Manuscript Details

Manuscript number	JTRP_2017_468_R1
Title	User satisfaction based model for resource allocation in bike-sharing systems
Article type	Full Length Article

Abstract

Over the past decade, the number of ongoing bike-sharing programs has remarkably risen. In this framework, operators need appropriate methodologies to support them in optimizing the allocation of their resources to globally enhance the bike-sharing program, even without massive and costly interventions on the existing configuration of the system. In this paper, we propose an optimization model able to determine how to employ a given budget to enhancing a bike-sharing system, maximizing the global user satisfaction. During the day, each bicycle station has a certain number of bikes that fluctuates according to the travel demand; it happens, however, that for certain time slots, the station is full or empty. Then, we propose to consider as key performance indicators the zero-vehicle time and the full-port time, that reflected respectively the duration of vehicle shortage and parking stall unavailability in the stations. Both these indicators, together with the lost users of the system, need to be kept to a minimum if the final aim is maximizing the customer satisfaction, i.e. not forcing the user to use other stations or turn/shift to other travel modes. We have analyzed the historical usage patterns of the bike-sharing stations, smoothing their trends (by wavelets), and operated a preliminary spatio-temporal clustering. Our model verifies the necessity of adding or removing racks to each station, setting at the same time the optimal number of bikes to allocate in them, and decide the eventual realization of further stations. Then, an application, both on a small test and a real-size network, is presented, together with a sensitivity analysis.

Keywords	resource allocation; bike-sharing system; spatio-temporal clustering.
Manuscript category	Special Issue Material
Manuscript region of origin	Europe

Corresponding Author	Leonardo Caggiani
Corresponding Author's Institution	Politecnico di Bari
Order of Authors	Leonardo Caggiani, Rosalia Camporeale, Mario Marinelli, Michele Ottomanelli

Submission Files Included in this PDF

File Name [File Type]

Cover letter.docx [Cover Letter]

Answers for reviewers.docx [Response to Reviewers (without Author Details)]

Highlights.docx [Highlights]

Title page.docx [Title Page (with Author Details)]

Full paper-FINAL.docx [Manuscript (without Author Details)]

Fig_2.tif [Figure]

FIGURE 3.tif [Figure]

FIGURE 4.tif [Figure]

FIGURE 5.tif [Figure]

FIGURE 6.tif [Figure]

https://www.sciencedirect.com/science/article/abs/pii/S0967070X17305322?via%3Dihub
Caggiani L, Camporeale R, Marinelli M, Ottomanelli M, User satisfaction based model for resource allocation in bike-sharing systems, Transport Policy, Volume 80, 2019, Pages 117-126, ISSN 0967-070X,
https://doi.org/10.1016/j.tranpol.2018.03.003.

Highlights

- We propose an optimization model able to enhance a bike-sharing system
- The objective function aims at maximizing user satisfaction under a given budget
- An application, both on a small test and on a real-size network, is presented

User satisfaction based model for resource allocation in bike-sharing systems

Leonardo Caggiani[1], Rosalia Camporeale, Mario Marinelli, Michele Ottomanelli
{leonardo.caggiani; rosalia.camporeale; mario.marinelli; michele.ottomanelli}@poliba.it

Dept. of Civil, Environmental, Land, Building Engineering and Chemistry (DICATECh)
Polytechnic University of Bari, Viale Orabona, 4 - 70125 Bari, Italy

[1] Corresponding author: leonardo.caggiani@poliba.it

User satisfaction based model for resource allocation in bike-sharing systems

Abstract. Over the past decade, the number of ongoing bike-sharing programs has remarkably risen. In this framework, operators need appropriate methodologies to support them in optimizing the allocation of their resources to globally enhance the bike-sharing program, even without massive and costly interventions on the existing configuration of the system.
In this paper, we propose an optimization model able to determine how to employ a given budget to enhancing a bike-sharing system, maximizing the global user satisfaction. During the day, each bicycle station has a certain number of bikes that fluctuates according to the travel demand; it happens, however, that for certain time slots, the station is full or empty. Then, we propose to consider as key performance indicators the zero-vehicle time and the full-port time, that reflected respectively the duration of vehicle shortage and parking stall unavailability in the stations. Both these indicators, together with the lost users of the system, need to be kept to a minimum if the final aim is maximizing the customer satisfaction, i.e. not forcing the user to use other stations or turn/shift to other travel modes. We have analyzed the historical usage patterns of the bike-sharing stations, smoothing their trends (by wavelets), and operated a preliminary spatio-temporal clustering. Our model verifies the necessity of adding or removing racks to each station, setting at the same time the optimal number of bikes to allocate in them, and decide the eventual realization of further stations. Then, an application, both on a small test and a real-size network, is presented, together with a sensitivity analysis.

Keywords: resource allocation; bike-sharing system; spatio-temporal clustering.

1. Introduction

Moving toward sustainable mobility, public bicycles schemes -better known as bike-sharing systems (BSSs)- have recently become increasingly popular for inner-city transportation. A BSS is a short-term bicycle rental service; it consists of a set of docking stations (i.e. pick-up and drop-off locations) usually scattered throughout an urban setting, together with a set of bicycles available to the system users. The principle is to provide individuals with a bicycle whenever they need it, without costs and responsibilities associated with bicycle ownership, leaving it behind once they reach the desired destinations.
Among their undeniable benefits, we could mention (Shaheen et al., 2010) the reduction of congestion and emissions, the individual financial savings, the health benefits due to physical activity and their support for multimodal transport connections (by acting as an effective 'last-mile' complement to other public transit systems).
Despite their success, one of the main issue that a BSS may experiment is the lack of resources: this happens when a user arrives at a station that has no bike available, or vice-versa if he/she finds a full station when returning the bicycle. Indeed, the allocation of resources (i.e. bicycles and racks) needs to be opportunely managed by BSS operators, to guarantee an efficient functioning of the system and a reliable alternative to the other means of transportation (Fricker and Gast, 2016).
The goal of this paper is to address this resource allocation problem, investigating the problem of strategically enhance a BSS under a predefined available budget, even without operate massive interventions on the existing system. To do so, we propose an optimization model which provides a spatio-temporal clustering of the usage pattern data related to the BSS stations and a subsequent maximization of the user satisfaction.
The method has been applied to a station-based BSS, but it could be easily adapted even to a free-floating one (Pal and Zhang, 2015), where docking stations and kiosk machines are not necessary, as the bicycles can be locked to ordinary

racks close-by the final trip destinations. Every district/zone can be treated as a bike-sharing station, and the resource distribution subsequent the optimization be done uniformly within the perimeter of each zone (see Caggiani et al., 2017 for further details).
The remainder of the paper is organized as follows. In the next section, the literature review is presented. Then, the proposed methodology that allows properly allocate resources in the system is presented. An application of the methodology to two networks of different size is enclosed, together with a sensitivity analysis and some final remarks about our main findings.

2. How to set up and analyze a bike-sharing system: Literature background

In this section, a literature review is given. The review presents the relevant studies that have been done covering the various facets involved in this study. For sake of clarity, we divided the review into three subsections. The first one is devoted to the network design of a station-based BSS. The second focuses on the associated indicators that have been used to assess the performance of the system. Finally, a summary of the main clustering techniques is reported.

2.1 Design and enhancing of BSSs

Recently, several research methods have been proposed and developed aiming at optimizing the bike-sharing system design and operation, considering as key decision variables the number of bikes (Sayarshad et al., 2012), the capacity and location of stations (Lin and Yang, 2011; García-Palomares et al., 2012; Angelopoulos et al., 2016) and/or the vehicles repositioning (Mahony and Shmoys, 2015 and references therein). These design decisions are commonly subject to restrictions and dependencies, such as the predicted user demand patterns, the synergies between the BSS and the operating transit system, the available budget, and so on. In Romero et al. (2012) proposed a model is used to optimize the location of docking stations in a public sharing bicycle system. They consider simultaneously private car and public bicycle transport modes, considering their interactions through the modeling of the modal split and the assignment of each mode's trips to the network. Martinez et al. (2012) focused on the case of Lisbon presented a BSS design model based on a heuristic, encompassing a Mixed Integer Linear Program (MILP). The model simultaneously optimizes the location of shared biking stations and the fleet dimension considering the bicycle relocation activities too. The allocation and optimization of the layout of the bicycle-sharing system inside the scenic spot and around its influencing are the topics of the paper by Guo et al. (2014). They proposed an optimization model and relevant solution algorithm based on the idea of cluster concept and greedy heuristic.
Since public investments in bike-sharing schemes are normally subject to a given budget, one of the main concern of the public authorities is to maximize the benefits at the design and implementation stages to make their investment as profitable as possible. As a matter of fact, different studies have considered the budget as an essential constraint to include in the formulation of their models. For example, Saharidis et al. (2014) proposed a mathematical formulation for the establishment of a bike-sharing network. Given the available budget and a set of candidate locations, their models select the number and the location of the stations, their capacity, and how many bikes should they have at the beginning of the day (service starting) to minimize the unmet travel demand. Recently, a more comprehensive methodology for the dynamic management of the free-floating BSSs is given by Caggiani et. al (2018) where they propose a new dynamic bike redistribution methodology that starts from the prediction of the number and position of bikes over a system operating area and ends with a relocation decision support system. The relocation process is activated at constant gap times in order to carry out dynamic bike redistribution, mainly aimed at achieving a high degree of user satisfaction and keeping the vehicle repositioning costs as low as possible.
Frade and Ribeiro (2015) proposed an optimization method to design a bike-sharing system based on the maximization of the covered demand and assuming the available budget as a constraint. In addition, it combines strategic decision (i.e. determining stations location and capacity, number of bikes in each of them) with operational decisions, such as the relocation of bicycles. The city of Coimbra is used as a testbed.
Also, Chen and Sun (2015) aimed at the minimization of the total travel time of all users, under the constraint of a certain investment budget, to guarantee that the needs for picking up and dropping off bikes could be satisfied. The

application to a numerical example shows how it is possible to determine not only the number and location of bike stations but also the number of bikes and parking lockers for each one of them.
However, if the BSS has been already implemented, there may be the necessity to allocate a limited amount of money to enhance its functioning, better adapting the system to its actual requirements. Therefore, this paper aims at establishing the most efficient way to invest a given budget in the determination of the optimal number of bikes and racks to add/subtract to each bike-sharing station of an operating system, looking primarily at the maximization of the user satisfaction. This more appropriate way to distribute the available resources could lead to an actual improvement of the situation without messing up with the actual configuration of the system.

2.2 Assessing the performance of a BSS

The design decisions of a bike-sharing scheme are made with concern for both total cost and service levels. Therefore, it becomes important to state the best way to measure the service quality of the system.
Both Yang et al. (2010) and Lin and Yang (2011) proposed to consider at the same time the coverage level (i.e. the fraction of total demand at both origins and destinations that is within some specified time or distance from the nearest rental station) and the availability rate of pick-up bike requests at stations. More recently, Neumann-Saavedra et al. (2016) have defined the service level as the percentage of successfully realized demand trips during a given time horizon. The mean parking time of bikes in a station and the rebalancing frequency of a station are used as key performance index in Benarbia and Labadi (2013) and Labadi et al. (2012), where a Petri-Nets based control model of Public BSS is proposed. The unsatisfied demand level is used by Angeloudis et al. (2014). The total relocation cost is used to measure the performances of the BSS in the works by Benchimol et al. (2011) and Nair et al. (2013). In the work by Kaspi et al. (2016) user dissatisfaction is used and measured by using a weighted sum of the expected shortages of bicycles and lockers at a single station.
However, the most accepted measure in literature seems to be the one that combines two possible situations of unsatisfied demand, namely: (1) a user needing a bike that finds the station empty; (2) a user returning a bike that finds the station full. These key performance indicators have been described by Kek et al. (2006 and 2009) and called zero-vehicle time (ZVT) and full-port time (FPT). When ZVT occurs, the station has no available bicycles and users requests at that station will be rejected. On the other hand, when FPT occurs, the station has no empty racks and users requesting to return her/his bike to that station will also fail. Both ZVT and FPT reduce the attractiveness of BSSs. From operator's point of view, ZVT implies a possible loss of revenue. From user's point of view, ZVT forces users to use other stations, or turn to other modes of travel, while FPT forces users to return the vehicles later or to another station, incurring additional usage cost. The same indicators (although not using the same acronyms) have also been used by Fricker and al. (2012), Alvarez-Valdes et al. (2014), and Fricker and Gast (2016). More specifically, these last authors used as service indicators the so-called proportion of problematic stations, that stands for the ones where either no bikes are available, or that are totally saturated.
In this paper, we decide to adopt ZVT and FPT to assess the performance of the bike-sharing system and reflect the user dissatisfaction.

2.3 Usage patterns and clustering techniques

One of the most important features that make a BSS successful and attractive is its ability to satisfy users' demand. This task could be challenging, as the bike-requests fluctuate according to a variety of factors, such as time of the day, the day of the week, weather conditions, and so on. Underlying these apparently random changing in the everyday demand, there are patterns that need to be identified, aiming at planning and managing the system most effectively, maximizing at the same time the level of customer satisfaction (Alvarez-Valdes et al., 2014).

In literature, clustering analysis - that aims at organizing a collection of different trends in a smaller number of homogeneous groups - has been widely used to explore the activity patterns connected to a bike-sharing system usage. They have turned out handy in dealing with BSS, as data collected on such systems are usually sizeable. Then, it results quite difficult to gain knowledge from them without a method able to supply a synthetic view of the fundamental information.

In particular, several studies have addressed usage patterns and their characterization focusing on the spatio-temporal correlation among data. Froehlich et al. (2009) provided a spatio-temporal analysis of Barcelona's shared bicycling system, identifying shared behaviors across stations, finding how these behaviors relate to location, neighborhood and time of the day. It is worth mentioning also Han et al. (2014), that correlated the historical usage records of Paris' bike sharing system at both spatial and temporal scale, integrating this analysis into forecasting goals, underlining how this represents a necessary information for accurately predicting bikes demand of each station. The work by Feng et al. (2017) deals with the prediction of the future availability of bikes at a bike station. They used the moment analysis of a population continuous-time Markov chain model with time-dependent rates with reference to the case of Santander Cycles in London.

Clustering methodologies have been extensively used in literature to explore the activity patterns related to a shared system usage and reveal communities of users, with a wide range of final goals. For example, some authors have shown how cluster analysis is capable of revealing groups of stations with a similar trend of rental and return activities during the day (Vogel et al., 2011). Zhou (2015) analyzed the case of Chicago BSS by defining a bike flow similarity graph and using a fastgreedy algorithm to detect spatial communities of biking flows. They also examined the temporal demands for bikes and docks using a hierarchical clustering method.

Recently, Caggiani et al. (2017a) proposed a bi-level method, able to aggregate at first temporally (making use of wavelets and hierarchical clustering) and after that spatially (through k-means clustering) patterns of available bikes in different zones of a city. With respect to the free-floating BSS Caggiani et al. (2017b) present a novel methodology for generating a flexible/dynamic zone clustering in order to define cost-efficient relocation strategies that allow to identify the optimal size and number of areas among which perform an effective and enhanced vehicle repositioning, reducing the necessity to move vehicles from one zone to another and, accordingly, shrinking the relocation costs. They show how the dynamic approach returns better results than the static ones. Other studies have conducted a spatio-temporal analysis of the bicycle station usage of bike-sharing systems, with real case studies application in Barcelona (Froehlich et al., 2009), Paris (Côme et al., 2014), London (Caggiani et al., 2017a).

The first step of the methodology suggested in this paper consists in spatiotemporally aggregate BSS stations, adopting a method similar to the one presented by Caggiani et al. (2017a), to obtain a clustering that represents the basis for the application of the subsequent optimization model. Further explanations can be found in the following section.

3. The proposed user satisfaction based model

In this section, we propose a methodology to calculate the optimal number of racks/bikes to allocate in each station of an operating BSS, aiming at enhancing its functioning, under the main constraint of a maximum budget to invest. At first, to reduce the amount of data to analyze in the subsequent phase, we operate a spatio-temporal clustering of the bicycle stations considering the similarities among their usage patterns. Then, we perform an optimization of the global service quality of the BSS, minimizing the time intervals in which a station is full or empty, and the total number of lost users.

In the following, a flowchart (Fig. 1) summarized the main steps of our approach. After that, a notation box presents symbols and notations that are introduced in the next subsections and adopted throughout the paper.

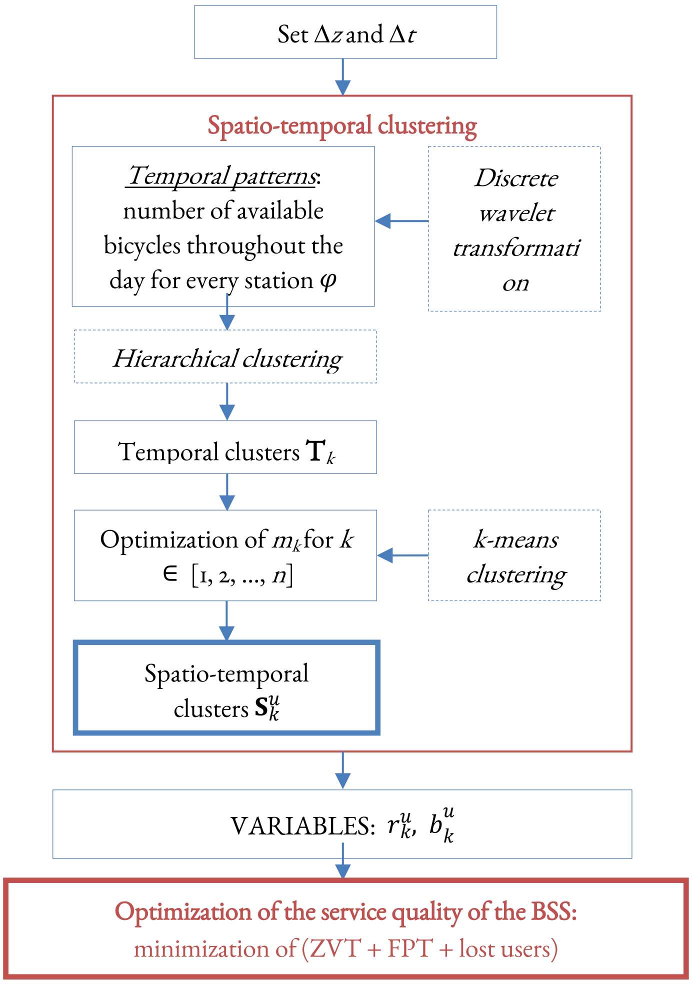


Fig.1 Flowchart of the user satisfaction based method to allocate resources in a BSS.

Spatio-temporal clustering notation

$\overline{\varphi}$	the total number of bike-sharing stations
φ	a generic station with $\varphi \in [1, 2, ..., \overline{\varphi}]$
Δz	the width of the significative period of operation of the BSS
Δt	the width of each time interval in which data are collected
n	the total number of temporal clusters vectors
T	the temporal cluster set
$\mathbf{T}_k$	the temporal cluster vector belonging to the set T with $k \in [1, 2, ..., n]$
w_k	the total number of $\mathbf{T}_k$ elements
t_k^y	a generic element of a temporal cluster vector $\mathbf{T}_k$ with $y \in [1, 2, ..., w_k]$
Ξ_k	the set of remaining t_k^y not included in the satisfactory clustering
S_k	the spatio-temporal cluster set associated with $\mathbf{T}_k$
m_k	the total number of spatio-temporal clusters
$\mathbf{S}_k^u$	spatio-temporal cluster vector belonging to the set S_k with $u \in [1, 2, ..., m_k]$

η_k^u	the total number of $\mathbf{S}_k^u$ elements
S_k^{uh}	a generic element of a spatio-temporal cluster vector $\mathbf{S}_k^u$ with $h \in [1, 2, ..., \eta_k^u]$
c_k^u	a generic centroid of a spatio-temporal cluster vector $\mathbf{S}_k^u$
wd	average user walking distance
$\overline{\vartheta}$	the total number of spatio-temporal cluster iterations
ϑ	a generic spatio-temporal cluster iteration, $\vartheta \in [1, 2, ..., \overline{\vartheta}]$
f_k^u	the total number of $S_{k'}^{uh}$, $\forall k' \neq k$, belonging to a temporal cluster $\mathbf{T}_{k'}$ inside the spatial boundary of the cluster $\mathbf{S}_k^u$
β	the maximum allowable value of b_k^u

Optimization of the service quality of the BSS notation

γ_1,γ_2	weight coefficients
U_L^{in}	lost users – incoming (no available bicycles)
U_L^{out}	lost users – outcoming (no available racks)
ZVT_k^u	the zero-vehicle time of the cluster $\mathbf{S}_k^u$
FPT_k^u	the full-port time of the cluster $\mathbf{S}_k^u$
c_b	the cost of a new bike
c_r	the cost of a new rack
c_{sh}	the cost of shifting one rack from a BSS station to another
c_φ	the cost of a new bike-sharing station facility
$\overline{b'}$	the total number of new bicycles to buy
$\overline{r'}$	the total number of racks to buy
$r'(k,u)$	racks to add to the cluster $\mathbf{S}_k^u$
$\overline{sh}$	the total number of rack shifts to operate
$\overline{\varphi'}$	the total number of new bike-sharing station facilities to build
B	the total available budget
b_k^u	the total available bicycles at the beginning of the day in $\mathbf{S}_k^u$
r_k^u	the total number of racks in the cluster $\mathbf{S}_k^u$
$\varphi_k^{u\varepsilon}$	a generic new bike-sharing station in $\mathbf{S}_k^u$ with $\varepsilon \in [1, ..., v]$
σ_1,σ_2	the minimum and maximum number of racks to allocate in a new station
μ	the maximum number of new stations in the cluster $\mathbf{S}_k^u$
δ_1	the minimum number of available bicycles at the beginning of the day in a new station $\varphi_k^{u\varepsilon}$
δ_2	the minimum number of available bicycles at the beginning of the day in the cluster $\mathbf{S}_k^u$
ρ	the maximum number of new racks r' to add in the cluster $\mathbf{S}_k^u$
ξ	the threshold associated with ZVT/FPT (usually equal to zero)

3.1 Spatio-temporal clustering method

The user satisfaction based methodology that we are presenting in this paper can be applied to any real cities/urban settings, in which a BSS has been already set up, and is regularly operating. Therefore, it follows that we are usually dealing with a considerable amount of data (big data) related to the BSS, that we need to understand and analyze to achieve a global enhancement of the system. This is the main reason why we suggest to operate a preliminary clustering to have a synthetic view of the information underlying the entire system.

Each bike-sharing station φ has its own trend of available bicycles, that fluctuates during the time. This number of bicycles, for each station, is collected every time interval Δt. The maximum value that could be reached at every time

step is equal to the total number of racks in that station (station completely full). On the contrary, if all the bicycles have been picked-up, no one is available to use, and at that moment the collected number will be zero.
Before performing our analysis, we need to select a significative period of operation of the BSS. We denote this interval as Δz. It may coincide with the last year/season/month of the functioning of the system (more recently collected data). Alternatively, the analyst could select a Δz corresponding to the latest period (season, month, group of weeks) with the highest bicycle requests, to have data linked to the temporal interval in which the system had been used more intensely. This choice can be operated to satisfy the user demand especially in high-demand periods (for example, non-rainy seasons).
Once Δz has been defined, we have at our disposal a database with the number of available bicycles for each station φ, collected every Δt: these are the station temporal patterns that we propose to cluster/aggregate. The preliminary spatio-temporal analysis of our methodology involves the definition of two main categories of clusters: temporal clusters (set T) and spatial clusters associated with each element $\mathbf{T}_k$ of the set T, called spatio-temporal clusters (set S_k).

Knowing the temporal patterns associated with each station ϕ and setting the total number of temporal clusters n, we can aggregate them according to their temporal trends. Therefore, we can assert that each station belonging to a given $\mathbf{T}_k$, $k \in [1, 2, ..., n]$, is a generic element t_k^y of it with $y \in [1, 2, ..., w_k]$. At first, a discrete wavelet transformation (Guan and Feng, 2004; Zhang et al., 2008) helps in the analysis of signals (in this case, they are the temporal trends representing the number of available bicycles in each station), as it has been done by Vlachos et al. (2003). Discrete wavelets can de-noise and compress the signals, and are often used as a preprocessing step before clustering (Antoniadis et al., 2013). Then, the filtered data are aggregated into a given number of temporal clusters $\mathbf{T}_k$, applying a hierarchical clustering methodology (Caggiani et al., 2017). At the end of this procedure, we obtain that each station belongs to a $\mathbf{T}_k$.

Next step aims at geographically aggregating groups of stations (Xu et al., 2013; Lee et al., 2014) belonging to the same T_k, creating a certain number of spatio-temporal clusters $\mathbf{S}_k^u$, $u \in [1, 2, ..., m_k]$, associated with each temporal one. We propose to operate this second clustering using a k-means algorithm. The k-means method is a widely-used clustering technique, whose goal is seeking to minimize the average squared distance between points in the same cluster. It is not capable to guarantee accuracy; however, thanks to its simplicity and computing speed, it is very appealing in practical applications (see MacQueen 1967 and Arthur and Vassilvitskii, 2007 for further details).
We are assuming that, for any user, choose to pick-up a bicycle from one station rather than another, provided that the walking distance between them is comparable with wd, is equivalent. Then, we can assume that every spatio-temporal cluster $\mathbf{S}_k^u$ should ideally have a size/maximum extension similar to the average distance wd that a user is willing to travel by walk, beginning the trip from his/her starting location (origin).
As a first approximation, the following bilevel optimization problem (to be repeated for each $k \in [1, 2, ..., n]$) is able to get a reasonable number of $\mathbf{S}_k^u$ related to each S_k:

$$\min m_k \tag{1}$$

$$\min \sum_{y=1}^{w_k} \sum_{c_k^u \in \mathbf{S}_k^u} \text{EuclideanDist}\,(t_k^y, c_k^u) \tag{2}$$

s.t.

$$\text{MaximumExt}(\mathbf{S}_k^u) \leq wd \tag{3}$$

$$\max f_k^u \leq \beta \tag{4}$$

The upper-level objective (1) aims at minimizing the total number m_k of $\mathbf{S}_k^u$ associated to each S_k. The lower level objective (2) represents the k-means optimization, that is, the minimization of the distance (in our case, Euclidean distance) between the positions of the centroids c_k^u of each spatio-temporal cluster, and the elements t_k^γ belonging to the temporal ones. Eq. (3) means that the maximum extension/size of each $\mathbf{S}_k^u$ has to be less than (or equal to) the average distance wd that a typical user is willing to travel by walk. The last constraint (4) forces the maximum value of the total number of $S_{k'}^{uh}$, $\forall\ k' \neq k$, belonging to temporal clusters $\mathbf{T}_{k'}$ inside the spatial boundary of the cluster $\mathbf{S}_k^u$ to be smaller or equal to a positive integer coefficient β. For example, the lower is the value given to β, the higher is the number of spatial clusters associated to S_k. The coefficient β should be conveniently calibrated according to the given case study. If it is set equal to 0, it leads to an unfeasible problem; on the other side, if it is too big, it will involve a lot of overlapping among areas of $\mathbf{S}_k^u$ belonging to different S_k.
The output of this optimization is the optimal m_k, that is the total number of spatio-temporal clusters $\mathbf{S}_k^u$ for each $k \in [1, 2, \ldots, n]$. The spatial boundary of each $\mathbf{S}_k^u$ is (usually) a polygon, having as vertices some of the elements t_k^γ (bike-sharing stations) belonging to T_k. We assume to consider these spatial clusters satisfactory if the number of these perimetric vertices is equal to or greater than 3 (that is, the minimum number that defines a polygon). However, it may happen that some $\mathbf{S}_k^u$ are made by only one or two elements t_k^γ; hence, for these remaining t_k^γ not included in the satisfactory clustering -and constituting the set Ξ_k-, we decide to repeat the optimization for a number of iterations equal to $\overline{\theta}$. This procedure is carried out to take advantage of an inherent feature of the k-means clustering, that is the arbitrary choice of the initial centers of each cluster (Arthur and Vassilvitskii, 2007). Each iteration ϑ selects different centers, and consequently there are more possibilities to further aggregate the remaining t_k^γ. In any case, it could still happen that, at the end of all the iterations, some spatial clusters are made by only one or two elements.
We can point out that this preliminary spatio-temporal clustering makes sense only for high-density BSSs, with stations located not too far away from each other. In urban contexts, where the distance between bike-sharing stations is averagely greater than wd (low-density BSSs), this first step can be skipped. The temporal trend of every station has to be considered, and the second and third steps of the methodology (subsections 3.2 and 3.3) can be applied straight away.
Finally, we can assert that, at the end of the spatio-temporal clustering, elements belonging to the same $\mathbf{S}_k^u$ result to be close to each other in space: it has been demonstrated by Vogel et al. (2011) and Côme et al. (2014) that temporal clusters belonging to the same category are usually adjacent spatially. Furthermore, stations that are neighboring to each other are likely to have a similar capacity to generate and attract potential users. The spatio-temporal $\mathbf{S}_k^u$ clusters are the starting point to keep on with the proposed methodology.

3.2 Optimization of the service quality of the BSS

In this subsection, we describe the aim of the proposed optimization approach which consists in appropriately allocating the bike-sharing related resources (namely, bicycles and racks) in a BSS under analysis. Assuming a given budget, we aim to enhance the BSS level of service (i.e., minimizing the lost users of the system, and the time intervals with empty or full stations).
As we are operating on the spatio-temporal clusters $\mathbf{S}_k^u$ previously defined, we can assert that the main variables of the problem are:

- racks/slots: optimal number of racks to allocate in each spatio-temporal cluster (sum of the racks of the bike-sharing stations belonging to each $\mathbf{S}_k^u$);
- bicycles: optimal numbers of bicycles to allocate in each spatio-temporal cluster $\mathbf{S}_k^u$ at the beginning of any operation day of the system (sum of the bicycles of the bike-sharing stations belonging to each $\mathbf{S}_k^u$).

Analyzing the historical trends of available bicycles in each spatio-temporal cluster $\mathbf{S}_k^u$, we can calculate for how many time-intervals Δt the spatio-temporal clusters have no available bicycles (ZVT_k^u), or vice-versa, no available racks (FPT_k^u). As explained before, these are the indicators that we have chosen to assess the quality of the service level of the system. The lowest is their value, the more efficiently the system is working.
At the same time, it is also important to keep down the number of lost users of the system. Considering the actual behavior of the BSS, it is not possible to identify the exact number of unsatisfied users that the system loses, and then

we consider as a proxy ZVT and FPT. However, supposing to change the number of racks and available bicycles at the beginning of the day in each spatio-temporal cluster, and assuming that the usage patterns are maintaining similar trends, we could generate new lost users U_L^{in} and U_L^{out} in the system: we have to minimize also their sum in order to guarantee the efficiency of the BSS. To better understand this concept, look at the following graphs (Fig. 2):

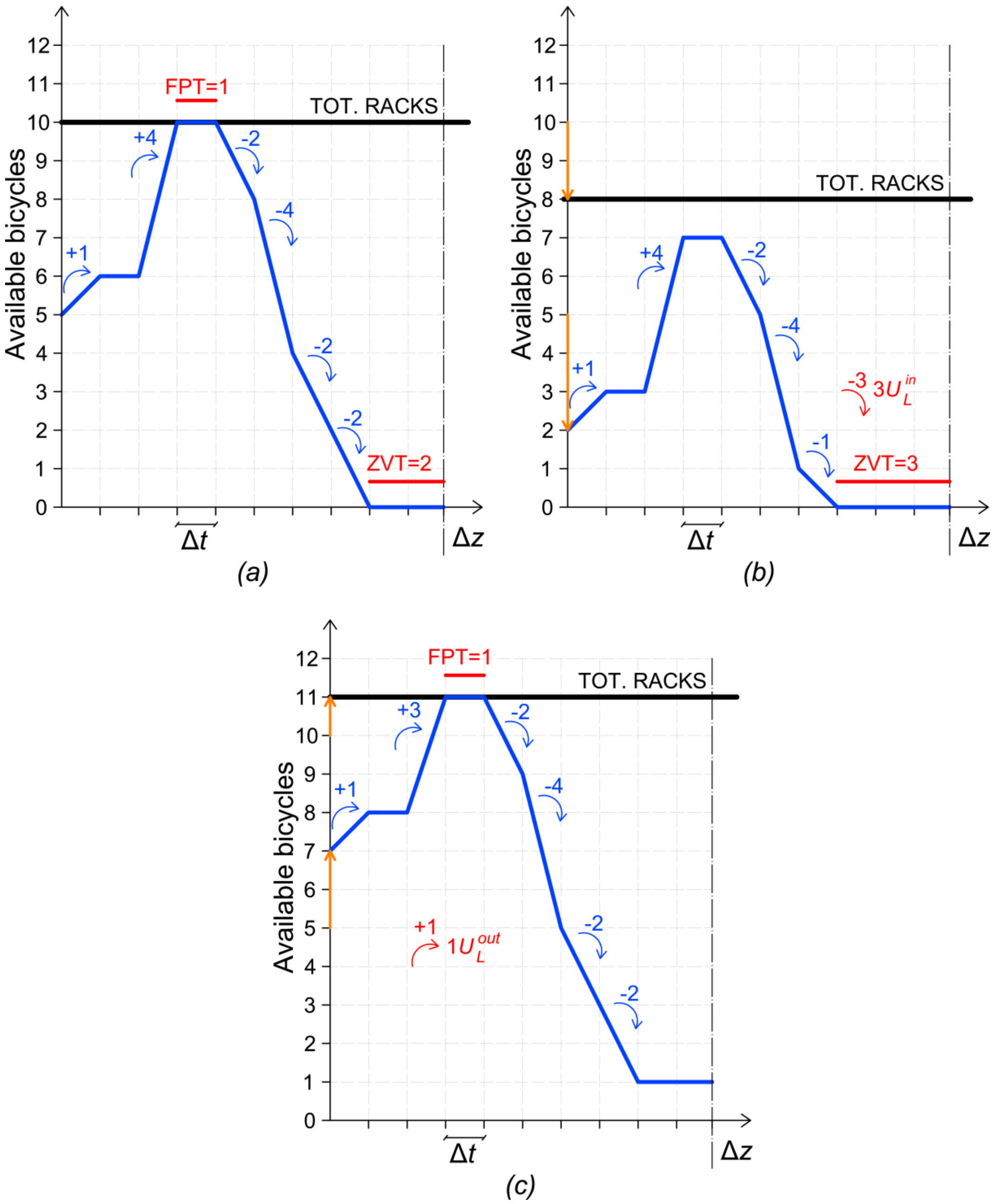


Fig.2 Example of calculation of ZVT, FPT, and lost users. (a) Current status; (b) Scenario 1; (c) Scenario 2.

Given a hypothetical spatio-temporal cluster $\mathbf{S}_k^u$ with its current trend of available bicycles (Fig.2a), we can see that it has a number of bikes at the beginning of the day equal to 5. During the considered time interval ($\Delta z = 10\ \Delta t$), we can see that the cluster has no available racks for one time-step Δt (i.e., $\text{FPT}_k^u = 1$), and no vehicles to pick-up for two time-steps ($\text{ZVT}_k^u = 2$). Let us suppose that the optimization suggests subtracting two racks at the cluster and allocate 3 bikes less at the beginning of the day (Fig. 2b). We can easily see that, keeping an analogous usage pattern, now FPT_k^u

= 0, ZVT_k^u = 3, and there are 3 lost users U_L^{in} in the system, not able anymore to pick-up a bike at that instant, since none is available.
Fig. 2c depicts another possible scenario. If we suppose to allocate two more bicycles (7 rather than 5) at the beginning of the day, adding one rack at the original configuration of the cluster, FPT_k^u=1, ZVT_k^u=0, and there is one lost user U_L^{out} in the system, not capable anymore of dropping his/her bike in the station as all the racks are full.
Having this in mind, we can now formulate our problem (Eqs. 5-11):

$$\min \sum_{k=1}^{n} \sum_{u=1}^{m_k} [\gamma_1 \big(U_L^{in}(k,u) + U_L^{out}(k,u)\big) + \gamma_2 \big(\text{ZVT}_k^u + \text{FPT}_k^u\big)] \tag{5}$$

s.t.

$$\overline{b'} \cdot c_b + \overline{r'} \cdot c_r + \overline{sh} \cdot c_{sh} + \overline{\varphi'} \cdot c_\varphi \leq B \tag{6}$$

$$\sigma_1 \leq r\big(\varphi_k^{u\varepsilon}\big) \leq \sigma_2 \tag{7}$$

$$v(k,u) \leq \mu(k,u) \tag{8}$$

$$b(\varphi_k^{u\varepsilon}) \geq \delta_1 \tag{9}$$

$$b_k^u \geq \delta_2 \tag{10}$$

$$r'(k,u) \leq \rho \tag{11}$$

$$b_k^u \leq r_k^u \tag{12}$$

The objective function (5) aims at minimizing the total number of lost users in the system, plus the zero-vehicle time ZTV and the full-port time FPT, for each spatio-temporal cluster $\mathbf{S}_k^u$. The analyst may assign two different weights γ_1 and γ_2 according to the components that he/she prefers to emphasize. The first constraint (6) is related to the available budget: there is a unit cost for each resource that it is possible to allocate in the BSS, namely bicycles, racks, shift of racks, and new stations. We consider the possibility to build new stations $\varphi_k^{u\varepsilon}$ in the spatio-temporal cluster $\mathbf{S}_k^u$ only if: there are suitable locations (ex. sidewalks, parks…) to place them at a convenient distance (*wd*) from the centroid c_k^u of the cluster; it is not possible to add any other racks to the existing stations; the new stations have a minimum and maximum number of racks respectively equal to σ_1 and σ_2 (7). The maximum number μ of new stations to allocate in each $\mathbf{S}_k^u$ varies according to the specific configuration of the spatio-temporal cluster (8); each of them needs to have at least δ_1 available bicycles at the beginning of the day (9). Furthermore, the bicycles b_k^u (to put at the beginning of the day in each $\mathbf{S}_k^u$) have to be at least equal or greater than a certain number δ_2 (10), and to each cluster $\mathbf{S}_k^u$ is not possible to add more than ρ new racks r' (11). The last one (11) is a consistency constraint, asserting that for each cluster $\mathbf{S}_k^u$ the number of available bicycles cannot overcome the number of racks.

At the end of this procedure, we obtain the optimal number of bicycles and racks to allocate in each spatio-temporal cluster. However, within each $\mathbf{S}_k^u$, there is a given number of bike-sharing stations: at first approximation, we propose to redistribute bikes and racks among the included station in a uniform way, without any additional constraint. We have pursued this criterion because, as we stated before, we are supposing that -for any user- choose to pick-up a bicycle from one station rather than another, provided that the walking distance between them is comparable with *wd*, is totally equivalent; then, no station has to be preferred to another if they are belonging to the same spatio-temporal cluster $\mathbf{S}_k^u$.

A further remark can be made looking at the usage patterns of each cluster $\mathbf{S}_k^u$, that we are assuming to follow a similar trend (Fig. 2) also after the enhancement of the BSS. Basically, we are pretending that the request of bicycles (bike-demand) is fixed. On the contrary, it could happen that, after the allocation of the novel resources, new potential users may be attracted by the system starting to use it (elastic demand). We suggest addressing this problem opportunely calibrating the thresholds ξ of both ZVT and FPT. As a matter of fact, it is true to assert that ZVT is the

zero-vehicle time, in which no vehicle is available in the cluster: this means that one vehicle is reckoned sufficient to satisfy the users' demand. However, if the decision maker/analyst believes that a higher number has to be assured in order to face a potential increase of bike-requests, (e.g., 3 vehicles), it can be possible to apply the same model setting ξ_{ZVT} equal to the number of vehicles to guarantee minus one (e.g., ξ_{ZVT} =3-1=2). This means that, in this way, each spatio-temporal cluster needs at least 3 vehicles to be self-sufficient and ensure a proper functioning of the BSS (the same applies to the FPT).
An alternative strategy to pursue could be artificially increasing the bike-requests in the network for every time interval, multiplying them by a certain number, to simulate a more intense use of the BSS.

4. Numerical application

This part of the paper concerns the application of the proposed method to a test network and to a real sized one. The aim is verifying the effectiveness of our optimization model, performing a sensitivity analysis with different budgets B and different thresholds ξ for ZVT and FPT to the test network. Then, a further application to a larger case study is presented, and the results discussed.

4.1 Test network case: results and sensitivity analysis

The user satisfaction based model that has been described in the previous section is here applied to a test network. The study area (1 km²) consists of 36 bike-sharing stations φ, whose coordinates have been generated perturbating the ones of the centroids of a 6×6 squares grid (see Fig. 3, where the number of racks and available bicycles for each φ are printed next to each station). Note that the proposed methodology could be applied to any kind of spatial configuration. On average, the distance between two stations is around 160 m, that is, comparable with the one detected in the city centers of big cities like Paris or London. Under these assumptions, the operating BSS can be considered a high-density one, and it makes sense to apply the preliminary spatio-temporal clustering (as reported in section 3.1). The acceptable walking distance wd of a typical user has been set equal to 250 m (Kabra, Belavina and Girotra, 2016).

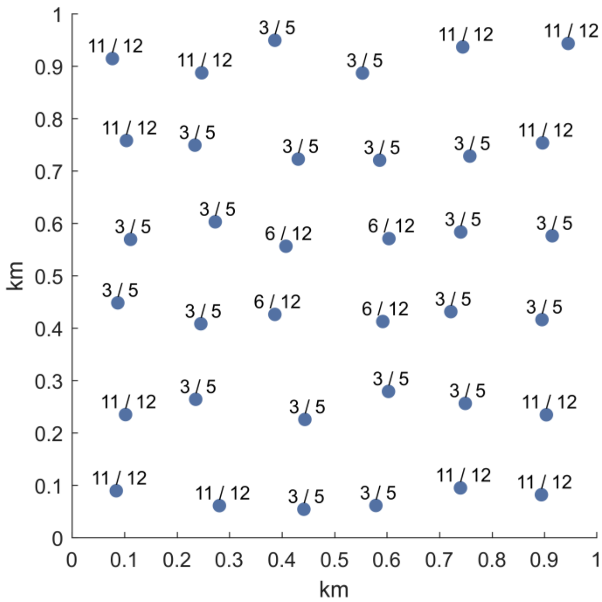


Fig.3 Test network with the indication of the 36 bike-sharing stations, and their respective available bicycles and racks.

Aiming at building a system as realistic as possible, we assume to have 4 different typologies of bike-sharing stations: central (with the highest demand), peripheral with a low demand request, peripheral with an average demand request, peripheral with a high demand request. Their temporal trends of available bicycles have been obtained thanks to the BSS simulator proposed by Caggiani and Ottomanelli (2012 and 2013), setting (for each station typology) two diverse levels of bike demand according to the days of the week. We set Δz = 14 days, and Δt = 5 minutes; 4 temporal clusters $\mathbf{T}_k$ and 10 spatio-temporal clusters $\mathbf{S}_k^u$ have been obtained, considering β (i.e. the maximum value of the total number of $S_{k'}^{uh}$, $\forall\, k' \neq k$, belonging to temporal clusters $\mathbf{T}_{k'}$ inside the spatial boundary of the cluster $\mathbf{S}_k^u$) smaller or equal to 1. Looking at Fig. 4, the stations (dots) have four distinct colors, corresponding to each temporal cluster; the 10 polygons (with asterisks indicating their centroids c_k^u), marked with a progressive ID number, are the resulting spatio-temporal clusters. Each $\mathbf{S}_k^u$ has a total number of racks that goes from 12 to 48 and a number of available bicycles between 11 and 33.

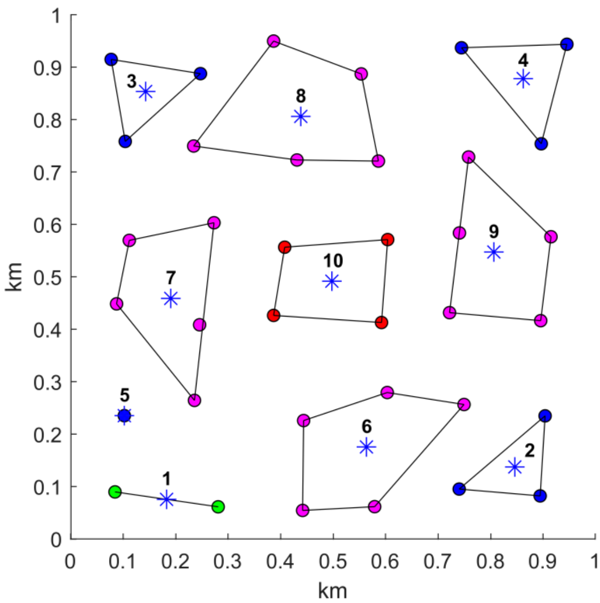


Fig.4 Spatio-temporal clustering of the bike-sharing stations.

Among the remaining parameters that we have to set before performing the optimization of the BSS level of service, there are the unit costs of the resources to allocate (c_b= 100€; c_r= 200€; c_{sh}= 100€; c_φ= 300€) and the various thresholds to be respected (σ_1=5; σ_2=15; μ=5; δ_1= δ_2=2; ρ=5). This means that we can build not more than 5 additional stations in every $\mathbf{S}_k^u$, and that each station can have 5 to 15 racks. At least 2 bicycles need to be available at the beginning of an operation day, both in a new station and in each spatio-temporal cluster; finally, not more than 5 new racks can be added in each $\mathbf{S}_k^u$. Both ξ_{ZVT} and ξ_{FPT} have been set equal to 0: this means that (respectively) one bicycle/one rack is reckoned sufficient to satisfy the users' demand. The total budget available to implement the optimal solution correspond to B=10000€.

The results in terms of bicycles, racks and new stations to build are summarized in Table 1.

Table 1. Resources allocated in each one of the 10 spatio-temporal clusters according to the proposed optimization model.

Cluster ID	1	2	3	4	5	6	7	8	9	10
Racks (starting values)	24	36	36	36	12	25	25	25	25	48

Bicycles (starting values)		22	33	33	33	11	15	15	15	15	24
New stations φ_k^u		0	0	1	0	0	0	0	0	0	0
Racks	$\mathbf{S}_k^u$	27	41	41	37	16	27	27	28	29	51
	φ_k^u	0	0	7	0	0	0	0	0	0	0
Bicycles	$\mathbf{S}_k^u$	24	34	33	36	13	16	16	16	16	25
	φ_k^u	0	0	5	0	0	0	0	0	0	0

Calculating the value of the objective function corresponding to the starting configuration (i.e. before performing the allocation of new resources), it results equal to 10070. As the lost users of the BSS cannot be computed at this stage, we can assert that this number is achieved summing both the ZVT and FPT time intervals of $\Delta t = 5$ minutes, in which respectively no bicycles/no racks are available.
At the end of the optimization -performed using a genetic algorithm-, using the budget B to enhance the functioning of the system, the objective function manages to reach the value of zero: this means that there are no time-steps with ZVT or FPT, neither expected lost users. This result can be guaranteed buying (globally) 18 new bicycles and 39 new racks, without transferring racks from one bike-sharing station to another, and building only one new station (spending 9900€, i.e. less than the available B), as deduced by looking at the data reported in Table 1.
It is now possible to further verify the achieved performance of the BSS with this resource allocation. To do so, we can consider another generic Δz constituted by 14 different days. We have generated again (with the BSS simulator, see Caggiani and Ottomanelli, 2012 and 2013) the temporal patterns of the stations during this time interval, considering analogous levels of bike-requests as input variables, but still with the randomness inherent in the simulation. The location of the stations is unchanged, while racks and bicycles are allocated as suggested by the model. Even in this case, the results are promising. The objective function value reaches a value equal to 7, still very close to zero (without the implementation, its value this time was corresponding to 9899). We can conclude that, with the considered budget, it is possible to substantially improve the functioning of the system.

After this preliminary result, we have conducted a sensitivity analysis. We want to verify if, varying the available budget, or the ZVT and FPT thresholds (i.e. supposing a potential future increase of bike-requests after the system optimization), our model still remains valid. The achieved results are summarized in Table 2.

Table 2. Results of the sensitivity analysis.

		$B = 2500€$			$B = 5000€$		
		$\xi = 0$	$\xi = 1$	$\xi = 2$	$\xi = 0$	$\xi = 1$	$\xi = 2$
O.F. starting		10070	15380	19537	10070	15380	19537
O.F. optimized	*best*	514	2386	5908	0	1227	3369
	median	755	3403	7066	77	1835	5592
O.F.' starting		9899	15144	19521	9899	15144	19521
O.F.' optimized	*best*	578	2528	6150	8	1293	3573
	median	820	3619	7339	88	1853	5756
Tot. costs (€)	*best (O.F.)*	2500	2500	2500	4900	5000	5000
New bicycles $\overline{b'}$		10	5	0	10	6	2
New racks $\overline{r'}$		6	9	5	19	22	24
Transfer racks $\overline{sh'}$		3	2	15	1	0	0
		$B = 10000€$			$B = 15000€$		

		$\xi = 0$	$\xi = 1$	$\xi = 2$	$\xi = 0$	$\xi = 1$	$\xi = 2$
O.F. starting		10070	15380	19537	10070	15380	19537
O.F. optimized	*best*	0	0	1744	0	0	434
	median	0	85	3737	0	0	1292
O.F.' starting		9899	15144	19521	9899	15144	19521
O.F.' optimized	*best*	0	0	1914	0	0	465
	median	0	120	3894	0	6	1379
Tot. costs (€)	*best (O.F.)*	6800	10000	9900	10800	12900	14900
New bicycles $\overline{b'}$		16	22	20	15	24	30
New racks $\overline{r'}$		26	39	38	45	51	55
Transfer racks $\overline{sh'}$		0	0	0	0	0	0

The *O.F. starting* is the value of the objective function before applying the optimization model (i.e. before the allocation of new resources); the *O.F. optimized*, instead, reports the best and the median values obtained carrying out 10 optimizations for each combination of B and ξ. The total costs and resources refer to the best-achieved values of this optimization (in the case of equality of the objective function values, we put in the table the ones with lower costs).

On the other side, O.F.' is related to another generic time-interval Δz, to verify the achieved performance of the BSS with the resulting resource allocation (as explained previously).

Looking at Table 2, we can state that as the available budget increases, better values of the objective function can be achieved. As expected, if we set $\xi > 0$, the sum of ZVT, FPT, and lost users is generally higher. A specific observation can be done comparing the total costs of the configuration B = 10000€ and $\xi = 0$, with the ones of B = 15000€ and $\xi = 0$. It seems that an optimal system status (with O.F.= 0 and O.F.'= 0) can be obtained in both configurations, but investing a different amount of money (6800€ instead of 10800€). We can avoid similar situations doing a sensitivity analysis varying the budget in a certain range comparable with the money available, in order to identify the best (and cheap) solution. However, we can observe that this seems to happen only with a high budget, i.e. sufficient to achieve an O.F.= 0.

4.2 Real-size test network: results of the optimization

The proposed user satisfaction based approach here has also been applied to a larger test network, to check its efficiency on a bigger reality. In this case, the study area (4 km^2) consists of 144 bike-sharing stations φ, whose coordinates have been generated perturbating the ones of the centroids of a 12×12 squares grid. The remaining initial considerations that have been done for the test network (subsection 4.1) applies here. We set Δz = 28 days, and Δt = 5 minutes; 4 temporal clusters $\mathbf{T}_k$ and 31 spatio-temporal clusters $\mathbf{S}_k^u$ have been obtained (Fig. 5). Each $\mathbf{S}_k^u$ has a total number of racks that goes from 20 to 70, and a number of available bicycles between 16 and 54. In this case, the total available budget for the implementation of an optimal solution correspond to B=30000€.

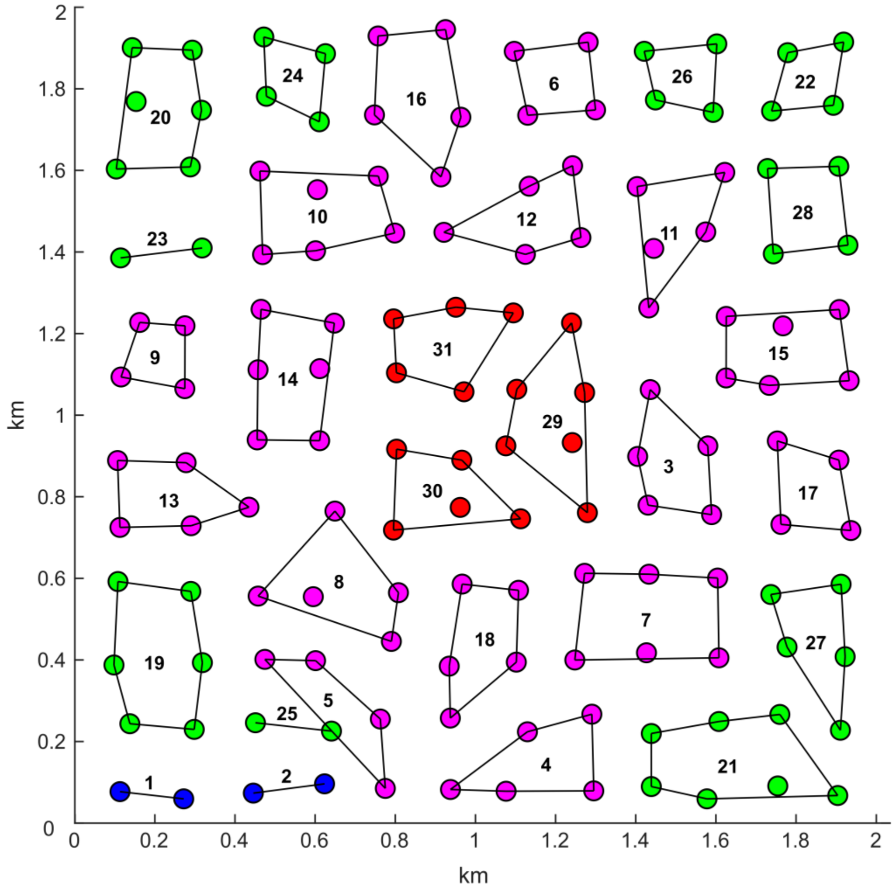


Fig.5 Spatio-temporal clustering of the bike-sharing stations.

The value of the objective function corresponding to the starting configuration (i.e. before performing the allocation of new resources) results equal to 56585. Even this time, as the lost users of the BSS cannot be computed, we can assert that this number is achieved summing both the ZVT and FPT time intervals of $\Delta t = 5$ minutes, in which respectively no bicycles/no racks are available. At the end of the optimization, using the budget B to enhance the functioning of the system, the objective function reaches the value of zero: this means that (again) there are no time-steps with ZVT or FPT, neither expected lost users. This result can be guaranteed buying (globally) 87 new bicycles and 66 new racks, transferring 37 racks from one bike-sharing station to another and building four new station (spending 26800€, i.e. less than the available B): this can be deduced looking at the data reported in Table 3.

Table 3. Resources allocated in each one of the 31 spatio-temporal clusters according to the proposed optimization model.

Cluster ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Racks (starting values)	20	20	50	50	40	40	60	50	40	60	50	50	50	60	60	50
Bicycles (starting values)	18	18	20	20	16	16	24	20	16	24	20	20	20	24	24	20
New stations φ_k^u	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0

Racks	S_k^u	24	25	48	55	45	38	65	55	38	56	52	51	40	53	54	49
	φ_k^u	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0
Bicycles	S_k^u	19	20	21	22	20	17	27	32	20	25	22	21	21	25	27	23
	φ_k^u	0	3	2	0	0	0	2	0	0	0	0	0	0	0	0	2

Cluster ID		17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Racks (starting values)		40	50	60	60	70	40	20	40	20	40	50	40	60	50	50
Bicycles (starting values)		16	20	54	54	63	36	18	36	18	36	45	36	36	30	30
New stations φ_k^u		0	0	0	0	1	0	1	0	0	0	0	0	0	1	0
Racks	S_k^u	42	55	64	57	75	42	25	45	24	43	51	44	64	55	52
	φ_k^u	0	0	0	0	5	0	5	0	0	0	0	0	0	5	0
Bicycles	S_k^u	21	23	56	55	61	40	16	40	21	39	49	38	39	33	31
	φ_k^u	0	0	0	0	4	0	3	0	0	4	0	0	0	2	0

We have verified the achieved performance of the BSS with this resource allocation for another generic Δz constituted by 28 different days (same consideration of section 4.1 applies here). Then, we have obtained an objective function value equal to 96 (sum of ZVT, FPT, and lost users). This result is a great achievement, considering that the starting value, in this case, was 57418. In Fig. 6, the convergence of the genetic algorithm used to perform the service level optimization is presented. The optimal solution is obtained after 775 generations using a population of 50 individuals and a maximum number of 1000 generations.

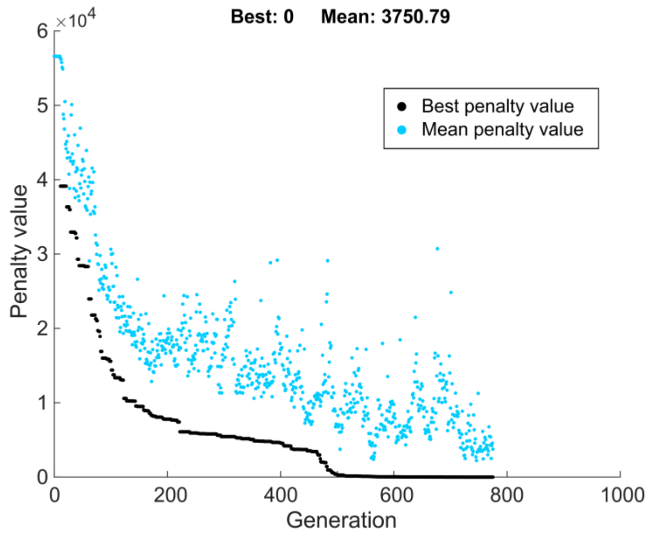


Fig. 6 Convergence of the genetic algorithm applied to solve the problem.

5. Conclusions and further research

In this paper, we proposed a user satisfaction based model to allocate resources (bicycles and racks) in an operating BSS, to enhance its functioning under (primarily) a budget constraint. We suggest performing a preliminary spatio-temporal clustering, which allows reducing the amount of collected data, returning a global and synthetic view of the functioning of the system.

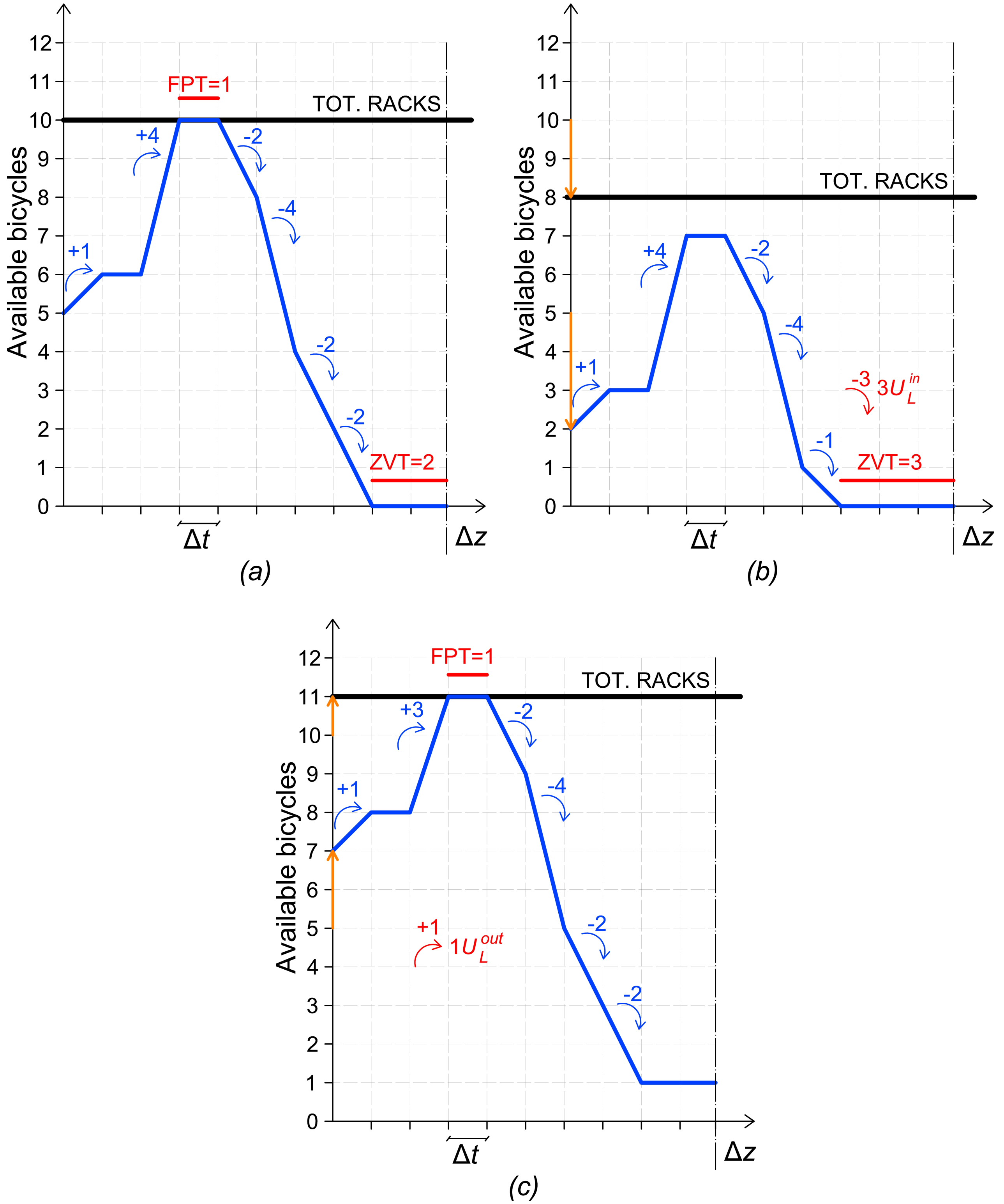
The optimization of the service quality of the BSS has been done from a user perspective: i.e., trying to minimize the eventuality that he/she does not find an available bicycle/rack when he/she needs it, and at the same time keeping low the number of lost users of the system. The achieved results are promising and seem to opportunely suggest the optimal way to invest the available budget granting a general improvement of the system. However, we can even notice that, although indirectly, this approach takes into account also the operator interests: assuring that ZVT and FTP are minimized, means at the same time that we are reducing the necessity to perform bicycle relocations in the system, essentially reducing the management costs of the BSS. Not secondarily, less lost users mean also a greater (potential) revenue.
Further research could explore the possibility of performing a temporal clustering of the station usage patterns according to the days of the week since a different behavior of the system has been widely recognized between weekdays and weekend. In this way, the number of bicycles to allocate in each station at the beginning of the day can be opportunely calibrated according to the real necessities of the BSS.

References

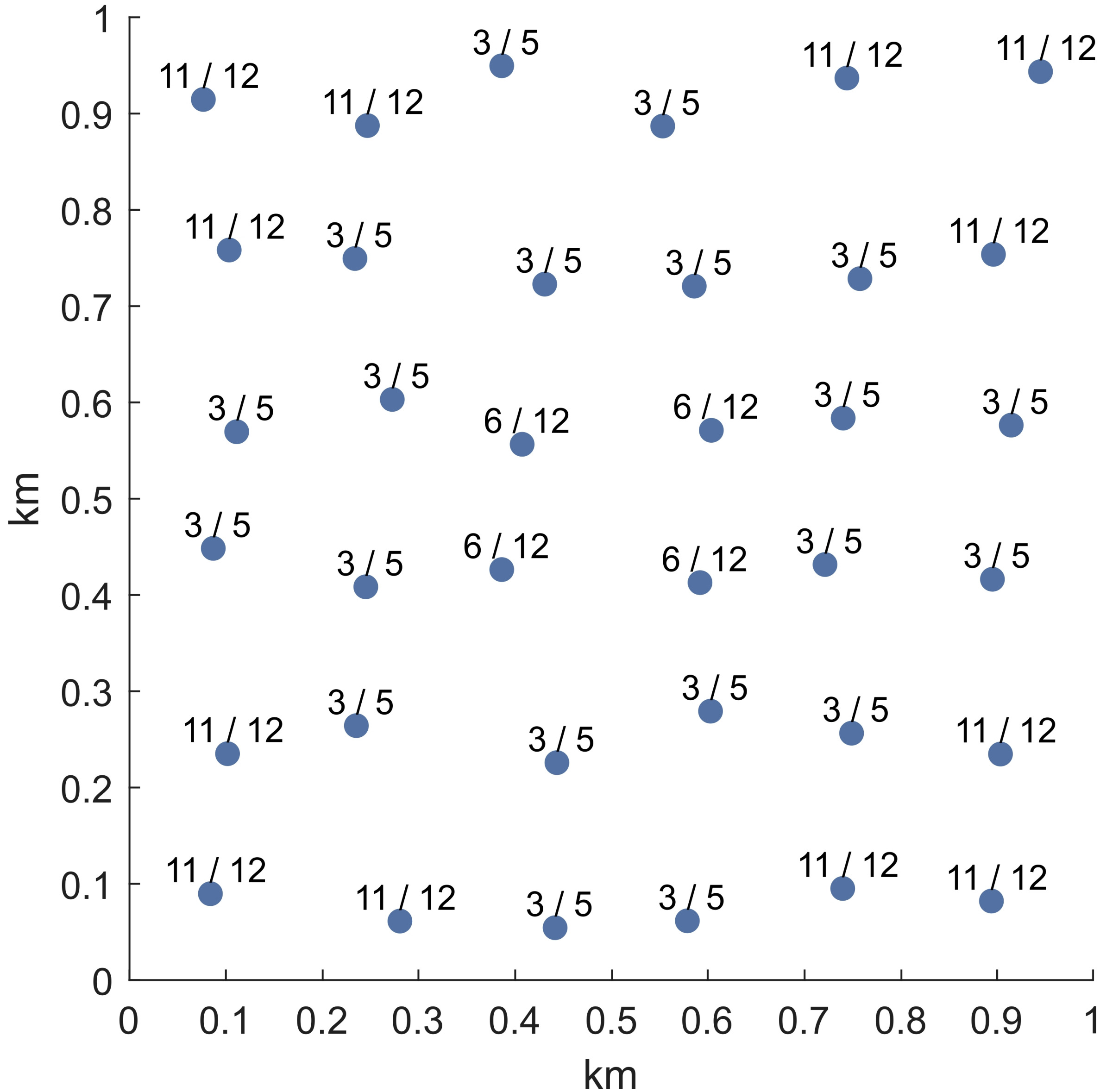
1. Alvarez-Valdes, R., Belenguer, J. M., Benavent, E., Bermudez, J. D., Muñoz, F., Vercher, E., and Verdejo, F. (2016). Optimizing the level of service quality of a bike-sharing system. *Omega*, 62, 163-175.
2. Angelopoulos, A., Gavalas, D., Konstantopoulos, C., Kypriadis, D., and Pantziou, G. (2016, November). An Optimization Model for the Strategic Design of a Bicycle Sharing System: A Case Study in the City of Athens. In *Proceedings of the 20th Pan-Hellenic Conference on Informatics* (p. 5). ACM.
3. Angeloudis, P., Hu, J., & Bell, M. G. H. (2014). A strategic repositioning algorithm for bicycle-sharing schemes. *Transportmetrica A: Transport Science*, 10, 759-774.
4. Antoniadis, A., Brossat, X., Cugliari, J., and Poggi, J. M. (2013). Clustering functional data using wavelets. *International Journal of Wavelets, Multiresolution and Information Processing*, 11(01), 1350003.
5. Arthur, D., and Vassilvitskii, S. (2007, January). k-means++: The advantages of careful seeding. In *Proceedings of the eighteenth annual ACM-SIAM symposium on Discrete algorithms* (pp. 1027-1035). Society for Industrial and Applied Mathematics.
6. Benchimol, M., Benchimol, P., Chappert, B., de la Taille, A., Laroche, F., Meunier, F., & Robinet, L. (2011). Balancing the stations of a self service "bike hire" system. *RAIRO - Operations Research*, 45, 37–61
7. Caggiani, L., and Ottomanelli, M. (2012). A modular soft computing based method for vehicles repositioning in bike-sharing systems. *Procedia-Social and Behavioral Sciences*, 54, 675-684.
8. Caggiani, L., and Ottomanelli, M. (2013). A dynamic simulation based model for optimal fleet repositioning in bike-sharing systems. *Procedia-Social and Behavioral Sciences*, 87, 203-210.
9. Caggiani, L., Ottomanelli, M., Camporeale, R., and Binetti, M. (2017a). Spatio-temporal Clustering and Forecasting Method for Free-Floating Bike Sharing Systems. In: *Advances in Intelligent Systems and Computing: Advances in Systems Science* 539, J. Swiatek, and J. M. Tomczak, eds., Springer International Publishing, Cham, 244-254.
10. Caggiani, L., Camporeale, R., and Ottomanelli, M. (2017b). A dynamic clustering method for relocation process in free-floating vehicle sharing systems. Transportation Research Procedia, 27, 278-285. doi: https://doi.org/10.1016/j.trpro.2017.12.146
11. Caggiani, L., R. Camporeale, M. Ottomanelli and W. Y. Szeto (2018). A modeling framework for the dynamic management of free-floating bike-sharing systems. *Transportation Research Part C: Emerging Technologies* 87, 159-182.
12. Côme, E., Randriamanamihaga, A., Oukhellou, L., (2014). Spatio-temporal usage pattern analysis of the Paris Shared Bicycle Scheme: a data mining approach, Transport Research Arena (TRA) 5th Conference: Transport Solutions from Research to Deployment

13. Chen, Q., and Sun, T. (2015). A model for the layout of bike stations in public bike-sharing systems. *Journal of Advanced Transportation*, 49(8), 884-900.
14. Côme, E., Randriamanamihaga, A., and Oukhellou, L. (2014) Spatio-temporal Usage Pattern Analysis of the Paris Shared Bicycle Scheme: A Data Mining Approach. In: *Transport Research Arena (TRA) 5th Conference: Transport Solutions from Research to Deployment*. Paris.
15. El-Assi, W., Mahmoud, M. S., and Habib, K. N. (2017). Effects of built environment and weather on bike sharing demand: a station level analysis of commercial bike sharing in Toronto. *Transportation*, 44(3), 589-613.
16. Feng, C., Hillston, J., & Reijsbergen, D. (2017). Moment-based availability prediction for bike-sharing systems. *Performance Evaluation*, 117, 58-74
17. Faghih-Imani, A., and Eluru, N. (2017). Examining the impact of sample size in the analysis of bicycle-sharing systems. Transportmetrica A: Transport Science, 13(2), 139-161.
18. Frade, I., and Ribeiro, A. (2015). Bike-sharing stations: A maximal covering location approach. *Transportation Research Part A: Policy and Practice*, 82, 216-227.
19. Fricker, C., Gast, N., and Mohamed, H. (2012, July). Mean field analysis for inhomogeneous bike sharing systems. In *Discrete Mathematics and Theoretical Computer Science*.
20. Fricker, C., and Gast, N. (2016). Incentives and redistribution in homogeneous bike-sharing systems with stations of finite capacity. *Euro journal on transportation and logistics*, 5(3), 261-291.
21. Froehlich, J., Neumann, J., and Oliver, N. (2009, July). Sensing and predicting the pulse of the city through shared bicycling. In *IJCAI* (Vol. 9, pp. 1420-1426).
22. García-Palomares, J. C., Gutiérrez, J., and Latorre, M. (2012). Optimizing the location of stations in bike-sharing programs: a GIS approach. *Applied Geography*, 35(1), 235-246.
23. Guan, L., and Feng, X. (2004). Research on Factors of the Signal De-Noising Effect Basing on the Wavelet Transform and the Matlab practice. In: *Automation & Instrumentation*, 6, 43-46.
24. Guo, T. Y., Zhang, P., Shao, F. & Liu, Y. S. (2014). Allocation optimization of bicycle-sharing stations at scenic spots. J. Cent. South Univ. 21, 3396–3403
25. Han, Y., Côme, E., and Oukhellou, L. (2014). Towards bicycle demand prediction of large-scale bicycle sharing system. In: *Transportation Research Board 93rd Annual Meeting* (No. 14-2637).
26. Kabra, A., Belavina, E., and Girotra, K. (2016). Bike-share systems: Accessibility and availability. *Chicago Booth Research Paper* No. 15-04.
27. Kek, A., Cheu, R., and Chor, M. (2006). Relocation simulation model for multiple-station shared-use vehicle systems. *Transportation Research Record: Journal of the Transportation Research Board*, 1986, 81-88.
28. Kaspi, M., Raviv, T., & Tzur, M. (2017). Bike-sharing systems: User dissatisfaction in the presence of unusable bicycles. IISE Transactions, 49(2), 144-158
29. Kek, A. G., Cheu, R. L., Meng, Q., and Fung, C. H. (2009). A decision support system for vehicle relocation operations in carsharing systems. *Transportation Research Part E: Logistics and Transportation Review*, 45(1), 149-158.
30. Labadi, K., Benarbia, T., Hamaci, S., & Darcherif, A.-M. (2012). Petri Nets Models for Analysis and Control of Public Bicycle-Sharing Systems. In *Petri Nets - Manufacturing and Computer Science*, 28.
31. Lee, C., Wang, D., and Wong, A. (2014). *Forecasting utilization in city bike-share program* (Vol. 254). Technical report, CS 229 2014 Project.
32. Lin, J. R., and Yang, T. H. (2011). Strategic design of public bicycle sharing systems with service level constraints. *Transportation research part E: logistics and transportation review*, 47(2), 284-294.
33. MacQueen, J. (1967, June). Some methods for classification and analysis of multivariate observations. In *Proceedings of the fifth Berkeley symposium on mathematical statistics and probability* (Vol. 1, No. 14, pp. 281-297).

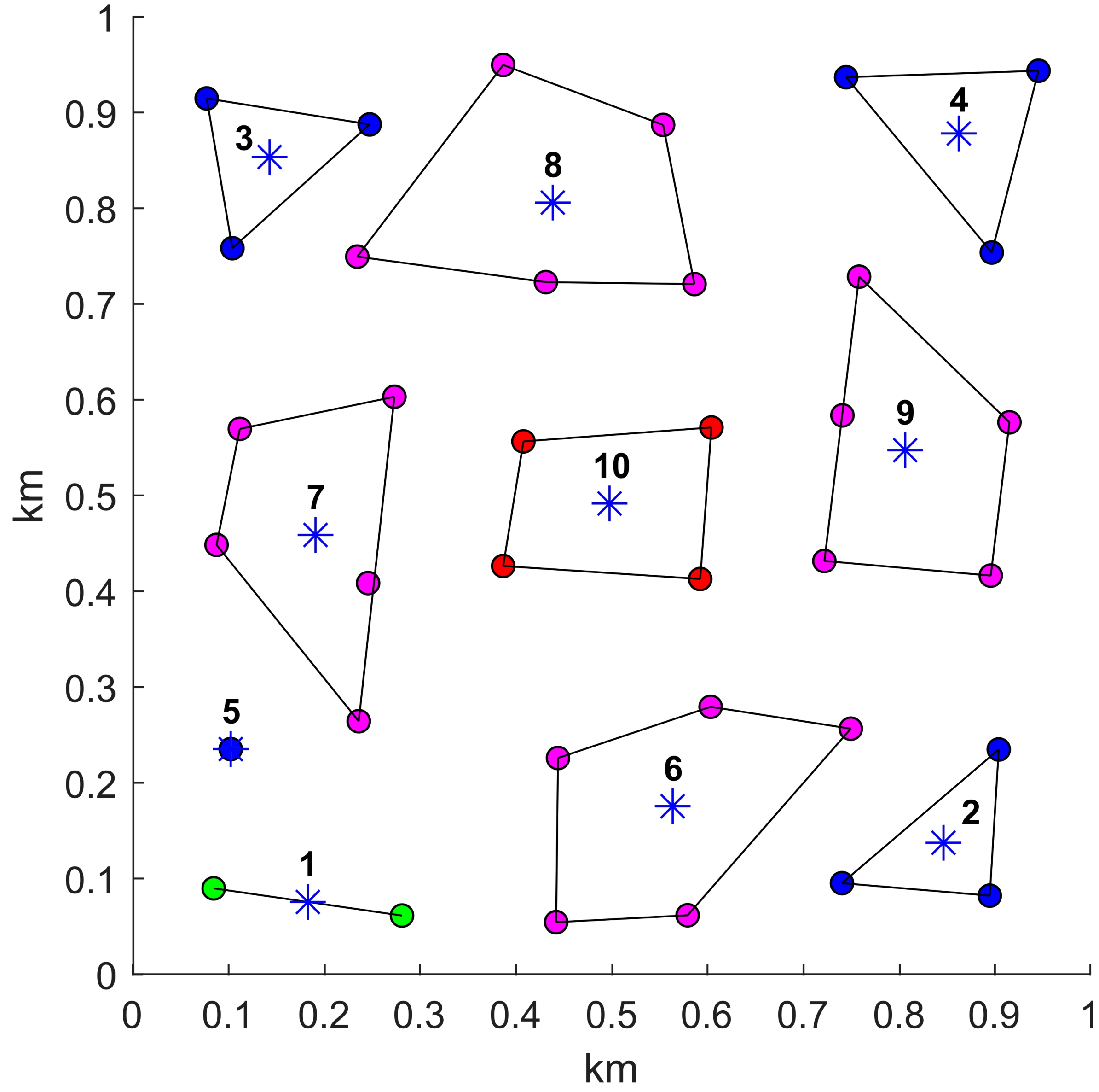
34. Martinez, L. M., Caetano, L., Eiró, T. & Cruz, F. (2012) An optimisation algorithm to establish the location of stations of a mixed fleet biking system: an application to the city of Lisbon. in Procedia - Social and Behavioral Sciences, 54, 513-524.
35. O'Mahony, E., and Shmoys, D. B. (2015, January). Data Analysis and Optimization for (Citi) Bike Sharing. In *AAAI* (pp. 687-694).
36. Nair, R., Miller-Hooks, E., Hampshire, R. C., & Bušić, A. (2013). Large-Scale Vehicle Sharing Systems: Analysis of Vélib'. *International Journal of Sustainable Transportation*, 7(1), 85–106
37. Neumann-Saavedra, B. A., Crainic, T. G., Gendron, B., Mattfeld, D. C., and Römer, M. (2016). Service Network Design of Bike Sharing Systems with Resource Constraints. In *International Conference on Computational Logistics* (pp. 352-366). Springer International Publishing.
38. Pal, A., and Zhang, Y. (2015) *Free-floating bike sharing: solving real-life large-scale static rebalancing problems.* Technical report, University of South Florida.
39. Randriamanamihaga, A. N., Côme, E., Oukhellou, L., and Govaert, G. (2014) Clustering the Vélib′ dynamic Origin/Destination flows using a family of Poisson mixture models, *Neurocomputing*, 141. 124-138,
40. Romero, J. P., Ibeas, A., Moura, J. L., Benavente, J. and Alonso, B. (2012) A simulation-optimization approach to design efficient systems of bike-sharing. in Procedia-Social and Behavioral Sciences 54, 646–655.
41. Saharidis, G. K. D., Fragkogios, A., and Zygouri, E. (2014). A multi-periodic optimization modeling approach for the establishment of a bike sharing network: A case study of the City of Athens. In *Proceedings of the International MultiConference of Engineers and Computer Scientists* (Vol. 2).
42. Sayarshad, H., Tavassoli, S., and Zhao, F. (2012). A multi-periodic optimization formulation for bike planning and bike utilization. *Applied Mathematical Modelling*, *36*(10), 4944-4951.
43. Shaheen, S., Guzman, S., and Zhang, H. (2010). Bikesharing in Europe, the Americas, and Asia. *Transportation Research Record: Journal of the Transportation Research Board*, 2143, 159-167.
44. Tran, T. D., Ovtracht, N., and d'Arcier, B. F. (2015). Modeling bike sharing system using built environment factors. *Procedia CIRP*, *30*, 293-298.
45. Vlachos, M., Lin, J., Keogh, E., and Gunopulos, D. (2003). A wavelet-based anytime algorithm for k-means clustering of time series. In *Proc. Workshop on Clustering High Dimensionality Data and Its Applications*.
46. Vogel, P., Greiser, T., and Mattfeld, D. C. (2011). Understanding bike-sharing systems using data mining: Exploring activity patterns. *Procedia-Social and Behavioral Sciences*, *20*, 514-523.
47. Xu, H., Ying, J., Wu, H., and Lin, F. (2013). Public bicycle traffic flow prediction based on a hybrid model. *Applied Mathematics & Information Sciences*, *7*(2), 667.
48. Yang, T. H., Lin, J. R., and Chang, Y. C. (2010, July). Strategic design of public bicycle sharing systems incorporating with bicycle stocks considerations. In *Computers and Industrial Engineering (CIE), 2010 40th International Conference on* (pp. 1-6). IEEE.
49. Zhang, Y., Thomas, T., Brussel, M., and Van Maarseveen, M. (2017). Exploring the impact of built environment factors on the use of public bikes at bike stations: case study in Zhongshan, China. *Journal of transport geography*, *58*, 59-70.
50. Zhang, Z., Zhang, P., Yin, Y., and Hou, L. (2008). Analysis on urban traffic network states evolution based on grid clustering and wavelet de-noising. In *Intelligent Transportation Systems, 2008. ITSC 2008. 11th International IEEE Conference on* (pp. 1183-1188). IEEE.
51. Zhou X. (2015) Understanding Spatiotemporal Patterns of Biking Behavior by Analyzing Massive Bike Sharing Data in Chicago. *PLoS ONE* 10(10): e0137922. https://doi.org/10.1371/journal.pone.0137922



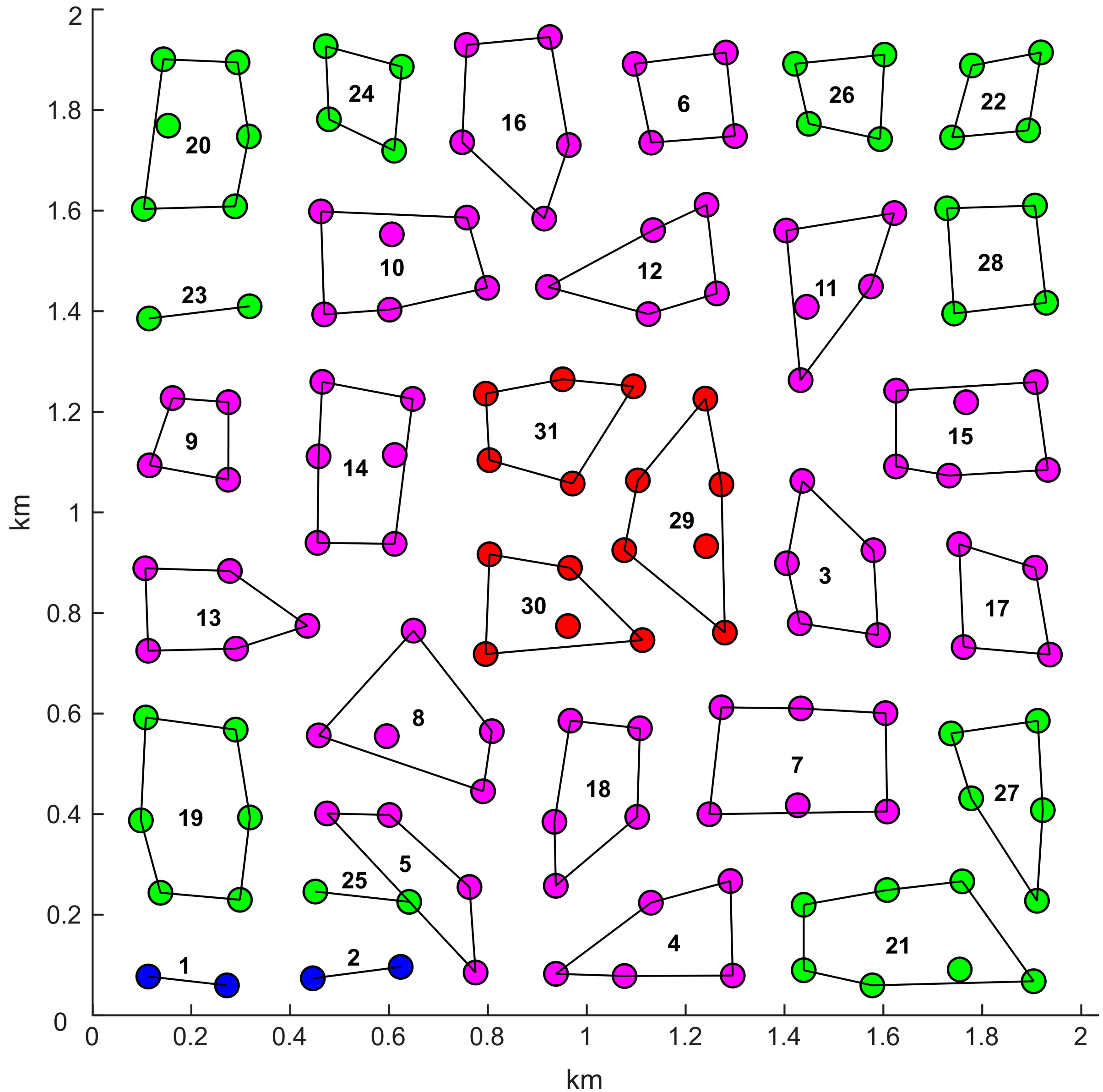
FPT=1
TOT. RACKS
Available bicycles
+1
+4
-2
-4
-2
-2
ZVT=2
Δt
Δz
(a)
TOT. RACKS
Available bicycles
+1
+4
-2
-4
-3 3U_L^in
-1
ZVT=3
Δt
Δz
(b)
FPT=1
TOT. RACKS
Available bicycles
+1
+3
-2
-4
+1 1U_L^out
-2
-2
Δt
Δz
(c)



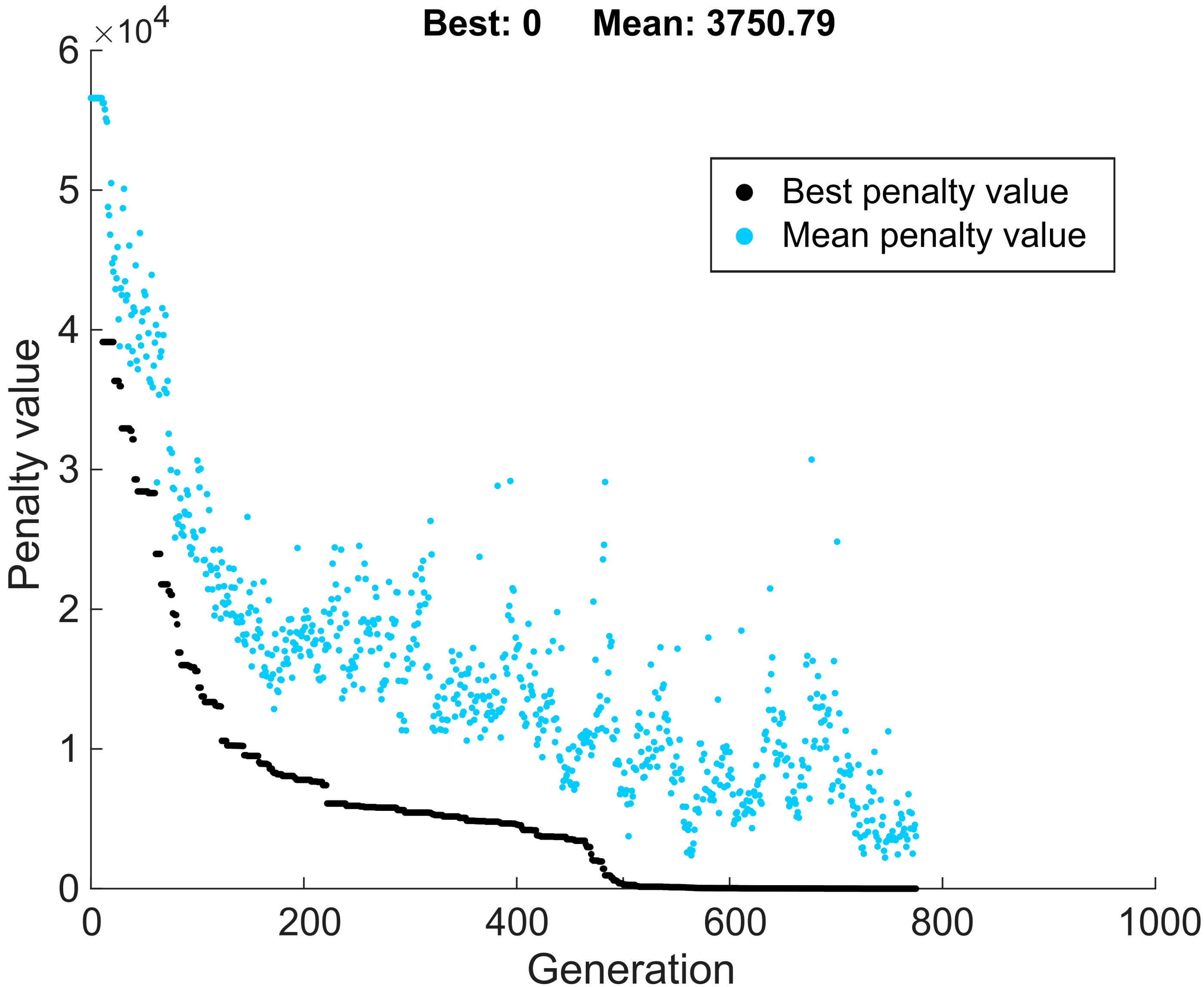
km
km
11 / 12
3 / 5
6 / 12



1
2
3
4
5
6
7
8
9
10
km
km
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



km
km
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31



Best: 0 Mean: 3750.79
Best penalty value
Mean penalty value
Penalty value
Generation