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Groundwater Levels in a Drained Beach in Long and Short Waves Conditions

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Abstract: Drainage of beaches is thought to be a soft engineering solution to counteract erosion. The present work aims to show the response of the groundwater table level when it is influenced by a drain inside the sand, in the vicinity of the shoreline. The knowledge of the water table dynamics helps to identify the role played by the drain, and consequently it is useful to address its function on the overall BDS functioning. The water table dynamics has been mathematically investigated, in presence of both long and short waves. An analytical solution for the Boussinesq equation with long waves and a drainage boundary condition within the porous medium is presented. A VOF numerical tool has been used to study the groundwater table fluctuations driven by the short waves swash. A comparison between the analytical solution and the numerical results in terms of groundwater level has been carried out. The numerical model reproduces the drain-induced dynamics inside the beach. The analytical model, on the other hand, although it is developed under the long wave hypothesis, seems to catch, at least qualitatively, the water table evolution in case of short period regular waves. It enables to think of it as a preliminary evaluation tool for the BDS design.

Keywords: groundwater dynamics, mathematical modeling, sandy beach drainage, soft engineering method

1 Introduction

The Beach Drainage System (BDS) is thought as a soft system to stabilize sediments from erosion. The BDS functioning relies on the interaction between coastal groundwater table, swash zone hydrodynamics and foreshore morphodynamics. Basically, it stands on the deployment of a series of buried drains that influence the groundwater level. The aim is to artificially act on the groundwater level, dropping it gravitationally. Due to the drainage, the layer of dry sand makes the sediment grains less prone to be mobilized by the waves. The lowered groundwater table induces the decrease of velocities during uprush and backswash, with respect to the undrained condition, enhancing the sediment deposition during the runup and limiting offshore transport during the rundown. On the other hand, it has been recognized the effect of the vertical fluid drag, which acts downward during the infiltration as an element of stabilization and vice versa during exfiltration. The functioning of this tool depends on several parameters, i.e. physic characteristics of the porous medium, waves characteristics, beach topography, BDS configuration and tidal components. Despite the encouraging starting results, its performances are not yet proven as an engineering solution to coastal erosion management (Turner et al., 1997). In fact, controversial performances of experimental (Damiani et al., 2011; Saponieri et al., 2018) and prototype scale (Bain et al., 2016; Ciavola et al., 2009) experiences have been observed so far. Even if the idea behind the BDS has been pursued for decades (Grant,1946; Chappell et al., 1979), its efficiency is not fully recognized yet. For the sake of simplicity, the whole problem could be split into two different aspects: the hydrodynamical correlation between the groundwater table and the incident waves, and the beach morphodynamics influenced by the presence of the drain.

This paper aims to investigate the response of the groundwater table in sandy beaches to different waves drivers by means of mathematical modeling. The knowledge of the interaction between drain and the different components of the coastal cross-shore forcing, will allow to better understand the BDS functioning needed to its design, and select its optimum configuration with respect to the forcing waves. Morphodynamics aspects are not considered herein. The main purpose is to investigate how the groundwater levels evolve in presence of a drain forced by both long and short waves. On one hand, numerical modeling has high computational costs, then it cannot be easily used to model the long duration of a tidal cycle. On the other hand, the analytical solution is not able to catch the transient short-waves propagation within the porous medium. Therefore, the phenomena that occur at the shoreline are characterized by different time scales from seconds to hours. Water table reacts showing in every case a decreasing perturbation as long as waves enter the beach. Considering an ideal vertical beach face, herein, the drain is modeled as a boundary condition in a finite length domain, without considering the design characteristics of the drainage pipe (i.e., discharge flow). The short-term groundwater level dynamics is further investigated by means of numerical approach. Indeed, the interaction between short waves during swash cycles and water table dynamics cannot be framed within the simplified hypotheses of tidal formulation. Several works deal with the water table elevation in the presence of a drain computed both for tides and swash, leaving out capillarity effects. They vary among the approaches and equations used. For instance, Saponieri and Damiani (2015) used an unsaturated model in static conditions based on the Richard's equation, Karambas and Ioannidis (2013) coupled a waves Boussinesq model with porous flow model for investigating the erosion at the beach. Herein it has been used a model capable to simulating water flow through the porous medium in presence of waves. The model, according to the author's knowledge, has never been used before to compute groundwater table in sandy beaches.

2 Analytical approach

The classical formulation for describing the unsteady motion of the water table inside an isotropic, homogeneous porous medium is the Boussinesq equation (Boussinesq, 1904). It comes from combining the Darcy's law with the continuity equation, with the Dupuit's assumption (Kovács, 2014). It means that every vertical flow is considered negligible: the vertical pressure is hydrostatic (Kirkham, 1958). The groundwater table dynamics inside a beach forced by a tidal oscillation can be then based on such assumptions (Nielsen, 1990). The simplest domain that can be considered is a vertical one, with an infinite length. At one boundary, the condition that simulates the tidal forcing is imposed by means of a harmonic oscillation of the water table: it stands for a linear wave with given amplitude, period and phase. At infinity the condition aims at limiting the value for the water head to the mean level, because every oscillation have damped. Fig. 1 depicts the sketch used to define such a problem.

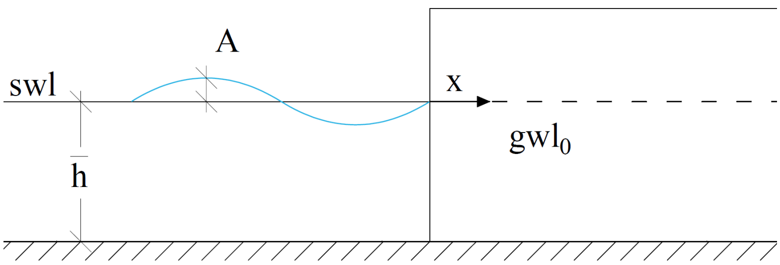

Fig. 1. The domain considered for the formulation of the problem ($\bar{h}$ is the aquifer depth, A is the wave amplitude, gwl_0 is the initial groundwater level. The origin of the axes is posed at the in correspondence of the intersection between the vertical side of the beach and the SWL).

The governing equation and boundary conditions read as follows:

$$\frac{\partial h}{\partial t} = \frac{K}{n}\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) \qquad x > 0, \qquad t > 0 \tag{1}$$

$$h(0,t) = \bar{h} + A sin(\omega t + \phi) \qquad x = 0, \qquad t > 0 \qquad (2)$$

$$\lim_{x \to \infty} h(x,t) , \qquad t > 0 \qquad (3)$$

where A is the wave amplitude, ω is the wave angular frequency, $\bar{h}$ is the aquifer depth corresponding to the static water level (SWL), ϕ is the phase shift, K is the permeability coefficient and n is the porosity.

In order to solve the equation, linearization has to be carried out, by considering the water table oscillation as negligible compared to the average aquifer depth (i.e. still water level), i.e. the tidal amplitude is small compared to $\bar{h}$. The solution read as follow (Kovács,1981):

$$h(x,t) = \bar{h} + Ae^{-\alpha x} sin(\omega t - \alpha x + \phi) \qquad (4)$$

$$\alpha = \sqrt{\frac{\omega n}{2K\bar{h}}}. \qquad (5)$$

The parameter α depends on the characteristics of the aquifer (i.e. depth $\bar{h}$, permeability K, porosity n) and on tidal forcing ω. The Eq. (4) indicates that as a wave enters the beach, an oscillation of the water table occurs as a damped wave: as the distance from the beach interface increases, its amplitude decreases, and it still maintains its wavy behavior. The dispersion relation is (Nielsen et al., 1997):

$$k^2\bar{h}^2 = i\frac{\omega n\bar{h}}{K}. \qquad (6)$$

where i is the imaginary unit.

In order to estimate the range of validity of the solution given by Eq. (4) in terms of distance from the boundary, the spatial variation of the decay rate in x-direction has been evaluated. As the distance from the interface increases, the effects of waves on the hydraulic head within the sand decreases. In such a condition, the groundwater level corresponds to the undisturbed condition, since the decay amplitude constitutes a small percentage of the amplitude at the boundary. Let the decay function q(x) be the exponential part of Eq. (4):

$$q(x) = Ae^{-\alpha x} \qquad (7)$$

The value x^* where the oscillation is small enough to consider no effects on the water table may be evaluated as:

$$x^*(\alpha, \varepsilon) = -\frac{ln\varepsilon}{\alpha} \qquad (8)$$

where ε is the (small enough) fraction with respect to the amplitude A. For different values of ε, the relation in Eq. (8) is represented in the Fig. 2. The quantities x^* and α are made dimensionless with regard, respectively, to the wave length $\sqrt{g\bar{h}}T$ and the amplitude A.

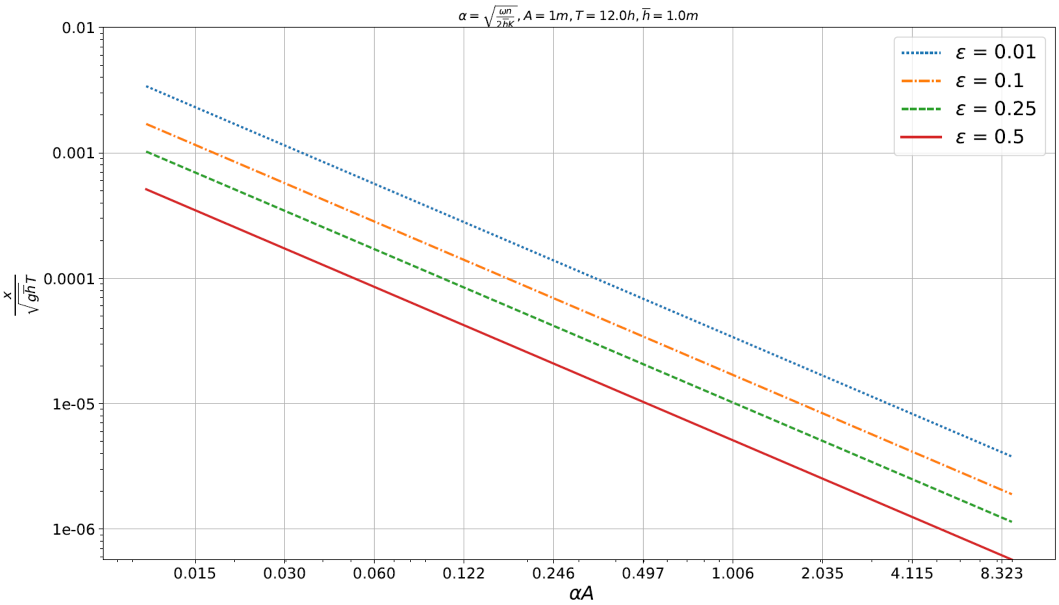

Fig. 2. Dimensionless graph of the relation $\alpha - x^*(\alpha, \varepsilon)$, changing ε properly. The plot has a bi-logarithmic scale.

The abacus in Fig. 2 is helpful to infer, according to the known parameters, the greatest distance from the shoreline inside the beach where installing a BDS could have any sense, since the wave maintains an amplitude that is greater that εA. As consequence of the forcing and aquifer characteristics, summarized by the parameter α, the present graph let read, given the amplitude of the wave what one is interested in, the distance where the water table oscillation starts to become negligible (i.e εA), namely not affected by waves motion. Nielsen et al. (1997) proposed another formulation for the groundwater oscillation due to waves, with the hypothesis of deep aquifer relaxed. It allows to take account for not properly long waves. Although the groundwater dynamics has been described by higher order theories, the intent here is to analytically reproduce the effect of a drain, nonetheless the simplified hypothesis and theory considered.

The analytical formulation of the problem makes use of the classical Boussinesq equation, with simplified boundary conditions. The drain is not considered itself: it is pretended to be modeled accounting for its effects on the water table only. It does not allow to investigate the dynamics of the drain (i.e. the drained discharge) but only how the drain affects the water table. The domain has finite length (L) equal to the distance of the drain from the shoreline. The formulation, proposed herein, lets the equation be solved analytically if the boundaries are studied as vertical, i.e. it does not account for the second order terms given by considering the slope beach (Nielsen, 1990). At one boundary the wave forcing is imposed as a sinusoidal function (Eq. 2), whereas at the rear boundary a given level is imposed. The constant head at the boundary is expressed as a depth with respect to the still water table level. The initial condition is imposed as a parabolic function, according to the Dupuit's formula for unconfined aquifer (Bear, 1972), that respects the boundary conditions. The domain configuration is depicted in Fig. 3 and the problem could be summarized as follow:

$$\frac{\partial \zeta}{\partial t} = \beta \frac{\partial^2 \zeta}{\partial x^2} \qquad 0 < x < L, \qquad t > 0 \tag{9}$$

$$\zeta(0,t) = A\sin(\omega t + \phi) \qquad x = 0, \qquad t > 0 \tag{10}$$

$$\zeta(L,t) = -p = d + A\sin\phi \qquad x = L, \qquad t > 0 \tag{11}$$

$$\zeta(x,0) = -d\frac{x^2}{L^2} + A\sin\phi \qquad 0 < x < L, \qquad t = 0 \tag{12}$$

where:

$$\beta = \frac{\bar{h}\,K}{n}$$

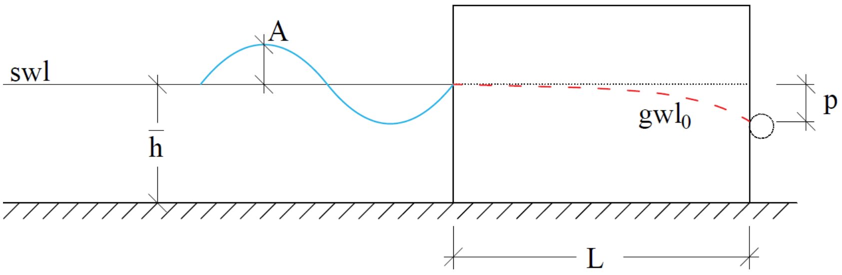

Fig. 3. Sketch of the domain, where the effects of the drain are taken into account. $\bar{h}$ is the aquifer depth, A is the wave amplitude, p is the drain depth below the undisturbed groundwater level, L is the domain length, gwl_0 is the initial condition for the groundwater level. In the figure the phase shift is assumed zero.

The Eq. (9) - Eq. (12) refer to the elevation ζ above the still water level (swl), rather than the absolute position of the groundwater table h. The solution is:

$$\zeta(x,t) = A\sin(\omega t + \phi) + \frac{x}{L}[-d - A\sin(\omega t + \phi)] + g(x,t) + f(x,t) \tag{13}$$

with:

$$g(x,t) = \sum_{n=1}^{\infty} \left\{ \frac{2[\,d(2 - 2\cos n\pi)]}{(n\pi)^3} + \frac{2A\beta\omega(\lambda_n^2 \cos\phi + \omega \sin\phi)}{n\pi(\omega^2 + \lambda_n^4)} \right\} e^{-\lambda_n^2 t} \sin\left(\frac{n\pi}{L}x\right) \tag{14}$$

$$f(x,t) = \sum_{n=1}^{\infty} -\frac{2A\beta\omega\lambda_n^2}{n\pi} \frac{[\omega \sin(\omega t + \phi) + \lambda_n^2 \cos(\omega t + \phi)]}{\omega^2 + \lambda_n^4} \sin\left(\frac{n\pi}{L}x\right) \tag{15}$$

where:

$$\lambda_n^2 = \beta\left(\frac{n\pi}{L}\right)^2 \tag{16}$$

is the n^{th} eigenvalue of the problem.

The solution could be compared with the one of Van de Giesen's et al. (1994), which takes into account the Laplace solution in the case of a forced unsteady drained problem. With no loosing of generality, for the discussion of the solution, the phase ϕ is posed as zero. The Eq. (13) maintains the oscillatory behavior in space and time, since the external forcing is still a sine function, but with respect to the solution in Eq. (4), this one presents a decay in time and also a superposition of modes that assures the decay in space. The transient phase is represented by the exponential time-dependent component, its effects concerns only the first instants of the phenomenon. The last term *f(x,t)* is instead the one given by the nonhomogeneous solution, dependent on the forcing period. Eq. (13) is a finite function series, unconverging at infinite. However, the solution of Eq. (13) can be evaluated by considering a limited number of modes in Eq. (14) and Eq. (15). For given time, the firsts two terms of Eq. (13) connect the boundaries linearly. The groundwater level oscillation given by these two terms varies in according to the forcing period. The last two terms depend on several factors, i.e. the parameters that characterize the aquifer β and hence the eigenvalues λ_n. The function *g(x,t)* is weakly dependent on the period of the forcing wave; it strongly depends, apart from the modes, on the length of the domain L, on time t and β. The overall descending behavior is led by the parameter λ_n: it can be stated that, since it depends inversely on L, the more the domain is wide the more the delay in time is needed to flatten to zero the function *g(x,t)*, once the parameter β is selected. In Fig. 4 the behavior of *g(x,t)* and *f(x,t)* is plotted, considering constant all the other parameters typical for sandy beaches ($\bar{h}$ = 3 m, T = 12 h, 5s k = 1.65^{-4} $^{m}/_{s}$, A=0.5 m, p=0.75 m) and two cases of domain length L = 1 m, 10 m. Every panel refers to different time scales, in according to the considered length and period. While the contribution of function *g(x,t)* drops in time, function *f(x,t)* still maintains an oscillating behavior, that depends on the wave period. Despite the introduced hypothesis of tidal waves, a comparison of the functions *g(x,t)* and *f(x,t)* between the case of short waves and long ones has been carried out. The Fig.4 shows the functions *f(x,t)* and *g(x,t)* for the different cases analyzed.

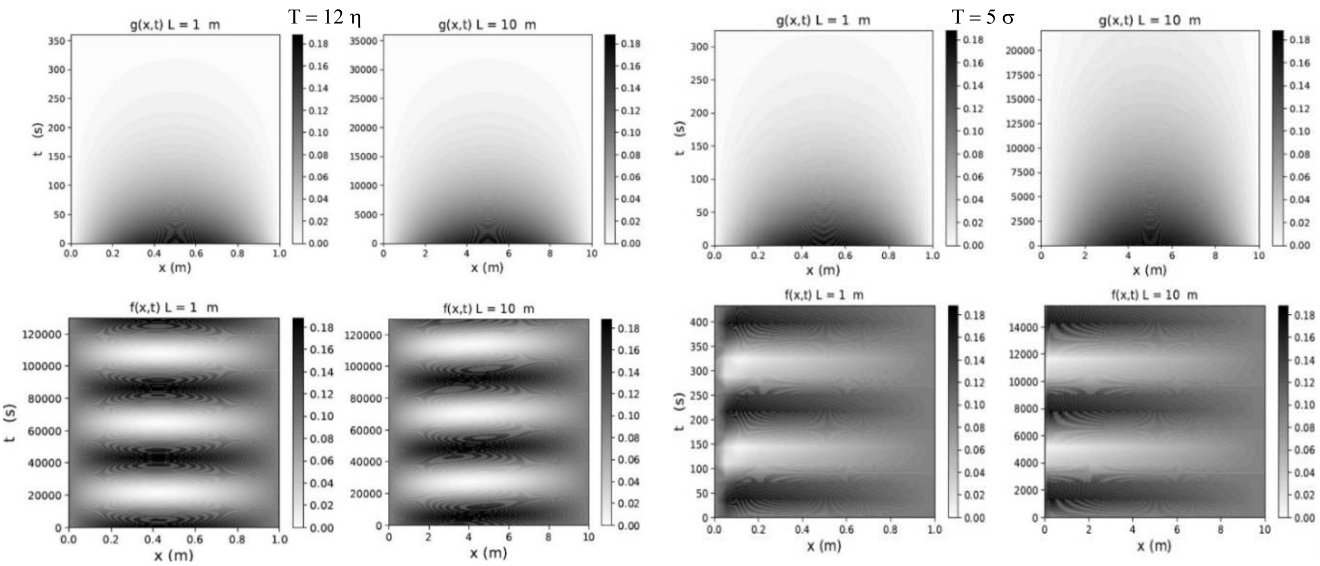

Fig. 4. The function *g(x,t)* (upper panels) and *f(x,t)* (lower panels) for a wave period of 12 h (left panels) and 5 s (right panels). *g(x,t)* is represented on the first rows of panels for two domain length. The different cases considered have a different time scale, in according to the time scale of phenomenon depending on the length and period considered.

As far the function *f(x,t)* concerns, the time scale has been chosen to capture the complete period of the functions. For short waves the time scale goes from minutes to hours for small domain (L = 1 m), whereas for long waves *f(x,t)* is not affected by the domain length and in both cases its period oscillation is of the order of hours. Reconstructing the solution, it could be worthy of interest to understand how the water table evolves according to this model. Having fixed an instant, the more it is large, the more the groundwater level in space tends to the straight line and the homogenous solution prevails. As already mentioned regard to *g(x,t)*, the smaller the domain is, the faster this behavior is manifested. For small time instants all the contributions count, and the solution depends on the period T as well as the length L.

3 Numerical approach

The interaction between short waves swash and water table cannot be framed inside the simplified hypothesis of tidal formulation, since it finds its theoretically foundation on the assumption of shallow aquifer and on the disregarding of vertical flow (Nielsen, 1997). With the same simplified domain of the analytical case, the problem has faced also by means of numerical modeling. It is worth reminding that only the hydrodynamics inside the sand is wanted to be understood. In this paper the numerical investigation of the hydrodynamic behavior inside the sand is carried out by means of a VOF numerical model, capable of computing flow through porous medium with active wave generation and absorption, within the OpenFoam frame (Higuera et al., 2013). The model uses VARANS and Darcy's law, i.e. the motion inside the sand is supposed to be laminar. An ideal vertical sandy beach (as the one considered in the analytical model) is modelled and forced by regular linear waves. Both the cases with and without the drain are investigated and compared. The 2D model replicates a wave tank 90 m long and 5 m high with a water depth h = 3 m. The beach, whose beach face is placed at 60 m from the tank boundary, is 4.6 m high. The generated regular waves are characterized by a period T = 5 s and a height H = 1m. Firstly, simulation with no drain was run. Then, the drain is modeled inside the porous domain. It is placed inside the beach at 2 m landward from the beach face and 0.75 m under the still water level, fixed at 3 m. The drain was modeled by means of a circular hole with atmospheric pressure imposed at its upper surface and by a wall boundary condition at its lower part in order to replicate the real case of a buried pipe inside the beach, whose upper part drains only. The pressure head gradient between the sand and the atmospheric pressure surface makes the water flow away the domain. The numerical validation for the sand grains size is out of the scope of this paper. Another noteworthy aspect is that no attention was paid to the dimension of the influence of the drain: a given diameter equal to 300 mm has been employed. In Fig. 5, a snapshot of the simulation is represented. A standard spectral analysis of the signal was carried out. The domain configuration in presence of the drain is depicted in Fig. 6. It shows also the position of the gauges where analyses were performed, both for the undrained and drained cases.

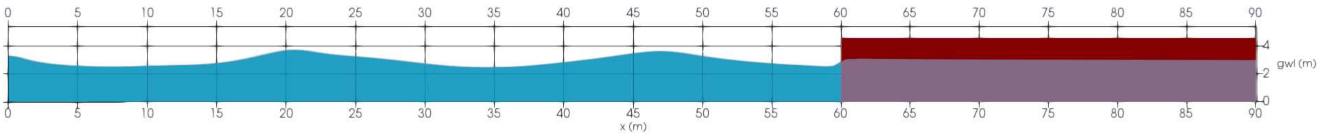

Fig. 5. The numerical domain in the undrained condition. The pink shaded area represents the sand phase, the blue one is the water and the red is the aquifer. The domain is 90 m of length, 5 m of high.

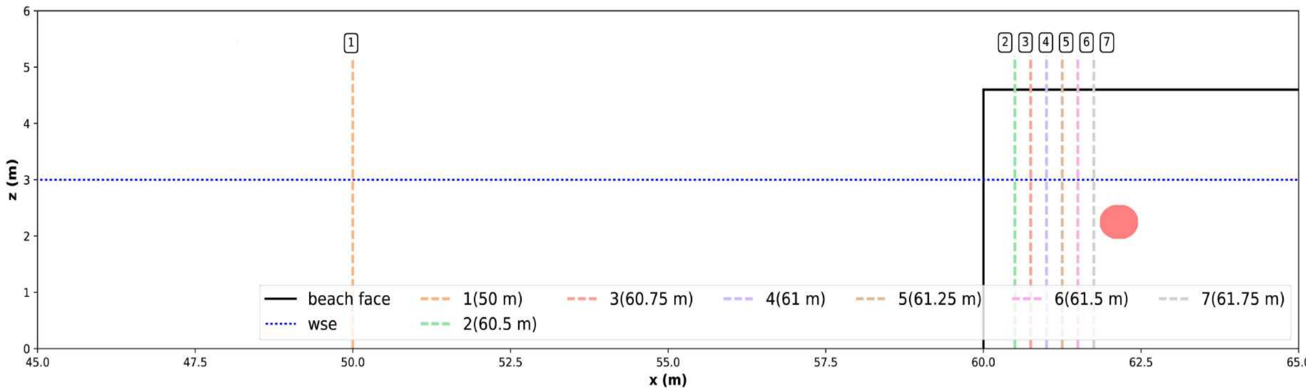

Fig. 6. Position of gauges inside the domain in the undrained case, where the spectral analysis is carried out. The red dot displays the position of the drain.

The results of the spectral analysis of the undrained case (Fig. 7) reveal that the waves propagate changing its amplitude, as expected from the analytical solution. Then, the numerical model does not seem to catch the period shift observed experimentally by Hegge and Masselink (1991).

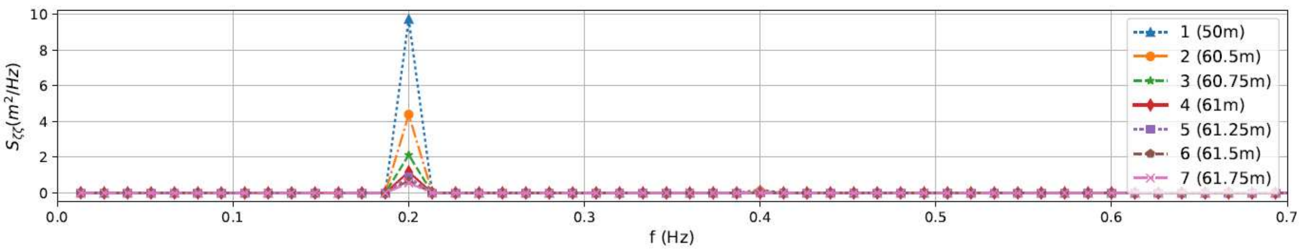

Fig. 7. Spectral energy density of the signal registered at various location, specified at the legend of the figure. The distances refer to the absolute distance from the origin of the tank, where the waves are generated. See Fig. 5 for the domain configuration. $S_{\zeta\zeta}$is the spectral energy density.

First, the analyses on the undrained case are performed (Fig. 8). This is due to the willing to understand how the different models work in computing the water table, according to their own basic hypothesis, no matter the characteristic and the position of the drain. In Fig. 8, the instantaneous (t=240 s) groundwater levels (upper panel) and the envelops (lower panel) are plotted for the numerical simulation, the equation Eq. (3) and the proposed by Nielsen et al. (1997), valid in intermediate waters. The particular instant was selected, being elapsed the transitory phase. The deviation between the numerical and the intermediate depth is acceptable for a distance from the beach face of about 0.5 m. Landward the difference increases. The numerical model shows a set up that has also been proven experimentally by Kang et al. (1994) for sandy beaches and Jacobsen et al. (2015) for porous structures. As expected, the solution for the long waves hypothesis shows no adequacy to represent short waves.

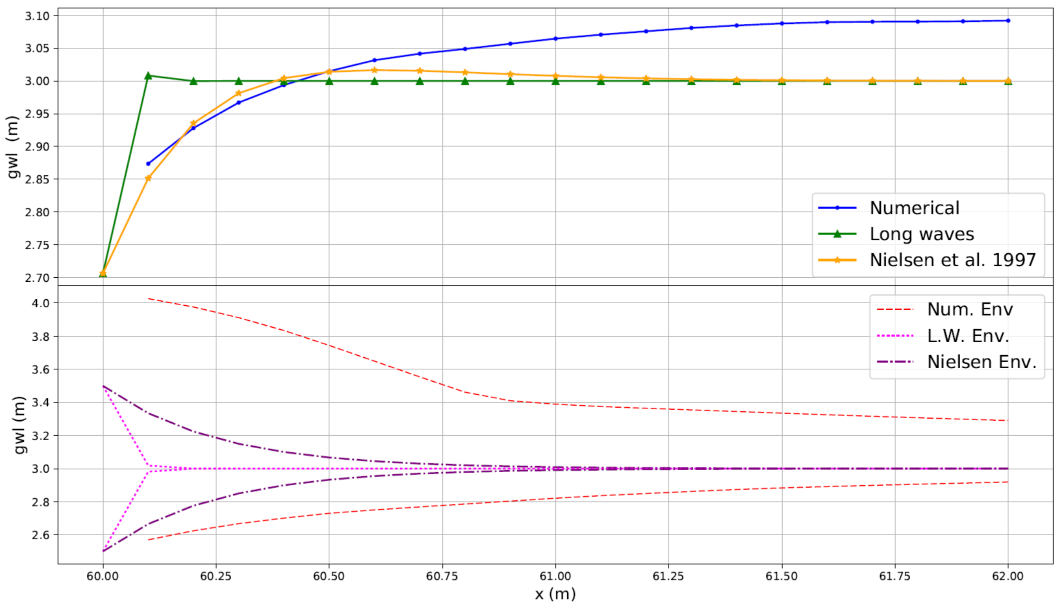

Fig. 8. Groundwater levels and their envelope for three different model in the undrained case: numerical and two analytical models. The legend labels stand for “Numerical Envelop”, “Long Waves Envelop” and “Nielsen et al., 1997 solution Envelop” respectively.

Fig. 9 shows the transmission coefficient in drained conditions. The coefficient is calculated as the ratio between the spectral amplitude inside the beach at different positions, and the one calculated offshore (gauge 1). The outcomes from Fig. 9 summarize the fact that the waves are damped gradually landward, with no lag in phase; similarly to the abacus in Fig. 2, Fig. 9 can help reading what distance a certain fraction of the initial amplitude is performed at.

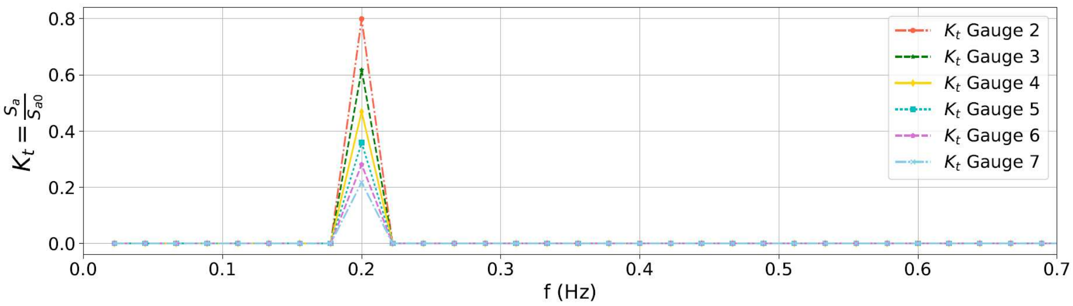

Fig. 9. Transmission coefficient K_t. S_a is the spectral amplitude of the signal corresponding to the gauges from 2 to 7 and S_{a0} is the spectral amplitude referred to the offshore gauge (gauge 1).

4 Discussion

From the gained results, it can be stated that the employed CFD tool can reproduce groundwater hydrodynamic forced by regular waves, at least from a qualitative point of view. Even though the advantages of a numerical tool, the analytical one is far more powerful to obtain general results, useful to define some design guidelines. For this reason, the analytical solution in Eq. (13), was forced to fit with waves whose period is T = 5 s, without modified the other parameters. It has to be stressed that, in order to correctly compare the two models, the condition at the interface between the water and sand phase has considered as equal to the analytical boundary condition. The results are shown in Fig. 10. In the upper panel the time series comparison between the two solutions is shown. In the bottom panel the envelope of the groundwater evolutions is represented regard to the drain distance from the beach face.

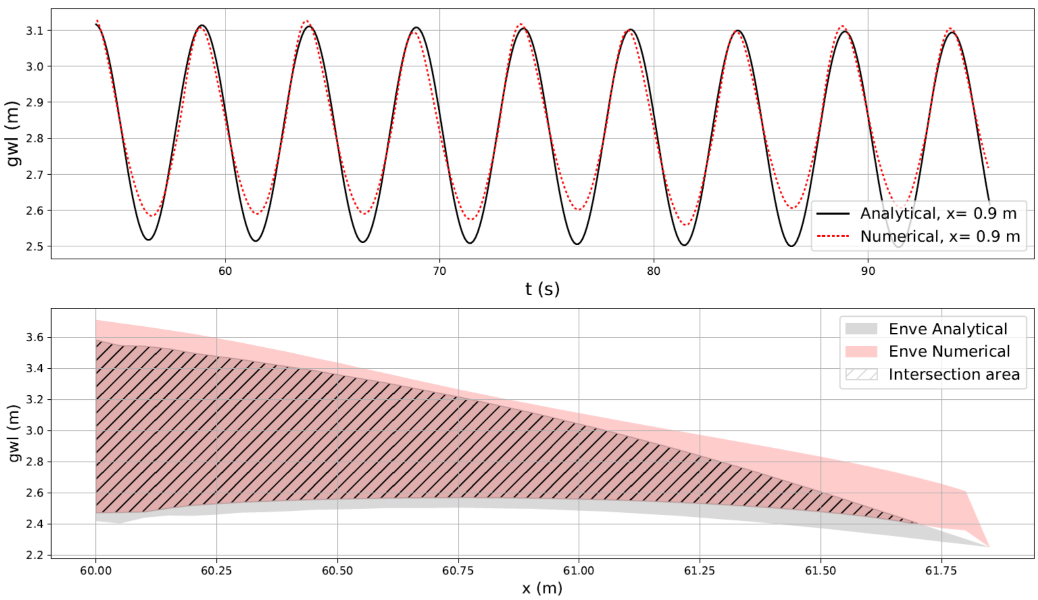

Fig. 10. Numerical and analytical results for short waves with the drain placed 2 m landward. The upper panel displays the groundwater level (gwl) oscillation at a given point (0.9 m landward from the beach face). The bottom panel shows the gwl envelops from the two models. The hatched area corresponds to the common envelop area between the twos.

5 Concluding remarks and ongoing research

An analytical solution for the linearized Boussinesq equation in presence of a drained boundary condition is presented. The solution is valid under long wave hypothesis (i.e. shallow aquifer). A numerical model was used to investigate the response of a sandy beach forced by short waves. The power of the analytical tool justifies the attempt to compare the analytical solution to the numerical results, even if out of its validity range. Then, the analytical solution was forced by means of waves of the same period of the numerical model and the main results are presented. The analytical model can catch the water table evolution inside the domain. The numerical shows a slight lower water level for troughs than the analytical, whereas neither of them shows a period shift. The numerical model accounts also for the waves propagation from the paddle to the sand phase, whereas the analytical considers the sand domain only: even if the instantaneous position of the groundwater table is not coincident due to the shift in time between the two models, their envelops coincide rather well. Although the analytical model is developed for an ideal vertical sandy beach, with no physical meaning, its solution fit rather well, at least qualitatively, with the numerical. It could be used as a preliminary tool to select the correct location of the drain at least in the preliminary stage of the design.

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