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Validation of a method for fluid flow rate measurement in square ducts by means of laser velocimetry

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Validation of a method for fluid flow rate measurement in square ducts by means of laser velocimetry

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Abstract. The paper deals with the introduction of an alternative method for the evaluation of the fluid flow rate through a square pipe, by directly measuring the flow mean velocity at a fixed location in the section area. The authors of this paper have previously introduced a way for the evaluation of the fluid flow rate, by measuring a single point flow velocity in square pipes. In this study, the authors perform a series of considerations aimed at the validation of the proposed methodology, by comparing it with prescribed procedures described by ISO 3966:2008 standard. In this standard a method for the evaluation of fluid flow rate (under specific fluid dynamic conditions) is stated from velocity measurements performed in a-priori fixed points (in a given test section and to be defined according to the cross-section shape), using a static Pitot-tube (or equivalent equipment). In this work, the authors validate the proposed simplified method against the Log-Thebycheff procedure (according to the same standard). A Laser Doppler Anemometer (LDA) has been used.

1. Introduction

In industrial and manufacturing processes, an accurate and precise measurement of the fluid flow rate is essential, [1].

For example, the operating condition of a water distribution network for the detection of possible leaks and pipe damages can be assessed only by means of an accurate fluid flow rate monitoring, [2], [3]

There exist several methods and devices for the measurement of the fluid flow rate in pipes. The most straightforward methodology takes advantage of flowmeters based on Coriolis, ultrasound and electromagnetic effects, [4], [5].

Another methodology consists in the measurement of fluid velocity in several points according to a cross-section grid and normed by EN 12599 and ISO 3966 standards and, then, averaging them accordingly, [6], [7]. The measurement of local fluid velocity can be made by means of Pitot tubes, hot wire anemometers, Laser Doppler Anemometers, [7], [8]

In this paper, the authors propose an alternative method for the evaluation of the fluid flow rate through square pipes, by means of a direct measurement of the flow velocity at a fixed point in the testing section, [9].

For this purpose, a pseudo-empirical model has been set up, allowing for the localization of such point. Therefore, the devised method has been validated by means of a series of experimental campaigns involving an appropriate test wind tunnel. A LDA has been utilized to measure the local velocities in the test section over a grid of points suitably chosen. The discrepancy between the estimated mean velocity estimated by means of the proposed method, and the one deriving from the tests is less than 1%.

2. Background

Many papers in the technical literature deal with the development of a simplified procedure for an accurate and precise estimation of fluid flow rate [6]-[9]. Experimental evidences reported in [10] show that, for turbulent flows and circular ducts, the ratio between the punctual speed measured at approximately 0.75 times the radius of the duct from its axis and the average speed in the section is, at different values of the Reynolds number, approximately unitary. Such result leads to the conclusion that the measurement of flow velocity at an appropriate point of the cross-section allows for the estimation of the fluid flow rate, for certain flow conditions.
From these considerations, the authors derive empirically a similar relation between the punctual speed and the average speed of a turbulent flow in ducts with square cross-section, exploiting the concept of equivalent circular cross section.
For turbulent and fully developed flows in circular ducts, it is worth to recall the so-called power law, which allows to estimate the punctual flow speed, according to the following relation.

$$\frac{v(r)}{v_0} = \left[1 - \frac{r}{R}\right]^{\frac{1}{n}} \Rightarrow \frac{\bar{v}}{v(r)} = \frac{(n+1)(2n+1)}{2n^2}\left[1 - \frac{r}{R}\right]^{\frac{1}{n}} \tag{1}$$

In equation 1, v refers to the point flow velocity, r the distance from the pipe axis, R the pipe radius, $\bar{v}$ the cross-section average velocity and n an empirical parameter ($n \approx 7$ for most industrial flows), that is influenced mainly by the Reynolds number [10].
For square ducts [11], the power law is still valid. Indeed the following series of relations hold.

$$\bar{v}S_{sqr} = \bar{v}S_{\Phi} \Rightarrow R_{eq} = \frac{L}{\sqrt{\pi}} \tag{2}$$

In equation 2, S_{Φ} and S_{sqr} refer to, respectively, cross-section area of the circular pipe and the equivalent square duct. L is the square edge length.

$$\frac{v(x)}{v_0} = \left[1 - \frac{\sqrt{\pi}x}{L}\right]^{\frac{1}{n}} \Rightarrow \frac{\bar{v}}{v(x)} = \frac{(n+1)(2n+1)}{2n^2}\left[1 - \frac{\sqrt{\pi}x}{L}\right]^{\frac{1}{n}} \tag{3}$$

Equation 3 is similar to equation 2. For square ducts, the term n must be recomputed, since it does not only depend on Reynolds number.

3. Experimental facility and instrumentation

The development and the validation of the pseudo-empirical model for square ducts, has been conducted by means of tests on a subsonic wind tunnel with these features:

- test volume square section plexiglass made, 150 mm side
- air flow rate controlled by an axial fan driven by electric motor.

Figure 1 shows the experimental facility employed. Figure 2a and figure 2b illustrate the LDA device pointing at the test section. The LDA used is in 2D configuration and it also comprised the transverse system.

The tunnel is an open circuit. The inlet nozzle has a contraction ratio of 4:1 (in terms of section sides ratio), while the test volume, at $0.600m$ from the nozzle exit, has a square

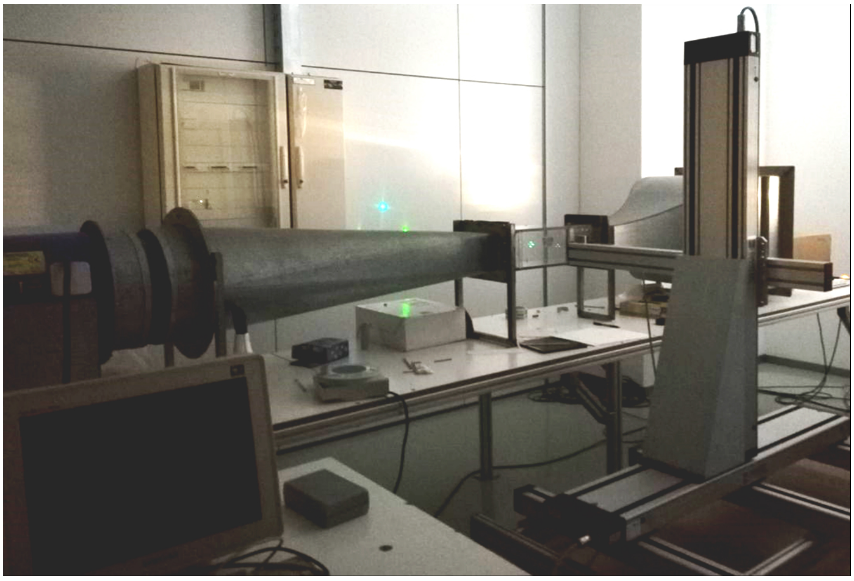
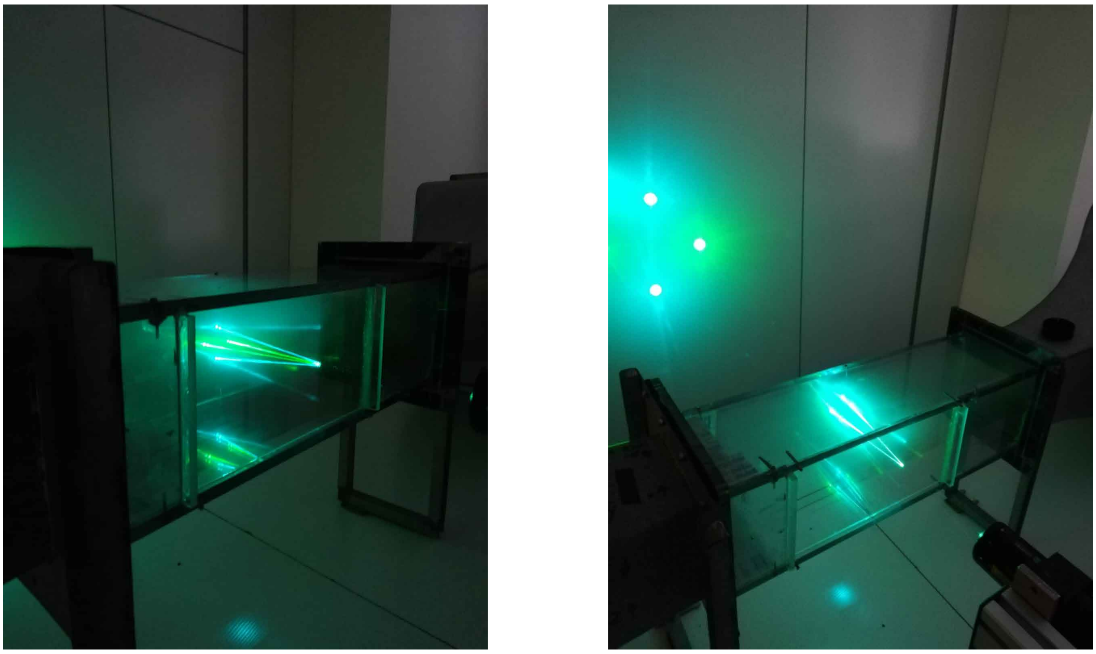


Figure 1: Testing subsonic wind tunnel.



(a) (b)

Figure 2: LDA system.

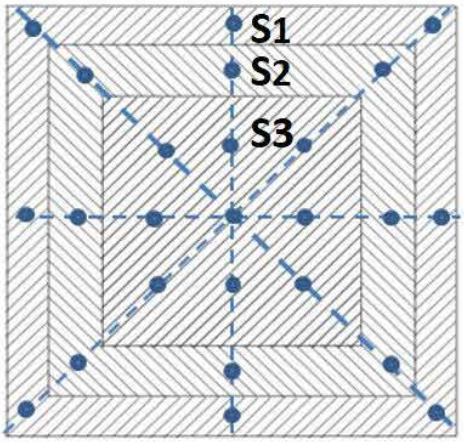


Figure 3: Testing points according to EN 12599.

transversal section. Its walls are plexiglass made, in order to ensure the optical access to the laser beam couples within the testing section. At the opposite side, an axial fan is connected driven by a three-phase asynchronous electrical engine ($2900RPM$ nominal speed), whose round speed is controlled by an inverter. A LDA anemometer, FlowLite 2D System by Dantec Dynamics, [15], has been employed having blue ($\lambda = 488nm$) and green (Laser YAG, $\lambda = 532nm$) beams combinations.
The pneumatic seeding generator utilized is fed by air extracted downstream of a pressure controller and located upstream of the inlet section of the nozzle, in axial position. The seeding is a solution of ethylene and paraffin oil in order to guarantee a better diffraction of light from the seeding particles and so a better SNR. Its flow rate was tuned through the air pressure feeding the generator.

4. Description of the procedure according to EN 12599

For the validation of the pseudo-empirical model, it has been initially considered the procedure described by standard EN 12599. According to the trivial method, the entire cross section is divided into sub-sections of equal area. The standard prescribes the number of required number of measurement points along each traverse direction, according to the desired uncertainty level and the velocity profile regularity. The mean velocity is obtained as the arithmetic mean of the punctual velocity measured at the centroids of the sub-areas S_i.

Here, it is considered the centroidal axes method. Since the flow profile is not symmetrical, the square ha been divided into 3 sections with equal areas. The centroids along the two pairs of orthogonal axes have been considered, in order to get better evaluations. Figure 3 shows the three sub-areas S_1, S_2 and S_3 and the measuring points. The measurements have been carried out for three different values of flow rate (3 different revolution frequencies controlled by the inverter, 10, 25 and $40Hz$). At each grid point, 5 replications have been set and a dataset of 2000 samples has been taken for each measurement. For each measurement point, the mean velocity has been estimated through the pooled statistics. The cross-section mean velocity has been estimated by group-averaging the point velocities in each sub-section.

4.1. Development of the pseudo-empirical model and results

In the plot shown in figure 4 it is shown the flow profile (along the axial direction) velocity measured by means of the LDA and averaged throughout the repetitions performed at each

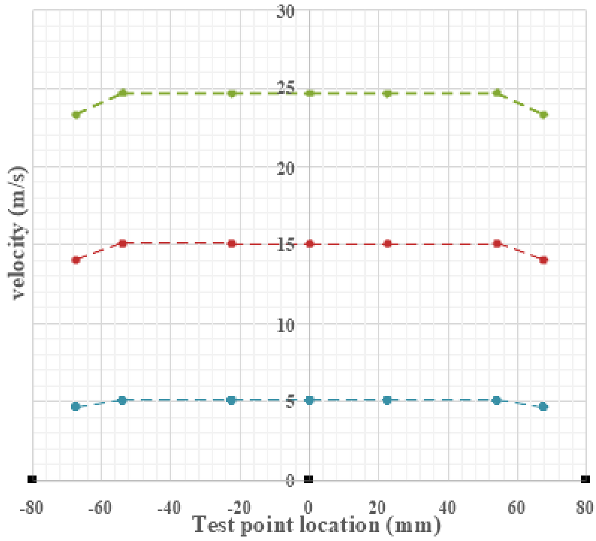


Figure 4: Point average velocities estimated experimentally at the three different fan speeds.

point. Each curve is related to each tested velocity.
Equation 4 allows for the estimation of term n in equation 3.

$$\ln\left[\frac{v(x)}{v_0}\right] = \frac{1}{n}\left[1 - \frac{\sqrt{\pi}x}{L}\right] \tag{4}$$

The resulting n values, for each tested flow speed, are reported in table 1.

Table 1: Tested fan speeds, Re numbers and the according n values.

Fan speed [RPM]	10	25	40
Re	56943	169535	278957
n	63	78	278957

For each air flow rate, the x-value where it is possible to estimate the cross-section mean velocity, has been estimated by means of equation 2. Such point is located at $x \approx 0.776L/2$. Having developed the pseudo-empirical model, the velocity profiles at each tested fan speed can be deduced, figure 5.

Table 2 points out the limited discrepancy existing between the flow average speed obtained experimentally and the one deduced from the developed pseudo-empirical model.

5. Description of the procedure according to ISO 3966

Another validation of the proposed model for square ducts is proposed. The validation has been performed by considering another measurement grid, according to Log-Tchebycheff rule (standard ISO 3966:2008). It minimizes the bias caused by the failure to account for losses at

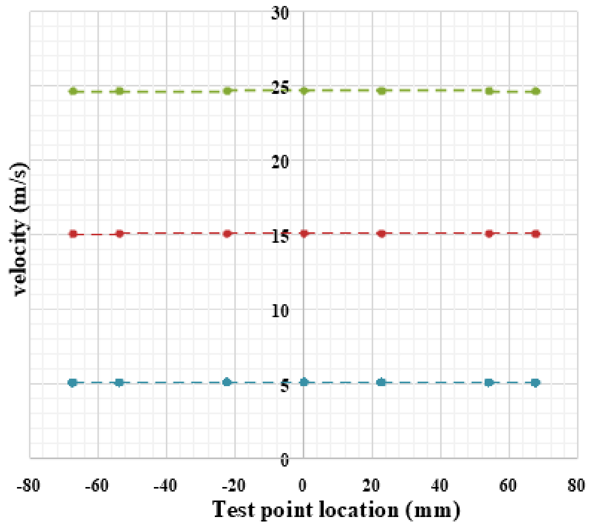


Figure 5: Point velocities estimated from the model at the three different fan speeds.

Table 2: Average velocity from developed model and from experimental tests comparisons.

	$\bar{v}$ Model [m/s]	$\bar{v}$ Exper. [m/s]	Error %
$10Hz$	4.90	4.95	1
$25Hz$	14.60	14.73	0.9
$40Hz$	24.05	24.24	0.8

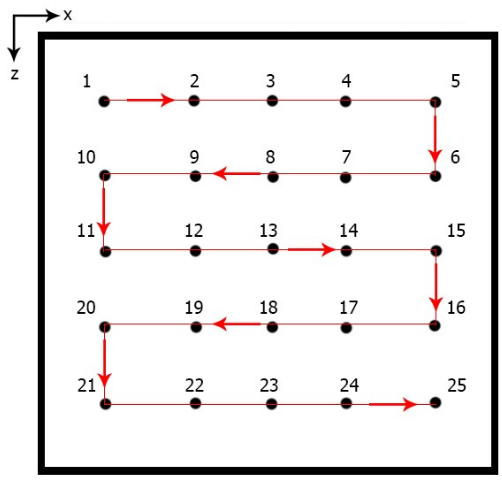


Figure 6: Test section with measuring points according to Log-Tchebycheff rule.

the duct wall. This error can occur when using the former method of equal sub-areas to traverse rectangular ducts.

Fig. 6 is a sketch of the test section with the measurement points underlined, Table 3 their distances from the origin.

Table 3: Distances of the measuring points from the origin (in mm), point 13, as indicated in figure 1

P_i	1	2	3	4	5
x	-65.18	-32.44	0	32.44	65.18
z	-65.18	-65.18	-65.18	-65.18	-65.18

P_i	6	7	8	9	10
x	65.18	32.44	0	32.44	65.18
z	-32.44	-32.44	-32.44	-32.44	-32.44

P_i	11	12	13	14	15
x	-65.18	-32.44	0	32.44	65.18
z	0	0	0	0	0

P_i	16	17	18	19	20
x	65.18	32.44	0	-32.44	-65.18
z	32.44	32.44	32.44	32.44	32.44

P_i	21	22	23	24	25
x	-65.18	-32.44	0	32.44	65.18
z	65.18	65.18	65.18	65.18	65.18

The same test bench has been used. In this case, the location of the axial fan has been modified, in order to get different air flow rates. Such a different setup is aimed at ensuring the correctness of the estimation of the location point where to measure the cross section mean flow velocity. In this case, the flow rates have been set at 10, 20 and 30 Hz. For each point, 2000 velocity samples have been acquired by means of the LDA system. The experimental mean velocity has been estimated by averaging the point mean velocity, by means of the pooled-statistics.

5.1. Development of the pseudo-empirical model and results

Figure 7 shows the experimental averaged velocity profiles along the duct axis for the three flow rates. The power exponent n for square ducts, is calculated by means of equation 4, using the experimental values (max velocity at the center). The n values are listed in table 4.

Table 4: Tested fan speeds, Re numbers and the according n values.

Fan speed [RPM]	10	20	30
Re	22419	33638	44837
n	18	19	23

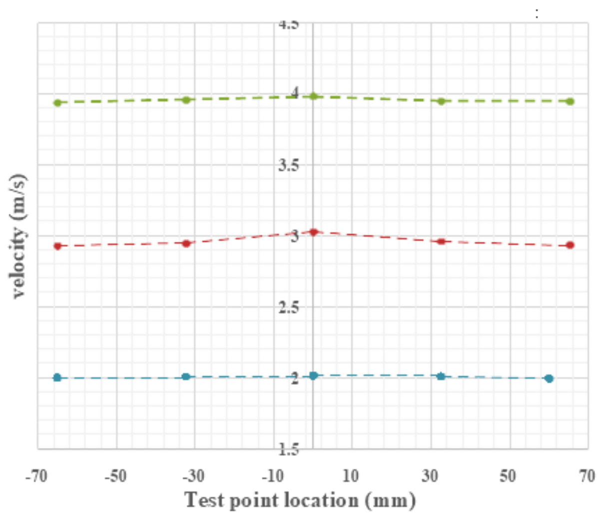


Figure 7: Point average velocities estimated experimentally for each tested fan speed.

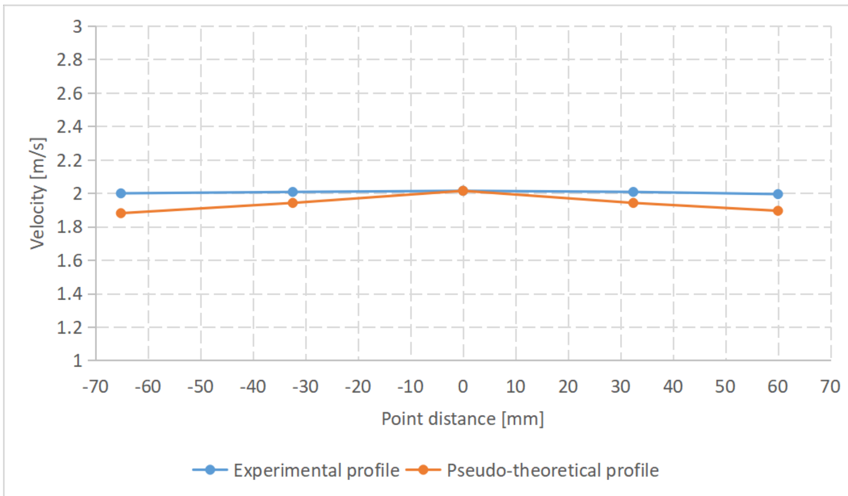


Figure 8: Experimental vs. pseudo-theoretical profile, 10 Hz.

Also in this case, according to experimental values got through the Log-Tchebycheff grid and equation 3, the x-value where it is possible to estimate the cross-section mean velocity is located at 0.776 $L/2$. Additionally, equation 3 has been used for the determination of the empirical velocity profiles for each tested fan speed.

Figure 8, figure 9 and figure 10 show, respectively, the comparisons between experimental and pseudo-empirical velocity profiles. The experimental profiles have been obtained from the LDA measurements (properly post-processed) and the pseudo-theoretical ones from the implementation of modified power-law equation for square ducts.

The discrepancy between the mean flow speed, according to ISO 3966:2008 and the one according to proposed method is less than 1%, with the immediate advantage of evaluating the mean velocity through only one measurement point.

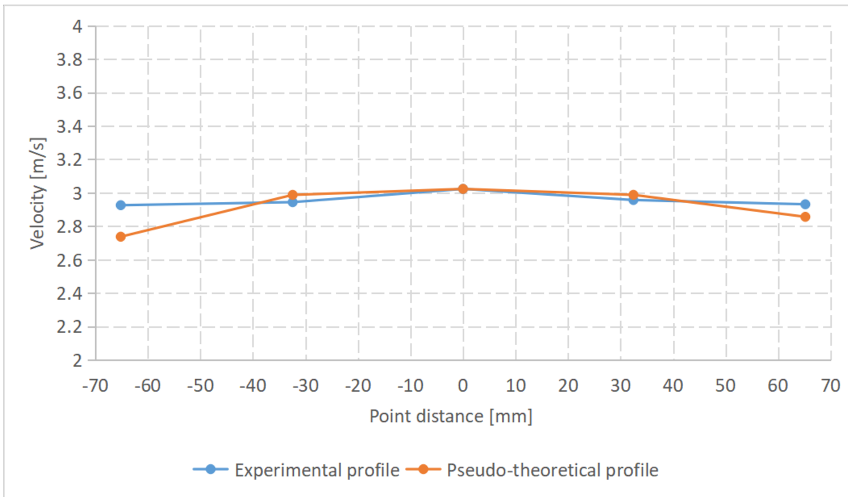


Figure 9: Experimental vs. pseudo-theoretical profile, 20 Hz.

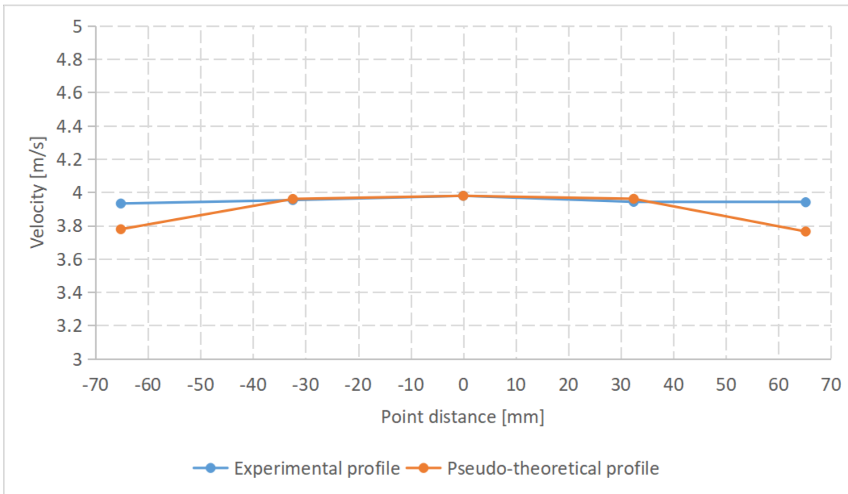


Figure 10: Experimental vs. pseudo-theoretical profile, 30 Hz.

Table 5: Results.

Frequency	$\bar{v}$ Model [m/s]	$\bar{v}$ Exper. [m/s]	Error %
10	1.998	2.004	0.3%
20	2.949	2.956	0.2%
30	3.975	3.949	0.6%

6. Conclusions

An alternative, easier and faster methodology based on a pseudo-empirical model has been developed for the evaluation of the cross-section average flow velocity through a square duct. It lies on the evaluation of a suitable point where the flow velocity equals the average one (for well determined flow conditions, as explained above). Such a formulation derives from the analogy with the circular cross sections, for which the standards EN 12599, ISO 7145 and BS 1042 estimate such position at $3/4R$, R being the radius of the section. The developed model has been validated by means of a series of measurements taken in a small subsonic wind tunnel whose test section has been properly designed. A laser Doppler anemometer has been utilized to measure the local velocities in the test section over a grid of points suitably chosen, in accordance with standards EN 12599 and ISO 3966. The comparisons between calculations and measurements have shown discrepancies of order of 1% for the tested flow velocity ranges. Such discrepancies are much lesser than the errors accepted in common practice.

Acknowledgments

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