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Abstract

Damping Reduction Factor plays a key role in scientific literature and Technical Codes, but till now existing formulations present differences and inconsistences probably because obtained by integration of real recorded events, thus sensitive to specific used data. This paper investigates the relation between damping reduction factor and earthquake duration by means of random vibration theory. A stochastic process, that is non-stationary and filtered, is used to model a seismic event. The modulation function is suitably chosen to describe earthquakes characterized by different durations. The stochastic process peak theory allows to calculate damping reduction factor after the definition of the probabilistic response of a simple linear visco-elastic oscillator. The variability with seismic duration for different soil conditions and damping ratios is investigated. The study points out that damping reduction factor is more sensitive to seismic duration in the range of high period and on rigid soil with respect to other conditions. The results show that, if damping ratio or effective duration values are increased, the damping reduction factor value diminishes.

Keywords: Ground Motion Duration, Damping Reduction Factor, Seismic response spectrum, Stochastic process, random vibration theory

1. Introduction

Serious seismic damage observed on structures and infrastructures systems up to today [1] can be prevented by means of retrofitting interventions if the capacity of these systems and the seismic demand are properly evaluated [2]-[6]. In structural seismic design, Damping Reduction Factor (DRF) represents an effective tool for design purposes to estimate the demand by response spectra characterized also by damping ratios different from 5% as in case of structures equipped with passive energy dissipation or isolation systems. DMF modifies the values of the conventional elastic spectral response with damping ratio equal to 5% to the values corresponding to a different damping level. It is defined as the ratio between the spectral ordinate at 5% of conventional damping and the spectral ordinate at a different level of damping.

DRF finds many applications to study the behavior of structures [7], especially for the ones equipped with passive energy dissipation or isolation systems [8]-[12]. In these situations, the DRF permits to estimate the variation of the structural response (displacements and forces) due to the high supplemental damping values [13]-[16]. In addition, for inelastic structures, DRF allows to calculate the maximum displacement demand from the one of an equivalent linear system [8]. For these and other reasons the DRF is particularly suitable for the seismic design of a structure since it provides a practical evaluation of the reduction of earthquake loads for effects of structural, non-structural and supplementary energy dissipation systems. For that reason, it is selected as a key parameter in the present study.

In past years, several studies for the formulation of the DRF have been carried out by many researchers, the outcomes of which have been adopted by the main seismic codes. Different expressions for the DRF can be found in literature. Up to today, the main research efforts have been oriented to the study of the response of a simple (elastic) SDOF system with viscous damping under seismic action [17]-[23]. As a consequence, the codes introduce a DRF that depends on the damping ratio only, whereas different authors [21]-[23] showed that various parameters may affect the DRF. There are two ways to classify the parameters that much influence the DRF: by non-structural parameters such as earthquake magnitude, ground motion duration (GMD), site conditions, epicentral distance, etc. or by structural parameters such as damping of the structure, natural vibration period, dissipation device properties (energy dissipation capacity), etc.. A very interesting topic is represented by the dependence of DRF on seismological parameters. This dependence is much evident considering local site conditions, source distance and magnitude of the earthquake [21]-[24]. In [25] a simulation procedure to estimate the DRF based on an artificial neural network has been developed. The effect of magnitude on the DRF is greater in case of large earthquakes as it was pointed out in different studies [22] for structures with natural periods greater than 0.5 sec. Attention should be placed in case of structures with shorter natural periods (< 0.5 sec) for which the magnitude can have also contrary effects [23]. Concerning the influence of seismological parameters on DRF, Bommer et al. [22] focused their studies on the GMD. The authors observed that it is possible to take into account the influence of magnitude and distance by studying the effect of the GMD and the number of cycles. Based on this observation, Stafford et al. [26] introduced significant equations that give the DRF for different damping ratios starting from the number of cycles and the GMD. In its research, Stafford concluded that a prediction model based on the GMD parameter could be used with difficulties as this parameter usually is not one of the parameters elaborated and directly available in earthquakes database. However, non - distinction between soil types is given by Stafford et al. [26]. Rosenblueth [27] suggested an equation to predict the influence of GMD on DRF and, in accordance with Stafford et al. [26], concluded that the influence of GMD appears negligible for earthquake GMD larger than about 20 sec.

The GMD finds a different definition in the models proposed by Stafford et al. [26] and Rosenblueth [27] but a very good match is observed for damping ratio equal to 10%. Some discrepancies are noticeable among the two models when damping ratio increases. The influence of GMD on DRF has been also investigated by Anbazhagan et al. [28]. The authors choose the pseudo-spectral acceleration to define the DRF and investigate how the DRF varies as a function of magnitude and GMD, distance of earthquake hypocenter, classification of site (soil type), period and damping. The dependence of DRF on the GMD was also analyzed by Daneshvar et al. [29] which concluded that DRF mainly depends on the GMD and the frequency content that are different for each record. Zhou et al. [30] studied how the DRF is affected by the effective GMD. The authors point out that greater values of the damping ratio or of the effective GMD produce smaller values of DRF. However, the GMD is function of distance and magnitude of ground motion and of soil type, that are parameters much more common and available in ground motion database. For that reason, it is usually preferable to define a function between these parameters and the DRF that indicates also a relation between DRF and GMD. However, in this study the authors do not include the type of soil in the evaluation of dependence of DRF with GMD.

Rezaeian et al. [31] propose a model to relate the DRF to the magnitude and distance to include in the model the high influence of GMD. The influence of different parameters on DRF is not the same in the cited studies because the sites are classified in different way and the selection of ground motions is performed by different criteria. So a stochastic process is adopted in the proposed study to overcome this difficulties.

In a previous study [32] the authors, by means of the random theory approach, investigated the effect of soil type on DRF and demonstrated that not only the predominant frequency of the seismic event but also the bandwidth of seismic signal affect the DRF. Since the study of the joint effect on DRF of parameters such as soil type and GMD has not performed in other studies, the authors will develop random vibration theory to analyse in a combined way the influence of earthquake duration and soil type on DRF. GMD is a key parameter in seismic design. Since the 1950s, the peak acceleration, frequency content and GMD are considered important parameters to design the structures but up to today the peak acceleration and frequency content are the only parameters used in design methods. The influence of GMD on DRF is investigated in the present study by means of the random vibration theory: a modulated filtered stochastic process is applied on a linear single degree of freedom (SDF) system and the peak theory of stochastic process is used to calculate the seismic spectrum in stochastic terms. The product of a time modulation function [33] [34] and a stationary filtered stochastic process gives a modulated non-stationary stochastic process. A series of two linear oscillators forced by a modulated white noise process permit to obtain the linear fourth order filter that is adopted in the procedure. A formulation correlating the modulating function and the GMD through the Arias intensity [42] is introduced to analyze the effects of the GMD on DRF. In this way, the stochastic dynamic response can be evaluated for different GMDs, properly defining the modulation function and therefore a sensitivity analysis on DRF can be carried out to evaluate how DRF changes as a function of different parameters. The proposed approach overcomes the limitations of the strategies based on seismic records of real events, because in these it is difficult to accurately identify the influence of different factors. This is pointed out by discrepancies between the various studies existing in the literature.

The main advantage of using a stochastic approach is, on the contrary, the possibility to represent seismic motion by a simple model defined by few parameters, but able to describe the most seismological characteristics of real earthquakes as content of energy, frequency and GMD. On the other hand, the proposed approach would require an assessment of the spectrum parameters themselves on the basis of seismic models that are more consistent with the seismic scenario.

The study is presented in the following sections: the stochastic model of the seismic acceleration is explained in section 2. The relation between the modulation function parameters and GMD is defined in section 3. The evaluation of DRF in stochastic terms is developed in section 4. The results of the sensitivity analysis developed considering different GMDs and soil conditions are shown and then discussed in section 5. A formulation for DRF evaluation useful for practical applications is proposed and compared with other existing formulations in section 6. Finally, the conclusions are given in section 7.

2. Stochastic Modelling of Seismic Motion

Seismic acceleration is assumed as a uniformly modulated non-stationary stochastic process that is calculated by the product of a time modulation function $\varphi(t)$ and a stationary process [35]. The stationary part of the process is described by the well known filtered process proposed by Clough et al. [36]: two linear oscillators in series, subjected to a modulated white noise process give a linear fourth order filter. Ground acceleration $\ddot{X}_g$ is given by:

$$\begin{cases} \ddot{X}_g(t) = -\omega^2{}_p X_p(t) - 2\xi_p\omega_p\dot{X}_p(t) + \omega^2{}_f X_f + 2\xi_f\omega_f\dot{X}_f(t) \\ \ddot{X}_p(t) + \omega^2{}_p X_p(t) + 2\xi_p\omega_p\dot{X}_p(t) = \omega^2{}_f X_f + 2\xi_f\omega_f\dot{X}_f(t) \\ \ddot{X}_f(t) + 2\xi_f\omega_f\dot{X}_f(t) + \omega^2{}_f X_f = -\phi(t)W(t) \end{cases} \quad (1)$$

where $W(t)$ is the white noise stochastic process, with Power Spectral Density function S_0; $X_f(t)$ is the first filter response, with frequency ω_f and damping ratio ξ_f, $X_p(t)$ is the second filter response with frequency ω_p and damping ratio ξ_p; $\phi(t)$ is the modulation function. The present research assumes the Jennings' modulation function [37], below reported:

$$\phi(t) = \alpha t e^{-\beta t} \quad \alpha, \beta > 0 \quad (2)$$

α, β being the parameters that describe the shape of the modulation function that will be selected in section 3.

3. Definition of modulation function considering the earthquake duration

Until now, GMD has been defined in different ways in literature [38] but the bracketed duration, the uniform duration and the significant duration are the most used. For a given curve that shows the values of the acceleration as a function of the time, the duration of the ground motion is a time interval. In the case of bracketed duration, a threshold value of acceleration (usually 0.05 g) is defined. The bracketed duration [39] is the time interval between the time corresponding to the first and the time corresponding to the last overrun of the defined threshold value of acceleration. The choice of the threshold value is different in literature and therefore this definition of GMD results subjective (it can be absolute or relative e.g. 10% of Peak Ground Acceleration (PGA)). The uniform duration [40], [41] is calculated as the sum of time intervals. During each time interval the acceleration values overrun the threshold value of the acceleration. This definition of GMD is explained in a way similar to the bracketed duration except for the interval between the thresholds. The disadvantages in the use of this GMD definition are: the dependence of the GDM on the chosen threshold acceleration value; the influence of small earthquakes recorded before or after the main earthquake record that could be included in the GMD evaluation. The effective duration is a preferable definition of GMD because it is the time interval necessary to release a given seismic energy content. The Arias intensity I_a [42] that considers the integral square of the ground acceleration, a measure of the energy content, is usually chosen to define this GDM. The Arias intensity is defined as:

$$I_a = \frac{\pi}{2g}\int_0^{T_t} \ddot{x}_g^{\,2}(t)dt \tag{3}$$

where $\ddot{x}_g(t)$ is the time history of the ground acceleration, g is the acceleration of gravity and T_t is the GMD of the record. The time intervals $T_{5\text{-}75}$ and $T_{5\text{-}95}$ between 5%-75% and 5%-95% of the Arias intensity (I_a) are respectively the two measures of significant duration most used in literature.

In this study, in order to analyze the influence of GMD on DRF, $T_{5\text{-}95}$ is considered, as it is one of the most common measure of GMD and it can be related to magnitude, distance and soil type. Different empirical formulation have been proposed in literature for effective duration [43]-[46] which consider the dependence on magnitude, distance and soil condition. The outcome was that effective duration increases with distance, magnitude and moving from rock to soft soil. Among these three influencing parameters, soil type has a larger influence than distance.

The present research deals with the evaluation of the influence of the effective duration on DRF for different soil conditions. To develop a suitable model for this analysis, a formulation correlating the modulating function and $T_{5\text{-}95}$ (effective duration) throughout the mean value of I_a is obtained. In this way, stochastic dynamic response can be evaluated for different GMDs, properly defining the modulation function. In order to achieve the correlation between GMD and modulation function, the two parameters α and β in Eq. 2 are obtained by an identification procedure. Introducing the time t_m where the modulation function exhibits its maximum value, parameters α and β are expressed as functions of this unknown parameter from the simultaneous equations:

$$\begin{cases} \phi(t_m) = 1 \\ \dfrac{d}{dt}\phi(t_m) = 0 \end{cases} \tag{4}$$

The two parameters α and β are then evaluated as function of t_m:

$$\beta = \frac{1}{t_m} \tag{5}$$

$$\alpha = \frac{e}{t_m} \tag{6}$$

The dimensionless time ratio $\tau = \frac{T_t}{t_m}$ can be introduced in Eq. (2), so obtaining:

$$\phi(\tau) = \tau e^{(1-\tau)} \tag{7}$$

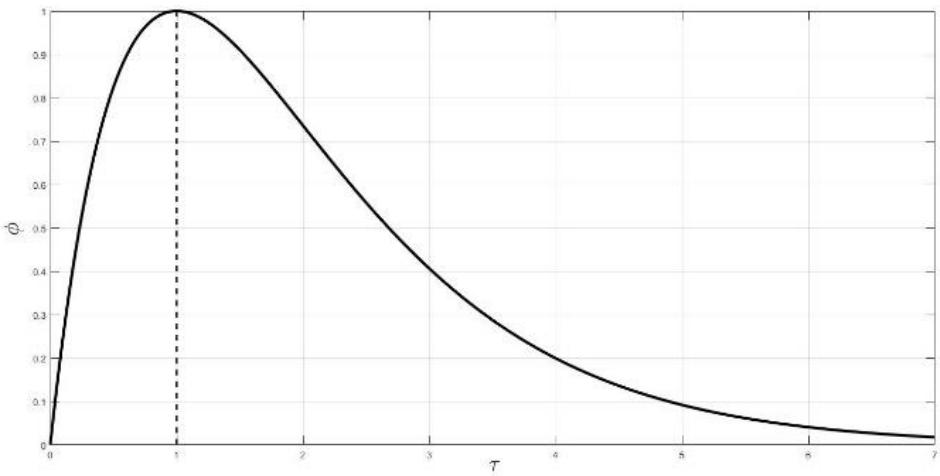

Figure 1: Modulation function (Φ) in dimensionless time (τ)

The time values at which 5 % and 95 % of the Arias intensity is reached can be used to evaluate the effective duration $T_{5\text{-}95}$. In stochastic terms, the mean value of I_a could be evaluated as:

$$\mu[I_a] = \frac{\pi}{2g}\int_0^{T_t} \left\langle \ddot{x}_g{}^2(t) \right\rangle dt = \frac{\pi}{2g}\sigma_{\ddot{x}_g}^2\ \psi_a(T_t) \tag{8}$$

where T_t is the total duration of the acceleration record, $\sigma_{\ddot{x}_g}^2$ is the variance calculated for the acceleration of the ground, $\left\langle \ddot{x}_g{}^2(t) \right\rangle$ denotes the expected value of the square of $\ddot{x}_g$ and $\psi_a(T_t)$ is defined by:

$$\psi_a(T_t) = \int_0^{T_t} \phi^2(t)dt \tag{9}$$

Now using Eq. (7), Eq. (9) becomes:

$$\int_0^{\rho} \phi(\tau)^2 \, d\tau = \frac{1}{4}\mathrm{e}^2 \left(1 - \mathrm{e}^{-2\rho}\left(1 + 2\rho\left(1+\rho\right)\right)\right) \tag{10}$$

where $\rho = \dfrac{T}{t_m}$. At the end of the phenomenon ($T = \infty$), it results:

$$\psi_a(\infty) = \int_0^{\infty} \phi(\tau)^2 \, d\tau = \frac{\mathrm{e}^2}{4} \tag{11}$$

So the dimensionless time ρ_k necessary to release the k% of the total I_a is the solution of the following equation:

$$\int_0^{\rho_k} \varphi(\tau)^2 \, d\tau = k \int_0^{\infty} \varphi(\tau)^2 \, d\tau \tag{12}$$

that means in implicit form:

$$\frac{1}{4}\mathrm{e}^2 \left(1 - k - \mathrm{e}^{-2\rho_k}\left(1 + 2\rho_k\left(1+\rho_k\right)\right)\right) = 0 \tag{13}$$

Equation (12) allows the definition of the ρ_5 and ρ_{95} values corresponding to the 5% and the 95% of the energy I_a calculated on the modulation function respectively. The values so obtained are:

$$\begin{aligned} \rho_5 &= 0.40884 \\ \rho_{95} &= 3.1478 \end{aligned} \tag{14}$$

Thus the values t_m corresponding to the selected effective duration $T_{5\text{-}95}$ can be determined by the definition of the ratio ρ:

$$\begin{aligned} T_5 &= 0.40884 t_m \\ T_{95} &= 3.1478 t_m \end{aligned} \tag{15}$$

Finally, the effective duration is $T_{5-95} = 2.7390 t_m$, so that it is possible to define t_m as:

$$t_m = \frac{T_{5-95}}{2.7390} \tag{16}$$

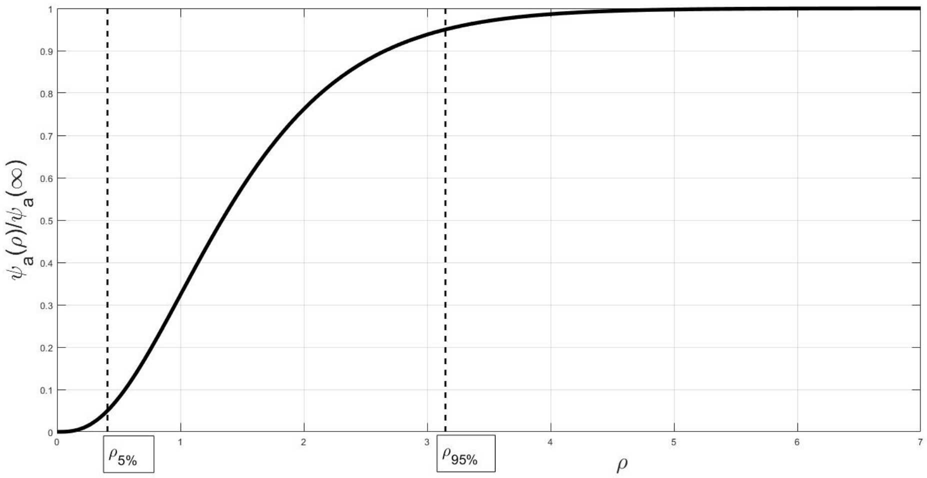

Figure 2: $\psi_a(\rho)/\psi_a(\infty)$ *in dimensionless time scale (ρ)*

4. Evaluation of Damping Reduction Factor by peak theory

In this section, the seismic ground acceleration action $\ddot{X}_g(t)$ given by Eq. (1) is applied on a simple linear-viscous SDOF system to evaluate the DRF in stochastic meaning. For this system, the motion equation is:

$$\ddot{X}_s(t) + 2\xi\omega\dot{X}_s(t) + \omega^2 X_s(t) = -\ddot{X}_g(t) \tag{17}$$

where X_s is system-ground relative displacement, ω is the natural frequency and ξ is the damping ratio:

$$\omega = \sqrt{\frac{k}{m}} \text{ and } \xi = \frac{c}{2\sqrt{km}}, \tag{18}$$

k and *m* being the system mass and stiffness, respectively.

In the state-space, the motion equation of the system becomes:

$$\dot{\mathbf{Z}}(\mathrm{t}) = \mathbf{A}\mathbf{Z}(\mathrm{t}) + \mathbf{F}(\mathrm{t}). \tag{19}$$

where **F** is the force vector and **Z** is the state-space vector:

$$\mathbf{Z} = \left(X_s, X_p, X_f, \dot{X}_s, \dot{X}_p, \dot{X}_f\right)^T \tag{20}$$

$$\mathbf{F} = \left(0, 0, 0, 0, 0, -\varphi(\mathrm{t})\,\mathrm{w}(\mathrm{t})\right)^T \tag{21}$$

Finally **A** is the state matrix:

$$\mathbf{A}=\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\omega^2 & \omega_p^2 & -\omega_f^2 & -2\xi\,\omega & +2\omega_p\xi_p & -2\omega_f\xi_f \\ 0 & -\omega_p^2 & +\omega_f^2 & 0 & -2\omega_p\xi_p & +2\omega_f\xi_f \\ 0 & 0 & -\omega_f^2 & 0 & 0 & -2\omega_f\xi_f \end{pmatrix} \tag{22}$$

The matrix Lyapunov differential equation [47]-[54] can be used to calculate the stochastic response of the system when it is excited by the non-stationary modulated Clough and Penzien stochastic process:

$$\dot{\mathbf{R}}(t)=\mathbf{A}\mathbf{R}(t)+\mathbf{R}(t)\mathbf{A}^T+\mathbf{B}(t) \tag{23}$$

where $\mathbf{R}(t)=\langle \mathbf{Z}\mathbf{Z}^T \rangle$ is the covariance matrix and $\mathbf{B}(t)$ is a matrix that is square and has all elements equal to zero except for the last one that assumes the value $2\pi S_0 \phi(t)^2$.

For the analyzed system, a definition of the displacement spectrum is:

$$S_d = \left| X_s(\mathrm{t}) \right|_{max} \tag{24}$$

The DRF (η) parameter permits to approximate the high damped elastic response spectrum (S_d) starting from the 5% damped one ($S_{d,\xi=5\%}$):

$$S_d = \eta S_{d,\xi=5\%} \tag{25}$$

where ξ is the damping ratio that is greater than 5% for the high damped system.

A seismic response spectrum gives the maximum displacement or acceleration response of a SDOF system when a recorded earthquake [4], [6], 5[55][61] is applied as a function of the natural period of the system. Different relations are obtained for different values of structural damping. From a stochastic point of view, the response spectrum is still a relation between the maximum acceleration or displacement response of a SDOF system, that is subjected to a ground motion, and the natural period of the SDOF system but the maximum response and the ground motion are considered in stochastic terms. Different values of structural damping again give different response spectra.

In the present paper the authors propose a procedure to calculate the DRF value starting from the definition of the stochastic displacement spectrum by mean of the peak theory. This theory assumes that the maximum response of a SDOF system in displacement $X_s^{\max}$ is the displacement value that is not overrun with a fixed value of the probability P^*_f. For this reason the analysis focuses on the evaluation of this maximum displacement that, from a mathematical formulation, is the displacement threshold *b* that will not be exceeded with a probability P^*_f during the system lifetime [62]. If this problem is analyzed for a generic process *X* and for a threshold *b*, the Vanmarcke formula [63] gives the probability that the process *X* exceeds the threshold *b*:

$$P(t,b) = 1 - \exp\left(-\int_0^t \alpha(\tau)d\tau\right) \tag{26}$$

where the expected decay rate $\alpha(\tau)$ is:

$$\alpha(\tau) = \nu_X(b,t)\frac{1-\exp\left[-\dfrac{\nu_R^+(b,t)}{\nu_X(b,t)}\right]}{1-\dfrac{\nu_X^+(b,t)}{\nu_X^+(0,t)}} \tag{27}$$

$\nu_X(b,t) = \nu^+{}_X(b,t) + \nu^-{}_X(b,t)$ being the expected rate of the response that exceeds the threshold. The up and down crossing expected rates are given by [64]:

$$\begin{aligned} \nu^+{}_X(b,t) &= \int_{\dot{b}}^{\infty} (\dot{x}-\dot{b})\, f_{X\dot{X}}(t,b;t,\dot{x})d\dot{x} \\ \nu^-{}_X(b,t) &= \int_{-\infty}^{\dot{b}} (\dot{b}-\dot{x})\, f_{X\dot{X}}(t,-b;t,\dot{x})d\dot{x} \end{aligned} \tag{28}$$

where $f_{X\dot{X}}(t,b;t,\dot{x})$ is the joint probability density function (JPDF) of X and $\dot{X}$.

In Eq. (14) the expected up-crossing rate of the envelope process *R(t)* is indicated with $\nu_R^+(b,t)$ and *R(t)* can be expressed as:

$$R(t) = \sqrt{X(\mathrm{t}) + \hat{\mathrm{X}}^2(\mathrm{t})} \tag{29}$$

where $\hat{\mathrm{X}}(\mathrm{t})$ is the Hilbert transform of $X(\mathrm{t})$.

In Eq. (27) $\nu_R^+(b,t)$ is obtained from Eq. (15) replacing $\dot{x}$ and $f_{X\dot{X}}(t,b;t,\dot{x})$ with $\dot{r}$ and $f_{R\dot{R}}(t,\mathrm{r};t,\dot{\mathrm{r}})$ respectively if the JPDF $f_{R\dot{R}}(t,\mathrm{r};t,\dot{\mathrm{r}})$ is available.

For convenience, the above crossing rates are evaluated in a normalized way by introducing the normalized variables:

$$Y(\mathrm{t}) = \frac{X(\mathrm{t})}{\sigma_x(\mathrm{t})} \tag{30}$$

$$Q(t) = \sqrt{Y(\mathrm{t}) + \hat{Y}^2(\mathrm{t})} = \frac{R(\mathrm{t})}{\sigma_x(\mathrm{t})} \qquad \kappa(\mathrm{t}) = \frac{b}{\sigma_x(\mathrm{t})} \tag{31}$$

$\sigma_x(\mathrm{t})$ being the standard deviation of *X* process and $\hat{Y}(\mathrm{t})$ the Hilbert transform of Y(t).

Since $\hat{Y}(\mathrm{t})$ and Y(t) result uncorrelated, Eqs. (15) become:

$$\nu^+{}_X(t,b) = \nu^+{}_Y(\mathrm{t},\kappa) = \omega_0\phi(\kappa)\left[\phi\left(\frac{\dot{\kappa}}{\omega_0}\right) - \frac{\dot{\kappa}}{\omega_0}\Phi\left(-\frac{\dot{\kappa}}{\omega_0}\right)\right] \tag{32}$$

$$\nu^-{}_X(t,b) = \nu^-{}_Y(\mathrm{t},\kappa) = \omega_0\phi(\kappa)\left[\phi\left(\frac{\dot{\kappa}}{\omega_0}\right) + \frac{\dot{\kappa}}{\omega_0}\Phi\left(-\frac{\dot{\kappa}}{\omega_0}\right)\right] \tag{33}$$

where ϕ is the standard normal density function, Φ is the standard normal distribution function and ω_0 is the standard deviation of $\dot{Y}(\mathrm{t})$, given by:

$$\begin{aligned}\omega_0^2 &= E\left[\dot{Y}^2(t)\right] = E\left[\frac{\sigma_X(t)\dot{X}(\mathrm{t}) - X(\mathrm{t})\dot{\sigma}_X(t)}{\sigma^2{}_X(t)}\right] \\ &= \frac{\sigma_{\dot{X}}(t)}{\sigma^2{}_X(t)} - 2\frac{\dot{\sigma}_X(t)}{\sigma^3{}_X(t)}c_{X\dot{X}}(t) + \frac{\dot{\sigma}_X^2(t)}{\sigma^2{}_X(t)}\end{aligned} \tag{34}$$

The functions $Q(t)$ and $\dot{Q}(\mathrm{t})$ are mutually independent in the case of a normal process, with the Rayleigh and normal distribution respectively given by:

$$f_Q(q) = q\exp\left(-\frac{q^2}{2}\right) \tag{35}$$

$$f_{\dot{Q}}(\dot{q}) = \frac{2}{\sqrt{2\pi\left(\omega_0^2 - \lambda^2\right)}}\exp\left(-\frac{1}{2}\frac{\dot{q}^2}{\left(\omega_0^2 - \lambda^2\right)}\right) \tag{36}$$

where $\lambda(t)$ is the covariance of $Y(t)$ and $\dot{\hat{Y}}(\mathrm{t})$. Since X(t) and $\hat{\mathrm{X}}(\mathrm{t})$ are uncorrelated, $\hat{\mathrm{X}}(\mathrm{t})$ and $\dot{\hat{\mathrm{X}}}(\mathrm{t})$ are uncorrelated too. Starting from the above equations, it is simple to introduce the relation:

$$\lambda(t)=E\left[Y(t)\dot{\hat{Y}}(\mathrm{t})\right]=E\left[\frac{X(t)}{\sigma_X(t)}\frac{\sigma_X(t)\dot{\hat{X}}(\mathrm{t})-\hat{\mathrm{X}}(\mathrm{t})\dot{\sigma}_X(t)}{\sigma^2{}_X(t)}\right]=\frac{c_{X\dot{X}}(t)}{\sigma^2{}_X(t)} \tag{37}$$

Then the evaluation of the up-crossing rate for the envelope is possible by:

$$\begin{aligned}&\upsilon_R^+(t,b)=\upsilon_Q^+(t,\kappa)=f_Q(\kappa)\int_{\dot{\eta}}^{\infty}(s-\dot{\kappa})\,f_{\dot{Q}}(s)ds=\\&\sqrt{\omega_0^2-\lambda^2}\kappa e^{-\frac{\kappa^2}{2}}\left[\phi\left(\frac{\dot{\kappa}}{\sqrt{\omega_0^2-\lambda^2}}\right)-\frac{\dot{\kappa}}{\sqrt{\omega_0^2-\lambda^2}}\Phi\left(-\frac{\dot{\kappa}}{\sqrt{\omega_0^2-\lambda^2}}\right)\right]\end{aligned} \tag{38}$$

If $\sigma^2{}_X(t), \sigma^2{}_{\dot{X}}(t)$, $c_{X\dot{X}}(t)$ and $c_{X\dot{\hat{X}}}(t)$ are available, then the crossing rates of $X(t)$ and $R(t)$ can be evaluated directly. The variation of the natural period of the SDOF allows to calculate the displacement spectrum. $X_{\max}^{P^*}(t)$ is the maximum displacement such that the probability that $X(t)$ will exceed the domain $[-X_{\max}^{P}, +X_{\max}^{P}]$ is equal to a given value P^*. This inverse problem can be solved by a numerical approach as described in [62].

5. Analysis results

In this section, a sensitivity analysis on DRF is carried out considering the effect of natural period, damping ratio, soil type and effective duration. More in details, different effective durations are considered, and modulation function parameters are identified by the procedure described in section 4. A wide range of GMD is considered, in order to cover all possible earthquakes. Different soil conditions are analyzed (soft, medium and rigid) by assigning the Clough and Penzien model parameters as reported in Table 1. In the analysis it is assumed $P^*=10^{-3}$.

Soil type	ω_p (rad/sec)	ξ_p	ω_c (rad/sec)	ξ_c
Rigid	15	0.6	1.5	0.6
Medium	10	0.4	1	0.6
Soft	5	0.2	0.5	0.6

Table 1 Filter parameters for different soil types

Figures 3 - 5 show the variability of DRF (η) for different soil conditions versus the system natural period $T_0 = 2\pi / \omega$. Each plot corresponds to an assigned value of the system damping ratio ξ. Six values of ξ are considered: 0.10, 0.15, 0.20, 0.40, 0.60 and 0.80. Vertical line defines the ground motion predominant period $T_f = 2\pi / \omega_f$. Figure 3 shows the results for a soft soil. Different colored lines correspond to different values of the effective duration T_{5-95}, which varies in the range 10.94s - 54.80s. The dependences between the parameters damping ratio, structural vibration period, soil type and GMD are represented by means of the plots in Figures 3 - 5. The DRF shows a period-dependent nature in Figure 3. It is worth noting that all curves show the same variability as T_0 varies and moves downwards and as the damping ratio increases in the considered range. The lowest values of DRF occurs when the natural period T_0 equals the predominant period of earthquake T_f. In detail, for $T_0 < T_f$ the DRF decreases with the increase of T_0 and reaches a minimum in T_f. Then, for $T_0 > T_f$ the DRF increases with the increase of the natural period T_0. The DRF tends to assume a unity value in correspondence of the lowest or highest values of period. This can be explained by the fact that the forces can be independent from the damping ratio in a very stiff or very flexible structure. It also emerges that DRF value decreases as damping ratio ξ increases. These results have been observed by Zhou et al. [30]. In addition, when period changes the DRF variation seems lower for low damping ratio. For example for ξ = 0.1 (Figure 3), DRF varies between 0.84 and 0.64 (for a GMD of 54.8 s), whereas for ξ =0.8, DRF varies between 0.57 and 0.12 (for a GMD of 54.8 s).

With regard to the variability of DRF with GMD, that is the main topic of this study, two different considerations can be carried out: firstly, it is noticed that as the GMD increases, the DRF decreases and the plots move downwards. This variability of DRF with GMD is in agreement with the studies from Bommer et al. [22] and Zhou et al. [30]. These authors observed a decrease of DRF when GMD increase. In addition, results show a larger variability of DRF with GMD in the range of high natural periods. The results of these studies make evident that the GMD has important effects on DRF and this should be considered in engineering implications. Secondly, the influence of earthquake GMD depends on damping ratio and it seems larger for the system with lower damping ratio and tends to reduce as the damping ratio increases. Later we will see that this is not always true.

Figures 4 and 5 show the results of the sensitivity analysis for medium and rigid soils. Firstly, it is observed that the soil type affects the DRF and more precisely DRF is larger for soft soil. This result agrees with the ones presented by Lin et al. [20]: the soil type has a significant effect on DRF especially for very stiff and rock sites. The aspects related to the influence of soil type on DRF have been discussed in a previous study by the authors to which reference is made [32]. With regard to the variability of DRF with earthquake effective duration that is the topic of this study, the same variability of the DRF, already observed in the case of soft soil, is observed for medium and rigid soils when GMD varies.

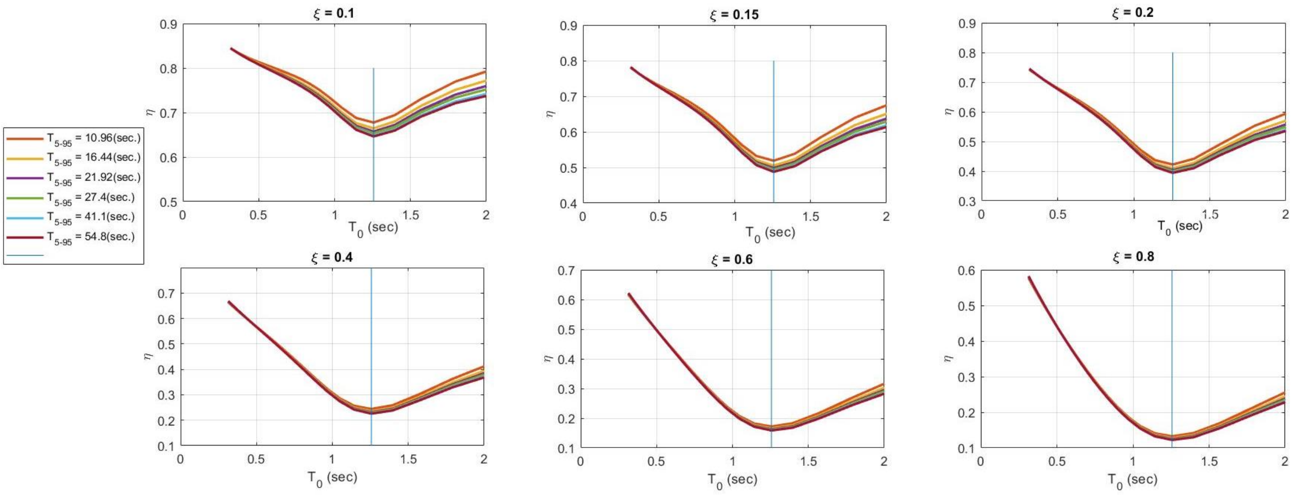

Figure 3 *Variability of DRF (η) with system natural period (T_0) for soft soil. The ground motion predominant period (T_f) is the blue line.*

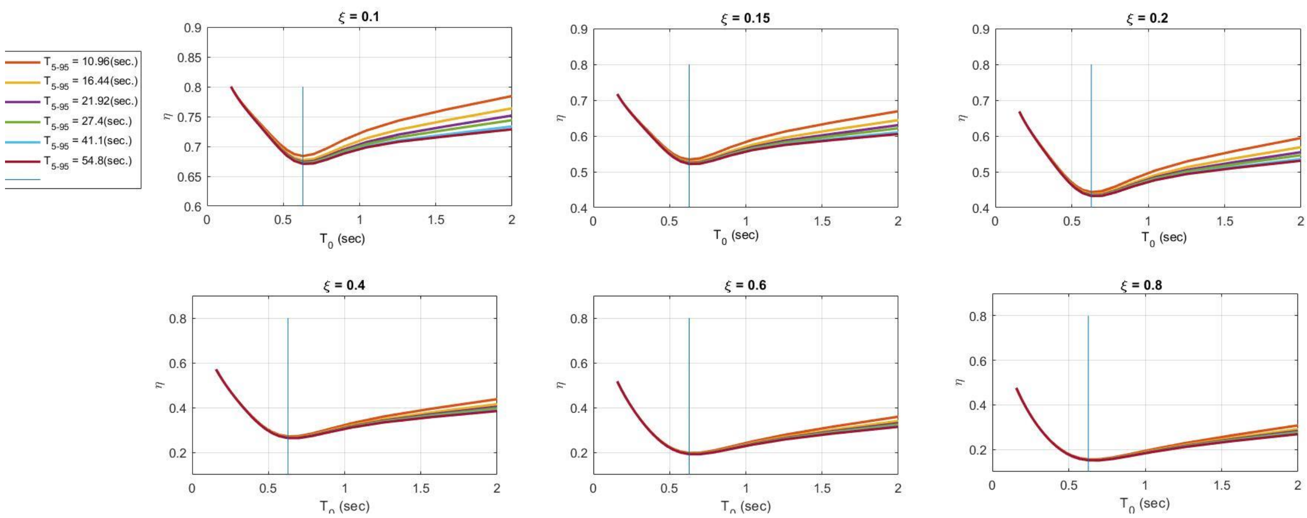

Figure 4 Variability of DRF (η) with system natural period (T_0) for medium soil. The ground motion predominant period (T_f) is the blue line.

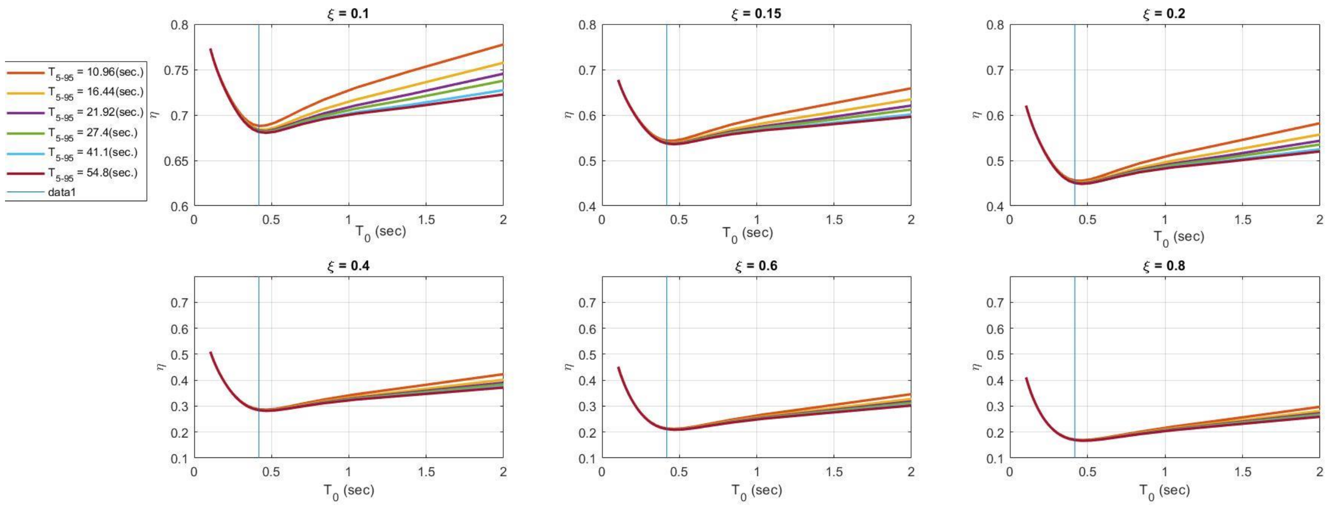

Figure 5 Variability of DRF (η) with system natural period (T_0) for rigid soil. The ground motion predominant period (T_f) is the blue line.

Moreover, the dependency of DRF on effective duration is more evident in these two cases with respect to soft soil. Also for medium and rigid soil it is observed that the variability of DRF is more evident for small values of structural damping (ξ =0.10 – 0.20) while tends to be negligible for greater values.

To better analyze the influence of natural period and damping ratio on the variation of DRF as a function of the effective duration, it is useful to analyze Figs. 6 - 8, representing the DRF (η) versus where the effective duration (T_{5-95}). Different colored lines correspond to different values of system damping ratio (ξ); the 4 sub-figures correspond to different system natural periods T_0 (T_0=0.5 sec, T_0=1 sec, T_0=1.5 sec, T_0=2 sec).

Firstly, the case of soft soil is analyzed. It is observed that earthquake GMD does not affect the DRF for system with low natural period (T_0=0.5 sec), except for earthquakes with a very short duration. If an exception is made for the curve corresponding to the lowest value of the damping ratio ξ =0.10, the curves are practically horizontal. When the natural periods of the system increase, a greater influence of the GMD on the DRF is observed, even for high values of damping ratio. The largest effects of the GMDs are for the system with natural period T=2 sec. In this case, the DRF varies between 0.82 and 0.72 (values evaluated for ξ =0.10). This variability is important since it leads to a 12% reduction of the spectral response and the implication in practical engineering applications are relevant. As before mentioned, a greater GMD value means a greater time window in which the seismic recording is analyzed. If the GMD increases, the number of cycles of the seismic event increases and so also damping effects on the response of the system increase. Consequently this produces [65] the decrease of DRF value. By observing in detail plots in Figs 6-8, one can observe that DRF variation with GMD is significant in the initial part and then it reaches gradually a steady state. Generally, it is observed that for a GMD greater than 20 sec the GMD of the excitation does not affect DRF. This result agrees with Stafford et al. [66]. This outcome is also consistent with the conclusions from Zhou et al [30]. The authors concluded that the maximum displacement curve shows a plateau without increasing further in the case of a system on which a higher number of cycles is applied. This study is based on the analysis of the harmonic excitation of SDOF and the results show that DRF is almost constant for each damping value. In addition, it is observed that, at all damping ratio values, the variation trends of DRF with the effective duration are consistent with each other.

The greatest variability of DRF is observed for T_0=2 sec. The same considerations can be made by observing the graphs in Figs 7 and 8, which refer to a medium and rigid soil, respectively. As already mentioned, for this last type of soil the greater variability of DRF with the earthquake GMD can be observed.

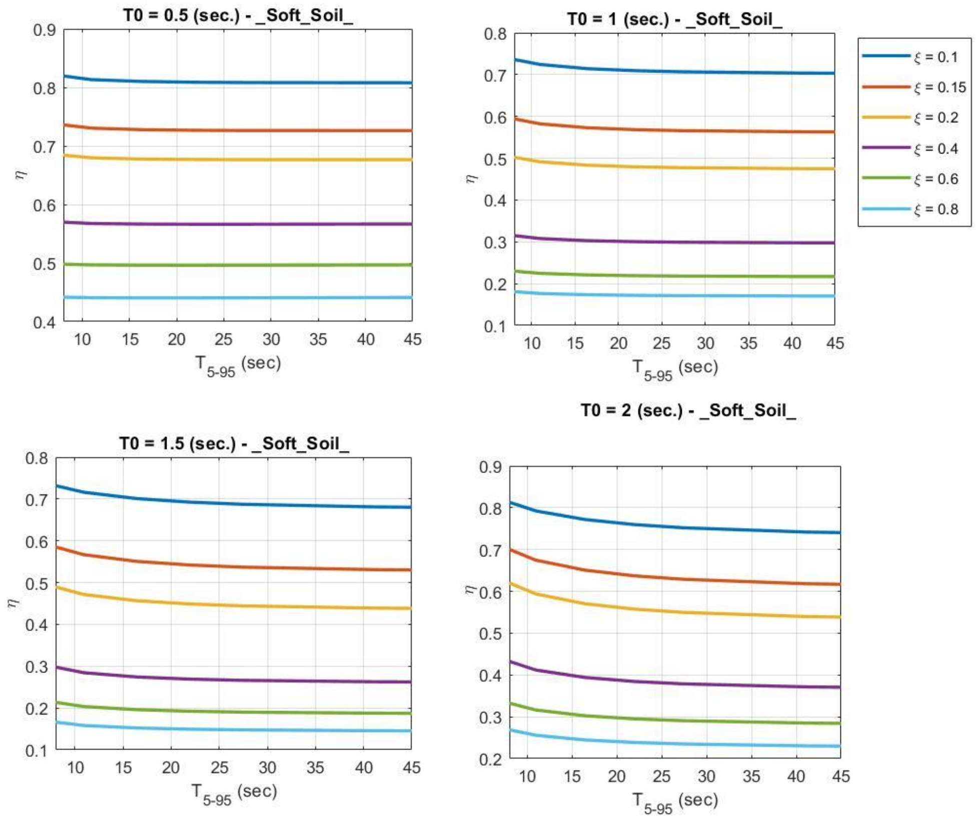

Figure 6 Variability of DRF (η) with earthquake effective duration ($T_{5\text{-}95}$) for soft soil; T_0 is the system natural period.

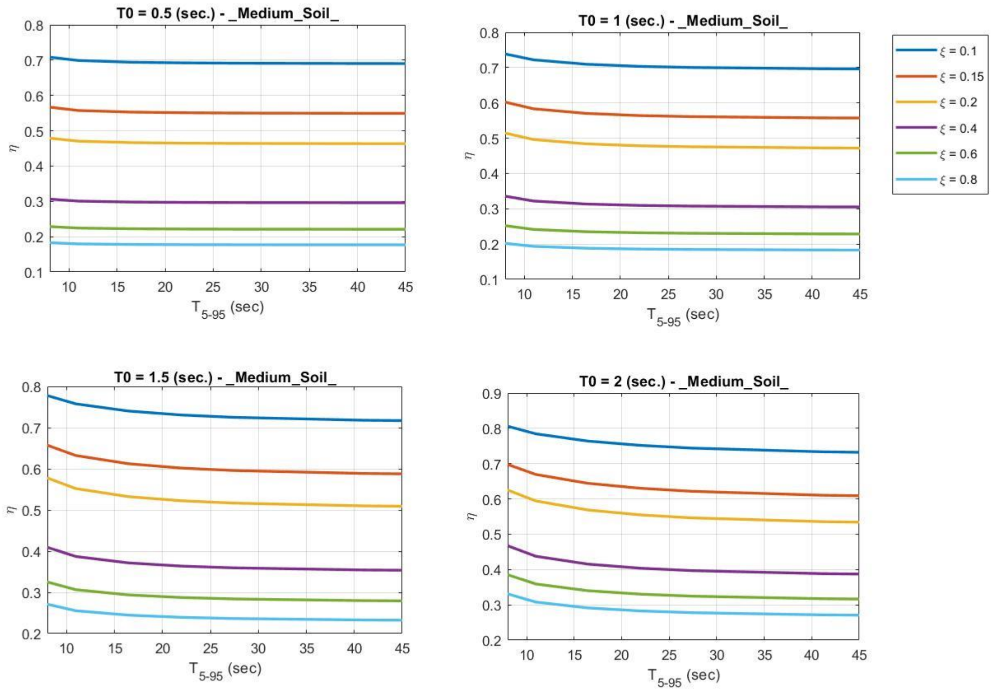

Figure 7 Variability of DRF (η) with earthquake effective duration ($T_{5\text{-}95}$) for medium soil; T_0 is the system natural period.

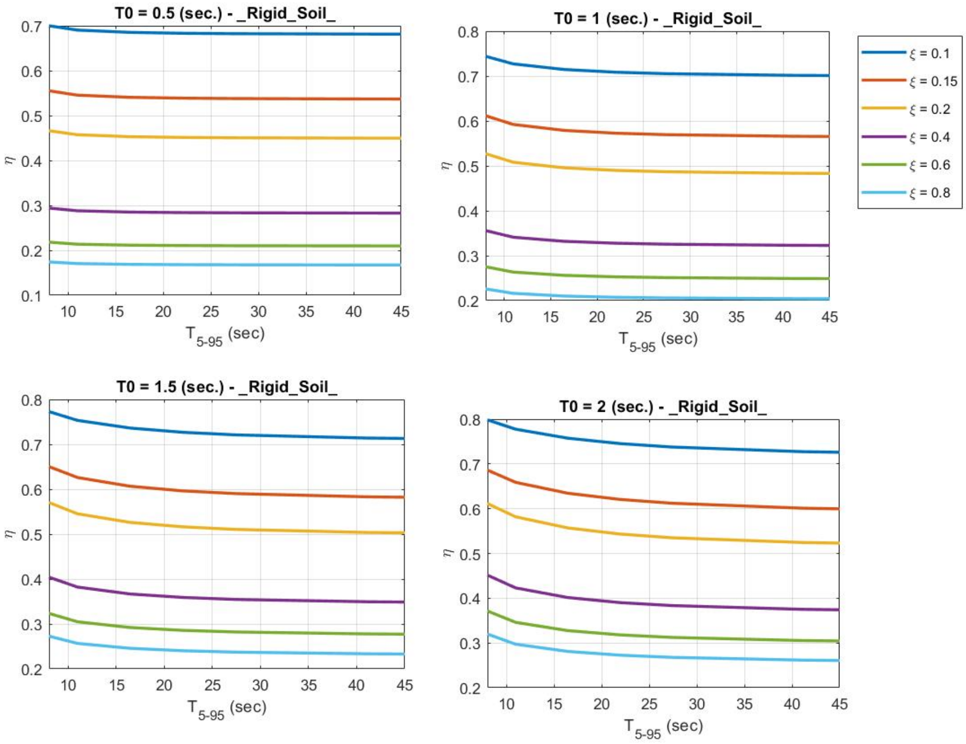

Figure 8 Variability of DRF (η) with earthquake effective duration ($T_{5\text{-}95}$) for rigid soil; T_0 is the system natural period.

In order to evaluate the amount of the DRF variation with GMD and therefore the engineering implications, in Figs 9 - 11 the maximum variability of DRF ($\Delta(\eta)$), that is the DRF value evaluated for the lower effective duration minus the DRF value evaluated for the larger effective duration, is shown. All soil conditions are considered. From Fig. 9 (soft soil), it can be firstly observed that the influence of the effective duration on DRF is more relevant in the range of high periods. The maximum variation equal to 0.11 is for T_0=2 s and ξ =0.15. For T_0=0.5 sec the variability of DRF with effective duration always decreases as the damping ratio increases, whereas as the system becomes more deformable a different behavior can be observed. In fact, as the damping ratio increases the effect of effective duration on DRF firstly increases and then decreases as the damping ratio grows up. This tendency is more evident for the system with the largest natural period considered in this study (T_0=2 sec). From Figures 10 and 11 (medium and rigid soil) an analogous behavior can be observed.

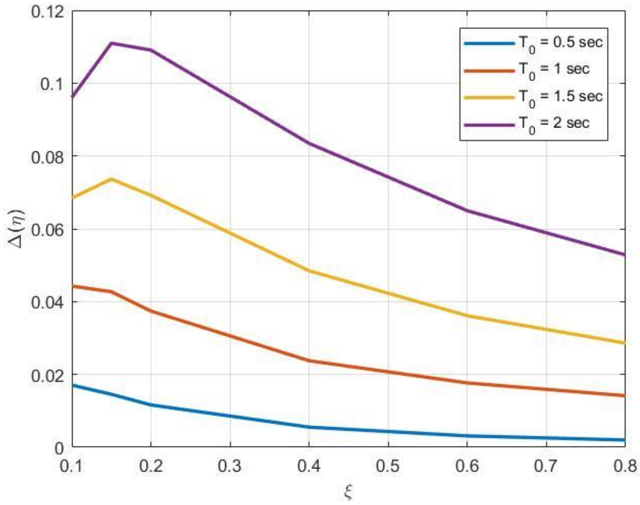

Figure 9 Maximum variability of DRF ($\Delta(\eta)$) with system damping ratio for soft soil; T_0 is the system natural period.

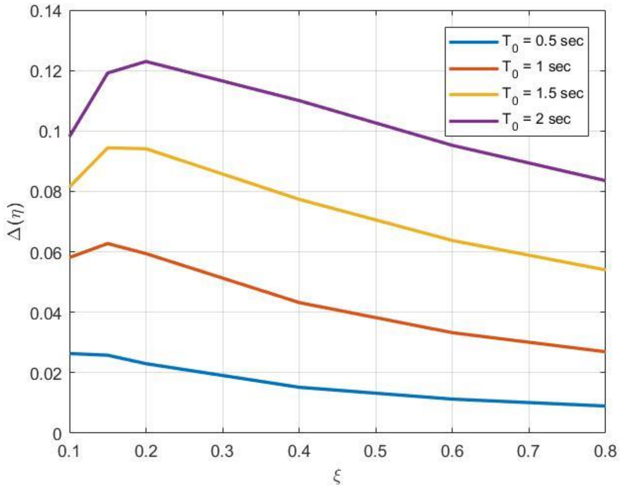

Figure 10 Maximum variability of DRF ($\Delta(\eta)$) with system damping ratio for medium soil; T_0 is the system natural period.

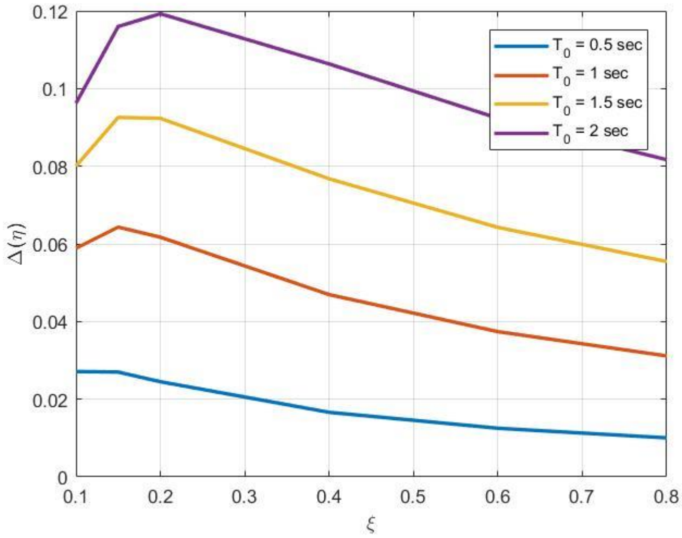

Figure 11 Maximum variability of DRF ($\Delta(\eta)$) with system damping ratio for rigid soil; T_0 is the system natural period.

Figure 12 shows the maximum DRF variability $\Delta(\eta)$ as damping ratio increases. The different colored lines correspond to the 4 structural periods analyzed, while the three symbols on the curves identify the three soil types. The variability of DRF with GMD is influenced by the natural periods of the system, by the damping ratio and by the soil type. The greater variability is for deformable systems on rigid soils and generally in the range of low damping ratio values, although a precise damping value at which the maximum variability is obtained can be identified. For rigid soil, the maximum DRF variability $\Delta(\eta)$ is 0.125 for $\xi = 0.2$.

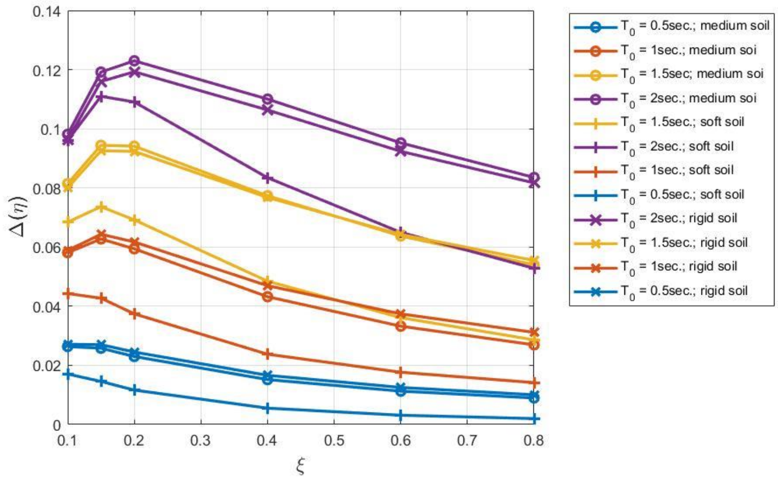

Figure 12 Comparison of maximum variability of DRF ($\Delta(\eta)$) with system damping ratio for different soils; T_0 is the system natural period.

6. Proposed DRF formulation

In this section a novel formulation for the DRF useful for practical application, which, in addition to the period and the damping, also accounts for the effects of GMD and soil characteristics is furnished:

$$DRF(\xi, T_0, \mathrm{T}_{5-95}) = \left(\frac{1 + \psi(T_0)\left(\dfrac{T_{5-95}}{T_o}\right)\xi}{1 + \psi(T_0)\left(\dfrac{T_{5-95}}{T_o}\right)0.05} \right)^{\vartheta(T_0)} \tag{39}$$

where

$$\psi(T_0) = \alpha e^{-\beta T_0} \tag{40}$$

$$\vartheta(T_0) = \chi + \delta T_0 \tag{41}$$

Nonlinear multiple regressions are carried out using a Matlab code [67] to acquire the relationship that assures the best-fit of the DRF with the influence of the soil type, GMD, damping ratio and natural period. The parameters in Eqs. (40) and (41) are given in Table 2.

In order to assess the efficiency of developed regression, first a comparison with results of stochastic analysis is shown. Figures 13-15 show the comparison between the proposed formulation and the results from the stochastic analysis for different soils. The dashed line is the proposed formulation while the continuous line is the result obtained by the stochastic analysis. A very good agreement is noticeable for rigid soil and medium soil whereas for soft soil some discrepancies can be observed in the range of high natural periods. For rigid soil, except for T_0=0.5 sec, the curves are practically coincident each other.

Table 2 Parameters in equations (40) and (41) for different soil types

Soil type	α	β	χ	δ
Rigid	16.3450	0.4626	0.4626	-0.6286
Medium	18.9182	0.5615	0.0677	-0.6259
Soft	171.8278	2.0195	-0.1364	-0.3302

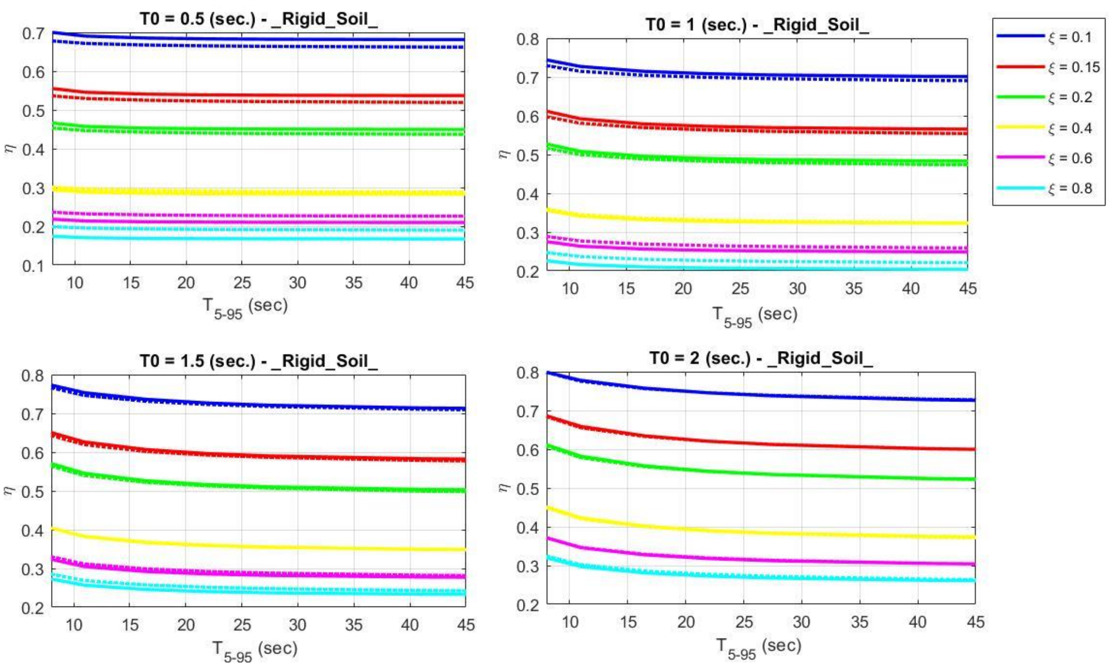

Figure 13 Comparison between the proposed formulation (Dashed line) and the results from stochastic analysis (continuous line) for DRF (η) in case of rigid soil; ξ is the damping ratio.

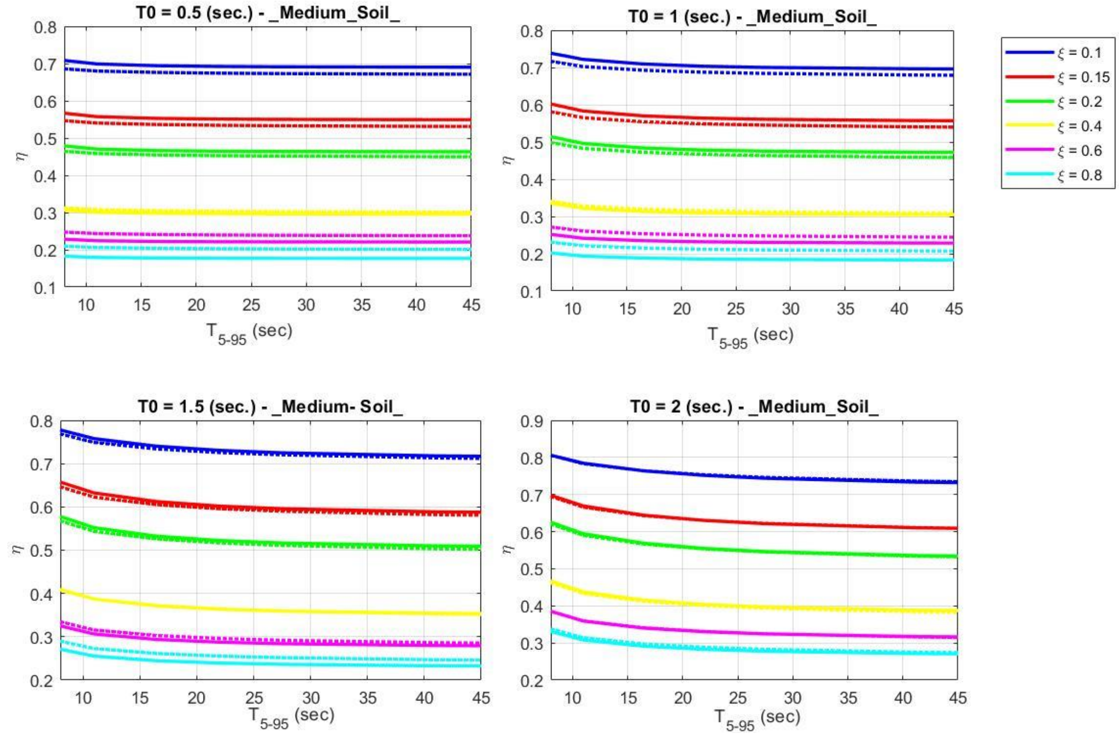

Figure 14 Comparison between the proposed formulation (Dashed line) and the results from stochastic analysis (continuous line) for DRF (η) in case of medium soil; ξ is the damping ratio.

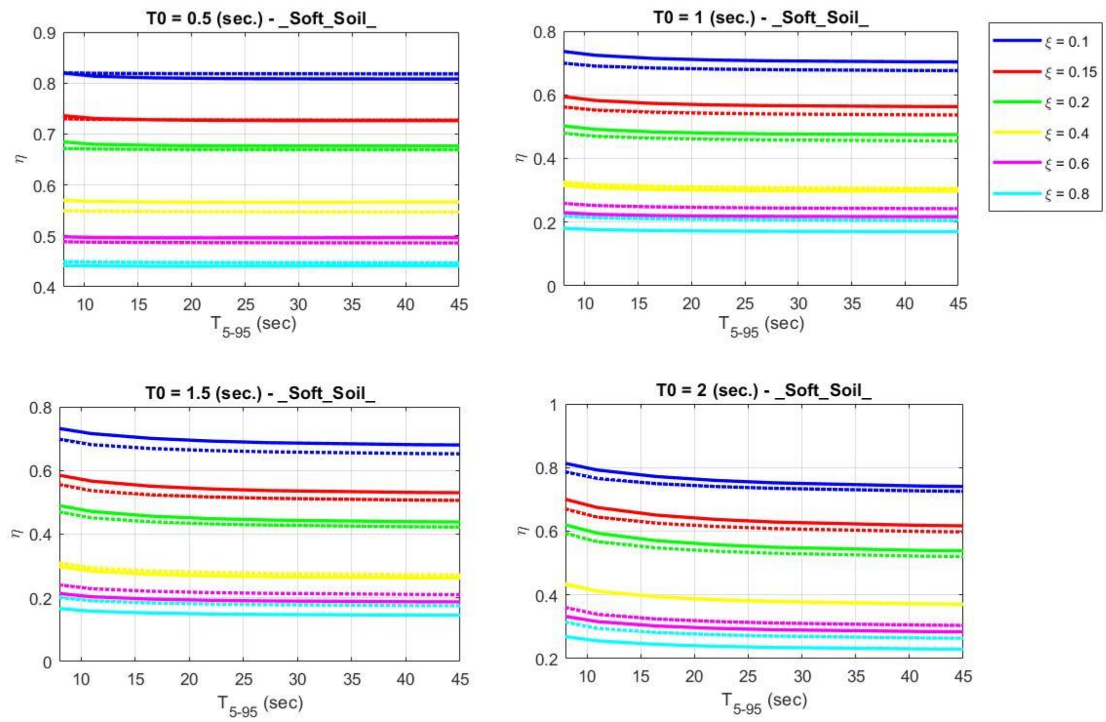

Figure 15 Comparison between proposed formulation (Dashed line) and results from stochastic analysis (continuous line) for DRF (η) in case of soft soil; ξ is the damping ratio.

Finally, it would be interesting to compare the proposed formulation with results performed by other studies. However as explained in the introduction, there are not studies which consider simultaneously the effect of soil type and GMD on DRF. Therefore, a possibility is to compare the results attained by proposed formulation with results reported in [69], in which only the influence of damping ratio, natural period and duration, but not type of soil, is considered.

The results of this comparison are shown for different soils in Figures 16-18. The comparison shows that in some cases the Zhou's formula overestimates the value of the DRF compared to the proposed formulation. This is generally true for low natural periods and for low damping. The formulations proposed, on the contrary, are in good agreement for T_0=2.0 s and for high damping ratio.

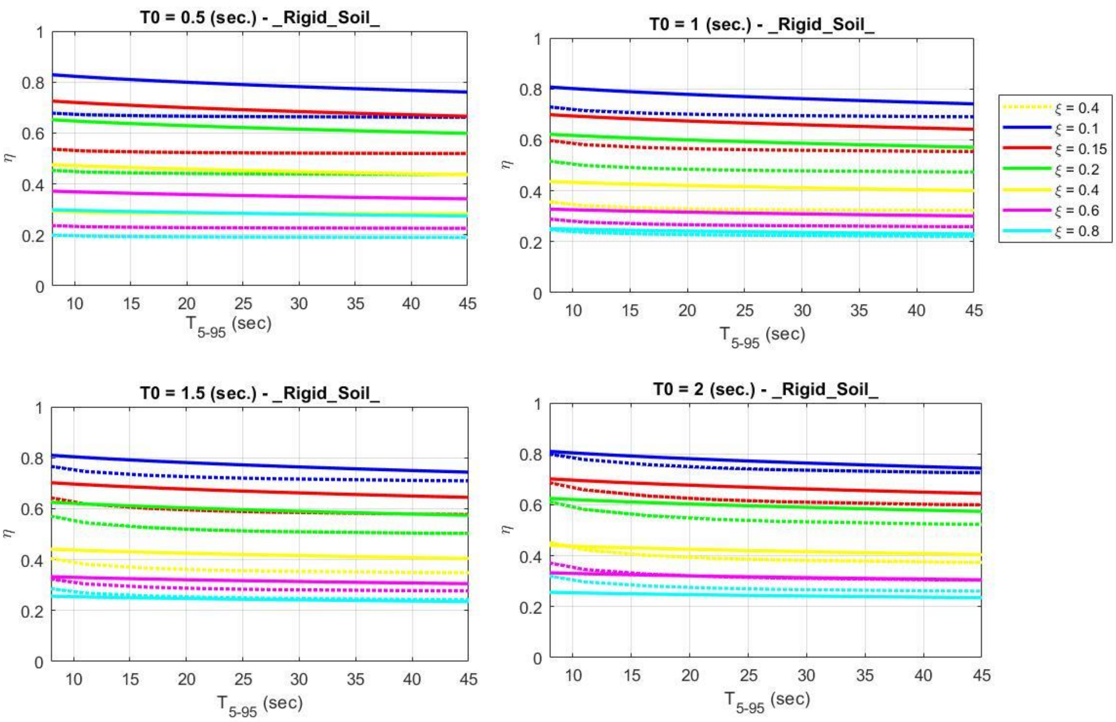

Figure 16 Comparison between proposed formulation (Dashed line) and results in [69] in case of rigid soil; ξ is the damping ratio.

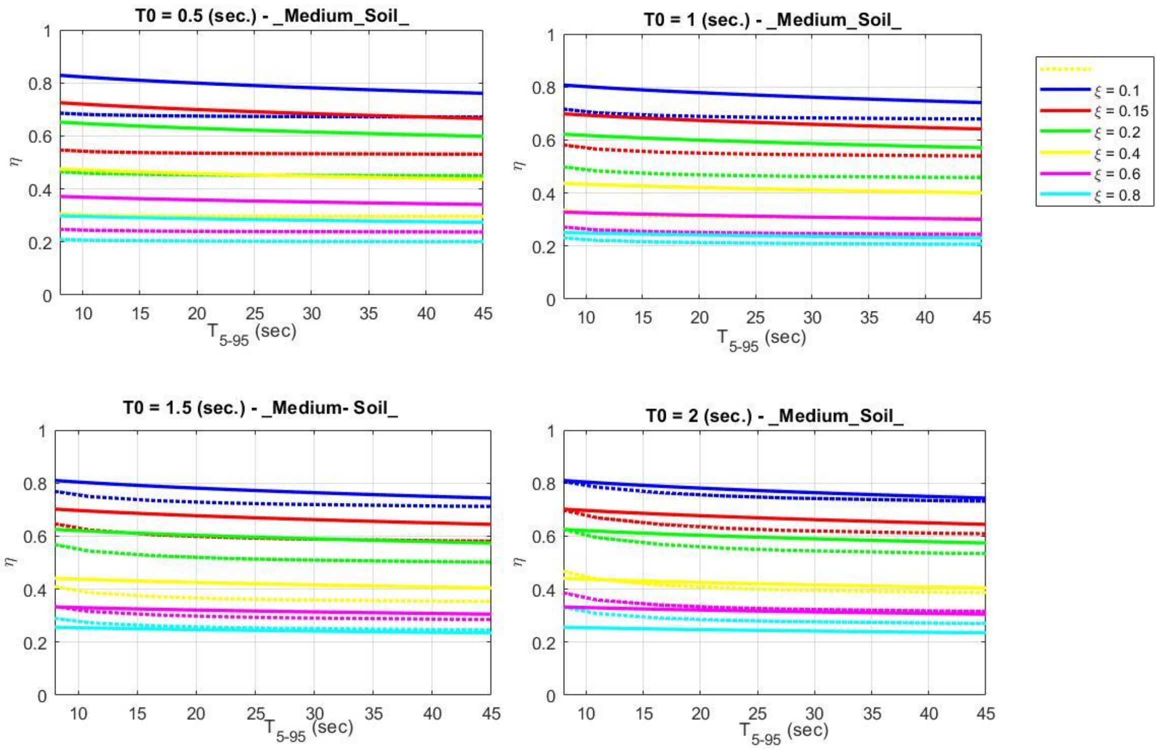

Figure 17 Comparison between proposed formulation (Dashed line) and results in [69] in case of medium soil; ξ is the damping ratio.

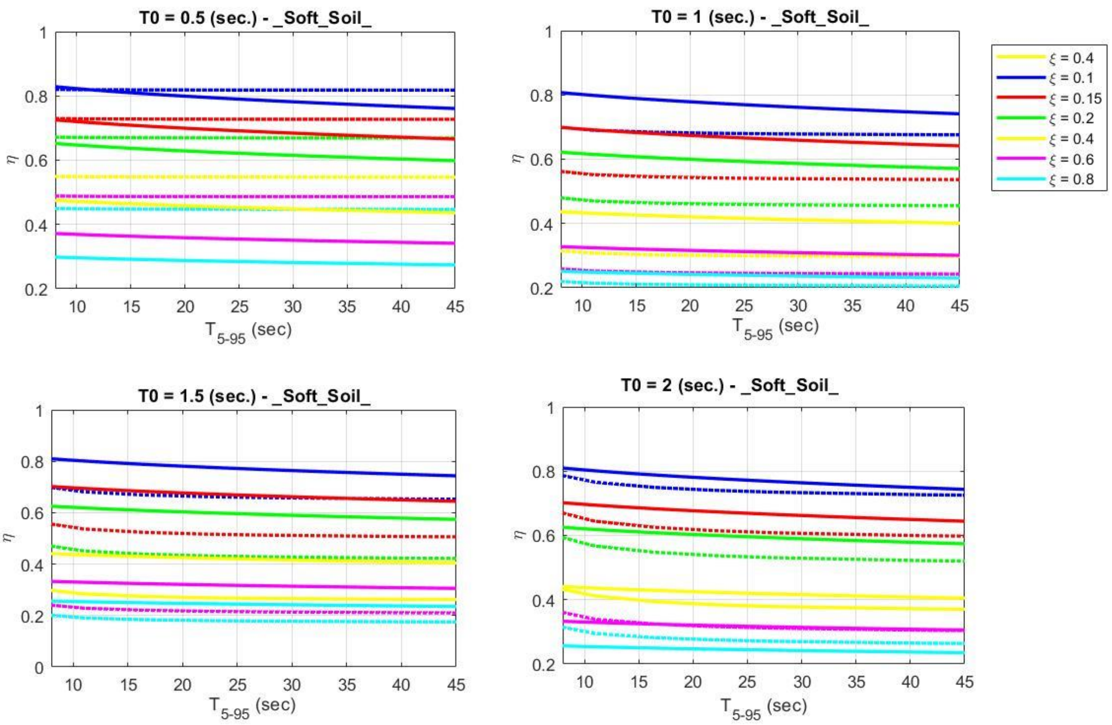

Figure 18 Comparison between proposed formulation (Dashed line) and results in [69] in case of soft soil

The results obtained from this work have a practical relevance in seismic engineering applications as they allow a more accurate evaluation of the DRF, which accounts also for soil type influence, with respect to other formulation existing in literature, and therefore it furnishes design forces on structures in a more accurate way. One should remind that in practical applications, the effective GMD can be evaluated by existing formulations [70] for a given earthquake scenario that is characterized by parameters such as magnitude, epicentral distance and soil type and therefore the DRF factor can be calculated by proposed formulation for a particular value of damping at any specific period of the structure.

7. Conclusions

In this study, a combined evaluation of DRF sensitivity to ground motion duration (GMD), soil type, damping ratio and natural period has been studied; the peak theory of stochastic processes is at the base of the proposed procedure. This theory models the seismic excitation by a non-stationary process characterized by means of a stationary predominant frequency and a given bandwidth. Parameters of modulation function have been identified in order to represent earthquake with different effective durations. The following conclusions can be given on the base of the obtained results. In detail:

- DRF decreases with the increasing of damping ratio. Moreover, DRF is significantly dependent on the vibration period, reaching a minimum for natural period equal to earthquake predominant period;
- the effective duration influences DRF: as effective duration increases, DRF decreases. However, this variability is evident in the first part of the effective GMD for reaching the steady state response in the final part. For GMDs larger than 20 s, the DRF tends to an asymptotic final value and there are no significant differences between them (the same result for different GMDs).
- the variability of DRF with GMD concerns especially deformable systems (in the study T_0=2 sec is considered) with low damping ratio and becomes negligible for rigid structures (T_0=2) with high damping ratio.
- the variability of DRF with GMD cis greater for small damping ratio (ξ =0.10) and reduces as the damping ration increases.
- DRF depends also on ground type and a larger sensitivity is observed for rigid soil. For example, for this soil the variation of DRF can amount to 0.125 and the implication on practical application can be relevant.

Based on conducted stochastic analyses a simple formulation for DRF evaluation, which accounts of effective duration, soil type, damping ratio and natural period of structure has been given. This study gives results with great relevance in structural applications because it emerges that as well as the earthquake GMD must be considered for damage structural evaluation it should be considered also in the evaluation of DRF to avoid an undervaluation of reducing factor of structural seismic forces. This aspect is more relevant for deformable structures on rigid soil and with low damping ratio. The proposed formulation for the DRF, which accounts in addition to the period and the damping also for the effects of the GMD and soil characteristics, results therefore useful for practical applications.

8. References

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