

# The Role of Phase Errors Distributions in Phased Array Systems Operations

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**Abstract:** A comprehensive study on the role of the phase errors distribution on the performances of the phased array systems has been led using a complete and behavioral model for radiation-pattern characteristics. The used model has many input parameters and it has a lot of features, such as parameters simulations with results analysis, unconventional two-dimensional color graph representation capability in order to show more clearly the results. The results of the study have been discussed and reported. The main achievement of this work is the demonstration that the RMS phase error is a valuable figure of merit of phased array systems but it is not sufficient to completely describe the behavior of a real system. Indeed, this work has shown how the phase errors distribution actually affects the performances of the phased arrays antennas.

**Key words:** Behavioral model, phased array, RMS phase error, phase errors distribution, antenna.

## 1. Introduction

Phased array antennas play an important role in many radar and airborne applications; their success is mainly due to high agility in reconfiguring pattern and quick steering capability. Many phased-array antennas are designed to use digital or digitalized phase shifters in which the phase shift varies in discrete steps rather than continuously; as a consequence of using digital phase shifter, the beam can be steered only in discrete steps and the granularity [1], defined as the finest realizable increment between adjacent beam positions, depends on the number of bits and the number of antennas; furthermore, the phase shifts produced by the phase shifter are not ideal, due to the fact that a phase shifter itself is affected by errors and non idealities and this implies that the radiation pattern characteristics are further altered. There exist different implementations of phased array systems and a simple classification can be made in terms of the used technique; in this case it is possible to mention the so-called distributed-type

phase shifter (DSPTs) based on the use of transmission lines, a second group is formed by the all-pass network phase shifter and the last group contains the phase shifters which use a vector sum of two or more orthogonal-phased signals, the so-called forward-type and reflective-type phase shifters (FTPSs, RTPSSs). Moreover, the phase array transceivers can be distinguished according to the Tx and Rx architectures; in the case of narrowband channel [2], the phase shift operation can be implemented at base band, RF, IF and LO domain. Each choice presents different design challenges and the main features affecting the radiation pattern characteristics performances are the insertion losses, the phase shift resolution (the number of bits in a digital phase shifter), the phase shift accuracy, and the number of radiators.

This paper reports a complete and innovative study on the real contribution of the phase accuracy to the performances of the phased arrays. Usually the phase accuracy is synthetically presented in terms of rms phase error defined as in Eq. (1) [3, 4]. Let's denote  $\theta_{\Delta i}$  the  $i$ -th error phase shift, so the rms phase error can be

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defined as

$$\Theta_{ARMS} = \sqrt{\frac{1}{N-1} \sum_{i=2}^N |\Theta_{\Delta i}|^2} \quad (1)$$

where the phase of one channel is chosen as the reference for other error calculations.

Nevertheless this figure of merit though being a significant feature of phase shifters, it is not satisfactory to completely describe the behavior of real phased array systems. Indeed as it has been shown in this work, an important role on determining the radiation-pattern characteristics is played by the phase errors distribution which can be defined as a vector with the length equal to the number of possible phase shifts and with the  $i$ -th element equal to the phase error corresponding to the  $i$ -th ideal phase shift.

Since a phase shifter with a low rms phase error is a challenging building block specially at high frequencies, it is easy to understand as the achievement presented in this study is an important key aspect to be taken into account designing a high performances phased array systems such as those used to achieve high directivity and high reduction of sidelobes for satellite, RADAR and wireless communications systems.

Therefore, the design of this kind of phase array transceivers asks for an advanced and complete model able to take into account, beside the classical parameters affecting the radiation-pattern characteristics (number of antennas, number of bits, etc.) also the non-idealities and aspects above described in terms of phase errors distribution. In this work this complete and advanced behavioral model, developed in MATLAB<sup>®</sup>, has been reported. This tool is able to predict antenna array performance in terms of beam shape, directivity, side lobe levels, main lobe deviation etc. considering also the phase errors distribution.

A further purposeful consequence deriving from the presented study is the importance, in these kinds of applications, to dispose of a phased array transceivers able to manage in a convenient way the phase accuracy. A possible valuable approach is to control the phase accuracy in the digital domain; recently a phased array

transmitter able to meet this need has been described in Ref. [5].

The paper is organized as follows: the model is described in Section 2; the analysis and results of the proposed study are presented in Section 3, while in the Section 4 the conclusions are given.

## 2. Model Description

The model has been developed in Matlab<sup>®</sup> environment, which is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numeric computation. The model implemented is very complete and it has more additional features not used for this study. Further details of the complete model are provided in the following subsections.

### 2.1 Array Configuration

The tool takes into account the array factors of linear arrays and it is based on basic antenna array theory. The array configuration is shown in Fig. 1 where the elements, considered as point sources, are positioned on the horizontal axis. The angle  $\theta$  represents the elevation angle which is the angle of the beam with respect to the  $z$  axis. The scan angle is the elevation angle imposed.

### 2.2 Type of Simulations

The model presented is very exhaustive, in fact it accepts as inputs, according to the chosen simulation type, the range of elevation angles for pattern

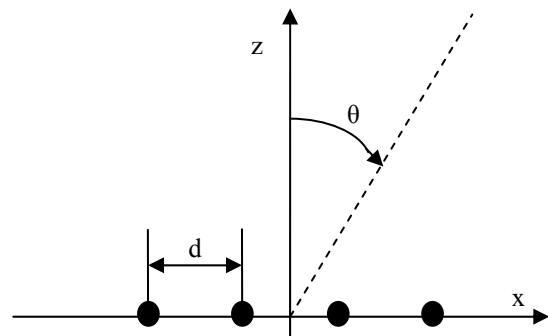


Fig. 1 Array configuration.

calculation, the number of antennas and the inter-element spacing between the antennas, the type of phase shifts fed at the antennas (ideal phase shift which simulates an ideal analog phase shifter, ideal quantization which simulates an ideal digital phase shifter, calculated real quantization which implements a simulated real digital phase shifter and finally real quantization obtained by inserting a vector of real digital phase shifter measurements which implements a real measured phase shifter), the wanted elevation angle (scan angle) or range of scan angles.

The program is organized in menu; first of all the user can choose the type of simulation among the following types:

(1) Variable number of antennas: it is a parametric simulation which has as parameter the number of the antennas. The required parameters are the antennas start number, the antennas stop number and the antennas increment number. The other parameters that have to be inserted are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan or start steering angle, the stop scan angle, the increment scan angle, the antenna spacing;

(2) Variable value of rms error: it is a parametric simulation which varies the rms error added to the ideal phase shifter in order to simulate a real phase shifter and to analyze the impact of the rms phase error introduced by a non-ideal phase shifter, on the radiation-pattern characteristic. This type of simulation, respecting the common practice, uses the rms phase error value as the only figure of merit of a phase shifter goodness while a more exhaustive study about this concept is achieved using the simulation option number 7 below described. In this case the parameters required are the number of simulation and the rms value to be used in those simulations. This is one of the most interesting features of the program because it gives the opportunity to investigate if there is a particular relation between the rms phase error of the phase shifter and some parameter of the radiation-pattern [6]. The other parameters required are

the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan, the stop scan angle, the increment scan angle, the number of the antennas, the inter-element antenna spacing;

(3) Variable value of inter-element spacing: it is a parametric simulation which has as variable the distance, in terms of wavelength, between the antennas on a linear distribution. The inputs required for this simulation are the inter-spacing start value, the stop value and the increment. The other parameters that have to be inserted are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan, the stop scan angle, the increment scan angle, the number of antennas;

(4) Variable number of bit: also in this case this is a parametric simulation which has as inputs the start number of bits, the stop number of bits and the increment. Such a simulation is the key point in those applications where a great precision is required during the scan. In fact, the more are the bits, the more is the scan precision till the ideal situation of analog phase shifter which could be assimilated to a phase shifter with infinite number of bits. So this type of simulation could be used as a dimension tool or as an instrument to study how effective is the increment of the number of bits and what is the impact on the radiation-pattern. The parameters required are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start, the stop scan angle, the increment scan angle, the number of antennas and the inter-element antenna spacing;

(5) Not parametric simulation: in this simulation there are no parameters and the simulation is carried out on a spread of desired scan angles. So the input parameters are the elevation start angle, the elevation stop angle, the elevation increment angle, the start scan angle, the stop scan angle, the increment scan angle, the number of antennas and the inter-element antenna spacing;

(6) Single simulation: it is the same of the previous simulation type with the difference that the simulation is carried on only a single specific scan angle;

(7) rms behavior study: this type of simulation is very important in studying the role of the effective phase error distribution on the beam shape. In fact for a generic phase shifter, it is often given the rms phase error as the only figure of merit, but this parameter does not represent the phase error distribution in an unambiguous way because there are infinite distributions with the same value of rms error, and different distributions can lead to significant different beam shapes. This simulation has as inputs the phase error vector distribution defined as the difference between the measured or simulated phase distribution and the ideal one; the simulation consists in combining the values of ideal phase shifts with the values of the phase errors in a different way for every simulation. The user can choose the type of this combination and in particular he can choose if the all permutations have to be used or a circular shift has to be used; the first option is only practical when the number of phase steps is small since the number of permutations rapidly increases with the phase steps requiring time and memory for the simulations. The other parameters that have to be inserted are the number of antennas, the inter-spacing antenna, the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan, the stop scan angle, the increment scan angle.

### 2.3 Round-off Techniques

For simulation type numbers 1, 3, 4, 5, 6 and 7, the user can choose the preferred round-off technique among the following ones:

(1) exact phase at each elements: this is the ideal situation represented by an ideal analog phase shifter. With this type of simulation the user can study the geometry of the array without worrying about the non idealities of the phase shifter;

(2) ideal quantization: most phase shifters are digital devices, or at least are digitally controlled. Therefore, only discrete values of phase shift are allowed, and they may not be the precise values

required; so this simulation takes into account the errors due to an ideal quantization;

(3) real simulated quantization: the digital phase shifts really fed to each element are not ideal and they depend on the phase shifter adopted. A figure of merit is the rms error of the phase generation of the phase shifter. Starting from the ideal quantization, a casual and random error distribution, with the imposed rms, is added to the phase shifts in order to obtain the simulated allowable phase states;

(4) measured values: it is possible to insert the real measurements made on a phase shifter in order to test the behavior of own phase shifter and the resultant array pattern.

### 2.4 Results Plot Types

When the simulations are finished, the user can plot the radiation-pattern obtained (or rather the radiation-patterns obtained) in various format: classical rectangular plot, classical polar plot or unconventional two-dimensional color graph representation in Cartesian coordinate system. Conventionally, radiation patterns are represented in two dimension (2D), in Cartesian or polar coordinate systems. In the Cartesian coordinate system, the magnitude of the radiated field, usually in decibels (dB), is indicated on the Y axis, whereas the angular parameter, the elevation angle  $\theta$  or the quantity  $u = \sin(\theta)$ , is on the X axis. In the polar coordinate system, the radius usually represents the magnitude of the radiated field and the angle represents the elevation angle. When comparison is required or a parametric simulation is carried out, several curves are plotted on the same graph or another dimension is added. If we use another axis as the third dimension, the extrapolation of the data is not so easy, considering that the three dimensional representation is actually two dimensional on a planar surface [6]. An attractive representation is then the use of a color parameter as the third dimension, which represents the variable or the parameter which depends on the user's interest. As a clarifying example, consider the case of a not

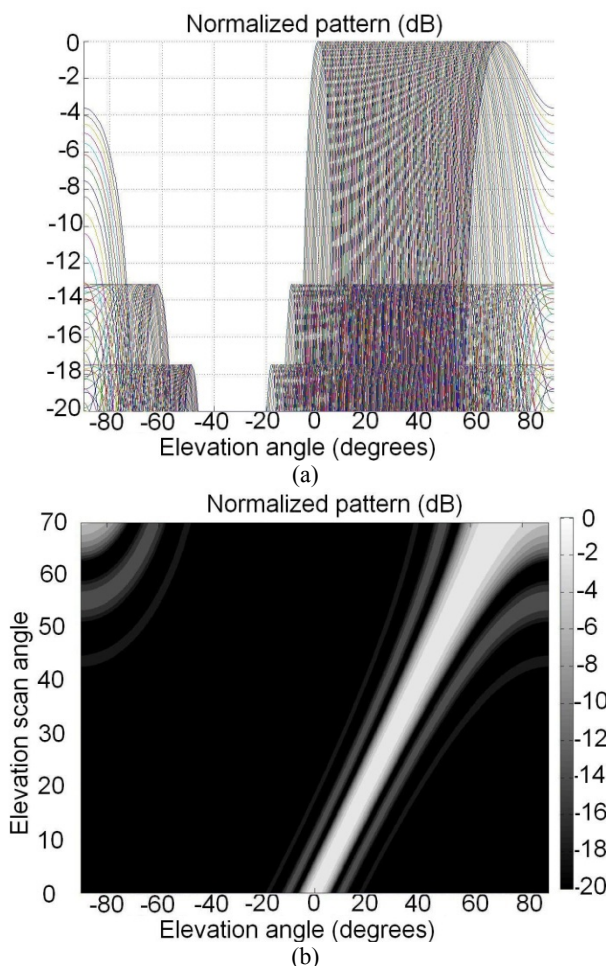
parametric simulation in which the user wants to study the array factor of a sixteen element array with the inter-element spacing equal to half a wavelength and exact phase at each element, over a range of scan angles; the elevation start angle is  $-90^\circ$ , the elevation stop angle is  $90^\circ$ , the elevation increment angle is  $0.2^\circ$ , the start scan angle is  $0^\circ$ , the stop scan angle is  $70^\circ$  and the increment scan angle is  $0.5^\circ$ . Two different representations of the same results are shown in Fig. 2, where Fig. 2a is the classical representation with the overlap of all simulation in different color and Fig. 2b is the proposed unconventional plot with the color scale in dB. In the latter, the wanted elevation scan angle is the parameter on the Y axis, while on the X axis there is the elevation angle and finally the color indicates the magnitude in dB. So each horizontal line corresponds

to the array factor calculated for the relative wanted elevation scan angle; in such a representation, it is easy to visualize the main lobe, sidelobe, nulls, and half-power-beamwidth (HPBW) variations when the scan angle changes and if there are some grating lobes [6]. Particularly, with the help of the black contour line at the  $-3$  dB level, the user can notice the widening of the HPBW with the increase in the scan angle. Referring to the Fig. 2a is quite impossible to extrapolate the results related to a single simulation.

It is clear the advantage in using the two dimensional color plot which gives more legibility to the graphical representation when, in general, a parametric or comparative simulation is required. This is only an example of the tool capabilities which depend on the characteristics the user wants to focus on. Another example of using the 2D color plot will be given in the next section.

### 3. RMS Effect on the Beam Shape

In this paragraph it is shown the importance that the effective errors distribution has on the beam shape, through a practical example. The measurements made on a real 6-bit phase shifter have been used. The considered phase shifter is that presented in Ref. [7]. This latter has a  $0.223^\circ$  rms phase error with a maximum and minimum phase error equal to about  $\pm 0.5^\circ$  as shown in Fig. 3. If we consider different prototypes of the same architecture, they have the same rms phase error approximately, but certainly they don't have the same errors distribution that is, in this context, the vector containing the phase errors associated to vector of the possible phase shifts. With the simulation number 7, called "Rms behavior study" explained in the Section 2 (with the option of circular shift due to the great number of possible phase shifts), we are able to study the beam shape variations related to 64 different phase error distributions with the same rms phase error. So the  $i$ -th simulation uses the vector of the phase errors inserted with a circular shift of exactly  $i$  places. As it is obvious the rms phase error of the distributions



**Fig. 2 (a) Overlapped simulations in a rectangular classical plot; (b) Unconventional color polar plot.**

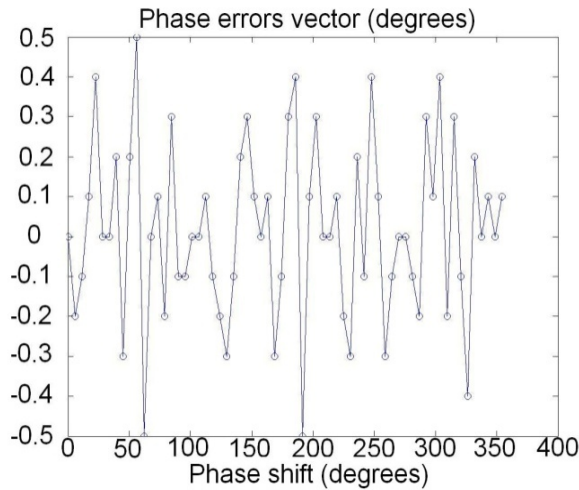


Fig. 3 Phase shifter error vector.

is the same for each simulation because the values of the phase errors are the same and only the arrangement in the vector is changed. The simulation setup is shown in Table 1, while in Figs. 4 and 5 the simulations are shown.

Fig. 4a represents the maximum lobe deviation as a function of the simulation; it means that for each simulation over all the imposed scan angles, only the maximum is considered. It can be noticed that each simulation has a different maximum value and the maximum excursion among these simulations is about 0.5 degrees. The same consideration are valid for Fig. 4b where it is represented the maximum Side Lobe Level (SSL) in relation to the number of simulations and for Fig. 4c where the maximum HPBW in relation to the number of simulations is represented; as it possible to note in Fig. 4b the excursion in the Maximum SSL is about 1.5 dB, while from the Fig. 4c the variation of the maximum HPBW is about 2 degrees.

A more exhaustive and complete representation of the simulation results are shown in Fig. 5a, Fig. 5b and Fig. 5c, where it has been used the unconventional color plot described in the previous section. In those type of plots all simulation results have been used without losing any important information.

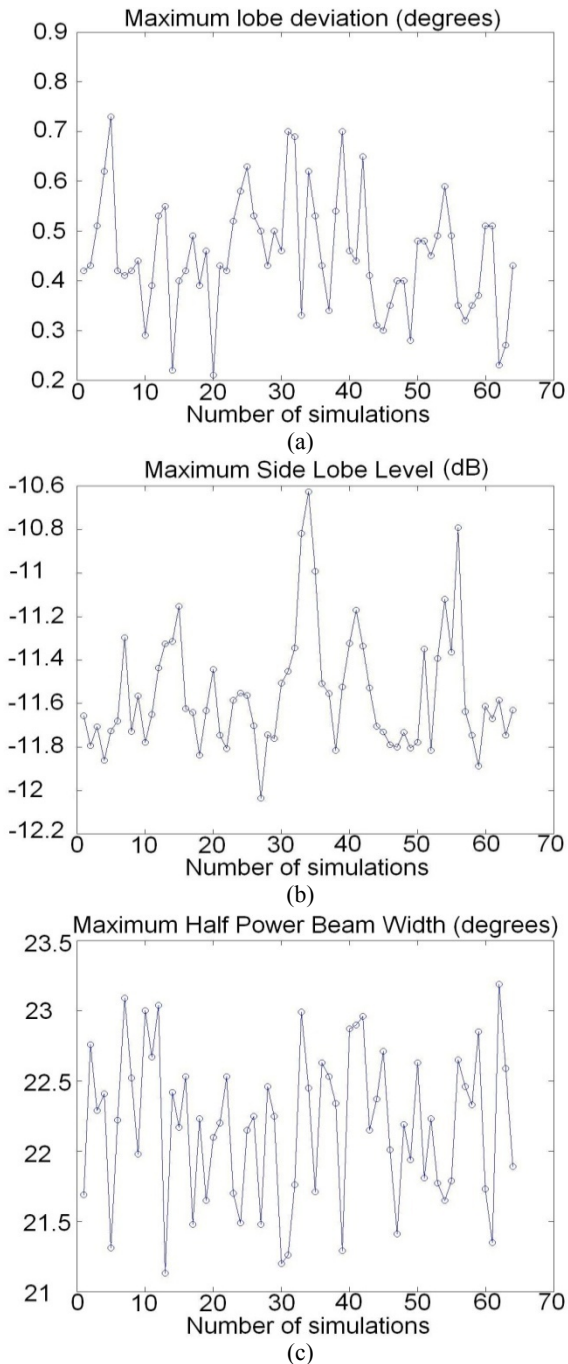
In Fig. 5a it is reported the lobe deviation as a function of the number of simulations: on the X axes there are the elevation scan angles in degrees, on the Y

Table 1 Summary of the simulation setup.

Parameter	Variable error vector
Elevation start angle (degree)	-90
Elevation stop angle (degree)	90
Elevation increment angle (degree)	0.001
Scan start angle (degree)	0
Scan stop angle (degree)	70
Scan increment angle (degree)	0.1
Number of antennas	16
Inter-element distance	0.5
Simulation type	circular
Number of simulation	64

axes there is the number of the simulations, the colorbar represents the value of the main lobe deviation in degrees. So on each horizontal line there are the lobe deviations related to the single simulation over the entire scan angles span. On each vertical line there are the lobe deviations related to the single elevation scan angle over all the simulations. In Fig. 5b it is represented the side lobe level as a function of the number of simulation: on the X axes there are the elevation scan angles in degrees, on the Y axes there is the number of the simulations, the colorbar represents the value of the side lobe level in dB. So on each horizontal line there are the side lobe level related to the single simulation over the entire scan angles span and on each vertical line there are the side lobe level related to the single elevation scan angle over all the simulations. In Fig. 5c it is represented half power beam width as a function of the number of simulations: on the X axes there are the elevation scan angles in degrees, on the Y axes there is the number of the simulations, the colorbar represents the value of the HPBW in degrees. So on each horizontal line there are the HPBW levels related to the single simulation over the entire scan angles span and on each vertical line there are the HPBW values related to the single elevation scan angle over all the simulations.

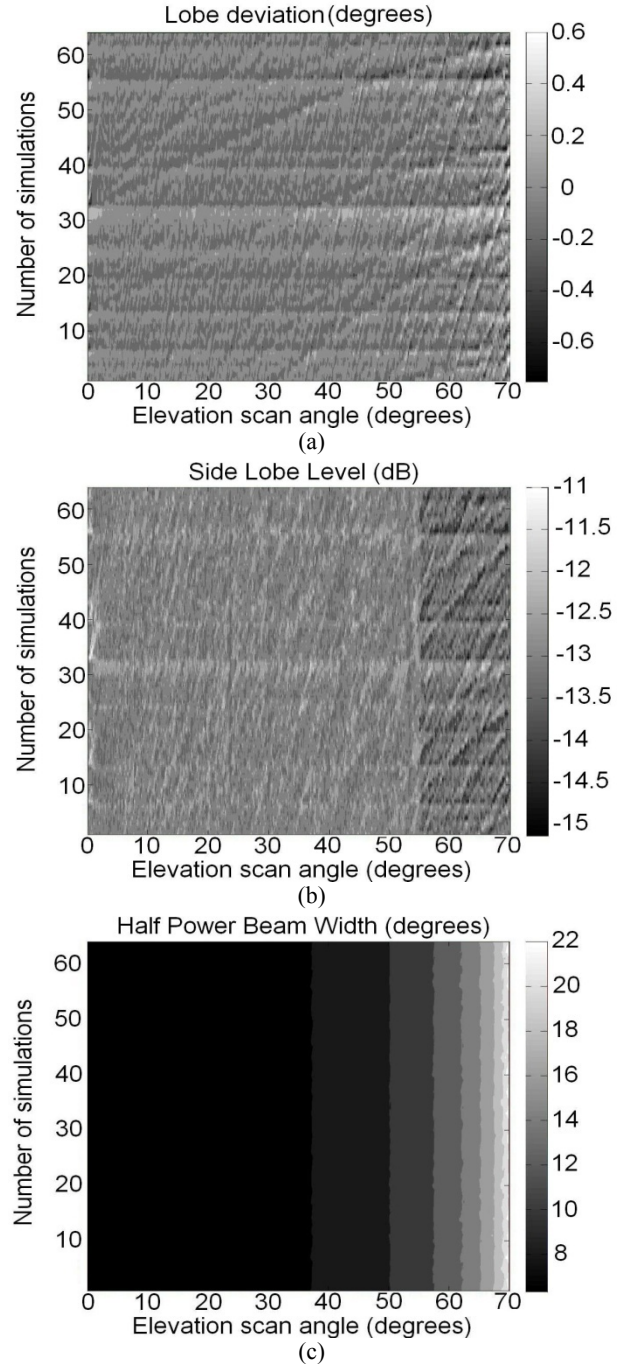
It is evident that the comparison among different simulations is very simple and it is given the opportunity to know also which are the critical angles. For example, it is easy to get the Fig. 4a from Fig. 5a, taking from the latter the value corresponding to the



**Fig. 4** Simulation results obtained changing the phase errors vector. (a) Maximum lobe deviation; (b) Maximum side lobe level; (c) Maximum half power bandwidth.

hottest color per each horizontal line. The same is for Fig. 4b derived from Fig. 5b and for Fig. 4c derived from Fig. 5c.

Those simulations demonstrate that the phase error rms value itself, even if, it is an important figure of merit, it is not enough because the beam shape characteristics



**Fig. 5** Simulation results obtained changing the phase errors vector. (a) Lobe deviation; (b) Side lobe level; (c) Half power bandwidth.

are strictly depending on the actual phase errors vector.

Ideally, if there were not any differences between the simulations, in Figs. 5a-5c we would have had the vertical lines exactly with the same colors; but in those figures there are evident changes between the simulations; in particular the lobe deviation and the

SLL have significant variations for high values of the elevation angles, for the same scan angle. The HPBW represented in Fig. 5c seems to be not changed varying the simulations since the graph is composed by vertical line of almost the same color; really it is not true and it is due to the great span of the HPBW represented which doesn't give the opportunity to appreciate little changes; in this case Fig. 4c shows us that also the HPBW value changes in relation to the real phase errors distribution.

As a consequence of these results, we can say that for a phase shifter a little rms phase error value is not the only key issue but a good phase shifter should have the phase error vector as flat as possible, that means that each phase error should be very similar to another. The ideal condition is a constant phase error over all the possible phase shifts.

#### 4. Conclusions

In this paper it has been demonstrated that the antennas array beam shape depends on the actual phase errors distribution and that a good phase shifter has to have the phase errors with a low rms value as constant as possible. The model presented is an optimum tool to be used in order to investigate how sensible an application is to the phase errors distribution and which parameters of the beam shape are mainly affected.

Moreover, the tool can be used for a yield analysis of sensitivity, simulating different realizations of the same architecture assuming that each phase shifter sample has a fixed rms phase error value but different phase errors vector.

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