimal routes
$a' \in \mathrm{A}$	arcs which constitute the optimal routes
u	number of optimal routes
D	district (traffic or travel demand zone)
b	number of walk access buffers to stops/stations in each district D

B_b	buffer b for each stop/station in each district D
y	socioeconomic variables
	Data and variables
Area	square kilometer spatial area
SL	service level measure
w_y	weight of the variable y
x_y	value of the variable y
Pop_D	total population in the district D
L_{min}	minimum length of any route in the transit network
L_{max}	maximum length of any route in the transit network
α	maximum allowed deviation (%) from the shortest path for any OD pair path
k_{tr}	maximum number of transfers in a path (number of vehicle changing)
u_{min}	minimum allowed number of routes in the network
u_{max}	maximum allowed number of routes in the network
h_{min}	minimum headway required for any route
h_{max}	maximum headway required for any route
W	maximum bus fleet size available for operations on the route network
P	capacity of vehicles operating on network
η	desired vehicle occupancy
δ	minimum percentage of the total demand to cover
β	maximum value for the equity constraint
p	number of nodes constituting the shortest path
Δt	number of nodes for each length interval $m \in M$
d_{tot}	total demand on the transit network
d_S	share of d_{tot} covered by routes in R directly (without transfers) or indirectly
Q	positive even number
z	overall social cost of the final transit network
$\gamma_1,\gamma_2,\gamma_3$	weights reflecting the relative importance of user cost, operator cost, and unsatisfied total demand cost
c_a	cost associated to arc a (in-vehicle travel time)
v_a	traffic flow on arc a
C_v	bus operating cost per hour (currency/vehicle/h)
C_m	value of time (currency/min)
C_d	value of each unsatisfied transit demand (currency/person)
O_v	operating time for bus running on any route (hour)
T_{rk}	round trip time of route r_k
h_{rk}	bus headway operating on route r_k (min/vehicle)
L_{rk}	overall length of route r_k

4. Model formulation

In this section, we explain in detail the proposed problem that allows determining the set of optimal routes, with associated costs and frequencies. In particular, to solve this optimization problem we use a GA solution approach. The objective function to be minimized corresponds to the overall social cost of the final transit network, assumed equal to the weighted sum of user, operator (the planner aims to make the best use of limited resources to optimize/improve the network performance) and unsatisfied demand (i.e., total travel demand excluding the transit demand served/satisfied by a specific network configuration) costs. They are represented respectively by the first, the second and third term in the sum given in Eq. (3):

$$\min z = \gamma_1 \cdot \left(\sum_{a'} c_{a'}(\mathfrak{R}') \cdot v_{a'} \right) + \gamma_2 \cdot \frac{C_v}{C_m} \cdot O_v \cdot \left(\sum_{k=1}^{u} \frac{T_{r_k}}{h_{r_k}(F')} \right) + \gamma_3 \cdot \frac{C_d}{C_m} \cdot (d_{tot} - d_S(\mathfrak{R}', F')) \tag{3}$$

subject to

$$u_{min} \leqslant u \leqslant u_{max} \quad \text{(numbers of routes)} \tag{3.1}$$

$$h_{min} \leqslant h_{rk} \leqslant h_{max} \quad \text{(headway feasibility)} \tag{3.2}$$

$$\left(\sum_{k=1}^{u} \frac{T_{r_k}}{h_{r_k}} \right) \leqslant W \quad \text{(fleet size)} \tag{3.3}$$

$$d_s \geqslant \delta d_{tot} \quad \text{(demand coverage)} \tag{3.4}$$

$$R\_Gini \leqslant \beta \quad \text{(equity)} \tag{3.5}$$

The input to the problem is the set of routes $\mathfrak{R}$ obtained at the end of the candidate route set generation procedure that is proposed in the relevant subsection. The two decision variables that can be identified are the generic two sets of routes and frequencies ($\mathfrak{R}'$, F′) that can potentially be selected while finding the optimal solution to the problem.

A solution to the problem is a pair ($\mathfrak{R}^*$, F*) where $\mathfrak{R}^* = \{r_1 \ldots r_u\}$, $\mathfrak{R}^* \subset \mathfrak{R}$, is the optimal set of routes, while $F^* = \{f_1 \ldots f_u\}$ is the optimal set of frequencies, where each f_k is a real number representing the inverse of the headway between subsequent vehicles on route r_k (headway).

The weights $\gamma_1,\gamma_2,\gamma_3$ are introduced in the objective function to reflect the tradeoffs between user costs, operator costs, and unsatisfied travel trip costs (Fan and Machemehl, 2011). They are dependent on the planners' experience and expert judgments. Different values of these weights may produce different optimal designs of the transit route network, still using the same proposed solution methodology. Operator costs are usually measured in monetary units, whereas user costs are measured in time spent in the system (minutes): that is the reason why we need the conversion factor between time and money (C_v/C_m).

Constraints refer typically to resources availability and practical guidelines. The first constraint (3.1) sets the minimum and the maximum number of routes, reflecting the fact that transit planners often set this range according to the fleet and the crew size. The second constraint (3.2) on the headway feasibility reflects the usage of policy headways. The third (3.3) is the fleet size constraint, that represents the resource limits of the transit company and guarantees that the optimal network pattern never uses more vehicles than available. The fourth constraint (3.4) specifies that d_S (the share of d_{tot} covered by routes directly or indirectly) has to be greater or equal to a specific percentage δ of the total transit demand. The last one (3.5) is the equity constraint whose specification proposal will be explained in detail in the next section. It ensures that the proposed revised Gini coefficient R_Gini associated to a transit configuration does not exceed the β value specified by the transit network planner.

4.1. Formulation of the equity constraint

One of the most innovative elements introduced in the solution of the proposed TNDP given in Eq. (3) consists, in the attempt to achieve equity goals by means of a new constraint to be added to the problem. It is important to clarify how we summarize both spatial/horizontal and social/vertical aspects into a single indicator capable of pushing towards the design of a fairer transit network.

The proposed indicator can be defined as *Revised Gini coefficient* (from this moment on we indicate it as 'R_Gini') calculated on the entire network. We propose to start from the original Gini's formulation, where it corresponds with the ratio of the area between the line of equality and the Lorenz curve, to the total area below the line of equality (Fig. 1).

However, there is a significant difference: if on the abscissa axis we still find the percentage of the resident population, on the y-axis we consider the level of transit supply weighted according to the public transport need index (W_SI_D, as specified in Eq. (4)), in order to look also at the percentage of disadvantaged people living in each travel demand zone D. In this way, the Lorenz curve associated with a given network represents the cumulative proportion of population against the cumulative proportion of the

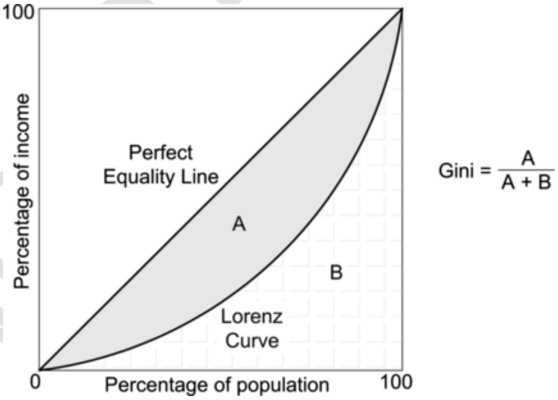


Fig. 1. Lorenz curve and Gini coefficient.

weighted transit service supply. Thus, the public transport need index is specified as:

$$W\_SI_D = \left(\sum_b \frac{Area_{Bb}}{Area_D} \cdot SL_{Bb} \right) \cdot \left(100 - \frac{(\sum_y w_y x_y) \cdot 100}{Pop_D} + 1 \right) \tag{4}$$

Our aim is to guarantee that the final layout of the transit service is the fairest possible compromise, according to both spatial distribution and social needs. Indeed, if we want simply to pursue a horizontal equity goal (i.e, SI_D), it would be sufficient to ensure that each zone has an even number of stops/stations and even service frequencies, commensurate to the number of the residents in that district. Instead, using the defined W_SI_D index to obtain the Lorenz curve, we wish to provide a more massive presence of stops/stations (so as to step up the sum of $Area_{Bb}$) and more frequent transit service (increasing SL_{Bb}) in the areas with a larger presence of disadvantaged people.

It is worthy to explain the meaning of the expression given in the second round-brackets of Eq. (4). It is equal to the complement to 100 of the disadvantaged population deduced by the PTN_D plus one (component added to prevent that the resultant product equals zero). As a matter of fact, the public transport need index is an input data of the problem, associated with the demographic composition of each district and thus unchangeable for the purposes of the global optimization. Therefore, the larger is the number of disadvantaged people in a given demand zone, the more the value of the correlated W_SI_D tends to decrease. Consequently, in order to guarantee to that penalized zone a level of transit supply able to compete with the one of the other districts in the network (in order to reach a global equity), the process of optimization works towards those 'editable' parameters related to the final configuration of the transit network (i.e., the number and the location of stops/stations and their level of service). Accordingly, the optimal solution coincides with a transit network capable of serving in a more widespread manner those areas that need it most.

It is worthy to underline that the value of β given in Eq. (3.5) should be selected carefully. On the one hand, if a lower value is selected (close to 0, corresponding to a perfectly even distribution), the constraint may lead to infeasibility trying to reach the highest possible degree of equity in the network, and therefore there might be no solution to the problem. On the other hand, if a higher value is selected, the constraint may be too loose to be active (i.e., to influence the solution), and the equity aspects may be neglected.

Notice that more than one public transportation system (i.e., bus, metro, train, and so on) can be present at the same time in the reality under analysis, each one respectively associated to its buffer area and its service level. An example of calculation of Supply Index (SI_D) when more than one transportation system coexists in the network can be found in Delbosc and Currie, 2011; on the other hand, the Public Transport Need (PTN_D) component is not affected by the considered number of transit systems.

5. Proposed solution method

As stated in the previous section, the input to the proposed TND problem is a set of potential optimal (candidate) routes $\mathfrak{R}$. In this section, we describe the candidate route set generation procedure and the relevant solution algorithm.The proposed solution method consists of two main components:

(a) a starting candidate route set generation procedure;
(b) a social costs minimization module with Genetic Algorithm (GA) solution approach to determine the optimal transit route set with the associated service frequencies, in compliance with all the constraints, including the equity one.

First of all, we need to set the main inputs of the problem. Namely, having a directed graph made of a certain number of nodes and arcs, we have to identify the traffic demand zones associated with the network. In a real size application, they could coincide with a single census district or with an aggregation of more of them. Established a centroid for each traffic zone, knowing the public transport Origin-Destination (OD) demand and the link costs associated with each arc in the network, the planner (or the transit agency) needs to set the remaining parameters. As a matter of fact, he/she has to determine, according to the size of the application and his/her expertise:

- the locations to be targeted as terminals, and a set of terminal pairs $t \in T$;
- the minimum and the maximum length of any route in the transit network, L_{min} and L_{max};
- the maximum percentage increase from the shortest path for any OD pair possible path, α;
- the maximum number of transfers in a path, k_{tr} (i.e., the number of line changing to reach a destination D from a given origin O).

With all these inputs, it is possible to generate all the existing routes between the terminals, to filter them according to a cost function (e.g. routes length) and their maximum deviation from the minimum cost (e.g. shortest) path, and to obtain a final set of feasible routes that represents one of the basic and fundamental input for the solution procedure.

Moving to the next step, the GA approach needs itself a series of parameters to be set, jointly with the above-mentioned set of feasible routes. Among them, we have to set:

- the minimum and the maximum allowed number of routes in the network, u_{min} and u_{max};
- the minimum and maximum required headway, h_{min} and h_{max};
- the maximum fleet size available for operations on the route network, W;
- the capacity of the vehicles operating on the network, P;
- the desired vehicle occupancy, η;
- the minimum percentage δ of the total demand to cover;
- the β value not to exceed for the equity constraint.

Applying the GA solution procedure, it is possible to determine the optimal transit route set (according to k_{tr}), the associated route frequencies and the related costs. The genetic algorithm provides a robust search as well as a near-optimal solution in a reasonable time: the simplicity of its working method jointly with its ability to find good solutions are two characteristics that make this method attractive to solve our optimization problem (see also Fan and Machemehl, 2011).

Note that our model does not estimate the access time; in average, we can consider reasonable a 5-min walk to reach a bus stop; in terms of physical distance, this corresponds to an access standard in urban areas of 400 m (Demetsky and Lin, 1982; Levinson, 1992; Federal Transit Administration, 1996; Ammons, 2001).In the following, the solution methodology is outlined in greater detail, to better clarify the underlying elements of the proposed model.

In order to generate all possible routes between each pair of terminals in the network, a way forward (Fan and Machemel, 2011) is the combined use of Dijkstra's shortest path algorithm (Abuja et al., 1993) and Yen's k-shortest path algorithm (Yen, 1971). By means of the first one, we are able to identify which is the shortest way (according to the predefined link costs) to connect a pair of nodes in the network. Then, through Yen's algorithm, it is possible to determine a list of all the alternative paths that connect each pair of terminals. We need to filter all these potential paths according to some of the parameters set in the input stage (i.e., L_{min}, L_{max}, α), discarding the ones that are too long or too short, and those that deviate from the shortest path over a certain threshold (i.e. α). After stripping from the list these unsuitable paths, we finally have only the feasible routes to use as input for the next step of the procedure.

In this section, however, we suggest an alternative model to generate the candidate route set, easily applicable in real size case studies. In order to facilitate the explanation of the method, the following flowchart (Fig. 2) summarizes the main steps of the suggested methodology.

Essentially, given a terminal pairs $t_k = (t'_k, t''_k) \in T$, $k \in [1 \dots \bar{t}]$, the main idea is to start from the shortest path between them. This shortest path is made by p nodes, belonging to the set N of the directed graph G. It is possible to find in the network a series of alternative paths able to connect the above mentioned two terminals, besides the shortest one. Through this method, we want to operate a selection of the best alternative paths able to satisfy to a greater extent the transit demand of the network. This selection is made establishing an interval (i.e. a certain number of nodes), called Δt. Adding progressively this Δt to the p nodes constituting the shortest path, we are able to obtain different ranges of length ($m \in M$) for the alternative paths. For each range we find, by means of a GA process, the best path in terms of transit demand satisfaction: as a matter of fact, the objective function to be minimized through the genetic algorithm is ($d_{tot} - d_S$).

Looking at the flowchart in Fig. 2, it is important to underline that the Q nodes selected by the GA must all be different from each other. The output gives us a set $\mathfrak{R}$ of candidate routes; we obtain a total number of routes equal to the product of the number of terminal pairs $\bar{t}$ and the number of the chosen length intervals $\bar{m}$. Each route belonging to the candidate set is the best in its length interval in satisfying the transit demand on the network.

6. Numerical applications

In this section, we propose a set of numerical applications considering a test network and then a case study on a real network. In particular, on the test network, a sensitivity analysis is carried out considering different parameters.

6.1. Sensitivity analysis

The proposed methodology has been first implemented on a test network, carrying out a sensitivity analysis to better clarify the correlation between equity and average costs on the network. The network, taken from Wan and Lo (2003), includes 5 travel demand zones (from A to E, identified by different patterns), 10 nodes and 19 undirected arcs (Fig. 3). The number reported on each link represents the link cost, c_a. There are nine OD pairs. The hourly OD demand is shown in Table 1.

For sake of simplicity, in this analysis, we will assume that all vertices of the infrastructure graph correspond to intersections of the network and could also be bus stops and origin and destination of the trips; this implies that the demand can be generated at any vertex and that walking arcs are then not considered. There is not a fixed set of terminals; consequently, each route generated at the end of the optimization could start and finish potentially in any node.

In order to generate the candidate route set, we have followed the method from previously mentioned literature, namely the combined use of Dijkstra's shortest path algorithm and Yen's k-shortest path algorithm, since the relatively small size of the net-

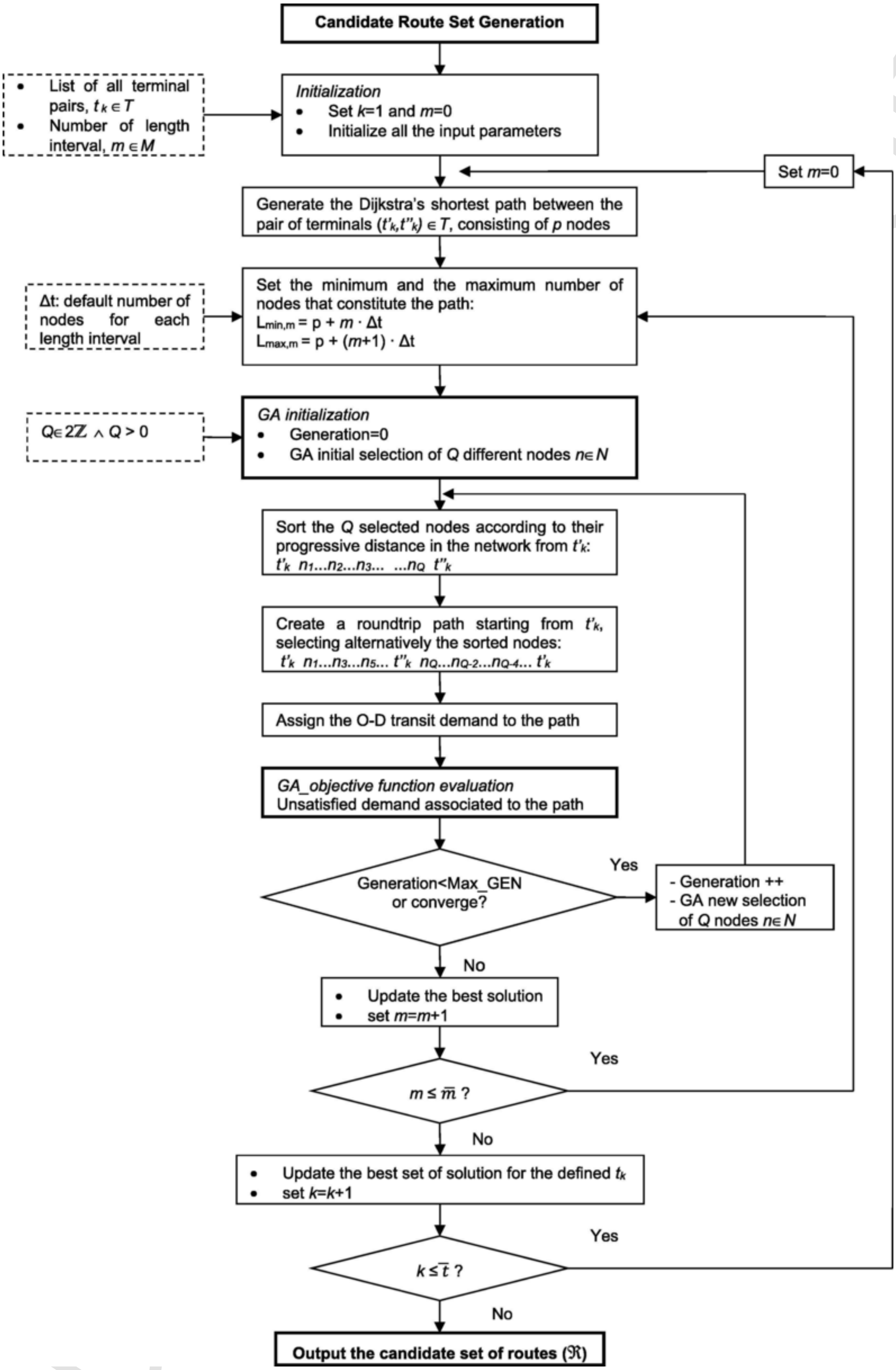


Fig. 2. Flowchart of the proposed candidate route set generation procedure.

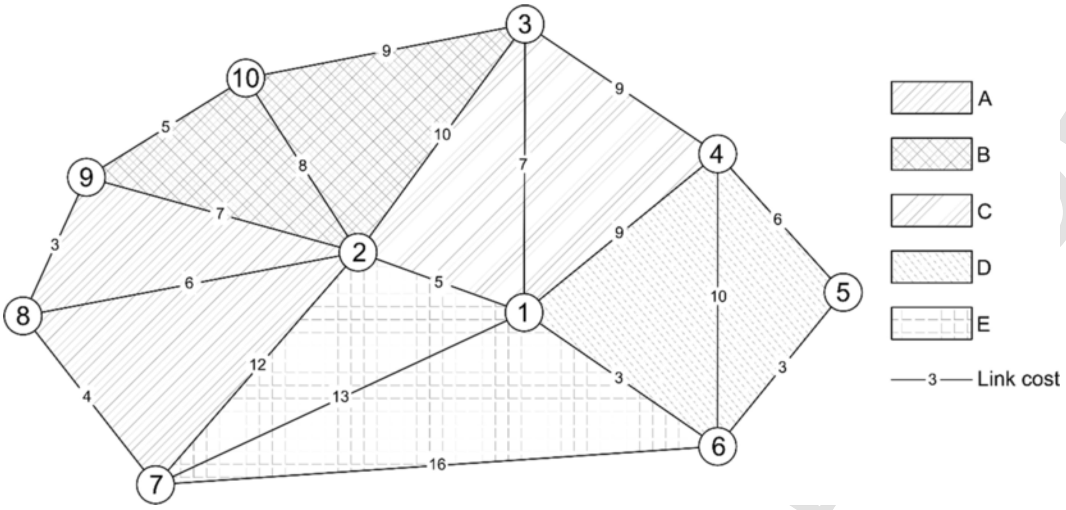


Fig. 3. Sensitivity analysis network (Wan and Lo, 2003).

Table 1
Origin-destination demand (pax/h).

OD pair	Demand	OD pair	Demand	OD pair	Demand
2→10	200	5→8	350	8→3	400
3→2	150	6→9	600	9→4	450
4→7	800	7→6	250	10→5	500

work. In other words, given the limited number of nodes and arcs, indeed, it is possible to easily enumerate all possible paths that connect each pair of nodes, and then filter them according to their length and α. We set the minimum length $L_{min} = 3$ km; the maximum deviation percentage allowed from the shortest path for any OD pair connection is set equal to 50%, consequently, we set $\alpha = 0.5$.

We assume that the three weights, γ_1, γ_2, γ_3, are equal to 1; consequently, in this application, we are assigning the same significance to all these components. We set C_v (i.e., per-hour operating cost of a bus [currency/vehicle/h]) equal to 150, C_d (i.e., value of each unsatisfied transit demand [currency/person]) equal to 10, and C_m equal to 1 (value of time [currency/min]).

We allow at most three transit lines operating in the network, i.e. $u_{max} = 3$. We set headway bounds equal to $h_{max} = 6$ min and $h_{min} = 20$ min (i.e., $f_{max} = 10$/h and $f_{min} = 3$/h), and a transit vehicle capacity of $P = 50$pax/bus (KFH Group, 2013). Finally, given the small size of the network, we neglect paths with more than one transfer ($k_{tr} = 1$), and not to impose the fleet size constraint, assuming that there is not a maximum threshold to the number of buses operating on the network. The minimum percentage δ of the total demand to cover is set equal to 0.7, meaning that we want to satisfy at least 70% of users asking for public transport service.

In order to infer the value of the proposed R_Gini equity constraint, corresponding to each possible route configuration, we need to calculate the weighted transit service supply index W_SI_D for each district D. We assume the length of the link 6–7 (the longest link of the experimental network) equal to 3 km. According to this assumption, each zone covers a total area ($Area_D$) roughly ranging from 1 km^2 to 2 km^2. Consistently, we suppose a population of about 2000–4000 inhabitants for each of them, guessing a population density coherent with the one of a medium size city center.

As clarified in the methodology section, a 5-min walk (roughly 400 m) to reach a bus stop is considered reasonable (Demetsky and Lin, 1982; Levinson, 1992; Federal Transit Administration, 1996; Ammons, 2001). Then, a radius of 400 m around each bus stop identifies the circle that defines its buffer area ($Area_{Bb}$). As service level measure SL_{Bb}, we consider the number of public vehicle arrivals per hour, assuming that we want to serve the hourly demand shown in Table 1. Time-dependent demand can be accounted for by considering time-dependent frequencies.

We accomplish our sensitivity analysis performing three different optimizations: the first one without the equity constraint (No_EQ), the second one only with the horizontal (spatial) constraint (EQ_h), the last one considering both horizontal and vertical equity (EQ_hv). In the No_EQ optimization, we calculate the Delbosc and Currie (2011) Gini coefficient (D&C_Gini), placing on the x-axis of the Lorenz curve graph the percentage of population, and on the y-axis the percentage of the transit supply index SI_D: in this way, we have an idea of the level of equity achieved without imposing any equity constraint on the network. The same D&C_Gini is computed for the EQ_h optimization, reflecting the spatial distribution of the transit service among the population. Only in the last optimization (EQ_hv) that considers both the horizontal and the vertical equity goals, our proposed R_Gini is calculated.

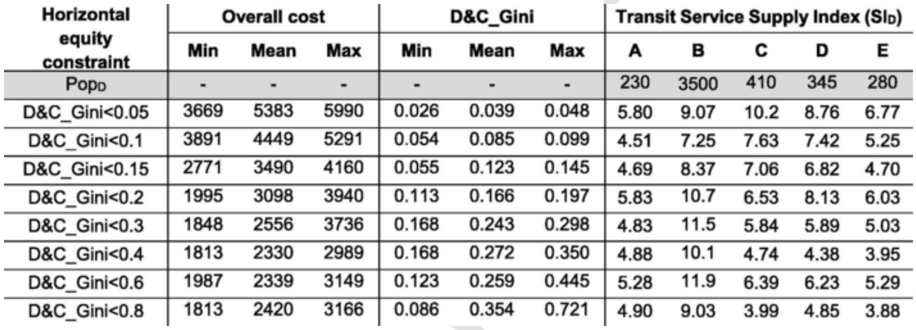
The key objective is to understand how much taking into account equity aspects since the planning stage of a public transportation network could affect the related global costs on it. Each set of optimizations is run through for 30 times. Consequently, the final values summarized in Tables 2, 3, 4a show the minimum, the average and the maximum value of the overall cost and Gini coefficient obtained for each group of optimizations. More specifically, in the EQ_hv optimization, we assume that only one district (A) has a certain percentage of the population belonging to a disadvantaged group; and we gradually increase this percentage (from 10% to 50%, with a 10% increasing step) carrying out 5 different groups of EQ_hv optimizations. Tables 4a and 4b included the results obtained just for one of these 5 configurations, namely the one with 30% of disadvantaged people living in district A. The values of SI_D and W_SI_D related to the columns of this district are highlighted in light gray. Results linked to the remaining 4 configurations of vulnerable people show a trend similar to the one shown by way of example in Tables 4a and 4b. Through this gimmick (the progressive increase of disadvantaged in one district), we force solutions in which district A is guaranteed with a larger number of routes and/or higher service frequencies according to the given percentage of disadvantaged people.

Note that the value assigned to β progressively increases from 0.05 to 0.8. Intervals are narrowed as they approach zero, i.e. the value of Gini coefficient representing the perfect equality. From the results given in the tables, we can easily observe that the more we seek to achieve a higher level of equity (both horizontal and horizontal & vertical) on the network, the more the overall costs rise. In a clear and immediate way, Fig. 4 depicts a surface representing the relation among overall medium costs, Gini fixed threshold β, and percentage of the disadvantaged population. We assess that, for the same value of β, it seems there is not an evident correlation between the percentage of disadvantaged people and costs, that fluctuate more or less around the same value. A possible explanation could be that the global amount of disadvantaged on the entire network is always too low compared to the total population living in all the districts, and it does not affect to a greater extent the final global costs.

Table 2
Results from the optimization without equity constraint (NO_Eq).

Overall cost			D&C_Gini		
Min	Mean	Max	Min	Mean	Max
16,830	23,548	31,105	0.1931	0.3234	0.4861

Table 3
Results from the optimization with the horizontal equity constraint (EQ_h).

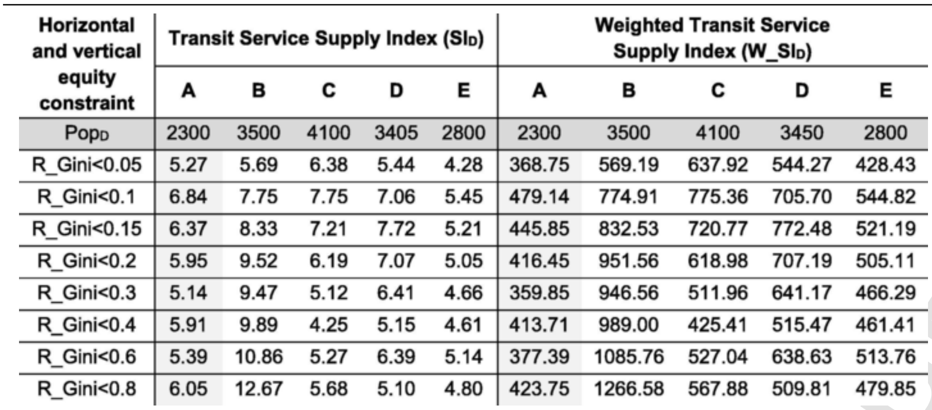


Horizontal equity constraint	Overall cost			D&C_Gini			Transit Service Supply Index (SI_D)				
	Min	Mean	Max	Min	Mean	Max	A	B	C	D	E
Pop_D	-	-	-	-	-	-	230	3500	410	345	280
D&C_Gini<0.05	3669	5383	5990	0.026	0.039	0.048	5.80	9.07	10.2	8.76	6.77
D&C_Gini<0.1	3891	4449	5291	0.054	0.085	0.099	4.51	7.25	7.63	7.42	5.25
D&C_Gini<0.15	2771	3490	4160	0.055	0.123	0.145	4.69	8.37	7.06	6.82	4.70
D&C_Gini<0.2	1995	3098	3940	0.113	0.166	0.197	5.83	10.7	6.53	8.13	6.03
D&C_Gini<0.3	1848	2556	3736	0.168	0.243	0.298	4.83	11.5	5.84	5.89	5.03
D&C_Gini<0.4	1813	2330	2989	0.168	0.272	0.350	4.88	10.1	4.74	4.38	3.95
D&C_Gini<0.6	1987	2339	3149	0.123	0.259	0.445	5.28	11.9	6.39	6.23	5.29
D&C_Gini<0.8	1813	2420	3166	0.086	0.354	0.721	4.90	9.03	3.99	4.85	3.88

Table 4a
Numerical results from the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 30% of disadvantaged people in district A. Overall cost and R_Gini.

Horizontal and vertical equity constraint	Overall cost			R_Gini		
	Min	Mean	Max	Min	Mean	Max
R_Gini < 0.05	34,746	51,233	63,528	0.0187	0.0433	0.0496
R_Gini < 0.1	33,660	44,016	56,577	0.0404	0.0824	0.9990
R_Gini < 0.15	26,865	34,683	41,700	0.0589	0.1210	0.1491
R_Gini < 0.2	19,576	31,235	39,441	0.0851	0.1669	0.1911
R_Gini < 0.3	17,515	26,877	35,326	0.1225	0.2396	0.2980
R_Gini < 0.4	18,900	23,837	28,500	0.1341	0.2794	0.3931
R_Gini < 0.6	18,130	22,864	28,656	0.1735	0.2857	0.5130
R_Gini < 0.8	18,130	21,635	28,210	0.1837	0.3106	0.6445

Table 4b
Numerical results from the optimization considering the horizontal and vertical equity constraint, Gini (EQ_hv) – 30% of disadvantaged people in district A. Transit Service Supply Index and Weighted Transit Service Supply Index.



Horizontal and vertical equity constraint	Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
	A	B	C	D	E	A	B	C	D	E
Pop_D	2300	3500	4100	3405	2800	2300	3500	4100	3450	2800
R_Gini<0.05	5.27	5.69	6.38	5.44	4.28	368.75	569.19	637.92	544.27	428.43
R_Gini<0.1	6.84	7.75	7.75	7.06	5.45	479.14	774.91	775.36	705.70	544.82
R_Gini<0.15	6.37	8.33	7.21	7.72	5.21	445.85	832.53	720.77	772.48	521.19
R_Gini<0.2	5.95	9.52	6.19	7.07	5.05	416.45	951.56	618.98	707.19	505.11
R_Gini<0.3	5.14	9.47	5.12	6.41	4.66	359.85	946.56	511.96	641.17	466.29
R_Gini<0.4	5.91	9.89	4.25	5.15	4.61	413.71	989.00	425.41	515.47	461.41
R_Gini<0.6	5.39	10.86	5.27	6.39	5.14	377.39	1085.76	527.04	638.63	513.76
R_Gini<0.8	6.05	12.67	5.68	5.10	4.80	423.75	1266.58	567.88	509.81	479.85

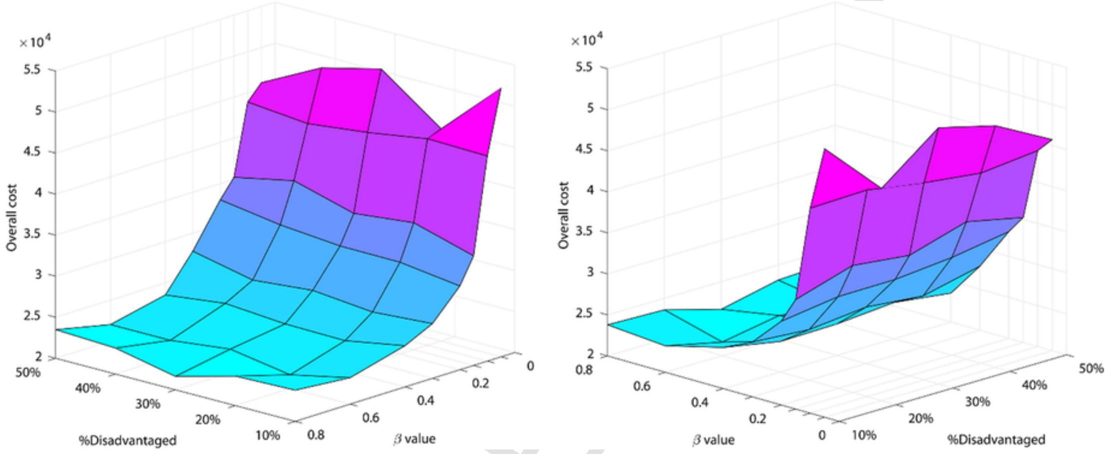


Fig. 4. Relation among equity constraint (R_Gini < β), overall medium cost on the network and percentage of disadvantaged population living in the district A.

An additional remark can be done looking at the values of SI_D and W_SI_D given in Tables 3 and 4b. The transit supply index SI_D reflects spatial/horizontal equity, so when it is in direct proportion to the number of people living in each district, it means that a fair degree of horizontal equity is achieved. As an example, in Table 3, for the sets of optimizations with D&C_Gini lower than 0.1 (where an optimal level of equity is reached), we observe that the greater SI_D values match with district C (4100 inhabitants), and gradually drop up to reach the lowest value for district A, with less population at all (2300 inhabitants). This does not happen in Table 4b, where the EQ_hv optimizations are performed trying to guarantee an amount of transit service proportional to both the spatial and social needs of the society. Therefore, although the values of W_SI_D are in proportion to the number of residents in each district, the corresponding values of SI_D show an unusually large value for the underpopulated district A. This occurs because district A has for sure fewer residents, but also a certain amount of disadvantaged people whose needs have to be taken into account granting them more bus stops and/or a greater frequency of the transit service, so requesting a bigger share of resources. Consequently, the 'apparent' disproportion that we can observe is due to the vertical equity component included in W_SI_D.

This concept is also expressed in Fig. 5, where it is possible to notice an imbalance in the SI_D Lorenz curve compared with the related W_SI_D Lorenz curve, closer to the line of perfect equity, at least for demanding values (R_Gini < 0.2) of the equity constraint. The difference between these two curves tends to be irrelevant as we relax the constraint.

In the following, a further sensitivity analysis has been performed, while varying the three weights, $\gamma_1,\gamma_2,\gamma_3$ in the proposed objective function (Eq. (3)). However, aiming at setting a consistent range of values to assign to them, we conduct a preliminary analysis looking at the final numerical values assumed by each one of the three addends in the sum (Eq. (3)) at the end of the previous sets of optimizations having the weights $\gamma_1,\gamma_2,\gamma_3$ all equal to 1 (see Tables 2, 3, 4a and 4b).

We acknowledge that, on average, the first component of the sum (3), corresponding to the user costs, is greater than the remaining two components. In particular, the second component (operator costs) results usually equal to a fourteenth of the first addend, while the third component (unsatisfied demand) is on average equal to a quarter of the first addend. Therefore, in order to change the level of mutual importance of the three elements constituting the objective function, in the performed sensitivity analysis associ-

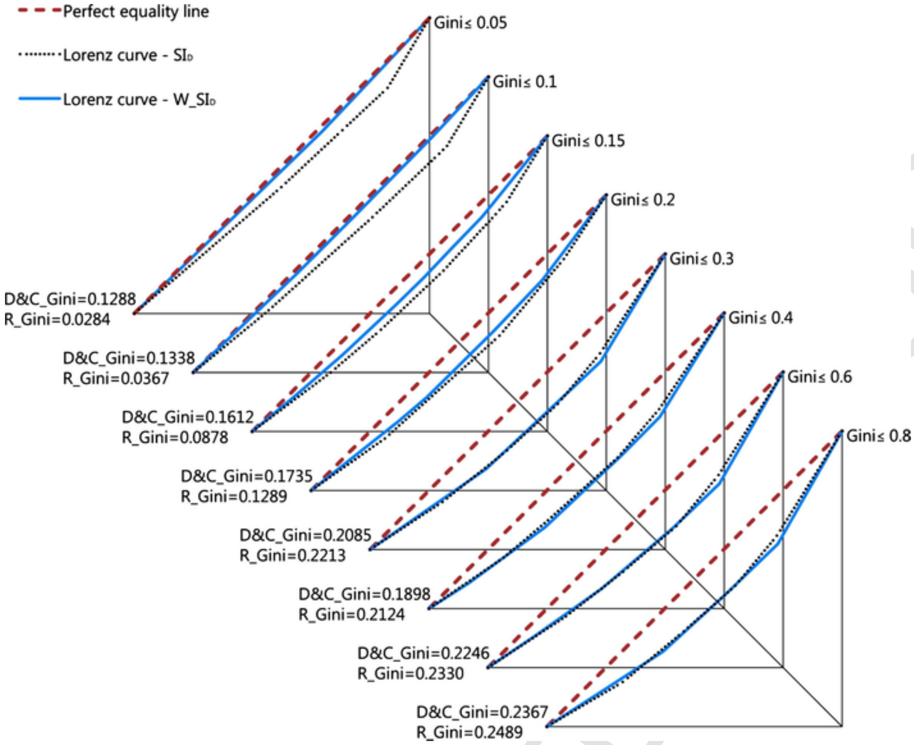


Fig. 5. Comparison between SI_D and W_SI_D Lorenz curves – 50% of disadvantaged people in district A.

ated to the γ weights, we decide to maintain γ_1 constant, and equal to one, while assigning to γ_2 the values of 7, 14 and 21, and to γ_3 the values of 2, 4 and 6. In this way, it has been possible to obtain 9 different weight combinations. Hence, the same set of optimizations that has been previously described has been repeated, considering these different combinations of the values of γ_1,γ_2,γ_3 , on the same test network (Fig. 3) The achieved results have been summarized in the following Tables 5, 6, and 7. The first line of each table (in italics) reports the results for γ_1,γ_2,γ_3 set equal to one, to allow a more immediate comparison among the final results.

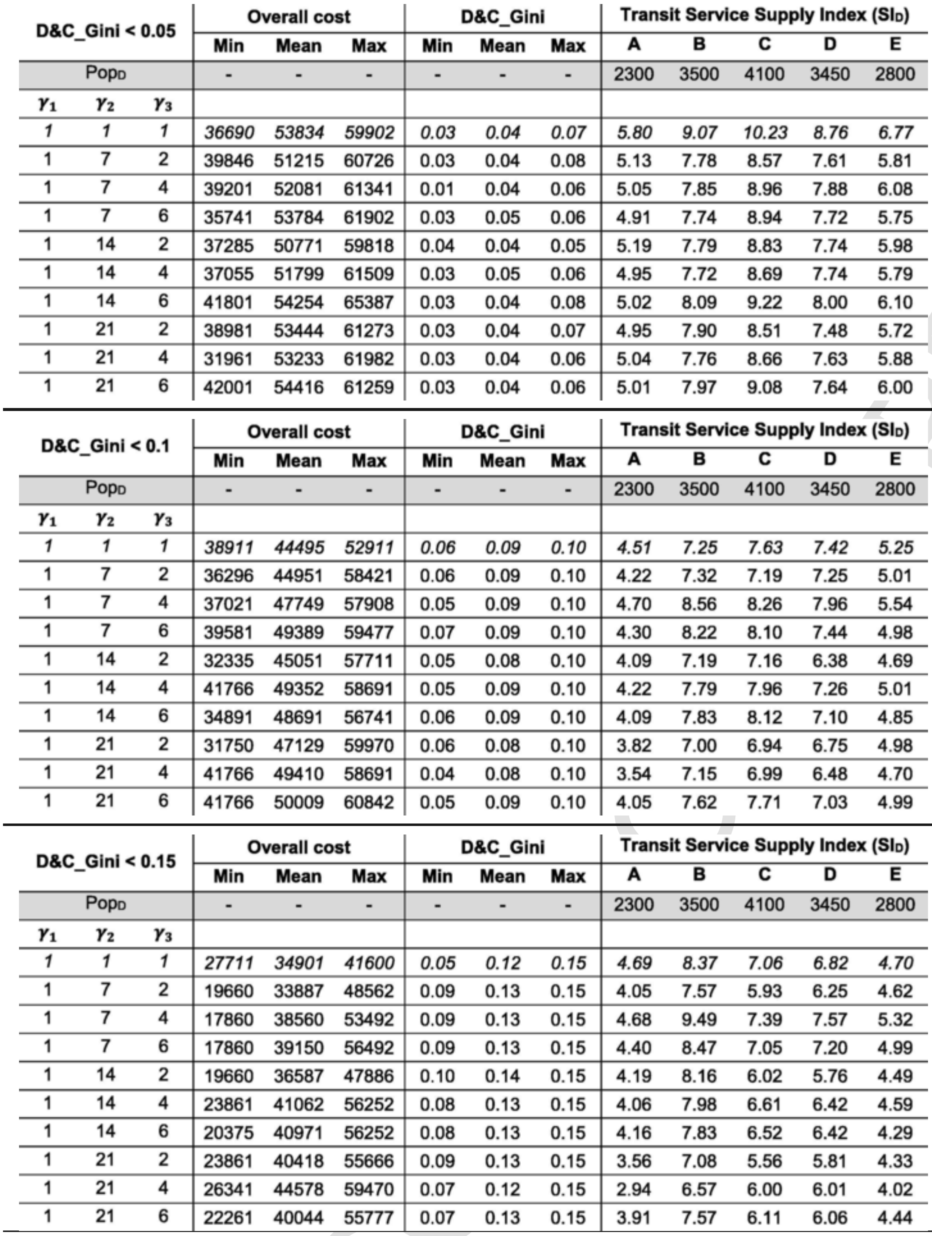
Looking at Table 5, that summarizes the results achieved at the end of the optimizations without equity constraint, we can notice that the overall cost on the network seem to progressively rise as increasing the weight associated to the operator cost (γ_2) from 7, to 14, to 21. In parallel, the minimum and average values of D&C_Gini steadily decrease. We may also observe that, while keeping constant γ_1 and γ_2, the best equity values on the network (i.e., the lowest D&C_Gini) are achieved with $\gamma_3 = 4$, that is the second highest value in the range we have tested (we allow γ_3 to be equal to 2, 4 or 6).

Basically, the same considerations about the overall cost and the equity indicator apply to the remaining two groups of optimizations (Tables 6 and 7). The only remarkable difference is in the D&C_Gini and R_Gini values, that follow the above-mentioned trend (decreasing while rising the weight of the operator cost γ_2) only when the imposed equity constraint on the network is not too nar-

Table 5
Results from the optimization without equity constraint (NO_EQ) and with variable weights γ_1,γ_2,γ_3 .

γ_1	γ_2	γ_3	Overall cost			D&C_Gini		
			Min	Mean	Max	Min	Mean	Max
1	*1*	*1*	*16,830*	*23,548*	*31,105*	*0.19*	*0.32*	*0.49*
1	7	2	18,775	23,002	35,956	0.21	0.31	0.57
1	7	4	18,775	24,982	40,735	0.20	0.31	0.42
1	7	6	18,775	25,195	40,600	0.12	0.31	0.53
1	14	2	20,485	27,387	41,420	0.12	0.26	0.38
1	14	4	20,485	28,162	40,812	0.14	0.26	0.39
1	14	6	24,316	29,695	44,545	0.16	0.25	0.33
1	21	2	23,680	28,186	41,766	0.10	0.24	0.50
1	21	4	24,716	33,523	43,457	0.10	0.22	0.35
1	21	6	24,316	32,549	46,291	0.10	0.24	0.44

Table 6
Results from the optimization with the horizontal equity constraint (EQ_h) and with variable weights.



D&C_Gini < 0.05			Overall cost			D&C_Gini			Transit Service Supply Index (SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3											
1	*1*	*1*	*36690*	*53834*	*59902*	*0.03*	*0.04*	*0.07*	*5.80*	*9.07*	*10.23*	*8.76*	*6.77*
1	7	2	39846	51215	60726	0.03	0.04	0.08	5.13	7.78	8.57	7.61	5.81
1	7	4	39201	52081	61341	0.01	0.04	0.06	5.05	7.85	8.96	7.88	6.08
1	7	6	35741	53784	61902	0.03	0.05	0.06	4.91	7.74	8.94	7.72	5.75
1	14	2	37285	50771	59818	0.04	0.04	0.05	5.19	7.79	8.83	7.74	5.98
1	14	4	37055	51799	61509	0.03	0.05	0.06	4.95	7.72	8.69	7.74	5.79
1	14	6	41801	54254	65387	0.03	0.04	0.08	5.02	8.09	9.22	8.00	6.10
1	21	2	38981	53444	61273	0.03	0.04	0.07	4.95	7.90	8.51	7.48	5.72
1	21	4	31961	53233	61982	0.03	0.04	0.06	5.04	7.76	8.66	7.63	5.88
1	21	6	42001	54416	61259	0.03	0.04	0.06	5.01	7.97	9.08	7.64	6.00

D&C_Gini < 0.1			Overall cost			D&C_Gini			Transit Service Supply Index (SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3											
1	*1*	*1*	*38911*	*44495*	*52911*	*0.06*	*0.09*	*0.10*	*4.51*	*7.25*	*7.63*	*7.42*	*5.25*
1	7	2	36296	44951	58421	0.06	0.09	0.10	4.22	7.32	7.19	7.25	5.01
1	7	4	37021	47749	57908	0.05	0.09	0.10	4.70	8.56	8.26	7.96	5.54
1	7	6	39581	49389	59477	0.07	0.09	0.10	4.30	8.22	8.10	7.44	4.98
1	14	2	32335	45051	57711	0.05	0.08	0.10	4.09	7.19	7.16	6.38	4.69
1	14	4	41766	49352	58691	0.05	0.09	0.10	4.22	7.79	7.96	7.26	5.01
1	14	6	34891	48691	56741	0.06	0.09	0.10	4.09	7.83	8.12	7.10	4.85
1	21	2	31750	47129	59970	0.06	0.08	0.10	3.82	7.00	6.94	6.75	4.98
1	21	4	41766	49410	58691	0.04	0.08	0.10	3.54	7.15	6.99	6.48	4.70
1	21	6	41766	50009	60842	0.05	0.09	0.10	4.05	7.62	7.71	7.03	4.99

D&C_Gini < 0.15			Overall cost			D&C_Gini			Transit Service Supply Index (SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3											
1	*1*	*1*	*27711*	*34901*	*41600*	*0.05*	*0.12*	*0.15*	*4.69*	*8.37*	*7.06*	*6.82*	*4.70*
1	7	2	19660	33887	48562	0.09	0.13	0.15	4.05	7.57	5.93	6.25	4.62
1	7	4	17860	38560	53492	0.09	0.13	0.15	4.68	9.49	7.39	7.57	5.32
1	7	6	17860	39150	56492	0.09	0.13	0.15	4.40	8.47	7.05	7.20	4.99
1	14	2	19660	36587	47886	0.10	0.14	0.15	4.19	8.16	6.02	5.76	4.49
1	14	4	23861	41062	56252	0.08	0.13	0.15	4.06	7.98	6.61	6.42	4.59
1	14	6	20375	40971	56252	0.08	0.13	0.15	4.16	7.83	6.52	6.42	4.29
1	21	2	23861	40418	55666	0.09	0.13	0.15	3.56	7.08	5.56	5.81	4.33
1	21	4	26341	44578	59470	0.07	0.12	0.15	2.94	6.57	6.00	6.01	4.02
1	21	6	22261	40044	55777	0.07	0.13	0.15	3.91	7.57	6.11	6.06	4.44

D&C_Gini < 0.2			**Overall cost**			**D&C_Gini**			**Transit Service Supply Index (SI_D)**				
			Min	**Mean**	**Max**	**Min**	**Mean**	**Max**	**A**	**B**	**C**	**D**	**E**
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3											
1	*1*	*1*	*19950*	*30985*	*39401*	*0.11*	*0.17*	*0.20*	*5.83*	*10.74*	*6.53*	*8.13*	*6.03*
1	7	2	17745	29075	44135	0.12	0.18	0.20	4.58	10.17	5.95	5.93	4.76
1	7	4	18430	32837	49306	0.12	0.19	0.20	4.72	9.86	6.04	5.56	4.81
1	7	6	18430	31855	47416	0.09	0.18	0.20	4.43	9.36	5.71	5.68	4.45
1	14	2	20030	34686	45795	0.12	0.17	0.20	3.81	6.92	4.94	5.17	3.74
1	14	4	23555	32782	48900	0.12	0.17	0.20	3.91	7.45	4.55	5.12	4.20
1	14	6	24680	36442	50176	0.12	0.17	0.20	3.78	7.95	5.65	5.21	4.06
1	21	2	24805	34000	43921	0.10	0.15	0.20	3.47	6.48	4.39	4.83	4.00
1	21	4	25166	38843	51026	0.09	0.15	0.20	3.70	6.99	5.23	5.27	3.76
1	21	6	25166	40869	56456	0.09	0.15	0.20	3.22	7.09	5.68	5.28	4.00

D&C_Gini < 0.3			**Overall cost**			**D&C_Gini**			**Transit Service Supply Index (SI_D)**				
			Min	**Mean**	**Max**	**Min**	**Mean**	**Max**	**A**	**B**	**C**	**D**	**E**
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3											
1	*1*	*1*	*18475*	*25566*	*37360*	*0.17*	*0.24*	*0.30*	*4.83*	*11.59*	*5.84*	*5.89*	*5.03*
1	7	2	18775	23578	39885	0.12	0.24	0.30	5.04	11.03	5.37	5.12	4.68
1	7	4	18775	26372	56266	0.12	0.24	0.30	4.68	10.57	5.17	4.93	4.30
1	7	6	18775	28470	43940	0.09	0.25	0.30	4.96	10.59	5.45	4.90	4.21
1	14	2	19800	28266	49481	0.12	0.23	0.30	4.07	7.40	3.68	4.32	3.79
1	14	4	22085	31064	43481	0.14	0.24	0.29	4.38	8.32	4.36	4.30	3.66
1	14	6	20485	31246	43950	0.12	0.23	0.29	4.11	7.67	4.18	4.25	3.64
1	21	2	23215	27566	41766	0.10	0.21	0.30	3.41	6.50	3.27	4.08	3.35
1	21	4	24316	32947	51192	0.10	0.22	0.30	3.44	6.99	3.60	4.02	3.40
1	21	6	24316	33196	45316	0.10	0.21	0.29	3.98	6.88	4.19	4.15	3.38

D&C_Gini < 0.4			**Overall cost**			**D&C_Gini**			**Transit Service Supply Index (SI_D)**				
			Min	**Mean**	**Max**	**Min**	**Mean**	**Max**	**A**	**B**	**C**	**D**	**E**
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3											
1	*1*	*1*	*18130*	*23309*	*29895*	*0.17*	*0.27*	*0.35*	*4.88*	*10.17*	*4.74*	*4.38*	*3.95*
1	7	2	18775	24578	33596	0.16	0.30	0.40	5.91	12.08	5.13	4.20	4.12
1	7	4	17515	24504	46295	0.18	0.29	0.40	5.99	12.80	5.91	4.63	4.27
1	7	6	18775	25796	41531	0.10	0.28	0.40	5.80	12.70	5.95	4.72	4.45
1	14	2	22750	27471	37336	0.12	0.24	0.39	4.37	7.78	3.69	3.85	3.61
1	14	4	22085	28927	40656	0.12	0.24	0.37	4.85	7.83	3.63	4.30	3.70
1	14	6	20485	29847	43450	0.12	0.24	0.32	4.30	8.61	4.21	4.11	3.73
1	21	2	24316	29083	41766	0.10	0.23	0.36	4.27	6.93	3.30	4.23	3.66
1	21	4	24316	32972	45937	0.10	0.19	0.30	3.65	6.84	4.16	4.57	3.50
1	21	6	24316	33442	49211	0.10	0.25	0.37	4.62	7.66	4.20	4.49	3.70

D&C_Gini < 0.6			**Overall cost**			**D&C_Gini**			**Transit Service Supply Index (SI_D)**				
			Min	**Mean**	**Max**	**Min**	**Mean**	**Max**	**A**	**B**	**C**	**D**	**E**
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3											
1	*1*	*1*	*19875*	*23394*	*31495*	*0.12*	*0.26*	*0.45*	*5.28*	*11.92*	*6.39*	*6.23*	*5.29*
1	7	2	18775	23596	38651	0.12	0.29	0.54	5.82	12.38	5.77	4.44	4.41
1	7	4	18775	24800	41521	0.23	0.32	0.42	6.12	11.58	4.60	4.32	4.15
1	7	6	18775	24481	35321	0.24	0.32	0.42	6.08	12.52	5.22	4.16	4.05
1	14	2	20485	26973	36216	0.21	0.28	0.39	4.97	8.85	3.51	3.80	3.70
1	14	4	22085	28427	39720	0.16	0.24	0.31	4.33	7.97	3.76	3.96	3.62
1	14	6	20485	29287	45745	0.16	0.27	0.32	4.50	9.21	4.43	3.89	3.60
1	21	2	22930	28691	41766	0.10	0.22	0.46	4.26	6.89	3.44	4.30	3.77
1	21	4	24316	31865	44945	0.10	0.24	0.36	4.64	7.81	4.04	3.92	3.43
1	21	6	24316	31971	48331	0.10	0.22	0.38	4.32	7.62	4.20	4.34	3.57

D&C_Gini < 0.8			Overall cost			D&C_Gini			Transit Service Supply Index (SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3											
1	*1*	*1*	*18130*	*24208*	*31666*	*0.09*	*0.35*	*0.72*	*4.90*	*9.03*	*3.99*	*4.85*	*3.88*
1	7	2	18775	23569	33596	0.12	0.30	0.42	6.36	11.33	4.49	4.56	4.24
1	7	4	18775	25293	43480	0.12	0.30	0.42	5.86	11.81	5.16	4.43	4.08
1	7	6	18775	25949	43480	0.12	0.31	0.42	6.45	12.79	5.59	4.48	4.07
1	14	2	22085	26839	38542	0.12	0.25	0.39	4.77	8.19	3.76	3.89	3.66
1	14	4	22085	29537	43850	0.12	0.23	0.33	4.61	8.31	4.27	4.34	3.77
1	14	6	22085	27985	39501	0.12	0.24	0.43	4.62	8.40	3.86	4.04	3.74
1	21	2	23495	28492	39151	0.11	0.26	0.50	4.12	6.65	2.77	3.90	3.38
1	21	4	24316	32587	52091	0.13	0.26	0.43	4.29	7.95	3.74	4.05	3.50
1	21	6	24316	33140	47907	0.10	0.23	0.38	3.74	7.50	4.19	4.46	3.40

row (i.e., for β values greater or equal than 0.3). On the other hand, for β values closer to zero (0.05, 0.1, 0.15, 0.2) the equity on the network do not seem to be consistently affected by the objective function weight variations.

Downstream of this sensitivity analysis, as far as concerns the achieved level of equity on the transit network, it appears that we cannot draw any specific conclusion in relation to the γ_3 tested variations, while a pattern can be clearly seen according to the γ_2 values. Particularly, giving a greater importance to the operator costs, it can be possible to achieve a higher level of horizontal (and vertical) equity on the network, although bearing larger overall costs.

As far as concerns the final values of SI_D and W_SI_D, the same considerations that have been done in relation to Tables 3 and 4b applies here. Moreover, looking at the objective function weight variations, the numerical values of SI_D and W_SI_D decrease while increasing γ_2: as a better global level of equity on the network is progressively achieved, the differences in (weighted) transit supply indexes between the zones appears to be smoothed out.

6.2. A case study: The city of Molfetta, Italy

The suggested methodology is here applied to a case study. We focus on the city of Molfetta of approximately 60,000 inhabitants, located in the South of Italy (Apulia region). The real transport systems network is modeled by a graph G (consisting of selected arcs and nodes), a set of link cost functions, an Origin-Destination matrix, and a transit network operating in the city. This provides the basis for comparison and validation of the proposed model.

The graph is made by 519 directed arcs and 210 nodes. We choose to be part of the network model only the streets of the city with an effective width sufficient to allow a bus to pass by easily, discarding the too narrow ones.

We divide the city into 28 zones, obtained aggregating different census districts (Fig. 6). We consider 200 m the radius of the walk access buffer around each bus stop so that two stops are distant from each other approximately 400 m. We make this choice trying to maintain the current distance between two following stops, as this is the level of accessibility required by the users to enjoy the transit service, according to recent surveys included in the Urban Master Plan of Sustainable Mobility of Molfetta. In particular, we assume that all those vertices falling on the bus routes can be considered bus stop locations. This choice is reasonable, as the average length of an arc of the network is 362 m; moreover, the (few) longer arcs are all located in the peripheral zones, where there is no need of additional stops given the average population density of the corresponding districts. An exception is constituted by those arcs shortest than 150 m, where just one of the two nodes that delimit each of them has been considered as a bus stop.

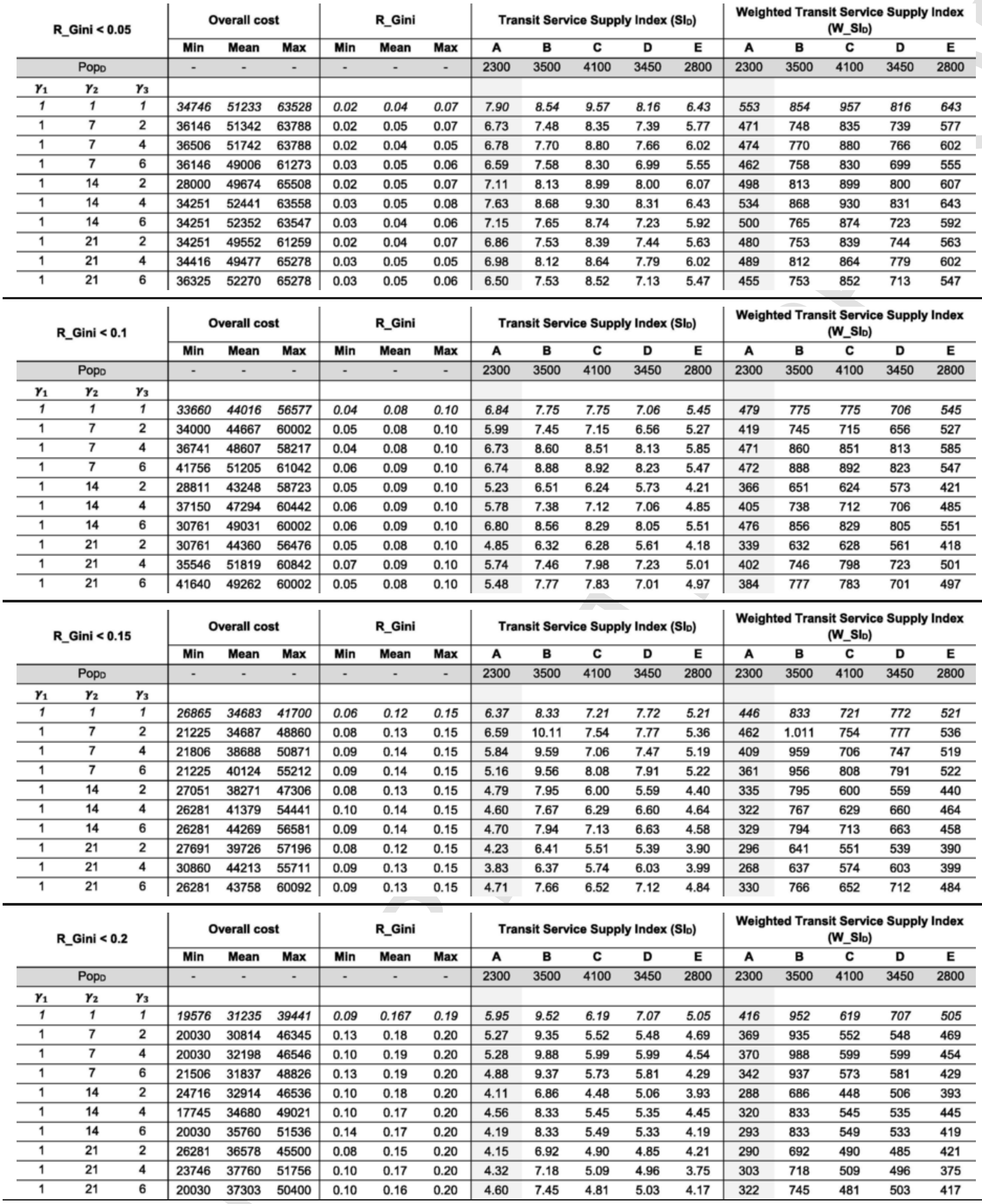
The demand produced (attracted) by a given zone, calculated by a four-step trip-based travel-demand model (Cascetta, 2009), is centered in centroids (black dots shown in Fig. 6) and considered as covered when a line passes by any place in the street network inside the zone and inside the destination (origin) zone, according to the capacity of the vehicles and their frequency. The total number of bus trips originating in or destined for each travel zone during the morning rush hour has been reported in Table 8.

Performing the proposed optimization, we aim at understanding if it is possible to obtain a configuration of the transit system able to satisfy to a greater extent the public transport demand, reaching a higher level of equity, i.e. serving in a widespread manner those zones with a larger percentage of disadvantaged people. According to the available census data, we include in this category unemployed, young (< 19 years old) and old (more than 65 years old) people, that are most likely to make use of the public transport (Table 9).

Currently, the public transportation system of Molfetta has 5 bus lines (Table 10), having a path roughly circular, with quite low-frequency bounds (i.e., $f_{max} = 1.43/h$ and $f_{min} = 1/h$), covering an average of 88 km per hour of operation of the system (Fig. 7).

In order to better understand the present state of the public transportation system in Molfetta, in terms of total costs supported by users and operator, unsatisfied demand and achieved level of equity, at first, we run the model giving as input the current 5 lines with their associated frequencies. As a matter of fact, the current configuration of the system is able to grant a R_Gini coefficient equals to 0.4216 and a percentage of unsatisfied demand of 40%.

Table 7
Numerical results from the optimization with the horizontal and vertical equity constraint, Gini (EQ_hv) – 30% of disadvantaged people in district A- with variable weights.

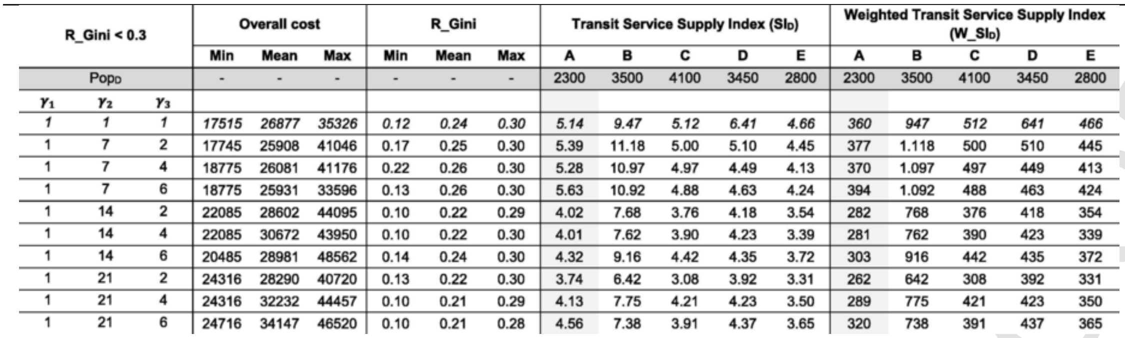


R_Gini < 0.05			Overall cost			R_Gini			Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3																
1	*1*	*1*	*34746*	*51233*	*63528*	*0.02*	*0.04*	*0.07*	*7.90*	*8.54*	*9.57*	*8.16*	*6.43*	*553*	*854*	*957*	*816*	*643*
1	7	2	36146	51342	63788	0.02	0.05	0.07	6.73	7.48	8.35	7.39	5.77	471	748	835	739	577
1	7	4	36506	51742	63788	0.02	0.04	0.05	6.78	7.70	8.80	7.66	6.02	474	770	880	766	602
1	7	6	36146	49006	61273	0.03	0.05	0.06	6.59	7.58	8.30	6.99	5.55	462	758	830	699	555
1	14	2	28000	49674	65508	0.02	0.05	0.07	7.11	8.13	8.99	8.00	6.07	498	813	899	800	607
1	14	4	34251	52441	63558	0.03	0.05	0.08	7.63	8.68	9.30	8.31	6.43	534	868	930	831	643
1	14	6	34251	52352	63547	0.03	0.04	0.06	7.15	7.65	8.74	7.23	5.92	500	765	874	723	592
1	21	2	34251	49552	61259	0.02	0.04	0.07	6.86	7.53	8.39	7.44	5.63	480	753	839	744	563
1	21	4	34416	49477	65278	0.03	0.05	0.05	6.98	8.12	8.64	7.79	6.02	489	812	864	779	602
1	21	6	36325	52270	65278	0.03	0.05	0.06	6.50	7.53	8.52	7.13	5.47	455	753	852	713	547

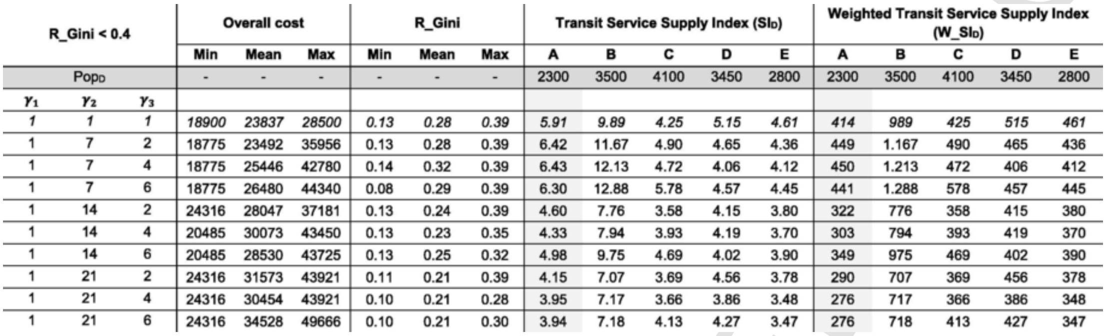
R_Gini < 0.1			Overall cost			R_Gini			Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3																
1	*1*	*1*	*33660*	*44016*	*56577*	*0.04*	*0.08*	*0.10*	*6.84*	*7.75*	*7.75*	*7.06*	*5.45*	*479*	*775*	*775*	*706*	*545*
1	7	2	34000	44667	60002	0.05	0.08	0.10	5.99	7.45	7.15	6.56	5.27	419	745	715	656	527
1	7	4	36741	48607	58217	0.04	0.08	0.10	6.73	8.60	8.51	8.13	5.85	471	860	851	813	585
1	7	6	41756	51205	61042	0.06	0.09	0.10	6.74	8.88	8.92	8.23	5.47	472	888	892	823	547
1	14	2	28811	43248	58723	0.05	0.09	0.10	5.23	6.51	6.24	5.73	4.21	366	651	624	573	421
1	14	4	37150	47294	60442	0.06	0.09	0.10	5.78	7.38	7.12	7.06	4.85	405	738	712	706	485
1	14	6	30761	49031	60002	0.06	0.09	0.10	6.80	8.56	8.29	8.05	5.51	476	856	829	805	551
1	21	2	30761	44360	56476	0.05	0.08	0.10	4.85	6.32	6.28	5.61	4.18	339	632	628	561	418
1	21	4	35546	51819	60842	0.07	0.09	0.10	5.74	7.46	7.98	7.23	5.01	402	746	798	723	501
1	21	6	41640	49262	60002	0.05	0.08	0.10	5.48	7.77	7.83	7.01	4.97	384	777	783	701	497

R_Gini < 0.15			Overall cost			R_Gini			Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3																
1	*1*	*1*	*26865*	*34683*	*41700*	*0.06*	*0.12*	*0.15*	*6.37*	*8.33*	*7.21*	*7.72*	*5.21*	*446*	*833*	*721*	*772*	*521*
1	7	2	21225	34687	48860	0.08	0.13	0.15	6.59	10.11	7.54	7.77	5.36	462	1.011	754	777	536
1	7	4	21806	38688	50871	0.09	0.14	0.15	5.84	9.59	7.06	7.47	5.19	409	959	706	747	519
1	7	6	21225	40124	55212	0.09	0.14	0.15	5.16	9.56	8.08	7.91	5.22	361	956	808	791	522
1	14	2	27051	38271	47306	0.08	0.13	0.15	4.79	7.95	6.00	5.59	4.40	335	795	600	559	440
1	14	4	26281	41379	54441	0.10	0.14	0.15	4.60	7.67	6.29	6.60	4.64	322	767	629	660	464
1	14	6	26281	44269	56581	0.09	0.14	0.15	4.70	7.94	7.13	6.63	4.58	329	794	713	663	458
1	21	2	27691	39726	57196	0.08	0.12	0.15	4.23	6.41	5.51	5.39	3.90	296	641	551	539	390
1	21	4	30860	44213	55711	0.09	0.13	0.15	3.83	6.37	5.74	6.03	3.99	268	637	574	603	399
1	21	6	26281	43758	60092	0.09	0.13	0.15	4.71	7.66	6.52	7.12	4.84	330	766	652	712	484

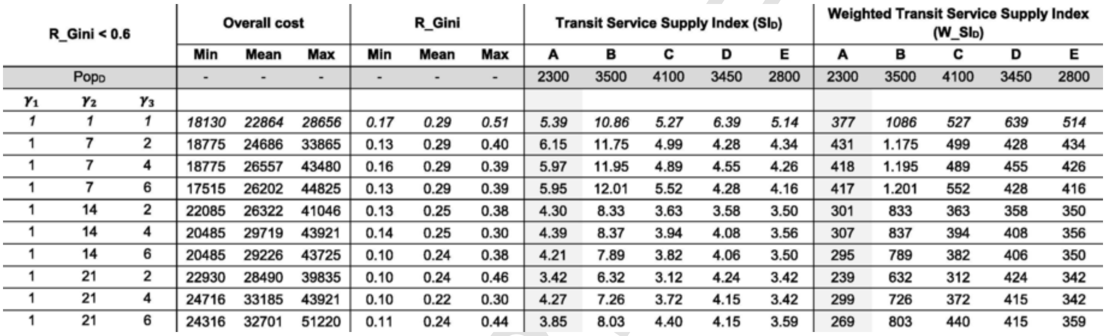
R_Gini < 0.2			Overall cost			R_Gini			Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3																
1	*1*	*1*	*19576*	*31235*	*39441*	*0.09*	*0.167*	*0.19*	*5.95*	*9.52*	*6.19*	*7.07*	*5.05*	*416*	*952*	*619*	*707*	*505*
1	7	2	20030	30814	46345	0.13	0.18	0.20	5.27	9.35	5.52	5.48	4.69	369	935	552	548	469
1	7	4	20030	32198	46546	0.10	0.19	0.20	5.28	9.88	5.99	5.99	4.54	370	988	599	599	454
1	7	6	21506	31837	48826	0.13	0.19	0.20	4.88	9.37	5.73	5.81	4.29	342	937	573	581	429
1	14	2	24716	32914	46536	0.10	0.18	0.20	4.11	6.86	4.48	5.06	3.93	288	686	448	506	393
1	14	4	17745	34680	49021	0.10	0.17	0.20	4.56	8.33	5.45	5.35	4.45	320	833	545	535	445
1	14	6	20030	35760	51536	0.14	0.17	0.20	4.19	8.33	5.49	5.33	4.19	293	833	549	533	419
1	21	2	26281	36578	45500	0.08	0.15	0.20	4.15	6.92	4.90	4.85	4.21	290	692	490	485	421
1	21	4	23746	37760	51756	0.10	0.17	0.20	4.32	7.18	5.09	4.96	3.75	303	718	509	496	375
1	21	6	20030	37303	50400	0.10	0.16	0.20	4.60	7.45	4.81	5.03	4.17	322	745	481	503	417



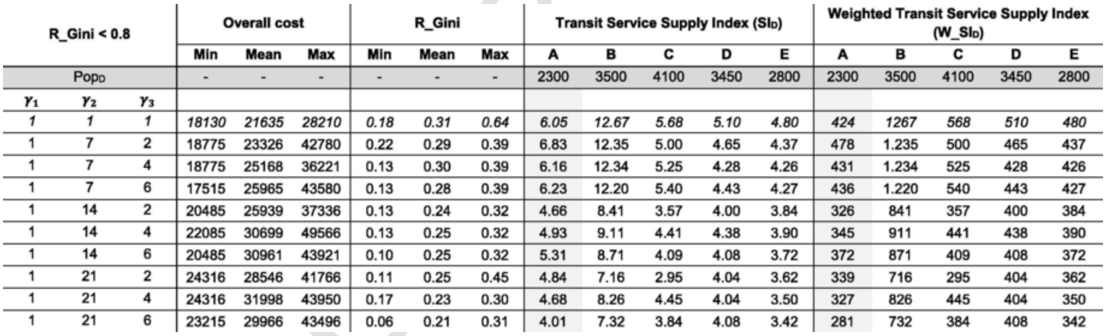
R_Gini < 0.3			Overall cost			R_Gini			Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3																
1	*1*	*1*	*17515*	*26877*	*35326*	*0.12*	*0.24*	*0.30*	*5.14*	*9.47*	*5.12*	*6.41*	*4.66*	*360*	*947*	*512*	*641*	*466*
1	7	2	17745	25908	41046	0.17	0.25	0.30	5.39	11.18	5.00	5.10	4.45	377	1.118	500	510	445
1	7	4	18775	26081	41176	0.22	0.26	0.30	5.28	10.97	4.97	4.49	4.13	370	1.097	497	449	413
1	7	6	18775	25931	33596	0.13	0.26	0.30	5.63	10.92	4.88	4.63	4.24	394	1.092	488	463	424
1	14	2	22085	28602	44095	0.10	0.22	0.29	4.02	7.68	3.76	4.18	3.54	282	768	376	418	354
1	14	4	22085	30672	43950	0.10	0.22	0.30	4.01	7.62	3.90	4.23	3.39	281	762	390	423	339
1	14	6	20485	28981	48562	0.14	0.24	0.30	4.32	9.16	4.42	4.35	3.72	303	916	442	435	372
1	21	2	24316	28290	40720	0.13	0.22	0.30	3.74	6.42	3.08	3.92	3.31	262	642	308	392	331
1	21	4	24316	32232	44457	0.10	0.21	0.29	4.13	7.75	4.21	4.23	3.50	289	775	421	423	350
1	21	6	24716	34147	46520	0.10	0.21	0.28	4.56	7.38	3.91	4.37	3.65	320	738	391	437	365



R_Gini < 0.4			Overall cost			R_Gini			Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3																
1	*1*	*1*	*18900*	*23837*	*28500*	*0.13*	*0.28*	*0.39*	*5.91*	*9.89*	*4.25*	*5.15*	*4.61*	*414*	*989*	*425*	*515*	*461*
1	7	2	18775	23492	35956	0.13	0.28	0.39	6.42	11.67	4.90	4.65	4.36	449	1.167	490	465	436
1	7	4	18775	25446	42780	0.14	0.32	0.39	6.43	12.13	4.72	4.06	4.12	450	1.213	472	406	412
1	7	6	18775	26480	44340	0.08	0.29	0.39	6.30	12.88	5.78	4.57	4.45	441	1.288	578	457	445
1	14	2	24316	28047	37181	0.13	0.24	0.39	4.60	7.76	3.58	4.15	3.80	322	776	358	415	380
1	14	4	20485	30073	43450	0.13	0.23	0.35	4.33	7.94	3.93	4.19	3.70	303	794	393	419	370
1	14	6	20485	28530	43725	0.13	0.25	0.32	4.98	9.75	4.69	4.02	3.90	349	975	469	402	390
1	21	2	24316	31573	43921	0.11	0.21	0.39	4.15	7.07	3.69	4.56	3.78	290	707	369	456	378
1	21	4	24316	30454	43921	0.10	0.21	0.28	3.95	7.17	3.66	3.86	3.48	276	717	366	386	348
1	21	6	24316	34528	49666	0.10	0.21	0.30	3.94	7.18	4.13	4.27	3.47	276	718	413	427	347



R_Gini < 0.6			Overall cost			R_Gini			Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3																
1	*1*	*1*	*18130*	*22864*	*28656*	*0.17*	*0.29*	*0.51*	*5.39*	*10.86*	*5.27*	*6.39*	*5.14*	*377*	*1086*	*527*	*639*	*514*
1	7	2	18775	24686	33865	0.13	0.29	0.40	6.15	11.75	4.99	4.28	4.34	431	1.175	499	428	434
1	7	4	18775	26557	43480	0.16	0.29	0.39	5.97	11.95	4.89	4.55	4.26	418	1.195	489	455	426
1	7	6	17515	26202	44825	0.13	0.29	0.39	5.95	12.01	5.52	4.28	4.16	417	1.201	552	428	416
1	14	2	22085	26322	41046	0.13	0.25	0.38	4.30	8.33	3.63	3.58	3.50	301	833	363	358	350
1	14	4	20485	29719	43921	0.14	0.25	0.30	4.39	8.37	3.94	4.08	3.56	307	837	394	408	356
1	14	6	20485	29226	43725	0.10	0.24	0.38	4.21	7.89	3.82	4.06	3.50	295	789	382	406	350
1	21	2	22930	28490	39835	0.10	0.24	0.46	3.42	6.32	3.12	4.24	3.42	239	632	312	424	342
1	21	4	24716	33185	43921	0.10	0.22	0.30	4.27	7.26	3.72	4.15	3.42	299	726	372	415	342
1	21	6	24316	32701	51220	0.11	0.24	0.44	3.85	8.03	4.40	4.15	3.59	269	803	440	415	359



R_Gini < 0.8			Overall cost			R_Gini			Transit Service Supply Index (SI_D)					Weighted Transit Service Supply Index (W_SI_D)				
			Min	Mean	Max	Min	Mean	Max	A	B	C	D	E	A	B	C	D	E
Pop_D			-	-	-	-	-	-	2300	3500	4100	3450	2800	2300	3500	4100	3450	2800
γ_1	γ_2	γ_3																
1	*1*	*1*	*18130*	*21635*	*28210*	*0.18*	*0.31*	*0.64*	*6.05*	*12.67*	*5.68*	*5.10*	*4.80*	*424*	*1267*	*568*	*510*	*480*
1	7	2	18775	23326	42780	0.22	0.29	0.39	6.83	12.35	5.00	4.65	4.37	478	1.235	500	465	437
1	7	4	18775	25168	36221	0.13	0.30	0.39	6.16	12.34	5.25	4.28	4.26	431	1.234	525	428	426
1	7	6	17515	25965	43580	0.13	0.28	0.39	6.23	12.20	5.40	4.43	4.27	436	1.220	540	443	427
1	14	2	20485	25939	37336	0.13	0.24	0.32	4.66	8.41	3.57	4.00	3.84	326	841	357	400	384
1	14	4	22085	30699	49566	0.13	0.25	0.32	4.93	9.11	4.41	4.38	3.90	345	911	441	438	390
1	14	6	20485	30961	43921	0.10	0.25	0.32	5.31	8.71	4.09	4.08	3.72	372	871	409	408	372
1	21	2	24316	28546	41766	0.11	0.25	0.45	4.84	7.16	2.95	4.04	3.62	339	716	295	404	362
1	21	4	24316	31998	43950	0.17	0.23	0.30	4.68	8.26	4.45	4.04	3.50	327	826	445	404	350
1	21	6	23215	29966	43496	0.06	0.21	0.31	4.01	7.32	3.84	4.08	3.42	281	732	384	408	342

The values related to the overall costs on the network, the current unsatisfied demand and the R_Gini coefficient achieved having as input the present public transport lines are shown in the *Current status* column of Table 13.

Starting from these values, we can define reasonable bounds associated with each constraint in the model, improving the actual public transport situation, without having excessive claims that would lead to the impossibility to converge to a feasible solution.

In order to apply the proposed solution methodology to the network of Molfetta, we need to set in advance some parameters, defining also the bounds associated with each constraint (summarized in Table 11).

First of all, we fix 4 terminals, according to the locations suitable to allocate the stalled vehicles during the downtime of the service, and 4 terminal pairs $t_k \in T$. We then implemented the method suggested in the flowchart (Fig. 2), considering as link cost in the calculation of the shortest paths the travel time on each arc of the network. At the end, we obtain a set of 20 ($\bar{l} \cdot \bar{m}$) candidate

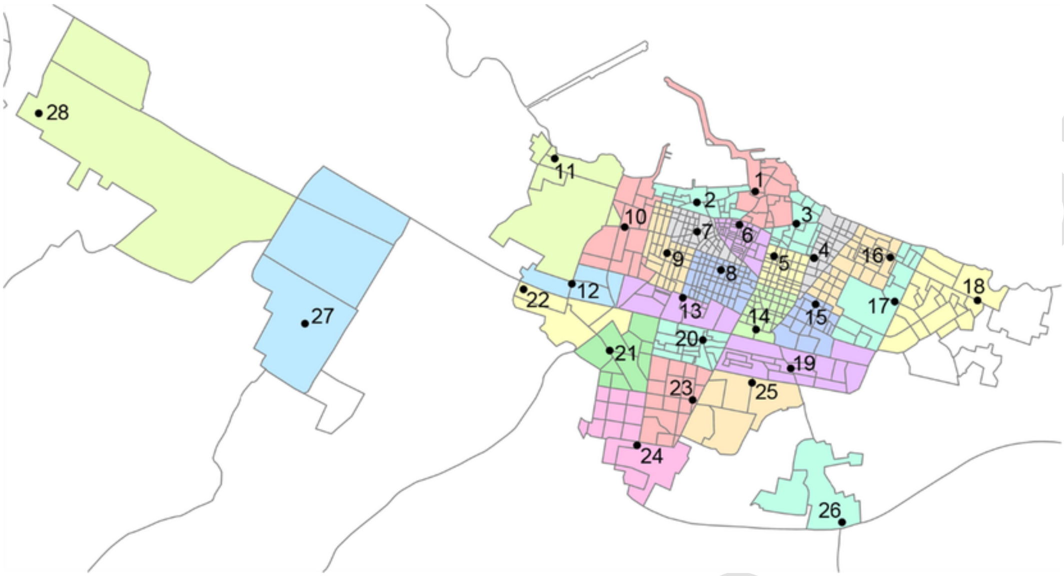


Fig. 6. Molfetta case study: traffic zones and centroid nodes.

routes, establishing 5 different length intervals $m \in M$ for each terminal pair $t_k \in T$. According to what explained in the previous section, in this way we are able to find the best paths to satisfy the transit demand for each route length interval, obtaining a set of candidate routes R that will be the input of the second part of the model.

We decide to neglect paths with more than one transfer ($k_{tr} = 1$) and to assume that the three weights $\gamma_1,\gamma_2,\gamma_3$, are equal to 1, giving the same significance to all these components. We set C_v (i.e., per-hour operating cost of a bus [currency/vehicle/h]) equal to 150, C_d (i.e., value of each unsatisfied transit demand [currency/person]) equal to 10, and C_m equal to 1 (value of time [currency/min]).

We allow at most 6 transit lines in the network, and at least 4 (i.e., $u_{min} = 4$ and $u_{max} = 6$), and we set headway bounds equal to $h_{max} = 60$ min and $h_{min} = 30$ min (i.e., $f_{max} = 2$/h and $f_{min} = 1$/h). We assume a maximum fleet size of 12 vehicles and transit vehicle capacity of $P = 50$ pax/bus, considering the vehicles currently used to perform the service.

The aim of the proposed model is to find (if it actually exists) a solution (i.e. routes configuration with associated frequencies) able to enhance the present situation of the public transportation network in Molfetta. Consequently, we set the remaining constraints to be satisfied according to our purpose: the minimum percentage δ of the total demand to cover is fixed equal to 0.97 (we are trying to satisfy as much as possible the current transit demand, allowing no more than 3% of users to be unsatisfied), while R_Gini has to be lower or equal to 0.38 (i.e., at least 10% less of the value achieved in the current status of the transit network).

The optimizations have been implemented using the MATLAB software. Computing times are of about 36 h (for each optimization) using a computer having an Intel Xeon E5-2687 W 3.10 GHz processor, and a 32 GB RAM. Although rather high, they may be considered acceptable, as this is not a problem whose solutions need to be calculated real-time. Furthermore, it can be possible to speed up the calculations, for instance parallelizing the code and/or using more appropriate programming languages.

We summarize in Table 12 the final results obtained at the end of the 30 performed optimizations. Among them, we identify as the optimal configuration of the system the one able to reach the lowest objective function value (i.e., the lowest overall costs on the network). Therefore, Table 13 shows the optimal configuration values of the system, that have been also compared with the values related the current status of the system.

The proposed solution for the public transportation system of Molfetta contemplates 5 bus lines (Table 14), covering an average of 96.37 km per hour of operation of the system (Fig. 8).

We can easily assert that, following the proposed methodology, we obtain a route configuration able not only to reduce the overall costs of the system, satisfying to a greater extent the transit demand on the network; but also, to guarantee a better spatial and social distribution of the service, reaching a higher level of horizontal and vertical equity on the network, expressed by means of the value of R_Gini.

We conduct a final sensitivity analysis while varying the three weights, $\gamma_1,\gamma_2,\gamma_3$ in the proposed objective function (Eq. (3)), as it has been previously done on the test network (Fig. 3). Nine different weight combinations have been set, running through each set of optimizations for 5 times for each combination, and the achieved results have been summarized in the following Table 15. Note that in the first line of the table the results for $\gamma_1,\gamma_2,\gamma_3$ set equal to one have been reported in italics, to allow a more immediate comparison.

Table 8
Number of bus trips for each travel zone during the morning rush hour.

Districts	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Produced trips	144	116	343	233	277	364	475	583	218	136	113	231	286	364
Attracted trips	49	449	322	243	61	82	206	303	201	37	188	446	148	205
Districts	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Produced trips	651	198	238	290	335	142	216	467	366	224	177	8	144	116
Attracted trips	747	493	780	90	93	573	27	268	259	195	20	220	700	49

Table 9
Census data, Molfetta.

Districts	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Pop_D	1564	1270	988	2737	1850	2214	2834	4043	4632	1711	1147	882	2050	2248
Unemployed	56	36	23	90	55	107	97	164	157	45	76	35	68	78
Young (<19)	312	223	134	527	318	469	624	803	976	328	215	168	351	419
Old (>65)	299	281	305	692	481	422	530	985	964	317	163	183	532	564
Districts	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Pop_D	3327	5245	1678	1986	2250	2967	1300	1438	3133	2747	1234	999	52	0
Unemployed	110	163	32	62	56	91	50	59	102	84	33	38	8	0
Young (<19)	492	905	247	328	348	443	171	349	682	456	352	264	13	0
Old (>65)	1064	1402	525	587	478	744	246	138	302	333	58	68	5	0

Table 10
Current public transport network in Molfetta.

Routes	Frequency (bus/h)	Length (km)	km per hour
1	1.20	12.93	15.52
2	1.20	13.86	16.63
3	1.15	13.74	15.80
4	1.43	13.75	19.67
5	1.00	20.67	20.67

Looking at Table 15, we can draw conclusions similar to those stated at the end of the sensitivity analysis performed on the test network. We note that the overall cost on the network of Molfetta seems to progressively rise as increasing the weight associated with the operator cost (γ_2) from 4, to 8, to 12. On the other hand, we do not observe a significative improvement in the value of the level of equity achieved on the network (R_Gini). We may deduce that the imposed equity constraint value ($\beta = 0.38$) can be considered narrow for the network under analysis, as the obtained trend shows the same behavior (being not consistently affected by the objective function weight variations) that has been observed on the test network for β values lower than 0.3.

At the end of this analysis, we may conclude that each specific network (with its associated transit lines configuration and distribution of advantaged/disadvantaged people) has its own maximum value of level of equity that can be potentially reached (i.e., minimum value of β that can be imposed to obtain feasible solutions). This could be an interesting aspect to investigate further, identifying how does it change the lowest achievable β value on transportation networks having different characteristics.

7. Conclusions

This paper focuses on the importance of applying the equity concept to a public transport network, intended as a fair and appropriate distribution of benefits and costs. Although it is common to find in the literature ex-post analysis regarding the pursued level of equity in a certain study area, or the socioeconomic characteristics that lead some categories of people to be excluded, previous attempts to incorporate both horizontal and vertical equity in the planning stage of a new public transportation system are lacking.

Therefore, we address this shortcoming elaborating a two-step method (starting candidate route set and optimal candidate route set generation) to quantitatively incorporate equity concepts inside the TNDP, by means of a constraint to the classical formulation based on a novel comprehensive equity indicator (R_Gini). The proposed model has been tested at first on a small network, carrying out a sensitivity analysis with the main purpose of understanding the correlation between the overall costs on the network and the pursued level of equity. This test confirms our expectations, that is a greater level of equity often means to bear more costs to be achieved. After this first analysis, we applied the proposed model to a real case study, an Italian city with an operating transit service. Our goal was to verify if it is possible to reach a better degree of horizontal and vertical equity on the network according to our assumptions and if this new configuration fits with the needs of both operator and users of the system. We find that it is possible, at least in this case, to find a better compromise, not only able to achieve the aforementioned equity aims, but also to allow a considerable saving of the associated costs.

Accordingly, we are firmly convinced that the attempt to integrate equity principles since the planning stage of a public transport network may be an added value to the design process that helps to ensure everyone a better service. It might be impossible or extremely expensive expect to reach an ‘ideal’ configuration; however, we have shown that it is feasible to improve consistently the current status of the service.

Testing the same model on different realities could help in the understanding of the effective benefits of the method, depending on the different distributions of the vulnerable categories of people in the area served by the transit services.

Uncited references

Kamruzzaman and Hine (2011), Lorenz (1905), Soltani and Ivaki (2011), Stanley (2004), Yigitcanlar et al. (2007), Zakowska and Pulawska (2014).

Acknowledgements

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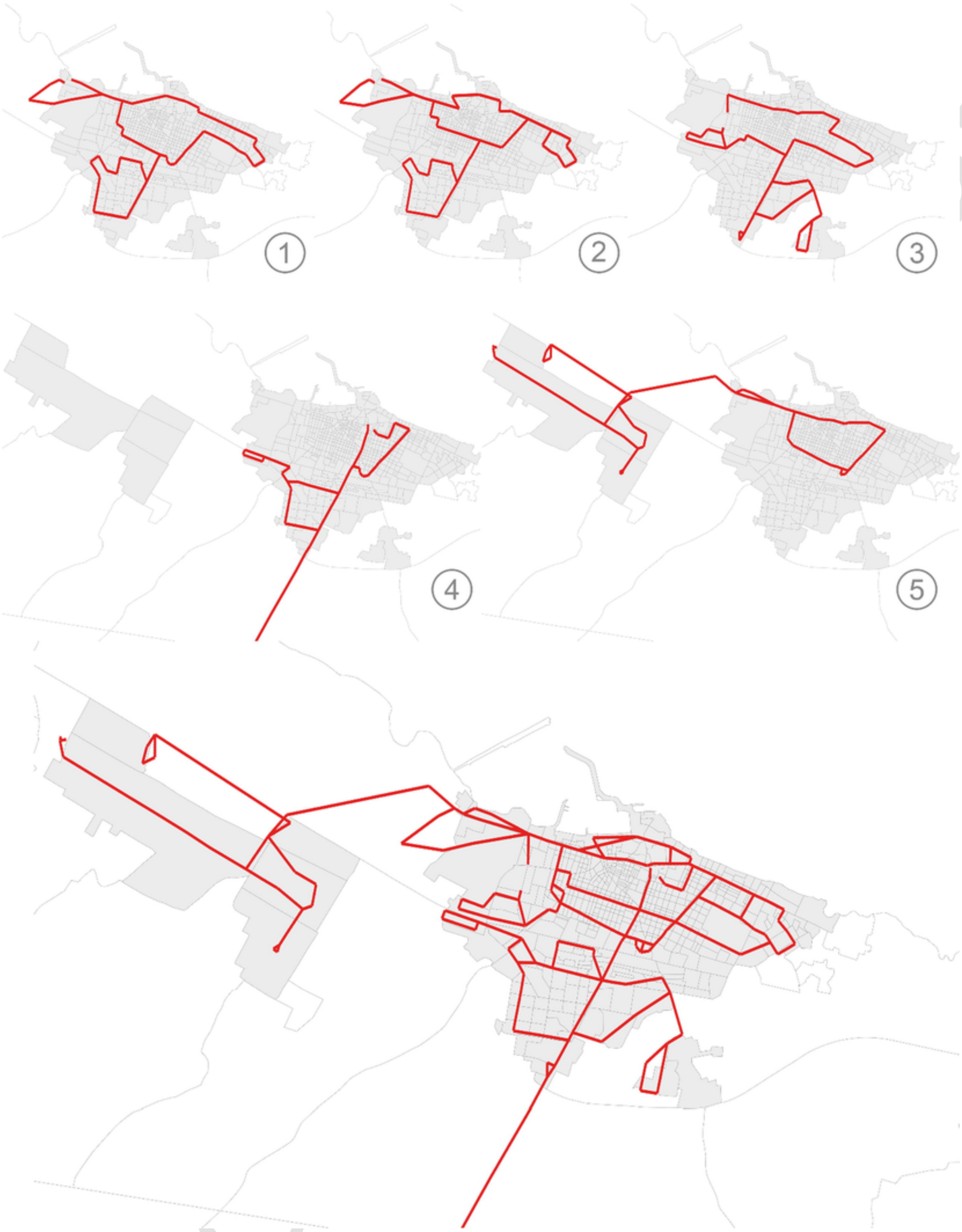

Fig. 7. Path covered by each one of the current routes, and by the set of routes as a whole.

Table 11
Set of constraints applied to the case study.

$4 \leq u \leq 6$	(numbers of routes)	(3.1)
$30 \leq h_{rk} \leq 60$	(headway feasibility)	(3.2)
$\left(\sum_{k=1}^{u}\frac{T_{r_k}}{h_{r_k}}\right) \leqslant 12$	(fleet size)	(3.3)
$d_s \geq 0.97 \cdot d_{tot}$	(demand coverage)	(3.4)
$R\_Gini \leq 0.38$	(equity)	(3.5)

Table 12
Numerical results from the optimization with the horizontal and vertical equity constraint, with weights $\gamma_1,\gamma_2,\gamma_3$ all equal to 1.

R_Gini < 0.38			Overall cost			R_Gini			User costs	Operator costs	Uns. Dem. costs
γ_1	γ_2	γ_3	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
1	1	1	23,943	35,704	51,471	0.321	0.346	0.278	30,646	3992	1066

Table 13
Comparison of results: current status and proposed solution for the transit network service.

	Current status	Proposed solution
Overall costs	78,053	23,944
User costs	44,855	18,969
Operator costs	3487	3720
Unsatisfied demand costs	29,711	1255
R_Gini coefficient	0.4216	0.3507
Unsatisfied demand	40%	1.7%

Table 14
Optimal transit route set.

Routes	Frequency (bus/h)	Length (km)	km per hour
1	1.45	16.69	24.14
2	1.25	9.14	11.40
3	1.41	13.88	19.60
4	1.43	17.69	25.24
5	1.55	10.30	16.00

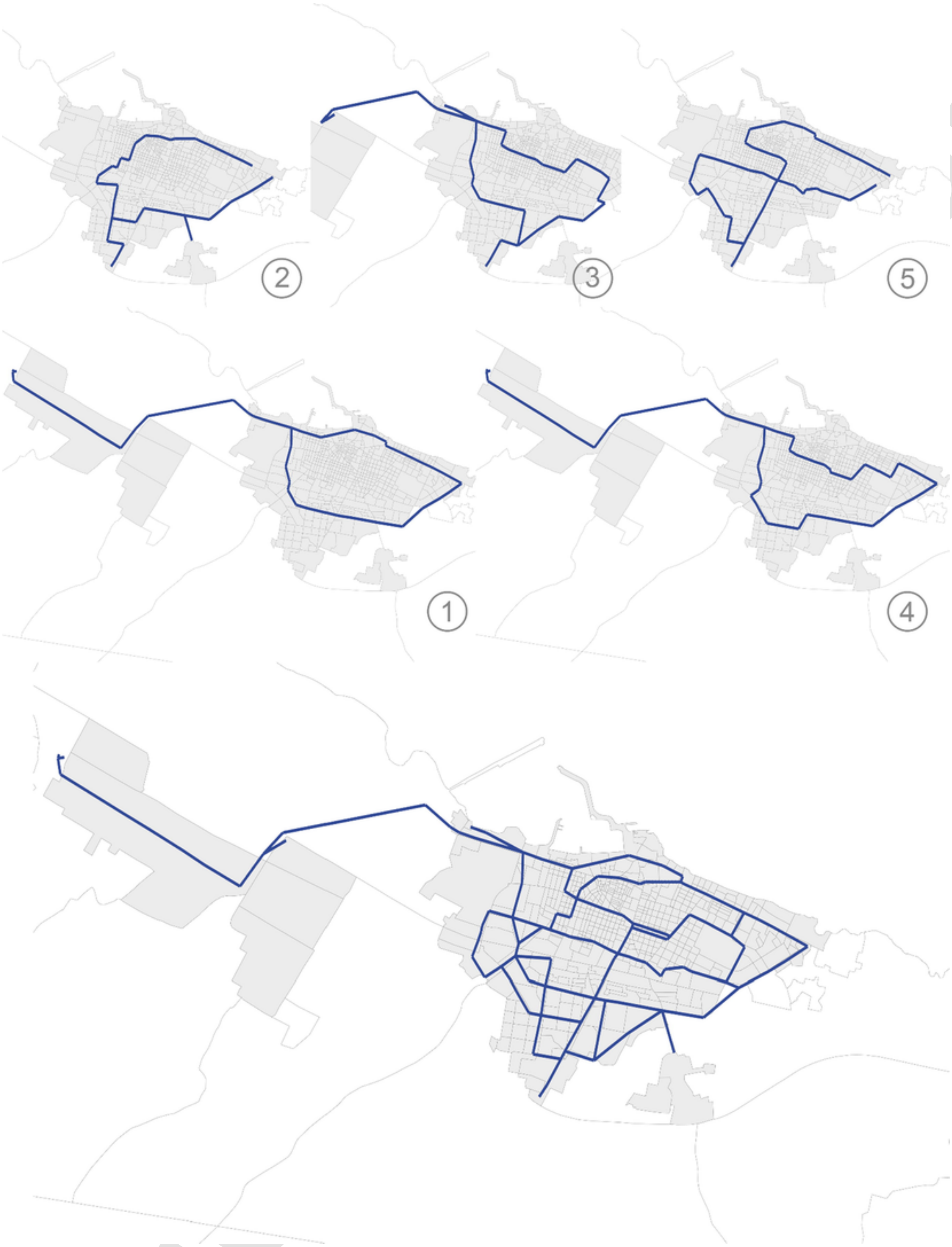

Fig. 8. Path covered by each one of the optimal routes, and by the optimal transit route set as a whole.

Table 15
Numerical results from the optimization with the horizontal and vertical equity constraint and with variable weights.

R_Gini < 0.38			Overall cost			R_Gini			User costs	Operator costs	Uns. Dem. costs
γ_1	γ_2	γ_3	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
1	*1*	*1*	*23,943*	*35,704*	*51,471*	*0.321*	*0.346*	*0.278*	*30,646*	*3992*	*1066*
1	4	15	31,724	34,448	35,896	0.33	0.34	0.35	29,443	4345	660
1	4	30	31,970	37,442	47,924	0.32	0.35	0.37	33,202	3940	301
1	4	45	28,027	38,006	43,348	0.30	0.31	0.32	33,751	4040	215
1	8	15	39,972	54,620	69,788	0.33	0.34	0.35	49,987	3856	777
1	8	30	39,359	45,549	49,308	0.34	0.36	0.37	40,742	3953	854
1	8	45	40,735	43,376	46,492	0.33	0.35	0.37	39,331	3955	90
1	12	15	40,170	48,067	57,972	0.31	0.34	0.37	43,091	4280	696
1	12	30	26,124	42,164	52,967	0.35	0.36	0.37	38,762	3130	272
1	12	45	41,119	49,237	64,626	0.31	0.35	0.38	45,648	3486	103

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