JOURNAL OF TRAFFIC AND TRANSPORTATION ENGINEERING (ENGLISH EDITION) 2018; 5 (6): 503–509

Available online at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/jtte

Original Research Paper

Volume/thrust optimal shape criteria for arches under static vertical loads

Giuseppe Carlo Marano [a,b], Francesco Trentadue [a], Rita Greco [c,*], Ivo Vanzi [d], Bruno Briseghella [b]

[a] DICAR, Department of Civil Engineering and Architecture, Technical University of Bari, Bari 70125, Italy
[b] College of Civil Engineering, Fuzhou University, Fuzhou 350108, China
[c] DICATECH, Department of Civil Engineering, Environment, Territory, Building and Chemical, Technical University of Bari, Bari 70125, Italy
[d] Department of Engineering and Geology, University of Chieti-Pescara "G. d'Annunzio", Pescara 65127, Italy

HIGHLIGHTS

- The case of a statically determinate plane arch under constant vertical load is analyzed.
- An analytical solution for the optimal shape is proposed.
- The weight of the arch total volume and the horizontal thrust are used as objective functions.

ARTICLE INFO

Article history:
Received 16 April 2018
Received in revised form
8 July 2018
Accepted 12 July 2018
Available online 8 October 2018

Keywords:
Plane arch
Vertical loads
Optimal shape
Volume/thrust objective functions
Analytical solution

ABSTRACT

Arches are widely used when large spans are necessary, e.g. to overpass large rivers, and further possess unquestioned aesthetics advantages. Their structural efficiency depends primarily on optimal material exploitation, i.e. minimization of internal stress eccentricity, and on minimization of structural material volume. An efficient structure, under these terms, further requires simpler and lighter scaffolding, contributing in minimizing construction costs.

Although arches have millenary use and many researches dealing with this typology are available in literature, there is still scope for design optimization. The proposed study is framed within this context. Investigation is limited to statically determinate plane arches under vertical load. The problem of finding the profile of an equal strength catenary subjected to its self-weight is spread out to the case of an inverted catenary of equal strength under its self-weight and an external constant load. In the first optimization step, constant normal stress is imposed at all sections, to maximize material exploitation, and the resulting arch centerline shape is computed in closed form. In the second step, the ensemble of foundations and arch is considered and optimized, taking the linear combination of arch weight and thrust as objective function. The linear combination is dependent on a single variable, and minima of the objective function (i.e. optimal geometric shape parameters) are computed and charted to be simply used in the design process.

* *Corresponding author*. Tel.: +39 08 0596 3875.
E-mail address: rita.greco@poliba.it (R. Greco).
Peer review under responsibility of Periodical Offices of Chang'an University.
https://doi.org/10.1016/j.jtte.2018.10.005

1. Introduction

Arches are inherently efficient structures: they are capable to transfer loads from the superstructure to the foundations (Wilson, 2005) with low flexibility and structural weight. When properly shaped, they provide, per unit material volume, the optimal solution to cross large spans and transfer high loads. Structural efficiency depends on the dominance of centered normal stress: smaller cross sections, as compared to beams, can be used (Allen and Zalewski, 2009; Marano et al., 2014; Wang and Wang, 2015). Internal stress eccentricity (i.e., large bending moments) or large shear stresses should be avoided, resulting in uneconomical design, sub-exploitation of building materials, unnecessary self-weight (Billington, 1982; Gohnert et al., 2013). Further design economy can be obtained via overall shape and cross section optimization to achieve specific objectives and satisfy given constraints (Trentadue et al., 2018). In many cases a key point is the minimization of structural volume, since arch self-weight is the largest component of the vertical load, accounting for about half the total.

Optimization is a key issue for good design. From the data on 55 arch bridges built during the twentieth century reported in Salonga and Gauvreau (2014), several empirical lessons may be learnt. The first one is that concrete arches (long span) consume, per unit length, higher material quantities as compared to post-tensioned concrete girder bridges (shorter span). This is an expected result, at least since arches are curved, whereas beams are not; however, post-tensioned concrete girder are not usable on large spans. The second lesson is that, for long span arch bridges, arch self-weight is about half the total vertical load. In fact vertical loads, per unit deck surface, may be roughly approximated as independent on span and equal to about 12 kN/m^2 (girder), 3 kN/m^2 (columns) and 5 kN/m^2 (traffic); contrarily arch self-weight strongly depends on the span length, approximately in a linear way, and ranges from 13 kN/m^2 (50 m span) to 31 kN/m^2 (300 m span). Ratios between arch self-weight and total load therefore range from about 0.4 (50 m span) to 0.6 (300 m span). Both lessons further motivate the search for optimal (less material consuming) solutions.

Further, structural optimization is an important design tool for shape selection, also from an architectural point of view (Adriaenssens et al., 2014; Briseghella et al., 2016; Fiore et al., 2016). The shape of an arch, deriving from the conceptual design stage, generally belongs to three main idealized configurations: circular, parabolic or catenary. On this topic, Lewis (2016) in a recent study shows how these forms can become moment-less arches when subject to certain specific load conditions: an inextensible chain that hangs under its own weight produces a catenary shape; a weightless chain carrying a uniformly-distributed load gives a parabola, and a uniform load applied normally to a weightless chain leads to a circular shape.

Structural optimization has been common for a long time in mechanical and aeronautical engineering. In civil engineering, it has been progressively adopted both for buildings and bridges (Allahdadian and Boroomand, 2010; Briseghella et al., 2013a, b; De Tommasi et al., 2015, 2016; Greco and Marano, 2016; Greco et al., 2016; Greco and Trentadue, 2013; Marano and Greco, 2011; Neves et al., 1995; Quaranta et al., 2014; Stromberg et al., 2011; Trentadue and Quaranta, 2013; Zordan et al., 2010).

Arches optimization traditionally departs from the concept of catenary. Catenary arches have the property of being stressed in pure compression, without bending moment or shear. A chain suspended between two points will form this unique curve, which is routinely used for arches, and sometime for shells (although this is not entirely correct due to bi-dimensional stiffness).

Catenary curves are typically referred to an ideal situation with uniform load and supports at the same level. A more general solution should however take into account non-homogeneous loading patterns and point loads and supports at uneven level (Briseghella and Zordan, 2015; Fiore and Monaco, 2009; Fiore et al., 2012, 2013).

Literature is abundant of studies dealing with arches and arches optimization, starting from the one from Budiansky et al. (1969). Optimal shaping of arches subjected to general loading is firstly dealt with by Farshad (1974). The study includes computation of the optimum center line shape and cross section area distribution. Mathematical techniques include the objective function with constraints; the objective function combines arch thrust, material volume, total arc length, and enclosed area. Tadjbakhsh (1981) determines the arch buckling load. Błachut and Gajewski (1981) define the optimal cross-sectional area function for catenary shape, considering transverse vibrating arches under external load. Bochenek and Gajewski (1989) analyze an elastic, plane, catenary circular arch, loaded by uniformly distributed radial pressure. The two lowest critical loads, for out-of-plane and in-plane buckling of the arch, are computed. The optimization problem is applied to the cross-section dimensions; the result is the minimization of the total arch volume under given external pressure and geometrical constraints. Serra (1994) examines the numerical problem of the optimal arch as a uniformly compressed structure subjected to static loads. A closed form solution for the optimal shape of a statically determined planar arch under uniform vertical loads is computed in Marano et al. (2014). The problem of a catenary under self-weight is extended to an inverted catenary under self-weight and constant vertical load. Wang and Wang (2015) find the closed form solution for a uniformly loaded arch.

This paper extends the results by Marano et al. (2014). The optimal arch shape is defined via minimization of structural volume and horizontal thrust, the latter being central in foundations optimization.

The problem is cast as follows: finding the optimal arch shape and cross section, that satisfy the condition of constant cross section stress. Loads are vertical, and include dead loads and self-weight. It should be noted that different catenaries correspond to different vertical loads distributions; the designer should therefore select a single load distribution, possibly coinciding with the one characterized by the highest intensity, and then check the design including the remaining load distributions.

2. Catenary arch optimal shape and section

The problem of finding the optimal arch shape (Marano et al., 2014) is here extended by searching the arch that minimizes a linear combination of the total weight, that is representative of the material cost, and of the arch thrust H, that represents the arch's foundation cost.

The search is focused on the optimal arch profile $y(x)$ and the cross section area $A(x)$ of a statically determinate arch (Fig. 1) subjected to self-weight and to a distributed external load $p(x)$, assigned for unit horizontal length. The arch shape is such that all cross sections are subjected to a unique constant stress $\overline{\sigma}$.

Generally speaking, in arches subjected only to vertical loads, a generic section with horizontal coordinate x is subjected to an horizontal internal force that, for equilibrium reasons, is always equal to the thrust H, while vertical internal force $V(x)$ and bending moment $M(x)$ are variable.

The arch is made up of a homogeneous material with constant specific weight γ, so that the arch self-weight for unit horizontal length $q(x)$ is given by

$$q(x)=\frac{\gamma A(x)}{\cos(\theta)(x)}=\gamma A(x)\left(1+y^{I}(x)^{2}\right)^{\frac{1}{2}} \tag{1}$$

where $\cos(\theta)$ (x) is the horizontal projection of an arch element

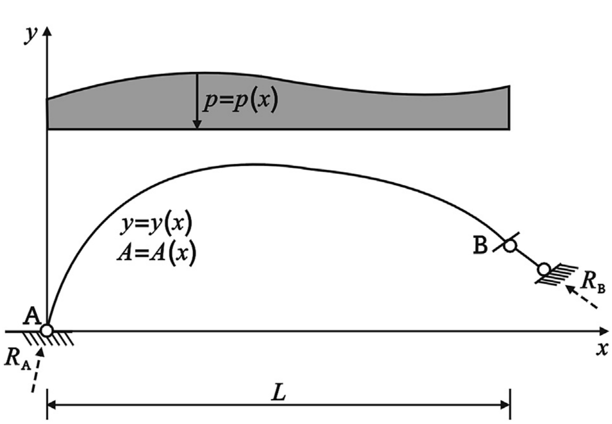

Fig. 1 – Arch structure with a variable section subject to a distributed vertical load $p = p(x)$.

with unit length, $y(x)$ is the vertical coordinate of the center of the generic section with horizontal coordinate x and the symbol $\bullet^{I}$ denotes the first derivative with respect to the independent variable x.

In an arch having all sections uniformly compressed, bending moment is equal zero in each section, so that it results

$$\begin{aligned}M(x)&=-\left[Hy-V_{A}x+\int_{0}^{x}\left[p+\gamma A(\tau)\left(1+y^{I}(\tau)^{2}\right)^{\frac{1}{2}}\right](x-\tau)\mathrm{d}\tau\right]\\&=0\quad x\in[0,L]\end{aligned} \tag{2}$$

where V_A is the vertical component of the reaction R_A in section A (Fig. 1) and τ is the integration variable.

Differentiation of Eq. (2) with respect to x gives the following expression

$$M^{I}(x)=-\left[Hy^{I}(x)-V_{A}+\int_{0}^{x}p\mathrm{d}\tau+\int_{0}^{x}\gamma A(\tau)\left(1+y^{I}(\tau)^{2}\right)^{\frac{1}{2}}\mathrm{d}\tau\right]=0 \tag{3}$$

Moreover, a further differentiation leads to

$$M^{II}(x)=-\left[Hy^{II}(x)+p+\gamma A(x)\left(1+y^{I}(x)^{2}\right)^{\frac{1}{2}}\right]=0 \tag{4}$$

where $\bullet^{II}$ denotes the second derivative with respect to the independent variable x.

This last equation, together with the following boundary conditions:

$$\begin{cases}M(0)=0\\M(L)=0\end{cases} \tag{5}$$

where L is the arch span, allows determining the arch shape such that a null bending moment and a null shear force are exerted in each cross section under the load p.

Further, the condition that each cross section is subject to a given constant axial compressive stress $\overline{\sigma}$ should be satisfied, so that

$$\overline{\sigma}=\frac{N(x)}{A(x)}=\frac{H}{A(x)\cos(\theta)(x)}=\frac{H\left(1+y^{I}(x)^{2}\right)^{\frac{1}{2}}}{A(x)} \tag{6}$$

Then the cross section area $A(x)$ is given by

$$A(x)=\frac{H}{\overline{\sigma}}\left(1+y^{I}(x)^{2}\right)^{\frac{1}{2}} \tag{7}$$

By substituting Eq. (7) into (6), Eq. (1) becomes

$$q(x)=\frac{\gamma}{\overline{\sigma}}H\left(1+y^{I}(x)^{2}\right)=\frac{H}{\overline{h}}\left(1+y^{I}(x)^{2}\right) \tag{8}$$

where the constant $\overline{h}=\overline{\sigma}/\gamma$ is the height of the column, subjected to its self-weight and made up of the same material of the arch, in which the compressive stress $\overline{\sigma}$ is reached at the base section.

By substituting Eq. (8) in Eq. (4) the following second order non-linear differential equation is obtained

$$\begin{cases}y^{II}(x)+\dfrac{p}{H}+\dfrac{1}{\overline{h}}\left(1+y^{I}(x)^{2}\right)=0\\M(0)=M(L)=0\end{cases} \tag{9}$$

Approximate solutions for Eq. (9) have been proposed by Serra (1994). In the following, an analytical solution of this equation is presented, in the case of a constant external load $p(x) = p$. At this aim, the solution of Eq. (9) is searched in the form

$$y(x) = \bar{h}\log(z(x)) \quad z(x) > 0 \tag{10}$$

By differentiating $y(x)$ with respect to x, and by substituting Eq. (10) in Eq. (9), it results

$$\bar{h}\frac{z^{II}(x)}{z(x)} - \bar{h}\left(\frac{z^{I}(x)}{z(x)}\right)^2 + \frac{p}{H} + \frac{1}{\bar{h}}\left[1 + \bar{h}^2\left(\frac{z^{I}(x)}{z(x)}\right)^2\right] = 0 \tag{11}$$

Finally, after some algebraic manipulation, the following system is obtained

$$\begin{cases} z^{II}(x) + \omega^2 z(x) = 0 \\ M(0) = M(L) = 0 \end{cases} \tag{12}$$

$$\omega = \sqrt{\frac{p}{H\bar{h}} + \frac{1}{\bar{h}^2}} = \frac{1}{\bar{h}}\sqrt{\frac{p\bar{h} + H}{H}} \tag{13}$$

Notice that the explicit solution of Eq. (9) can be written as

$$y(x) = \bar{h}\log(z(x)) = \bar{h}\log[C\cos(\omega(x - x_c))] \tag{14}$$

where the parameters C and x_c depend on the boundary conditions. Since the following analysis refers only to symmetric shape arches, it results $x_c = L/2$. Further, referring to Fig. 1 and to Eq. (2), it emerges that since $y(0) = 0$ the condition $M(0) = 0$ is always satisfied, meanwhile the condition $M(L) = 0$ is implied by the symmetry. Finally, $y(0) = 0$ implies $z(0) = 1$ and then

$$C = \frac{1}{\cos\left(\frac{\omega L}{2}\right)} \tag{15}$$

Then, the optimal arch profile $y(x)$ and the cross section area $A(x)$ are

$$\begin{cases} y(x) = \bar{h}\log\left[\dfrac{\cos\left(\omega\left(x - \frac{L}{2}\right)\right)}{\cos\left(\frac{\omega L}{2}\right)}\right] \\ A(x) = \dfrac{H}{\bar{\sigma}}\left[1 + \bar{h}^2\omega^2\tan(\omega(x - L/2))^2\right]^{\frac{1}{2}} \end{cases} \tag{16}$$

Since the constant C in Eq. (14) has to be positive in order to ensure the existence of the solution given by Eq. (10), the condition $\omega L < \pi$ has to be satisfied and then

$$L \le \tilde{L} = \frac{\pi}{\omega} = \pi\bar{h}\sqrt{\frac{H}{p\bar{h} + H}} \tag{17}$$

where $\tilde{L}$ represents an upper bound for the arch span L (Marano et al., 2014).

It is worth noting that in the case of null applied load, Eq. (17) furnishes $L \le \tilde{L} = \pi\bar{h}$, showing that the maximum span of an optimal arch cannot be greater than π times the height of the column subjected to its self-weight and made up of the same material of the arch, in which the compressive stress $\bar{\sigma}$ is reached at the base section.

Moreover the rise f of the arch at its crown section is given by

$$f = -\bar{h}\log\left[\cos\left(\frac{\omega L}{2}\right)\right] \tag{18}$$

Finally, it is worth to underline that the optimal thrust H remains indeterminate in the set of feasible solutions defined by Eq. (16), so that the optimal solution minimizing a chosen objective function can be searched within this set.

3. Optimal design problem

As already stated, our preliminary design aims at minimizing a linear combinations of the arch total weight and of the arch thrust H. It is useful to consider the ratio of these quantities and the total external load pL to obtain a dimensionless objective function. For the following developments, two dimensionless parameters are introduced

$$\begin{cases} \alpha = \dfrac{L}{\tilde{L}} = \dfrac{\omega}{\pi}L \\ \eta = \dfrac{L}{\bar{h}} = \dfrac{\gamma L}{\bar{\sigma}} \end{cases} \tag{19}$$

Thus α is the ratio between the span of the arch and the maximum feasible span and η relates the span with the mechanical properties of the material. Notice that the existence condition given by Eq. (17) implies $\alpha \le 1$. Referring to Eqs. (16) and (19), the arch weight can be expressed as

$$W = \int_0^L q(x)dx = \frac{H}{\bar{h}}\int_0^L \left[1 + \bar{h}^2\omega^2\tan(\omega x - L/2)^2\right]dx$$
$$= pL\left(\frac{2\pi\alpha\tan\left(\frac{\pi\alpha}{2}\right)}{\pi^2\alpha^2 - \eta^2} - 1\right) \tag{20}$$

Meanwhile, as a consequence of Eq. (20), the arch thrust can be determined as

$$H = \frac{\bar{h}p}{\bar{h}^2\omega^2 - 1} = \left(\frac{\eta}{\pi^2\alpha^2 - \eta^2}\right)pL \tag{21}$$

Since our analysis is limited only to finite positive values of the thrust ($0 \le H \le +\infty$), Eq. (21) implies that $\eta/\pi > \alpha$.

The objective function is thus

$$\Phi_\psi(\alpha, \eta) = \frac{1}{pL}(W + \psi H)$$
$$= \left(\frac{2\pi\alpha\tan\left(\frac{\pi\alpha}{2}\right)}{\pi^2\alpha^2 - \eta^2} - 1\right) + \Psi\left(\frac{\eta}{\pi^2\alpha^2 - \eta^2}\right) \quad \frac{\eta}{\pi} < \alpha < 1 \tag{22}$$

where the positive parameter ψ considers the incidence of the cost of foundation on the total cost. The objective function Φ_ψ is shown in Fig. 2 for $\psi = 0.2$. Notice that Φ_ψ tends to an infinite value on the boundaries of the feasible domain. It is worth to note that the boundary $\alpha = 1$ corresponds to arches with an infinite rise f and the boundary $\alpha = \pi/\eta$ corresponds to arches exerting an infinite thrust H.

Though the above choice of the design parameters allows to clearly represents the feasible domain of this optimum problem, from an engineering point of view it is more convenient to consider as design parameter the dimensionless rise $\tilde{f} \equiv f/L$. To this end, from Eqs. (18) and (19), it derives

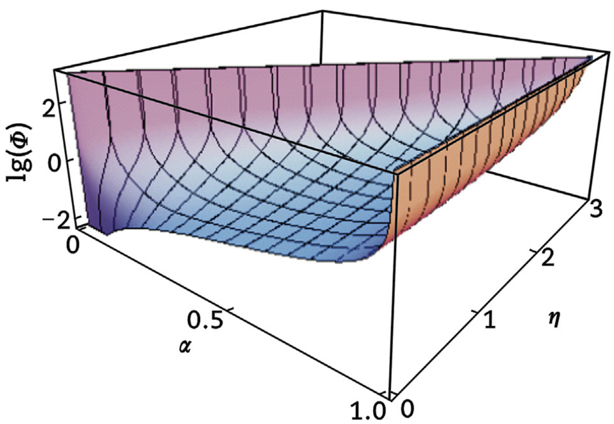

Fig. 2 – Graph of the function $\Phi_\psi(\alpha, \eta)$ for $\psi = 0.2$.

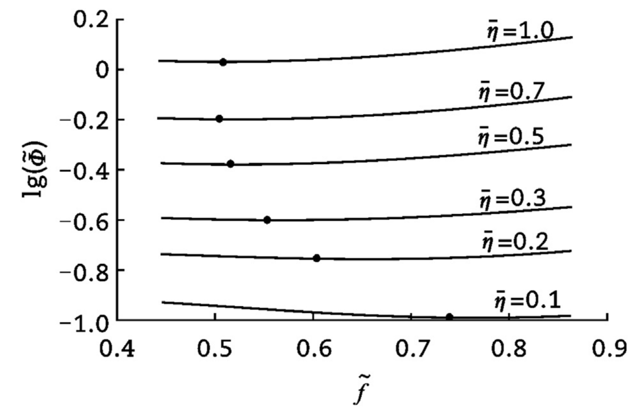

Fig. 4 – Graph of the function $\tilde{\Phi} = \tilde{\Phi}_\Psi(\tilde{f}, \overline{\eta})$ for $\overline{\eta} = \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0\}$, $\psi = 0.2$.

Eq. (23). So that the objective function can be rewritten as Eq. (24).

$$\alpha = \alpha\left(\eta, \tilde{f}\right) = \frac{2}{\pi}\arccos\left[\exp\left(-\eta\tilde{f}\right)\right] \tag{23}$$

$$\tilde{\Phi}_\Psi\left(\tilde{f}, \eta\right) = \Phi_\Psi\left(\alpha\left(\eta, \tilde{f}\right), \eta\right) \tag{24}$$

By adopting these new design parameters, the objective function becomes independent from the applied load p. In Fig. 3 the graph of the objective function $\tilde{\Phi}_\Psi(\tilde{f}, \eta)$ for $\psi = 0.2$ is reported. It can be noted that the objective function always decreases with respect to the design parameter η relating the span L to the mechanical properties of the material.

In Fig. 4 the curves $\tilde{\Phi} = \tilde{\Phi}_\Psi(\tilde{f}, \overline{\eta})$ are drawn for $\overline{\eta} = \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0\}$. The minimum point of each curve is marked by a bold dot and defines the optimal value $\tilde{f}_{opt}$ of the dimensionless rise

$$\tilde{f}_{opt} = \arg\min_{\tilde{f}}\left\{\tilde{\Phi}_\Psi\left(\tilde{f}, \overline{\eta}\right)\right\} \tag{25}$$

It emerges that $\tilde{f}_{opt}$ varies in an almost strict range.

Based on the above considerations, the following design procedure can be proposed: after fixing the material parameters γ and $\overline{\sigma}$, the span L and the relative cost of the foundation

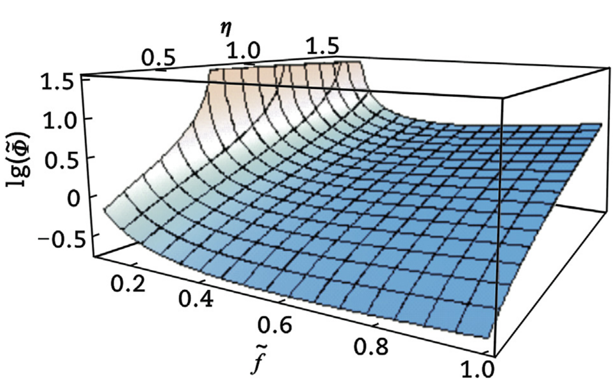

Fig. 3 – Graph of the function $\tilde{\Phi} = \tilde{\Phi}_\Psi(\tilde{f}, \eta)$ for $\psi = 0.2$.

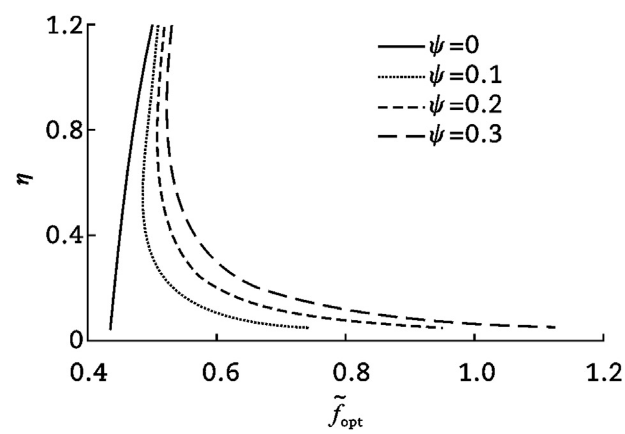

Fig. 5 – Optimal dimensionless rise $\tilde{f}_{opt} = \tilde{f}(\eta, \Psi)$.

by the parameter Ψ, the optimal value of the dimensionless rise $\tilde{f} = f/L$ can be determined as $\tilde{f}_{opt} = \tilde{f}(\eta, \Psi)$.

Fig. 5 shows the optimal dimensionless rise $\tilde{f}_{opt}$. Values of the span L in the interval [40 m, 250 m], a specific weight $\gamma = 25$ kN/m^3 and a working stress $\overline{\sigma}$ in the interval [5 MPa, 10 MPa] are considered in the analysis. Therefore values of the parameter η in the interval [0.1, 1.2] are obtained.

It is worth to underline that the optimal dimensionless rise $\tilde{f}_{opt}$ here determined is independent from the applied load p. Moreover, when the cost of foundation is neglected ($\Psi = 0$), the optimal dimensionless rise always decreases with respect to η, while the other cases exhibit a more complex behaviour. However, limiting our analysis to the higher range of values of η ($\eta \geq 0.3-0.4$), the optimal dimensionless rise varies within a rather strict range.

4. Conclusions

In the present work an analytical solution for the optimal shape of a plane statically determined arch subject to a constant vertical load has been presented. The classical problem of finding the profile of an equal strength catenary subjected to its self-weight has been extended to the case of an inverted catenary of equal strength subject to both its self-weight and an external applied constant load. The weight of the arch total volume and the lateral thrust were used as objective function, in order to include two different aspect of arch construction costs, that is self-weight and foundation rigidity. Some sensitivity analyses to obtain optimal solutions in a dimensionless formulation were also carried out, at the aim to define some practical rules useful for predesign. More precisely, the proposed analysis allows to obtain the optimal value of the arch dimensionless rise after properly fixing the material properties, the span length and the foundation relative cost.

Conflicts of interest

The authors do not have any conflict of interest with other entities or researchers.

Acknowledgments

This work is framed within the research project “OptArch – 689983, H2020-MSCA-RISE-2015/H2020-MSCA-RISE-20“.

REFERENCES

Allen, E., Zalewski, W., 2009. Form and Forces: Designing Efficient, Expressive Structures. John Wiley & Sons, New York.

Adriaenssens, S., Block, P., Veenendaal, D., et al., 2014. Shell Structures for Architecture: Form Finding and Optimization. Routledge, New York.

Allahdadian, S., Boroomand, B., 2010. Design and retrofitting of structures under transient dynamic loads by a topology optimization scheme. In: 3rd International Conference on Seismic Retrofitting, Tabriz, 2010.

Billington, D.P., 1982. Thin Shell Concrete Structures. McGraw-Hill, New York.

Briseghella, B., Fenu, L., Feng, Y., et al., 2016. Optimization indexes to identify the optimal design solution of shell-supported bridges. Journal of Bridge Engineering 21 (3), 04015067.

Briseghella, B., Fenu, L., Feng, Y., et al., 2013a. Topology optimization of bridges supported by a concrete shell. Structure Engineering International 23 (3), 285–294.

Briseghella, B., Fenu, L., Lan, C., et al., 2013b. Application of topological optimization to bridge design. Journal of Bridge Engineering 18 (8), 780–800.

Briseghella, B., Zordan, T., 2015. An innovative steel-concrete joint for integral abutment bridges. Journal of Traffic and Transportation Engineering (English Edition) 2 (4), 209–222.

Błachut, J., Gajewski, A., 1981. On unimodal and bimodal optimal design of funicular arches. International Journal of Solids and Structures 17 (7), 653–667.

Bochenek, B., Gajewski, A., 1989. Multimodal optimization of arches under stability constraints with two independent design functions. International Journal of Solids and Structures 25 (1), 67–74.

Budiansky, B., Frauenthal, J.C., Hutchinson, J.W., 1969. On optimal arches. Journal of Applied Mechanics 36 (4), 880–882.

De Tommasi, D., Marano, G.C., Puglisi, G., et al., 2015. Optimal complexity and fractal limits of self-similar tensegrities. Proceedings of the Royal Society A: Mathermatical, Physical and Engineering Science 471 (2184), 20150250.

De Tommasi, D., Marano, G.C., Puglisi, G., et al., 2016. Morphological optimization of tensegrity-type metamaterials. Composites Part B: Engineering 115, 182–187.

Farshad, M., 1974. On general conservative loading of naturally curved and twisted funicular rods. International Journal of Solids and Structures 10 (9), 985–991.

Fiore, A., Marano, G.C., Greco, R., et al., 2016. Structural optimization of hollow-section steel trusses by differential evolution algorithm. International Journal of Steel Structures 16 (2), 411–423.

Fiore, A., Monaco, P., 2009. POD-based representation of the alongwind Equivalent Static Force for long-span bridges. Wind and Structures 12 (3), 239–257.

Fiore, A., Monaco, P., Raffaele, D., 2012. Viscoelastic behaviour of non-homogeneous variable-section beams with post-poned restraints. Computers and Concrete 9 (5), 375–392.

Fiore, A., Foti, D., Monaco, P., et al., 2013. An approximate solution for the rheological behavior of non-homogeneous structures changing the structural system during the construction process. Engineering Structures 46, 631–642.

Gohnert, M., Fitchett, A., Bulovic, I., et al., 2013. Structurally efficient housing using natural forms. Journal of the South African Institution of Civil Engineers 55 (3), 96–102.

Greco, R., Marano, G.C., Fiore, A., 2016. Performance–cost optimization of tuned mass damper under low-moderate seismic actions. The Structural Design of Tall and Special Buildings 25 (18), 1103–1122.

Greco, R., Marano, G.C., 2016. Optimum design of viscous dissipative links in wall-frame systems. Structural Design of Tall and Special Buildings 25 (9), 412–428.

Greco, R., Trentadue, F., 2013. Structural reliability sensitivities under non-stationary random vibrations. Mathematical Problems in Engineering 2013, 426361.

Lewis, W.J., 2016. Mathematical model of a moment-less arch. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 472 (2190), 20160019.

Marano, G.C., Trentadue, F., Petrone, F., 2014. Optimal arch shape solution under static vertical loads. Acta Mechanica 225 (3), 679–686.

Marano, G.C., Greco, R., 2011. Optimization criteria for tuned mass dampers for structural vibration control under stochastic excitation. Journal of Vibration and Control 17 (5), 679–688.

Neves, M.M., Rogrigues, H., Guedes, J.M., 1995. Generalized topology design of structures with a buckling criterion. Structural Optimization 10 (2), 71–78.

Quaranta, G., Fiore, A., Marano, G.C., 2014. Optimum design of prestressed concrete beams using constrained differential evolution algorithm. Structural and Multidisciplinary Optimization 49 (3), 441–453.

Salonga, J., Gauvreau, P., 2014. Comparative study of the proportions, form, and efficiency of concrete arch bridges. Journal of Bridge Engineering 19 (3), 4013010.

Serra, M., 1994. Optimal arch: approximate analytical and numerical solutions. Computer and Structures 52 (6), 1213–1220.

Stromberg, L.L., Beghini, A., Baker, W.F., et al., 2011. Application of layout and topology optimization using pattern gradation for the conceptual design of buildings. Structural and Multidisciplinary Optimization 43 (2), 165–180.

Tadjbakhsh, I.G., 1981. Stability and optimum design of arch-type structures. International Journal of Solids and Structures 17 (6), 565–574.
Trentadue, F., Quaranta, G., 2013. Limit analysis of frictional block assemblies by means of fictitious associative-type contact interface laws. International Journal of Mechanical Sciences 70, 140–145.
Trentadue, F., Marano, G., Vanzi, I., et al., 2018. Optimal arches shape for single-point-supported deck bridges. Acta Mechanica 229 (5), 2291–2297.
Wang, C.Y., Wang, C.M., 2015. Closed-form solutions for funicular cables and arches. Acta Mechanica 226 (5), 1641–1645.
Wilson, 2005. Practical Design of Concrete Shell. Monolithic Dome Institute, Texas.
Zordan, T., Briseghella, B., Mazzarolo, E., 2010. Bridge structural optimization through step-by-step evolutionary process. Structural Engineering International 20 (1), 72–78.

Giuseppe Carlo Marano is associate professor at the Department of Civil Engineering and Architecture of the Technical University of Bari where actually teaches "Structural Design" and distinguished professor at the College of Civil Engineering, Fuzhou University, China. His research focuses on seismic engineering, structural optimization and Genetic Algorithm.

Francesco Trentadue is associate professor of Structural Mechanics at the Politecnico di Bari. He graduated with a Bachelor's and Master's Degree from Bari University, Italy, and a PhD from Firenze University, Italy. His main research activities have been focused on optimal structural design and stability of elastic structures.

Rita Greco is associate professor at the Department of Civil Engineering, Environmental, Territory, Building and Chemical of Technical University of Bari where actually teaches "Structural Design". She obtained the PhD in structural engineering from the University of Naples "Federico II" in 1998. Her research focuses on seismic engineering, structural optimization and Genetic Algorithm.

Ivo Vanzi is full professor in Design of Earthquake Resistant Structures at the University of Chieti-Pescara "G. d'Annunzio", member of the Italian Higher Council of Public Works, and member of the Technical and Scientific Council of Emilia-Romagna. He is author of more than 100 scientific papers on structural and earthquake engineering. His research interests include structural and system reliability, bridge design, structural retrofitting of industrial installations.

Bruno Briseghella is distinguished professor and dean of the College of Civil Engineering, Fuzhou University, China, and founding director of the "Sustainable and Innovative Engineering Research Center". He graduated with a Bachelor's and Master's Degree from Padova University, Italy, and a PhD from Trento University, Italy. His main research activities have been focused on bridge and structural design, integral abutment bridges, monitoring and retrofit of bridges, earthquake engineering and composite steel-concrete structures, both from the theoretical and experimental point of view.