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# A decision-making tool for energy efficiency optimization of street lighting

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## Abstract

This paper develops a multi-criteria decision making tool to support the public decision maker in optimizing energy retrofit interventions on existing public street lighting systems. The related literature analysis clearly highlights that, to date, only a few number of studies deal with the definition of optimal decision strategies complying with multiple and conflicting objectives in the planning of street lighting refurbishment. To fill this gap, we propose a decision making tool that allows deciding, in an integrated way, the optimal energy retrofit plan in order to simultaneously reduce energy consumption, maintain comfort, protect the environment, and optimize the distribution of actions in subsystems, while ensuring an efficient use of public funds. The presented tool is applied to a real street lighting system of a wide urban area in Bari, Italy. The obtained results highlight that the approach effectively supports the city energy manager in the refurbishment of the street lighting systems.

**Keywords:** Energy efficiency management, public street lighting, multi-criteria optimization.

## 1. Introduction

Pursuing energy-efficient improvements has become mandatory at all levels of the public administration, not only for environmental sustainability reasons, but also since the prediction of energy consumption accounts for a global increase of almost 40% by the year 2030 [1]. Not surprisingly, the improvement of energy efficiency is at the basis of the worldwide significant trend towards smart city researches and projects [2-5]. Referring to the actions that can be undertaken, the recommendations for energy efficiency by the International Energy Agency (IEA) cover seven different priority areas: buildings, appliances, lighting, transport, industry, energy utilities and cross-sectorial issues [6-7]. Within these areas, public (predominantly street) lighting contributes to about 2.3% to the global electricity consumption. Hence, energy-efficient programmes in this field are very welcome, since the possibilities for energy savings in street lighting are numerous and some of them even enable reductions in electricity consumption of more than 50%. This explains the growing attention reserved by policy makers to energy consumption for urban street lighting in the energy and economic balance of many cities, as the increasing commitment of city authorities towards energy efficiency and green energy for public lighting systems demonstrates [8]. Municipal planners and

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engineers search for opportunities for effective energy-efficient street lighting management [9-10]. In response to this need, several researchers have investigated tools to support administrations in finding optimal street lighting solutions that minimize energy consumption.

The literature on energy efficiency and consumption reduction for street lighting installations may be divided into three main streams of studies. The first refers to the optimization of the street lighting system design, aiming at finding the best combination of design parameters (e.g., height of lighting units, inclination of the same units, overhang of lighting poles, inter-distance between poles, etc.) [11], so as to maximize the overall illuminance uniformity and installation efficiency [12-14], etc. The second stream includes studies focusing on the technological development of lamps and luminaires, as mentioned in [15-20]. The third one includes the study and development of systems for the control of lamps and luminaires that, under the project conditions, allow the reduction of power consumption, e.g., the optimal use of equipment that allows the variation in the luminous flux as a function of time and traffic conditions on the roads [21-28].

Although the recalled studies solve several important issues related to the energy-efficient street lighting management, they lack on two main aspects. First, focusing on the solution of a specific issue, they do not support the decision-maker in defining an optimal retrofit strategy comprising a mix of interventions on existing street lighting systems. Second, studies already developed in the fields are mainly mono-objective. Instead, the interventions on public street lighting has by definition multiple and conflicting objectives: reducing energy consumption while maintaining the same energy services, keeping the comfort and quality of life, protecting the environment, ensuring the energy supply and promoting sustainability in its use, and minimizing the necessary public resources. The overall aim of road lighting, in fact, is to reproduce lighting conditions that provide a safe and comfortable environment for the driver and pedestrians during the night hours. The effective use of road lighting helps protecting drivers/pedestrians and improving traffic, while providing economic benefits. When lighting levels are optimal, the system provides enough illumination to minimize the number of accidents, maintaining at the same time minimum energy consumption [13].

With the aim of filling the highlighted gap in the literature on street lighting decision-making, this paper develops a multi-criteria optimization tool able to support the public decision maker in selecting the proper mix of retrofit interventions to be taken on an existing street lighting system in order to reduce energy consumption, maintain the required comfort and quality of life, protect the environment, and simultaneously optimize the distribution of actions in subsystems, while ensuring an efficient use of public funds.

The authors of this paper have previously addressed in a hierarchical way the overall problem of energy governance of smart cities, which includes several decision panels at the same time (e.g., buildings and public street lighting). We proposed the general architecture of an urban control center, to be used by policy makers for the smart city energy governance in [29-30], based on a bi-level programming architecture. The urban control center proposed in [29-30] includes two levels: an upper decision unit takes care of the overall smart city manager preferences and guidelines, while several lower level decision units or panels are related to the different urban subsystems that may be affected by specific energy efficiency and retrofit policies. Moreover, in [31] we presented a preliminary work on the overall structure of the decision panel for the public street lighting. Our work in [31] considers the street lighting decision panel in the wider hierarchical

architecture of the urban control center, with a focus on the overall energy governance of the smart city. Contrarily, in this paper the perspective is that of the energy manager of the street lighting system, rather than that of the city policy maker. Hence, in this paper we detail and extend the problem statement provided in [31]. In [31] a limited set of specific decision criteria and retrofit actions are addressed and a customized formulation of few performance indicators is presented, resulting in a contingent optimization model that cannot comply with diverse situations and general needs encountered by energy managers. Moreover, the model proposed in [31] does not take into account any metric concerning the allocation of the optimal retrofit actions among the various street lighting subsystems. To address such issues, in this paper we define a more comprehensive optimization model considering different categories of generalizable decision criteria (including criteria associated to the implementation and mode of intervention) and providing a general formulation for performance indicators in terms of parameterized sets of retrofit actions (whose amount could be eventually enlarged in accordance with decision maker choice). This new perspective has the advantage of providing the street lighting manager with a decision making model that is compatible with different street lighting contexts, namely to contexts characterized by diverse objectives, constraints, and performance as well as implementation criteria. Therefore the contributions of this paper may be summarized as follows:

- From a theoretical point of view, this paper contributes to the literature on energy-efficient street lighting management, which lacks multi-objective studies for identifying the optimal retrofit strategy comprising a mix of interventions on existing street lighting systems.
- We define a comprehensive optimization model considering different categories of objectives, constraints, and performance as well as implementation criteria.
- Finally, from a practical point of view, the proposed tool, addressing a multi-criteria optimization problem, provides the decision maker with an effective tool for screening optimal solutions.

The rest of the paper is organized as follows. Section 2 formulates the problem statement and describes the scheme of the proposed multi-criteria decision tool for selecting the optimal actions to pursue the energy efficiency optimization of a given street lighting system. The decision model and the optimization algorithms are also presented. Then, Section 3 presents the case study and results of the application of the proposed decision-making tool. Finally, Section 4 provides some conclusions along with the future research directions.

## **2. The problem formulation**

The proposed decision making tool aims at helping Decision Makers (DMs) select the optimal actions to take in order to improve the performance of a given street lighting system against a set of conflicting criteria within a given budget. Hence, the problem statement may be described as follows.

A scheme of the multi-criteria decision process is shown in Fig. 1. Following [5], [32], all activities in the decision process are divided into two macro-phases: a first part (the decision design at the top of Fig. 1) that comprises in turn the diagnosis (i.e., the subsystems' status acquisition activities) as well as the so-called identification of criteria (i.e., the choice of characteristics or qualities that the DM intends to pursue in the

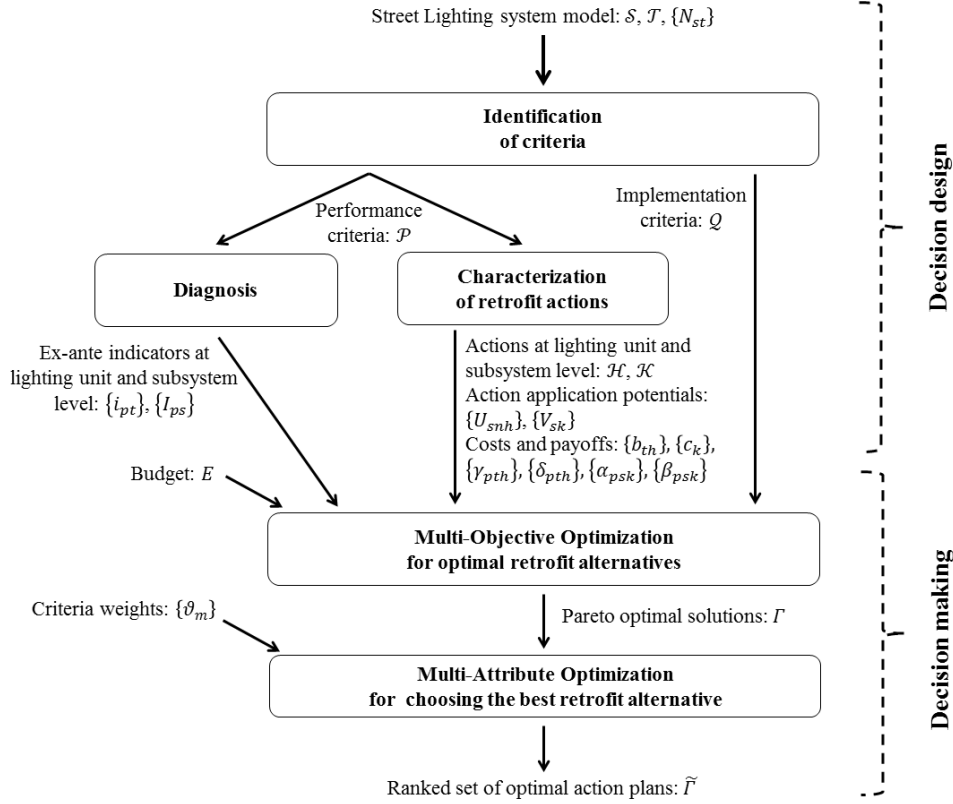


Fig. 1 Scheme of the proposed decision-making process.

street lighting retrofit) as the so-called characterization of retrofit actions (i.e., the identification of convenient retrofitting measures activities), and a second phase which implements the actual multi-criteria analysis of the possible actions (the decision making at the bottom of Fig. 1). While the high-level structuring of the decision making into multi-objective optimization and retrofit alternatives ranking is common to [5], [32], where we define a multi-criteria decision making tool to determine an optimal energy retrofit plan for a portfolio of buildings, we highlight that distinctive and unique features differentiate the decision making for the retrofit of a street lighting system from the decision making for the retrofit of a buildings stock.

Before introducing the mathematical formulation of the problem, we highlight that Table I provides the basic notation used in the paper.

Table I The adopted notation

Parameters and indices	Description
$N$	Number of lighting units totally deployed in the street lighting system
$n$	Generic lighting unit
$\mathcal{T}$	Set of lighting unit type
$T$	Number of lighting unit types
$t$	Generic lighting unit type in $\mathcal{T}$
$\tau_n$	Type related to the $n^{\text{th}}$ lighting unit
$\mathcal{S}$	Set of zone lighting subsystems
$S$	Number of zone lighting subsystems

Table I The adopted notation (continued)

Parameters and indices	Description
$s$	Generic zone lighting subsystem in $\mathcal{S}$
$N_{st}$	Number of lighting units of the $t^{\text{th}}$ type deployed in the $s^{\text{th}}$ subsystem
$N_s$	Number of lighting units totally deployed in the $s^{\text{th}}$ subsystem
$\mathcal{P}$	Set of performance criteria
$P$	Number of performance criteria
$p$	Generic performance criterion in $\mathcal{P}$
$\mathcal{Q}$	Set of implementation criteria
$Q$	Number of implementation criteria
$q$	Generic implementation criterion in $\mathcal{Q}$
$i_{pt}$	Ex-ante performance indicator of the $t^{\text{th}}$ type of lighting unit with respect to the $p^{\text{th}}$ criterion
$I_{psn}$	Ex-ante performance indicator of the $n^{\text{th}}$ lighting unit in the $s^{\text{th}}$ subsystem with respect to the $p^{\text{th}}$ criterion
$I_{ps}$	Ex-ante performance indicator of the $s^{\text{th}}$ street lighting subsystem with respect to the $p^{\text{th}}$ criterion
$I_p$	Ex-ante performance indicators of the whole street lighting system with respect to the $p^{\text{th}}$ criterion
$\mathcal{H}$	Set of actions at individual lighting unit level
$H$	Number of actions at individual lighting unit level
$h$	Generic action at individual lighting unit level in $\mathcal{H}$
$\mathcal{K}$	Set of actions at subsystem level
$K$	Number of actions at subsystem level
$k$	Generic action at subsystem level in $\mathcal{K}$
$\gamma_{pth}$	Multiplicative unitary payoff of the $h^{\text{th}}$ action on the $t^{\text{th}}$ type lighting units with respect to the $p^{\text{th}}$ criterion
$\delta_{pth}$	Additive unitary payoff of the $h^{\text{th}}$ action on the $t^{\text{th}}$ type lighting units with respect to the $p^{\text{th}}$ criterion
$U_{snh}$	Application potential of the $h^{\text{th}}$ action related to the $n^{\text{th}}$ lighting unit in the $s^{\text{th}}$ subsystem
$W_{sth}$	Application potential of the $h^{\text{th}}$ action related to lighting units of the $t^{\text{th}}$ type in the $s^{\text{th}}$ subsystem
$b_{th}$	Unitary cost for implementing the $h^{\text{th}}$ action on lighting units of the $t^{\text{th}}$ type
$\alpha_{psk}$	Multiplicative unitary payoff that the $k^{\text{th}}$ action produces on the $s^{\text{th}}$ subsystem with respect to the $p^{\text{th}}$ criterion
$\beta_{psk}$	Additive unitary payoff that the $k^{\text{th}}$ action produces on the $s^{\text{th}}$ subsystem with respect to the $p^{\text{th}}$ criterion
$V_{sk}$	Application potential of the $k^{\text{th}}$ action in the $s^{\text{th}}$ subsystem
$c_k$	Unitary cost for implementing the $k^{\text{th}}$ action
$E$	Budget allocated to the retrofit actions plan of the whole street lighting system
$\Gamma$	Set of Pareto optimal solutions
$G$	Number of Pareto optimal solutions
$X_g^*$	Generic Pareto optimal solution in $\Gamma$
$g$	Index of the generic Pareto optimal solution in $\Gamma$
$M$	Number of ranking criteria
$m$	Generic ranking criterion
$\vartheta_m$	Weight of the $m^{\text{th}}$ ranking criteria in TOPSIS method
$\tilde{F}$	Ranked set of alternatives
$J_g$	Index of the generic Pareto optimal solution in $\tilde{F}$
Decision variables	Description
$u_{snh}$	quantity of the $h^{\text{th}}$ action to be applied to the $n^{\text{th}}$ lighting unit in the $s^{\text{th}}$ subsystem
$w_{sth}$	quantity of the $h^{\text{th}}$ action to be applied to the overall set of the $t^{\text{th}}$ type lighting units in the $s^{\text{th}}$ subsystem
$v_{sk}$	quantity of $k^{\text{th}}$ action to be applied to the $s^{\text{th}}$ subsystem
$X_g^*$	Generic Pareto optimal solution in $\Gamma$
Indicators functions	Description
$I'_{psn}$	Post-retrofit performance indicator of the $n^{\text{th}}$ lighting unit in the $s^{\text{th}}$ subsystem with respect to the $p^{\text{th}}$ criterion
$I'_{ps}$	Post-retrofit performance indicator of the $s^{\text{th}}$ subsystem with respect to the $p^{\text{th}}$ criterion
$I'_p$	Post-retrofit performance indicator related to the whole street lighting system with respect to the $p^{\text{th}}$ criterion
$J'_q$	Indicator related to the $q^{\text{th}}$ implementation criterion
$I_p^*$	Utopia point related to the $p^{\text{th}}$ performance criterion
$J_q^*$	Utopia point related to the $q^{\text{th}}$ implementation criterion

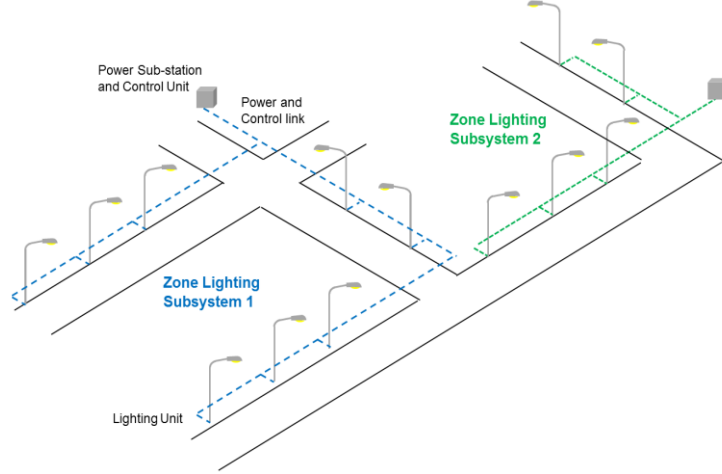


Fig. 2 Example of street lighting system composed by two zone lighting subsystems.

### 2.1 The system model

We refer to a generic urban area street lighting system that includes several lighting units, a power distribution system (composed by electric lines and power substations), and a command and control system (composed by communication links and control base stations). We assume that the number of deployed lighting units is  $N$  and their types belong to the set  $\mathcal{T} = \{1, \dots, t, \dots, T\}$  whose cardinality is  $T = |\mathcal{T}|$ . Of course, the lighting units of the  $t^{\text{th}}$  type are characterized by the same electric features (e.g., energy consumption) and photometric quantities (e.g., luminous flux). Street lighting systems are commonly widespread, with a large number of lighting points and an extensive power and communication network. Hence, given the geographic distribution of lighting equipment over an urban area, we assume that the street lighting system is subdivided in a certain number of subsystems (Fig. 2). Each of these, denoted as zone lighting subsystems (named simply subsystems in the remainder), is responsible for reproducing lighting conditions in the set of neighboring road or street segments pertaining to the related urban zone. We denote the set of zone lighting subsystems in the given urban area as  $\mathcal{S} = \{1, \dots, s, \dots, S\}$  whose cardinality is  $S = |\mathcal{S}|$ . The lighting units of a given zone lighting subsystem are energized by their own power substation that is also provided with a control unit aimed at automatically turn lights on or off by a timer switch. Finally, we denote by  $N_{st}$  ( $\forall s = 1, \dots, S, \forall t = 1, \dots, T$ ) the number of lighting units of the  $t^{\text{th}}$  type deployed in the  $s^{\text{th}}$  subsystem and with  $N_s = \sum_{t=1}^T N_{st}$  the number of lighting units totally deployed in the  $s^{\text{th}}$  subsystem.

### 2.2 The decision design phase

The decision design phase (see Fig. 1) is performed by the decision analyst in conjunction with street lighting operators. This phase of the decision process includes various steps of data collection, analysis, and modeling: basically, it aims at understanding and defining metrics and models that can be used to preventively estimate the impacts of potential modifications on the street lighting system performance.

### 2.2.1 Identification of criteria

Primarily, having in mind the energy efficiency and environmental sustainability as overall goals of the decision making, the identification of specific *performance criteria* is required. After a joint analysis and walk-through surveys, street lighting managers together with technical experts identify a set  $\mathcal{P} = \{1, \dots, p, \dots, P\}$  of  $P = |\mathcal{P}|$  criteria aiming at the assessment of the performance of the given street lighting system. For instance, the related literature commonly used performance criteria include: energy consumption, lighting pollution, and the so-called color rendering index [33-35]. These performance criteria are employed to carry out both the ex-ante and ex-post assessments of a retrofit plan. Consequently, they provide metrics on the basis of which decision makers can judge the value of the design and establishment of actions having multi-sectorial impacts.

Furthermore, the decision maker may require other type of criteria that are neither related to the performance and energy behavior of the given street lighting system not to the specific nature of candidate retrofit actions, but are only associated to their implementation and mode of intervention. For instance, one of these so-called *implementation criteria* may be the optimal allocation of retrofit actions among the various street lighting subsystems in accordance with a homogeneity or priority order metric. These implementation criteria are denoted as a set  $\mathcal{Q} = \{1, \dots, q, \dots, Q\}$  whose cardinality is  $Q = |\mathcal{Q}|$ .

Once all criteria have been identified, the next steps of the decision design phase are the street lighting diagnosis and the evaluation of renovation and energy efficiency actions. Since both these steps are devoted to analyzing the current (ex-ante) state of the system and the (ex-post) effect that the retrofit plan may potentially have on it, they are addressed only with respect to the performance criteria in set  $\mathcal{P}$ , as detailed in the following subsections.

### 2.2.2 Diagnosis with respect to performance criteria

The street lighting diagnosis aims at evaluating the general state of the street lighting system with respect to the performance criteria in set  $\mathcal{P}$  selected in the first part of the Diagnosis phase, e.g., energy consumption, light pollution, drivers comfort, etc. In particular, the diagnosis is conducted for all the subsystems and on a lighting unit per lighting unit basis. Thus, let  $I_{psn}$  be the performance indicator of the  $n^{\text{th}}$  lighting unit in the  $s^{\text{th}}$  subsystem with respect to the  $p^{\text{th}}$  criterion. The performance indicator of the  $s^{\text{th}}$  street lighting subsystem with respect to the  $p^{\text{th}}$  criterion, denoted as  $I_{ps}$ , is consequently determined as follows:

$$I_{ps} = \sum_{n=1}^{N_s} I_{psn}, \forall p = 1, \dots, P, \forall s = 1, \dots, S. \quad (1)$$

Clustering the lighting units into types, (1) could be equivalently written as:

$$I_{ps} = \sum_{n=1}^{N_s} i_p \tau_n = \sum_{t=1}^T N_{st} i_{pt}, \forall p = 1, \dots, P, \forall s = 1, \dots, S \quad (2)$$

where  $\tau_n$  indicates the type of the of the  $n^{\text{th}}$  lighting unit and  $i_{pt}$  denotes the performance indicator of the  $t^{\text{th}}$  type of lighting unit with respect to the  $p^{\text{th}}$  criterion.



Having evaluated the performance indicators related to all the subsystems, the resulting value of the  $p^{\text{th}}$  criterion on the whole street lighting system, denoted as  $I_p$ , is computed as follows:

$$I_p = \sum_{s=1}^S I_{ps}, \forall p = 1, \dots, P. \quad (3)$$

### 2.2.3 Characterization of retrofit actions with respect to performance criteria

Subsequently, the preventive evaluation of the renovation and energy efficiency actions is performed with respect to the performance criteria selected in the first phase. This step requires an applicability and feasibility study that is conducted specifically for each subsystem. In fact, each of these may exhibit unique technical, architectural, and structural characteristics, and customized retrofit options must be individually investigated. The considered retrofit measures may be either structural or equipment changes. We remark that operational changes (e.g., set-point optimization, etc.) are hereby not taken into account because they are beyond the scope of the paper. The outcome of the evaluation of renovation and energy efficiency actions is the actual list of possible actions that may be implemented in the given urban area lighting system. Each determined action is successively characterized from three perspectives: its application potential (i.e., the estimation of a metric related to the action implementation), unitary cost (since the cost of each action is here simply modeled in accordance with a linear pricing model, as the product between the unitary cost and the related implemented action quantity), and retrofit payoff parameters (i.e., parameters related to the post-retrofit value of indicator for each performance criterion).

In particular, we identify the following two categories of retrofitting actions:

- Actions at individual lighting unit level –  $\mathcal{H}$ -type: this set includes all actions that have impact on an individual lighting unit and we denote it by  $\mathcal{H} = \{1, \dots, h, \dots, H\}$  whose cardinality is  $H = |\mathcal{H}|$ . An example of action of this category is the replacement of a lamp (or of a whole luminaire) of a given lighting unit.

For a unitary implementation of the  $h^{\text{th}}$  action of  $\mathcal{H}$ -type, the post-retrofit value of the performance indicator of the  $n^{\text{th}}$  lighting unit of  $\tau_n$  type with respect to the  $p^{\text{th}}$  criterion is defined in accordance with a linear model as follows:

$$I'_{psn} = (1 - \gamma_{p\tau_n h}) i_{p\tau_n} - (\delta_{p\tau_n h}), \quad (4)$$

$$\forall p = 1, \dots, P, \forall s = 1, \dots, S, \forall n = 1, \dots, N_s.$$

where  $\gamma_{p\tau_n h}$  and  $\delta_{p\tau_n h}$  denote the multiplicative and additive unitary payoffs, respectively. Note that, throughout this paper, the apex symbol in variable superscript denotes that the indicator value is relative to the ex-post status (i.e., after the implementation of the retrofit actions). In case a performance indicator has a range whose upper (lower) level stands for poor (excellent) performance, we assume that  $\gamma_{pnh}$  and  $\delta_{pnh}$  are positive (negative) if the  $h^{\text{th}}$  action produces benefit (detriment) on the beneficiary lighting unit, respectively. For instance, assume that the  $p^{\text{th}}$  criterion indicates the energy consumption and the  $h^{\text{th}}$  action consists in replacing the lamp of the  $t^{\text{th}}$  type by a new one that

fulfills the same requirements about road surface luminance while its consumption is less than  $\delta_{pth}$  [kWh]. In this case,  $\delta_{pth}$  and  $\gamma_{pth} = 0$  define the retrofit payoff parameters that characterize the  $h^{\text{th}}$  action applied to the  $n^{\text{th}}$  beneficiary lighting unit (i.e.,  $I'_{psn} = i_{pt} - \delta_{pth}$ ).

The application potential of the  $h^{\text{th}}$  action of  $\mathcal{H}$ -type related to the  $n^{\text{th}}$  lighting unit in the  $s^{\text{th}}$  subsystem is denoted  $U_{snh}$ . The unitary cost for implementing the  $h^{\text{th}}$  action of  $\mathcal{H}$ -type related to the  $n^{\text{th}}$  lighting unit of  $\tau_n$  type is denoted by  $b_{\tau_n h}$ .

- Actions at subsystem level -  $\mathcal{K}$ -type: this set includes actions that have impact on a whole subsystem and is denoted by  $\mathcal{K} = \{1, \dots, k, \dots, K\}$  whose cardinality is  $K = |\mathcal{K}|$ . An example of action of this type is the installation of a remote control station for dimming all the lighting units in a given subsystem.

For a unitary implementation of the  $k^{\text{th}}$  action of  $\mathcal{K}$ -type, the post-retrofit value of the performance indicator of the  $s^{\text{th}}$  subsystem with respect to the  $p^{\text{th}}$  criterion is defined in accordance with a linear model as follows:

$$I'_{ps} = (1 - \alpha_{psk})I_{ps} - (\beta_{psk}), \forall p = 1, \dots, P, \forall s = 1, \dots, S. \quad (5)$$

where  $\alpha_{psk}$  and  $\beta_{psk}$  denote the multiplicative and additive unitary payoffs, respectively. In case a performance indicator has a range whose upper (lower) level stands for poor (excellent) performance, we assume that  $\alpha_{psk}$  and  $\beta_{psk}$  are positive (negative) if the  $k^{\text{th}}$  action produce benefit (detriment) on the beneficiary subsystem. For instance, assume that the  $p^{\text{th}}$  criterion indicates the upright luminous flux and the  $k^{\text{th}}$  action consists in installing a dimming device that reduces the power level by 15%. In this case,  $\alpha_{psk} = 0.15$  and  $\beta_{psk} = 0$  define the retrofit payoff parameters that characterize the  $k^{\text{th}}$  action applied to the  $s^{\text{th}}$  beneficiary subsystem (i.e.,  $I'_{ps} = 0.85I_{ps}$ ).

The application potential of the  $k^{\text{th}}$  action of  $\mathcal{K}$ -type in the  $s^{\text{th}}$  subsystem is denoted by  $V_{sk}$ . The unitary cost for implementing the  $k^{\text{th}}$  action of  $\mathcal{K}$ -type is denoted as  $c_k$ .

### 2.3 The Decision making phase

The second phase of the decision process consists in the actual decision making and is a responsibility of the decision maker, i.e., the public administration planner or street lighting manager. This phase comprises two subsequent steps. The first one consists in the definition of the Multi-Objective Optimization (MOO) problem. The solution of such a problem provides a set of Pareto-optimal strategies, also called non-dominated solutions, constituting the so-called Pareto frontier [36]. In order to choose the best alternative among the determined non-dominated solutions, different approaches may be followed. Traditional methods base the choice on experts' intuition or preference. Alternatively, a second step optimization procedure is applied, based on the use of one the well-known Multi-Attribute Decision Making (MADM) procedures that provide a ranking of the obtained retrofit strategies (see for instance [37]).

### 2.3.1 The Multi-Objective Optimization model

A MOO problem is defined in order to determine the Pareto frontier, i.e., the set of all the possible optimal retrofit strategies. The decision model relies on several decision variables representing the choices on actions. For this purpose, the following types of decision variables are considered.

- Decision variables that reflect the choices regarding the retrofit actions of  $\mathcal{H}$ -type. Let  $u_{snh}$  be a variable representing the quantity of the  $h^{\text{th}}$  action to be applied to the  $n^{\text{th}}$  lighting unit in the  $s^{\text{th}}$  subsystem. This is a non-negative integer variable that is upper bounded by the related application potential  $U_{snh}$  as follows:

$$u_{snh} \in \{\mathbb{N}\}, 0 \leq u_{snh} \leq U_{snh}, \forall s = 1, \dots, S, \forall n = 1, \dots, N_s, \forall h = 1, \dots, H. \quad (6)$$

- Decision variables that reflect the choices regarding the retrofit actions of  $\mathcal{K}$ -type. Let  $v_{sk}$  be a variable representing the quantity of  $k^{\text{th}}$  action to be implemented in the  $s^{\text{th}}$  subsystem. This is a non-negative integer variable that is upper bounded by the related application potential  $V_{sk}$  as follows:

$$v_{sk} \in \{\mathbb{N}\}, 0 \leq v_{sk} \leq V_{sk}, \forall s = 1, \dots, S, \forall k = 1, \dots, K. \quad (7)$$

The application of retrofit actions to the street lighting system impacts both on the indicators related to performance criteria in set  $\mathcal{P}$  and on the indicators related to implementation criteria in set  $\mathcal{Q}$ . We assume that all these indicators have a range whose upper level stands for poor performance, while its lower level indicates excellent performance.

Further, while the indicators related to performance criteria in set  $\mathcal{P}$  may be formulated in a similar way as functions of decision variables and retrofit action parameters, this is not possible for the indicators related to implementation criteria in set  $\mathcal{Q}$ . These may have a different formulation depending on their meaning, which is specified by the DM on a case-by-case basis.

In the sequel we provide a general model for the estimate of ex-post indicators related to performance criteria  $\mathcal{P}$ , formulated as a function of decision variables and retrofit action parameters.

The application of retrofit actions of  $\mathcal{H}$ -type to the  $n^{\text{th}}$  beneficiary lighting unit of  $\tau_n$  type in the  $s^{\text{th}}$  subsystem provides the  $p^{\text{th}}$  criterion index with a decrease equal to the cumulated estimated payoffs, i.e.:

$$I'_{psn}(\{u_{snh}\}) = \left(1 - \sum_{h=1}^H \gamma_{p\tau_n h} u_{snh}\right) i_{p\tau_n} - \left(\sum_{h=1}^H \delta_{p\tau_n h} u_{snh}\right), \quad (8)$$

$$\forall p = 1, \dots, P, \forall s = 1, \dots, S, \forall n = 1, \dots, N_s.$$

Subsequently, the overlapping application of retrofit actions in  $\mathcal{K}$  to the  $s^{\text{th}}$  subsystem together with the application of the retrofit actions of  $\mathcal{H}$ -type to the  $n^{\text{th}}$  beneficiary lighting unit of  $\tau_n$  type in that subsystem provides the  $p^{\text{th}}$  criterion index with a decrease equal to the cumulated estimated payoffs, i.e.:

$$I'_{ps}(\{u_{snh}\}, \{v_{sk}\}) = \left(1 - \sum_{k=1}^K \alpha_{psk} v_{sk}\right) \left(\sum_{n=1}^{N_s} I'_{psn}\right) - \left(\sum_{k=1}^K \beta_{psk} v_{sk}\right), \quad (9)$$

$$\forall p = 1, \dots, P, \forall s = 1, \dots, S.$$

Combining (8) and (9), we obtain the formulation of ex-post indicators related to the  $s^{\text{th}}$  subsystem:

$$\begin{aligned}
& I'_{ps} (\{u_{snh}\}, \{v_{sk}\}) = \\
& = \left( 1 - \sum_{k=1}^K \alpha_{psk} v_{sk} \right) \left( \sum_{n=1}^{N_s} \left( 1 - \sum_{h=1}^H \gamma_{p\tau_n h} u_{snh} \right) i_{p\tau_n} - \left( \sum_{h=1}^H \delta_{p\tau_n h} u_{snh} \right) \right) - \left( \sum_{k=1}^K \beta_{psk} v_{sk} \right), \quad (10) \\
& \quad \forall p = 1, \dots, P, \forall s = 1, \dots, S.
\end{aligned}$$

In each subsystem, it is possible to combine choices related to lighting units of the same type into clusters. This allows replacing the large set of  $N_s H$  non-negative integer decision variables  $\{u_{snh}\}$ , one for each lighting unit, with a smaller set of  $TH$  non-negative integer decision variables  $\{w_{sth}\}$ , one for each cluster identified by index  $t$ :

$$w_{sth} \in \{\mathbb{N}\}, 0 \leq w_{sth} \leq W_{sth}, \forall s = 1, \dots, S, \forall j = 1, \dots, T, \forall h = 1, \dots, H. \quad (11)$$

where  $W_{sth} = \sum_{n=1}^{N_s} U_{snh}$  denotes the application potential of the  $h^{\text{th}}$  action to be applied to the lighting units of the  $t^{\text{th}}$  type in the  $s^{\text{th}}$  subsystem. Decision variable  $w_{sth}$  represents the quantity of the  $h^{\text{th}}$  action to be applied to the overall set of lighting units of the  $t^{\text{th}}$  type in the  $s^{\text{th}}$  subsystem. From a computational point of view, the integer variables  $w_{sth}$  provide a structure that both reduces the dimensionality of and guides the search through the combinatorial lighting unit actions state space. In addition, clustering reduces the number of equations and variables since all relations now apply over the smaller number of clusters rather than the full set of individual units.

Clustering by the lighting unit type, the formulation of (10) changes as follows:

$$\begin{aligned}
& I'_{ps} (\{w_{sth}\}, \{v_{sk}\}) = \\
& = \left( 1 - \sum_{k=1}^K \alpha_{psk} v_{sk} \right) \left( \sum_{t=1}^T \left( 1 - \sum_{h=1}^H \gamma_{pth} w_{sth} \right) N_{st} i_{pt} - \left( \sum_{h=1}^H \delta_{pth} w_{sth} \right) \right) - \left( \sum_{k=1}^K \beta_{psk} v_{sk} \right), \quad (12) \\
& \quad \forall p = 1, \dots, P, \forall s = 1, \dots, S.
\end{aligned}$$

Finally, the ex-post estimate of the  $p^{\text{th}}$  indicators related to the whole street lighting system is given by the following formula:

$$I'_p (\{w_{sth}\}, \{v_{sk}\}) = \sum_{s=1}^S I'_{ps} (\{w_{sth}\}, \{v_{sk}\}), \forall p = 1, \dots, P. \quad (13)$$

Regarding the indicators related to implementation criteria in set  $\mathcal{Q}$ , as previously discussed, we do not provide any specific formulation for them, but we denote the indicator related to the  $q^{\text{th}}$  implementation criterion as a generic function of the decision variables  $\{w_{snh}\}$  and  $\{v_{sk}\}$  characterizing the retrofit plan, namely  $J'_q (\{w_{snh}\}, \{v_{sk}\})$ .

The main constraint in the choice of the decision variables is the financial resources limitation. Hence, calling  $E$  the overall budget allocated to the retrofit actions plan of the whole street lighting system, the following inequality must be verified:

$$\sum_{s=1}^S \left( \sum_{t=1}^T \sum_{h=1}^H b_{th} w_{sth} + \sum_{k=1}^K c_k v_{sk} \right) \leq E. \quad (14)$$

Finally, the MOO problem is defined as the problem of determining the  $S(TH + K)$  decision variables  $\{w_{sth}\}$  and  $\{v_{sk}\}$  that minimize the indicators related to the  $P + Q$  criteria not exceeding the limited resource:

$$\begin{aligned} \min_{\{w_{sth}\}, \{v_{sk}\}} & \left( I'_1(\{w_{sth}\}, \{v_{sk}\}), \dots, I'_P(\{w_{sth}\}, \{v_{sk}\}), J'_1(\{w_{sth}\}, \{v_{sk}\}), \dots, J'_Q(\{w_{sth}\}, \{v_{sk}\}) \right) \\ \text{s.t.} & (7)-(11)-(14) \end{aligned} \quad (15)$$

The decision problem (15) is a multi-objective combinatorial optimization problem. It is generally a non-linear integer MOO problem since at least the products of two variables such as  $w_{sth}v_{sk}$  in the objective functions  $I'_p$  ( $\forall p = 1, \dots, P$ ) are present due to (12). Hence, depending on the actual formulation of indices  $J'_q$  ( $\forall q = 1, \dots, Q$ ), problem (15) could fall within a well-defined class of non-linear problems. For instance, if  $J'_q$  ( $\forall q = 1, \dots, Q$ ) are linear functions, (15) would reduce to be a bilinear MOO problem. Finally, we remark that the overall problem in (15) has generally  $P + Q$  objective functions, a number of  $S(K + TH)$  integer variables, 1 inequality constraint, and  $2S(K + TH)$  bounding constraints. Designating  $X_g^*$  as one of the determined Pareto optimal solutions, the Pareto solutions set is denoted as follows:

$$\Gamma = \{X_g^*\}, \quad \forall g = 1, \dots, G \quad (16)$$

where  $G$  is the cardinality of the set of Pareto optimal solutions  $\Gamma$ .

### 2.3.2 The Multi-Attribute Optimization

After the solution of the MOO problem (15), the best retrofit alternative has to be selected among the determined Pareto optimal solutions set (16). The choice of the best alternative among the determined solutions may be done by different approaches. Among these, methods based on expert knowledge or preference are preferred when the dimension of the solutions' set is small. Contrarily, when the set size is very large, a MADM technique is preferred in order to determine a ranking of the obtained retrofit strategies [38-41]. MADM is an approach that allows for choosing an option from a set of alternatives, which are characterized in terms of their attributes [38], [42]. The DM may express or define a ranking of the attributes in terms of importance/weights. MADM aims at obtaining the optimum alternative that has the highest degree of satisfaction for all of the relevant attributes [43].

In this paper, to solve the MADM problem, one of the most classical MADM methods and widely accepted for identifying a solution from a finite large set of alternatives is used, namely the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). It is based on the assumption that the best alternative is the one with the shortest distance from the so-called Ideal Solution (IS) and consequently the farthest distance from the so-called Negative Ideal Solution (NIS).

The TOPSIS method requires in input a  $G \times M$  decision matrix  $D$ , where  $M$  is the number of criteria upon which the solution ranking has to be based. Note that the ranking may be performed on the basis of all or part of the  $P + Q$  indicators considered in the first part of optimization model or by way of different criteria from the ones considered to solve the MOO problem. Hence, the generic element  $d_{gm}$  of the decision matrix  $D$ , with  $g = 1, \dots, G$  and  $m = 1, \dots, M$ , represents the  $m^{\text{th}}$  indicator value of the  $g^{\text{th}}$  MOO solution  $X_g^*$  of

problem (15). The method also requires some cardinal attribute importance weights of the alternatives with respect to the criteria. Thus, a weight  $\vartheta_m$ , with  $m = 1, \dots, M$ , is associated by the DM to each of the ranking criteria in order to model the importance degree of the  $m^{\text{th}}$  criterion in the ranking of the different retrofit alternatives. The ranking criteria weights are assigned so that the sum of all such weights is unitary. TOPSIS consists of the following steps [44].

*Step 1. Constructing the normalized decision matrix.* Determine each element  $\delta_{gm}$  of the  $G \times M$  normalized decision matrix  $\Delta$  as follows:

$$\delta_{gm} = \frac{d_{gm}}{\sqrt{\sum_{g=1}^G d_{gm}^2}}, \forall g = 1, \dots, G, \forall m = 1, \dots, M. \quad (17)$$

*Step 2. Constructing the weighted normalized decision matrix.* Determine the  $G \times M$  weighted normalized decision matrix  $\Omega$ , whose element is computed as follows:

$$\omega_{gm} = \delta_{gm}\vartheta_m, \forall g = 1, \dots, G, \forall m = 1, \dots, M. \quad (18)$$

*Step 3. Determining the ideal and negative ideal solutions.* Define the ideal solution (IS) as the solution with performance indicators given by the row vector  $\Omega_{max} = (\omega_{max,1}, \dots, \omega_{max,m}, \dots, \omega_{max,M})$ , where  $\omega_{max,m} = \max(\omega_{1m}, \dots, \omega_{gm}, \dots, \omega_{Gm})$  with  $m = 1, \dots, M$ . Moreover, define the NIS as the ideal solution associated to performance indicators of the row vector  $\Omega_{min} = (\omega_{min,1}, \dots, \omega_{min,m}, \dots, \omega_{min,M})$ , where  $\omega_{min,m} = \min(\omega_{1m}, \dots, \omega_{gm}, \dots, \omega_{Gm})$  with  $m = 1, \dots, M$ .

*Step 4. Calculating the separation distances.* Calculate the separation distance  $\sigma_{max,g}$  from the IS of each alternative  $X_g^*$  with  $g = 1, \dots, G$  as follows:

$$\sigma_{max,g} = \sqrt{\sum_{m=1}^M (\omega_{gm} - \omega_{max,m})^2}, \forall g = 1, \dots, G \quad (19)$$

Moreover, determine the separation distance  $\sigma_{min,g}$  of  $X_g^*$  with  $g = 1, \dots, G$  from the NIS as follows:

$$\sigma_{min,g} = \sqrt{\sum_{m=1}^M (\omega_{gm} - \omega_{min,m})^2}, \forall g = 1, \dots, G \quad (20)$$

*Step 5. Calculating the relative closeness of alternatives to the ideal solution.* Determine the closeness  $Cl_g$  to the NIS of each alternative  $X_g^*$  with  $g = 1, \dots, G$  as follows:

$$Cl_g = \frac{\sigma_{min,g}}{\sigma_{max,g} + \sigma_{min,g}}, \forall g = 1, \dots, G \quad (21)$$

*Step 6. Ranking alternatives.* The ranked set of alternatives is represented by the ordered set  $\tilde{\Gamma}$  defined as:

$$\tilde{\Gamma} = \{X_{j_1}^*, \dots, X_{j_g}^*, \dots, X_{j_G}^*\}, \quad (22)$$

where all the elements of the set  $\tilde{\Gamma}$  are arranged according to the decreasing order of the closeness value  $Cl_g$  associated to the  $g^{\text{th}}$  solution for  $g = 1, \dots, G$ . Hence,  $X_{j_1}^*$  is the best retrofit alternative and  $X_{j_G}^*$  is the worst one.

Table II Enumeration of lighting units

Description	Parameter	Unit	Value									
			1	2	3	4	5	6	7	8	9	10
Subsystem	$s$	-	1	2	3	4	5	6	7	8	9	10
Number of lighting units – type $t = 1$	$N_{s1}$	-	0	0	0	0	0	1	5	52	5	27
Number of lighting units – type $t = 2$	$N_{s2}$	-	44	33	29	26	30	3	23	54	29	45

Table III Ex-ante values of performance indicators

Description	Parameter	Unit	Value	
Lighting unit type	$t$	-	1	2
Lighting unit performance indicator – criterion $p = 1$	$i_{1t}$	[kWh/yr]	660	1,100
Lighting unit performance indicator – criterion $p = 2$	$i_{2t}$	[lm]	10	10
Lighting unit performance indicator – criterion $p = 3$	$i_{3t}$	-	35	40

### 3. Case study

In this section, we use a case study from the street lighting system of Bari, the capital city of the Apulia region (southern Italy), to test the developed model and to show its applicability using real-world data. Moreover, various experiments in different scenarios of analysis are conducted to demonstrate the usefulness and flexibility of our proposed approach.

#### 3.1 Setup of experiments

The proposed decision-making tool is applied to the energy retrofit of the street lighting system of a wide area of Bari. Bari is currently engaged in implementing a series of actions targeted at the reduction of CO<sub>2</sub> and the increase of energy efficiency, characterized by the use of smart city enabling technologies [5]. In this context, we describe the application of the proposed optimization model in supporting the city energy manager to solve the following decision problem: identifying a set of optimal retrofit actions for improving the street lighting in a given urban area (energy retrofitting actions planning) given a pre-defined budget  $E = \text{€}100,000.00$ .

In the given urban area, the lighting system is composed by  $S = 10$  lighting subsystems that make use of  $N = 316$  lighting units of  $T = 2$  types: Table II enumerates the lighting units for each type in each subsystem.

##### 3.1.1 Decision design

After a joint analysis and walk-through surveys conducted with street lighting managers and technical experts, we identify the following  $P = 3$  performance criteria:

- $p = 1$  - Energy Consumption. This criterion provides a measurement of the annual amount of energy required for supplying with energy the whole street lighting system.
- $p = 2$  - Uplight Luminous Flux – This criterion provides a measurement of the direct sky glow caused by the entire street lighting system.
- $p = 3$  - Color Rendering Index – This criterion provides a measurement of the color rendering of electric lights in the entire street lighting system.

We assume that the set  $Q$  of implementation criteria include only one criterion ( $Q = 1$ ) aimed at characterizing the optimal allocation of actions on different subsystems.

- $q = 1$  - Retrofit Actions Allocation Index - This criterion provides a measurement of the distribution degree of actions in subsystems, namely, the number of retrofit actions allocated to a subsystem. In cases of urban infrastructure refurbishment, the distribution of actions is usually concentrated in few subsystems, either for esthetic reasons (e.g., avoiding to have simultaneously new and old lighting units in the same road) or operational reasons (e.g., spread interventions would complicate the action plan, by increasing time and costs in the implementation phase and reducing the service level to road users) [45].

Hence, an on-site audit activity follows. Each of the lighting subsystems undergoes a diagnosis phase in order to assess the specific characteristics of interest. Having assessed the performance indicators of each type of lighting units (i.e.,  $i_{pt}$ ,  $\forall t = 1, \dots, T$ ), the diagnosis phase ends the assessment of the ex-ante status of the entire street lighting system through the application of (2) and (3). Table III reports the outcomes of the diagnosis phase conducted for each subsystem and each lighting unit type (i.e., the ex-ante values of performance indicators).

Subsequently, the evaluation of renovation and energy efficiency measures is performed. We identify three retrofitting actions, which we successively characterize as regards application potential, cost and payoffs. The first one has an impact on an individual lighting unit (i.e., it is of  $\mathcal{H}$ -type) and the other two actions have an impact on the whole subsystem (i.e., they are of  $\mathcal{K}$ -type).

- $h = 1$  - Replacement of luminaires - We assume that the luminaire of each  $t^{\text{th}}$  type lighting unit may be replaced by a new one that fulfills the same requirements of the old one about road surface luminance while being more efficient (i.e., reducing the energy consumption), more environmentally respectful (i.e., limiting the luminous flux above the horizon), and more comfortable (i.e., accurately reproducing colors). The decision variables that reflect the choices regarding this retrofit action are non-negative integer variables denoted as  $\{w_{st1}\}$ :

$$w_{st1} \in \{\mathbb{N}\}, 0 \leq w_{st1} \leq N_{st}, \forall s = 1, \dots, S, \forall j = 1, \dots, T. \quad (23)$$

The application potential  $W_{st1}$  for luminaire replacement action simply coincides with the number of lighting units of  $t^{\text{th}}$  type in the  $s^{\text{th}}$  subsystem (i.e.,  $W_{st1} = N_{st}$ ). The unitary cost of replacement of  $t^{\text{th}}$  type lighting unit is indicated as  $b_{t1}$  [€/pc]. Let  $\delta_{1t1}$ ,  $\delta_{2t1}$  and  $\delta_{3t1}$  be the reduction of energy consumption [kWh/yr], the reduction of uplight luminous flux [lm] and the improvement of the color index of the replacing lamp, respectively; we assume that there are no multiplicative payoffs for this action ( $\gamma_{pt1} = 0$ ,  $\forall p = 1, \dots, P$ ,  $\forall t = 1, \dots, T$ ).

- $k = 1$  - Installation of energy harvesting modules - We assume that each lighting unit may be equipped with a system by which energy is derived from external natural sources, captured, stored and used at a convenient time. For instance, solar-powered lighting may be obtained by installing a solar panel or photovoltaic cell that collects the sun energy during the day, stores it in a rechargeable gel cell battery, and energizes the lamp during the night using the stored energy in the rechargeable battery [33]. Depending both on the renewable source production volume and on the battery storage capacity, the energy harvesting module reduces the energy demand from the grid distribution. The



decision variables that reflect the choices regarding this retrofit action are non-negative integer variables denoted as  $\{v_{s1}\}$ .

$$v_{s1} \in \{\mathbb{N}\}, 0 \leq v_{s1} \leq N_s, \forall s = 1, \dots, S. \quad (24)$$

Hence, the application potential  $V_{s1}$  of installing energy harvesting modules in the  $s^{\text{th}}$  subsystem coincides with the number of lighting units in the  $s^{\text{th}}$  subsystem (i.e.,  $V_{s1} = N_s$ ). The unitary cost of an energy harvesting modules is indicated as  $c_1$  [€/pc]. Let  $\beta_{1s1}$  be the estimated annual amount of energy provided by the single energy harvesting module [kWh/yr] and let  $\beta_{2s1} = \beta_{3s1} = 0$  be the reduction of uplight luminous flux [lm] and the improvement of the color index; we assume that there are no multiplicative payoffs for this action ( $\alpha_{ps1} = 0, \forall p = 1, \dots, P$ ).

- $k = 2$  - Installation of dimming devices - We assume that all the lamps already deployed or to be installed in each lighting subsystem are dimmable and thus the related control station of may be equipped with a dimming device to reduce the luminous output of the lamp to more suitable levels when traffic flows are low or at off peak times. Note that lamps equipped with such a dimming device can be turned up to full power when needed. Based on an evaluation of vehicular traffic flow conditions in the street segments pertaining to each lighting subsystem, light dimming levels are determined for the different time slots in the operating periods. The decision variables that reflect the choices regarding this retrofit action are binary variables denoted as  $\{v_{s2}\}$ .

$$v_{s1} \in \{\mathbb{N}\}, 0 \leq v_{s1} \leq V_{s2}, \forall s = 1, \dots, S. \quad (25)$$

The application potential  $V_{s2}$  is unitary (i.e.,  $V_{s2} = 1$ ). The cost of the installation of a dimming device is  $c_2$  [€/pc]. The application of this action produces a multiplicative payoff only on the performance indicators related to the energy consumption and light pollution ( $\alpha_{1s2} = \alpha_{2s2}$  is the relative total annual energy saving factor of the  $s^{\text{th}}$  subsystem equipped with a dimming device with respect to the original deployment;  $\alpha_{3s2} = 0$ ); we assume that there are no additive payoffs for this action ( $\beta_{ps2} = 0, \forall p = 1, \dots, P$ ).

Tables IV to VI report the payoffs and costs referred to the three considered retrofitting actions.

Table IV Payoffs and costs for action  $h = 1$  at individual lighting unit level

Description	Parameter	Unit	Value	
Lighting unit type	$t$	-	1	2
Additive payoff – criterion $p = 1$	$\delta_{1t1}$ ,	[kWh/yr]	260	380
Additive payoff – criterion $p = 2$	$\delta_{2t1}$ ,	[lm]	5	5
Additive payoff – criterion $p = 3$	$\delta_{3t1}$ ,	-	-25	-20
Multiplicative payoff – for criterion $p = 1,2,3$	$\gamma_{pt1}$	-	0	
Unitary cost	$b_{t1}$	[€/pc]	1,300.00	1,500.00

Table V Payoffs and costs for action  $k = 1$  at subsystem level

Description	Parameter	Unit	Value
Additive payoff – criterion $p = 1$ , for each subsystem $s = 1, \dots, 10$	$\alpha_{1s1}$	[kWh/yr]	240
Additive payoff – criterion $p = 2$ , for each subsystem $s = 1, \dots, 10$	$\alpha_{2s1}$	[lm]	0
Additive payoff – criterion $p = 3$ , for each subsystem $s = 1, \dots, 10$	$\alpha_{3s1}$	-	0
Multiplicative payoff – for criterion $p = 1,2,3$ , for each subsystem $s = 1, \dots, 10$	$\beta_{ps1}$	-	0
Unitary cost	$c_1$	[€/pc]	500.00

Table VI Payoffs and costs for action  $k = 2$  at subsystem level

Description	Parameter	Unit	Value									
			1	2	3	4	5	6	7	8	9	10
Subsystem	$s$	-	1	2	3	4	5	6	7	8	9	10
Multiplicative payoff – criterion $p = 1$	$\alpha_{1s2}$	-	0.25	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.2
Multiplicative payoff – criterion $p = 2$	$\alpha_{2s2}$	-	0.25	0.25	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.2
Multiplicative payoff – criterion $p = 3$	$\alpha_{3s2}$	-	0									
Additive payoff – for criterion $p = 1,2,3$	$\beta_{ps2}$	[kWh/yr]	0									
Unitary cost	$c_2$	[€/pc]	800.00									

From (12) and (13) we get the formulation of ex-post value of indicators related to the previously defined performance criteria  $p = 1,2,3$ , noting that  $I'_1$  and  $I'_2$  must be minimized and  $I'_3$  must be maximized:

$$\begin{aligned}
I'_1(\{w_{st1}\}, \{v_{s1}\}, \{v_{s2}\}) &= \sum_{s=1}^S (1 - \alpha_{1s2} v_{s2}) \left( \sum_{t=1}^T N_{st} i_{1t} - \delta_{1t1} w_{st1} \right) - \beta_{1s1} v_{s1} \\
I'_2(\{w_{st1}\}, \{v_{s1}\}, \{v_{s2}\}) &= \sum_{s=1}^S (1 - \alpha_{2s2} v_{s2}) \left( \sum_{t=1}^T N_{st} i_{2t} - \delta_{2t1} w_{st1} \right) \\
I'_3(\{w_{st1}\}, \{v_{s1}\}, \{v_{s2}\}) &= \sum_{s=1}^S \sum_{t=1}^T N_{st} i_{3t} - \delta_{3t1} w_{st1}
\end{aligned} \tag{26}$$

Finally, the indicator related to the previously defined implementation criterion  $q = 1$ , i.e., the Retrofit Actions Allocation Index, is defined as the variance between the level of intervention on subsystems and a constant target level of intervention. The level of intervention on subsystems is defined as the ratio of the number of actual planned actions and the number actions that can be potentially implemented, weighted by the actions costs. For the sake of simplicity, the level of intervention on subsystems that we assume is a constant target level refers to the case when no actions are planned on all subsystems (i.e., non-intervention). This indicator must be maximized in order to concentrate the distribution of actions in few subsystems.

The formulation is the following:

$$J'_1(\{w_{st1}\}, \{v_{s1}\}, \{v_{s2}\}) = \frac{1}{S} \sum_{s=1}^S \left( \frac{\sum_{t=1}^T (w_{st1} b_{t1}) + v_{s1} c_1 + v_{s2} c_2}{\sum_{t=1}^T (W_{st1} b_{t1}) + V_{s1} c_1 + V_{s1} c_2} - 0 \right)^2. \tag{27}$$

### 3.1.2 Decision making

Having defined the performance and implementation indicators in (26) and in (27), respectively, the MOO problem may be defined as follows:

$$\begin{aligned}
&\min_{\{w_{st1}\}, \{v_{sk}\}} ( I'_1(\{w_{st1}\}, \{v_{s1}\}, \{v_{s2}\}), I'_2(\{w_{st1}\}, \{v_{s1}\}, \{v_{s2}\}), \\
&\quad -I'_3(\{w_{st1}\}, \{v_{s1}\}, \{v_{s2}\}), -J'_1(\{w_{st1}\}, \{v_{s1}\}, \{v_{s2}\}) ) \\
&\text{s.t. (23), (24), (25), and} \\
&\quad \sum_{s=1}^S \left( \sum_{t=1}^T b_{t1} w_{st1} + c_1 v_{s1} + c_2 v_{s2} \right) \leq E.
\end{aligned} \tag{28}$$

We note that  $J'_1$  is a quadratic function,  $I'_3$  is a linear function, and  $I'_1$  and  $I'_2$  are bilinear functions. Consequently, (28) is a quadratic integer MOO problem [46]. Note that, in accordance with the scenario previously described in the section, the MOO problem presents 4 objective functions, 30 integer variables and 10 binary variables, 60 bounding constraints and 1 inequality constraint.

The Pareto optimal solutions obtained solving (28) may be subsequently ranked. To show the flexibility of the proposed technique, we consider two alternative rankings, with the DM using two different sets of ranking criteria according to the following cases:

- case A: the ranking is based on  $M = 4$  criteria that are exactly coincident with the criteria adopted in the multi-objective optimization, i.e. 1) energy consumption ( $I'_1$ ), 2) upright luminous flux ( $I'_2$ ), 3) color rendering index ( $I'_3$ ), and 4) allocation index of retrofit actions ( $J'_1$ ).
- case B: the ranking is based on  $M = 3$  criteria that ignore the implementation criterion ( $J'_1$ ) and include only the performance criteria ( $I'_1$ ,  $I'_2$ , and  $I'_3$ ).

In both cases, the DM assigns the same importance to the ranking criteria, i.e., values of weights assigned to the ranking criteria are equal to  $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta_4 = 1/4$  for case (a) and  $\vartheta_1 = \vartheta_2 = \vartheta_3 = 1/3$  for case (b).

### 3.2 Solution approach and model implementation

The proposed decision-making tool is implemented in the MATLAB environment making use of a graphical user interface (GUI) to allow the interaction of the DM. The GUI asks DM to fill in the decision design parameters (e.g., total budget, decision criteria, number of retrofit actions, costs, payoffs, application potentials, etc.) as well as to define the set of ranking criteria and their associated importance. Hence, the ranking and optimization results of the proposed decision-making tool are displayed in the GUI and stored in a spreadsheet file. Finally, the GUI enables the run of the core part of decision-making tool, consisting in two sequentially-called algorithms –both of them implemented in the MATLAB environment- for computing the Pareto optimal solutions of (28) and determining the ranking results (22).

As for the ranking of the Pareto optimal solutions, the MATLAB implementation straightforwardly follows the TOPSIS steps described in subsection 2.3.2. As for the MOO problem (28), we describe the resolution approach in the sequel. In fact, since (28) is a multi-objective integer quadratic knapsack optimization, it may be solved by means of several techniques [47]. Classical solution approaches to multi-objective optimization problems such as (28) can be characterized by high time complexity when they are used for finding multiple solutions, since they have to be applied many times to determine the Pareto frontier. Since our problem is to be solved off-line, the time complexity and computing efficiency are not real issues in this kind of evaluation. Hence, we use a classical approach. In particular, we choose a simple augmented  $\varepsilon$ -constraint (SAUGMECON) method [48], a variant of the well-known  $\varepsilon$ -constraint method that can be properly used to produce the complete Pareto set of multi-objective integer programming problems. With the SAUGMECON method, all the nondominated solutions can be efficiently found. Differently from traditional  $\varepsilon$ -constraint method, no weakly Pareto optimal solutions are generated. Furthermore, thanks to several

innovative acceleration mechanisms, the SAUGMECON speeds up the whole process by avoiding redundant iterations [48]. The MOO problem is thus implemented in the MATLAB environment with the Optimization Toolbox and using the SCIP (Solving Constraint Integer Programs) solver [49], supplied with the OPTI Toolbox [50].

### 3.3 Numerical analysis and discussion

As a first outcome, Table VII reports the utopia points related to the MOO problem (28). These values - obtained optimizing single objectives individually and independently from the others - concisely demonstrate the competitiveness of the decision criteria and the effectiveness of the proposed approach in providing the decision maker with a set of alternative solutions that present an optimal trade-off between the various competing criteria.

In the considered case study, the Pareto frontier contains more than 1,000 optimal solutions. For the sake of brevity, we report the results about the objective functions of the top ten ranked solutions. In particular, for the top ten ranked solutions, Figs. 3a and 3b illustrate the indicators improvement (measured in terms of absolute difference between ex-ante and ex-post indicators value) normalized with respect to utopia points in case A and B, respectively. It is apparent that a higher improvement of a criterion typically corresponds to a lower improvement achieved by the other ones. Comparing action plans in Figs. 3a and 3b, we highlight that the Retrofit Actions Allocation Index is clearly higher in case A (where the retrofit actions distribution criteria is taken into account) than in case B (where this criteria is neglected). Consequently, this implies that the optimal solutions are composed by actions whose distribution is concentrated in fewer subsystems in case A than in case B. To show this, in Figs. 4a and 4b we illustrate in detail the allocation of the planned retrofit actions (as a ratio between the number of planned actions and the number of actions that can be potentially implemented, for all the four retrofitting action types) on the subsystems in the top three ranked solutions, in case A and B, respectively. Moreover, from Figs. 4a and 4b we note that both in cases A and B the most frequent optimal action plan concerns the dimming installation, approximately for all the subsystems, while the luminaries replacement and the energy harvesting module installation are applied in fewer subsystems. Furthermore, in no action plan of case B any subsystem is completely refurbished; on the contrary, in all the shown solutions of case A a couple of subsystems are about totally retrofitted. This evident remark is present also in the subsequently ranked solutions but these further results are omitted for the sake of brevity.

Table VII Utopia points

		Ex-post lighting system energy consumption [kWh/year]	Ex-post Lighting system uplight luminous flux [lm]	Ex-post Lighting system color rendering index [-]	Retrofit actions allocation index [-]
Types of solution	min $I'_1$	$I'_1^*=193,330$	2,447	12,190	0.0358
	min $I'_2$	222,930	$I'_2^*=2,167$	13,940	0.0628
	max $I'_3$	288,240	2,780	$I'_3^*=14,090$	0.0510
	max $J'_1$	277,440	2,874	13,260	$J'_1^*=0.2677$

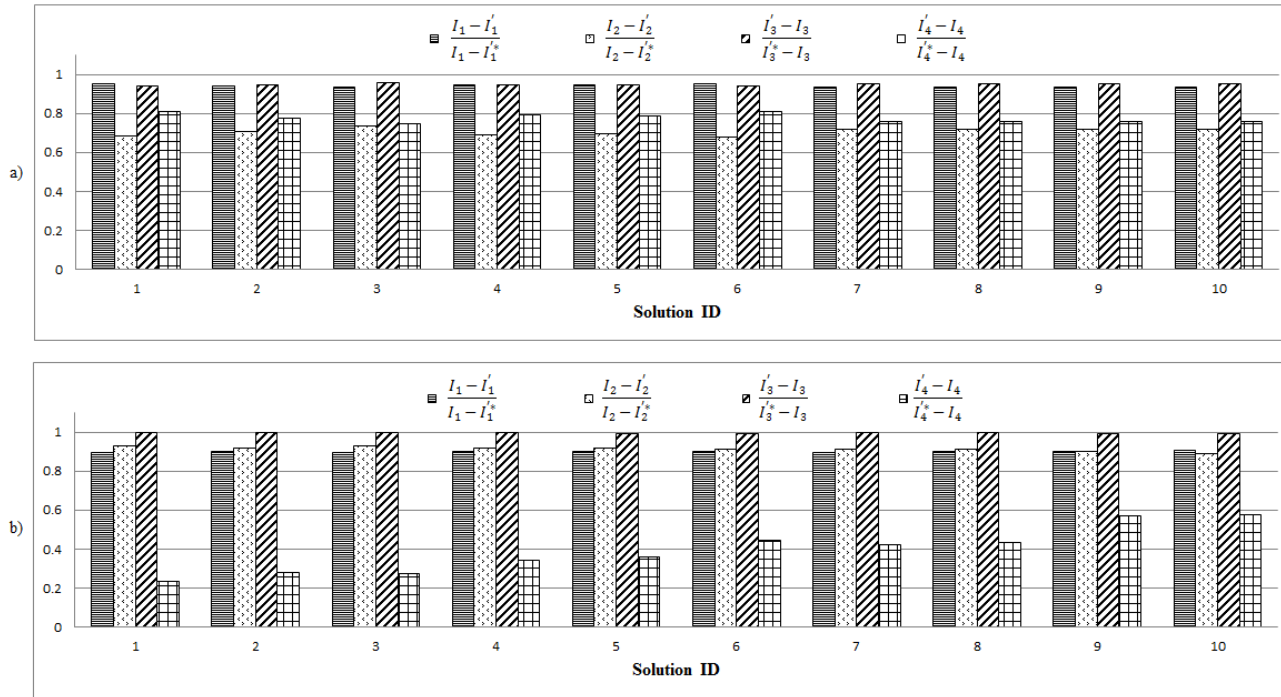


Figure 3 – Top ten ranked Pareto optimal solutions - Indicators value improvement normalized with respect to utopia points: a) TOPSIS based on both performance and implementation criteria (case A); b) TOPSIS based only on performance criteria (case B).

As a side finding about the overall two-step decision making tool, we observe that in all presented simulations the total run time to determine the Pareto optimal solutions set and rank them is around 1 hour, on a PC equipped with a 2.4 GHz Intel Core 2 Duo CPU and 4 GB RAM.

Finally, we wish to highlight that the proposed tool is effective in concurrently obtaining the optimization of the defined criteria and the optimal allocation of resources among the street lighting subsystems. To demonstrate this, a further analysis of the case study is conducted. As a reference scenario, a simplistic strategy for determining actions plan is considered:

- case C: sorting the subsystems on the basis of the number of actions that can be potentially implemented from the smallest to the largest one, planning from the highest to the lowest cost actions in the smallest subsystem and going to the subsequent larger ones until all actions are covered by the budget.

Table VIII reports the values of indicators under this simplistic strategy (third column) and the best ranked solution obtained in case A and B (first and second column, respectively).

As expected, since the simplistic strategy basically aims at completely retrofitting the smallest subsystems, in case C the Retrofit Actions Allocation Index reaches a high value (tending towards the utopia point). At the same time, the results' comparison demonstrates that the indicators related to the performance criteria in case A and B are better than those achieved by the simplistic strategy. Obviously, a similar study may be conducted for any other a-priori retrofit action planning among the subsystems and for any other choice of the criteria and actions.

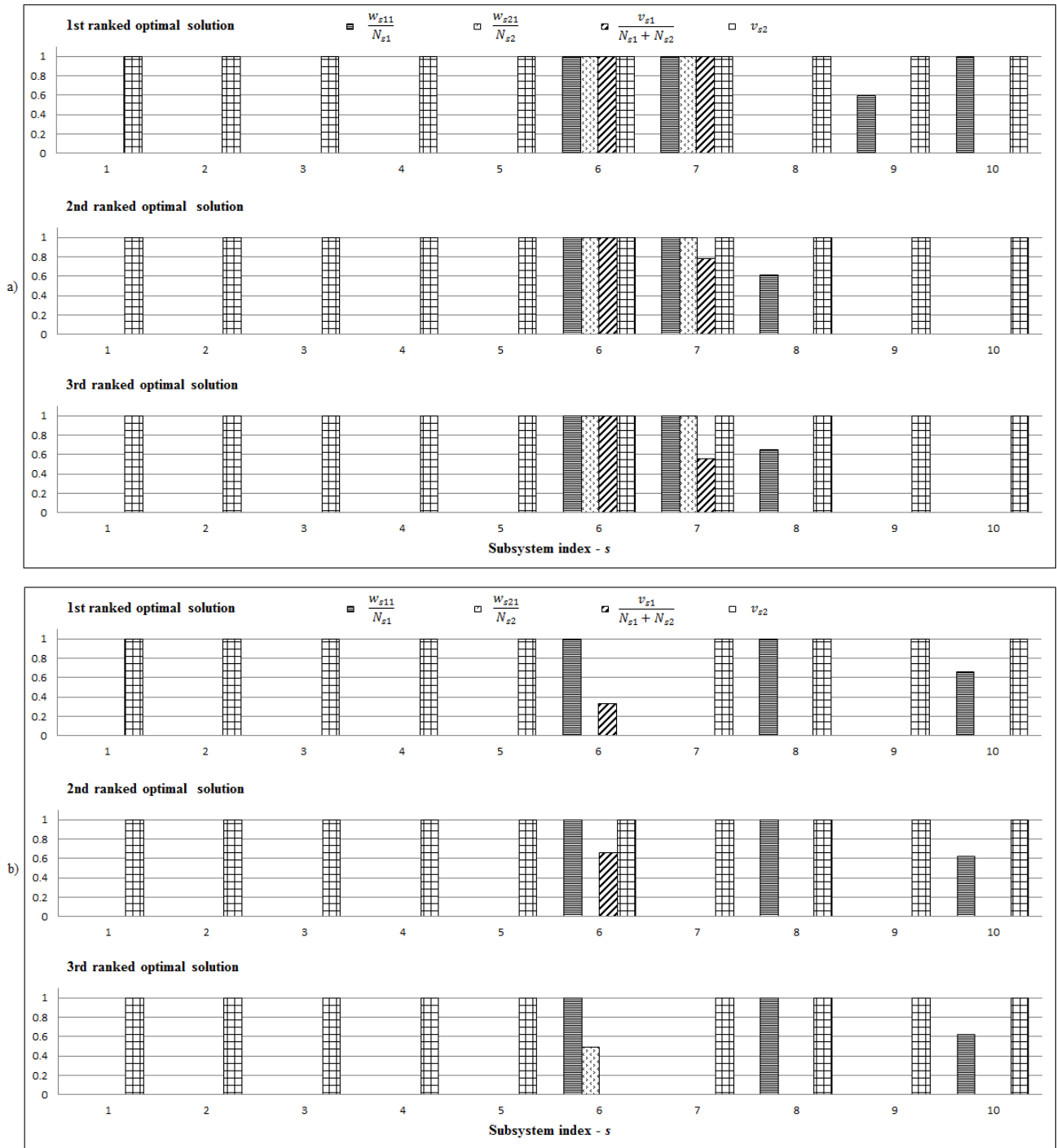


Figure 4 - Top three ranked Pareto optimal solutions - Distribution of planned retrofit actions among subsystems: a) TOPSIS based on both performance and implementation criteria (case A); b) TOPSIS based only on performance criteria (case B).

Table VIII Ex-post values of performance indicators in case A, B, and C

Description	Parameter	Unit	Value		
			Case A	Case B	Case C
Ex-post Lighting system energy consumption	$I_1'$	[kWh/yr]	217,682	223,002	270,435
Ex-post Lighting system uplift luminous flux	$I_2'$	[lm]	2,223	2,168	2,868
Ex-post Lighting system color rendering index	$I_3'$	-	13,490	13,965	13,120
Retrofit actions allocation index	$J_1'$	-	0.2176	0.0629	0,2636

## 4. Conclusions

This paper develops a multi-criteria decision making tool supporting the public decision maker in optimally selecting a set of retrofit interventions to be taken in an integrated way on an existing street lighting system of a wide urban area in order to reduce energy consumption, maintain the required comfort and quality of life, protect the environment, and simultaneously optimize the distribution of actions in subsystems, while ensuring an efficient use of public funds.

The contribution of the research is twofold. From a theoretical point of view, it contributes to the literature on energy-efficient street lighting management, which lacks multi-objective studies for identifying the optimal retrofit strategy comprising a mix of interventions on existing street lighting systems. In this sense, the model, defining and solving a multi-criteria optimization problem, may be applied to different street lighting contexts, namely to contexts characterized by diverse objectives, constraints, and performance as well as implementation criteria. It is able to easily include new and competing objectives, which can descend from new strategies formulated by the energy managers. From a practical point of view, the tool, addressing a multi-criteria optimization problem, provides the decision maker with an effective tool for screening optimal solutions.

The main limitation of the presented approach is related to the assessment of the impact of each action on the selected criteria, whose outcome may be not deterministic as we assume in this paper. In order to overcome this limitation, future research will be devoted to modeling uncertainties that affect the estimation of optimization model parameters.

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