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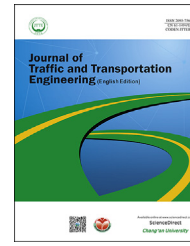

Original Research Paper

New few parameters differential evolution algorithm with application to structural identification

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HIGHLIGHTS

- New schemes for both mutation and crossover operators of standard differential evolution algorithm (DEA).
- An adaptive mutation operator is proposed.
- A crossover operator replacing binomial formulation is proposed.

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ABSTRACT

Differential evolution algorithm (DEA) is a stochastic, population-based global optimization method. In this paper, we propose new schemes for both mutation and crossover operators in order to enhance the performances of the standard DEA. The advantage of these proposed operators is that they are "parameters-less", without a tuning phase of algorithm parameters that is often a disadvantage of DEA. Once the modified differential evolutions are presented, a large comparative analysis is performed with the aim to assess both correctness and efficiency of the proposed operators. Advantages of proposed DEA are used in an important task of modern structural engineering that is mechanical identification under external dynamic loads. This is because of the importance of using a "parameters-less" algorithm in identification problems whose characteristics typically vary strongly case by case, needing of a continuous set up of the algorithm proposed. This important advantage of proposed optimizers, in front of other identification algorithms, is used to develop a computer code suitable for the automatic identification of a simple supported beam subject to an impact load, that has been tested both using numerical simulations and real standard tests dynamic. The results point out that this algorithm is an interesting candidate for standard applications in structural identification problems.

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1. Introduction

The differential evolution algorithm (DEA) is an interesting soft computing technique for solving complex optimization problems proposed by Storn and Price (1997) whose positive features are attracting the interest of several researches in the field of the applied sciences. Differently from traditional evolutionary algorithms, this optimizer is completely self-organizing and requires few lines of code in most of the existing programming languages. Moreover, its functionality requires a small set of embedded control parameters, which makes it easy to use for non-experts. Although its simplicity, DEA has shown remarkable performances in solving hard optimization problems so that its popularity is expected to grow rapidly. The general structure of the DEA shares similar features with other evolutionary algorithms, such as genetic algorithm (GA), initially proposed by Holland (1975). DEA is different in handling distance and direction information to move from the population at the current generation toward the next generation in virtue of a constructive cooperation between individuals.

There are two ways to enhance the performances of the standard DEA. One way is to look for new operators (or hybridizations) for DEAs. For instance, Kaelo and Ali (2006) proposed two new DEAs. The first version modifies the original mutation rule by including a tournament selection. The second version is a DEA using the best vector with localization. Das et al. (2009) described a class of DEA-based variants which utilizes the concept of neighborhood for each population member in order to balance the exploration and exploitation abilities of DEAs. Omran et al. (2009) presented the barebones DEA which is a hybridization between barebones PSOA and DEA. A second (and maybe fundamental) way to improve the standard DEA deals with the definition of the control parameters. The most common procedure (originally adopted for the standard DEA) is based on parameter tuning, a commonly practiced approach in evolutionary computation in good values control parameters are selected after a "learning stage". Unfortunately, this operation is not that simple, and generally needs "experts" who know how to use algorithms, reducing strongly the appeal and the advantages of such methodologies. For instance, the user may assign the numerical values by analyzing the results carried out from trial-and-error runs. This strategy requires an appropriate knowledge of the optimizer to obtain the expected performances quickly and, therefore, a high degree of interaction is needed between the user and the adopted optimizer. A solution may be that of employing another algorithm such as an artificial neural network. However, even this choice is not immune from criticism as it increases considerably the level of complexity making it less attractive to non-experts in the field of soft computing methodologies. Once the use or machine-based learning stage is completed, the optimizer seeks the optimal solution on the basis of the values obtained without further modifications.

In the fields of civil and mechanical engineering, DEAs have been applied for solving structural optimization problems (Hull et al., 2006; Jármai et al., 2003), but only few researches have explored new variants for handling constrained optimization problems (Wang and Li, 2009). Most recently, DEAs have been applied for parameter/tolerance design problems (Rout and Mittal, 2010), mechanical systems identification (Quaranta et al., 2014a; Tang et al., 2008) and health monitoring (Casciati, 2008). The main goal of this paper is to suggest new schemes "parameters-less" for both mutation and crossover operators for DEA, whose performances are of the same order of other DEA. The main advantages of the proposed scheme is an easiest and widest applicability in real problems particularly sensible to parameters setup, to increase the appeal of DEA for those real problems. To demonstrate the efficiency of the new operators, an extensive numerical investigation is conducted to evaluate the correctness of the proposed differential evolution based optimizers. To achieve this goal, a first comparative analysis deals with the optimization of thirteen well known benchmark test functions (Neculai, 2008) over small and medium search spaces: in doing this, five standard differential evolution algorithms and four particle swarm-based optimizers, including a chaotic particle swarm optimization algorithm (PSOA) are taken into account. Subsequently, another comparative analysis has been performed by taking into account more recent adaptive DEAs.

In this study an important problem of structural engineering is treated as practical application of proposed DEA scheme, which regards the impact load identification. In structural monitoring, in fact, it is crucial to know the external loads that act on structural elements or mechanical components. With the knowledge of the loads, we can assess damage in terms of the strength, fatigue and reliability of structures. In several circumstances, dynamic loads can be directly measured by means of opportune sensors. Nonetheless, numerical methods for load identification are highly welcome when a direct measure is complicated, i.e., because of extremely large magnitudes of loads for a very short time period (impact loads). A quite typical way for solving load identification problems deals with the formulation (and resolution) of a single-objective optimization problem (Marano et al., 2006, 2007a, 2007b, 2008). In doing this, the first step is the mathematical definition of the loading model: an opportune parameterization of the dynamic load allows the reconstruction of the time-dependent load history to be more efficient, since the point-by-point time domain based identification of the loading process is much more difficult and less reliable (Greco et al., 2015, 2016; Marano et al., 2009a, 2009b, 2010; Trentadue et al., 2018). Following this way, combining with a forward model, which characterizes the dynamic response of the structure subject to a known dynamic force, the unknown parameters depicting the dynamic loads can be identified by minimizing the difference between the computed analytical responses and the actually measured responses. Unfortunately, a typical problem arising from the optimization-based formulation of the load identification process is that the resulting problems can be non-linear and multi-modal. Differential information (i.e., gradients) or initial information (i.e., starting values of the unknown parameters to be identified), which is required by traditional optimization approaches, may be difficult and time-consuming to calculate. Therefore, free-gradient algorithms with global exploration capabilities may be explored to overcome this difficulties.

Given the inherent difficulties in solving optimization-based load identification problems, several researchers have adopted GA-based numerical strategies (Marano et al., 2009c, 2011). For instance, the solution procedure proposed in (Martin and Doyle, 1996) combines frequency-domain problem solving using the spectral element method with a genetic-based heuristic iterative search technique. Flores et al. (2007) formulated an optimization problem in which the objective function (OF) represents the difference between the measured modal characteristics of the loaded structure and their finite element counterparts. The loading parameters (magnitude, position and direction) were assumed as unknown variables of this optimization problem and a heuristic technique known as the life-cycle model was used to resolve the problem. The life-cycle model is a heuristic numerical technique that considers different stages of evolution to reflect natural phases, as experienced in life. This strategy uses two nature based methods (GA and PSOA).

2. Standard differential evolution algorithm

A general single-objective problem in a continuous real domain is considered. A general formulation of such a problem is as follow.

$$\min_x \{f(x)\} \quad \text{s.t. } x^l \le x \le x^u \tag{1}$$

where $x = \{x_1, \cdots, x_j, \cdots, x_n\}$ is a set of real parameters, $x^l = \{x_1^l, \cdots, x_j^l, \cdots, x_n^l\}$ and $x^u = \{x_1^u, \cdots, x_j^u, \cdots, x_n^u\}$ are lower and upper bounds of x, respectively. The solution that minimizes the objective function $f(x)$ (OF) is denoted as x^*.

The floating-point code is the most appropriate choice when problem variables are continuous because it allows for the representation of the precision of the machine and requires a reduced time of elaboration because code/decode processes are not required. The use of the floating-point code is typical in several identification problems, for instance Tang et al. (2008) or Monti et al. (2010), and is suitable for engineering applications. However, combinatorial or mixed-integer optimization problems are also common in engineering. Although some methods have been proposed to overcome the issue of integer or discrete variables when using a floating-point code, for instance Lampinen and Zelinka (1999), it is always advisable to adopt appropriate numerical strategies to consider realizable solutions. Therefore, the strategies discussed in this paper are not suitable for solving combinatorial or mixed-integer optimization problems. Another relevant issue deals with the existence of constraints. The original DEA was usually applied to unconstrained continuous optimization problems was the case with many other evolutionary algorithms. Several methods exist to incorporate a constraints handling technique into originally unconstrained evolutionary algorithms (Coello, 2002).

2.1. *Mutation operators*

The standard version of the DEA uses the differences between randomly selected individuals as the source of random variations for a third individual referred to as the target vector. Trial solutions are generated by adding weighted difference vectors to the target vector. This process is dubbed as mutation operator and its main goal is to enhance diversity in the current population as well as "to move" the individuals in such a way a better result (Fiore et al., 2016a). By computing the differences between two individuals randomly chosen from the population, the algorithm estimates the gradient in that zone rather than in a single point of the search space. Consider ${}^k x_i = \{{}^k x_{i1}, \cdots, {}^k x_{ij}, \cdots, {}^k x_{in}\}$ the ith individual (with $i = 1, \cdots, N$) at generation k. The initial population ${}^0 x_i$, for $i = 1, \cdots, N$, is defined by generating randomly the collection of N solutions within the specified search space. In this study, the Latin hypercube sampling technique has been iteratively used to generate the best initial population with minimum correlation between samples (Tang, 1993). During the generation $k+1$, for each individual ${}^k x_i$, a mutation vector ${}^{k+1} v_i$ is computed by means of one of the following alternatives.

$${}^{k+1} v_i = {}^k x_{r1} + F^1({}^k x_{r2} - {}^k x_{r3}) \tag{2}$$

$${}^{k+1} v_i = {}^k x_{\text{best}} + F^1({}^k x_{r1} - {}^k x_{r2}) \tag{3}$$

$${}^{k+1} v_i = {}^k x_i + F^2({}^k x_{\text{best}} - {}^k x_i) + F^1({}^k x_{r1} - {}^k x_{r2}) \tag{4}$$

$${}^{k+1} v_i = {}^k x_{\text{best}} + F^2({}^k x_{r1} - {}^k x_{r2}) + F^1({}^k x_{r3} - {}^k x_{r4}) \tag{5}$$

$${}^{k+1} v_i = {}^k x_{r1} + F^2({}^k x_{r2} - {}^k x_{r3}) + F^1({}^k x_{r4} - {}^k x_{r5}) \tag{6}$$

where r_1, r_2, r_3, r_4 and r_5 denote integers randomly selected within the set $\{1, \cdots, i\text{-}1, i, i+1, \cdots, N\}$ such that $r_1 \ne r_2 \ne r_3 \ne r_4 \ne r_5$, the individual ${}^k x_{\text{best}}$ is the best performer in the population at the generation k, the coefficients F^1 and F^2 are the so-called mutation coefficients and are real positive constants and they are real positive constants whose typical values are in the range [0.40,1.00], and 0.50 is in our numerical applications.

2.2. *Crossover operator*

The crossover follows the mutation phase. For each mutated vector ${}^{k+1} v_i$, a trial vector ${}^{k+1} u_i$ (offspring) is generated by using the following Eq. (7) so-called binomial crossover.

$${}^{k+1} u_{ij} = \begin{cases} {}^{k+1} v_{ij} & w \le p^c \text{ or } j = \text{randint}(1, n) \\ {}^k x_{ij} & \text{otherwise} \end{cases} \tag{7}$$

where w is a random number generated by using the uniform probability density functions in the range [0,1], j is the gene under consideration and n is a integer number, parameter p^c is the probability of crossover and it assumes values between 0 and 1, randint $(1,n)$ is a randomly integer selected within the set $\{1, \cdots, n\}$ and is adopted to ensure that at least one parameter is taken into account for constructing the vector ${}^{k+1} u_i$. Typically, the probability of crossover p^c is much more sensitive to the problem's property and complexity, such as the multimodality, whereas the mutation constants regulate the convergence speed. It was found that $p^c = 0.5$ is a good choice (for instance in Kaelo and Ali (2006) and its references).

2.3. Selection

The selection operator employs a very simple one-to-one competition scheme between ${}^{k+1}u_i$ and ${}^{k+1}x_i$ as follow.

$$ {}^{k+1}x_i = \begin{cases} {}^{k+1}u_i & f({}^{k+1}u_i) < f({}^{k+1}x_i) \\ {}^{k}x_i & \text{otherwise} \end{cases} \tag{8} $$

Therefore, the winner ${}^{k+1}x_i$ in the selection stage is the best performer between the parent individual ${}^{k}x_i$ and its trial one ${}^{k+1}u_i$. The output of this operator is a new population for the next generation, unless a stopping criteria has not been fulfilled. The evolutionary search will terminate once a maximum number of iterations L is achieved.

2.4. Preserving the feasibility of the solution

An opportune strategy is needed to ensure the feasibility of the obtained solutions (Fiore et al., 2016b; Quaranta et al., 2014b). This means obtaining the fulfillment of the lower and upper bounds of the search space could be in the optimization problem Eq. (1). To achieve the goal, the mutation operator can be repeated until a feasible solution is obtained. Some authors adopt penalty based methods in which a penalized OF value is assigned to the unfeasible solutions. However, both strategies do not ensure the obtainment of a feasible solution. The assignment of the penalty model and the numerical definition of the penalty weight are not an easy task. Usually, the introduction of new control parameters are required in the algorithm.

3. Optimization-based strategy for identification problems

Engineering analysis can be broadly categorized as direct and inverse analysis. Direct analysis for structural systems aim to predict structural response (output) for given excitation (input) and known system parameters (Fiore et al., 2013a, 2016c; Resta et al., 2013), whereas inverse analysis deals with identification of structural parameters based on given input and output (I/O) information (Colapietro et al., 2013; Fiore and Marano, 2017). The latter may be termed as "structural identification" and falls within the larger domain of system identification. Structural identification can be applied to update or calibrate structural models so as to better predict response and achieve more cost-effective designs. It can also be used for structural health monitoring and damage assessment in a nondestructive way by tracking changes in pertinent structural parameters. This is especially useful for identifying structural damage caused by natural actions, such as earthquakes. For structural control applications, identification of actual parameters is essential for effective control. From a computational point of view, structural identification presents a very challenging problem, particularly when the system involves a large number of unknown parameters. In addition to accuracy and efficiency, robustness is an important issue for selecting the identification strategy. Presently, the main hurdle is the lack of a robust and intelligent computational strategy to identify parameters and the given limited number of sensors and inevitable noise in reality. Koh et al. (2003) suggest that a good identification method should fulfill the following features.

(I) The method should not require a good start-point (good initial values for the investigated parameters) in order to converge to the correct solution.
(II) The identification strategy should preferably be not too sensitive to noise.
(III) A useful property requires a good identification when handling incomplete measurements. On its part, this characteristic deals with two possible issues.
  (i) It is not necessary to have measurements at all degrees of freedom (DOFs).
  (ii) It is indispensable to suppose that, in most cases, a unique response typology is monitored (generally, accelerations). When this happens, it is absolutely preferable to operate on the available set of data directly without any type of mathematical manipulation (integration or derivation).

Identification methods proposed until now can be classified by means of different criteria. To begin with, there are frequency and time domain methods, parametric and non-parametric methods, deterministic and non-deterministic methods, classical and non-classical methods. Non-classical methods in structural identification (artificial neural networks, genetic algorithms, genetic programming, differential evolution algorithms, particle swarm optimization algorithms, ant colony based algorithms) are continuously gaining attention in virtue of their robustness and efficacy (especially in real structures) and are still under investigation. This is imputable to the lack of accurate and detailed mathematical formulations, so that in many circumstances their convergence cannot be formally demonstrated a-priori but only via "trial-and-error" validations.

4. Proposed operators for differential evolution algorithms

The functionality of the above soft computing based techniques for systems identification depends on a quantity of control parameters (acceleration factors and inertia weight for PSOAs; mutation coefficients and probability of crossover for DEAs). Consequently, a parameters tuning stage is needed to improve (if possible) the final result. Unfortunately this operation is not so simple. For instance, the user may assign the numerical values by analyzing the results carried out from numerical sensitivity analyses: this strategy requires an appropriate knowledge of the optimizer to achieve the expected performances as soon as possible, and therefore a high degree of interaction between the user and the adopted optimizer is needed. Alternatively, this end can be achieved by employing another algorithm (i.e., a neural network can be developed for this goal). However, also this choice is not totally immune from criticisms since it introduces a considerable level of complexity and thus its use becomes less attractive for non-experts in the field of soft computing methodologies. Therefore, the usability and the autonomy of soft computing techniques in mechanical systems identification can be sensibly improved looking for numerical strategies whose functionality requires adaptive or self-adaptive control parameters. An optimal result will be the use of free-parameters operators.

4.1. *Proposed adaptive mutation operator*

The first modification of the standard DEA concerns the mutation operator. In detail, the proposal deals with the following revised version of Eq. (4).

$$ {}^{k+1}v_i = {}^{k}x_i + {}^{k}F_{r3,i}({}^{k}x_{r3} - {}^{k}x_i) + {}^{k}F_{r1,r2}({}^{k}x_{r1} - {}^{k}x_{r2}) \quad \frac{k}{L} \le \kappa \tag{9} $$

$$ {}^{k+1}v_i = {}^{k}x_i + {}^{k}F_{best,i}({}^{k}x_{best} - {}^{k}x_i) + {}^{k}F_{r1,r2}({}^{k}x_{r1} - {}^{k}x_{r2}) \quad \frac{k}{L} > \kappa \tag{10} $$

The mutation coefficients are calculated as follows.

$$ {}^{k}F_{r3,i} = \max\left\{\left|\frac{f({}^{k}x_{r3}) - f({}^{k}x_i)}{{}^{k}f_{max} - {}^{k}f_{min}}\right|, 0.5\right\} \tag{11} $$

$$ {}^{k}F_{r1,r2} = \begin{cases} \max\left\{\left|\dfrac{f({}^{k}x_{r1}) - f({}^{k}x_{r2})}{{}^{k}f_{max} - {}^{k}f_{min}}\right|, 0.5\right\} & \dfrac{k}{L} \le \kappa \\ \left|\dfrac{f({}^{k}x_{r1}) - f({}^{k}x_{r2})}{{}^{k}f_{max} - {}^{k}f_{min}}\right| & \dfrac{k}{L} > \kappa \end{cases} \tag{12} $$

$$ {}^{k}F_{best,i} = \left|\frac{{}^{k}f_{min} - f({}^{k}x_i)}{{}^{k}f_{max} - {}^{k}f_{min}}\right| \tag{13} $$

in which

$$ \begin{cases} {}^{k}f_{min} = \min\limits_{i=1,\cdots,N}\{f({}^{k}x_i)\} \\ {}^{k}f_{max} = \max\limits_{i=1,\cdots,N}\{f({}^{k}x_i)\} \end{cases} \tag{14} $$

The process for generating new solutions using Eqs. (9) and (10) is represented in Fig. 1 over a generic bi-dimensional real search space. Based on the proposed scheme, the mutation occurs in two distinct manners. The first one takes place when $k/L \le \kappa$ and its goal is to help the exploration of the search space. In this effort, the weighted difference vectors in Eq. (9) only involve randomly selected individuals and the mutation coefficients are forced to be greater than 0.5, Eq. (11) and the first equality in Eq. (12). When $k/L > \kappa$, the mutation scheme proposed in Eq. (10) is performed. In this case, the current best individual ${}^{k}\mathbf{x}_{best}$ is taken into account. Fundamentally, the goal of this alternative scheme is to keep track of the current best performer within the population. Moreover, an improved exploitation can be achieved by removing the lower bounds for the adopted mutation coefficients (therefore numerical values less than 0.5 are accepted this time). There is not rigid separation between exploration and exploitation because the numerical values of the scale factors are dynamically adjusted during the evolutionary search. For instance, if the exploration of the search space is not concluded for $k/L \le \kappa$, then the numerical values of the mutation parameters in Eqs. (12) and (13) remain sensibly large and the global recognition is not penalized. It should be observed that the numerical values of the mutation coefficients are always less than 1 and this upper bound is in agreement with most of literature (see the above discussion about the scale factors for the standard DEA). Similarly, both lower bounds in Eqs. (11) and (12) are also compatible with the suggestions provided by current state of practice. A good value for κ has been carried out from numerical investigations based on the solution of the benchmark optimization problems presented in this paper. The numerical analysis demonstrate that a good value for κ should be selected within 0.40 and 0.60. In this paper, $\kappa = 0.50$ and this numerical value is proposed as a

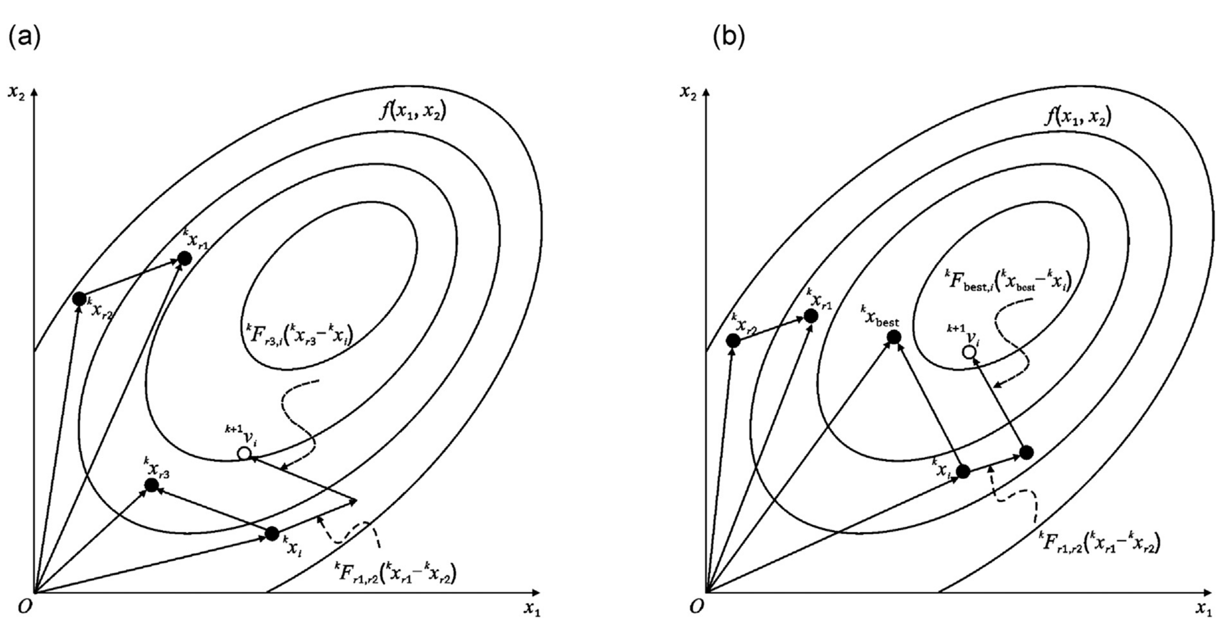

Fig. 1 – Bi-dimensional example of an objective function showing its contour lines and the process for generating new solutions by means of the proposed mutation operator. (a) $k/L \le \kappa$. (b) $k/L > \kappa$.

starting choice for further applications of the proposed mutation operator.

4.2. Proposed crossover operator

The second modification on the standard DEA deals with the crossover operator. In this case, the binomial scheme Eq. (7) is replaced with the following one.

$$^{k+1}u_i = {}^{k+1}w_i \times {}^{k}x_i + (\mathbf{1} - {}^{k+1}w_i) \times {}^{k+1}v_i \quad (15)$$

where $^{k+1}w_i$ is a vector whose n components are random numbers generated by using the uniform probability density functions in the range [0,1]. The symbol "×" denotes the term-by-term vector multiplication and $\mathbf{1} = (1_1, \cdots, 1_j, \cdots, 1_n)$.

Fig. 2 provides a graphical visualization of the effects of the proposed crossover scheme also with reference to a generic bi-dimensional search space. The candidate results of the binomial crossover in Eq. (7) are the vertex points of the hypercube described by $^{k+1}v_i$ and $^{k+1}x_i$, (see the triangular markers in Fig. 2 for a bi-dimensional search space). These two solutions marked with a triangular symbol are also candidate solutions of the crossover operator presented in Eq. (15) when $^{k+1}w_{ij} \to 0$ or $^{k+1}w_{ij} \to 1$ for each $j = 1, \cdots, n$. However, unlike the binomial crossover, the proposed one allows the exploration of the inner space bound by the hypercube (Fig. 2). It is evident that the probability of reproduction is not required to perform the proposed crossover. Thus, this is a free-parameter operator.

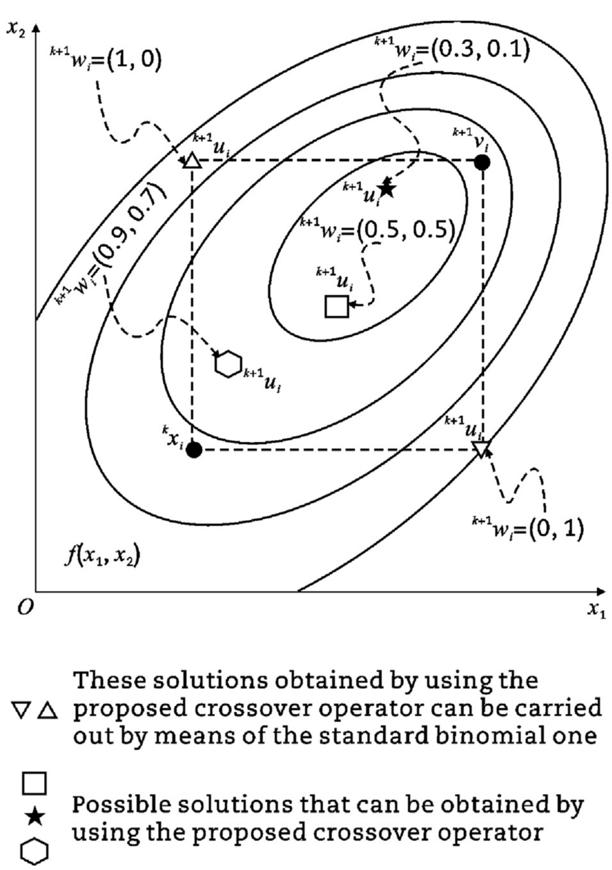

Fig. 2 – Some solutions that may be carried out by means of the proposed crossover operator. Two solutions (denoted with triangular markers) can also be obtained by performing a standard binomial scheme.

Table 1 – Statistical results for "Branin" $-f(x^*) = 0.397887$ (population size 20, number of generations 40, number of variables 2).

Algorithm	Min	Max	Mean	Std
DEA01	0.3979	0.3980	0.3979	2.9369e-5
DEA02	0.3979	0.3979	0.3979	2.2368e-8
DEA03	0.3979	0.3979	0.3979	1.2955e-6
DEA04	0.3979	0.3979	0.3979	1.4190e-6
DEA05	0.3979	0.3984	0.3980	1.1834e-4
DEA06	**0.3979**	**0.3979**	**0.3979**	**5.6854e-6**
PSOA01	0.3979	0.3979	0.3979	2.0286e-6
PSOA02	0.3979	0.3981	0.3979	4.0300e-5
PSOA03	0.3979	0.3979	0.3979	8.8494e-8
PSOA04	0.3979	0.4009	0.3987	6.3956e-4

Bold represents DEA06 is compared with other results.

5. Comparison with standard differential evolution algorithms and several swarm intelligence based optimizers

Comparative numerical analyses have been performed to estimate the performances of the proposed optimizer. This numerical study involves the class of the standard DEAs (the complete list is given in Appendix). Particle swarm based optimizers are also taken into account to include a different paradigms in the numerical competition. The list of the adopted PSOAs is given in Appendix. To protect the cohesion of the swarm, the velocities of the particles are forced to be in absolute value, less than a maximum velocity given by $(x^u - x^l)/2$. In the PSOA02, the maximum velocity is limited to the dynamic range of the particle on each dimension (Eberhart and Shi, 2000). A craziness operator has been performed on the velocity of some particles with the aim of increasing the direction diversity in the swarm (Fourie and Groenwold, 2002). The probability of craziness is equal to 0.05 for all the PSOAs and it is assumed to be a constant value.

Table 2 – Statistical results for "Haupt-1" $-f(x^*) = -18.5547$ (population size 30, number of generations 60, number of variables 2).

Algorithm	Min	Max	Mean	Std
DEA01	−18.5547	−18.5547	−18.5547	1.0337e-08
DEA02	−18.5547	−18.5547	−18.5547	1.8257e-14
DEA03	−18.5547	−18.5547	−18.5547	6.0333e-08
DEA04	−18.5547	−18.5547	−18.5547	1.1540e-10
DEA05	−18.5547	−18.5542	−18.5547	7.6339e-05
DEA06	**−18.5547**	**−18.5540**	**−18.5547**	**1.5485e-04**
PSOA01	−18.5547	−18.5547	−18.5547	4.4627e-07
PSOA02	−18.5547	−18.5532	−18.5547	2.2281e-04
PSOA03	−18.5547	−18.5547	−18.5547	2.0082e-08
PSOA04	−18.5546	−18.4972	−18.5485	0.0095

Bold represents DEA06 is compared with other results.

Table 3 – Statistical results for "Haupt-2" $-f(x^*) = -345.3599$ (population size 30, number of generations 60, number of variables 2).

Algorithm	Min	Max	Mean	Std
DEA01	−345.3599	−345.3599	−345.3599	4.1856e-07
DEA02	−345.3599	−211.4547	−342.6818	18.9371
DEA03	−345.3599	−345.3599	−345.3599	8.9519e-07
DEA04	−345.3599	−345.3599	−345.3599	1.6688e-07
DEA05	−345.3599	−345.3573	−345.3598	5.1078e-04
DEA06	**−345.3599**	**−345.3599**	**−345.3599**	**1.4066e-05**
PSOA01	−345.3599	−345.3595	−345.3599	7.4264e-05
PSOA02	−345.3599	−211.4547	−337.0930	28.7569
PSOA03	−345.3599	−252.2501	−343.4977	13.1677
PSOA04	−345.3417	−340.2211	−344.5178	1.0775

Bold represents DEA06 is compared with other results.

Several numerical comparative analyses are presented to assess, both, accuracy and reliability of the proposed soft computing based non-classical identification technique.

The numerical competition is performed over thirteen benchmark test functions from the standard set of optimization problems available in the specialized literature (Neculai, 2008). The test functions reflect different degrees of complexity. A complete list of information about the benchmark test functions adopted for this numerical investigation is given in the Appendix and results are given in Tables 1–13. The optimization problems are solved fifty times by using five DEAs, four PSOAs, and the modified DEAs proposed in this study, and the results are recorded. The initial population is different for each run. Statistical results are carried out from the recorded numerical analyses and they are presented in the following Tables. For each method, the best (Min), the worst (Max) and the average (Mean) value, as well as the standard deviation (Std), are calculated over the fifty simulated runs. The number of evaluations is equal for all optimizers (equal to $N \times L$), so it is not taken into consideration for the comparative analyses.

Table 4 – Statistical results for "Shaffer" $-f(x^*) = 0$ (population size 30, number of generations 60, number of variables 2).

Algorithm	Min	Max	Mean	Std
DEA01	0.0014	0.0108	0.0085	0.0028
DEA02	< 1e-021	0.0099	0.0059	0.0044
DEA03	< 1e-021	0.0099	0.0079	0.0032
DEA04	1.8938e-04	0.0098	0.0085	0.0028
DEA05	0.0018	0.0237	0.0099	0.0023
DEA06	**1.4514e-05**	**0.0097**	**0.0086**	**0.0030**
PSOA01	< 1e-021	0.0097	0.0078	0.0039
PSOA02	2.6540e-04	0.0099	0.0088	0.0026
PSOA03	< 1e-021	0.0097	0.0049	0.0049
PSOA04	< 1e-021	0.0236	0.0094	0.0037

Bold represents DEA06 is compared with other results.

Table 5 – Statistical results for "Six-hump Camel" $-f(x^*) = -1.0316$ (population size 30, number of generations 60, number of variables 2).

Algorithm	Min	Max	Mean	Std
DEA01	−1.0316	−1.0316	−1.0316	3.6868e-06
DEA02	−1.0316	−1.0316	−1.0316	4.6403e-16
DEA03	−1.0316	−1.0316	−1.0316	4.0126e-08
DEA04	−1.0316	−1.0316	−1.0316	1.3290e-08
DEA05	−1.0316	−1.0314	−1.0316	3.1209e-05
DEA06	**−1.0316**	**−1.0316**	**−1.0316**	**1.5049e-06**
PSOA01	−1.0316	−1.0316	−1.0316	9.5837e-09
PSOA02	−1.0316	−1.0316	−1.0316	7.3008e-06
PSOA03	−1.0316	−1.0316	−1.0316	1.3192e-10
PSOA04	−1.0316	−1.0235	−1.0298	0.0018

Bold represents DEA06 is compared with other results.

Table 6 – Statistical results for "Ackley" $-f(x^*) = 0$ (population size 150, number of generations 300, number of variables 30).

Algorithm	Min	Max	Mean	Std
DEA01	1.909034	2.632981	2.307796	0.208676
DEA02	2.27e-07	19.859370	5.209168	8.717906
DEA03	3.23e-07	6.33e-05	5.46e-06	1.19e-05
DEA04	2.489280	19.854420	12.86839	4.759546
DEA05	19.420500	19.966770	19.89776	0.096700
DEA06	0.688304	3.126793	1.818985	0.604219
PSOA01	0.000573	1.899835	0.609118	0.661950
PSOA02	19.954970	19.962750	19.95868	0.001205
PSOA03	0.004908	1.971800	0.381879	0.558306
PSOA04	2.791422	3.693946	3.308441	0.253601

Table 7 – Statistical results for "Cosine Mixture" $-f(x^*) = -0.1n$ (population size 50, number of generations 150, number of variables 20).

Algorithm	Min	Max	Mean	Std
DEA01	−1.9895	−1.9230	−1.9596	0.0135
DEA02	−2.0000	−1.7044	−1.8788	0.1144
DEA03	−2.0000	−1.7044	−1.9793	0.0668
DEA04	−1.9969	−1.8032	−1.9722	0.0350
DEA05	−0.9692	−0.4032	−0.6691	0.1356
DEA06	**−1.9997**	**−1.9919**	**−1.9983**	**0.0013**
PSOA01	−2.0000	−1.4086	−1.8491	0.1594
PSOA02	−1.9994	−1.2605	−1.7279	0.1839
PSOA03	−1.9994	−1.3859	−1.8445	0.1304
PSOA04	−1.9791	−1.0783	−1.6603	0.1871

Bold represents DEA06 is compared with other results.

Table 8 – Statistical results for "Goldstein and Price" $-f(x^*) = 3$ (population size 30, number of generations 60, number of variables 30).

Algorithm	Min	Max	Mean	Std
DEA01	3.0000	3.0046	3.0004	0.0009
DEA02	3.0000	3.0000	3.0000	0.0000
DEA03	3.0000	3.0001	3.0000	0.0000
DEA04	3.0000	3.0002	3.0000	0.0000
DEA05	3.0000	3.0244	3.0048	0.0061
DEA06	**3.0000**	**3.0005**	**3.0001**	**0.0001**
PSOA01	3.0000	3.0000	3.0000	0.0000
PSOA02	3.0000	3.0004	3.0001	0.0001
PSOA03	3.0000	3.0000	3.0000	0.0000
PSOA04	3.0006	3.3587	3.0521	0.0724

Bold represents DEA06 is compared with other results.

Table 9 – Statistical results for "Griewank" $-f(x^*) = 0$ (population size 100, number of generations 200, number of variables 30).

Algorithm	Min	Max	Mean	Std
DEA01	1.7422	2.1580	1.9504	0.1349
DEA02	0.0000	0.0295	0.0080	0.0081
DEA03	0.0000	0.0173	0.0020	0.0047
DEA04	1.0767	1.3189	1.1886	0.0582
DEA05	33.4896	66.6038	48.6475	7.2088
DEA06	**1.4768**	**2.7829**	**1.9628**	**0.3406**
PSOA01	0.0061	0.1256	0.0524	0.0301
PSOA02	0.0837	0.9074	0.4211	0.2376
PSOA03	0.1532	1.0047	0.5416	0.2096
PSOA04	1.3043	1.9978	1.6526	0.1746

Bold represents DEA06 is compared with other results.

Table 10 – Statistical results for "Rastrigin" $-f(x^*) = 0$ (population size 400, number of generations 800, number of variables 30).

Algorithm	Min	Max	Mean	Std
DEA01	106.2080	138.7823	126.2862	8.2822
DEA02	6.9647	61.5221	18.9021	12.3493
DEA03	84.5703	104.8653	94.6072	5.2677
DEA04	112.0754	162.4771	138.6558	10.5792
DEA05	146.9282	184.6684	171.3315	8.7585
DEA06	**6.1583**	**25.1300**	**14.0261**	**4.9777**
PSOA01	11.9395	36.8134	22.6851	5.9318
PSOA02	28.8538	102.7326	52.4476	17.3502
PSOA03	0.9950	44.8992	19.8384	8.3240
PSOA04	26.6269	86.8123	52.4240	15.1265

Bold represents DEA06 is compared with other results.

Looking at statistical results carried out from bi-dimensional test functions, the differences between investigated optimizers are of negligible significance. This is certainly true for the Haupt-1 problem (Table 2), Shaffer problem (Table 4) and Six-hump Camel problem (Table 5). Moderate differences exist for the Branin problem (Table 1) in which the maximum values of the OF carried out by DEA06 and PSOA04 are higher than the optimal ones. DEA02, PSOA02, PSOA03 and PSOA04 are not so constant in the simulation runs performed to solve the Haupt-2 problem (Table 3). In fact, it can be observed that there is a sensible difference between the maximum value of the OF carried out and its optimal value. Consequently, no ideal performances were found in terms of mean value and standard deviation.

DEA06 has not statistically significant difference with DEA01 for the Griewank problem (Table 9), with DEA02 for the Rastrigin problem (Table 10), with PSOA02 for the Goldstein and Price problems (Table 8) and finally with PSOA04 for the Sphere problem (Table 12), and has been outperformed by DEA03 (Table 9). In all other problems DEA06 has been statistically significant differences. In the resolution of the Ackley problem (Table 6), DEA03 is the best performer, but the behavior of PSOA01 and PSOA03 are also very good. The performances obtained by DEA06 are good enough, but not the best. On the contrary, DEA05, DEA 02 and PSOA02 provide the worst results for this test function. Although the proposed DEA06 does not provide the best minimum value of the OF for the Cosine Mixture problem (Table 7), it is the best optimizer for this test function because maximum value, mean value and standard deviation value calculated over fifty simulation runs are sensibly better than the others. In the case of the Zakharov problem (Table 13), it can be observed that the best performances are achieved by DEA02 and DEA03. In the resolution of Sphere problem (Table 12) DEA06 does not obtained a good performance, it has not statistically differences with PSOA04, while PSOA02, PSOA03 and DEA04 are the best optimizers. In the resolution of Goldstein and Price problem (Table 8) DEA06 has not statistically difference with PSOA02, but all the optimizers provide the solution. The proposed DEA06 is the best competitor for the resolution of the Rastrigin problem (Table 10) and Schwefel problem (Table 11). In both these cases, only the proposed operators enable us to find the optimal solutions. On the contrary, standard DEAs and all the PSOAs are not able to achieve the optimal region of the search space.

6. Impact load identification on a simply supported elastic beam as a case study

It is crucial to know the external loads that act on structural elements or mechanical components in structural monitoring. Load damage can be assessed in terms of the strength, fatigue and reliability of structures. In some circumstances, dynamic loads can be directly measured by means of opportune sensors. Numerical methods for load identification are highly welcome when obtaining a direct measure is complicated in the course of dealing with extremely large magnitudes of loads during a very short period of time (impact loads).

The formulation and resolution of a single-objective optimization problem is a typical way of solving load identification problems. In doing so, the first step is the mathematical definition of the loading model. An appropriate parameterization of the dynamic load allows for a more efficient reconstruction of the time-dependent load history. A point-by-point time domain based identification of the loading process is much more difficult and less reliable. Combined to a forward

Table 11 – Statistical results for "Schwefel" $-f(x^*) = -418.9829n$ (population size 300, number of generations 800, number of variables 30).

Algorithm	Min	Max	Mean	Std
DEA01	−7669.99	−7283.76	−6993.03	178.60780
DEA02	−8732.77	−7755.95	−7213.97	341.28850
DEA03	−6955.44	−6550.02	−6262.62	195.33900
DEA04	−8287.73	−7495.91	−6788.09	516.47710
DEA05	−7317.86	−6723.83	−6377.10	208.52330
DEA06	**−12126.80**	**−11079.30**	**−7050.18**	**1289.14100**
PSOA01	−7932.19	−7438.44	−7097.05	173.61890
PSOA02	−7326.34	−6694.82	−6492.72	188.81350
PSOA03	−6974.06	−6587.57	−6137.41	184.86010
PSOA04	−5476.52	−5423.01	−5417.67	15.07789

Bold represents DEA06 is compared with other results.

Table 12 – Statistical results for "Sphere" $-f(x^*) = 0$ (population size 150, number of generations 300, number of variables 30).

Algorithm	Min	Max	Mean	Std
DEA01	3.48100	7.452200	5.128900	1.068100
DEA02	3.90e-014	5.121e-013	1.742e-013	1.189e-013
DEA03	6.34e-013	2.245e-012	1.220e-012	4.58e-013
DEA04	0.15950	0.577700	0.328200	0.121000
DEA05	1.4263e+003	2.2484e+003	1.7940e+003	0.198e+0037
DEA06	**21.73490**	**107.502400**	**56.299700**	**20.778500**
PSOA01	4.37e-07	0.000606	3.36e-05	0.000110
PSOA02	7.15e-06	0.000917	0.000152	0.000169
PSOA03	6.34e-05	0.003309	0.000925	0.000785
PSOA04	20.30052	81.861310	52.617470	14.206280

Bold represents DEA06 is compared with other results.

Table 13 – Statistical results for "Zakharov" $-f(x^*) = 0$ (population size 100, number of generations 300, number of variables 20).

Algorithm	Min	Max	Mean	Std
DEA01	0.1788	0.5548	0.3211	0.0917
DEA02	7.8972e-13	2.5143e-11	5.9014e-12	5.9037e-12
DEA03	1.1186e-10	2.4279e-09	3.9736e-10	3.7205e-10
DEA04	0.0025	0.0111	0.0069	0.0026
DEA05	24.1523	151.4827	92.7521	29.7932
DEA06	**1.0471e-04**	**0.0022**	**7.9752e-04**	**4.9738e-04**
PSOA01	1.9617e-07	1.6268e-05	3.4076e-06	3.6366e-06
PSOA02	7.2631e-05	0.0251	0.0011	0.0035
PSOA03	1.6771e-05	0.0019	4.3732e-04	3.7322e-04
PSOA04	2.1543	102.1635	20.4623	19.3724

Bold represents DEA06 is compared with other results.

model which characterizes the dynamic response of the structure subject to a known dynamic force, the unknown parameters of the dynamic loads can be identified by minimizing the difference between the computed analytical responses and the actually measured responses. Unfortunately, a problem arising from the optimization-based formulation of the load identification process is that the results can be nonlinear and multi-modal. Differential information (i.e., gradients) or initial information (i.e., starting values of the unknown parameters to be identified) required by traditional optimization approaches may be difficult to calculate and time-consuming. Therefore, gradient-free algorithms with global exploration capabilities may be explored to overcome these difficulties.

The impact load identification on a simply supported elastic beam has been chosen as a case study given that degree of complexity involved is not so high as to jeopardize this effort. The problem has been formalized as a single-objective optimization problem in Wang and Chiu (2003) and has been solved by means of a standard GA in Hashemi and Kargarnovin (2007). The experimental results obtained from a laboratory test can be easily reproduced, given its simplicity.

6.1. Problem formulation

In considering a uniform, simply supported beam with linear elastic behavior, the layout of the system is represented in Fig. 3 in which l is the length of the beam. It represents a widespread structural scheme for bridge structures (Fiore et al., 2012, 2013b).

The beam is subject to an impact force, P, acting on $\xi = \xi_P$ and the dynamic response is measured in $\xi = \xi_S$ with an appropriate sensor. Under the Euler-Bernoulli hypothesis, the equation of the motion $y(\xi,t)$ of the beam solves the following differential equations.

$$EI\frac{\partial^4 y(\xi,t)}{\partial \xi^4} + \frac{\partial}{\partial t}Cy(\xi,t) + \rho A\frac{\partial^2 y(\xi,t)}{\partial t^2} = P\delta(t)\delta(\xi - \xi_p) \tag{16}$$

where E, C and ρ are, respectively, the elastic modulus, the damping constant and the density of the beam, I and A are, respectively, the moment of inertia and the area of the transversal section, the symbol $\delta(\cdot)$ is the Dirac's delta. The right side of Eq. (16) represents an impact force, P, applied to the abscissa ξ_p. $\delta(t)$ is functional to the instantaneous load, while $\delta(\xi-\xi_p)$ is its application at a single point.

The necessary and sufficient boundary conditions are

$$\begin{cases} y(\xi = 0, t) = 0 \\ \left.\dfrac{\partial^2 y(\xi,t)}{\partial \xi^2}\right|_{\xi=0} = 0 \end{cases} \tag{17}$$

$$\begin{cases} y(\xi = l, t) = 0 \\ \left.\dfrac{\partial^2 y(\xi,t)}{\partial \xi^2}\right|_{\xi=l} = 0 \end{cases} \tag{18}$$

According to the above presentation, the impact load identification can be formalized as an inverse problem based on Eq. (16). It is assumed that the displacement response of the beam subject to the unknown impact force is measured at $\xi = \xi_S$, namely $y^m(\xi_S, t_{sample})$ in which t_{sample} is the sampling instant time. Moreover, $y^e(\xi_S, t_{sample}|x)$ is the estimated response from the forward model given by Eq. (16). Vector x collects the load parameters, that is $x = (\xi_P, P)$. Thus, the impact load identification problem is formalized as a single-objective optimization problem, Eq. (1), whose OF is

$$f(x) = \frac{\sum_{sample=1}^{N_{sample}} \left(y^m(\xi_S, t_{sample}) - y^e(\xi_S, t_{sample}|x)\right)^2}{N_{sample}} \tag{19}$$

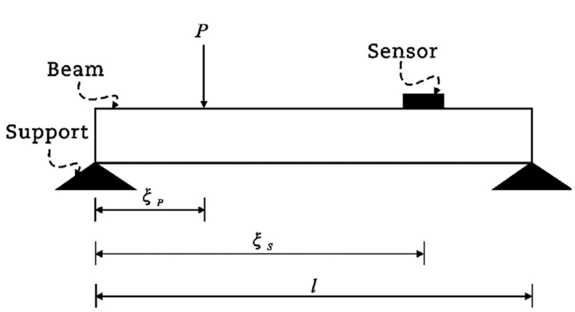

Fig. 3 – General layout of the impact force prediction problem for a simply supported beam.

where N_{sample} is the total length of the records. The final optimal solution, x^*, of the optimization problem provides the impact location and its magnitude.

6.2. Numerical application

Initially, a simulated experiment was conducted with the aim of verifying the correctness of the adopted optimization-based identification strategy and to assess the effectiveness of the proposed DEAs. The physical properties of the investigated beam, as well as the "true values" for both impact force position and magnitude, are listed in Table 14. The impact load prediction problem is solved fifty times by using the five standard DEAs (DEA01-DEA05), four PSOAs (PSOA01-PSOA04), and the modified DEAs proposed in this study (DEA06/DEA07). The lower and the upper bounds for this simulated experiments are x^l = (0.00 m, 0.1 N) and x^u = (3.00 m, 10 N), respectively. The statistical results are listed in Table 15 and a presentation of the convergence histories of the proposed DEA06 is represented in Fig. 4. It can be observed that the best optimizer is DEA02, but the proposed DEA06 provides more constant identification results. In fact, the latter's statistics in terms of max/mean/std values are the best (Table 14). In the search for a compromise between accuracy and algorithm reliability, the DEA06 proves to be the most adequate optimizer for this application. It is interesting to observe that this result is coherent with those previously obtained for the optimization of some bi-dimensional benchmark test functions.

Table 14 – Physical and geometrical properties of the beam.

Property	Value
Length of the beam (m)	3
Base of the transversal section (m)	0.04
Width of the transversal section (m)	0.02
Thickness of the transversal section (m)	0.002
Density (aluminum) (kg/m^3)	2700
Elastic modulus (aluminum) (N/m^2)	70×10^9
Damping ratio (lightly damped beam) (%)	1
Sensor position (m)	2.55
Exact impact force position (m)	0.85
Exact impact force magnitude (N)	5.00

Table 15 – Statistical results for the impact force identification problem.

Algorithm	Best	Worst	Mean	Std
DEA01	0.0033	0.3342	0.0547	0.0589
DEA02	8.3501e-6	0.0016	2.1798e-4	3.3147e-4
DEA03	0.0034	0.9286	0.0819	0.1361
DEA04	0.0016	0.1323	0.0203	0.0230
DEA05	0.0079	0.9512	0.3189	0.2399
DEA06	**9.1652e-05**	**0.0012**	**1.9092e-4**	**2.8798e-4**
PSOA01	1.1850e-4	0.0125	0.0016	0.0019
PSOA02	2.9116e-4	0.0477	0.0109	0.0103
PSOA03	8.2832e-6	0.0113	0.0011	0.0019
PSOA04	0.0164	0.3233	0.1374	0.0722

Bold represents DEA06 is compared with other results.

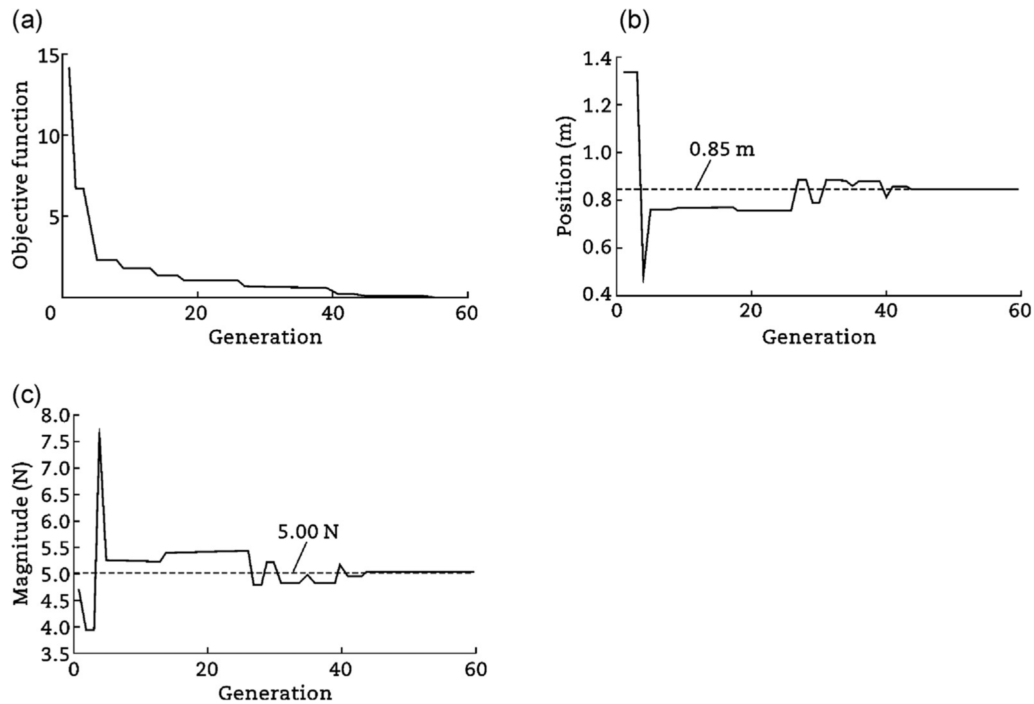

Fig. 4 – Convergence histories of the impact force prediction problem using DEA06. (a) Objective function. (b) Position. (c) Magnitude.

7. Experimental application

Evolutionary algorithms have shown remarkable performances in solving several identification problems using noisy signals without the use of pre-filtering techniques for noise reduction. It has been recognized that said resistance against noise contamination depends on the implicit parallelism of these algorithms (Monti et al., 2010). This interesting feature can be easily assessed experimentally. A simple, low-cost laboratory test has been conducted with the aim of validating the obtained results using real data. The layout of the performed experiment is shown in Fig. 5. The beam is an aluminum alloy element and a single accelerometer with MEMS technology is installed on it. Specifically, the dynamic response is measured by a three-dimensional Kionix KXM52 accelerometer. The linear output range of this accelerometer is ± 2g (g is the gravity acceleration), its sensitivity is equal to 660 mv/g and the noise level is 35 $\mu g/\sqrt{\text{Hz}}$ for two axes and 65 $\mu g/\sqrt{\text{Hz}}$ for the third axis. By limiting the frequency band to 500 Hz, the RMS noise intensity is respectively 0.989 and 1.838 mg. A simple hammer is adopted to reproduce an impact force acting on the beam and the vertical accelerations are resolved by adopting a sampling frequency equal to 1000 Hz. The elaboration unit is a laptop in which a MATLAB SIMULINK program carries out a realtime visualization of the recorded accelerations, as well as the corresponding fast Fourier transforms (FFTs) which are very useful for an instantaneous control of the experiment (Leonardis, 2009). Once acquirement of the dynamic response has terminated, the identification problem can be

Fig. 5 – Experimental test.

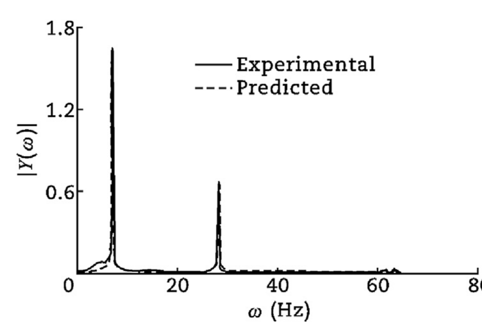

Fig. 6 – Comparison between experimental and predicted Fourier transforms.

solved by using several evolutionary algorithms. The DEA06 was adopted for this experimental application based on the above numerical simulations. The solution of the impact load prediction problem by using the proposed DEA06 shows that the experimental and predicted FFTs match well (Fig. 6). Therefore, the final identification obtained can be considered very satisfactory.

8. Conclusions

The present study dealt with the potentialities of DEA, a stochastic, population-based global optimization method. New schemes for both mutation and crossover operators were proposed at the aim to improve the performances of the standard DEA. In order to prove the efficiency of the new operators, an extensive numerical investigation was firstly carried out, by comparing the corresponding optimization results with thirteen well known benchmark test functions. The results showed that proposed DEA06 is the best competitor for the resolution of the Rastrigin problem and Schwefel problem. In both these cases, only the proposed operators enable us to find the optimal solutions. On the contrary, standard DEAs and all the PSOAs are not able to achieve the optimal region of the search space. In other benchmark test functions, DEA06 has not statistically significant difference with standard DEAs whereas in other problems DEA06 has been statistically significant differences and its performances obtained are good enough, but not the best. Successively the good performances of the proposed DEAs were confirmed by a numerical application regarding a simply supported beam, also experimentally validated. Also in this case the proposed DEA06 performed well because the final identification obtained can be considered very satisfactory.

Conflicts of interest

The authors do not have any conflict of interest with other entities or researchers.

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Appendix

(1) List of DEAs
- DEA01 – A DEA whose mutation operator is given by Eq. (2) and with binomial crossover Eq. (7).
- DEA02 – A DEA whose mutation operator is given by Eq. (3) and with binomial crossover Eq. (7).
- DEA03 – A DEA whose mutation operator is given by Eq. (4) and with binomial crossover Eq. (7).
- DEA04 – A DEA whose mutation operator is given by Eq. (5) and with binomial crossover Eq. (7).
- DEA05 – A DEA whose mutation operator is given by Eq. (6) and with binomial crossover Eq. (7).

Based on the mutation operators – and on the crossover operator Eq. (15), the following modified DEAs have been analyzed:

- DEA06 – A DEA whose mutation operator is given by – and whose crossover operator is given by Eq. (15).

The numerical values for the involved control parameters are the following: $F^1 = 0.50$, $F^2 = 0.50$ and $p^c = 0.50$. They are the same for any generation and for any DEA.

(2) List of PSOA
- PSOA01 – A PSOA with inertia weight, social and cognitive factors. A linearly decreasing inertia weight has been adopted in which the initial value and the final one are respectively 0.9 and 0.4. Moreover, linearly generation-dependent models have been adopted for the cognitive factor with values between 2.5 and 0.5 as well as, with values between 0.5 and 2.5.
- PSOA02 – A PSOA with constriction factor. Using this optimizer, the cognitive and the social factors are assumed both equal to 2.05 (constant value).
- PSOA03 – A PSOA based on the use of chaotic maps (so-called chaotic PSOA). A Logistic map is used for the inertia weight but scaled in the range 0.4–0.9. The Zaslavskii map is adopted for both cognitive and social factors but scaled in the range 0.5–2.5.
- PSOA04 – A PSOA with passive congregation. The inertia weight linearly decreases between 0.9 and 0.7. The cognitive factor, the passive congregation factor and the social are equal to 0.5 (constant value).

(3) Benchmark test functions
- Ackley

$$f(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{j=1}^{n}x_j^2}\right) - \mathrm{xp}\left(\frac{1}{n}\sum_{j=1}^{n}\cos(2\pi x_j)\right) + 20 + \mathrm{e}$$

$$-32 \le x_j \le 32; x^* = \{0, \cdots, 0\}; f(x^*) = 0$$

- Branin

$$f(x) = \left(x_3 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$$

$$-5 \le x_1 \le 10; 0 \le x_2 \le 15$$

$$x^* = \{-\pi, 12.275\}\wedge\{\pi, 2.275\}\wedge\{9.42478, 2.475\}$$

$$f(x^*) = 0.397887$$

- Cosine Mixture

$$f(x) = \sum_{j=1}^{n}x_j^2 - 0.1\sum_{j=1}^{n}\cos(5\pi x_j)$$

$$-1 \le x_j \le 1;\ x^* = \{0, \cdots, 0\}; f(x^*) = -0.1n$$

- Goldstein and Price

$$f(x) = \left[1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)\right] \cdot \left[30 + (2x_1 - 3x_2)^2 (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)\right]$$

$$-2 \le x_1, x_2 \le 2; x^* = \{0, -1\}; f(x^*) = 3$$

- Griewank

$$f(x) = 1 + \frac{1}{4000}\sum_{j=1}^{n} x_j^2 - \prod_{j=1}^{n} \cos\left(\frac{x_j}{\sqrt{j}}\right)$$

$$-600 \le x_i \le 600; x^* = \{0, \cdots, 0\}; f(x^*) = 0$$

- Haupt-1

$$f(x) = x_1 \sin(4x_1) + 1.1x_2 \sin(2x_2)$$

$$0 \le x_i \le 10; x^* = \{9.039, 8.668\}; f(x^*) = -18.5547$$

- Haupt-2

$$f(x) = -e^{-0.2\sqrt{x_1^2+x_2^2}+3(\cos(2x_1)+\sin(2x_2))}$$

$$-5 \le x_1, x_2 \le 5; x^* = \{0, 0.7687\}; f(x^*) = -345.3599$$

- Rastrigin

$$f(x) = 10n + \sum_{j=1}^{n}\left[x_j^2 - 10\cos(2\pi x_j)\right]$$

$$-5 \le x_j \le 5; x^* = \{0, \cdots, 0\}; f(x^*) = 0$$

- Shaffer

$$f(x) = 0.5 + \frac{\sin\left(\sqrt{x_1^2 + x_2^2}\right)^2 - 0.5}{(1 + 0.001(x_1^2 + x_2^2))^2}$$

$$-100 \le x_1, x_2 \le 100; x^* = \{0, 0\}; f(x^*) = 0$$

- Schwefel

$$f(x) = -\sum_{j=1}^{n} x_j \sin\left(\sqrt{|x_j|}\right)$$

$$-500 \le x_j \le 500; x^* = \{420.9867, \cdots, 420.9867\}$$

$$f(x^*) = -418.9829n$$

- Six-hump Camel

$$f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$$

$$-5 \le x_1, x_2 \le 5$$

$$x^* = \{0.08983, -0.7126\} \wedge \{-0.08983, 0.7126\}$$

$$f(x^*) = -1.0316$$

- Sphere

$$f(x) = \sum_{j=1}^{n} x_j^2$$

$$-100 \le x_j \le 100; x^* = \{0, \cdots, 0\}; f(x^*) = 0$$

- Zakharov

$$f(x) = \sum_{j=1}^{n} x_j^2 + \left(\sum_{j=1}^{n} \frac{j}{2} x_j\right)^2 + \left(\sum_{j=1}^{n} \frac{j}{2} x_j\right)^4$$

$$-5 \le x_j \le 5; x^* = \{0, \cdots, 0\}; f(x^*) = 0$$

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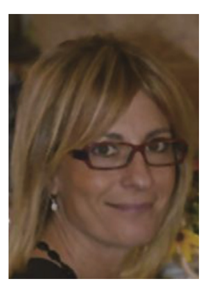

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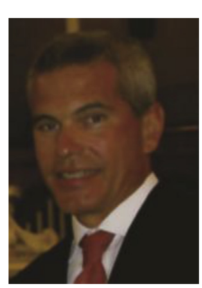

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