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A dynamic clustering method for relocation process in free-floating vehicle sharing systems

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Abstract

Recently, vehicle sharing emerged as a new type of mobility service. In particular, if these systems happen to be free-floating, their operating area is typically located within the city and vehicles could be picked-up and parked in any permitted spot, and not only at predetermined stations. This specific feature enables everyone to pick-up and drop-off a rented vehicle close to his demand points, with no need to visit a station before or after the ride, granting greater flexibility for users together with the opportunity of a shorter trip. Free-floating systems, due to their inherent characteristics, are becoming more popular; however, at the same time, they involve additional operational challenges especially in facing the relocation processes, namely displacing vehicles from areas with higher concentration toward those with a higher request of the service.
Similar to the station based ones, free-floating vehicle shared-use systems deal with significant fluctuations in demand, depending on day/time and area of a city. Therefore, we suggest a methodology for generating a dynamic zone clustering in order to define cost-efficient relocation strategies. The aim of the proposed flexible clusterization is identifying the optimal size and number of areas among which perform an effective and enhanced vehicle repositioning, reducing the necessity to move vehicles from one zone to another and, accordingly, shrinking the relocation costs. The proposed method is applied to a test case study, in order to verify the accuracy of the suggested model.

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Keywords: vehicle sharing; free-floating; vehicle relocation; clustering techniques.

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1. Introduction

In the last decades, a lot of changes have taken place in the urban transportation (Jorge et al., 2014). Although private vehicles offer a significant accessibility, the high levels of congestion, pollution and nonproductive time for travelers still represent an undesired outcome (Schrank et al., 2010). These issues are mitigated by mass transit systems (such as buses, trains, etc.) which, however, have inevitably disadvantages related to the pre-set stops locations and times -coverage and schedule inflexibility-, with consequent lack of personalization (Murakami et al., 2005).

Alternative strategies are needed to address these issues. In this framework, systems and methods for sharing a fleet of vehicles among a plurality of users have proved to be a valuable option (Murakami et al., 2005). In a shared system, a certain number of vehicles is normally guaranteed in several designated parking areas. Each user can pick-up a vehicle located next to the starting point of his/her trip, and return it to the parking area nearest to his/her destination. Generally, these shared systems can be classified as "non-floating" (also known as station-based) and "free-floating" (Boyacı et al., 2015). If in a non-floating system users have to use predefined stations in which pick-up or drop-off the vehicle, in a free-floating system more flexibility and spontaneity is allowed. This happens because free-floating systems define a geofence in which it is possible to hire and return the vehicle very close to the demand points, without having the necessity to pass by a station before or after the ride (Herrmann et al., 2014). Usually, free-floating shared systems allow savings on start-up costs (i.e. avoiding construction of stations and kiosk machines) in comparison to the non-floating (traditional and station-based) ones (Pal and Zhang, 2015), but usually they lead to additional operational challenges (Kortum and Machemehl, 2012).

In this paper, we focus on free-floating shared systems (namely, bike or car-sharing), where vehicles are dispersed in different demand areas. In this framework, during the day significant fluctuations in travel demand (due to weather conditions, time of the day and holidays/weekends) can be observed. Sometimes there is a vehicle overcrowding in certain zones, and a lack of available vehicles in others, at the time the users need them (Herrmann et al., 2014). These imbalances of supply and demand can be resolved/mitigated only with an appropriate reallocation strategy (Reiss and Bogenberger, 2015), namely a transfer of vehicles from zones with high accumulation to areas where the shortage is experienced (Boyacı et al., 2015). It has to be noted that bicycles' relocation is easier than cars'one, as up to 60 bikes can be carried together by a single relocator, while cars have to be moved individually or by a car transporter that implies high costs; this makes the pursued strategy even more important (Weikl and Bogenberger, 2013).

Mainly, all the approaches adopted to relocate the shared fleets can be grouped into two categories according to who actually performs the relocation: user-based and operator-based strategies. Whereas the formers are based on bonus models that want to encourage customers to relocate a vehicle themselves after their trip (Di Febbraro et al., 2012), the latter are conducted by the operators, adopting optimization approaches or simulation models (e.g. see Jorge et al., 2014, and Angeloudis et al., 2014). User-based relocation strategies are advantageous from a financial point of view and environmentally sustainable (Weikl and Bogenberger, 2013): as vehicles are driven directly by users, no additional trips are conducted. On the other side, operator-based relocation strategies are based on interventions established by the system manager and executed by the vehicle-sharing provider itself. It is worthy to mention Reiss and Bogenberger (2015), that in order to apply their operator-based strategy to a bike-sharing system, have divided the operating area of the free-floating system into a certain number of zones, that in a way could be interpreted as stations.

Aiming at carrying on a cost-efficient relocation strategy, in this work we suggest a methodology to generating a dynamic zone clustering for the free-floating systems under analysis. The main purpose of a clustering analysis is organizing a collection of different patterns into a smaller number of homogeneous groups, without any prior knowledge. Clustering methodologies have been extensively used in literature to explore the activity patterns related to a shared system usage and reveal communities of users, with a wide range of final goals. For example, some authors have shown how cluster analysis is capable of revealing groups of stations with a similar trend of rental and return activities during the day (Vogel et al., 2011). Other studies have conducted a spatio-temporal analysis of the bicycle station usage of bike-sharing systems, with real case studies application in Barcelona (Froehlich et al., 2009), Paris (Côme et al., 2014), London (Caggiani et al., 2017). However, to the best of our knowledge, it seems that in the literature pertaining to shared transportation systems the zoning (preparatory to the repositioning of vehicles) of a given area is assumed to be fixed and unchangeable during the day. No specific studies about a flexible/dynamic clusterization method appear to have been put forward. With our work, we try to fill this apparent gap in the literature, suggesting an alternative method to be applied to a free-floating vehicle-shared system (FFVSS).

In the following section, our dynamic clustering methodology is applied to a FFVSS aiming to enhance the cost-efficiency of operator-based relocation operations. The proposed model is then tested on a network, accompanied by a sensitivity analysis, to prove its efficacy and usefulness. The first findings are promising, as they show how, adopting a flexible vehicle repositioning during the day, it seems to be possible to reduce the occurrence of those system configurations that require the performing of a relocation.

2. Proposed dynamic clustering methodology

The novelty of this paper consists in the suggestion of a flexible/dynamic strategy in carrying out the zoning of a FFVSS, that allows to look at the system from a different perspective.

As a matter of fact, we start from the assumption that a static zoning within a day could lead to an excess of relocation processes performed on the system, with the (misleading) idea of being actually improving its overall performance and satisfying a greater number of users. Have a look at the example depicted in Fig. 1 to better understand this claim. We have to deal with a small area, divided into 8 zones/districts, where a FFVSS is operating. We want to study the condition of this system for a given time interval, in order to understand if, at that moment, a relocation strategy needs to be implemented or not to enhance its global functioning. The number reported inside each zone denotes how many vehicles are (at that moment) available to any potential user. Every zone of this FFVSS could be seen as a station (in a station-based sharing-system), that aggregates/contains (inside its borders) a number of vehicles. Assuming that the zoning on the left of Fig.1 has been fixed a priori, we can observe that two clusters can be identified. In the upper one, globally 6 vehicles are available (1+4+0+1); the cluster below, on the contrary, has no usable vehicle. It seems that a relocation could be helpful to move, at least one vehicle, from the above cluster to the 'needy' one below. Have a look, instead, at another possible clusterization of the same system depicted on the right of Fig. 1: in this case, both clusters have at least one vehicle ready to be picked-up by a user that wants to start his/her trip. Consequently, if the decision maker believes that (at least) one vehicle for each cluster could be counted as satisfactory for a proper functioning of the system, no relocation operations need to be performed.

1	4
0	1
0	0
0	0

1	4
0	1
0	0
0	0

Fig. 1. Two different clusterizations applied to the same FFVSS configuration.

It has to be specified that what we have just said it is true if both the possible clusterizations illustrated by Fig.1 have a cluster extension α_y comparable with the average distance w that a typical user is willing to walk to reach a free-floating vehicle. Note that the same applies to a station-based vehicle-sharing system with a distribution/density on the territory sufficient to ensure that the average distance between two stations is commensurate with w.

Having this small example in mind, we can move on in detail the methodology that we want to suggest. We propose to use, as a performance indicator of our FFVSS, the zero-vehicle-time (ZVT) (Kek et al., 2009). When ZVT occurs, a zone (or station, in a station-based system) is without any available vehicle; then, a customer requesting for vehicles at that moment in that zone will be rejected/unsatisfied. Basically, it can be asserted that, from the operator's point of view, ZVT implies a potential loss of income; from the user's point of view, ZVT binds him/her to move to another zone to pick-up a vehicle or alternatively forces him/her to change travel mode. Then, to assure a satisfactory running of a FFVSS, a possible strategy to adopt is trying to globally minimize the ZVT.

Therefore, we propose the adoption of the following bi-level optimization model (from Eq. (1) to Eq. (5)), to determine the dynamic clusterization to be associated with each time interval Δz when ZVT occurs (during a typical operation day), in order to effectively find out when the system needs a proper vehicle relocation, or when (on the contrary) a repositioning is unnecessary. Consider that, before performing the optimization, the study area must be divided into zones having whatsoever shape, but with a size that allows them to be appropriately clusterized (i.e. a cluster obtained by the aggregation of two of them needs to have a maximum spatial width less than or equal to the average user walking distance w). In the following, a notation table is presented in order to summarize and define the symbols adopted throughout the paper.

Notations	
Δt	width of time (sub)interval
Δz	width of a total operation time interval, discretized in k subintervals Δt, with $k \in [1, 2, \ldots, n]$
$\bar{\varphi}$	total number of centroids related to each zone
φ	generic centroid, with $\varphi \in [1, 2, \ldots, \bar{\varphi}]$
y	total number of clusters (decision variable)
$\bar{y}$	positive integer coefficient corresponding to the minimum admissible number of clusters
C_y	generic cluster, with $y \in [\bar{y}, \ldots, \bar{\varphi}]$
c_y	generic centroid of cluster C_y, with $y \in [\bar{y}, \ldots, \bar{\varphi}]$
α_y	maximum spatial width of each cluster C_y
w	average distance that a user is willing to walk to pick-up a free-floating vehicle

$$\min \sum_{k=1}^{n} \sum_{j=1}^{y} \mathrm{ZVT}_{kj}(y) \tag{1}$$

$$\min \sum_{j=1}^{y} \sum_{\varphi \in C_y} \mathrm{EuclideanDist}\,(c_y, \varphi) \tag{2}$$

s.t.

$$y \geq \bar{y} \tag{3}$$

$$y \leq \bar{\varphi} \tag{4}$$

$$\alpha_y \leq w \tag{5}$$

The upper-level objective (1) aims at minimizing the sum of the ZVTs related to each cluster in a given time interval Δz, discretized in k subintervals Δt. The lower level objective (2) represents the k-means optimization (see MacQueen, 1967, and Arthur and Sergi, 2007 for further details) that is, the minimization of the distance (in our case, Euclidean distance) between the positions of the centroids φ of each zone, and the centroids c_y of the clusters C_y. Equation (3) means that the total number of clusters y has to be greater than (or equal to) a positive integer coefficient $\bar{y}$; at the same time, it has to be lower or equal to the total number of zones $\bar{\varphi}$ in the study area (4). Finally, the maximum spatial width α_y of each cluster C_y needs to be lower than w, i.e. the average distance that a user is willing to walk to pick-up a free-floating vehicle. It is worthy to underline that to carry out the k-means optimization is required to set up the so-called 'replicates', that is, the number of times to repeat the clustering using a new initial cluster centroid positions. If usually the selected replicate is the one that returns the minimum Euclidean distance, in the suggested model we decide to rather choose the one associated with the lowest ZVT value.

Furthermore, it is true to assert that ZVT is the zero-vehicle time, in which no vehicle is available in a cluster –this means that one vehicle is reckoned sufficient to satisfy the users' demand. However, if the decision maker believes that a higher number has to be assured (e.g., 3 vehicles), we can apply the same model setting ZVT equal to the number of vehicles to guarantee minus one (e.g., ZVT=3-1=2): this means that, in this way, each cluster needs at least 3 vehicles to be self-sufficient and ensure a proper functioning of the FFVSS.

3. Numerical application

In this section, we apply the suggested methodology to a study area of 1.2 km x 1.2 km of extension. This area is composed of 36 square zones, with a side length equal to 0.2 km (grid of 6x6 zones).

We assume that in this area a free-floating bike-sharing system (FFBSS) is operating. A further assumption is that a typical user is willing to cover a maximum distance of about 630 meters by walk to reach the bicycle closest to the origin of his/her trip. This means that, in the described context of the study, the clusterization process can aggregate groups of zones having only four possible configurations, in order to have an area extension suitable for everybody: a square of 2x2 zones; an L shape composed of 3 zones; a rectangle made by two adjacent zones; a rectangle made by three adjacent zones. If no aggregation results possible, the cluster will coincide with a single zone (Fig. 2).

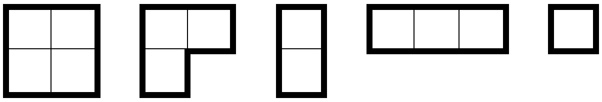

Fig. 2. Eligible cluster configurations.

The bike-sharing system simulator proposed by Caggiani and Ottomanelli (2012 and 2013) has been used to represent and model the FFBSS under analysis, pretending that the centroids of each zone coincide with a hypothetical bike-sharing station. According to the simulator, the operating day is divided into discrete time intervals. For each time interval Δt and for each station, given the pick-up bicycle demand (number of bicycles picked-up), it simulates the destination choice in order to assess the arrival time for each user. The choice model is based on the relative origin/destination attractiveness and on the nature of the trip (one-way or round trip). At the beginning of each interval Δt, the number of bicycles is updated by considering the in-out user's flow (turn-over). For further details refer to Caggiani and Ottomanelli (2012 and 2013). To sum up, we know how many bicycles there are (at any time interval) in the system (we assume that these obtained values represent the forecasted trends for each zone during the day), and which is the demand level (i.e. bicycle request) during the day. In this numerical application, we assume to have 212 bicycles in the study area, and a total number of daily bicycle requests of 1532. At the beginning of the day, all the bicycles are available; during the day, part of them is not immediately usable, being ridden by the users of the system. As an example, Fig. 3(a) shows the available bycicles in each one of the 36 square zones during the first Δt of the day (00:00 a.m. – 00:05 a.m.); Fig. 3(b) is representative of the status of the system during the interval 10:00 a.m. – 10:05 a.m., and Fig. 3(c) is the demand matrix (number of bicycle requests in each zone) in the same time step. For a given Δt, in those zones in which the pick-up bicycles demand results unsatisfied (namely, where the number of bicycles available is not sufficient to meet the demand), we suppose that the user walks to a nearby zone, according to the maximum distance he is willing to walk. If he is not able to find any bicycle within the area having as radius his/her walking distance, he becomes a lost user for the system.

9	9	4	4	9	9
9	4	4	4	4	9
4	4	6	6	4	4
4	4	6	6	4	4
9	4	4	4	4	9
9	9	4	4	9	9

a)

9	8	0	4	2	8
9	1	0	0	2	6
0	0	21	15	8	3
3	5	13	16	4	9
5	1	0	4	0	2
0	0	1	0	3	3

b)

0	1	1	0	0	0
0	0	0	0	0	0
0	0	0	0	0	1
0	1	0	0	0	0
0	0	0	1	0	0
0	0	0	0	0	0

c)

Fig. 3. An example of bicycle matrices (a and b) and a demand matrix (c).

The aim of the application presented in this section is twofold. The first goal consists in comprehending the advantages deriving from a dynamic spatial clusterization of the study area compared with a traditional static one (the final ZVT values achieved are presented and discussed). The second one is to understand what happens if the level of bicycle request/congestion of the FFBSS increases (carrying out a preliminary sensitivity analysis).

3.1. ZVT analysis comparing static and dynamic clusterization methods

In the described study area, we simulate the behavior of the FFBSS, calculating the associated ZVT for an entire day (that is, Δz = 24 hr) running 3 different tests. The ZVT value indicates the total number of time steps of 5 minutes in which a cluster has zero available bicycles (as we set the minimum threshold able to guarantee an acceptable functioning of the system equal to 1). As an example, if (at a given time step) the study area is divided into 10 clusters, and 3 out of these clusters do not have any available bicycle for users to pick-up, then ZVT=3 for that time step (i.e., for those 5 minutes). The global (daily) ZVT is achieved summing the corresponding values of the partial ZVTs calculated every 5 minutes. The minimization of ZVT has been done using genetic algorithms; the number of replicates for the k-means optimization has been set equal to 50. -Test (1): Static (fixed) clusterization. The 36 zones are aggregated in 9 square clusters, each one made by 2x2 zones.

-Test (2): Static (fixed) clusterization, calculated during the day applying (only one time, Δt = 24 hr) the proposed optimization (Eqs. 1-5). The allowed cluster shapes are those shown in Fig. 2.

-Test (3): Dynamic clusterization, applied every Δt = 5 min. We obtain a total number of clusters y and an optimal configuration of clusters associated to each Δt. The minimization of ZVT (dynamic clustering method) runs for a given Δt only if at least one zone out of 36 has no bicycle available. Otherwise, the system is clearly working properly, there is no need to apply the suggested model, and the cluster configuration remains the same of the preceding Δt.

Figure 4, with an example of clustering results obtained for a given time step for the 3 test cases, makes easier the understanding of the proposed dynamic clusterization and the associated benefits. Although the number of bicycles in every zone remains exactly the same, in the configuration proposed by Test (3) no cluster has zero vehicles, then no relocation needs to be performed.

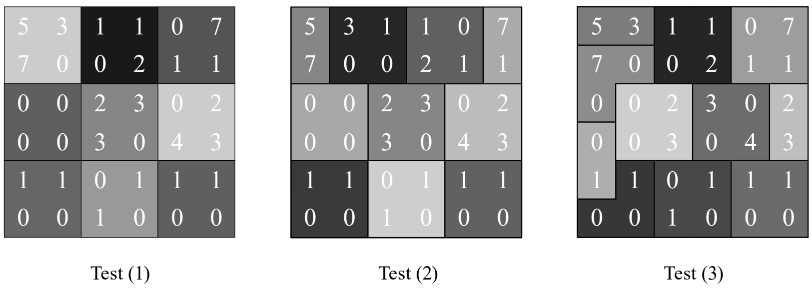

Fig. 4. Clustering results at 4:20 pm for the three test cases.

Looking at the results presented in Table 1, we note that the more we set smaller time intervals in which to perform the ZVT minimizations, the more the value of ZVT at the end of the day seems to shrink. There is a global difference of 23 steps (=130-107) between the ZVT calculated in the static zoning, and the ZVT obtained at the end of the dynamic clusterization process. This number (23) means that for 115 min (23x5min) the FFBSS is already working in a satisfactory way, any user can easily reach a free-floating bicycle by walk, and there is no need to perform a relocation to enhance the current situation. The reduction of ZVT obtained by our dynamic clusterization methodology does not derive from any bicycle repositioning: it represents only a strategy to look at the FFBSS from a different point of view, realizing that (maybe) fewer users than we expected are experiencing a drawback deriving from an unbalanced functioning of the system. Then, we can assert that part of the relocation operations that could be suggested looking at the static zoning of the system could turn to be unnecessary.

Table 1. ZVT (number of 5 minute-steps) calculated adopting different zone clusterization methods, and corresponding total minutes.

	Test(1)	**Test(2)**	**Test(3)**
ZVT	130 (650 min)	115 (575 min)	107 (535 min)

A further observation could be done about the forecasted trends of the number of bicycles during the day for each zone, that basically constitute one of the inputs of our methodology. It is important to say that, when dealing with forecasted data, there is always a certain degree of uncertainty associated with the prediction. In this context, we are assuming that the forecasted trends are reliable and that they are actually going to come true. We are not carrying out a comparison among different methods of prediction, risking to 'corrupt' the outcomes of our approach and losing sight of our goal. We want just to prove that, under certain hypothetical/forecasted conditions, a dynamic clusterization could allow savings associated with a lower number of performed repositions.

3.2. Sensitivity analysis

The sensitivity analysis presented in this subsection aims at verifying what happens if the bicycle demand as origin in the network changes, varying accordingly the level of congestion of the FFBSS. We believe that the efficacy of our method is strictly dependent to the picked-up bicycles demand distribution, and to the number/density of bikes in the territory; then, carrying on this preliminary sensitivity analysis is fundamental.

We hypothesize to perform the 3 tests for 4 different demand scenarios, with an increasing users' bicycle request (maintaining unchanged the total available bicycles in the system, 212). The total daily number of bicycle requests (demand level) is respectively equal to 1532 requests, 2280 requests, 2766 requests and 4742 requests for each demand level DL(1), DL(2), DL(3), DL(4). For each DL, 10 simulations have been done, running 10 times the bike-sharing system simulator proposed by Caggiani and Ottomanelli (2012 and 2013), thus obtaining different trends related to the number of bicycles during the day for each zone. In this way, we can prove that, under various starting conditions, the proposed dynamic clusterization method still maintains its validity. Table 2 reports the results achieved:

Table 2. Sensitivity analysis of ZVT values (number of 5 minute-steps) obtained for 4 levels of pick-up bike demand.

Simulation number		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	Test(1)	130	116	107	146	68	98	176	121	154	112
DL(1)	**Test(2)**	115	116	107	146	68	98	176	121	154	112
	Test(3)	107	104	106	136	59	91	167	101	142	95
	Test(1)	381	340	357	309	333	377	329	327	343	329
DL(2)	**Test(2)**	381	340	357	309	333	377	329	327	343	329
	Test(3)	381	335	351	306	336	377	324	324	336	322
	Test(1)	313	277	363	407	374	323	331	339	383	315
DL(3)	**Test(2)**	313	277	363	407	374	323	331	339	383	315
	Test(3)	299	269	358	399	370	318	317	337	378	307
	Test(1)	498	465	467	428	524	558	509	489	451	501
DL(4)	**Test(2)**	498	465	467	428	524	558	509	489	451	501
	Test(3)	489	465	463	426	518	548	502	483	454	495

Looking at Table 2, we can assert that the results obtained in Test (3) -i.e. using the suggested dynamic clusterization method- seem to be promising, as they reach a ZVT (almost) always lower than in the static zoning configuration. Anyhow, the efficacy of our model is strictly related to the spatio-temporal bicycle distribution; then, it can happen that for certain specific configurations it does not lead to better results than the traditional approach. However, the model works better for a lower demand level (see DL (1) in Table 2), as with an increasing bicycle request (keeping, at the same time, unaltered the total number of free-floating bicycles in the system), inevitably, it becomes progressively more difficult to find ways to improve ZVT.

Furthermore, it has not been noticed any valuable difference between the static square zoning -Test (1)- and the static clusterization obtained applying our method on a time step coinciding with the entire day (Test (2), with Δt = 24 hr), as their values, almost always, tend to overlap. This is probably related to the fact that we are considering forecasts equal to the observed demand. Probably, in a dynamic situation (real case study), Test (2) and Test (3) may have a more advantageous behavior.

4. Conclusions and further research

Dealing with free-floating vehicle-sharing systems often involves additional operational challenges, in particular with regard to the relocation processes. With this paper, we are proposing an optimization method that leads to the generation of a dynamic zone clustering, in order to perform only the necessary and effective repositioning operations that actually allow enhancing the global functioning of the system.

The application to a test case study and a preliminary sensitivity analysis have demonstrated the effectiveness of our approach in relation to the ZVTs achieved during the day if compared to a more traditional static zoning. One of the first outcomes of our analysis, as an example, has revealed a global difference of 23 time steps between the ZVT calculated in the static zoning, and the one obtained at the end of our dynamic/flexible clusterization process. Basically this means that, without adopting the proposed methodology, 23 zones (globally, during an operation day) would have supposed to have zero available vehicles, requiring a higher number of (unnecessary) repositionings and leading to more costs to bear by operators, and more traffic and congestion associated to the relocation trucks. This has proven to be true also with a different (increased) level of bicycle requests. Ongoing research is dealing with the inclusion of the proposed dynamic clusterization method in the general framework of operation of a FFVSS, in order to simulate the relocation process and verify if the actual number of relocated vehicles is lower than in the static clusters scenario.

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