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# Performance prediction model of multistage centrifugal Pumps used as Turbines with Two-Phase Flow

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## Abstract

Pump as Turbines (PaTs) can be used not only in hydraulic power generation but also in chemical processes, such as refinery, where fluids containing dissolved or undissolved gases or volatiles can be expanded from a higher to a lower pressure level for energy recovery. As the gas contained in the fluid is released from the solution during expansion, the flow rate increases and additional energy is delivered with respect to the case of incompressible flow. This higher power output is very attractive. In this work, a theoretical approach is proposed in order to predict the PaT performance with a two-phase flow whose expansion characteristics are known.

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*Keywords:* Pump as Turbine; PaT; Centrifugal Pump; Two-phase flow; Performance prediction model.

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## 1. Introduction

Nowadays, the increasing energy demand represents a priority issue to be faced on social, economic, political and technical point of views. For a sustainable development, renewable energy sources should be preferred to the conventional ones. In many industrial plants, Pumps as Turbines (PaTs) could be an important renewable energy technology, contributing to create smart grids, rural electrification, sustainable industrial development as well as the

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reduction of greenhouse gas emissions and deforestation. A PaT is a conventional pump used in reverse mode, as a turbine, in order to recover energy wasted otherwise. Due to the complexity of developing customized turbines, the use of pumps operating in reverse mode can represent a practical solution, in consideration of the wide range of specific speed numbers and available standard sizes of pumps.

PaTs can be used not only in hydraulics but also in chemical processes (gas washing plants, ammonia synthesis, reverse osmosis, mines cooling, oil supply systems). For example, Laux [1] studied reverse-running multistage pumps as energy recovery turbines in oil supply systems; Apfelbacher et al. [2] studied the application of a PaT in a reduction pressure station in Aachen (Germany) and PaTs with two phase flows [3], Gopalakrishnan [4] focused on PaTs for the process industry and Bolliger examined pumps as turbine in reverse osmosis plants [5] and in a gas washing plant [6].

## Nomenclature

### Symbols

$a_c$	Coriolis acceleration [m/s]
$A$	cross section area [m <sup>2</sup> ]
$G$	mass flow rate [kg/s]
$N$	rotational speed [rpm]
$N_{st}$	number of stages
$P$	power [W]
$p$	pressure [Pa]
$Q$	volumetric flow rate [m <sup>3</sup> /s]
$R$	gas constant [J/(kg K)]
$T$	temperature [K]
$u$	tangential velocity [m/s]
$x$	mass vapour fraction [-]
$Y$	specific work [J/kg]

### Greek letters

$\alpha$	volume vapour fraction [-]
$\eta_y$	hydraulic efficiency [-]
$\lambda$	power coefficient [-]
$\rho_L$	liquid density [kg/m <sup>3</sup> ]
$\rho_v$	vapour density [kg/m <sup>3</sup> ]
$\rho_v^*$	non-dimensional vapour density [-]
$\psi$	pressure coefficient [-]
$\varphi$	flow coefficient [-]

### Subscripts / Superscript

$1P$	single phase
$2P$	two-phase
$1$	outlet section of the machine / stage
$2$	inlet section of the machine / stage
$mix$	mixture
$ref$	reference condition
$T$	multistage turbine
$st$	stage

In these last various chemical engineering processes, fluids containing dissolved or undissolved gases are expanded from a higher to a lower pressure level. If the energy released by this process is sufficiently large, it may be worthwhile to expand the fluid in a pump operating as a turbine. A two-phase flow is established in the turbine if the liquid fluid

contains free gas and dissolved gases are released from the solution or part of the liquid evaporates during the expansion. As the gas contained in the fluid is expanded, additional energy is delivered with respect to the case of incompressible flow. This increase of power output is very interesting in terms of energy management improvement in process industries. Interest in pumps as turbines with two-phase flow dates back to the early 80's when the first technical reports on theoretical and experimental studies appear in literature [1-6]. However, these models with simple assumptions show a limited applicability.

In this framework, a literature survey of prediction models of two-phase PaT performance has been conducted. In this study, a 6-stage centrifugal pump operating as turbine has been used as case study in order to develop a theoretical model, which could help in the prediction of the performance of a pump as turbine operating with two-phase flow, whose properties are known. Moreover, the fluid is considered with no vapour phase at the inlet section of the machine, whereas during the expansion both the mass vapor fraction,  $x$ , and the volume vapor fraction,  $\alpha$ , can increase.

## 2. Multi-phase flows in turbomachinery

In many processing engineering fields, both pumps and turbines can deal with multiphase flows. Two general kind of multiphase flow can be identified, namely disperse flows and separated flows. Disperse flows consist of finite particles, drops or bubbles (the disperse phase) distributed in a connected volume of the continuous phase. On the other hand, separated flows consist of two or more continuous streams of different fluids separated by interfaces [7]. A persistent theme throughout the study of multiphase flows is the need to model and predict the detailed behavior of those flows and the phenomena that they manifest in turbomachines.

The capability of a centrifugal pump to convey a two-phase mixture depends in the first place on whether gas and liquid form a homogenous mixture or to what extent the two phases separate. Fine gas bubbles dispersed in a liquid can be considered as a quasi-homogenous mixture. The bubbles are entrained by the liquid flow, but there is some slip between the phases, which causes additional losses. Slip and losses due to exchange of momentum are much higher than in bubbly flow. With larger gas volume fractions, bubbly flow is no longer possible, since small bubbles tend to coalesce to form larger gas accumulations. Body forces in rotating runners are effective in determining the flow patterns, which have an impact on the energy transfer. Indeed, the fluid in the runner channel is subject to a strong Coriolis acceleration  $\bar{a}_c = 2 \times \bar{\omega} \times \bar{w}$ . The Coriolis force acts perpendicularly to the direction of the relative velocity of the liquid phase and a stratified flow develops if a sufficient amount of a gas is present.

Flow pattern prediction and interpretation in an impeller with two-phase flow is more difficult than with single-phase flow and it is difficult to establish general rules. Indeed, pumps and turbines show different behaviors when two-phase flows occur. In pumps, Coriolis forces transport liquid to the pressure side, while buoyancy effects move the gas to the suction side of the blades of centrifugal impellers. Phase separating effects grow with increasing flow rate because the pressure gradient from the pressure to the suction side (buoyancy) as well as the Coriolis acceleration become stronger with increasing of flow rate. With larger gas volume fractions bubbly flow is no longer possible, since small bubble tend to coalesce to form larger gas accumulations, which can cause the “gas-locked” condition. In addition, in radial pumps gas bubbles travel against a strong centrifugal pressure gradient towards the outlet. All of these aspects cause a deterioration of pump performance and the appearance of operating instabilities [8]. Differently, in a turbine a pressure difference in the flow direction is established between the inlet and the exhaust nozzle. Consequently, the buoyancy effect tends to accelerate the gas in the direction of the flow, avoiding the gas-locking, even with high gas content. Apart from a possible Mach number limit, phase separation problems do not restrict the stable operation of a turbine: the gas is always blown through the machine, albeit with high losses and possibly little useful work [9].

## 3. Literature survey

The interest in pumps as turbine with two-phase flow starts during the 80's with some technical reports on first theoretical and experimental studies in order to predict the two phase performance for safety assessments of nuclear power plants. All of these models were based on the main assumption of the homogeneous flow in which two phases could be sufficiently well mixed and therefore the disperse particle size sufficiently small to eliminate any significant relative motion.

Gülich [9] was one of the first to study this issue. He tested a three-stage pump in reverse mode with an air/water mixture and he proposed a model to predict the performance in turbine mode during two-phase flow conditions. The model considers an isothermal flow and correlates the two-phase performance to the single-phase operating conditions by means of two empirical factors,  $f_\psi = \psi_{2P}/\psi_{1P}$  and  $f_\eta = \eta_{2P}/\eta_{1P}$ , which depends on the void fraction  $\alpha$ :

$$f_\psi = \frac{\psi_{2P}}{\psi_{1P}} = 1 + 0.45\alpha \quad (1)$$

$$f_\eta = \frac{\eta_{2P}}{\eta_{1P}} = 1 - 0.55\alpha - \alpha^3 \quad (2)$$

Gülich assumes that up to a void fraction  $\alpha$  of about 80%, the gas mass fraction is negligible compared to that of the liquid, and the temperature remains roughly constant. In addition to this, effects of liquid evaporation or gas desorbing are neglected and both the phases are considered separately. As a consequence of these assumptions, the isothermal specific work from the inlet pressure,  $p_2$ , to the outlet pressure,  $p_1$ , of each stage is computed by summing the contribution of both phases, as stated in equation 3. The liquid and the gas contributions are weighted by the corresponding mass fractions.

$$Y_{isoth.} = (1 - x) \frac{p_2 - p_1}{\rho_l} + xRT \ln \frac{p_2}{p_1} + \frac{c_2^2 - c_1^2}{2} \quad (3)$$

Once the specific isothermal work is computed, Gülich proposes to calculate the equivalent head per stage with single-phase liquid flow by means of the empirical factor  $f_\psi$ :

$$\psi_{1P} = \frac{\psi_{2P}}{f_\psi} = \frac{2 Y_{isoth.}}{N_{st} u_2^2 f_\psi} = \frac{2 Y_{isoth.}}{N_{st} u_2^2 (1 + 0.45\alpha)} \quad (4)$$

By means of  $\psi_{1P}$ , the flow coefficient,  $\varphi$ , and the efficiency,  $\eta_{1P}$ , can be read from the single-phase characteristics. Using these values, the volumetric flow rate,  $Q$ , the efficiency  $\eta_{2P}$  and the power  $P_{2P}$  can be calculated:

$$\begin{cases} Q = A_2 u_2 \varphi \\ \eta_{2P} = \eta_{1P} f_\eta = \eta_{1P} (1 - 0.55\alpha - \alpha^3) \\ P_{2P} = G Y_{isoth.} \eta_{2P} \end{cases} \quad (5)$$

Hamkins et. al [3] carried out an experimental characterization of an eight-stage pump running as turbine in a petrochemical plant under real production conditions. The flow within the turbine was in a two-phase condition from the inlet to the outlet. The exit void fraction ranged from 0.35 to 0.40 for these tests. They predicted the characteristic curve by treating the mixture as a homogeneous compressible fluid.

One of the most recent work in this field is by Visser [10], who computes the average density of the 2-phase mixture stage by stage and treats it as an incompressible mixture during each single-stage work calculation. First of all, the void fraction is experimentally known as function of the pressure. The total pressure drop is divided stage by stage in order to know the void fraction as function of the inlet and the outlet pressure of each stage. Once the mean void fraction of the stage is known, it is possible to calculate the density of the homogeneous mixture,  $\rho_{mix}$ . Then, the total work is obtained by summing the single specific works of each stage. Finally, it is possible to calculate the output power of the turbine by multiplying the specific work by the total mass flow rate and the turbine efficiency in two-phase flow conditions. The efficiency is obtained by means of the empirical correlation proposed by Gülich, as a function of the average void fraction. All of these models assume that the equilibrium gas volume fraction is established instantaneously.

The formation of gas bubbles requires time because the gas molecules must diffuse through the liquid and form a bubble. The equilibrium conditions in which two-phase density is a unique function of pressure,  $\rho_{mix}(p)$ , can only be established after a finite time. Gopalakrishnan [4] applied the analysis of Payvar [11] and determined that the bubble development time is of the same order of magnitude as the residence time in the turbine. The model calculates the

bubble growth history in hydraulic turbines. The main assumptions are that the two-phase flow is homogeneous and one-dimensional, the pressure distribution is known a priori and the gas volume rate increases because of the growth of bubble and because of the generation of new bubbles at all solid surfaces in contact with the fluid stream.

#### 4. Proposed model

In order to design a two-phase flow multistage PaT, the manufacturer needs to know the available mass and volumetric flow rates, the power required, the flow properties ( $x$ ,  $\alpha$ ,  $\rho_v^* = \rho_v/\rho_L$ ) at a reference temperature  $T = T_{ref}$ , the maximum and the minimum pressures of the process. Usually, manufacturing companies test their machines in single phase (generally water). Hence, in order to evaluate the behavior under two-phase flow, they need a global performance correction tool.

In this work, a multistage centrifugal pump with 6 equal stages has been considered as a case study. In Figure 1a the characteristics of all the single stages are reported in terms of non-dimensional parameters, with the assumptions that  $\eta_y^{(st)} = \eta_{y,T,1P}$ ,  $\varphi_{1P}^{(st)} = \varphi_{T,1P}$  and  $\psi_{1P}^{(st)} = \psi_{T,1P}/N_{st}$ . Moreover, Figure 1b shows flow properties (vapour density,  $\rho_v$ , vapour mass fraction,  $x$ ), as function of the pressure value during the expansion ( $p_{ref} = 190$  bar). Once the mass fraction,  $x$ , the liquid density,  $\rho_L$ , and the vapour density,  $\rho_v$ , are known, it is possible to compute the vapour volume fraction,  $\alpha = x\rho_L/(x\rho_L + (1-x)\rho_v)$ .

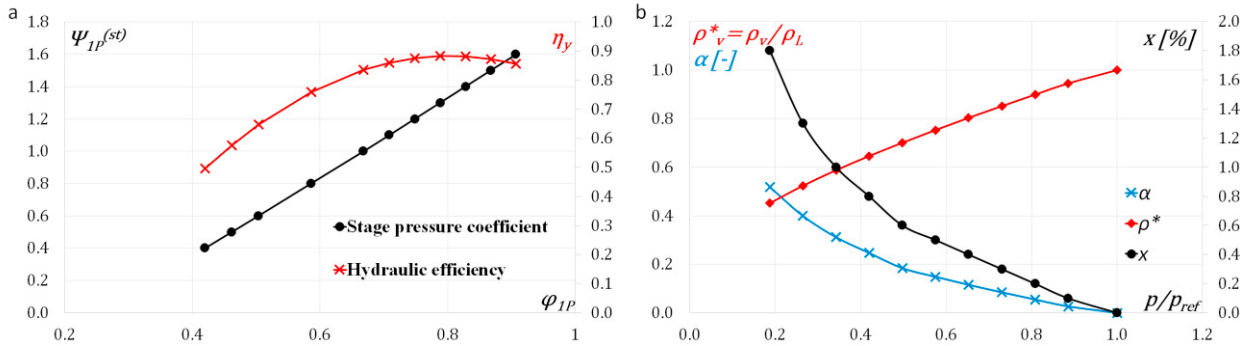


Fig. 1. (a) Experimental stage pressure coefficient,  $\psi_{1P}^{(st)}$ , and hydraulic efficiency,  $\eta_y$ , vs. one-phase flow coefficient,  $\varphi_{1P}$ , in turbine operation mode; (b) values of the considered two-phase flow properties.

Figure 2 shows the algorithm of the proposed model. This calculation is performed simultaneously on each stage. The subscripts 2 and 1 are respectively the inlet and the outlet sections of each stage. For initialization, the turbine pressure drop,  $\Delta p_T = (p_{in} - p_{out})$ , is equally divided among each stage,  $\Delta p^{(st)} = \Delta p_T/N_{st}$ . The value of  $p_{out}$  is critical in order to avoid too high velocities. Although PaTs can handle significant vapour volume fractions, the proposed model can handle  $\alpha$  lower than 50%, otherwise bulk velocities become too high and comparable to sound speed. In such case, the hypothesis of incompressible flow across each stage is no more valid. This problem is particularly important in the last stage.

Once all the stage pressure drops  $\Delta p^{(st)}$  are determined,  $p_2^{(st)}$  and  $p_1^{(st)}$  of each stage are known and it is possible to obtain from the correlation in Figure 1b the vapour mass fraction,  $x$ , the vapour density,  $\rho_v$ , and the vapour volume fraction,  $\alpha$ , at inlet and outlet section of the stage ( $x_2^{(st)}$ ,  $x_1^{(st)}$ ,  $\rho_{v,2}^{(st)}$ ,  $\rho_{v,1}^{(st)}$ ,  $\alpha_2^{(st)}$ ,  $\alpha_1^{(st)}$ ). In order to compute the pressure levels across each turbine stage, the following conditions are imposed:

$$\begin{cases} p_2^{(1st)} = p_{in} \\ p_2^{(st)} = p_1^{(st-1)} \\ p_1^{(st)} = p_2^{(st)} - \Delta p^{(st)} \\ p_1^{(Nst)} = p_{out} \end{cases} \quad (7)$$

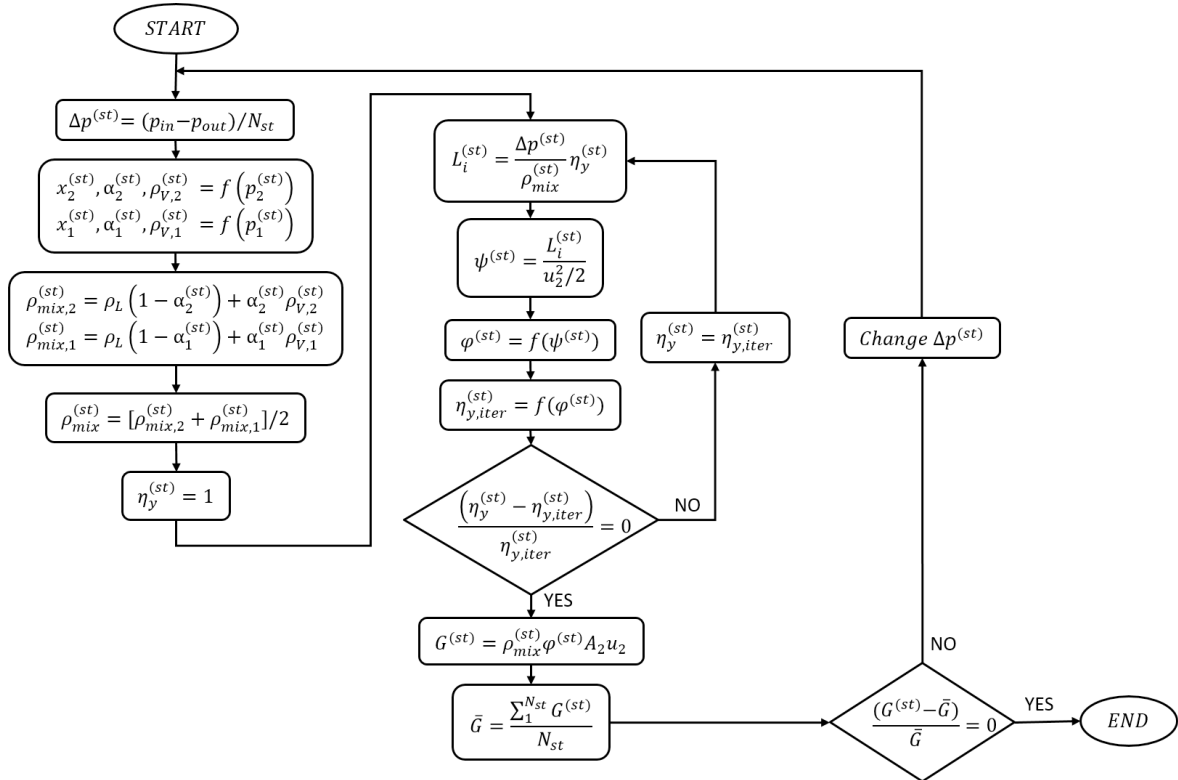


Fig. 2. Flow chart of the proposed model.

Considering a homogeneous mixture, it is possible to calculate the mixture density at both inlet,  $\rho_{mix,2}^{(st)}$ , and outlet,  $\rho_{mix,1}^{(st)}$ , of each stage by means of equation 8:

$$\rho_{mix,i}^{(st)} = (1 - \alpha_i^{(st)})\rho_L + \alpha_i^{(st)}\rho_{v,i}^{(st)} \quad \forall i = 1,2 \quad (8)$$

Under the hypothesis of relative low vapour volume fraction,  $\alpha$ , the mean mixture density of the stage,  $\rho_{mix}^{(st)}$ , can be computed as the average of the two mixture densities.

$$\rho_{mix}^{(st)} = \frac{(\rho_{mix,1}^{(st)} + \rho_{mix,2}^{(st)})}{2} \quad (9)$$

At this point, the stage pressure coefficient can be calculated ( $\eta_y^{(st)} = 1$ , for initialization), as follows:

$$\psi_{st} = \frac{L_i^{(st)}}{u_2^2/2} = \frac{2\Delta p^{(st)}\eta_y^{(st)}}{\rho_{mix}^{(st)}u_2^2} \quad (10)$$

Entering with  $\psi_{1P}^{(st)} = \psi^{(st)}$  on the characteristics curves of Figure 1a,  $\varphi = f(\psi^{(st)})$  is obtained and hence the new hydraulic efficiency,  $\eta_y^{(st)} = f(\varphi)$ . This calculation is iterated until convergence of  $\eta_y$ . Successively, the mass flow rate is calculated by means of equation 11:

$$G^{(st)} = \rho_{mix}^{(st)}Q^{(st)} = \rho_{mix}^{(st)}\varphi A_2u_2 \quad (11)$$

where  $A_2$  and  $u_2$  are respectively the cross section and the tangential velocity at the inlet of the runner. Since during expansion the value of  $\rho_{mix}^{(st)}$  for each stage is different, the mass flow rates,  $G^{(st)}$ , for each stage result to differ from each other.

This means that a new stage pressure drop distribution has to be evaluated until the mass conservation across the entire PaT is satisfied. In the iterative process, in order to estimate the new stage pressure drop distribution, it has been supposed that the relative error on the pressure drop  $e_{\Delta p}^{(st)} = (\Delta p_{new}^{(st)} / \Delta p_{old}^{(st)} - 1)$  is the opposite to the relative error on the mass flow rate  $G^{(st)}$  with respect to the mean mass flow rate  $\bar{G}$ .

**5. Results and discussion**

Figure 3 shows the comparison between the characteristic curve of the investigated 6-stages PaT under one phase operation and the predicted performance curves under two-phase flow for different pressure drops and different outlet pressure, in terms of turbine pressure coefficient,  $\psi_T$ , and turbine power coefficient,  $\lambda_T = 2P / (\rho_L A_2 u_2^3)$ . Each curve has been obtained at constant rotational speed,  $N$ , constant outlet pressure,  $p_{out}$ , and by increasing the pressure upstream the turbine. Pressure values have been selected in order to guarantee that the flow coefficient in each stage remains within its operating range ( $0.4 < \varphi < 0.9$ , as depicted in Figure 1a). Indeed, the increase of the volumetric flow rate, when two-phase flows are considered, involves an increase of  $\psi_T$  and  $\lambda_T$  respectively of 40% and 20% on average. The minimum value of  $p_{out}$  is 35 bar (when  $p_{in}$  is 130 bar), otherwise the vapor volume fraction,  $\alpha$ , becomes too high (greater than 50%) and the flow coefficient of the last stage becomes higher than the upper allowable limit ( $\varphi = 0.9$ ). Figure 3 points out that the reduction of the outlet pressure,  $p_{out}$ , causes two opposite effects: on one side, it involves a higher work output but, on the other side, a reduction of the operating range. In the case with  $p_{out} = 60$  bar, the maximum inlet pressure is  $p_{in} = 190$  bar. This means that the increase of the outlet pressure allows on an increase of the the inlet pressure.

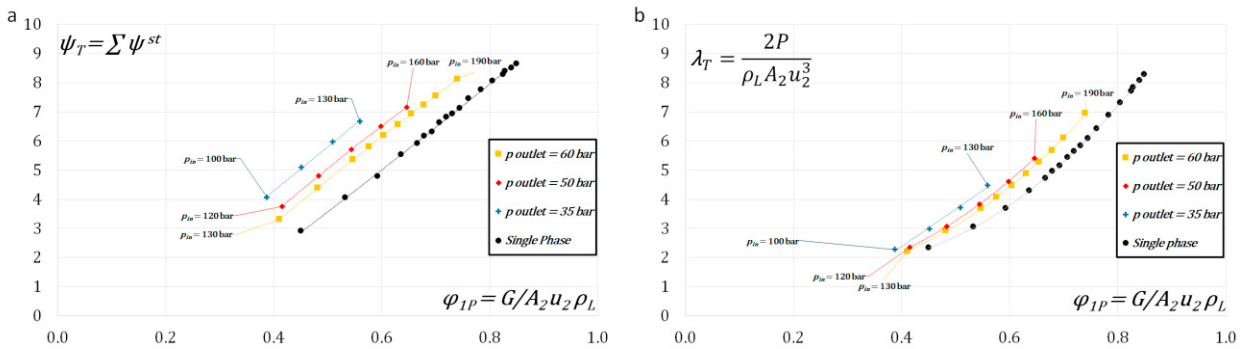


Fig. 3. Comparison of the predicted performance curve with two-phase flow and the experimental curve with water in terms of pressure coefficient,  $\psi_T$ , vs. flow coefficient,  $\varphi_{1P}$ , (a) and power coefficient,  $\lambda$ , vs. flow coefficient,  $\varphi_{1P}$ , (b)

Figure 4 shows how the operating points of each stage change during expansion. In this case, the condition with  $p_{in} = 160$  bar and  $p_{out} = 60$  bar has been considered.

During the expansion, the operating points of each stage, identified with  $\psi^{(st)}$ ,  $\varphi$  and  $\eta_y$ , remain on the non-dimensional characteristic of the machine and move towards higher flow coefficient and pressure coefficient, due to the decrease of the mean mixture density. Moreover, the effect of the increase of the volume vapour fraction is highlighted by the increase of the steps between two consecutive operating points from the first to the last stage.

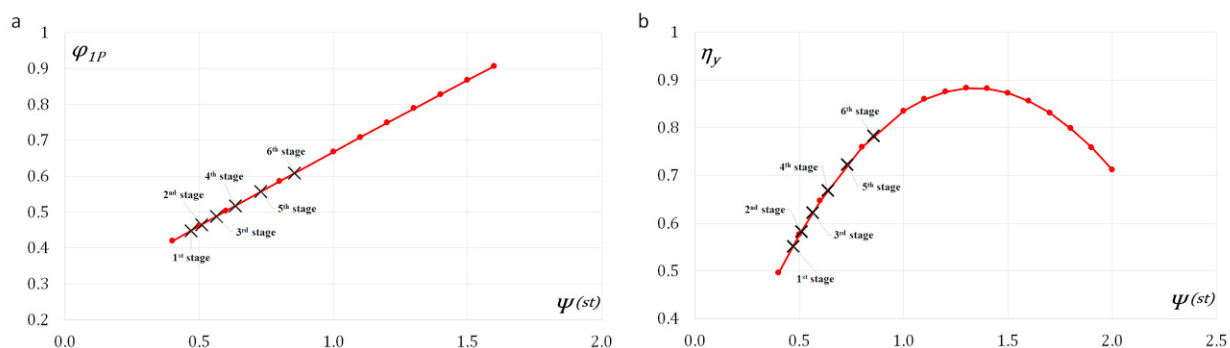


Fig. 4. Change of operating points stage by stage ( $p_{in} = 160$  bar and  $p_{out} = 60$  bar)

## 6. Conclusion

A literature survey has been conducted in order to investigate the behavior of centrifugal pumps operating in reverse mode with two-phase flows. The first papers in the literature can be dated back to the early '80. These models are essentially based on the assumption of a homogenous flow, which is in two-phase condition from the inlet to outlet section of the machine. In this work a fluid with the vapour phase, which is not present at the inlet section of the machine and increases during the expansion is considered.

A 6-stages centrifugal pump operating as turbine has been used as a case study in order to develop a theoretical model, which predicts the performance operating with two-phase flow. Companies usually test their machines in single phase (generally water). By starting from the characteristic curves provided by the company, non-dimensional curves have been obtained  $\psi = f(\varphi)$  and  $\eta_y = f(\varphi)$ . Thanks to these curves, is possible to model the behavior of the same machine operating with a two-phase mixture, whose properties are known. The advantage of the proposed model is that it does not rely on experimental correlations based on a limited number of samples, which could limit its range of applicability. Although experimental data are not available, the model has been proposed as an engineering tool which can help in the initial prediction of the performance of a PaT operating with a two-phase flow.

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