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# A Decision Support Approach for Postal Delivery and Waste Collection Services

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**Abstract**—This paper presents an Urban-Decision Support System (U-DSS) devoted to manage in a unified framework the logistic services of the smart cities, such as postal delivery and waste collection services. The U-DSS architecture is proposed by describing its main components. In particular, the paper focuses on the core of the U-DSS, i.e., the model component that provides the solutions of a general vehicle assignment and routing optimization problem with the aim of minimizing the length of the routes and satisfying time and capacity constraints. In order to solve the vehicle routing problems in reasonable time, a two-phase heuristic algorithm is proposed based on a clustering strategy and a farthest insertion heuristic for the solution of a Travelling Salesman Problem. The applicability of the proposed U-DSS is enlightened by comparing the proposed heuristic algorithm solutions with the Mixed Integer Linear Programming problem solutions of the postal delivery and waste collection services. Moreover, the discussion of the real case studies of the city of Bari (Italy) assesses the proposed approach.

**Note to Practitioners**—The motivation of this work is to design an Urban-Decision Support System (U-DSS) for use by decision makers in the offline optimal planning of the vehicle assignment and routing in different services, such as postal delivery and waste collection. The U-DSS provides solutions to the decision makers with threefold important key features. First, the modelled and solved vehicle routing problem is general and can take into account several landfills, set of vehicles with different capacities and speeds, several shifts and routes satisfying capacity and time constraints. Second, the travelling times are obtained by the routes and the distances determined by Google Map Application Programming Interface, the traffic and weather forecasts. Third, the solutions are obtained in extremely short times, even if the dimensions of the problem are very large.

Future research aims at extending the U-DSS by considering dynamic routing and modifying in real time the planning of the routes with a suitable use of the modern Information and Communication Technologies.

**Index Terms**—Logistics, Decision Support System, Vehicle Routing Problem, Heuristic Algorithms.

## I. INTRODUCTION

ONE emerging and challenging issue in Smart Cities is the city logistics. Every urban dedicated management system, such as traffic management, waste collection and postal

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delivery, is very complex and socially and environmentally sensitive: it requires dedicated and intelligent support from the decision making process point of view. The increasing availability of Information and Communication Technologies (ICTs) allows the development of models and novel decision making strategies to optimize the information process and utilization for the Decision Makers (DMs).

Among the most critical services to be managed in the smart cities, the Postal Delivery (PD) and Waste Collection (WC) services are considered core problems in the city logistics [1]. They are often modelled as Vehicle Routing Problems (VRP) that are largely addressed in the related literature (see for instance among the others the review papers [1], [2], [3] and [4]).

In particular, the management of the PD system involves locations, network design, vehicle routing and crew assignment [5]. Multi-depot vehicle routing in city VRP is considered by [6] and [7] that study a vehicle routing and crew scheduling problem for the Australian post. However, the authors deal with just some case studies by using a heuristic algorithm. Paper [8] proposes a hybrid method for route generation based on the automatic developing of the delivery scheduling system. The method combines experts know-how and various optimization algorithms such as Genetic Algorithms. In addition, the authors in [9] address the VRP in the mail delivery context: they propose a hybrid algorithm that combines the Ant Colony Optimization meta-heuristics with two local optimization heuristics. Paper [10] considers the problem of borough postal transportation and propose a lexicographic order linear programming model to minimize transportation costs based on the minimization of fixed costs: an optimal solution is derived using a genetic algorithm. In a successive work [5], the authors construct a postal express mail network that can realize the fastest transfer of mails with the lowest cost: the problem is solved by a swarm intelligent scheme. In [11] a modular decision support framework is proposed in order to support fleet and logistics managers in the reporting of routing decision, based on shortest path criterion. However, the authors focus on a scenario made of a single vehicle multi-stops routing problem.

The WC Routing Problem (WCRP) is one of the critical management problems to be solved in order to obtain an efficient and quick WC. It concerns the optimal design of routes to be performed by a fleet of vehicles with limited capacities to serve a set of customers. In the field of the WCRP different approaches and strategies are presented in order to deal with the complexity and the large dimensions of the real systems (see for instance paper [1] for a complete and discussed review): i)

genetic algorithms [12]; ii) heuristic algorithms based on arc or node routing problems [13], iii) swarm intelligence methods [14] [15] and ants heuristics [16]; iv) simulation approaches [17]. In particular, paper [12] develops methodologies based on genetic algorithms to find the best route for collecting solid waste in cities. In addition, the authors in [13] deal with a capacitated arc routing problem and consider several routes assigned to each truck. Paper [16] proposes an ant colony heuristic, based on node routing problem, to determine the solution of an urban WC problem in the municipality of Sant Boi de Llobregat, within the metropolitan area of Barcelona (Spain). However the authors assume that each route starts and ends at the depot and no time constraints are considered. On the other hand, paper [17] presents an innovative modeling framework based on simulation to validate the solution of a WC problem. In addition, paper [18] proposes an ant colony optimization algorithm that takes in account time window constraints and several landfills: the authors analyze a WCRP considering a set of bins to be served, a number of facilities where the collected waste must be disposed off and an infinite number of vehicles stationed in a single depot. Furthermore, a set of contributions considers time windows constraints [14], [18], [15] [19] and a more complex problem complicated by several time windows and inter-arrival time constraints at each customer point is addressed in [20].

Besides the large number of contributions about the VRP problems presented in the related literature, few authors propose a decision support approach able to deal with various types of urban logistics problems. An intelligent Decision Support System (DSS) is described in [21] to support the management of urban infrastructures, such as waterworks and sewage. The system defines the requirements and functionalities developed to improve the delivery, performance and coordination of municipal services. Paper [13] proposes a DSS with several components including one for prediction of waste amounts and one for arc routing of trucks. The article [22] presents a user-friendly web-based spatial DSS aimed at generating optimized vehicle routes for multiple vehicle routing problems that involve serving the demand located along arcs of a transportation network. The DSS incorporates Google Maps, a database, a heuristic and an ant-colony meta-heuristic developed to generate routes and detailed individual vehicle route maps. However, the proposed DSS is devoted only to the WC and optimizes separately each town district and does not face the overall city system.

Hence, the development of efficient solution methods for city logistics that can include traffic data, fast response and ITS tools is an open and huge problem.

This paper aims at providing contributions in this context and presents an Urban-DSS (U-DSS), devoted to manage in a unified framework the PD and WC services. The U-DSS architecture is proposed by describing its main components: the data component, the interface component and the model component.

In particular, the proposed U-DSS provides the optimal routes for a general Vehicle Assignment and Routing (VAR) problem to minimize the length of the routes and satisfy time and capacity constraints. It is well known that these kinds of

problems are NP-hard and can be solved in reasonable time only if the dimension of the system is quite limited. Hence, we solve the considered VAR problems by a two-phase heuristic algorithm: the first phase is based on a clustering strategy and the second phase is based on a farthest insertion heuristic for the solution of the Travelling Salesman Problem (TSP).

The proposed general algorithm is specified for the PD and WC services. Moreover, in this paper we present the Mixed Integer Linear Programming (MILP) models of the two services: comparing the algorithm solutions with the optimal solutions provided by the MILP, we show that the heuristics gives good solutions and can be used to solve large and complex problems in short time.

In order to show the applicability of the proposed U-DSS, we consider the PD and the WC services of the city of Bari (Italy).

The paper is organized as follows. Section II introduces the U-DSS architecture and Section III describes in detail the VAR module of the model component by proposing a general heuristic algorithm. Moreover, Sections IV and V present two instances and customized applications of the VAR module for the PD and WC services, respectively: two case studies enlighten the U-DSS applicability. Finally, Section VI discusses the concluding remarks.

## II. URBAN-DECISION SUPPORT SYSTEM ARCHITECTURE

In this section the U-DSS architecture and its main components are briefly described. The U-DSS is developed following user centered [23] and model based [24] approaches: the first activity consists on the identification of the user needs and requirements through a series of interviews; then the second activity consists on building the U-DSS modules giving emphasis on mathematical models and optimization techniques.

The structure of the proposed U-DSS consists of three main components: the Data Component (DC), the Model Component (MC) and the Interface Component (IC). Fig. 1 shows a simple scheme of the U-DSS architecture by enlightening the connections among them.

The DC collects the data and the information necessary to the U-DSS services: historical data and real time data. In the cases of the VAR problems, the historical data base collects the sites to be served, the demand of each site, the number and the dimension of the vehicles, the time length of the shift. The real time data are provided by the IC: real time information are received by the modern ICT tools about the traffic, the whether conditions, peaks of the demand and unusual situations.

Combining historical data with real-time data makes it possible to refine inputs for next decisions or select inputs that better fit with a particular scenario. Real-time information, for instance travel times along routes detected by GPS, can be stored in the DC in order to update the historical data and forecast more suitable system parameters for different scenarios.

On the other hand, the MC is the core of the U-DSS and contains the models, algorithms and rules needed to provide decision support for the users. In particular, in the U-DSS the MC is composed by a module dedicated to solve the

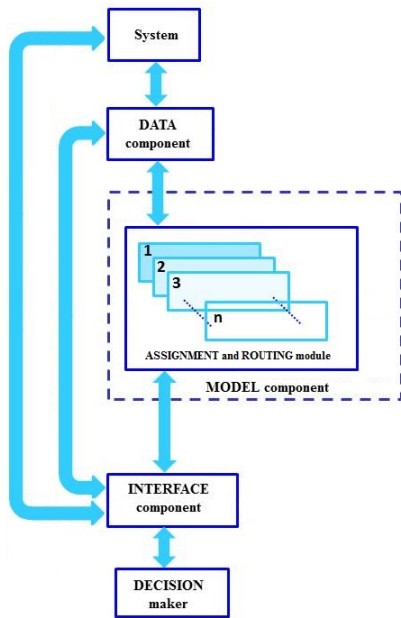


Fig. 1. The scheme of the U-DSS architecture.

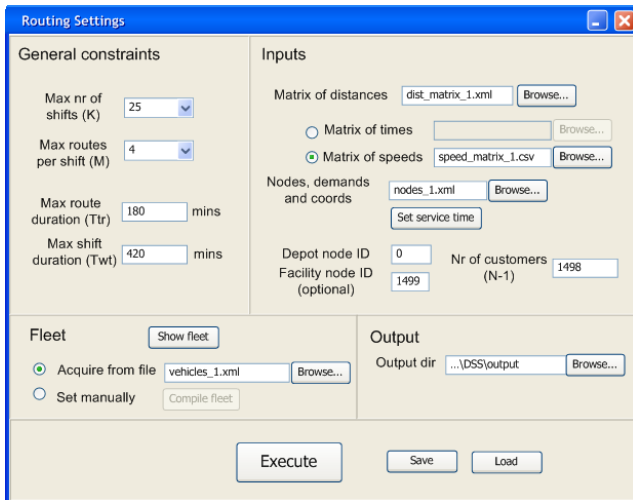


Fig. 2. IC input display.

VAR problems. In the next section, the module is described in detail by proposing a general heuristic algorithm that can be customized to be applied to different services based on vehicle routing problems.

Finally, the IC allows the effective interaction of the user with the real system: it is responsible of the communication and interaction among the MC, the DC, the real-world and the users. The IC main tasks are: i) setting the information needed for the MC to operate and determine the optimal solution; ii) displaying the solutions provided by the MC; iii) connecting the U-DSS with the real-time data sources. For instance, Fig. 2 shows the panels that the user has to fill in order to provide the necessary information and Fig. 3 shows some possible U-DSS text and graphic outputs of the IC.

For application in real-time context, U-DSS receives unpre-

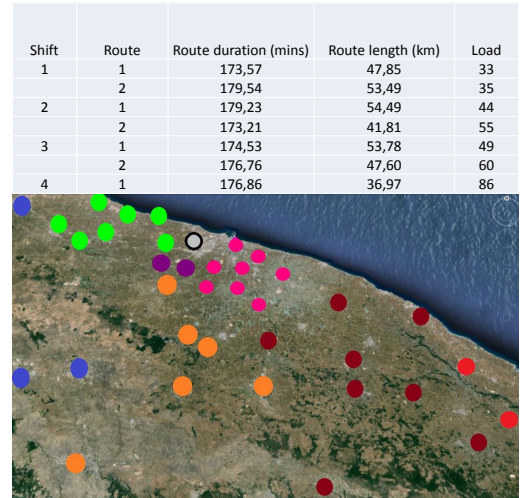


Fig. 3. IC solution display.

dictable events such as street interruptions, accidents, blocking, etc., and the input data of the MC can be modified accordingly. Two cases may occur: i) the event happens before the daily service, hence a new solution of the complete VAR problem is determined; ii) the event happens when some shifts have already started, then the system description is modified in order to set up the new scenario by considering only the not yet travelled routes.

### III. VEHICLE ASSIGNMENT AND ROUTING MODULE

In this section the MC of the proposed U-DSS is described: it considers the general problem of optimizing the routes of a set of vehicles that have to visit a set of customers by minimizing the total travelled distance, under capacity and time constraints. Our effort is focused on presenting a general model that can provide solutions for different urban logistics problems: suitable instances and customized applications of the proposed module can be realized in the MC.

#### A. Vehicle Assignment and Routing Problem Description

Let consider a set of vehicles  $U$  that operate collection or delivery activities subject to time and capacity constraints. During a work shift, each vehicle can follow several routes that may be limited by a maximum of  $M$  routes per shift. In order to associate a shift to a vehicle, we simply denote by  $k$  the shift performed by vehicle  $u_k \in U$  of capacity  $C^k$ . In addition, each route starts from a depot, follows the assigned path and, when the vehicle attains capacity or time limits, it goes to an intermediate or final destination that can coincide with the depot. In this site the workers can perform an operation and/or make a stop before starting a new route. Each route time has to be less than or equal to  $T_r$  time units and the shift, composed of a set of routes performed by the same vehicle, is limited by the working time of  $T_w$  time units.

Let  $G = (V, A)$  be a directed fully connected graph, where  $V = \{v_0, v_1, \dots, v_N\}$  is the set of nodes and  $A$  is the set of arcs. In particular, node  $v_0$  is the depot and node  $v_N$  represents

a facility, that can be a node with specific role such as a landfill in waste collection system or the depot itself in typical VRPs. Moreover, each node  $v_i$  with  $1 \leq i \leq N - 1$  is a customer characterized by a demand  $q_i > 0$  and a service time  $p_i > 0$ . Then,  $\bar{V} = \{v_1, v_2, \dots, v_{N-1}\}$  denotes the customer set. In addition, each arc  $a_{ij} \in A$  corresponds to the shortest path from  $v_i$  to  $v_j$  of length  $c_{ij}$  and  $t_{ij}^k$  is the time spent by vehicle  $u_k$  to perform the path of length  $c_{ij}$ .

The  $m$ -th route of the  $k$ -th shift is described by the following elements:

- set  $V_m^k \subseteq \bar{V}$  contains the customers visited by in the  $m$ -th route of the  $k$ -th shift;
- function  $\vec{V}_m^k: \{1, 2, \dots, |V_m^k| + 2\} \rightarrow V_m^k \cup \{v_0, v_N\}$ , where  $\vec{V}_m^k(j) = v_i$  means that the  $j$ -th node visited during the  $k$ -th shift in route  $m$  is  $v_i \in V_m^k \cup \{v_0, v_N\}$ , for  $j = 1, \dots, |V_m^k| + 2$ . Note that  $\vec{V}_m^k(2) \in \bar{V}$  denotes the first node of route  $m$  visited after  $v_0$  or  $v_N$  and  $\vec{V}_m^k(1) \in \{v_0, v_N\}$ ,  $\vec{V}_m^k(|V_m^k| + 2) \in \{v_0, v_N\}$  are the first and the last nodes of the route, respectively.

Symbol  $|\cdot|$  denotes the cardinality of set  $(\cdot)$ .

Moreover, we denote  $\bar{m} \geq 2$  the last route in the shift and  $\bar{j}$  the total number of nodes visited in the  $m$ -th route (i.e.,  $\bar{j} = |V_m^k| + 2$ ).

Now, the overall duration of the route described by  $\vec{V}_m^k$  is computed as follows:

$$B_m^k = \sum_{j=1}^{\bar{j}-1} (p_{\vec{V}_m^k(j)} + t_{\vec{V}_m^k(j), \vec{V}_m^k(j+1)}^k). \quad (1)$$

We remark that  $B_m^k$  includes the overall travelling times and the service times of the  $m$ -th route in the  $k$ -th shift.

In addition, we define  $Q_m^k$  the overall load of vehicle  $u_k$  at the end of the  $m$ -th route:

$$Q_m^k = \sum_{j=1}^{\bar{j}} q_{\vec{V}_m^k(j)}. \quad (2)$$

At this point it is possible to define the *feasible routes* as follows:

**Definition 1.** We say that route  $m$  performed by vehicle  $u_k$  is *feasible* if  $Q_m^k \leq C^k$  and  $B_m^k \leq T_r$ .

Moreover, the total duration of the routes in a shift must be less than or equal to  $T_w$ , i.e.:

$$\hat{B}^k = \sum_{i=1}^{\bar{m}} B_i^k + t_{N0}^k \leq T_w. \quad (3)$$

In other words, the duration of the shift takes into account the durations of the shift routes  $B_i^k$  and of the final return to the depot  $t_{N0}^k$ .

The aim of the VAR problem consists of designing a set of feasible routes with minimum total length such that:

- $\vec{V}_m^k(j) = \begin{cases} v_0 & \text{if } m = 1 \text{ and } j = 1 & (4.1) \\ v_0 & \text{if } m = \bar{m} \text{ and } j = \bar{j} & (4.2) \\ v_N & \text{if } 1 \leq m < \bar{m} \text{ and } j = \bar{j} & (4.3) \\ v_N & \text{if } 1 < m \leq \bar{m} \text{ and } j = 1 & (4.4) \\ v_i \in \bar{V}, & \text{otherwise} & (4.5) \end{cases} \quad (4)$

for each  $k$ ;

- $\forall v_i \in \bar{V} \exists$  only one triple of values  $(m, k, j)$  such that  $\vec{V}_m^k(j) = v_i$ , i.e., each node is visited exactly once.
- $\bar{m} \leq M$ , i.e., the number of routes performed by each vehicle is less than or equal to  $M$ .

More clearly, the two conditions (4.1) and (4.2) impose that the first route of the each shift departs from the depot and the last route of each shift returns to the depot, respectively. On the other hand, the two conditions (4.3) and (4.4) constraint each route, but the last one, to end to  $v_N$  and each route, but the first one, to start from  $v_N$ . In all the other cases the  $j$ -th visited node is a customer.

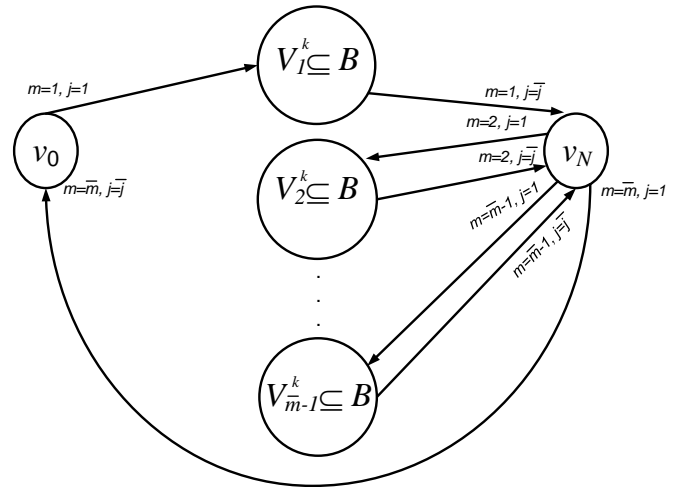


Fig. 4. Routes structure.

Figure 4 depicts the described conditions about the routes structure and the necessary notations are summarized as follows:

- $U$  set of vehicles
- $\bar{V}$  set of customers
- $N - 1$  number of customers
- $K$  maximum number of shifts
- $q_i$  demand of node  $v_i \in V$
- $p_i$  service duration at node  $v_i \in V$
- $C^k$  capacity of vehicle  $u_k \in U$  performing the  $k$ -th shift
- $m$  the route index in the shift
- $c_{ij}$  length of the shortest path from  $v_i$  to  $v_j$
- $t_{ij}^k$  time to go from  $v_i$  to  $v_j$  by vehicle  $u_k$
- $T_r$  maximum route time of each vehicle
- $T_w$  maximum working time of each vehicle

$V_m^k$	set of nodes visited in route $m$ of the $k$ -th shift
$\vec{V}_m^k$	function describing the $m$ -th route of the $k$ -th shift: $\vec{V}_m^k(1), \dots, \vec{V}_m^k(\bar{j})$ gives the sequence in which the nodes of $V_m^k$ are visited
$Q_m^k$	total load of cluster $V_m^k$ defined by (2)
$B_m^k$	duration of route $\vec{V}_m^k$ defined by (1)
$\hat{B}^k$	duration of the $k$ -th shift defined by (3).

### B. Vehicle Assignment and Routing Algorithm

The VAR module of the U-DSS is based on a heuristic algorithm able to solve the problems characterized by the assumptions and conditions described in the previous subsection.

In particular, a two-phase heuristic algorithm is proposed for each shift: the first phase is based on a *clustering strategy* and the second phase is based on a *farthest insertion heuristic* for the solution of the TSP. More precisely, farthest insertion heuristic searches for the not yet inserted node whose minimum distance to all the inserted nodes is minimum. Then the selected node is placed into the tour at the point causing the shortest increase of the total length [25]. The presented strategy, known in the related literature as *cluster first, route second* is successfully applied in the considered cases where the capacity constraints are more stringent than the time constraints.

Let us consider the  $m$ -th route of the  $k$ -th shift. Before the application of the algorithm, the vehicles  $u_k \in U$  are sorted in descending order of their capacity  $C^k$ .

*Phase 1: Clustering strategy.* This phase is devoted to build the node set  $V_m^k$  that collects customers positions to be visited in the  $k$ -th shift in its  $m$ -th route. In this phase the cluster corresponding to  $V_m^k$  is determined by considering only the vehicle capacity constraints: the cluster is built starting from the farthest node from the garage or the fulfil not yet served in the  $k$ -th shift (the seed node). Successively, the nearest not served node to the seed is added to the cluster and so on. Hence, the cluster is completed when no other node can be added because of the capacity limit of the vehicle  $u_k$ .

The basic idea behind the clustering strategy is to consider first the farthest nodes which require a long time to be reached. Indeed, starting from the farthest node in the clusters construction gives a solution in which residual nodes (to be covered by the last clusters) are close among themselves and close to garage, so they can be served with a short route. This intuition is supported by the literature (see for instance [25]). *Phase 2: Routing strategy.* Considering the built cluster  $V_m^k$ , the routing is determined by the *farthest insertion heuristic* [25]. If the constraints about the times  $T_r$  and  $T_w$  are satisfied, then the  $m$ -th route of the shift  $k$  is completed. Otherwise, the last customer position inserted in the cluster is dropped and the TSP is solved again. Such a procedure is iteratively executed until the constraints on  $T_r$  and  $T_w$  are satisfied. Then at this point the cluster is closed. If the constraints on  $T_w$  or  $T_r$  do not permit to create a new cluster (and the related route), then we close the  $k$ -th shift.

Such a procedure is iterated for each shift until all the nodes of  $\bar{V}$  are included in one and only one route.

Now, the heuristic algorithm is presented by specifying the steps of the two phases. The following additional sets are introduced:

$U^U \subseteq U$	set of used vehicles
$U^{NU} \subseteq U$	set of not used vehicles
$V^S \subseteq \bar{V}$	set of served nodes
$V^{NS} \subseteq \bar{V}$	set of not served nodes

#### Phase 1. Clustering strategy

##### Step 0. Initialization.

**Set**  $U^{NU} = U$ ,  $U^U = \emptyset$ ,  $V^S = \emptyset$ ,  $V^{NS} = \bar{V}$ ,  $k = 0$ .

##### Step 1. New shift

**Set**  $k = k + 1$

**If**  $k \leq K$  **then set**  $m = 1$ .

**else go to Step 9**

**End**

##### Step 2. Choosing vehicle

**Select**  $u_k \in U^{NU}$  s.t.  $C^k = \max_{u_i \in U^{NU}} C^i$ .

##### Step 3. Determining the seed of the $m$ -th cluster

Let  $v_s \in V^{NS}$  be the farthest node from the depot node  $v_0$  not yet included in any cluster (not yet served).

**Select**  $v_s \in V^{NS}$  s.t.  $c_{0s}^k = \max_{j: (v_j \in V^{NS})} \{c_{0j}^k\}$

**set**  $V_m^k = \{v_s\}$

##### Step 4. Populating the cluster with the nearest node

Select the nearest not served node  $v_n \in V^{NS}$  to the seed  $v_s$ :

**Select**  $v_n \in V^{NS} \setminus V_m^k$  s.t.  $c_{sn}^k = \min_{j: (v_j \in V^{NS})} \{c_{sj}^k\}$

##### Step 5. Checking capacity constraints

**If**  $q_n + Q_m^k \leq C^k$  **then**

**set**  $V_m^k = V_m^k \cup \{v_n\}$  **and go to Step 4.**

**End if**

*Phase 2. Solve TSP considering the nodes of the cluster and checking time constraints.*

##### Step 6. Solving TSP considering the nodes of the cluster

**Set**  $\bar{j} = |V_m^k| + 2$ ,

**Select**  $\vec{V}_m^k(1)$  and  $\vec{V}_m^k(\bar{j})$  according to equations (4.1) and (4.2).

**Determine**  $\vec{V}_m^k(2), \dots, \vec{V}_m^k(\bar{j}-1)$  and  $B_m^k$  as solution of the *farthest insertion heuristic* described in [25] and applied to the set  $V_m^k$ .

##### Step 7. Checking time constraints.

**Determine**  $\hat{B}^k$

**If**  $\{B_m^k > T_r \vee \hat{B}^k > T_w\}$  **then**

**select**  $v_z \in V_m^k$  s.t.  $v_z = \max_{j: (v_j \in V_m^k)} c_{sj}^k$

**set**  $V_m^k = V_m^k - \{v_z\}$

**go to Step 6.**

**End if**

##### Step 8. Checking shift closure conditions

**Set**  $V^S = V^S \cup V_m^k$

```

set  $V^{NS} = V^{NS} \setminus V_m^k$ 
If  $V^{NS} = \emptyset$  then go to Step 9.
End If
If  $\{V_m^k = \emptyset \vee m \geq M - 1\}$  then
  If  $U^{NU} \neq \emptyset$  then
    call POST_FIT_PROCEDURE
    set  $U^{NU} = U^{NU} \setminus \{u_k\}$ 
    set  $U^U = U^U \cup \{u_k\}$ 
    and go to Step 1
  else go to Step 9
  End if
else set  $m = m + 1$  and go to Step 3.
End if

```

**Step 9. END**

The POST\_FIT\_PROCEDURE looks for the smallest vehicle for the in-closure shift. Note that the procedure can be disabled whether the fleets are homogeneous.

**POST\_FIT\_PROCEDURE:**

```

select  $u_h \in U^{NU}$  s.t.  $C^h = \min\{C^l \mid C^l \geq Q_i^k\}$ 
   $\forall i = 1, \dots, m\}$ 
If  $\{\exists u_h \wedge C^h < C^k\}$  then
  switch  $u_k$  with  $u_h$ 
End if

```

At Step 0 the sets  $U^{NU}$ ,  $U^U$ ,  $V^S$ ,  $V^{NS}$  and  $k$  are initialized. If  $k \leq K$ , then at Step 1 the algorithm considers the first route  $m = 1$  of the new  $k$ -th shift, otherwise the algorithm stops.

At Step 2 the vehicle  $u_k$  with the biggest capacity is assigned to the shift.

Step 3 determines the seed of the cluster, i.e., the customer from which the composition of the cluster starts. The algorithm chooses the farthest customer position from the depot.

At Steps 4 and 5 the cluster is populated with the customer positions nearest to the seed. More precisely, Step 4 chooses the next node  $v_n$  and Step 5 checks the vehicle capacity constraint for the considered  $m$ -th route. If the capacity constraint is satisfied, then the new customer position  $v_n$  is included in the cluster and the procedure goes to Step 4 in order to determine a new position to be included; otherwise the cluster is completed.

When the cluster is formed, the procedure solves the TSP by considering only the cluster nodes and starting and ending according to equations (4.1) and (4.2). Indeed, Step 6 determines the optimal routing  $\vec{V}_m^k$  as a solution of the farthest insertion heuristic.

Step 7 checks both the  $T_r$  and  $T_w$  constraints. If one of the two constraints is not satisfied, then the algorithm removes from the cluster the farthest node from the seed and goes to Step 6 in order to resolve again the TSP.

Step 8 updates the sets  $V^S$  and  $V^{NS}$  and decides if there are the conditions to complete the  $m$ -th route or the shift: if it is not possible to add nodes to the  $m$ -th route for the time constraints then the shift is completed.

After the shift construction, Step 8 checks if the vehicle  $u_k$  is the best choice for the shift. Indeed, if there is a different available vehicle  $u_h$  that can perform the shift with a lower capacity than the  $u_k$  capacity, then the algorithm associates  $u_h$  to the shift instead of  $u_k$ . We remark that it is not necessary to give the limit  $M$ : if  $M$  is very high, than the procedure returns as an output the number  $\bar{m}$  of the routes performed by each vehicle.

For the sake of clarity, Fig. 5 shows the UML activity diagram [26] enlightening the steps of the proposed heuristic algorithm.

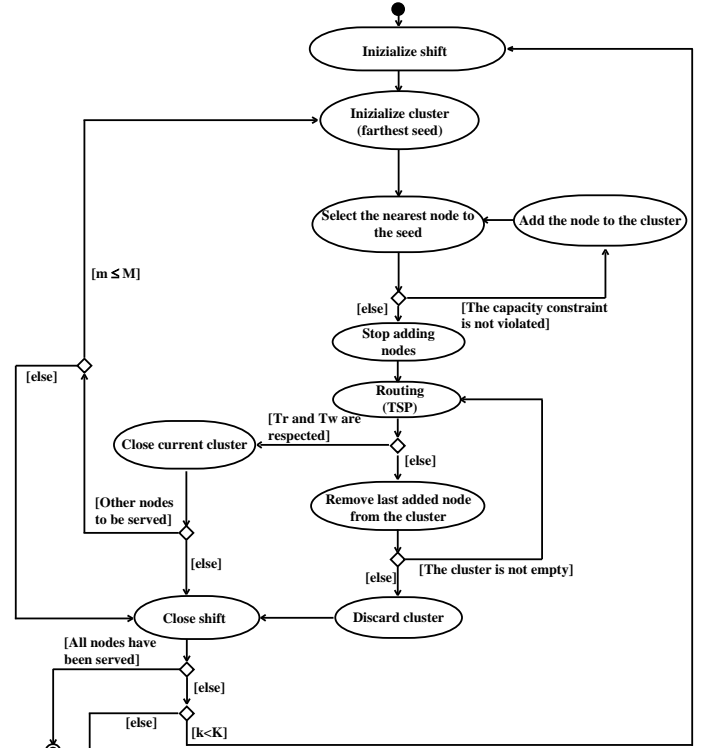


Fig. 5. UML Activity Diagram of the Assignment and Routing Algorithm.

IV. POSTAL DELIVERY PROBLEM

A. Mathematical Formulation

This section describes the configured VAR module of the U-DSS devoted to the PD for the city of Bari (Italy). The main activity of the PD system is the distribution of the postal products from the regional Post Automation Center (PAC) to the borough Postal Delivery Centers (PDCs).

The postal network is modelled by a fully connected directed graph  $G$  where the nodes are the PDCs and the depot is the PAC. The delivery service is performed by a single route for each shift ( $M = 1$ ) in which every vehicle starts from and ends to the PAC (i.e.,  $v_N = v_0$ ).

Now, the mathematical formulation to solve the VAR is presented in order to assess the proposed heuristic algorithm. The choice of the variables and the constraint structure is inspired by [27].

Decision variables:

$$x_{ij}^k = \begin{cases} 1, & \text{if vehicle } u_k \text{ visits in sequence nodes } v_i \text{ and } v_j \\ 0, & \text{otherwise.} \end{cases}$$

Dependent variables:

$$\begin{aligned} Q_i^k & \text{ total load of vehicle } u_k \text{ when it leaves node } v_i \in V; \\ S_i^k & \text{ starting time of the mail delivery of vehicle } u_k \text{ at} \\ & \text{node } v_i \in \bar{V}; \\ S_N^k & \text{ working time of vehicle } u_k. \end{aligned}$$

The PD problem can be formulated as the following Mixed Integer Linear Programming (MILP) problem.

$$\min \sum_{k=1}^K \sum_{i,j:a_{ij} \in A} c_{ij} x_{ij}^k \quad (5)$$

subject to:

$$\sum_{k=1}^K \sum_{j:a_{ij} \in A} x_{ij}^k = 1 \quad \forall v_i \in \bar{V} \quad (6)$$

$$\sum_{j:a_{0j} \in A} x_{0j}^k = 1 \quad \forall u_k \in U \quad (7)$$

$$\sum_{j:a_{iN} \in A} x_{iN}^k = 1 \quad \forall u_k \in U \quad (8)$$

$$\sum_{i:a_{i0} \in A} x_{i0}^k = 0 \quad \forall u_k \in U \quad (9)$$

$$\sum_{i:a_{ij} \in A} x_{ij}^k - \sum_{j:a_{ji} \in A} x_{ji}^k = 0 \quad \forall u_k \in U, \forall v_j \in \bar{V} \quad (10)$$

$$x_{ij}^k = 1 \Rightarrow Q_j^k = Q_i^k + q_j \quad \forall u_k \in U, \forall a_{i,j} \in A \quad (11)$$

$$x_{ij}^k = 1 \Rightarrow S_j^k \geq S_i^k + p_i + t_{ij}^k \quad \forall u_k \in U, \forall a_{i,j} \in A \quad (12)$$

$$0 \leq S_N^k \leq T_w \quad \forall u_k \in U \quad (13)$$

$$0 \leq Q_i^k \leq C^k \quad \forall u_k \in U \quad \forall v_i \in V \quad (14)$$

$$x_{ij}^k \in \{0, 1\} \quad (15)$$

The objective function (5) minimizes the length of the routes performed by all the vehicles in each work shift. Constraints (6) impose that each  $v_i \in \bar{V}$  is served exactly once. The constraints (7)-(8) guarantee that every vehicle starts the route from the depot and ends it at the node  $v_N = v_0$ . Constraints (9) avoid that vehicles come back to  $v_0$  after visiting  $v_i \in \bar{V}$  and (10) ensure the flow conservation. Constraints (11) and (12) update the vehicle capacity and determine the arrival time of vehicle at each  $v_i \in V$ , respectively. Finally, (13) and (14) impose time and capacity constraints, respectively.

The outputs of the formulation are the routes assigned to the vehicles and the number of the vehicles necessary to perform the PD.

## B. Validation and Results

The performances of the MILP formulation and the heuristic algorithm are compared and tested by large sets of randomly generated instances. In this section we discuss six sets of them with five instances for each set. The characteristics of the data set are described in Table I, where the values of  $N$  and  $K$  are chosen so that the MILP formulations can be solved in reasonable time.

The instances are generated by determining at random the values of  $q_i$  and  $c_{ij}$  according to a uniform distribution into the intervals given in Table I. In addition, we consider a fleet composed by vehicles respectively of capacities 20, 15 and 10 Handling Units (HUs), which is a capacity unit represented by the yellow box normally used for the mail delivery in Italy. In particular, the fourth, fifth and sixth column of Table I indicate the name of the vehicles associated to the corresponding capacities.

In order to determine the travel times  $t_{ij}^k$ , we assume an average speed of 50 km/h. Moreover, the service time  $p_i$  associated with  $v_i \in B$  is 15 minutes and the vehicle working time is  $T_w = 360$  minutes.

The MILP problem and the heuristic algorithm are implemented in Matlab environment on a PC equipped by a 3.40 Ghz Intel i7-3770 with 8 GB of memory, in single-thread mode. In particular, the MILP problem is solved by using the Gnu Linear Programming Kit [28].

TABLE I  
CHARACTERISTICS OF DATA SETS FOR MAIL SERVICE

Sets	N [positions]	K [shifts]	20 [HU]	15 [HU]	10 [HU]	$q_i$ [bins]	$c_{ij}$ [km]
1	7	4	$u_1$	$u_2, u_3$	$u_4$	1-5	60-70
2	7	4	$u_1$	$u_2, u_3$	$u_4$	2-7	65-80
3	7	4	$u_1$	$u_2, u_3$	$u_4$	3-9	70-100
4	8	4	$u_1$	$u_2, u_3$	$u_4$	1-5	60-70
5	8	4	$u_1$	$u_2, u_3$	$u_4$	2-7	65-80
6	8	4	$u_1$	$u_2, u_3$	$u_4$	3-9	70-100

Table II summarizes the results that are obtained by allowing a maximum of 1000 seconds of CPU time for the solution of each instance: the objective function values of the MILP formulation (OPT), the objective function values of the heuristic algorithm (UB) and the CPU times to obtain them (CPU-OPT and CPU-UB, respectively).

Moreover, the last column of Table II reports the optimality gap (named *Gap*) expressed as follows:

$$Gap = (UB - OPT)/OPT * 100. \quad (16)$$

In particular, for the instances where the CPU time is equal to 1000 seconds, the optimal solution is not reached and the value of the best feasible obtained solution is reported. The results of Table II show that the value of the optimality gap averaged on the 30 instances is equal to 1.79 %, with a



TABLE II  
COMPARISON BETWEEN MILP PROBLEM AND HEURISTIC ALGORITHM

Inst.	OPT	UB	CPU-OPT	CPU-UB	Gap
1.1	620.44 km	637.50 km	561 s	13 ms	2.75 %
1.2	621.61 km	636.01 km	332 s	11 ms	2.32 %
1.3	636.54 km	641.10 km	464 s	14 ms	0.72 %
1.4	632.14 km	632.14 km	527 s	12 ms	0.00 %
1.5	637.89 km	643.76 km	180 s	12 ms	0.92 %
2.1	775.75 km	778.48 km	340 s	14 ms	0.35 %
2.2	778.10 km	790.50 km	75 s	14 ms	1.59 %
2.3	762.09 km	772.69 km	480 s	15 ms	1.39 %
2.4	779.70 km	779.70 km	110 s	13 ms	0.00 %
2.5	775.59 km	776.94 km	68 s	14 ms	0.17 %
3.1	888.86 km	931.50 km	38 s	13 ms	4.80 %
3.2	861.79 km	873.17 km	23 s	11 ms	1.32 %
3.3	891.83 km	932.36 km	38 s	14 ms	4.54 %
3.4	854.98 km	862.17 km	62 s	13 ms	0.84%
3.5	853.14 km	869.85 km	66 s	14 ms	1.96%
4.1	695.26 km	708.22 km	1000 s	16 ms	1.86 %
4.2	696.62 km	699.42 km	1000 s	15 ms	0.40 %
4.3	686.71 km	693.95 km	1000 s	14 ms	1.05 %
4.4	687.75 km	707.89 km	796 s	13 ms	2.93 %
4.5	688.46 km	698.96 km	1000 s	14 ms	1.53 %
5.1	837.12 km	848.61 km	1000 s	15 ms	1.37 %
5.2	846.69 km	856.20 km	1000 s	16 ms	1.12 %
5.3	834.72 km	861.66 km	895 s	16 ms	3.23 %
5.4	828.48 km	828.48 km	923 s	13 ms	0.00 %
5.5	853.73 km	838.62 km	1000 s	12 ms	0.35 %
6.1	967.71 km	975.00 km	1000 s	13 ms	0.75 %
6.2	939.35 km	989.21 km	763 s	13 ms	5.31 %
6.3	956.05 km	981.45 km	1000 s	14 ms	2.66%
6.4	933.38 km	971.98 km	273 s	12 ms	4.14%
6.5	943.87 km	974.96 km	553 s	11 ms	3.29%

maximum value of 5.31 % and a variance of 2.27 %. Moreover, for 3 instances the two solutions coincide. In conclusion, the proposed heuristic algorithm obtains very good solutions in all considered instances and runs in less than 17 ms. Hence, the presented algorithm can be used to solve large and complex problems in short time.

C. Case Study of a Postal Delivery System

In this section, the U-DSS PD module is assessed by considering the case study of Bari. The network is modelled by the graph shown in Fig. 6 that depicts the locations of the nodes without showing the arcs for the sake of clarity. Hence, the network is composed by  $N = 63$  nodes, where 61 nodes represent the PDCs and the node  $v_0 = v_N$  is the PAC that is symbolized by a red triangle in Fig. 6. So, the graph  $G$  is composed by  $(N + 1)^2 - (2N) = 3843$  directed arcs.

The values of the elements  $c_{ij}$  and  $t_{ij}^k$  are determined on the basis of the results obtained by Google Maps Application Programming Interface. The evaluation of these parameters is crucial for the routes optimization: indeed, they can be affected by the traffic congestion, the weather forecast, the number of street lanes, the slope of the streets, the considered period of the year and of the day.

Moreover, the capacity constraints depend on the vehicle  $u_k$  used in the  $k$ -th shift. In particular, the considered system has  $K = 15$  vehicles with the capacities reported in Table III: vehicles  $u_1, \dots, u_6$  have capacity equal to 20 HUs,  $u_7, \dots, u_{12}$

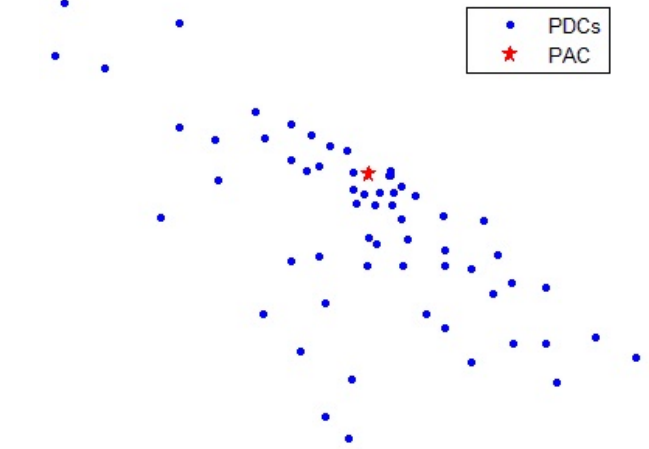


Fig. 6. The node locations of the postal delivery system of the case study.

have capacity equal to 15 HUs and  $u_{13}, u_{14}, u_{15}$  have capacity equal to 10 HUs. The service duration in each node  $v_i \in \bar{V}$  is equal to  $p_i = 15$  minutes, while the service duration in the PAC is  $p_0 = p_N = 0$ . The demand of each PDC daily changes: the values of the demand of each node that is considered in the case study is reported in Table IV. Since only one route can be performed in each shift, it holds  $T_r = T_w = 360$  minutes.

TABLE III  
VEHICLE FLEET

$u_k$	$C^k$ (HU)
$u_1 - u_6$	20
$u_7 - u_{12}$	15
$u_{13} - u_{15}$	10

TABLE IV  
PDC DEMAND OF THE CASE STUDY (IN HU)

$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$	$v_7$	$v_8$	$v_9$	$v_{10}$	$v_{11}$
5	3	3	3	2	2	3	2	3	2	2
$v_{12}$	$v_{13}$	$v_{14}$	$v_{15}$	$v_{16}$	$v_{17}$	$v_{18}$	$v_{19}$	$v_{20}$	$v_{21}$	$v_{22}$
3	3	6	3	3	5	2	3	7	1	5
$v_{23}$	$v_{24}$	$v_{25}$	$v_{26}$	$v_{27}$	$v_{28}$	$v_{29}$	$v_{30}$	$v_{31}$	$v_{32}$	$v_{33}$
6	5	1	6	1	3	1	6	1	2	3
$v_{34}$	$v_{35}$	$v_{36}$	$v_{37}$	$v_{38}$	$v_{39}$	$v_{40}$	$v_{41}$	$v_{42}$	$v_{43}$	$v_{44}$
1	2	1	2	3	3	1	1	1	4	1
$v_{45}$	$v_{46}$	$v_{47}$	$v_{48}$	$v_{49}$	$v_{50}$	$v_{51}$	$v_{52}$	$v_{53}$	$v_{54}$	$v_{55}$
1	3	3	1	1	1	1	1	1	1	1
$v_{56}$	$v_{57}$	$v_{58}$	$v_{59}$	$v_{60}$	$v_{61}$					
1	5	1	1	2	1					

The solution of the case study is summarized in Table V and Table VI. Let us remark that the first route is performed by the vehicle  $u_7$  of capacity 15 HU because of the post fit procedure. Indeed, at first the heuristic algorithm tries to perform the route with the biggest available vehicle  $u_1$ . However, due to time constraints, the vehicle capacity is not used completely (14 HUs of 20), hence the algorithm switches  $u_1$  with the vehicle  $u_7$  of 15 HUs. In this way a better utilization of the vehicles is obtained.

TABLE V  
ROUTE CHARACTERISTICS OF THE SOLUTION

Route	duration (min)	length (km)	load (HU)	vehicle/capacity
1	349.45	364.86	14	$u_7/15$ HU
2	296.92	313.84	18	$u_1/20$ HU
3	340.28	348.58	15	$u_8/15$ HU
4	341.53	322.43	19	$u_2/20$ HU
5	345.57	268.75	13	$u_9/15$ HU
6	335.52	234.89	18	$u_3/20$ HU
7	359.37	222.49	15	$u_{10}/15$ HU
8	317.72	154.65	18	$u_4/20$ HU
9	291.63	93.34	20	$u_5/20$ HU
10	66.15	28.10	2	$u_{13}/10$ HU
Sums	<b>3044.13</b>	<b>2351.92</b>	<b>152</b>	

TABLE VI  
ROUTE NODES OF THE SOLUTION

Route	$\vec{V}_1^k$
1	$v_0 v_{38} v_{60} v_{61} v_{57} v_{39} v_0$
2	$v_0 v_{14} v_{26} v_{21} v_{22} v_0$
3	$v_0 v_{33} v_{34} v_{35} v_{16} v_{30} v_0$
4	$v_0 v_{23} v_{24} v_{17} v_{58} v_{32} v_0$
5	$v_0 v_5 v_{13} v_{27} v_{28} v_{29} v_3 v_0$
6	$v_0 v_{20} v_{25} v_{45} v_{43} v_{12} v_{44} v_{59} v_0$
7	$v_0 v_{50} v_{11} v_{48} v_{31} v_{15} v_{49} v_{47} v_{46} v_0$
8	$v_0 v_6 v_{36} v_{56} v_{55} v_{19} v_4 v_7 v_{37} v_8 v_0$
9	$v_0 v_{53} v_{51} v_{42} v_{40} v_{41} v_{10} v_{18} v_2 v_9 v_1 v_0$
10	$v_0 v_{52} v_{54} v_0$

Remark that in route 10 (see Table V) the capacity of the vehicle is under utilized: this happens because there are not vehicles with capacity smaller than 10 HUs, and route 10 visits the last two residual nodes, near the depot. The DM could decide to assign those nodes to some of the other routes, by applying them an extra-time. The clusters are graphically represented in Fig. 7 with different colors: the red star represents the depot. For the sake of clarity, in Fig. 7 the connections with the depot are not depicted.

## V. WASTE COLLECTION PROBLEM

### A. Waste Collection System Description and Matematical Formulation

In this section, the configured VAR module of the U-DSS devoted to a WC system is described. We consider a maximum number  $K$  of shifts and a set of  $K$  identical vehicles  $U = \{u_1, u_2, \dots, u_K\}$ : each vehicle  $u_k \in U$  can collect

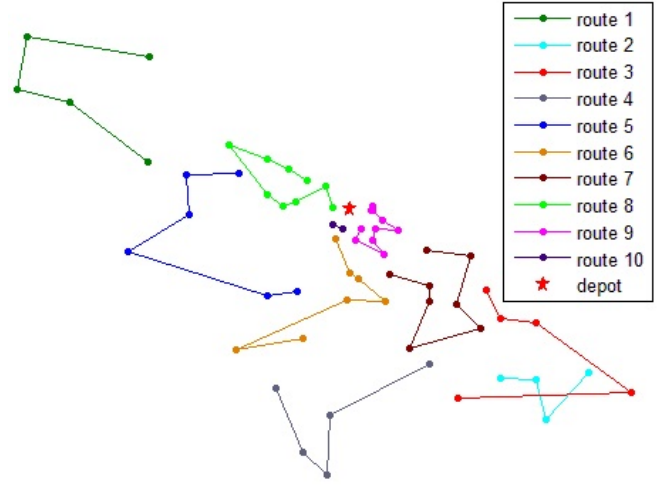


Fig. 7. The clusters obtained by the heuristic algorithm

$C^k$  bins. Every day a team of workers drives the vehicles to perform the waste collection subject to time and capacity constraints. During a work shift, each driver of the team can perform several routes for a maximum of  $M$ .

The first travel starts from the depot, follows the assigned route by collecting the bin waste and, when the truck is full or the time limit is attained, it has to stop at the landfill. In this site the workers unload the waste, make a stop and then start a new new route from the landfill. On the basis of the company specifications, the work shift goes to an end by an empty travel from the landfill to the garage. Each route time has to be less than or equal to  $T_r$  minutes and the shift, composed of a set of routes, is limited by the working time of  $T_w$  minutes.

Fig. 8 shows the UML activity diagram of the vehicle shift highlighting the actions of drivers and vehicles.

The problem is minimizing the length of all the routes performed by the vehicles in each work shift. Each route is given as a sequence of visited nodes with the indication of the arrival times.

Now, a mathematical formulation of the considered VAR is presented on the basis of the model in [27] but also providing several routes in each shift. Hence, the decision and dependent variables are slightly modified with respect to the MILP (5-15).

Decision variables:

$$x_{ij,m}^k = \begin{cases} 1, & \text{if vehicle } u_k \text{ visits in sequence nodes } v_i \text{ and } v_j \\ & \text{during routemofshiftk} \\ 0, & \text{otherwise.} \end{cases}$$

Dependent variables:

- $Q_{i,m}^k$  load collected by vehicle  $u_k$  when it leaves node  $v_i \in V$  during route  $m$
- $S_{i,m}^k$  starting time of the pick up operation of vehicle  $u_k$  at node  $v_i \in \bar{V}$  during route  $m$
- $S_{N,m}^k$  arrival time to node  $v_N$  of route  $m$  performed by  $u_k$

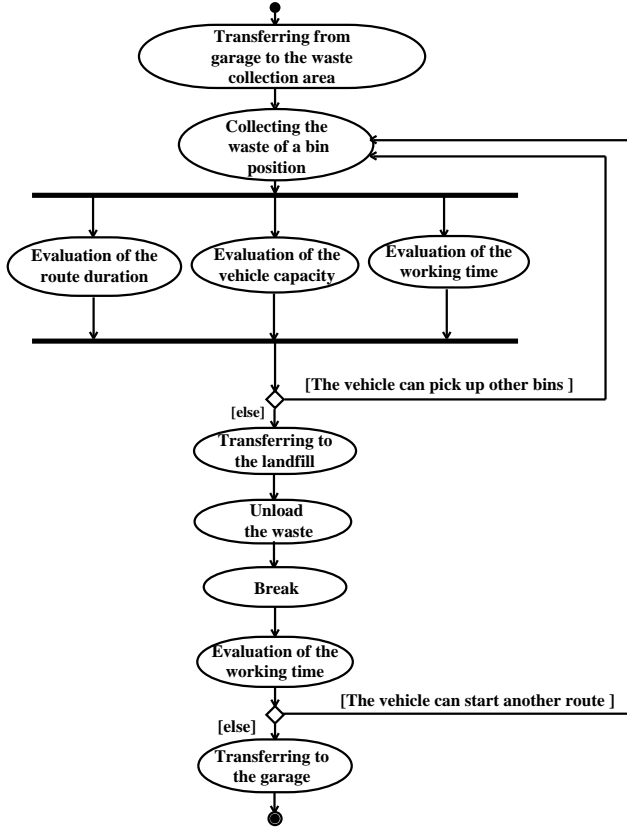


Fig. 8. Activity diagram of the vehicle during a shift.

$S_{0,m}^k$  working time of vehicle  $u_k$  during route  $m$

The considered VAR consists of designing a set of feasible routes with minimum total length such that:

- each node is visited by exactly one vehicle;
- the first route starts from  $v_0$  and ends to  $v_N$ ;
- the last (empty) route starts from  $v_N$  and ends to  $v_0$ ;
- the remaining routes start from and end to  $v_N$ ;
- the number of routes performed by each vehicles is less than or equal to  $M$ .

Now, the considered VAR can be formulated by the following MILP problem.

$$\min \sum_{k=1}^K \sum_{m=1}^M \sum_{i,j:a_{ij} \in A} c_{ij} x_{ij,m}^k \quad (17)$$

subject to:

$$\sum_{m=1}^M \sum_{k=1}^K \sum_{j:a_{ij} \in A} x_{ij,m}^k = 1 \quad \forall v_i \in \bar{V} \quad (18)$$

$$\sum_{j:a_{0j} \in A} x_{0j,1}^k = 1 \quad \forall u_k \in U \quad (19)$$

$$\sum_{m=2}^M \sum_{j:a_{0j} \in A} x_{0j,m}^k = 0 \quad \forall u_k \in U \quad (20)$$

$$\sum_{j:a_{Nj} \in A} x_{Nj,m}^k \leq 1 \quad \forall u_k \in U, m = 2, \dots, M \quad (21)$$

$$\sum_{i:a_{iN} \in A} x_{iN,m}^k \leq 1 \quad \forall u_k \in U, m = 1, \dots, M \quad (22)$$

$$\sum_{i:a_{ij} \in A} x_{ij,m}^k - \sum_{i:a_{ij} \in A - \{a_{j0}\}} x_{ji,m}^k = 0 \quad (23)$$

$$\forall v_j \in \bar{V}, \forall u_k \in U, \forall m$$

$$\sum_{i:a_{iN} \in A} x_{iN,m}^k - \sum_{j:a_{Nj} \in A} x_{Nj,m+1}^k = 0 \quad (24)$$

$$\forall u_k \in U, m = 1, 2, \dots, M - 1$$

$$\sum_{m=1}^M x_{N0,m}^k = 1 \quad \forall u_k \in U \quad (25)$$

$$x_{ij,m}^k = 1 \Rightarrow S_{j,m}^k \geq S_{i,m}^k + p_i + t_{ij}^k \quad (26)$$

$$\forall u_k \in U, \forall a_{i,j} \in A \text{ s.t. } i \neq N, \forall m$$

$$x_{Nj,m}^k = 1 \Rightarrow S_{j,m}^k \geq S_{N,m-1}^k + p_N + t_{Nj}^k \quad (27)$$

$$\forall u_k \in U, \forall j \text{ s.t. } a_{N,j} \in A, \forall m > 1$$

$$x_{ij,m}^k = 1 \Rightarrow Q_{j,m}^k = Q_{i,m}^k + q_j \quad (28)$$

$$\forall u_k \in U, \forall a_{i,j} \in A \text{ s.t. } i \neq N, \forall m$$

$$x_{Nj,m}^k = 1 \Rightarrow Q_{j,m}^k = q_j \quad (29)$$

$$\forall u_k \in U, \forall j \text{ s.t. } a_{N,j} \in A, \forall m > 1$$

$$0 \leq S_{N,1}^k \leq T_r \quad \forall u_k \in U \quad (30)$$

$$0 \leq S_{N,m}^k - S_{N,m-1}^k - p_N \leq T_r \quad \forall u_k \in U, \forall m > 1 \quad (31)$$

$$0 \leq S_{0,m}^k \leq T_w \quad \forall u_k \in U, \forall m \quad (32)$$

$$0 \leq Q_{i,m}^k \leq C^k \quad \forall u_k \in U, \forall v_i \in V, \forall m \quad (33)$$

$$x_{ij,m}^k \in \{0, 1\} \quad (34)$$

The objective function (17) minimizes the length of the routes performed by all the vehicles in each work shift. Constraints (18) impose that each node  $v_i \in \bar{V}$  has to be served exactly once. The constraints (19)-(21) guarantee that every vehicle starts the first route at the garage while the other routes begin from the disposal site. Moreover, constraints (22) state that each vehicle ends its route at the landfill site and constraints (23) ensure the flow conservation avoiding that the vehicles come back to the garage after having served a node  $v_i \in B$ . Constraints (24) connect the end of the route  $m$  to the start of the route  $m + 1$ . Constraints (25) impose that the last route of the shift has to be performed from the disposal site to the garage.

Constraints (26) and (27) determine the arrival time at node  $v_i \in V$  and constraints (28) and (29) update the loads  $Q_{i,m}^k$ . On the other hand, (30)-(33) impose the time and capacity constraints.

Note that the non-linear constraints (26)-(29) can be linearised by a big number formulation [29].

### B. Validation and Results

The performances of the MILP formulation and the heuristic algorithm are compared and tested by nine sets of randomly generated instances. Table VII reports the data sets chosen to compare the two approaches: the values of  $N$  and  $K$  are selected so that the MILP formulations can be solved in reasonable time. Note that the instances of the sets 1-3, 4-6 and 7-9 generate 720, 891 and 1440 variables, respectively. Moreover, the sets 1-3 as well as the sets 4-6 and 7-9 differ for the demand in each pick up position and for the distance between the nodes: these parameters are generated with uniform probabilities between the values shown respectively in the forth and fifth columns of Table VII.

In order to determine the route times  $t_{ij}^k$  between two consecutive pick up sites, we assume a speed of 40 km/h in the following cases: i) the vehicle goes from the depot to the first not served pick up point; ii) the vehicle goes from the landfill to the first not served pick up point; iii) the vehicle comes back to the landfill from the last served pick up point; iv) the vehicle comes back to the garage from the landfill. In all the other cases the average speed is assumed equal to 20 km/h.

Moreover, the service time  $p_i$  associated with  $v_i \in B$  is determined on the basis of the number of bins located in the considered pick up position: the vehicle simultaneously loads two bins in 1.5 minutes. On the other hand, the unloading time at the landfill is  $p_N = 30$  minutes and includes the compulsory work break. Moreover it holds  $q_0 = q_N = 0$  and  $p_0 = 0$ .

Summing up, the route times  $t_{ij}^k$  for  $a_{ij} \in A$  and the service times  $p_i$  for  $v_i \in V$  are determined by the following relations:

$$t_{ij}^k = \begin{cases} \frac{c_{ij}-60}{20000}, \forall i, j \text{ with } i \neq 0, N \text{ or } j \neq 0, N \\ \frac{c_{ij}-60}{40000}, \text{ otherwise} \end{cases} \quad (35)$$

$$p_i = \begin{cases} 0, & \text{if } i = 0; \\ 30, & \text{if } i = N; \\ 1.5 \lceil \frac{q_i}{2} \rceil, & \text{otherwise.} \end{cases} \quad (36)$$

Table VIII shows the results that are obtained by allowing a maximum of 1000 seconds of CPU time for the solution of each instance.

In particular, we remember that for the instances where the CPU time is equal to 1000 seconds the optimal solution has been not reached and the value of the best feasible obtained solution is reported. The results of Table VIII show that the value of the optimality gap (16) averaged on the 45 instances is equal to 5.93 %, with a maximum value of 17.86 %. Moreover, it is important to note that *Gap* is less than 10 % for 37 instances, it is less than 5% for 25 instances and for 2

TABLE VII  
CHARACTERISTICS OF DATA SETS

Sets	N [positions]	K [vehicles]	$q_i$ [bins]	$c_{ij}$ [kilometers]
1	6	3	30-40	20-30
2	6	3	35-45	15-25
3	6	3	30-50	10-40
4	7	3	30-40	20-30
5	7	3	35-45	15-25
6	7	3	30-50	10-40
7	8	4	30-40	20-30
8	8	4	35-45	15-25
9	8	4	30-50	10-40

instances the two solutions coincide. Considering only the 33 optimal solutions, the *Gap* is less than 10 % for 25 instances. The *Gap* of the instances 4.5 and 6.1 is justified by the fact that the MILP problem solution provides three vehicles, while the heuristic algorithm provides four vehicles: this difference determines the largest gap between the two solutions.

In conclusion, the proposed heuristic algorithm obtains good solutions in the most of the considered instances and runs in less than 31 ms.

### C. Case Study of a Waste Collection System

In this subsection the case study of WC problem of a neighborhood of Bari is described and solved. The maximum number of shifts is  $K = 24$  that is the number of the shifts performed in the current waste management. Moreover, 24 identical vehicles form the fleet with capacity  $C^k = 90$  bins for  $k = 1, \dots, 24$ . During a work shift, each driver of the team can perform a maximum number of  $M = 4$  routes and it holds  $T_r = 180$  and  $T_w = 420$  minutes.

The waste collection system of the considered neighborhood consists of 2,790 bins distributed over  $N - 2 = 1,498$  pick up positions. The graph  $G$  is of 1,500 nodes and  $(N + 1)^2 - (N + 1) = 2,248,500$  arcs. In order to perform a realistic evaluation of the routes, each value of  $c_{ij}$  is increased of 0.15 km and 0.30 km for each left-turn and U-turn, respectively. Each parameter  $t_{ij}$  is determined by the sum of the following terms: the vehicle average speed obtained by (35) and some penalties due to weather and traffic conditions, rash hours, condition and slope of the street.

The solution of the considered WC obtained by the heuristic algorithm is shown in Table IX. It is apparent that 18 shifts and 40 routes are necessary to perform the waste collection. More precisely, most of the shifts are composed of two routes, not considering the fixed empty route from the landfill to the garage at the end of the shift. Although these "end-shift" routes are omitted in Table IX, total duration and total length take them in account (each of them has a duration of about 7 minutes for a length of 4.5 kilometers).

TABLE VIII  
COMPARISON BETWEEN MILP PROBLEM AND HEURISTIC ALGORITHM

Inst.	OPT	UB	CPU-OPT	CPU-UB	Gap
1.1	324.67 km	337.36 km	24 s	26 ms	3.91 %
1.2	326.62 km	336.44 km	29 s	23 ms	3.01 %
1.3	327.77 km	335.43 km	25 s	28 ms	2.34 %
1.4	296.78 km	302.26 km	12 s	23 ms	1.85 %
1.5	336.92 km	336.92 km	25 s	23 ms	0.00 %
2.1	270.55 km	276.12 km	21 s	25 ms	2.06 %
2.2	274.48 km	277.08 km	18 s	25 ms	0.95 %
2.3	250.38 km	260.09 km	25 s	23 ms	3.88 %
2.4	273.56 km	291.48 km	22 s	25 ms	6.55 %
2.5	260.55 km	281.83 km	18 s	26 ms	8.16 %
3.1	262.48 km	270.42 km	6 s	23 ms	3.02 %
3.2	294.96 km	319.06 km	14 s	26 ms	8.17 %
3.3	228.17 km	228.17 km	6 s	20 ms	0.00 %
3.4	258.15 km	261.31 km	26 s	23 ms	1.25 %
3.5	267.62 km	281.27 km	13 s	23 ms	5.10 %
4.1	376.36 km	386.56 km	102 s	23 ms	2.71 %
4.2	369.89 km	387.03 km	265 s	29 ms	4.63 %
4.3	379.81 km	386.00 km	232 s	26 ms	1.89 %
4.4	364.07 km	375.68 km	84 s	24 ms	3.19 %
4.5	374.37 km	433.83 km	166 s	28 ms	15.77 %
5.1	320.66 km	345.12 km	98 s	31 ms	7.63 %
5.2	314.91 km	342.94 km	120 s	29 ms	8.91 %
5.3	309.72 km	348.84 km	29 s	26 ms	12.63 %
5.4	312.38 km	352.76 km	41 s	27 ms	12.93 %
5.5	323.08 km	363.32 km	137 s	25 ms	12.45 %
6.1	369.20 km	435.14 km	11 s	26 ms	17.86 %
6.2	353.40 km	369.41 km	30 s	26 ms	4.53 %
6.3	338.60 km	393.52 km	32 s	27 ms	16.22%
6.4	305.49 km	336.14 km	8 s	27 ms	10.03%
6.5	314.21 km	349.04 km	13 s	25 ms	11.08%
7.1	417.08 km	434.45 km	1000 s	26 ms	4.16 %
7.2	414.14 km	422.70 km	1000 s	26 ms	1.95 %
7.3	421.70 km	431.66 km	1000 s	27 ms	2.36 %
7.4	441.12 km	462.74 km	843 s	29 ms	4.90 %
7.5	442.35 km	476.02 km	748 s	31 ms	7.61 %
8.1	349.10 km	366.66 km	1000 s	28 ms	5.03 %
8.2	344.15 km	357.50 km	1000 s	29 ms	3.88 %
8.3	367.87 km	377.50 km	1000 s	28 ms	2.62 %
8.4	363.33 km	378.44 km	1000 s	26 ms	4.16 %
8.5	348.70 km	357.70 km	1000 s	28 ms	2.58 %
9.1	351.98 km	378.11 km	1000 s	28 ms	7.42 %
9.2	373.88 km	404.99 km	1000 s	27 ms	8.55 %
9.3	383.16 km	418.24 km	725 s	31 ms	9.15%
9.4	403.37 km	428.82 km	1000 s	31 ms	6.31%
9.5	413.37 km	428.56 km	1000 s	28 ms	3.67%

In order to show the effectiveness of the proposed algorithm, the obtained solution is compared with the current waste management of the case study where 24 shifts composed of 2 routes (not considering the last route) are now performed. Hence, the proposed solution allows saving 6 shifts and 8 collecting routes with significant economic and environmental benefits.

### VI. CONCLUSION

This paper presents a decision support approach for optimizing delivery and collection services in a metropolitan area. To this aim an Urban-Decision Support System (U-DSS) is designed to support decision makers about the work shifts organization by minimizing the travel times of the vehicle routes in the shifts. Then, in order to deal with real cases, a two-phase heuristic algorithm is proposed based on

TABLE IX  
SOLUTION OBTAINED BY THE HEURISTIC ALGORITHM

$u_k$	$m$	duration [minutes]	length [kilometers]	Loaded bins [bins]
1	1	173.57	47.85	33
	2	179.54	53.49	35
2	1	179.23	54.49	44
	2	173.21	41.81	55
3	1	174.53	53.78	49
	2	176.76	47.60	60
4	1	176.86	36.97	86
	2	173.82	44.24	59
5	1	178.45	39.35	72
	2	173.39	43.09	55
6	1	173.15	42.83	57
	2	178.04	37.10	89
7	1	144.14	26.44	89
	2	178.17	34.40	87
8	1	168.82	34.58	90
	2	179.31	37.36	90
9	1	149.13	28.30	90
	2	165.96	34.60	89
10	1	173.79	28.12	90
	2	140.77	25.15	86
11	1	156.42	24.20	90
	2	161.57	29.49	89
12	1	115.67	19.20	89
	2	132.01	24.01	90
	3	74.28	17.55	25
13	1	140.41	23.68	89
	2	118.49	20.63	89
	3	63.54	17.41	26
14	1	140.86	25.06	90
	2	175.04	25.96	84
15	1	136.94	22.34	90
	2	143.10	20.82	90
	3	41.93	12.33	19
16	1	154.19	23.75	90
	2	121.53	19.06	90
	3	45.76	13.33	18
17	1	155.61	26.08	87
	2	152.21	24.93	88
	3	14.15	7.43	4
18	1	81.19	15.70	48
Total		3986.5	923.2	2419

a clustering strategy and a farthest insertion heuristic for the Travelling Salesman Problem.

In particular, Postal Delivery (PD) and Waste Collection (WC) services are considered, each of them with their own peculiarities, but also with common aspects that allow designing a common framework.

While the strategy and the heuristic algorithm are the same for both services, in order to validate the algorithm for different applications, two Mixed Integer Linear Programming formulations are presented: in any case the aim is minimizing the total length of all the planned routes.

For the PD service, an heterogeneous vehicle fleet is in-

volved: the applied heuristic algorithm allows not only minimizing the vehicles routing but also using vehicles of suitable dimensions.

About the WC service, a complex and constrained problem is addressed: i) capacity and time constraints are taken into account; ii) a network with a large number of bins characterized by different demand and pick up times is considered; iii) each vehicle can perform several routes during a daily shift.

Comparing the presented approach with the solutions proposed in the related literature, we point out the following features:

- a unified framework to solve different Vehicles Assignment and Routing problems is considered;
- the problem is general and can take into account several landfills, different vehicles with different capacities and speed;
- the routes can form several shifts satisfying time and capacity constraints;
- the travelling times are obtained by the routes and the distances determined by Google Map Application Program Interface, the traffic and weather forecasts;
- the solutions are obtained in extremely short times, even if the dimensions of the problem are very large.

Future research will focus on extending the U-DSS by considering dynamic routing in order to modify in real time the planning of the routes.

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