

## Derived distribution of floods based on the concept of partial area coverage with a climatic appeal

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**Abstract.** A new rationale for deriving the probability distribution of floods and help in understanding the physical processes underlying the distribution itself is presented. On the basis of this a model that presents a number of new assumptions is developed. The basic ideas are as follows: (1) The peak direct streamflow  $Q$  can always be expressed as the product of two random variates, namely, the average runoff per unit area  $u_a$  and the peak contributing area  $a$ ; (2) the distribution of  $u_a$  conditional on  $a$  can be related to that of the rainfall depth occurring in a duration equal to a characteristic response time  $\tau_a$  of the contributing part of the basin; and (3)  $\tau_a$  is assumed to vary with  $a$  according to a power law. Consequently, the probability density function of  $Q$  can be found as the integral, over the total basin area  $A$ , of that of  $a$  times the density function of  $u_a$  given  $a$ . It is suggested that  $u_a$  can be expressed as a fraction of the excess rainfall and that the annual flood distribution can be related to that of  $Q$  by the hypothesis that the flood occurrence process is Poissonian. In the proposed model it is assumed, as an exploratory attempt, that  $a$  and  $u_a$  are gamma and Weibull distributed, respectively. The model was applied to the annual flood series of eight gauged basins in Basilicata (southern Italy) with catchment areas ranging from 40 to 1600 km<sup>2</sup>. The results showed strong physical consistence as the parameters tended to assume values in good agreement with well-consolidated geomorphologic knowledge and suggested a new key to understanding the climatic control of the probability distribution of floods.

### 1. Introduction

The theoretical derivation of flood probability distributions is a matter of great interest for hydrologists as an attractive tool for flood frequency analysis in regions where sufficient or reliable direct measurements of streamflow are lacking. Furthermore, it helps in providing physical support to the classic statistical analysis and remains a significant way to improve knowledge of the climatic and geomorphologic processes leading to the generation of extreme floods.

*Klemes* [1987] ascribes the principal differences between observed flood series to the mechanisms underlying the generation of floods. In addition, a remarkable influence of the climate on the variability of the statistical behavior of floods was clearly identified by *Farquharson et al.* [1992]. They analyzed flood data relative to arid and semiarid areas spread all over the world (162 sites in Africa, Iran, Jordan, Saudi Arabia, Russia, and the United States), in comparison with other flood data recorded in the United Kingdom, in humid and temperate climates. In arid areas a significant homogeneity between the observed frequency curves was recognized.

All the theoretical models deriving flood distributions owe a tribute to *Eagleson* [1972], who first depicted a complete framework to describe the physical mechanisms underlying the probability distributions of rainfall-generated floods. In his model the kinematic wave theory was used to account for the

surface runoff. The intensity and the duration of rainfall were expressed as exponentially distributed and reciprocally independent, and the infiltration process was schematized by way of a constant loss rate. Other kinematic wave-based models are those of *Shen et al.* [1990] and *Cadavid et al.* [1991], who investigated, within the same framework, some situations not completely developed by *Eagleson* [1972].

*Hebson and Wood* [1982] and *Wood and Hebson* [1986] used the geomorphologic instantaneous unit hydrograph theory [*Rodriguez-Iturbe and Valdes*, 1979; *Rodriguez-Iturbe et al.*, 1982] to model the basin response. *Diaz-Granados et al.* [1984] introduced the Philip's equation to model infiltration. *Moughamian et al.* [1987] tested the *Hebson and Wood* [1982] and *Diaz-Granados et al.* [1984] methods and noticed unsatisfying performances in both cases.

*Sivapalan et al.* [1990] investigated the case of partial contributing areas controlled by different mechanisms such as Hortonian infiltration excess and Dunne's saturation excess [*Leopold*, 1974]. They assumed that the rainfall intensity was gamma-distributed and used the geomorphologic unit hydrograph to account for the basin response. *Raines and Valdes* [1993] used the Soil Conservation Service curve number method to model infiltration. *Kurothe et al.* [1997] accounted for the bivariate probability density function of rainfall intensity and duration, which were supposed to be negatively correlated.

It seems to us that the various quoted authors who have gone through *Eagleson's* [1972] model have always limited their action to improve either the rainfall statistic model or the

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basin response function. In other words, they only modified some parts of the model, leaving unchanged the general framework. The first exception is given by *Gottschalk and Weingartner* [1998] where the peak runoff is synthetically linked to the rainfall volume by means of a runoff coefficient which is considered to vary stochastically according to a beta function.

In this paper the basic idea arises from *Eagleson's* [1972] paper where he strongly highlighted the important role of the partial area contributing to the direct runoff. In that paper this area was identified as a stochastic variable dependent on storm intensity, duration, and size as well as on antecedent surface and subsurface conditions like vegetation and soil moisture. Only because of the difficulty in handling a trivariate joint distribution, did Eagleson consider the contributing area as a basin feature, being independent of the rainfall variables.

In our approach the partial area  $a$  which contributes to the flood peak is considered as a stochastic variable. In addition, the "storm duration," here intended as the rainfall duration most significant for flood peak generation, is treated as a variable dependent on it. In particular, given a certain  $a$ , the lag time  $\tau_a$ , depending on the contributing area  $a$ , is supposed to be the duration of the storm "responsible" for the flood peak. This lag time is here intended as the lag of direct runoff centroid to effective rainfall centroid.

Such a scheme allows for simplifying the routing model, at the price of the increased complexity needed to model the marginal distribution of the runoff contributing area. In this paper it is also suggested that this distribution may be somehow related to the long-term climatic characteristics of the basin.

The paper begins with a general description of the method proposed to derive the cumulative distribution function of floods. The method is based on the combination of the probability density functions of the contributing areas with that of the peak runoff per unit contributing area. Next, the whole group of governing parameters involved in the proposed distribution is surveyed focusing on their physical interpretation, and an expression for the index flood is derived. In section 3, the application of the model to eight annual flood series recorded in southern Italy suggests that the flood distribution parameters that have a stronger physical sound are sensitive to the long-term climatic features of the basin. Then, on the basis of a dimensionless formulation of the derived distribution, a preliminary sensitivity analysis also highlights such a climatic control.

## 2. Theory

In the presence of a storm of any size, duration, and intensity, whatever process of transformation of effective rainfall into runoff is considered or whatever model for that transformation is adopted, the peak of direct streamflow is always

$$Q = u_a a, \quad (1)$$

where  $a$  is the peak contributing area, that is, the partial area contributing to the peak flow, and  $u_a$  is the discharge per unit (contributing) area at the flood hydrograph peak.

The "contributing area" accounts for both the facts that the storm area is finite and that the direct runoff comes from only a fraction of the area covered by a storm. Moreover, for short-duration storms the "peak contributing area" is also controlled by the routing process. It is sometimes as low as 5% [*Betson*, 1964].

Considering  $u_a$  and  $a$  as two stochastic dependent variables, it is possible to derive the cumulative distribution function of peak streamflow integrating the joint density function  $g(u_a, a)$  over the region  $R(q)$  within which the peak streamflow, depending on  $u_a$  and  $a$  as in (1), is smaller than  $q$ .

$$G_Q(q) = \text{prob}[Q < q] = \iint_{R(q)} g(u_a, a) du_a da. \quad (2)$$

Analogously, we get the exceedance probability function of the peak streamflow  $Q$ ,  $G'_Q(q)$  integrating the same joint density function  $g(u_a, a)$  over the region  $R'(q)$ , within which  $Q$  is greater than  $q$ .

$$\begin{aligned} G'_Q(q) &= \text{prob}[Q \geq q] = 1 - G_Q(q) \\ &= \iint_{R'(q)} g(u_a, a) du_a da. \end{aligned} \quad (3)$$

As already stated,  $u_a$  and  $a$  are mutually dependent; thus the joint density function  $g(u_a, a)$  may be calculated as

$$g(u_a, a) = g(u_a|a)g(a) = g(a|u_a)g(u_a), \quad (4)$$

where  $g(u_a|a)$  is the distribution of the average runoff per unit area given the same contributing area,  $g(a|u_a)$  is the distribution of the peak contributing area conditional on the unit runoff, while  $g(a)$  and  $g(u_a)$  are the respective marginal distributions.

Substituting (4) into (3), we get

$$G'_Q(q) = \int_0^A \int_{q/a}^\infty g(u_a|a)g(a) du_a da \quad (5)$$

or

$$G'_Q(q) = \int_0^\infty \int_{q/u}^A g(a|u_a)g(u_a) da du_a. \quad (6)$$

Thus the domain  $R(q)$  in (3) becomes explicit as the contributing area  $a$  may assume only values lower or equal to the total area  $A$  of the basin, and then, in (5), for any  $a$ ,  $u_a$  must be greater or equal to  $q/a$ , while in (6),  $a$  must range between  $q/u_a$  and  $A$ . In principle, either (5) or (6) could be exploited; in the following, consistent with the assumptions which we will make in sections 2.1 and 2.2, we will use (5), which can also be expressed as

$$G'_Q(q) = \int_0^A G'_{u_a|a}\left(\frac{q}{a}, a\right)g(a) du_a da, \quad (7)$$

which may lead to different formulations for the  $G'_Q$  as different hypotheses are assumed for  $G'_{u_a|a}$ , that is, the exceedance probability function of  $u_a$  given  $a$  and  $g(a)$ .

### 2.1. Contributing Areas

Let us refer to floods produced by runoff coming from an area that may represent only a portion of the entire basin. This happens for the following arguments. Thinking of the basin area as composed of some independent subbasins, the storm may affect only one or a few of them. Furthermore, within any of these subbasins the conditions needed to produce surface runoff, and eventually significant through flow, are not reached

everywhere during the flood event. Therefore the contributing area is composed of the sum of the partial areas belonging to the contributing subbasins. In other words, the flood peak is given by the superposition of flows coming from a random number of subcatchments involved by the storm according to the storm size and movement. This number depends on the basin size relative to two characteristic space scales, namely, the storm and rain-cell sizes, hereafter indicated as  $S_s$  and  $S_c$ , respectively. In fact, for very small basins, whose area is comparable to or less than  $S_c$ , there is a high probability that the entire catchment contributes to the peak flood with space-time homogeneous rainfall. Whereas for basins whose area ranges between  $S_c$  and  $S_s$ , it is much more likely that  $a$  is given by the sum of separate subcatchment contributing areas. This is due to different timing and duration with respect to which separate portions of the basin are involved by the rain cells and to different response times of these portions.

We assumed that the probability density function (pdf) of the peak contributing area is a gamma distribution for any  $a$  smaller than  $A$ . Thus letting  $P_A$  be the finite discrete probability that the whole catchment is contributing to the flood peak, we get

$$g(a) = \frac{1}{\alpha\Gamma(\beta)} \left(\frac{a}{\alpha}\right)^{\beta-1} \exp\left(-\frac{a}{\alpha}\right) + \delta(a-A)P_A, \quad (8)$$

where, indicating with  $\gamma(\cdot, \cdot)$  the incomplete gamma function [e.g., *Gradshteyn and Ryzhik*, 1980, p. 940],

$$\begin{aligned} P_A &= \text{prob}[a = A] = \gamma(A/\alpha, \beta) \\ &= \int_A^\infty \frac{1}{\alpha\Gamma(\beta)} \left(\frac{a}{\alpha}\right)^{\beta-1} \exp\left(-\frac{a}{\alpha}\right) da, \end{aligned} \quad (9)$$

while the impulsive function  $\delta(\cdot)$  is the generalized function defined by

$$\int_{-\infty}^{\infty} \varphi(x)\delta(x) dx = \varphi(0), \quad (10)$$

so that

$$\int_{-\infty}^{\infty} P_A\delta(a-A) da = P_A. \quad (11)$$

Basically, the choice of the gamma distribution is a working hypothesis. Nevertheless, it can be justified when it is reminded that this function arises as the distribution of the sum of  $\beta$  stochastic (independent) variables exponentially distributed with equal mean value  $\alpha$ .

This justification helps in giving a meaning to the distribution parameters. In fact, without any loss of generality, we may think of any flood peak as being due to the superposition of flows coming from a number of subbasins which can be differently involved by the storm. In addition, it is always possible to postulate that the subbasins that may provide runoff have comparable sizes, so that it is consequent to suppose, for modeling purposes, that their contributing areas have equal mean. They can also be thought of as being independent of each other, with the warning that in the eventuality of lack of independence, attention should be paid to the meaning of  $\beta$ .

The number of these subbasins is lower bounded by unity for very small basins. Instead, for basins whose area ranges be-

tween  $S_c$  and  $S_s$ , it is possible to think of  $\beta$  as the number  $N_\omega$  of subbasins of Horton order immediately smaller than that of the whole basin. In fact, these subbasins are, on average, of comparable sizes and may consequently be modeled by the same value of  $\alpha$ . For larger catchments,  $\beta$  could be even greater than  $N_\omega$  because of the probable superposition of discharges coming from different subbasins covered by more than one storm. Incidentally, it may be interesting to remember that, according to well-consolidated geomorphologic knowledge,  $N_\omega$  tends to be invariant at any scale and assumes values ranging between 3 and 5 in nearly all cases [*Horton*, 1945]. According to *Gupta and Waymire* [1983], its expected value is close to 4.

However, the estimation of the gamma parameters could be left to direct observation, and it is pointed out that even other assumptions for the probability distribution of  $a$  could be made, especially if local information is available to support them.

## 2.2. Peak Runoff per Unit Area and Flood Distribution

Let us make the following basic assumptions: (1) The peak discharge per unit contributing area  $u_a$  can be linearly related to the total amount of excess rainfall occurring on the contributing part of the basin in a duration equal to a characteristic response time  $\tau_a$  of the area itself; within this duration the areal and temporal variability of the rainfall intensity can be neglected thanks to the basin storage effect. (2) The duration  $\tau_a$  depends on the size of the contributing area only and is, on average, related to it by a power law. (3) The probability distribution of the areal rainfall depth which may occur in a fixed duration presents all the moments of order higher than one independent of the duration, whereas its mean scales with the duration according to a power law.

The first assumption is quite often invoked as a first-order approximation and leads to a rational type formula. Accordingly, the duration  $\tau_a$  is assumed as the so-called critical duration (different from the flow equilibrium duration), that is, the rainfall duration which leads, for a given return period, to the maximum peak discharge. To support the assumption, we refer to *Fiorentino et al.* [1987], who showed that under different choices of the basin hydrologic response function, the critical duration approaches the lag time. This is particularly true when the shape parameter  $c'$  of the intensity-duration function (idf) is in the range  $(-0.5, -0.7)$ . Here the idf is intended such that the rainfall intensity is proportional to the rainfall duration to the power  $c'$ . Moreover, with particular regard to the hydrologic response functions more close to the reality, the ratio  $\xi$  of the peak direct runoff to the time-averaged excess rainfall intensity in the critical duration appeared very stable. In fact,  $\xi$ , which is a routing factor, was found to vary conditionally on  $c'$  in a narrow range (0.6, 0.8) with an average value close to 0.7.

The assumption that the time variability of the rainfall intensity can be neglected within the critical duration is supported by the above quoted observation that this duration is often close to the lag time. In fact, according to that and owing to the smoothing effect provided by the routing of excess rainfall through the basin, intensity fluctuations within the rainfall duration should not be significant. Moreover, since the lag time tends to scale, as is well known, with the basin area by a power law, the second assumption is also supported.

The third assumption is usually very accurate for rainfall duration ranging from more or less 1 hour to a little more than

1 day. Thus it can be confidently applied when analyzing catchments with area ranging from a few tenths to some thousands square kilometers. It is still reasonable out of (but not too far from) this range. In the range considered, in fact, the pdf moments different from the first are controlled by climatic features, for example, the average humidity conditions and the probability of occurrence of extreme rainfall. These quantities can be considered event features and then considered independent of the within-the-event rainfall characteristics. Moreover, in this range the rainfall idf is often expressed as a simple power law.

Let us make an additional assumption: (4) The excess rainfall can be derived by finding the difference between the rainfall depth and the total loss amount.

This is mainly based on the first assumption, according to which, within the contributing area and during the production of runoff, the storage and infiltration losses can be clustered into a loss factor  $f_a$ , dependent on the contributing area  $a$ . We express this quantity as the ratio of the total water losses to the rainfall duration. Thus the peak hydrograph can be predicted without discriminating between the initial adsorption and infiltration. The coefficient  $f_a$  is strongly dependent, at the event scale, on the antecedent moisture conditions of the watershed, while it is expected to be much more stable with respect to the basin permeability and to its degree of vegetation. However, we believe that in the frequency domain,  $f_a$  can confidently be assumed as mainly controlled by the expected basin status at the timing of a heavy storm.

All that being stated, we may write:

$$u_a = \xi(i_{a,\tau} - f_a), \tag{12}$$

where  $i_{a,\tau}$  is the space-time averaged intensity of the total rainfall which occurs in a duration  $\tau_a$  over the area  $a$  contributing to the peak discharge  $Q$ . Hence, following the aforementioned assumptions, it is possible to state that the probability distributions of  $u_a$  and  $i_{a,\tau}$  are mutually related. In addition, the pdf of  $u_a$  given  $a$  will exploit the above-quoted relationships existing between the expectation of  $i_{a,\tau}$ ,  $\tau_a$ , and  $a$ .

As a distribution of  $i_{a,\tau}$ , we assume, consistent with the exploratory nature of the paper, a Weibull pdf which is well suited to account for the frequently observed skewness of rainfall intensity. It can be written as

$$g(i_{a,\tau}) = \frac{k}{\theta_{a,\tau}} i_{a,\tau}^{k-1} \exp\left(-\frac{i_{a,\tau}^k}{\theta_{a,\tau}}\right), \tag{13}$$

with

$$\theta_{a,\tau} = E[i_{a,\tau}^k] = \{E[i_{a,\tau}]/\Gamma(1 + 1/k)\}^k \tag{14}$$

Accordingly, the cumulative distribution function (cdf) of  $i_{a,\tau}$  is written as

$$G(i_{a,\tau}) = 1 - \exp\left(-\frac{i_{a,\tau}^k}{\theta_{a,\tau}}\right). \tag{15}$$

Then, the cdf of  $i_{a,\tau}$ , given  $i_{a,\tau}$  greater than  $f_a$ , is

$$G(i_{a,\tau}|i_{a,\tau} > f_a) = 1 - \exp\left(-\frac{i_{a,\tau}^k - f_a^k}{\theta_{a,\tau}}\right). \tag{16}$$

Following (16), because of the monotonic relation between  $u_a$  and  $i_{a,\tau}$  (12), the cdf of  $u_a$  is

$$G(u_a|i_{a,\tau} > f_a) = 1 - \exp\left[-\frac{\left(\frac{u_a}{\xi} + f_a\right)^k - f_a^k}{\theta_{a,\tau}}\right]. \tag{17}$$

In addition, let  $E[i_{a,\tau}]$  be related to  $a$  by a simple power law:

$$E[i_{a,\tau}] = i_1 a^{-\varepsilon}, \tag{18}$$

where  $i_1$  is the mean areal rainfall intensity referred to the unit area and  $\varepsilon$  is a scaling factor that is somehow dependent on both the exponents of the idf and of the power law relating  $\tau_a$  to  $a$ .

The local infiltration process may be modeled by means of the Philip's equation:

$$f_i = \frac{1}{2} S_i t^{-1/2} + c, \tag{19}$$

where  $S_i$  is the sorptivity and  $c$  is the gravitational rate of infiltration. Integrating (19) with respect to time, we get the cumulated infiltration volume at time  $t$ :

$$F_i = S_i t^{1/2} + ct, \tag{20}$$

which, divided by  $t$ , gives the mean infiltration rate:

$$\bar{f}_i = S_i t^{-1/2} + c. \tag{21}$$

Inserting the relation between the lag time and the contributing area,

$$\tau_a = \tau_1 a^\nu, \tag{22}$$

and neglecting the gravitational term and passing from the local scale to that the contributing area, we get

$$f_a = f_1 a^{-\varepsilon'}, \tag{23}$$

where  $f_1$  is a constant value and  $\varepsilon' = \nu/2$ ;  $\varepsilon'$  tends to 0.25 when considering for the exponent  $\nu$  of (22) the classic value 0.5. However, because of the uncertainty about both the gravitational term and the exponent of the Philip's equation when used at the basin scale, one should expect that the best estimate of  $\varepsilon'$  could be significantly different from  $\nu/2$ .

The exceedance probability function of  $u_a$ , given  $a$ , is obtained by (17), (14), (18), and (23) as

$$G'(u_a|i_{a,\tau} > f_a) = \exp\left\{-\frac{(u_a/\xi + f_1 a^{-\varepsilon'})^k - (f_1 a^{-\varepsilon'})^k}{[i_1 a^{-\varepsilon}/\Gamma(1 + 1/k)]^k}\right\}. \tag{24}$$

By means of (8) and (24), (7) becomes

$$G'_Q(q) = \int_0^A g(a) \exp\left\{-\frac{[q/(\xi a) + f_1 a^{-\varepsilon'}]^k - (f_1 a^{-\varepsilon'})^k}{[i_1 a^{-\varepsilon}/\Gamma(1 + 1/k)]^k}\right\} da, \tag{25}$$

where  $g(a)$  is given by (8).

Since  $Q$  is the peak direct streamflow, the flood peak is found as

$$Q_p = Q + q_0, \tag{26}$$

with  $q_0$  as a base flow.

Under the hypothesis of Poissonian occurrence of independent flood events the cdf of the annual maximum values of  $Q_p$  is

$$F_{Q_p}(q_p) = 1 - \frac{1}{T} = \exp[-\Lambda_q G'_Q(q_p)], \tag{27}$$

where  $T$  is the recurrence interval in years and  $\Lambda_q$  is the average annual number of independent peak streamflow events.

Finally, after (8), (25), and (26), (27) becomes

$$F_{Q_p}(q_p) = \exp \left\{ -\Lambda_q \left[ \int_0^A g(a) \cdot \exp \left[ -\frac{((q_p - q_0)/(\xi a) + f_1 a^{-\varepsilon'})^k - (f_1 a^{-\varepsilon'})^k}{(i_1 a^{-\varepsilon}/\Gamma(1 + 1/k))^k} \right] da \right] \right\}. \quad (28)$$

**2.3. Governing Parameters**

Summarizing, the quantities that have been recognized as responsible for controlling the probability distribution of annual floods can be grouped as (1) those affecting the probability distribution of contributing areas, that is,  $\alpha$  and  $\beta$  as in (8); (2) the ones regarding the probability distribution of areal excess rainfall intensities given a certain contributing area, namely,  $i_1, f_1, \varepsilon, \varepsilon', \xi,$  and  $k$  as in (24); (3) the base discharge  $q_0$  above which the flood sequence can be schematized as a compound Poisson process; and (4) the annual rate  $\Lambda_q$  of this process.

Although the meaning of these parameters has been clarified in sections 2.1 and 2.2, it is useful to discuss the way they can be reconnected, when needed, to other measurable and physically interpretable quantities. Furthermore, something may be said about their typical range and their stability.

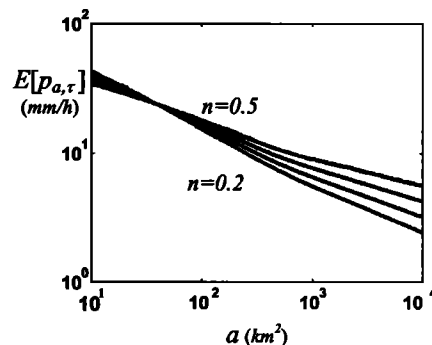
With regard to the first group of parameters most have been discussed in section 2.1 with respect to  $\beta$ . Regarding  $\alpha$ , that is, the ratio of the mean contributing area to  $\beta$ , one can argue that its ratio to the basin area  $A$  should be strongly controlled by the runoff-generating mechanism and by the climate. In fact, this ratio is expected to be lower in humid basins where a saturation excess mechanism is likely to prevail. This will be hereafter confirmed in section 3. In arid zones, where the vegetation is usually scarce or absent, the mean contributing area may also strongly depend on the watershed geology.

The parameters ascribed to the second group can further be discriminated into three subcategories. In particular,  $i_1, \varepsilon,$  and  $k$  depend on the space-time-frequency rainfall pattern,  $\xi$  is a routing coefficient which, as referred to in section 2.2, shows high stability, and  $f_1$  and  $\varepsilon'$  are related to the infiltration and adsorption mechanisms. With regard to the first subcategory the necessary information may follow from knowledge of the space-time rainfall process which, however, can still be considered as an area open for research development. However, a way to draw simplified information about the main features of this process is provided by empirical formulae relating the areal to the point precipitation. Moreover, when the probability distribution of the annual maximum rainfall is known, one can exploit the relationships existing between the annual maxima and the base process. Incidentally, the use of information provided by the annual maxima rainfall process could be seen as more appropriate than investigating the base rainfall mechanism when inferring extreme flood distribution, as in this case.

Let  $p_t$  be the annual maximum of the at-site rainfall intensity averaged on the duration equal to  $t$ ;  $p_t$  varies with  $t$  according to the idf, then its expected value can be written as

$$E[p_t] = p_t t^{n-1}. \quad (29)$$

In (29),  $n - 1$  is equal to the coefficient  $c'$  defined in section 2.2.



**Figure 1.** Expectation of the annual maximum value of the areal rainfall intensity in the critical duration  $\tau_a$  versus the area  $a$ .

The mean rainfall intensity scales with the catchment area and duration in a quite complex manner [e.g., *Rodriguez-Iturbe and Mejia, 1974; Ranzi and Bacchi, 1994*]. However, according to the aim of this paper, it may be sufficient to refer to the simple and classic method of the areal reduction factor proposed by the U.S. Weather Bureau, invoked here as by *Eagleson [1972]*. Thus letting  $p_{a,\tau}$  be the annual maximum of the rainfall intensity averaged in time over  $\tau_a$  and in space over  $a$ ,

$$\frac{E[p_{a,\tau}]}{E[p_t]} = 1 - \exp(-1.1\tau_a^{0.25}) + \exp(-1.1\tau_a^{0.25} - 0.004a), \quad (30)$$

with  $\tau_a$  expressed in hours and  $a$  in square kilometers. As it will be shown below, this method allows for preservation of the simple scaling relationship of (29) with regard to  $E[p_{a,\tau}]$  too. In fact, as a simple exercise, we substituted (22) and (29) into (30) using, in equation (29),  $p_1 = 25$  (mm hr<sup>-1</sup>) and  $n$  ranging from 0.2 to 0.5, values typically observed in Basilicata (southern Italy). Also, in (22), let  $\tau_1 = 0.16$  (h km<sup>-1</sup>) and  $\nu = 0.5$ , values which account for the lag time spatial variability in most basins of the same region.

Consequently, the curves shown in Figure 1, where  $E[p_{a,\tau}]$  is plotted versus the basin area  $a$ , are achieved. Although these curves clearly present a scaling break at the value  $a = 500$  km<sup>2</sup>, it should be pointed out that before and after that point the slope does not change dramatically. In addition, the exponent of the relative power laws stays in a very narrow range between  $-0.1$  and  $-0.3$ .

This result supports the basic assumption leading to (18) because  $E[p_{a,\tau}]$  is linearly related to  $E[i_{a,\tau}]$  in a number of compound Poisson processes. In our case we assumed the independent rainfall to be distributed as Weibull; hence the relative compound Poisson process accounting for the distribution of the annual maxima is the so-called power extreme value distribution [e.g., *Villani, 1993*]. In such a case the relationship between the mean value  $E[i_{a,\tau}]$  of the exceedances in the base process and the mean value  $E[p_{a,\tau}]$  of annual maxima is

$$E[p_{a,\tau}] = \Lambda E[i_{a,\tau}] S_\Lambda, \quad (31)$$

with

$$S_\Lambda = \sum_{j=0}^{\infty} \frac{(-1)^j \Lambda^j}{j!(j+1)^{(1/k+1)},} \quad (32)$$

**Table 1a.** Observed Basins and Their Main Features With Particular Regard to the Length, the Moments, and Parameters of the Annual Series

Site	$A$ , km <sup>2</sup>	$N$	$E[Q_p]$ , m <sup>3</sup> s <sup>-1</sup>	$C_v$	$Ca$	$\Lambda_q$	$\Lambda_p$
Bradano—San Giuliano	1657	17	507	0.79	1.03	2.9	21
Bradano—Ponte Colonna	462	32	202	0.76	1.21	4.0	21
Basento—Menzena	1382	24	401	0.63	1.57	6.6	21
Basento—Gallipoli	853	38	353	0.63	2.25	8.5	21
Agri—Tarangelo	511	32	189	0.49	0.75	16.8	21
Sinni—Valsinni	1140	22	555	0.56	2.42	19.1	21
Basento—Pignola	42	28	35	0.43	1.1	19.6	21
Sinni—Pizzutello	232	19	255	0.51	0.75	31.0	32

Definitions are as follows:  $A$ , basin area;  $N$ , length;  $E[Q_p]$ , mean annual flood;  $C_v$ , coefficient of variation;  $Ca$ , skewness of annual flood series;  $\Lambda_q$ , mean annual number of independent floods; and  $\Lambda_p$ , mean annual number of independent storms.

where  $\Lambda$  is the annual mean number of rainfall occurrences that exceed a given intensity threshold. It can be shown that  $E[i_{a,\tau}^k]$  is independent of that threshold, while  $\Lambda$  depends on it. In particular, assuming the threshold equal to zero,  $\Lambda$  coincides with the mean number of independent annual rainfall events  $\Lambda_p$ , and the expectation of  $i_{a,\tau}$  may be calculated as in (31) replacing  $\Lambda$  with  $\Lambda_p$ .

As frequently observed, the value of  $\Lambda_p$  may be considered, at least at an hourly timescale, as independent of  $\tau$  [e.g., *National Environmental Research Council*, 1975]. Therefore, following (31),  $E[i_{a,\tau}]$  may be supposed to scale with  $a$  as in (18), where, according to Figure 1,  $\varepsilon$  should range between  $-0.1$  and  $-0.3$ .

Consistent with (12), the mean annual number of independent floods  $\Lambda_q$  must equal the mean annual number of exceedances over a threshold equal to  $f_a$ . Then, according to the distribution adopted for  $i_{a,\tau}$ , the independence of  $E[i_{a,\tau}^k]$  from  $f_a^k$  allows writing the following relation between  $\Lambda_p$  and  $\Lambda_q$  as

$$\Lambda_q = \Lambda_p \exp\left(-\frac{f_a^k}{E[i_{a,\tau}^k]}\right), \quad (33)$$

which also gives

$$\log(\Lambda_p/\Lambda_q) = f_a^k/\theta_{a,\tau} \quad (34)$$

The ratio at the right-hand side of (33) depends on the contributing area  $a$  that, in turn, is related to the critical rainfall duration. Therefore, since  $\Lambda_q$  is independent of the rainfall duration, (33) supports the assumption  $\varepsilon = \varepsilon'$  when  $\Lambda_p$  is constant with respect to duration too. A convenient dimensionless parameter related to  $f_a$  is

$$f^* = f_a/E[i_{a,\tau}]. \quad (35)$$

This parameter represents a global loss coefficient that is independent of the contributing area  $a$  and of the rainfall duration as demonstrated by the following relationship derived by use of (33) and (14)

$$f^* = \frac{[-\log(\Lambda_q/\Lambda_p)]^{1/k}}{\Gamma(1+1/k)} \quad (36)$$

#### 2.4. Index Flood

On the basis of the above equations, we can now introduce an index flood  $Q_I$ , defined as

$$Q_I = E[u_{E[a]}]E[a]\Lambda_q S_{\Lambda_q} + q_0. \quad (37)$$

$Q_I$  is as closer to the mean annual flood  $Q_m$  as  $u$  and  $a$  are less dependent on each other and as the probability distribution of  $u$  given  $a$  is better described by a Weibull law.  $Q_I$  represents a consistent approximation for  $Q_m$ , whose exact expression is not easy to derive because of the complexity of (28). Anyway, the fact that  $Q_I$  provides a good approximation for  $Q_m$  is supported by a number of numerical experiments presented in section 3.2.

In (37),  $E[u_{E[a]}]$  can be obtained once  $E[u_a]$  is calculated accordingly to (17):

$$\begin{aligned} E[u_a] &= E[\xi(i_{a,\tau} - f_a)|i_{a,\tau} > f_a] \\ &= \int_{f_a}^{\infty} \frac{k}{\theta_{a,\tau}} (i_{a,\tau} - f_a) i_{a,\tau}^{k-1} \exp\left(-\frac{i_{a,\tau}^k - f_a^k}{\theta_{a,\tau}}\right) di_{a,\tau} \end{aligned} \quad (38)$$

Then, integrating by parts,

$$E[u_a] = \xi \left[ i_{a,\tau} \exp\left(\frac{f_a^k}{\theta_{a,\tau}}\right) \frac{\gamma(1+1/k, f_a^k/\theta_{a,\tau})}{\Gamma(1+1/k)} - f_a \right]. \quad (39)$$

Thus (37) becomes

$$Q_I = \xi \left\{ i_1 \frac{\gamma[1+1/k, \log(\Lambda_p/\Lambda_q)]}{\Gamma(1+1/k)} - f_1 \right\} \Lambda_q S_{\Lambda_q} r^{1-\varepsilon} A^{1-\varepsilon} + q_0 \quad (40)$$

where  $r$  is the ratio of the mean contributing area to the total basin area  $A$ , that is,

$$r = E[a]/A. \quad (41)$$

This parameter is related to basin features such as climate and vegetation as shown in section 3.

### 3. Validation

An analysis aimed to test whether the proposed distribution is able to reproduce real-world data with parameter values consistent with the proper physical interpretation is reported here. The proposed distribution was applied to the annual flood series of eight gauged basins in Basilicata (southern Italy), with at least 15 years of reliable observations, where floods were practically rainfall-generated only.

#### 3.1. Estimation Procedure

All the distribution parameters needed for the analysis were drawn from available studies, except the contributing area ratio  $r$  as in (41), which we used for calibration purposes. This parameter is related to  $\alpha$  as in (8) through (41), given the well-known property of the gamma distribution:

$$\alpha = E[a]/\beta. \quad (42)$$

All the available studies were based on annual maximum data, with regard to both rainfall and floods. The main features of the basins are listed in Table 1a, where  $N$ ,  $E[Q_p]$ ,  $C_v$ , and  $Ca$  are length, sample average, coefficient of variation, and skewness of the annual flood series, respectively. In Table 1a the basin area  $A$  and the mean annual number of independent floods  $\Lambda_q$ , as well as of independent storms  $\Lambda_p$ , are also listed. In particular,  $\Lambda_q$  and  $\Lambda_p$  were taken from *Iacobellis et al.* [1998], who used the generalized extreme value distribution [Jenkinson, 1955] for fitting floods and the two-component

extreme value distribution [Rossi *et al.*, 1984] for analyzing rainfall annual maxima.

The values of parameters  $p_1$  and  $n$ , shown in Table 1b, were derived by Claps and Straziuso [1996], who applied a kriging procedure in order to achieve basin representative values starting from the available at-site information.

The values of the basin lag time  $\tau_A$  (see Table 1b) were drawn, with regard to basins 1, 2, 4, 5, and 7, from Rossi [1974], who recognized a local stability of  $\tau_A$  for return periods greater than 10 years. For basins 3, 6, and 8 we used the empirical relationship  $\tau_A = 0.16\sqrt{A}$  ( $\tau_A$  in hours and  $A$  in square kilometers), fitted to basins of Bradano, Basento, and Sinni [Dipartimento di Ingegneria e Fisica dell'Ambiente, 1998].

With regard to  $k$ , we used annual maximum data of 78 gauging stations located within the entire region. In particular, we decided not to deeply detail the regional analysis, and we put  $k = 0.8$ , equal to the overall average of the  $k$  at-site estimates obtained from the annual daily rainfall series.

The base flow  $q_0$  (see Table 1b) was given a tentative value equal to the average monthly flow observed in January and February, which are the months with the highest probability of flood occurrence. Finally, for  $\beta$  as in (8),  $\xi$  as in (12),  $\varepsilon$  and  $\varepsilon'$  as in (18) and (23) respectively, we decided, following the above discussions, to assume all over the region and as a sufficiently good first-order approximation,  $\beta = 4$ ,  $\varepsilon = \varepsilon' = 0.25$ , and  $\xi = 0.7$ . Here  $f^*$  was estimated by use of (36).

For estimation purposes, in order to overcome the uncertainty in dealing with the parameters  $i_1$  and  $f_1$ , we substituted the following for (18) and (23):

$$E[i_{a,\tau}] = i_A(a/A)^{-s} \quad (43)$$

$$f_a = f_A(a/A)^{-\varepsilon'} \quad (44)$$

Thus  $i_A$  was calculated by use of (29), (30), (31), and (32) by putting  $t = \tau_A$  in (29),  $a = A$  in (30), and  $\Lambda = \Lambda_p$  in (31) and (32). The loss coefficient  $f_A$  was derived by (33) taking the quantities at the right-hand side as referring to the whole watershed.

### 3.2. Results

The estimated values of  $r$  are shown in Table 4 along with those of  $f_A$  and  $f^*$ ; in subsection 3.3 we will focus on the climate influence on these parameters. In Table 2 the theoretical values of  $Q_I$  as from (37), of the mean ( $E[Q_p]_c$ ) and of the coefficient of variation ( $Cv_c$ ), which were calculated by nu-

**Table 2.** Calculated Values of Mean Annual Flood, Index Flood, and Coefficient of Variation

Site	$A$ , km <sup>2</sup>	$E[Q_p]_c$ , m <sup>3</sup> s <sup>-1</sup>	$Q_I$ , m <sup>3</sup> s <sup>-1</sup>	$Cv_c$
Bradano—San Giuliano	1657	523	564	0.85
Bradano—Ponte Colonna	462	201	213	0.77
Basento—Menzena	1382	434	465	0.61
Basento—Gallipoli	853	355	370	0.58
Agri—Tarangelo	511	199	198	0.50
Sinni—Valsinni	1140	556	541	0.48
Basento—Pignola	42	37	36	0.49
Sinni—Pizzutello	232	239	226	0.44

Abbreviations are as follows:  $Q_I$ , index flood;  $E[Q_p]_c$ , calculated mean annual flood; and  $Cv_c$ , calculated coefficient of variation.

merical integration based upon the proposed distribution, are displayed.

The good agreement between  $E[Q_p]_c$  and  $Cv_c$  and  $E[Q_p]$  and  $Cv$ , respectively (see Table 1a) stems trivially because of the performed calibration. However, it is worth mentioning that because of the accordance between  $E[Q_p]_c$  and  $Q_I$ , the choice of the proposed index flood is validated.

Figures 2 and 3 show the found cdfs of the peak flood and the contributing area pdfs, respectively. In Figure 2 we used the plotting position suggested by Cunnane [1978]:

$$P_i = (i - 0.4)/(N + 0.2).$$

Figures 2 and 3 clearly display that the theoretically derived distribution is able to describe the main statistical features of the observed annual flood series. This is not a trivial result if one notes that, despite the numerous parameters used in the distribution, only  $\Lambda_q$  and  $r$  were estimated by using the flood data.

### 3.3. Climatic Control

As pointed out by T. Dunne (quoted by Leopold [1974]), in humid and semihumid basins, where it is likely to find highly vegetated zones, one can observe that, depending on conditions, only the part of a basin that is near the channels contributes to surface runoff, and the rest of the basin area, farther away, makes no contribution. This contributing area expands and contracts depending on the surface and subsurface conditions such as vegetation and soil moisture at the time prior to the flood event. Accordingly, in such basins the area that contributes to overland runoff depends on the antecedent soil conditions and on the storm rainfall depth. The spatial extension of the storm plays a role too. In semiarid zones, according to the Horton runoff generation model, in the presence of intense rainfall all parts of a drainage area contribute to overland flow. In such a case the antecedent soil conditions are crucial to allowing activation of overland flow, but, once surface runoff has begun, the contributing area is rather controlled by the storm extension.

In order to distinguish objectively between areas of different climatic characteristics, we used the simple index:

$$I = \frac{h - E_p}{E_p}, \quad (45)$$

where  $h$  is the mean annual rainfall depth and  $E_p$  is the mean annual potential evapotranspiration.  $E_p$  was calculated according to Turc [1961]. This index allows the climate classification

**Table 1b.** Observed Basins and Their Main Features With Particular Regard to the Basin Lag, the Intensity-Duration Function Parameters, and the Base Discharge

Site	$A$ , km <sup>2</sup>	$\tau_A$ , hours	$p_1$ , mm hr <sup>-1</sup>	$n$	$q_0$ , m <sup>3</sup> s <sup>-1</sup>
Bradano—San Giuliano	1657	7.1	23.52	0.289	10
Bradano—Ponte Colonna	462	4.3	22.20	0.283	5
Basento—Menzena	1382	6.0	21.48	0.315	25
Basento—Gallipoli	853	4.8	20.41	0.315	25
Agri—Tarangelo	511	8.9	21.56	0.362	10
Sinni—Valsinni	1140	5.6	23.13	0.405	45
Basento—Pignola	42	2.9	21.00	0.311	1.5
Sinni—Pizzutello	232	2.4	21.56	0.362	15

Definitions are as follows:  $\tau_A$ , basin lag time;  $p_1$  and  $n$ , intensity-duration function parameters; and  $q_0$ , base flow.

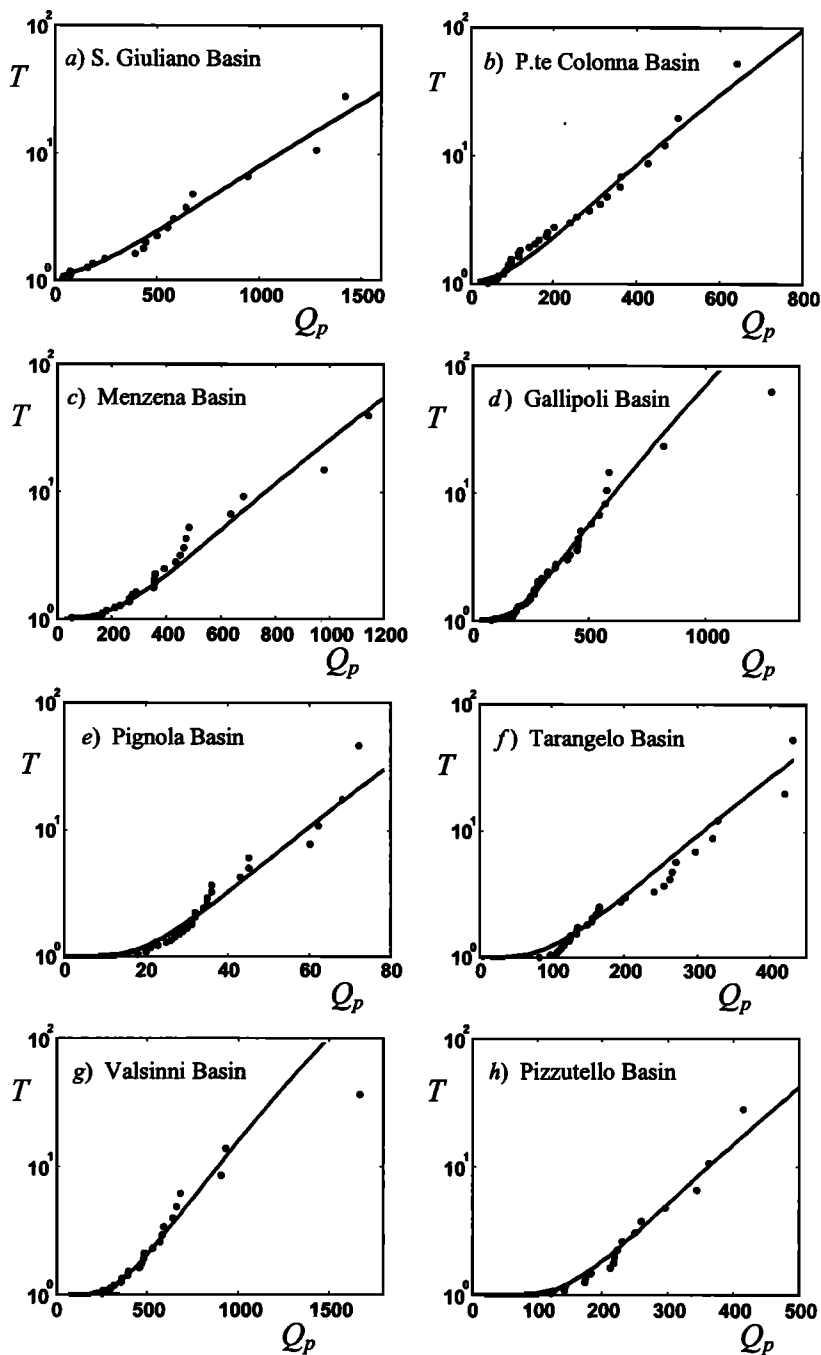


Figure 2. Peak flood derived distribution fitted to data.  $T$  is in years and  $Q_p$  in  $m^3/s$ .

shown in Table 3. The sign of  $I$  discriminates between humid (positive) and arid (negative) basins.

The values of  $I$  for the observed basins are shown in Table 4; they allow for roughly splitting the entire region into two parts: semiarid and humid. In fact, the climatic index assumes negative values on the Bradano River basins only and always remains positive elsewhere. It is higher than 0.3 for those basins characterized by higher mean altitude and a greater degree of vegetation and forested hillslopes. More particularly, one can note that all the parameters in Table 4 show a clear dependence on the climatic index.

First, as already recognized by *Iacobellis et al.* [1998], the  $I$  index seems to crucially influence the reduction of  $\Lambda_q$  with

respect to  $\Lambda_p$ . This is consistent with the meaning of parameters  $f_A$  and  $f^*$  which, according to (36), control the ratio  $\Lambda_q/\Lambda_p$ .

In fact, within the humid areas, where the probability of finding humid antecedent moisture conditions is high, the loss factor  $f_A$  should decrease with  $I$ . In addition, the ratio  $\Lambda_q/\Lambda_p$  should tend to grow with  $I$  and approach unity in hyperhumid climate.

Actually, a certain role may also be played by the mean value of the exceedances in the base rainfall process (as suggested by (35)). However, with regard to this issue, not too much can be said here since this quantity does not vary sensibly throughout the examined basins. The way the ratio  $\Lambda_q/\Lambda_p$  and the parameters  $f_A$  and  $f^*$  depend on  $I$  is shown in Figure 4.



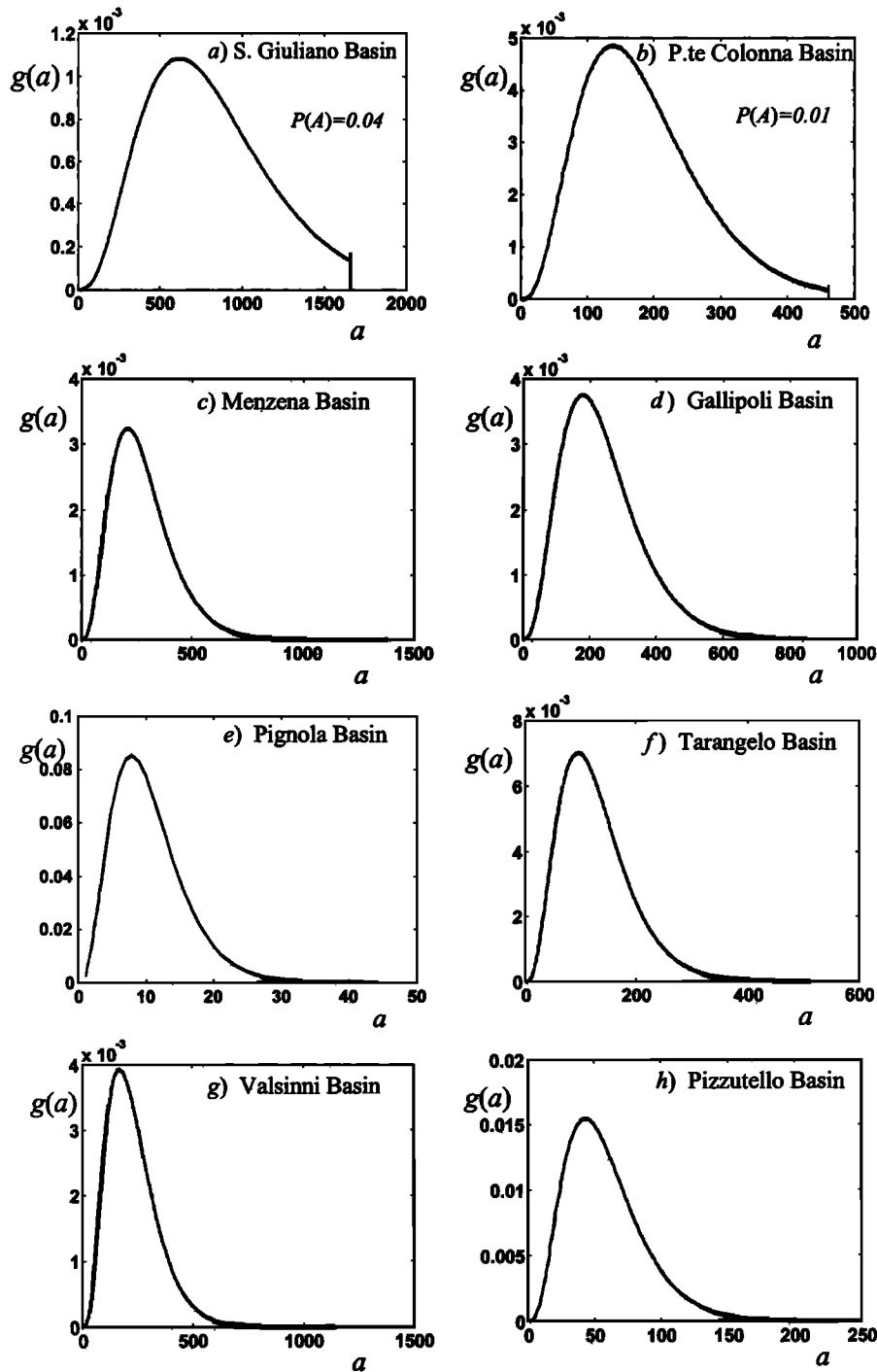


Figure 3. Probability density functions of the contributing area (square kilometers).

It is so emphasized that the climate directly interferes with the hydrological losses, as these are strongly dependent on the characteristic moisture conditions. In other words, the basin aridity produces a greater initial adsorption capacity, which causes the low yield in the number of flood events compared to the number of storms. Instead, in humid basins, which are characterized by the presence of permanently saturated areas near the streams, any precipitation event tends to produce runoff, as also pointed out by Leopold [1997, p. 42].

The climate influence can also be noted with regard to the parameter  $r$  (Table 4). In fact, the expected fraction of the

basin that contributes to flood peak is quite higher (about 0.5) for the arid Bradano River basin. Instead, humid basins have lower  $r$  values in the range 0.2–0.3. This could be related to the way the long-term climate tends to constrain the activation of a certain runoff generation mechanism. Since the storm size may become as large as the entire basin, in arid zones the finite probability that the whole watershed contributes to the flood peak (see (9)) cannot be neglected. Instead, in humid basins the presence of highly vegetated areas favors a saturation excess mechanism which allows direct runoff only in those saturated parts, usually small, near the streams.

**Table 3.** Climate Classification

Climate	<i>I</i>
Hyperhumid	$1.0 \leq I$
Humid	$0.2 \leq I < 1.0$
Subhumid	$0.0 \leq I < 0.2$
Dry subhumid	$-0.2 \leq I < 0.0$
Semiarid	$-0.4 \leq I < -0.2$
Arid	$-0.6 \leq I < -0.4$

*I* is climatic index.

**4. Dimensionless Cumulative Distribution Function and Preliminary Sensitivity Analysis**

Let us introduce the following dimensionless quantities:

$$Q_p^* = \frac{Q_p}{Q_I}, \quad q_0^* = \frac{q_0}{Q_I}, \quad a^* = \frac{a}{rA}, \quad \alpha^* = \frac{\alpha}{rA} = \frac{1}{\beta}. \quad (46)$$

Inserting these quantities into (25), along with the parameter *f*\* defined by (36), we get a dimensionless expression for the exceedance probability function of the peak streamflow *Q*<sub>*p*</sub>:

$$G_{Q_p^*}^*(q_p^*) = \int_0^{1/r} g(a^*) \cdot \exp \left\{ -\Gamma(1 + 1/k)^k \left[ \left( \eta \frac{q_p^* - q_0^*}{a^{*(1-\varepsilon)}} + f^* \right)^k - f^{*k} \right] \right\} da^*, \quad (47)$$

with

$$g(a^*) = \frac{1}{\alpha^* \Gamma(\beta)} \left( \frac{a^*}{\alpha^*} \right)^{\beta-1} \exp \left( -\frac{a^*}{\alpha^*} \right) + \delta \left( a^* - \frac{1}{r} \right) \gamma \left( \frac{1}{r\alpha^*}, \beta \right), \quad (48)$$

where  $\delta(\ )$  and  $\gamma(\ )$  were introduced in (8) and (9), respectively. In (47),  $\eta$  is given by

$$\eta = \frac{\Lambda_q S_{\Lambda_q}}{1 - q_0^*} \left\{ \frac{\Lambda_p}{\Lambda_q} \frac{\gamma[1 + 1/k, \log(\Lambda_p/\Lambda_q)]}{\Gamma(1 + 1/k)} - f^* \right\}. \quad (49)$$

Finally, the dimensionless distribution of annual floods is readily achieved as

**Table 4.** Parameters Controlled by Climate and Climatic Index

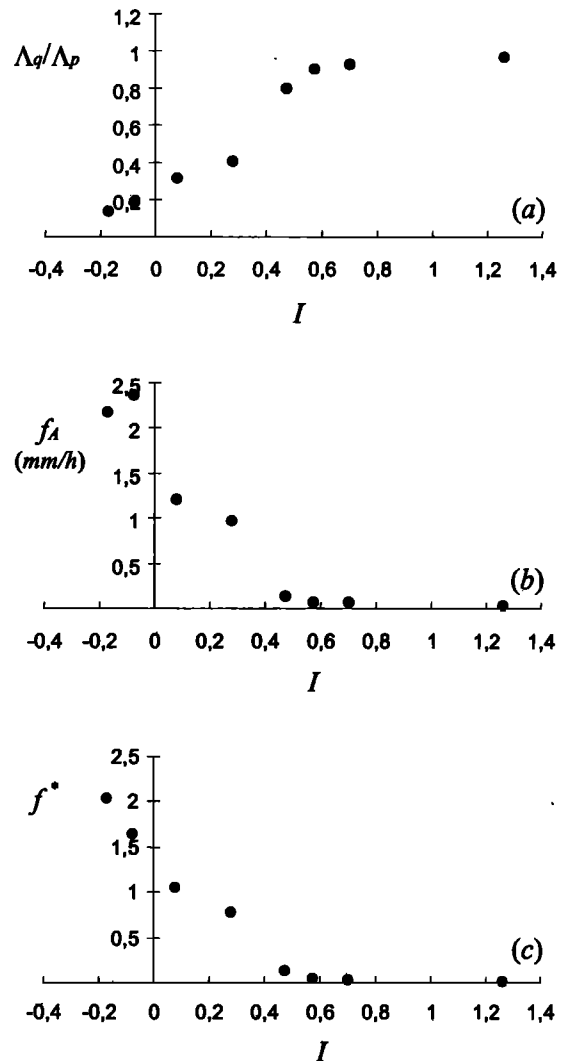
Site	<i>A</i> , km <sup>2</sup>	<i>r</i>	$\Lambda_q/\Lambda_p$	<i>f</i> <sub><i>a</i></sub> , mm/hr <sup>-1</sup>	<i>f</i> *	<i>I</i>
Bradano—San Giuliano	1657	0.50	0.14	2.170	2.040	-0.17
Bradano—Ponte Colonna	462	0.50	0.19	2.360	1.645	-0.08
Basento—Menzena	1382	0.20	0.32	1.210	1.050	0.08
Basento—Gallipoli	853	0.28	0.41	0.975	0.776	0.28
Agri—Tarangelo	511	0.27	0.80	0.141	0.136	0.47
Sinni—Valsinni	1140	0.20	0.91	0.070	0.046	0.57
Basento—Pignola	42	0.30	0.93	0.069	0.032	0.70
Sinni—Pizzutello	232	0.26	0.97	0.027	0.012	1.26

Parameters are defined in sections 2.2 (*f*<sub>*a*</sub>, *a* = *A*), 2.3 ( $\Lambda_q$ ,  $\Lambda_p$ , and *f*\*), and 2.4 (*r*).

$$F_{Q_p^*}(q_p^*) = 1 - \frac{1}{T} = \exp [-\Lambda_q G_{Q_p^*}^*(q_p^*)]. \quad (50)$$

This theoretical dimensionless distribution makes it possible to look at some interesting scaling properties of its moments and to investigate significant climatic effects on the flood distribution.

The sensitivity analysis of the derived distribution has been carried out with regard to two climatic conditions. The first is representative of humid regions and is characterized by a low *f*\* value and consequently by a high  $\Lambda_q/\Lambda_p$  ratio. The second one, which is more typical of arid zones, presents higher values of *f*\* (then, lower  $\Lambda_q/\Lambda_p$ ). The following working parameter values were used: in the first case, *f*\* = 0.13,  $\Lambda_q$  = 16, and  $\Lambda_p$  = 20 ( $\Lambda_q/\Lambda_p$  = 0.8) and, in the second, *f*\* = 1.60,  $\Lambda_q$  = 4, and  $\Lambda_p$  = 20 ( $\Lambda_q/\Lambda_p$  = 0.2). The analysis was carried out with regard to both configurations, starting from the following arbitrary set of the remaining parameters: *k* = 0.8,  $\beta$  = 4,  $\varepsilon$  = 0.3, *r* = 0.5, and *q*<sub>0</sub>\* = 0. These values were changed one at a time within the following ranges: *k* = 0.6–1,  $\beta$  = 1–5,  $\varepsilon$  = 0.1–0.6, *r* = 0.1–0.7, and *q*<sub>0</sub>\* = 0–0.2.



**Figure 4.** (a) The ratio  $\Lambda_q/\Lambda_p$ , (b) the loss factor *f*<sub>*a*</sub>, and (c) the global loss coefficient *f*\* versus the climatic index, within the observed region. See Table 3 for climate classification.

**Table 5.** Parameters Used in the Analysis of Sensitivity to  $k$  in a Humid Climate and an Arid Climate

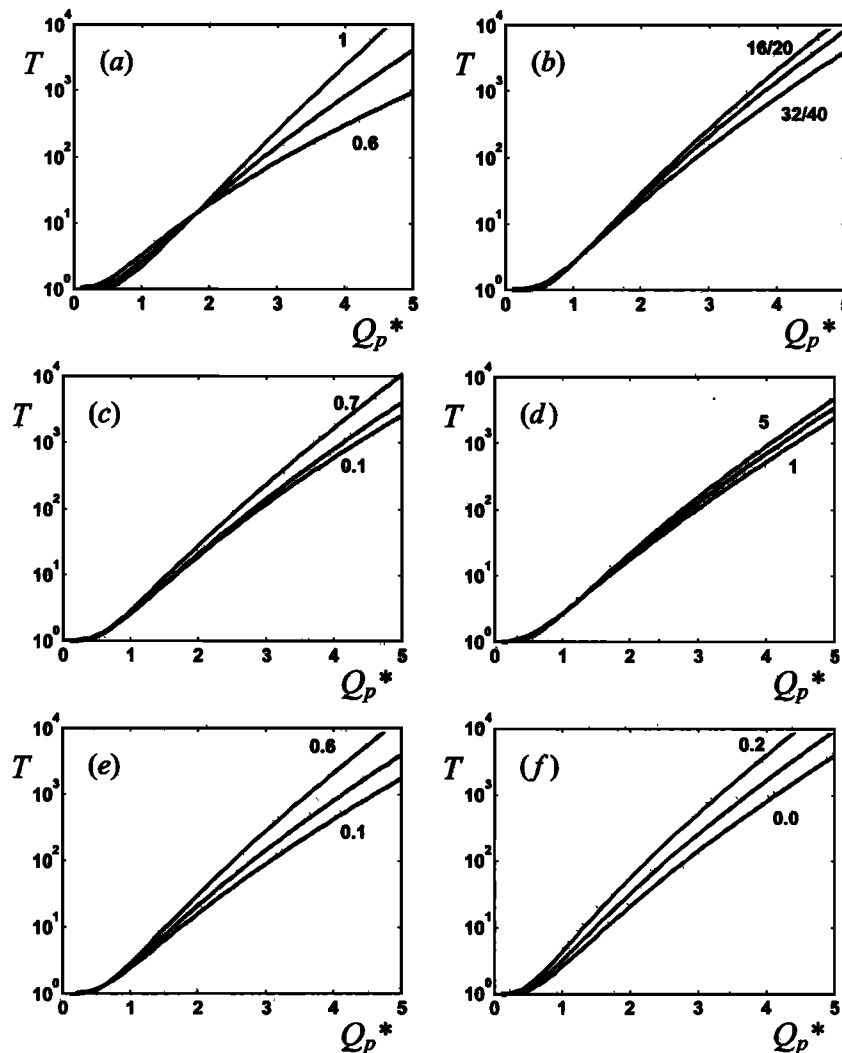
$k$	$\Lambda_q$	$\Lambda_p$	$\Lambda_q/\Lambda_p$
<i>Humid Climate (<math>f^* = 0.13</math>)</i>			
0.6	16	23.5	0.68
0.8	16	20.0	0.80
1	16	18.3	0.87
<i>Arid Climate (<math>f^* = 1.60</math>)</i>			
0.6	4	21.8	0.18
0.8	4	20.0	0.20
1	4	19.8	0.20

The parameters  $\Lambda_q$  and  $\Lambda_p$  were also varied, keeping their ratio constant; in particular, in the first case we put  $\Lambda_q/\Lambda_p = 16/20, 24/30,$  and  $32/40$  (all ratios = 0.8), and in the other case,  $\Lambda_q/\Lambda_p = 4/20, 6/30,$  and  $8/40$  (all ratios = 0.2). This was not possible while investigating the case of variable  $k$ , since, according to (36), fixing  $f^*$  and  $k$ , does not allow preservation of the ratio  $\Lambda_q/\Lambda_p$ . In this case we chose such values of  $\Lambda_p$  leading to the selected values of  $f^*$  (see Table 5).

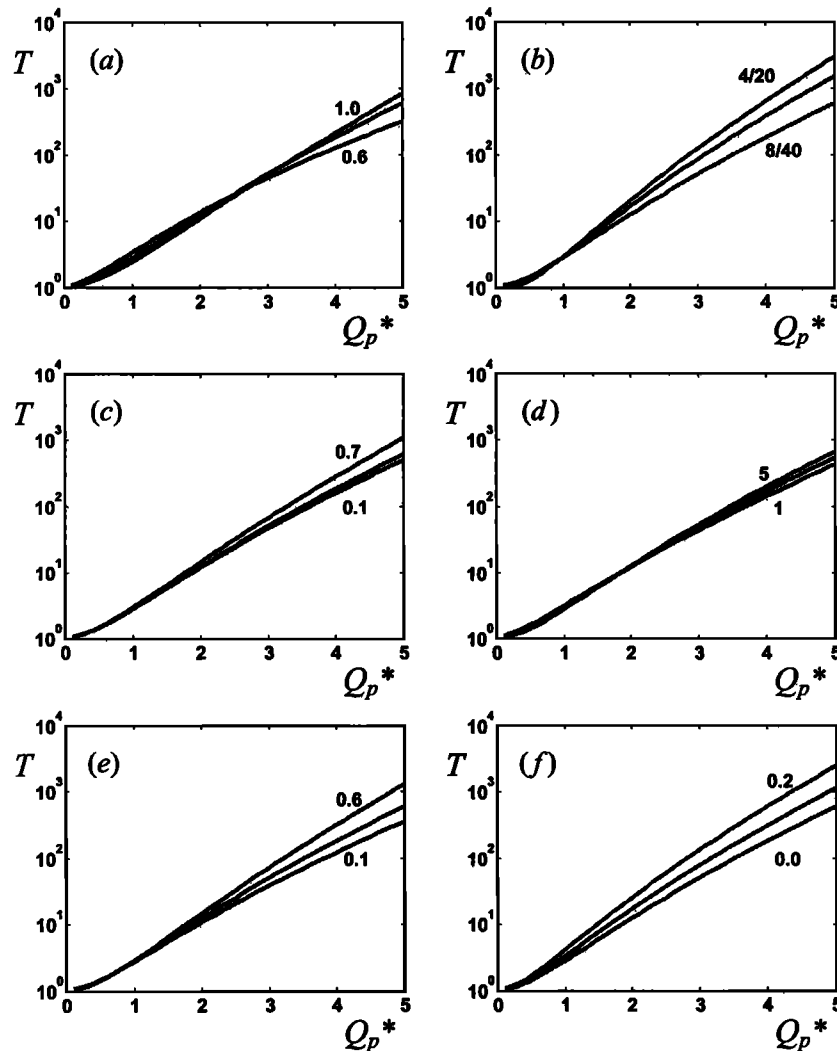
Figures 5 and 6 show the derived dimensionless frequency curves with reference to the humid and arid areas, respectively. It is possible to observe the different sensitivity of the cdfs to the parameters in the two climates. In fact, as also frequently observed, flood distributions relative to arid climates are steeper than others. Furthermore, in comparison it seems that in humid climate the cdfs are more sensitive to the parameter values, with particular regard to the Weibull shape parameter  $k$ . This could be explained by the observation that in arid areas the scarcity of flood events makes the process mainly random thus hiding the influence of physical features.

In Figure 7 the way the mean of the dimensionless distribution varies with  $\Lambda_q$  within the above-defined range of variability of  $k, \beta, \varepsilon, r,$  and  $q_0^*$ , and for  $f^*$  ranging between 0.1 and 2, is displayed. Figure 7 highlights that the calculated values of the mean are close to unity, so validating the use of the index flood  $Q_I$  as an approximation for the mean annual flood.

Thus, on the basis of the definition of the index flood it is also possible to speculate about the scale features of  $Q_I$  with respect to the basin area. In fact, dividing  $Q_I$  by the catchment area  $A$  and neglecting the role of the base flow, (40) suggests



**Figure 5.** Dimensionless cumulative probability function of  $Q_p^* = Q_p/Q_I$  in humid climate: (a)  $k = 0.6-0.8-1$ ; (b)  $\Lambda_q/\Lambda_p = 16/20-24/30-32/40$ ; (c)  $r = 0.1-0.5-0.7$ ; (d)  $\beta = 1-3-5$ ; (e)  $\varepsilon = 0.1-0.3-0.6$ ; and (f)  $q_0^* = 0-0.1-0.2$ .



**Figure 6.** Dimensionless cumulative probability function of  $Q_p^* = Q_p/Q_I$  in arid climate: (a)  $k = 0.6-0.8-1$ ; (b)  $\Lambda_g/\Lambda_p = 4/20-6/30-8/40$ ; (c)  $r = 0.1-0.5-0.7$ ; (d)  $\beta = 1-3-5$ ; (e)  $\varepsilon = 0.1-0.3-0.6$ ; and (f)  $q_0^* = 0-0.1-0.2$ .

that this ratio varies with  $A$  according to a power law with the exponent equal to  $-\varepsilon$ . Since the probable values of  $\varepsilon$  range, as seen before, between 0.1 and 0.3, such a result is in fairly good agreement with real-world observations [e.g., *Robinson and Sivapalan, 1997*].

## 5. Conclusions

A derived distribution of flood frequency is proposed with clear reference to the climate influence. It arises from the analysis of the stochastic features of rainfall and of basin hydrologic response. The theoretical scheme is founded on the identification of a probabilistic model of the basin response, which accounts for both the geomorphoclimatic features of the basin and the spatial distribution of rainfall. The distribution seems to be able to reproduce the observed patterns of flood annual maxima by means of a number of physically based parameters in a wide range of climatic conditions.

The influence of stochastic and pseudodeterministic factors influencing the process of generation of floods is highlighted taking into account in a synthetic manner some crucial elements involved in processes like the mechanisms of runoff generation.

This model identifies the combined role played by climatic and physical factors at the basin scale. Conversely, it indicates the influence of climate on the water loss processes and sheds more light on the way to identify the relative importance of climate, lithology, and land use. It also highlights that the probability distribution of floods is controlled by the expected values of some quantities, for example, contributing area and soil moisture conditions, whose high randomness is significant at the event scale only.

Moreover, the influence of the runoff generation mechanism on the probability distribution of the contributing areas is revealed. Thus the need for a deeper knowledge of conditions at which a certain mechanism tends to prevail at the basin scale clearly arises. Further research with regard to the spatial variability of rainfall is also recommended. Therefore field experiments aimed at better understanding the physical processes underlying flood generation processes are also advised, paying particular attention to the role played by the long-term climate and degree of vegetation of the basin.

In this paper the influences of nonlinear processes of flood generation were not taken into account. However, we should

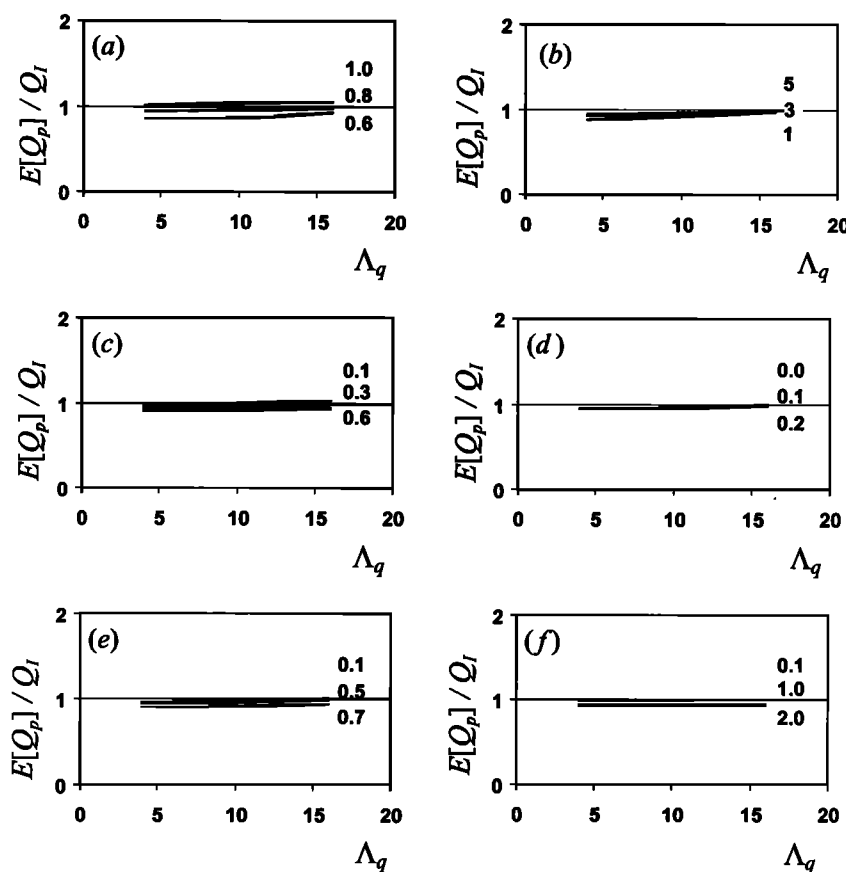


Figure 7. Sensitivity of the expected value of  $Q_p^* = Q_p/Q_I$  to parameters: (a)  $k = 0.6-0.8-1$ ; (b)  $\beta = 1-3-5$ ; (c)  $\varepsilon = 0.1-0.3-0.6$ ; (d)  $q_0^* = 0-0.1-0.2$ ; (e)  $r = 0.1-0.5-0.7$ ; and (f)  $f^* = 0.1-1.0-2.0$ .

remark that these influences may play a crucial role in amplifying the skewness of flood series, mainly in humid basins. In fact, in these watersheds, sensible skew may be induced by possible activation of the Horton type mechanism in very severe storms. In addition, a smoother nonlinear behavior may be related to the catchment response time  $\tau_a$ , which tends to decrease as the rainfall intensity increases. Both these effects should be less important in arid basins. In fact, dry soils allow only high-intensity rainfall to produce floods, and scarce vegetation tends to make the runoff generation mechanism less dependent on the rainfall intensity. The developed model, besides suggesting remarkable hints for further scientific research and unlike the merely statistical flood frequency methods, succeeds in highlighting the differences, rather than the similarities, in the physioclimatic features of the basins.

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## References

- Betson, R. P., What is watershed runoff?, *J. Geophys. Res.*, 69(8), 1541–1552, 1964.
- Cadavid, L., J. T. B. Obeysekara, and H. W. Shen, Flood-frequency derivation from kinematic wave, *J. Hydraul. Eng.*, 117(4), 489–510, 1991.
- Claps, P., and E. Straziuso, Analisi regionale delle piogge brevi in Basilicata (in Italian), internal report, Dip. di Ing. e Fis. dell'Ambiente, Univ. della Basilicata, Italy, Potenza, 1996.
- Cunnane, C., Unbiased plotting positions—A review, *J. Hydrol.*, 37, 205–222, 1978.
- Diaz-Granados, M. A., J. B. Valdes, and R. L. Bras, A physically based flood frequency distribution, *Water Resour. Res.*, 20(7), 995–1002, 1984.
- Dipartimento di Ingegneria e Fisica dell'Ambiente, Valutazione delle Piene in Basilicata (in Italian) report, Univ. della Basilicata, Potenza, Italy, 1998.
- Eagleson, P. S., Dynamics of flood frequency, *Water Resour. Res.*, 8(4), 878–898, 1972.
- Farquharson, F. A. K., J. R. Meigh, and J. V. Sutcliffe, Regional flood frequency analysis in arid and semi-arid areas, *J. Hydrol.*, 138, 487–501, 1992.
- Fiorentino, M., F. Rossi, and P. Villani, Effect of the basin geomorphoclimatic characteristics on the mean annual flood reduction curve, in *Proceedings of the IASTED International Conference, Modeling and Simulation*, pp. 1777–1784, Univ. of Pittsburgh, Pittsburgh, Pa., 1987.
- Gottschalk, L., and R. Weingartner, Distribution of peak flow derived from a distribution of rainfall volume and runoff coefficient, and a unit hydrograph, *J. Hydrol.*, 208, 148–162, 1998.
- Gradshteyn, I. S., and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic, San Diego, Calif., 1980.
- Gupta, V. K., and E. Waymire, On the formulation of an analytical approach to hydrologic response and similarity at the basin scale, *J. Hydrol.*, 65, 95–123, 1983.
- Hebson, C., and E. F. Wood, A derived flood frequency distribution using Horton order ratio, *Water Resour. Res.*, 18(5), 1509–1518, 1982.
- Horton, R. E., Erosional development of streams and their drainage basins: Hydrophysical approach to quantitative geomorphology, *Geol. Soc. Am. Bull.*, 56, 275–370, 1945.

- Iacobellis, V., P. Claps, and M. Fiorentino, Sulla dipendenza dal clima dei parametri della distribuzione di probabilità delle piene (in Italian), in *Proceedings of the XXVI Convegno di Idraulica e Costruzioni Idrauliche*, vol. II, pp. 213–224, Univ. of Catania, Catania, Italy, 1998.
- Jenkinson, A. F., The frequency distribution of the annual maximum (or minimum) of meteorological elements, *Q. J. R. Meteorol. Soc.*, *81*, 158–171, 1955.
- Klemes, V., Hydrological and engineering relevance of flood frequency analysis, *Proceedings of the International Symposium on Flood Frequency and Risk Analysis: Application of Frequency and Risk in Water Resources*, pp. 1–18, D. Reidel, Norwell, Mass., 1987.
- Kurothe, R. S., N. K. Goel, and B. S. Mathur, Derived flood frequency distribution for negatively correlated rainfall intensity and duration, *Water Resour. Res.*, *33*(9), 2103–2107, 1997.
- Leopold, L. B., *Water: A Primer*, pp. 50–54, W. H. Freeman, New York, 1974.
- Leopold, L. B., *Water, Rivers and Creeks*, Univ. Sci., Books, Sausalito, Calif., 1997.
- Moughamian, M. S., D. B. McLaughlin and R. L. Bras, Estimation of flood frequency: An evaluation of two derived distribution procedures, *Water Resour. Res.*, *23*(7), 1309–1319, 1987.
- Natural Environment Research Council, Flood studies report, in *Hydrologic Studies*, vol. II, p. 15, London, 1975.
- Raines, T. H., and J. B. Valdes, Estimation of flood frequencies for ungauged catchments, *J. Hydraul. Eng.*, *119*(10), 1138–1154, 1993.
- Ranzi, R., and B. Bacchi, Analysis and forecasting of rainfall fields using radar, in *Advances in Distributed Hydrology*, edited by I. Becchi et al., pp. 327–346, Water Resour. Publ., Highlands Ranch, Colo., 1994.
- Robinson, J. S., and M. Sivapalan, An investigation into the physical causes of scaling and heterogeneity of regional flood frequency, *Water Resour. Res.*, *33*(5), 1045–1059, 1997.
- Rodriguez-Iturbe, I., and J. M. Mejia, On the transformation from point rainfall to areal rainfall, *Water Resour. Res.*, *10*(4), 729–735, 1974.
- Rodriguez-Iturbe, I., and J. B. Valdes, The geomorphologic structure of hydrologic response, *Water Resour. Res.*, *15*(6), 1409–1420, 1979.
- Rodriguez-Iturbe, I., M. Gonzales-Sanabria, and R. L. Bras., A geomorphoclimatic theory of the instantaneous unit hydrograph, *Water Resour. Res.*, *18*(4), 877–886, 1982.
- Rossi, F., Criteri di similitudine idrologica per la stima della portata al colmo di piena corrispondente ad un assegnato periodo di ritorno (in Italian), in *Proceedings of the XIV Convegno di Idraulica e Costruzioni Idrauliche*, vol. II, pp. 235–261, Univ. of Naples, Naples, Italy, 1974.
- Rossi, F., M. Fiorentino, and P. Versace, Two-component extreme value distribution for flood frequency analysis, *Water Resour. Res.*, *20*(7), 847–856, 1984.
- Shen, H. W., G. J. Koch, and J. T. B. Obeysekera, Physically based flood features and frequencies, *J. Hydraul. Eng.*, *116*(4), 495–514, 1990.
- Sivapalan, M., E. F. Wood, and K. J. Beven, On hydrologic similarity, 3, A dimensionless flood frequency model using a generalized geomorphologic unit hydrograph and partial area runoff generation, *Water Resour. Res.*, *26*(1), 43–58, 1990.
- Turc, L., Estimation of irrigation water requirements, potential evapotranspiration: A simple climatic formula evolved up to date, *Ann. Agron.*, *12*, 13–14, 1961.
- Villani, P., Extreme flood estimation using Power Extreme Value (PEV) distribution, in *Proceedings of the IASTED International Conference, Modeling and Simulation*, edited by M. H. Hamza, pp. 470–476, Univ. of Pittsburgh, Pittsburgh, Pa., 1993.
- Wood, E. F., and C. S. Hebson, On hydrologic similarity, 1, Derivation of the dimensionless flood frequency curve, *Water Resour. Res.*, *22*(11), 1549–1554, 1986.

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