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ScienceDirect

Transportation Research Procedia 37 (2019) 59–66

Transportation Research Procedia

www.elsevier.com/locate/procedia

21st EURO Working Group on Transportation Meeting, EWGT 2018, 17th – 19th September 2018, Braunschweig, Germany

An urban bikeway network design model for inclusive and equitable transport policies

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Abstract

This study suggests an optimization framework to plan and design a network of bike lanes in an urban context, based on equity principles and subject to a given available budget. The novelty of the proposal consists in an objective function that aims at minimizing the existing inequities among different population groups in terms of accessibility/opportunity to the bikeways. The proposed methodology represents a reliable decision support system tool that could help transport authorities/managers to select the priority areas of their future investments related to the cycling infrastructures. To prove the effectiveness and value of the methodology, an application with relevant analysis to a test case study is presented.

Keywords: Bikeways planning; equity and social inclusion; accessibility; network design; Theil coefficient.

1. Introduction

In recent decades, the challenges associated with sustainability are becoming crucial in transportation planning and consequently in several research fields. Overall, bicycles represent a valuable contribution to the achievement of an efficient and sustainable transport system. At the same time, cyclists belong to one of the most vulnerable groups of road users (Dondi et al., 2011). This is the reason why, in this framework, planning and design cycling facilities and infrastructures is a primary requirement to foster sustainable mobility.

The goal of this paper is to propose a bike lane network design model (i.e. a model that allows identifying the optimal layout of bikeways in the network) to support the planning of cycling networks in built-up urban areas. The

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10.1016/j.trpro.2018.12.166

methodology takes into account the existing inequalities among users/cyclists from both spatial and social standpoints: spatial, due to the city district in which they reside; and social, due to the categories to whom they belong according to their gender, age, ethnicity, income, etc. Particularly, we focus on inequalities in the accessibility to the bike lane network, considering as optimal a configuration in which the main destinations throughout the city can be reached, from a given origin, by cycling on a continuous system of bikeways. Indeed, bike lanes have proven to be an effective way to improve cycling safety and comfort (Pucher and Buehler, 2008), globally contributing to the promotion of bicycles as everyday sustainable mode of mobility in urban areas.

The largest amount of previous research has mainly referred to the network design problem of roadways and transit systems, often overlooking the non-motorized travel modes. In outline, the general methodologies to approach these problems are similar, although the set of related constraints may change according to the context under analysis. However, if road network design problems are mostly related to the manipulation of the existing network (new city construction is not that frequent in the real world), public transport and bicycle-related network design studies mostly deals with new configurations, or complete reconfigurations of the existing networks (Elshafei, 2006; Farahani et al., 2013). Recent studies seem to be paying an increasing attention in finding the best approach to design an optimal network of bike lanes (Caggiani et al., 2018a). Among the others, we want to mention the bi-level optimization suggested by Mesbah, Thompson, and Moridpour (2012), who acknowledged the compromise that has to be reached between private car users and bike users when planning the links of the network on which a bike lane may be introduced. Worthy to cite is also the research of Lin and Yu (2013), whose model considers a complete set of constraints, such as bikeway type, monetary budgets, and path continuities, together with the value ranges of decision variables. Sometimes, the bikeway network design problem has been also seen as part of the strategic design of a public bike-sharing system: this category of models, for instance, usually aims at determining number and location of bike-sharing stations together with the structure of the bike paths that connect the stations to each other (Lin and Yang, 2011).

However, the equity concern does not appear in the panorama of bikeway planning until a few years ago, and mainly in indirect ways (i.e. developing low-stress networks able to provide an adequate mobility of each user type, look at Scrivener, 2015), or in form of general recommendations, highlighting the importance to address the equity issue as practical and ethical requirement of the bike planning process (Jackson, 2017). An interesting example of equity evaluation (ex-post) of a cycling network can be found in Wang and Lindsey (2017), that used two measures (the Gini coefficient and the loss of accessibility to jobs via bikeways) aiming at assessing both the spatial and social equity related to the bikeway distribution in Minneapolis, Minnesota.

In the above-described bike planning framework, our research contribution seeks to fill the gap that apparently exists in the literature, namely proposing a way to plan and design progressively (over the time, according to the available budget) a bike lane structured and connected system, taking directly into account equity principles *since the preliminary (ex-ante) scenarios and evaluations*. In particular, to the best of our knowledge, the novelty of our approach consists in expressing an objective function for this bikeway network design problem that contains explicitly an equity expression. In fact, usually, in other network design contexts, when the equity issue has been considered, it has been always specified as a constraint to the optimization problem.

The remaining part of the paper has been organized as follows: at first, we present the concepts of accessibility and equity previously discussed in the literature. Then, we introduce the proposed urban bikeway network design model, clarifying the meaning that we attribute to accessibility and equity in this framework. At last, a numerical application to a test case study proves the advantages that can be related to the adoption of such a model in a real context. Final remarks and future research directions conclude the paper.

2. Accessibility and equity concepts in literature

The accessibility concept plays a fundamental role in the transport planning scene. Transport accessibility can be defined as the extent to which land-use and transport systems enable individuals (or groups of people) to reach activities/opportunities in the network (workplaces, shops, public transport stations and stops, health facilities, etc.) using a (combination of) transport mode(s) (Geurs and Van Wee, 2004). In this study, an accessibility measure has been used to quantify the degree to which desired locations can be reached by cycling on a connected bikeway network.

On the other hand, the equity concept, in this context, refers to a fair distribution of costs and benefits among the members of a society (Litman, 2017). If an example of transportation cost is represented by the travel time/distance necessary to reach a certain destination, benefits can range from general enhancements in mobility and accessibility to reduced costs for the network users (Bills and Walker, 2017; Caggiani et al., 2018b).

Planners need to consider, when discussing a specific cycling related investment, whether it is equitably distributed, properly understanding and addressing its impacts on the population. Here we focus on a combination of the two concepts of equity and accessibility, using a bike lane accessibility measure to understand how equitably prioritize feasible bikeways projects that would improve the global quality of the service.

2.1. How to use Theil index to assess equity in the accessibility

Among the existing equity indicators, we want here to theoretically describe the Theil coefficient (Theil, 1967), since it is part of the bikeway network design model that we are going to present in the next section. It derives from the concept of information theory and originally aimed at quantifying the level of disorder within a distribution of income; however, it has been as well applied in economics to assess transportation equity. Supposing that y is the variable whose equity in distribution among population needs to be assessed, the following Eq. (1) can be considered as the general Theil index T formulation:

$$T = \frac{1}{P_T} \sum_{j=1}^{P_T} \frac{y_j}{\bar{y}} \cdot \ln\left(\frac{y_j}{\bar{y}}\right) \tag{1}$$

where P_T is the total population; y_j is the value of the variable associated with each individual j; $\bar{y}$ is the average per capita value of the variable in the study area. Theil's measure falls between 0 in the case of perfect equality (high association between measures) and ln(j) for perfect inequality.

Theil coefficient is probably slightly difficult to interpret if compared to other measures; however, it is considered the most sensitive in measuring changes at the end of a resources distribution, and effective when stating the equity differences existing between subgroups of population. This is mainly due to its the feature of being perfectly decomposed into its components (*within* and *between* any arbitrarily defined population subgroup), without residual terms: this characteristic makes possible to study and discuss not only the total level of equity reached on the entire cycling network but also its inner perspectives, looking at what is happening to each spatial and/or social group of the involved bike users. In particular, the Theil *between* component T_B can be expressed as follows (Eq. 2):

$$T_B = \sum_{i=1}^{m} \left(\frac{p_i}{P_T}\frac{y_i}{\bar{y}}\right) \cdot \ln\left(\frac{y_i}{\bar{y}}\right) \tag{2}$$

Looking at Eq. (2), m is the number of groups i of population (they can be constituted by groups of individuals living in different geographical districts, or having different socio-economic features); p_i corresponds to the number of people belonging to the group i; while y_i is the average per capita value of the variable associated to the group i. The contribution that is given by each group i (that is, each term in the sum) can be either positive or negative. In the former case, the status of group i contributes to increasing inequality, in the latter instead it improves equity. Note that although the global Theil T (*between* + *within* components) remains always positive overall (as the positive contributions are always higher than the negative ones), the Theil *between* itself (T_B) can essentially be either a positive or a negative number.

It is important to comprehend the meaning of the above-mentioned Theil components, to better understand the methodology described in the next section. Let us assume to have m = 2 groups i to study, for instance 'old' and 'young' people. If the Theil *within* component assesses the inequities inside each group (i.e., the inequities within the individuals belonging to the 'young' group, and the inequities within those that are part of the 'old' group); the Theil *between* component is able to capture the differences in equity between 'young' and 'old' people. For the purposes of our research, the Theil between component is the most meaningful, as allows to understand which is the group that bears more costs/inequities compared to the others, and to what extent its share of resources is unfair.

3. Proposed urban bikeway network design model

When dealing with projects of bikeway implementation on the territory, possible limited availability of resources may force planners to realize only part of the planned cycling interventions/constructions on the network. This may lead to the realization of disconnected paths, not able to guarantee an effective improvement of the users' experience, and moreover, it can generate inequities in the accessibility to the cycling infrastructure (and to opportunities) among various groups of the population. The main idea of this study is to support transport planners/authorities that have to face these strategical decisions, suggesting them a viable way to progressively realize a continuous bikeways project on the territory, assuring that the bicycle accessibility of disadvantaged districts/population groups could be adequately taken into account during the steps of the effective implementation of the project on the network.

In the following, symbols and notations used in the paper are presented.

A_{Bw}	bikeway accessibility
D	set of origin/destination centroid nodes/districts
o	origin of the cycling trips/zone centroid in the network, $o = \{1, 2, \ldots, k\}$, $o \in D$
d	destination of the cycling trips/zone centroid in the network, $d = \{1, 2, \ldots, k\}$, $d \in D$
R	maximum acceptable distance between o and d [meters]
M	set of valid alternative paths that connect o to d
μ	generic valid alternative path that connects o to d, $\mu = \{1, 2, \ldots, \rho\}$, $\mu \in M$
$\overline{od}_\mu$	length of the generic path μ in the network that connects o to d [meters]
$bw\,(\overline{od}_\mu)$	length of bikeways alongside the path $\overline{od}_\mu$ [meters]
T_B	Theil between index
P_T	total population in the study area
i	group of individuals, $i = \{1, 2, \ldots, m\}$
p_i	number of individuals belonging to the group i
H	set of connected bikeway paths h, with $h = \{1, 2, \ldots, s\}$
$c(h)$	costs associated with the implementation of the bikeway paths h
B	total available budget (equivalent to the total meters of bikeway network that can be realized) [meters]
B_{min}	minimum budget to be used, equal to $\alpha \cdot B$ [meters], with $\alpha \leq 1$ and $\alpha \in \Re^+$

3.1. Definition of bikeway accessibility

Our methodology aims at assessing the level of equity in the accessibility to the bikeway network across a population, using the *between* component of the Theil coefficient (Theil, 1967). The first step consists in specifying the meaning of the word 'accessibility' in this context.

Considering a certain study area, we propose to divide it into k zones/districts, identifying for each of them a centroid – i.e. origin o and destination d of all the cycling trips of the people living in that area. From every centroid, it could be possible to reach the others by bike if they fall within an acceptable (by bicycle) distance radius R. We suppose that, for each acceptable pair of origin-destination (o,d), the users could travel following the shortest path that connects them, or one of the possible alternative k-paths calculated using the Yen's algorithm (Yen, 1971). We assume that an alternative k-path could be preferred by the cyclist (and marked as 'valid') if it is not longer than the 125% of the corresponding shortest path between the same (o,d) pair. It results that we assume possible to cycle between each (o,d) pair following more than one alternative path μ (the shortest one plus the valid k-Yen ones). Therefore, we define the bikeway accessibility $A_{Bw,\, o\rightarrow d}$ for an (o,d) pair as follows (Eq. 3):

$$A_{Bw,o\rightarrow d} = \max\left(\frac{bw(\overline{od}_\mu)\cdot 100}{\overline{od}_\mu}\right) \qquad \mu = \{1, 2, \ldots, \rho\}, \mu \in M \tag{3}$$

This means that, for each alternative path μ, we calculate the percentage of it that has bike lanes alongside the road. Then, we select the path with the maximum coverage, and we assume that this coverage corresponds to the bikeway accessibility between that (o,d) pair.

Going one step further, a global indicator of bikeway accessibility $A_{Bw,o}$ can be calculated for each origin zone o (Eq. 4):

$$A_{Bw,o} = \sum_{d=1}^{k} \frac{A_{Bw,o\to d}}{\overline{od}} \tag{4}$$

We assume $A_{Bw,o}$ to be our indicator of the accessibility to the bikeway infrastructure for the people residing in the zone o.

3.2. Formulation of an equitable bikeway network design model

The proposed bikeway network design model aims at the minimization of the inequities *between* groups of individuals, residing in a certain study area. For sake of clarity, we assume to deal with two main categories/population groups, that we call *advantaged* and *disadvantaged*. This 'disadvantage', for instance, may correspond to a specific income class of people or can be understood according to age, ethnicity, and so on.

The goal of the proposed model is to minimize the difference in accessibility to the bikeway infrastructure between these two population groups. The novelty is that the objective function (Eq. 5) explicitly considers equity: it coincides with the minimization of the *between* component of the Theil coefficient T_B calculated on the network. The variable (y in Eqs. 1 and 2) whose equity in distribution among the population has to be assessed is the bikeway accessibility $A_{Bw,o}$, as defined by Eq. 4.

$$\min \sum_{i=1}^{m} \left(\frac{p_i}{P_T} \cdot \frac{A_{Bw,i}(h)}{\bar{A}_{Bw}(h)}\right) \cdot \ln\left(\frac{A_{Bw,i}(h)}{\bar{A}_{Bw}(h)}\right) \tag{5}$$

$$s.t.$$

$$B_{min} \le c(h) \le B \tag{6}$$

$$h = \{1, 2, \dots, s\}, h \in H \tag{7}$$

The main constraint to the problem (Eq. 6) is the budget B available to the transport authorities to realize a bikeway project on the network. Decision variables are the connected cycling paths h (part of a global bikeways project that might be implemented on the territory) that could be potentially built: these paths belong to a specific set H (Eq. 7), established a priori by the transport planner according to the network features. Note that, in case of partial paths (h) overlapping, the costs associated with the implementation of the bikeway on each arc are computed only once. Moreover, the minimum budget B_{min} to be used is equal to $\alpha \cdot B$, with $\alpha \le 1$. B_{min} should be quite close to the total budget B in order to fully take advantage of the available resources, aiming at guaranteeing the maximum possible bike lane coverage on the territory.

The construction of each bikeway path h on the network guarantees a connected and meaningful set of bike lanes on the territory, although not complete. With a sufficient available budget, it can be possible to realize all those paths $h \in H$ that together constitute a complete bikeway project for the case study under analysis. The suggested model helps in finding the most equitable compromise among the available options, selecting a subgroup of bikeway paths that allows, without exceeding the total budget, to realize an equitable bike lanes system in the network.

4. Numerical application

In this section, the suggested methodology has been applied to a test network, having a total area of approximately 2.3 km^2 (look at the figure included in Table 2). It consists of 44 links and 20 nodes, each of which is the centroid (i.e. origin and destination of every cycling trip) of a corresponding district. Table 1 summarizes the demographic attributes of each zone, that is, the residing total population and the percentage of disadvantaged individuals (scattered throughout the territory, but mostly residing in the peripheral districts of the network) over the total.

Table 1. Total population and share of disadvantaged individuals for each district.

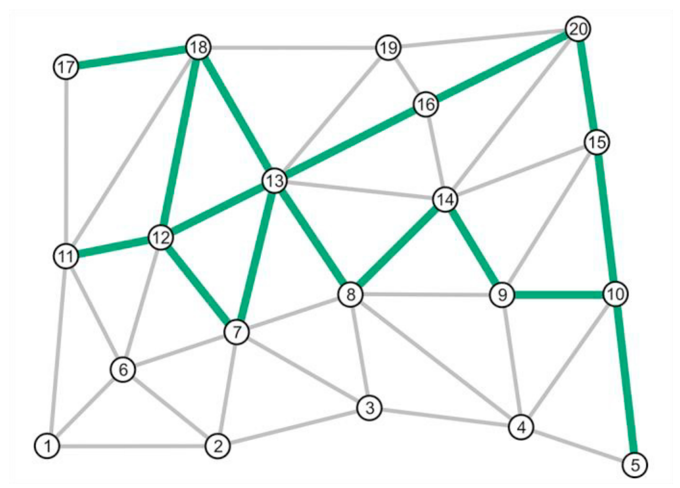
District number	1	2	3	4	5	6	7	8	9	10
Total population	135	102	254	301	287	132	118	130	143	145
Disadvantaged/Tot. pop.	59.3%	56.9%	58.3%	61.5%	59.2%	30.3%	21.2%	11.5%	29.4%	60.7%
District number	11	12	13	14	15	16	17	18	19	20
Total population	210	223	312	280	384	320	298	203	190	210
Disadvantaged/Tot. pop.	58.6%	29.1%	9.6%	10.4%	61.5%	30.6%	60.4%	59.6%	59.5%	60.5%

We assume to have a budget B equal to 6600 [meters]; setting $\alpha = 0.95$, we want to guarantee the minimum budget B_{min} to be used being very close to the total available amount of resources. On average, this range of budget allows realizing 5 different paths h on the territory. We set 48 different bikeway paths $h \in H$, each consisting of 2 to 5 connected arcs in the network. This implies that the number of possible paths combinations to be selected in order to define the optimal solution is rather high.

Therefore, the best approach for this kind of problem could be a heuristic or meta-heuristic optimization technique. More specifically, we propose the use of a genetic algorithm (GA) in order to find a feasible solution to our optimization problem, identifying which bikeway realization should be prioritized according to the available budget, minimizing the arising inequities in the accessibility between advantaged and disadvantaged individuals. Note that in this study we are suggesting to solve the problem using a GA; however, further methods, such as local search algorithms, could be explored and compared in future works in order to understand which one is the most suitable in solving the proposed optimization.

The solution (GA chromosome) consists of a binary string, having a length equal to the number of alternative bikeway paths (48 in this study). The unitary elements in the string correspond to the optimal bike lane paths that should be realized. The fitness function to minimize has been defined equal to the Theil between that measures the equitable distribution of the bikeway accessibility A_{Bw} among the different groups of individuals (Eq. 5). The population size is set equal to 200, the maximum number of generation equal to 1000; the algorithm stops, before reaching the maximum generation number, if the average relative change in the best fitness function value over 50 generations is less than or equal to 10E-12. In total, we have performed 100 iterations. The genetic operators used to generate offspring are the Stochastic uniform selection, the Scattered crossover, and the Gaussian mutation (look at The Mathworks, 2017 for further details).

Table 2. Optimal bikeways layout over 100 iterations (B=6600).

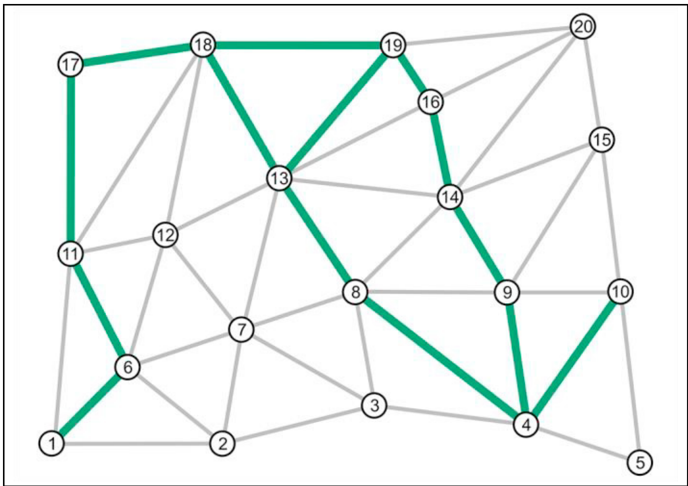


	MEDIAN over 100 iterations
$T_B(A_{Bw,i})$	1.8×10^{-9}
Total length [m]	6449
Coverage	36.4%
	BEST SOLUTION
$T_B(A_{Bw,i})$	1.1×10^{-13}
Total length [m]	6353
Coverage	35.9%
Paths selected (ID number)	17 (Nodes: 5 - 10 - 15 - 20) 21 (Nodes: 11 - 12 - 13 - 16 - 20) 25 (Nodes: 17 - 18 - 13 - 8) 30 (Nodes: 18 - 12 - 7 - 13) 42 (Nodes: 15 - 10 - 9 - 14 - 8)

Table 2 reports the results of the performed optimizations. The optimal paths (best solution) have been both depicted graphically (by thicker green arcs, while the remaining arcs in the network have been plotted gray and thinner) and reported with their ID number (over the 48 possible alternatives), together with the corresponding sequence of nodes. Looking at Table 2, we can notice that the Theil between referred to the bikeway accessibility T_B ($A_{Bw,i}$) scores a very low value, extremely close to zero. If, on the one hand, limited claims can be made about the absolute values of this Theil component, what matters in this analysis is the possibility to compare these values for different scenarios.

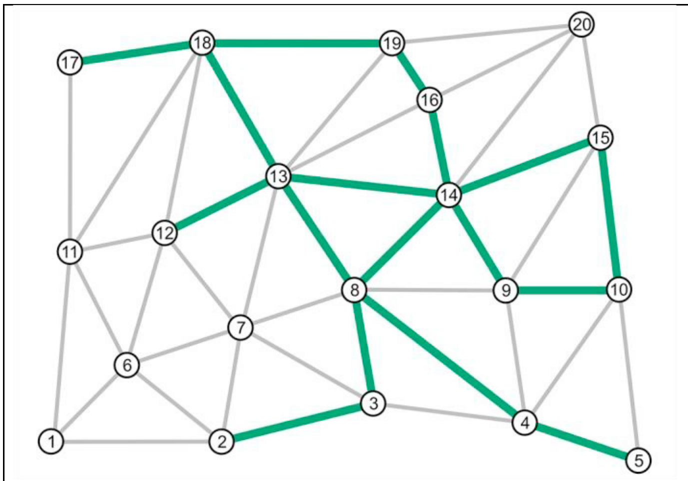
Using the same pool of 48 bikeway paths, and performing 100 additional iterations (this time aiming at the maximization of T_B ($A_{Bw,i}$) on the territory), we can have a better understanding of the existence of different combinations of paths that may lead to an increased level of inequities on the network. We have found that the highest achieved value of T_B ($A_{Bw,i}$) -i.e., higher inequities on the network- corresponds to 0.0123, while the median over 100 iterations is 0.0049. This means that applying the proposed approach (Eqs. 5 to 7) is actually possible to increase the level of bikeway accessibility on a predefined network, alleviating the disparities between different groups of individuals.

Table 3. Optimal bikeways layout with disadvantaged individuals concentrated in zone 1 (B=6600).



MEDIAN over 100 iterations	
T_B ($A_{Bw,i}$)	3.0×10^{-9}
Total length [m]	6429
Coverage	36.3%
BEST SOLUTION	
T_B ($A_{Bw,i}$)	1.8×10^{-13}
Total length [m]	6476
Coverage	36.6%
Paths selected (ID number)	2 (Nodes: 1 - 6 - 11 - 17) 14 (Nodes: 4 - 8 - 13 - 19) 15 (Nodes: 4 - 9 - 14 - 16 - 19) 22 (Nodes: 11 - 17 - 18 - 19) 25 (Nodes: 17 - 18 - 13 - 8) 31 (Nodes: 18 - 19 - 16 - 14) 46 (Nodes: 10 - 4 - 8 - 13 - 18)

Table 4. Optimal bikeways layout with disadvantaged individuals concentrated in zone 3 (B=6600).



MEDIAN over 100 iterations	
T_B ($A_{Bw,i}$)	4.2×10^{-9}
Total length [m]	6415
Coverage	36.2%
BEST SOLUTION	
T_B ($A_{Bw,i}$)	3.1×10^{-14}
Total length [m]	6572
Coverage	37.1%
Paths selected (ID number)	6 (Nodes: 2 - 3 - 8 - 13 - 18) 19 (Nodes: 5 - 4 - 8 - 14 - 15) 26 (Nodes: 17 - 18 - 19 - 16) 33 (Nodes: 19 - 16 - 14 - 15) 42 (Nodes: 15 - 10 - 9 - 14 - 8) 45 (Nodes: 10 - 9 - 14 - 13 - 12)

A sensitivity analysis has been carried out, assuming a different distribution of disadvantaged population across the districts. In particular, it has been supposed zone 1 (Table 3) and zone 3 (Table 4) have a residing population mainly composed of disadvantaged individuals (i.e. Disadvantaged/Total Population > 95%), while the remaining zones have been assumed almost entirely composed by what we defined as advantaged ones (Disadvantaged/Total Population < 1%). Aiming at minimizing the differences in inequities between the two groups, we expect the disadvantaged zones to be at least partially connected with the nearby ones by bikeway paths. In both cases, the available budget has been set equal to 6600, as in the original optimization round (Table 2). Looking at Tables 3 and 4, we can notice that the final bikeway configurations are made by a selection of arcs that aims at balancing the cycling accessibility opportunities between the two groups, avoiding one of them to bear more inequities. At least on one of the arcs converging in those supposed to be the most disadvantaged zones of the network (respectively, zone 1 and 3 in Tables 3 and 4), it is suggested to build a bikeway.

5. Conclusions and further research

The results achieved applying the proposed methodology to a test network seem to be promising: it is possible to allocate bike lane paths on the network reducing the accessibility disparities between different social groups of individuals. The proposed optimization model may fairly contribute to the promotion of cycling, suggesting to the transport planners that are evaluating possible bike lanes design scenarios which one could better satisfy the cycle accessibility needs of citizens.

This can be of extreme importance since urban planners often have to deal with a limited amount of available money, that makes not realistic to realize on the network a unique and connected project all in one step. This method might assist them in determining those portions of bikeways that have higher priority to be realized, particularly trying to avoid the generation of further inequities across categories of population. Further research developments entail the attribution of different weights to different cycling (*o,d*) pairs, in order to prioritize the interventions also according to the attractiveness of the zones/cycling paths. Moreover, an additional sensitivity analysis while varying the available budget could be performed, before testing the method on a pilot/real network.

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