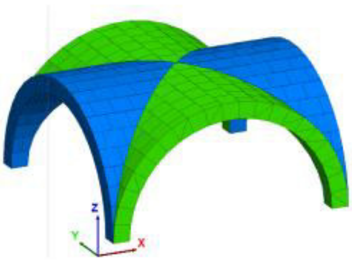
DEM modelling and experimental analysis of the static behavior of a dry-joints masonry cross vaults

Graphical Abstract

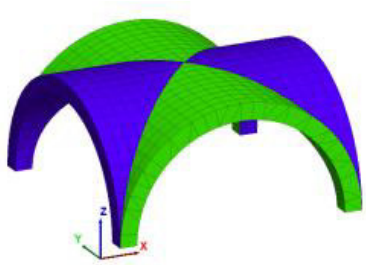
GRAPHICAL ABSTRACT

NUMERICAL ANALYSIS OF A CROSS VAULT WITH DIFFERENT MASONRY PATTERN: a) orthogonal weaving; b) parallel weaving

a)

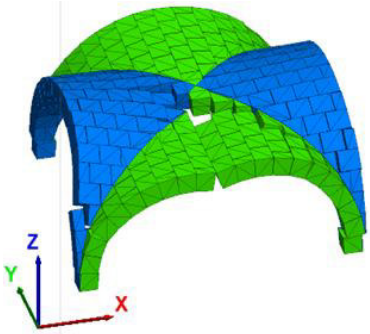


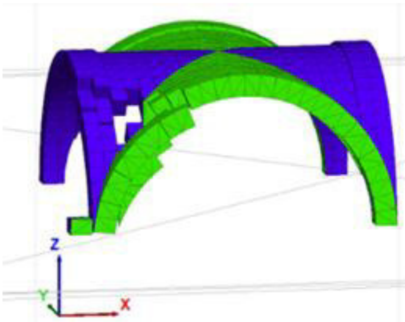
b)



Horizontal displacement, diaonal displacement (xy direction), vertical displacement at one support.

For ex.: Horizontal displacement at one support, numerical results for both cases, respectively:

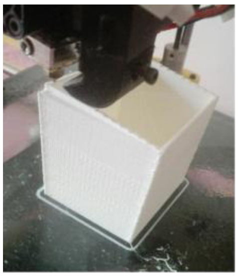




EXPERIMENTAL BEHAVIOR OF A CROSS VAULT WITH ORTHOGONAL WEAVING

Each block has been obtained after different steps:

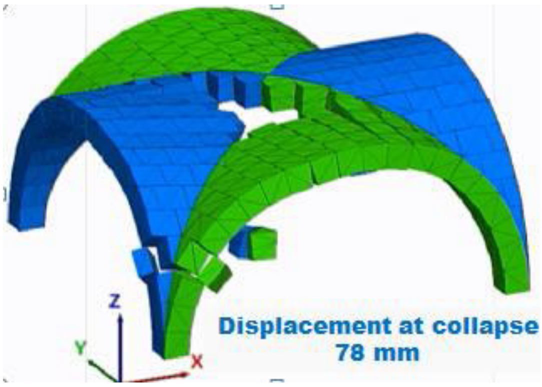
1) CAD design;
2) 3D printing of the hole skin of the block with a thickness of 2 mm;
3) Filling with a concrete mix (Figure 8).



Manufacturing steps of the specimens' blocks.

Horizontal (x-direction) collapse test: Arch-mechanism from an experimental point of view:

Comparison for the diagonal collapse test between the experimental study and the numerical analysis (by 3DEC based on DEM analysis) on a vault with the same geometric and mechanical characteristics of the real vault. Limit state before collapse.


Z
Y
X
Displacement at collapse
78 mm

DEM modelling and experimental analysis of the static behavior of a dry-joints masonry cross vaults

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Abstract. The purpose of this paper is to study the dynamic and static three-dimensional behavior of a dry-assembled masonry cross vault, through the comparison of Distinct Element Modelling results and laboratory tests' results on a physical model obtained by mean of 3D printing. The work consists of two phases: the first one compares two numerical models of a cross vault built with different masonry patterns (parallel, orthogonal); the second phase deals with a comparison between the static behavior of the computational and the real scaled models (1m x1m) of the same cross vault, tested at one support collapse. The study focuses on three principal aspects: (i) to evaluate the three-dimensional mechanism of the cross vault, (ii) to determine the support displacement's magnitude that leads to its collapse and (iii) to evaluate the ability of computational methods to predict the experimental results. The results obtained from the numerical and the experimental tests have been compared in order to give general specifications on the behavior of these types of vaults.

Keywords: *Masonry Vaults, Collapse, DEM (Distinct Element Modelling), 3D-printing, Scale model testing.*

1. INTRODUCTION

In recent years, considering the high percentage of historic-monumental buildings, their age of constructions and the damages due to the even more frequent earthquakes in Italy (i.e. L'Aquila, 2009, Amatrice 2016, 2017), more and more attention is being paid to the vulnerability and structural safety of these kinds of buildings. To this goal, nowadays, there are many research activities aimed at identifying even more reliable modeling procedures, and durable and less invasive recovery techniques for historical buildings [1,2].

At the same time, however, most of these experimental researches are focused on structural analyses of the vertical components of buildings (walls) rather than on the study of the shell behavior (vaults). But given that vaults and domes are architectural elements necessary from a structural point of view to ensure the transfer of the forces to the supports their behavior significantly influences the overall building response in terms of strength and stiffness, both in the static and dynamic fields. The investigation of their dynamic behavior under earthquake excitation (stress and deformation states) is a fundamental issue for effective structural interventions [3].

Studies on masonry vaults developed over time are numerous and rely on different methodologies ranging from simplified methods, such as the non-interacting arched scheme, to the more complex methods that are based on the finite element analysis, or computational approach based on the well-known analogy between the equilibrium of arches and that of hanging strings or cables working in tension [4], up to the analysis of the distinct elements modeling and through a Differential Variational Inequalities (DVI) formulation specifically developed for the 3D discrete elements method [5].

Coming back to the XV century and before, when some geometric indications of the "rule of art" of vaults were first defined, the respect of the geometric proportions and the experience of building masters were the only tools available [6,7].

One of the first historical personalities interested in the behavior of arches was Leonardo da Vinci [8,9], he defined, for the first time, an approach to the study of the static of an arch here considered as a system of two bars.

At the beginning of '700, Philippe De la Hire in his *Traité de Mecanique* [10], provided interesting elaborations on the staticity of masonry structures, tracing a sort of collapse calculus.

During the nineteenth century, starting from Luis Navier (1785-1836), the problem was faced in a different way, considering the material characteristics and their strengths rather than the geometry and the form.

Based on the studies of Navier, a first study on pressure curves was carried out by Eduard Henry Méry in 1840, who proposed a graphical method, still used nowadays for small arches [12]. The Méry's method consists in the construction of the pressure curve relative to the load system corresponding to the individual blocks of the arch (Figure 1).

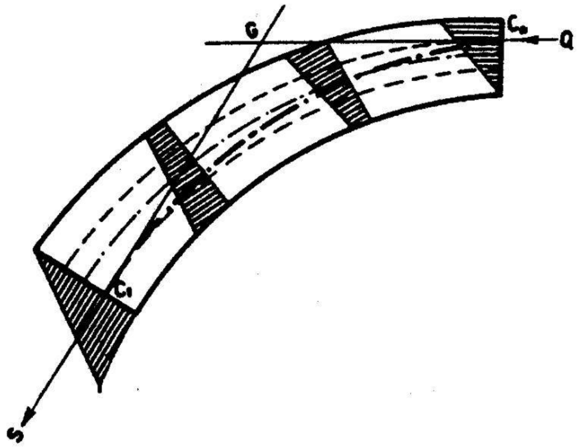

Figure 1. Pressure Curve (Méry)

It was only later, with Luigi Menabrea (1808-1896) and Alberto Castigliano (1847-1884) that the modern theory of limit analysis came out. It still today represents one of the best mathematical tools to understand the mechanics of masonry arches and vaults. The limit analysis was best dealt with by Professor Heymann (1982) too [13]. In particular, the theorem of the uniqueness of the limit analysis leads to foreseeing pressure lines associated to the collapse mechanism, i.e. in each point where the funicular touches the extrados or intrados, a plastic hinge appears, and this means that the collapse of the structure can take place when the fourth plastic hinge first appears.

Beside the boundary analysis studies, the membrane theory of thin vaults was developed [14,15]; it allows to evaluate the stress state of a vault considered to be a flexural rigid membrane, subjected only to tangent membrane stresses in its plane. It can be considered a valid tool for studying the state of the previous cracking stress, but does not account for the scrolling effects between the blocks, which can only become negligible in the case of thin shells [16].

Until now, there are several methods of analysis available: the method of Heyman's pressure line in 1966 [14], and the membrane theory [15]. Another analytical tool is the finite element method (FEM), first utilized for civil structures in the '70s. This method is widely utilized for the numerical analysis of ancient masonry structures [17] and for arches and vaults too [18,19].

However, FEM presents some problems in masonry modeling; the main ones are described below:

- Definition of the type of elements for the modeling (mono-dimensional, two-dimensional or three-dimensional elements).

- Uncertainty in the mechanical characterization: in the choice of the constitutive law of the material (elastic-linear, elastic-plastic, elastomeric, resistance or non-traction resistance material), in the values of Young's Modulus, E, Poisson's Coefficient, υ, etc., in the evaluation of anisotropy and material inequality and, above all, in the impossibility to know the load history.

- The geometry is already deformed.

- Difficulty in taking into account the discontinuities (joints, cavities, rift).

The difficulty of correctly detecting these values and their variability within the structure often makes it difficult the interpretation of the results. Application of the method to masonries requires "equivalent" modeling approaches, such as lowering the stiffness in order to simulate the nonlinear behavior of a masonry.

Also, it must be considered that the masonry is an anisotropic material, consisting of two materials, the blocks (ashlars) and the mortar. If one can define a regular block arrangement, a macroscopic approach, which involves homogenization, is a powerful tool for structural analysis. But when the homogenization process is approximate, for the heterogeneity of the materials and the behavior, this type of modeling is inadequate, especially for the estimation of the seismic vulnerability of historical buildings. In this case, a discontinuous modeling, in which distinctly blocks and joints are considered with their bond constraints is certainly a more rigorous approach that returns more accurate results, especially at a local level [20,21]. In the distinct elements modeling (DEM), in fact, the blocks and joints are considered distinctly with their respective constitutive laws, considering the actual arrangement of the components of the masonry, resulting in a more accurate approach that returns better results [22-24].

An application of the method for vaults was proposed by Lengyel and Bagi [25]. By comparing FEM and DEM modellings, it was studied the importance of ribs on the mechanical behavior of a cross vault. The efficacy of DEM applied to a masonry with respect to FEM analysis was also demonstrated. DEM approach made it possible to observe the openings of the cracks with sliding areas.

In particular, the study of mechanism formation in arches made of few blocks by [26] showed that the hinges always appear only in the joint between the blocks. The system, in fact, reaches the minimum energy configuration in this situation because the creation of a crack inside the block is much more expensive in terms of energy. This performance is radically different from the behavior of continuous arches or of arches made of a lot of blocks.

Another interesting research was proposed by Van Mele et al. [27]. The main objective was to evaluate the resistance capacity to differential displacements of a cross vault, to test the reliability of DEM analysis and to sustain the importance of experimental validation of the computational model. Tests on scale models are an effective tool for analysis, they aim to better understand the three-dimensional behavior of structures and to test the reliability of theoretical results.

With this background, the research here proposed aims to study the static behavior of a cross vault through computational models of distinct elements and a physical model obtained through 3D printing; its ability to withstand differential displacements and to determine the amount of displacements that lead the structure to collapse is also evaluated [28-31].

In particular, the first phase of the present research consists of numerically modeling and analyzing two distinct elements models utilizing the software 3DEC of Itasca Consulting Group [32]. Two vaults have been modelled, they have the same mechanical characteristics and dimensions (width and thickness), but different wall texture. In one model, the bricks were arranged with orthogonal weaving, in the other with parallel weaving. The main purpose is to compare the resistance of the vault to differential displacements (vertical, horizontal and diagonal) when masonry cross vaults are built with different wall textures. In the second phase, the research is advanced with laboratory tests on dry-assembled scaled models. By testing the specimens at different displacements of one support and comparing the results with the ones numerically obtained, the ultimate goal is to validate DEM analysis and to define the reliability and accuracy of the results.

2. NUMERICAL ANALYSIS OF A CROSS VAULT WITH DIFFERENT MASONRY PATTERN

The distinct elements method (DEM) was utilized to study the behavior of a cross vault subject to a displacement in correspondence of a support (in the vertical, longitudinal and diagonal directions). This analysis is useful to understand the three-dimensional behavior of the vault when it is built with different pattern arrangements, i.e. orthogonal weaving and parallel weaving.

A vault is built with an orthogonal weaving when the blocks are arranged in rows according to the generators of the cross vault (Figure 2a).

A vault is built with a parallel weaving when the blocks are arranged in rows according to the direction of the barrel vaults that generate the cross vault (Figure 2b).

The geometry is obtained from the intersection of two barrel vaults, with a radius equal to 2.5m and thickness of 0.35m. The plane projection is squared based, with a total development of 5x5 m (Figure 2).

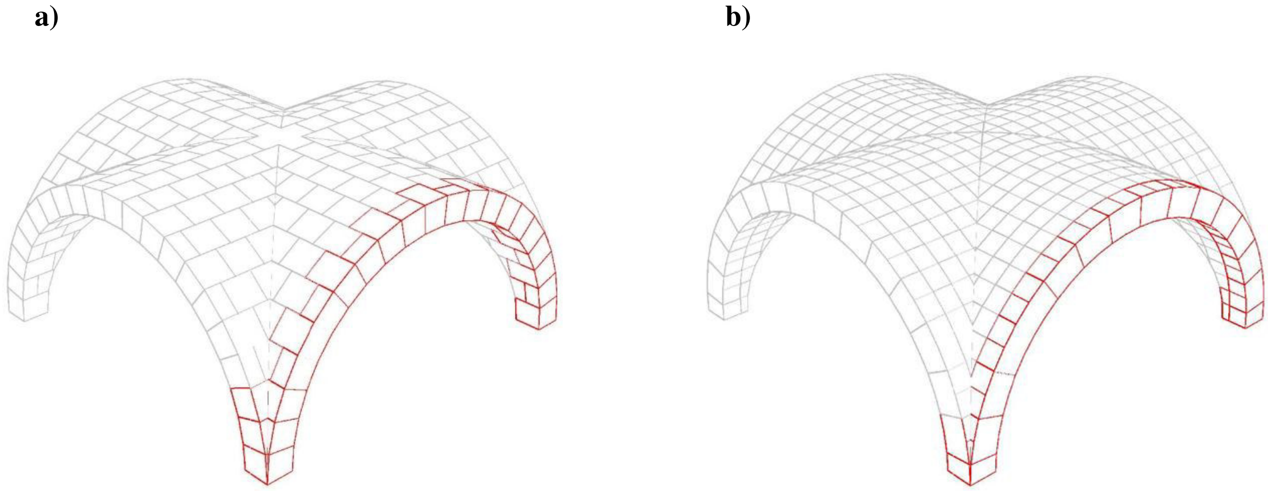

Figure 2. Computational model with a) orthogonal weaving and b) parallel weaving.

Assignment of properties is required for blocks and joints. In particular, it has been assumed the hypothesis of rigid blocks and the Mohr-Coulomb criterion for the joints' modeling [32].

Table 1 shows the mechanical characteristics of the vault.

Table 1. Mechanical characteristics assumed for the vault.

Density, ρ	**Normal stiffness (jkn)**	**Shear stiffness (jks)**	**Friction angle Φ (jfric)**
2200 kg/m^3	900 GPa	900 GPa	32°

In the absence of specific tests, the friction coefficient values proposed in the literature are used, that is in the range μ_s=0.3÷0.8. In this case, an angle of 32° (mean value for masonry) corresponding to μ_s=0.62 has been assumed [33].

Moreover, supporting blocks have boundary constraints (fixed), but one of them is free to move in the direction investigated. Yielding displacements were applied at low speeds (0.01 m/s) in order to reproduce the quasi-static behavior and to allow the model to re-establish the static equilibrium at each increment of the displacement [27].

During the test, the followings have been determined:

- deformation and collapse mechanisms;

- magnitude of differential displacements at the fall of the 1st block and in correspondence of the collapse of the entire vault (the collapse is considered occurred when the fall of 50% of the blocks is reached).

2.1 Displacement of one support

- Cross vault with orthogonal weaving

In correspondence of a displacement of one support in the in-plane horizontal direction (x-direction), the vault showed a typical arch collapse mechanism with three alternate and asymmetrical hinges (extrados-intrados). Another important damage mechanism, directly associated to the horizontal in-plane response of the structure, is the typical diagonal crack; other minor sliding phenomena are visible close to the supports (Figure 3).

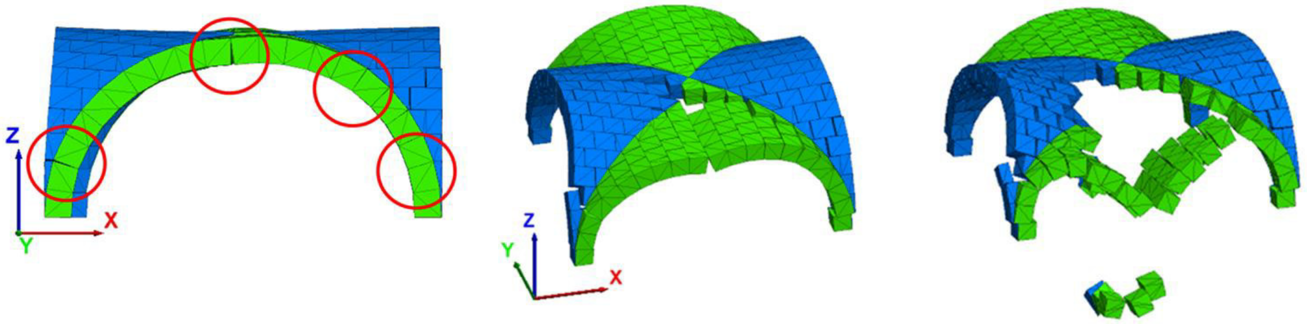

Figure 3. Displacement in x-direction: damage mechanism.

Similarly to the previous case, analysis of the support displacement in the diagonal direction has highlighted the presence of the typical collapse mechanism of an arch with three hinges, asymmetrical and alternating between intrados and extrados. There is a symmetry of the hinges along the diagonal of the vault, i.e. opposite webs have the same hinges but with inverted signs. The symmetry of the mechanism is evident, with the presence of slidings near the diagonal and perpendicular to the direction of the displacement (Figure 4).

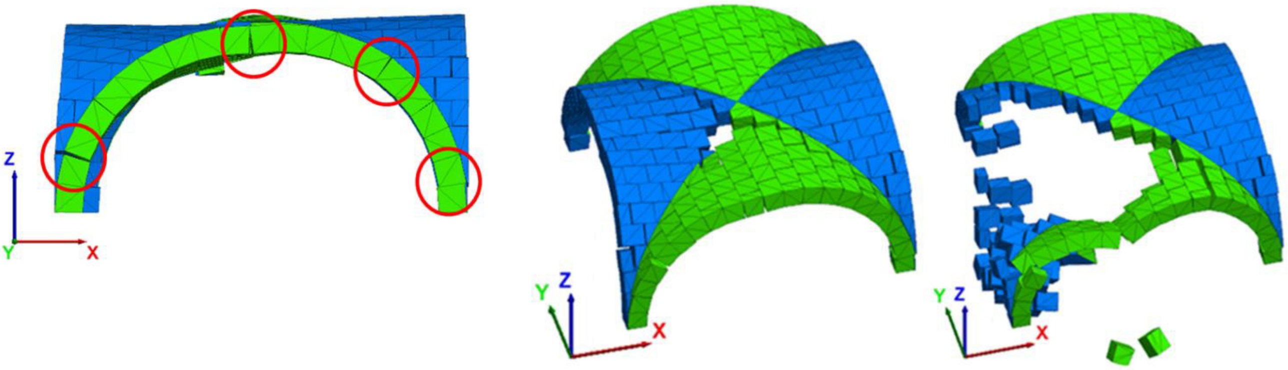

Figure 4. Diagonal displacement in xy-direction: damage mechanism.

The presence of hinges along the length of the arch resulting in a collapse observed in the case of in-plane displacements, represents the development and increase of the bending stresses. An increment of the bending stress is due to the eccentricity between the pressure line and the arch media line. Until when the resultant of the forces remains internal to the middle third of the section, the stresses are still compression ones; when the resultant is applied outside the middle third, the section partializes and cracks appear, usually located along the joints between the blocks [34].

In the test with a vertical displacement at one support, the collapse mechanism is different; on the sides of the vault involved in the vertical displacement in-plane distortions (plane xz and plane yz), slidings of blocks along the diagonal and the activation of the collapse mechanism for relative slidings of the blocks occur (Figure 5).

By monitoring the mechanisms it is observed that, in general, when a reliable point of support for the vault loses its effectiveness, the vault search for alternative resistant mechanisms [35].

Table 2 shows the values of the different displacements in the three directions investigated when the fall of the first block and the collapse of the structure occur.

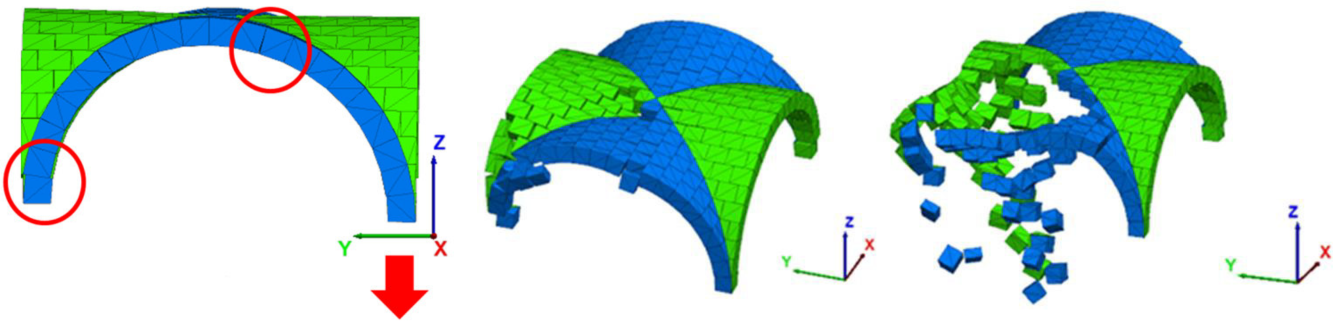

Figure 5. Vertical displacement: damage mechanism.

Table 2. Differential displacements.

Displacement direction	**1° Block Fall (m)**	**Collapse (m)**
Transverse	0.4187	0.4349
Diagonal	0.2698	0.4090
Vertical	0.564	0.5761

- Cross vault with parallel weaving

In the case of a vault with parallel weaving the vault behaves like a series of side by side arches constrained by the geometry and the boundary conditions [36]. In general, the arches don't work alone, but they are embedded within other closing elements; for this reason in-plane constraints were assigned in order to simulate the confinement of the structure.

The size of the vault (5m) and, therefore, its excessive weight, and the lack of ribs proved to be elements of vulnerability for the structure; they could compromise the equilibrium of the vault and cause problems at the beginning of the numerical simulation (Figure 6).

a) **b)** **c)**

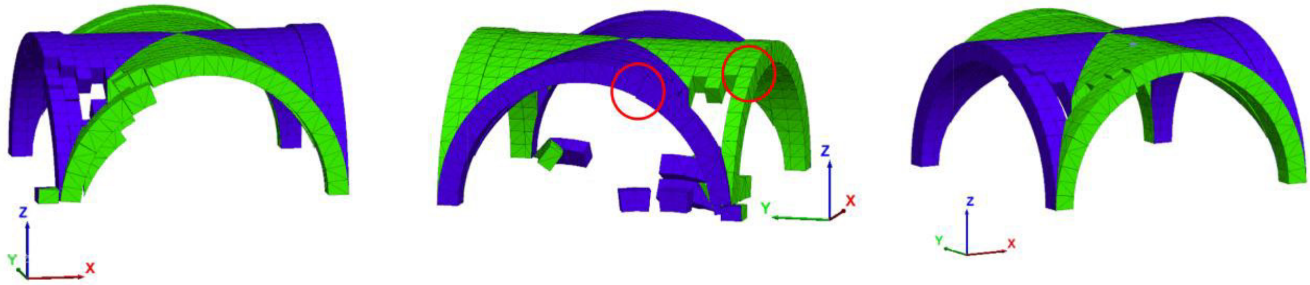

Figure 6. Damage mechanisms: a) horizontal (x-direction), b) diagonal (xy-direction) and c) vertical.

In all the three cases (vertical, diagonal, and horizontal displacement of the support) a partial collapse of the vault for relative slidings of the blocks are observed, and also the attribution of lateral constrains is such as to create a new resistant mechanism.

Table 3 collects the values of the differential displacements for the three cases investigated.

Table 3. Differential displacement.

Displacement direction	**1° Block Fall (m)**	**Collapse (m)**
Transverse	0.3661	0.3661
Diagonal	0.00389	0.350
Vertical	0.004278	0.235

2.2 Response to different friction coefficients

In the case of vaults with orthogonal weaving a study on the friction angle of the masonry was carried out.

More in detail, it was considered the horizontal displacement with the aim of observing the mechanism that is activated, recording the magnitude of the ultimate displacement that leads the structure to collapse, and identifying the law that links this displacement to the friction angle. A series of friction coefficient values μ_s=0.3÷0.8 provided by the literature for masonry have been considered (Table 4).

From the analysis it has been observed that the first two values of the friction coefficient are so low that the vault is not statically verified, highlighting slidings between the first and the second blocks.

At μ_s=0.5 the vault starts to work, and the collapse outlines the typical arched mechanism with four asymmetrical hinges, alternating between the intrados and the extrados. For such friction coefficients, in function of other boundary constrains, geometry, block organization, and mechanical characteristics of the masonry, a linear law was defined (Figure 7).

Table 4. Displacement values recorded for different friction coefficients.

μ_s	ϕ	s_{ult} (m)
0.3	17°	0.01
0.4	21.8°	0.013
0.5	26.6°	0.34
0.6	31°	0.43
0.7	35°	0.55
0.8	38.6°	0.67

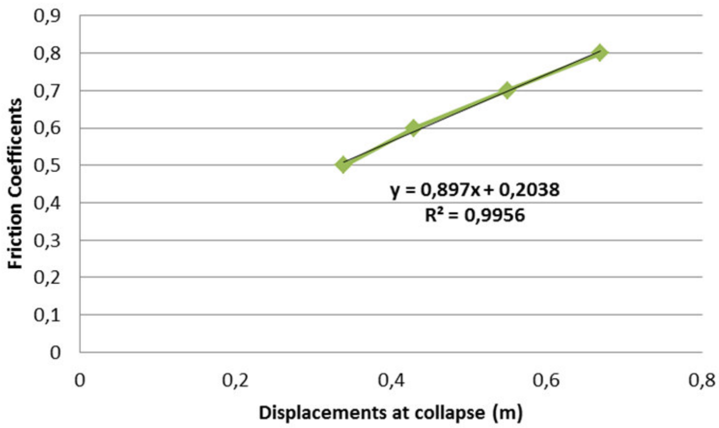

Figure 7. Displacements for different friction coefficients.

3. EXPERIMENTAL BEHAVIOR OF A CROSS VAULT WITH ORTHOGONAL WEAVING

In this second phase of the research, an orthogonal weaving cross vault has been realized and tested from an experimental point of view. Its geometry is generated by the intersection of two barrel vaults with a radius of 0.5m, a total development of 1x1m, and a thickness of 0.07m.

The entire vault is made of 195 blocks, mostly cubic ones with a side of about 5 cm. Among them 32 are half blocks, 10 are special pieces for each diagonal and one key block.

Each block has been obtained after different steps:

1) CAD design;
2) 3D printing of the hole skin of the block with a thickness of 2 mm;
3) Filling with a concrete mix (Figure 8).

The skin is realized with polylactic acid, while the concrete utilized to fill the blocks is made with 25% of sand and chalk and 75% of slaked.

Figure 8. Manufacturing steps of the specimens' blocks.

The density of the material was obtained from the weight of 15 pieces (n), considering the mean square deviation as in the following relation:

$$\rho = \frac{\Sigma \frac{m}{V}}{n} \pm \sigma_{sqm} = 649.50 \pm 36.97 \text{ Kg/m}^3 \qquad (6)$$

where:

m = mass of each block,
V = volume of each block,

n = number of blocks (n=15).
σ_{sqm} = mean square deviation.

The friction angle of the material was obtained by the following test. By providing different axial loads, a pull force was applied to the central block confined between two disks of the same material, and the magnitude of the force that led to the sliding of the block itself was recorded (Figure 9).

The formula that has been applied to determine the internal friction coefficient and the friction angle is (7):

$$\mu = \frac{\sum \frac{F_O}{N}}{n} \pm \sigma_{\text{sqm}} = 0.3266 \pm 0.1923 \Leftrightarrow \varphi = \text{arctg}\,(0.3266 \pm 0.1923) = 18° \qquad (7)$$

where:

F_o is the horizontal force,
N is the normal force,
μ is the internal friction coefficient of the material, corresponding to the friction angle φ.

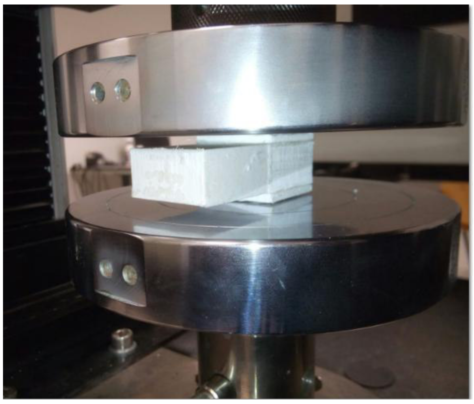

Fig.9 Test Structure.

In the assembly phase, a polystyrene structure manufactured with a thermal cutting technique by means of a numerical control machine, was utilized as a support. The blocks have been dry-assembled, they are able to work in contrast to each other without the help of a binder material (Figure 10). This assembly technology allowed the repeatability of the tests with relative ease of execution.

a) **b)**

Figure 10. a) Assembly of the arch and b) disassembly of the polystyrene support.

3.1 Differential displacements on the vault specimen

Differential displacements were impressed at one support of the vault by a mechanical system connected to the base block. Using a comparator it was possible to record the displacement value when the vault collapsed. In addition, the positioning of cameras in multiple angles allowed to observe the collapse mechanisms.

Three experimental tests were performed for a vertical, diagonal and horizontal direction yielding displacements at one support. Each time the test started from different boundary conditions (due to the assembly of the structure). For this reason, the mechanisms and magnitude of the ultimate displacements are sometimes different from one test to another. To influence the ultimate collapse displacement's measure, there are of course millimetric imbalances, which are not easily to measure, and that have been obtained during the assembly of the structure and disassembly of vault's support, although the care and accuracy utilized in these phases.

In the vertical direction, the relative displacements of the blocks near the support and the slidings along the diagonal, were the principal cause of the collapse.

The collapse mechanism was also characterized by an out-of-plane displacement of the support (Figure 11).

The maximum displacements at collapse are shown in Table 5.

Figure 11. Vertical collapse test: Limit state before collapse.

Table 5. displacements at collapse [mm].

	In-plane horizontal displacement at the support	Vertical displacement at the support	Diagonal displacement at the support
1° test	16	14	76.5
2° test	13	27	48
3° test	25.5	47.5	73

In the case of in-plane horizontal displacements (x-direction), the second and third simulations worked better than the first one. In general, in both tests, it was defined the arch mechanism with three alternate hinges, intrados-extrados, at the side directly affected by the displacement (Figure 12). Nevertheless, the vault collapsed due to a relative sliding problems between the blocks close to the support and not for activation of an arch-mechanism (Figure 13).
This aspect is consistent with the numerical results obtained in sect. 2.2. In this section, in fact, it was pointed out that having too low values of the friction between the blocks means to not have typical arched mechanism with asymmetrical hinges.

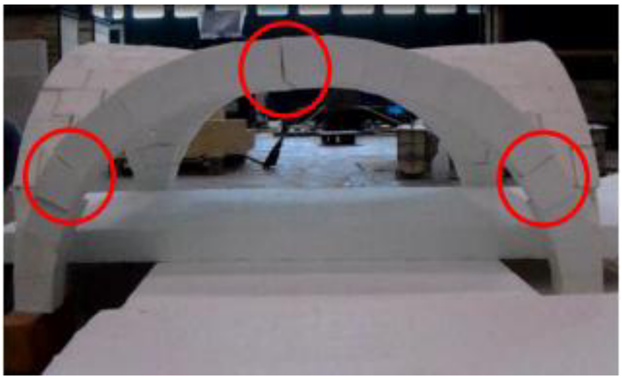

Figure 12. Horizontal (x-direction) collapse test: Arch-mechanism.

Figure 13. Horizontal (x-direction) collapse test: Limit state before collapse.

During the diagonal collapse test (Table 5), except for the second test, which showed the premature fall of one block, in the first and third tests it was noticed an arch mechanism at three alternate hinges, on both sides of the vault that discharge directly on a yielding support. Sliding between blocks along the diagonal were also noticed. Unlike all other cases, in this case a partial fall of the structure occurred, starting from the blocks near the key block, and then following a symmetrical mode with respect to the same diagonal (Figure 14).

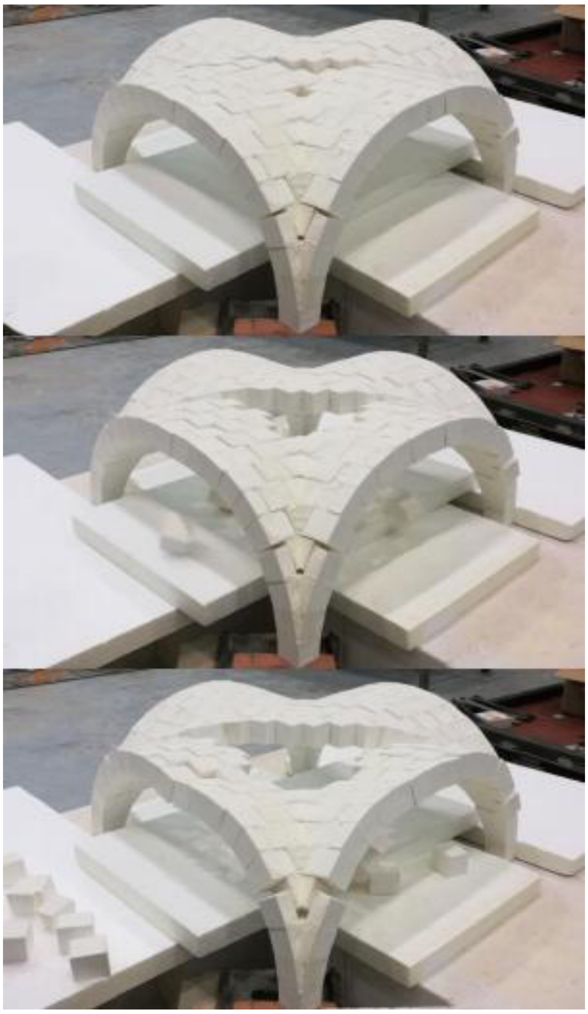

Figure 14. Diagonal collapse test: collapse mechanism.

3.2 Numerical study of differential displacements at the support

A numerical model of the vault has been realized for comparison purposes with the specimen vault. It has the same geometry and mechanical characteristics of the specimen.

The rigid blocks basic hypothesis was assumed, and the density and friction angle were obtained from the values derived from laboratory measurements (Table 6).

Table 6. Mechanical characteristics of the specimen material.

Density, ρ	**Normal stiffness (jkn)**	**Shear stiffness (jks)**	**Friction angle Φ (jfric)**
650 kg / m^3	900 GPa	900 GPa	18°

According to the experimental tests, differential displacements at one support were applied at low speeds. During the tests the mechanisms were monitored and the values that bring the structure to collapse were recorded. The following analysis allows a comparison with the laboratory results and the validation of the Distinct Element Modeling method.

In the case of vertical yielding displacement tests, the mechanism is characterized by relative slidings along the diagonal perpendicular to the direction of failure, and by the presence of the 3-hinge arch mechanism on the two sides directly involved in the mechanism. The collapse of the structure appeared after a displacement of 45 mm, showing a mechanism that was not comparable to the one obtained during the experimental tests (Figure 15).

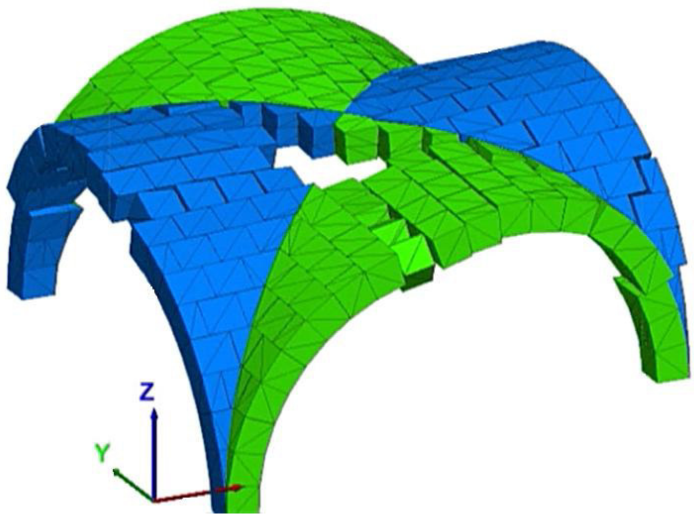

Figure 15. Vertical collapse test: Limit state before collapse.

In the case of horizontal displacements, the kinematic system is characterized by the presence of a 3-hinged arch mechanism and by important relative slidings of the blocks along the diagonal. The collapse took place for a displacement of 56 mm and the activation of a 3-hinged arch mechanism (Figure 16). In the laboratory, instead, while observing this mechanism, the vault collapsed for other problems of instability, i.e. the presence of local hinges.

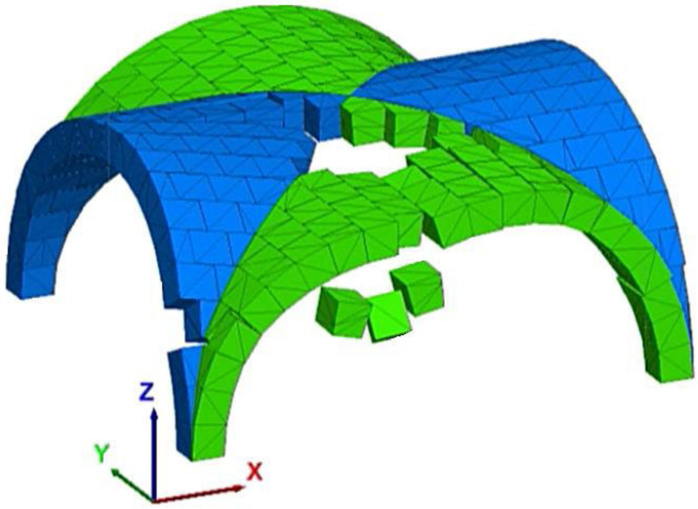

Figure 16. Horizontal direction collapse test: Limit state before collapse.

The diagonal collapse displacement is well defined for both the mechanism and the magnitude of the displacement itself. Similarly to the experimental tests, a symmetry of the displacement with respect to the diagonal involved in the support displacement was observed, recording a displacement value equal to 78 mm, quite comparable with the experimental tests.

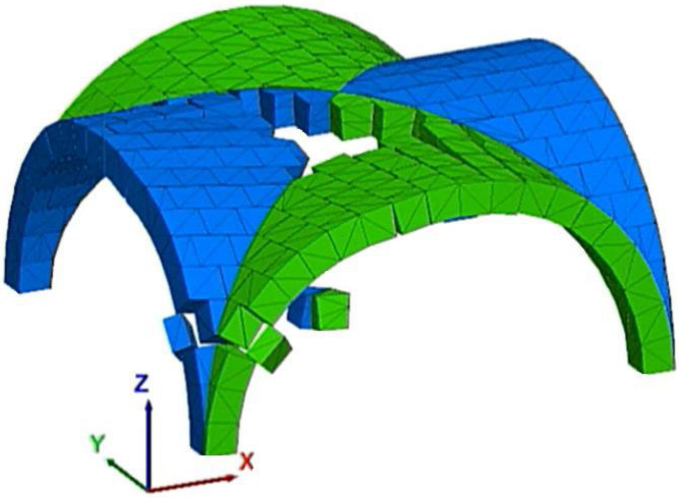

Figure 17. Diagonal collapse test: Limit state before collapse.

4. DISCUSSION

From the numerical results first obtained on the two vaults differently assembled according to the two different masonry patterns (orthogonal and parallel) it has been possible to notice that their behavior is not comparable, at least in the static field. From the results it is noticed that the parallel pattern masonry vault does not comply with the expected results (Table 7). The size of the structure, the lack of robustness of the ribs, the absence of side reinforcements are considered to be the elements of vulnerability for this type of masonry assembly.

Table 7. Comparison computational model results at collapse.

Displacement direction	**Collapse displacement (mm)**	
	Orthogonal weaving	Parallel weaving
Vertical	43.49	36.61
Horizontal	40.90	35.0
Diagonal	57.61	23.5

Nevertheless, for both patterns, is observed that the hypothesis of collapse of two parts of the structure does not prevent the vault to find an alternative equilibrium configuration. It is obtained with a change of the symmetrical arch mechanism set on the ribs, to a system of arches that takes the form of a "tripod", set on the three strength points able to support it. So, it can be affirmed that the cross vaults have considerable flexibility and adaptability of the resistant mechanisms, even to find an equilibrium in conditions of serious instability, such as the collapse of a large portion.

From the results of the experimental tests and the numerical comparisons, it has been noticed an absence of compatibility of the mechanisms and entities of the ultimate displacements recorded in the case of vertical and horizontal support displacements. It is justified by the fact that the computational models have a perfect geometry [23]. However, often the numerical results overestimate the experimental results due to the errors that inevitably happen during the tests. The geometric and mechanical perfection of the numerical models, in fact, is almost impossible to get in a real model.

The absence of an electronic control system of the movement's speed and the presence of slight dislocations at the beginning of the test are conditions that vary from case to case and are not evaluable, they are the cause of the presence of local hinges. Moreover, the geometric imperfections of 3D printed blocks cause an imperfect contact of the surfaces and distribute non-uniform friction coefficients between the different blocks.

From these considerations and comparing on a case-by-case basis the values obtained in the laboratory with those obtained from 3DEC (Table 8), it can be affirmed that both the computational and the experimental studies attest the good capability of the masonry vaults to stand differential displacements at the supports.

With reference to the case of the diagonal displacement, the method is quite reliable in quantifying the structure's capacity and in predicting the collapse mechanism.

Table 8. Comparison of experimental and computational model results.

Displacement direction	**Collapse displacement (mm)**			
	Physical model			3DEC
Vertical	16	13	25.5	45
Horizontal	14	27	47.5	56
Diagonal	76.5	48	73	78

5. CONCLUSIONS

The present study consisted in the modeling of two kinds of cross-vaults by distinct elements, one with parallel weaving and one with orthogonal weaving. In a second phase of the research a 1mx1m vault made with orthogonal weaving has been realized and tested in laboratory for different yielding displacements at one support. The results gave the possibility to evaluate the three-dimensional mechanism of the cross vault, to determine the displacement magnitudes that lead to collapse, and to evaluate the ability of computational methods to predict the experimental results.

This research also wants to highlight that, given the heterogeneous nature of a masonry and given the different possibilities of assembly of the blocks, the distinct elements modeling and structural analysis methodology are to be preferred than the traditional methods of modeling. The latter, in fact, by mean of a discretization of the object of the study, are often not able to make a distinction among its components. DEM, instead, is a more accurate modeling approach because masonry is considered with its actual components' arrangement and better results are obtained.

The method is also valid for the comparison with the experimental results obtained by testing a cross vault specimen subjected to displacements at one support.

Computational simulations allowed to quantify the load-bearing capacity of the vault and to describe the different kinematics of the structure. With due consideration of the case, the results proved to be quite comparable. Specifically for the case of diagonal failure, less than in the other two directions, the experimental laboratory model showed a premature collapse for the presence of local hinges of some edge blocks.

In summary two important results are obtained from this research:

1) Comparing numerical models of vaults with different patterns the vault's behavior is certainly influenced by the weaving of blocks;
2) Comparing a real model of a vault with a computational model the results are comparable and the distinct element method is an optimum method of modeling for the study of masonry structures and the interpretation of the results.

Future implications of the following research can certainly be the computational study of additional masonry equipment (herringbone pattern) and to complete the experimental part with a dynamic simulation on laboratory specimens.

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