Prediction of the fundamental frequencies and modal shapes of historic masonry towers by empirical equations based on experimental data

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Abstract.

The adequate knowledge of the modal characteristics (natural frequency, modal shapes and damping) constitutes an important start point to carry out a reliable dynamic structural analysis. In the case of historic masonry towers, in particular, due to their geometric and structural characteristics they can be considered as typical and repetitive structures and predictive empirical laws can be generated. In this paper the performances of some formulas proposed in literature (even only for generic masonry towers) have been assessed on a group of case studies through the Mean Squared Error (MSE). Different results have been found for both bounded and isolated towers. Moreover, to improve to robustness and reliability of the prediction, new optimized functions, obtained minimizing the MSE of the linear regression of a model of exponential law, have been developed. The results, compared using the determination coefficient "*r*", have shown a good capability of the new proposed laws to predict the fundamental frequency for the historic masonry towers. Finally, some correlation regarding the estimation of the higher modes have been highlighted.

Keywords: *Structural Monitoring, Historic Masonry Towers, Natural Frequencies, Empirical Formula, Optimization Procedures*

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1. Introduction

As it is well known, Italy is a country with a wide, ancient and important building heritage and, unfortunately, a high seismic risk [1]. As a consequence, during the last decades a great attention has been devoted to the necessity of protecting this patrimony. A correct seismic assessment and design of risk mitigation interventions, however, need a careful study of the structure.

On the other side, due to the historical character of these buildings, all the necessary information for the definition of a deep knowledge of them are usually unavailable, and the possibility of conducting classical tests is limited to the ones that are non-destructive. In the case of architectural heritage, in fact, destructive tests are hardly carried out; therefore, structural monitoring techniques based on dynamic monitoring of the structures and output-only modal identification techniques become very useful to get the modal parameters of the structure in operational conditions. To this aim, Operational Modal Analysis (OMA) is an efficient method to be utilized in these cases; it allows to know the modal parameters of a structure in a non-invasive way, and can be applied not only to historic and cultural buildings but also to modern ones [2]. Through a process, which is simple and accurate, it is possible then to acquire the ambient vibrations in situ, in order to later estimate the modal shapes of vibration, the natural frequencies and the modal damping ratios based on the processing of the acquired data [3]. Thus, the dynamic identification through operational input is also a solution to evaluate the characteristics of the materials and the boundary conditions of the structure, in order to establish reliable numerical models, through procedures of model updating. Apart from the dynamic identification via finite element 3D modeling, some analysis have been also carried out on a linear simplified model of the tower constituted in a vertical cantilever beam [4].

The techniques, developed for a model updating process and also utilized in some recent studies [5]**Error! No bookmark name given.-**[20], estimate the unknown mechanical properties of the materials and/or the boundary constraints by comparing the identified and the numerical modal parameters and minimizing specific objective functions.

In this context, the so-called "*output-only methods*", are widely spread in the field of monitoring of historical structures, as they may be applied to operationally-induced vibrations that have the advantage of being compatible with the ordinary service of the structure, and of reducing the costs connected with the test setup, as the installation of shakers or actuators is not requested.

OMA methodology is particularly suited to slender structures such as towers, campaniles, antennas, chimneys, mosques. Indeed, thanks to their geometrical shape, the noise-to-signal ratio becomes very low especially at the top and so this allows to extract important structural information from the acquired accelerations. This type of construction is generally distinguishable according to its prevalent vertical development and constitutes a relevant part of the Italian cultural and artistic heritage. It is also specified in the Italian guidelines "*Evaluation and reduction of seismic risk of the cultural heritage with reference to the technical standards for construction in DM 14/01/2008*" (DPCM 09/02/2011) that the towers, if subjected to vibrations, even of low intensity, generally produce very sharp signals, easily recordable.

These structures are widespread in Central Europe and therefore, for their conservation, it was necessary refined techniques of reinforcement for restoring their performance. In the past, these problems were neglected because the presence of any crack was regarded as part of a normal stage in which the structure is poured. In recent years, however, the sudden collapse of some masonry buildings has prompted researchers to study the response of these structures to the seismic effects and heavy loads. For this reason, a deep knowledge of the dynamic properties is crucial to assess the effective seismic vulnerability of historic towers. Moreover, due to their repetitive and proportional structural features, during the years different Authors have analyzed and proposed models of empirical formulas able to predict the fundamental frequency. Very useful formulas are reported in the standards for construction of Italy [21]and Spain [22]. They have been applied, in particular, to carry out simplified structural analysis able to perform a fast and reliable evaluation of the seismic demand through an empirical determination of such fundamental frequency/period. In the formula of the Italian code the only requested parameter is the total height of the building, while in the Spanish one also the minor side length of the base plan has to be inserted in the formula. It is worth to notice that in both cases only the evaluation of geometrical features are needed, so that these laws are very easy to manage. However, they have been developed for generic masonry structures that, of course, should include also the historic masonry towers. In general, the empirical predictive functions proposed in literature have a marked exponential behavior that decreases the frequency as the total height increases. For example, in Shakya et al [4] the exponent and amplitude of an exponential model are defined on the basis of the specific masonry structure like towers or minarets. Other formulations have been defined for structures located in specific regions, as in the case of the ones proposed by Ranieri et al. [23] and Faccio et al. [24].

The present paper seeks to face and improve the issue of a rapid evaluation of the modal characteristics for masonry historic towers. Indeed, in some situations, the knowledge of these properties leads to a reduction of cost and time that constitute one of the priorities for the responsibles of the cultural heritage's management. After the introduction (section 1), in the second paragraph a wide review of the most important case-studies regarding the analysis of the dynamic behavior for historic masonry towers has been carried out. It has been faced especially through dynamic experimental tests or structural monitoring but also developing and updating 3D finite element modeling. In general, the instrumental setup is composed by an accelerometer wired network and the measured data have been processed using the well-known and consolidated procedures working in both time and frequency domains. The data collected in section 2 have been analyzed and compared with the numerical results provided by the empirical laws introduced above. In particular, these exponential laws, have been compared using the Mean Squared Error (MSE) between the experimental and predictive fundamental frequencies. Moreover, the latter has been applied to two different sets of towers: bounded and isolated. For the first group it has been estimated the percentage of height constrained only taking into account the information found in the literature. The results appeared good enough even though they are not specific laws for historic masonry towers. For this reason, in section 4 some new empirical laws more suited for masonry towers have been proposed. They are based on minimizing the MSE of their linear regression. Some considerations are also given about the modal characteristics of the higher modes and their modal shapes and dampings. Finally, in the conclusions some possible suggestions to improve the actual results are highlighted.

2. Experimental applications

The towers are among of the most important cultural heritage to be preserved for their historical relevance in the actual society. During the medieval period this type of structure had especially a territory's defensive and control purpose. For these reasons the height is the principal dimension

and, moreover, they show a massive structure with very few and small openings. In Figure 1a and Figure 1b two examples of typical masonry towers are reported, while in Table 1 a list of the main cases investigated in recent years by many researches is reported. A lot of structures belong to the Medieval Period (476 A.C. – 1492 A.C.) and other architectures are referred to the Modern Period (1492 A.C. – 1948 A.C.). Notwithstanding this long period (more than 1000 years) the construction technology and the type of material is more or less the same. It seems correct to summarize and organize the common features, in particular from geometrical and mechanical points of view.
The analysis of the scientific publications and researches highlights a common procedure to assess the structural behavior of historic towers:

1. *Construction's identification*: location (focusing the attention to the zone's risks), geometric relief of the building, visual identification of elements and materials (particular attention to the construction details and interconnections), historical overview;

2. *Experimental tests*: they can be divided into static or dynamic tests; to the first class they belong techniques such as laser scanning, ultrasonic, georadar or flat-jack tests, acoustic emission, static monitoring, while in the second one Ambient Vibration Testing (AVT, in general performed in one or two days) and long-term dynamic monitoring can be found.

3. *Signal processing and system identification*: in this step a handling of the acquired data is carried out. The main objective of this point is to extract relevant information useful to improve the representativeness of the numerical model.

4. *Data interpretation*: different issues can be faced using the updated models: seismic vulnerability, damage assessment or non-linear analysis.

The vibration-based experimental test is one of the most useful tool able to provide a huge amount of information. Indeed, in the scientific papers mentioned in Table 1, the majority of the Authors used AVTs. Of course, the most important sensors employed during this typology of tests are accelerometers. Most of the utilized accelerometers are wired (Figure 1d), even if in some recent studies [25] wireless sensors have been proposed (Figure 1c), which have the advantage of reducing the installation time for the dynamic test setup. Not all Authors declare the type of sensor used to carry out the tests but it is worth to notice that a careful choice of the sensor's performance is fundamental to obtain useful information from the data processing. Among the more stable and robust accelerometers there are the piezoelectric accelerometers which take advantage by the piezoelectric material's properties able to generate an electric charge when they are subjected to a variable force. Other important types of accelerometers are the servo and force-balance ones that offer good performances especially at low frequencies (below 1 Hz). They solve the piezoelectric limitations but they are bulky and heavy; moreover, for their high sensitivity they have also a high cost.

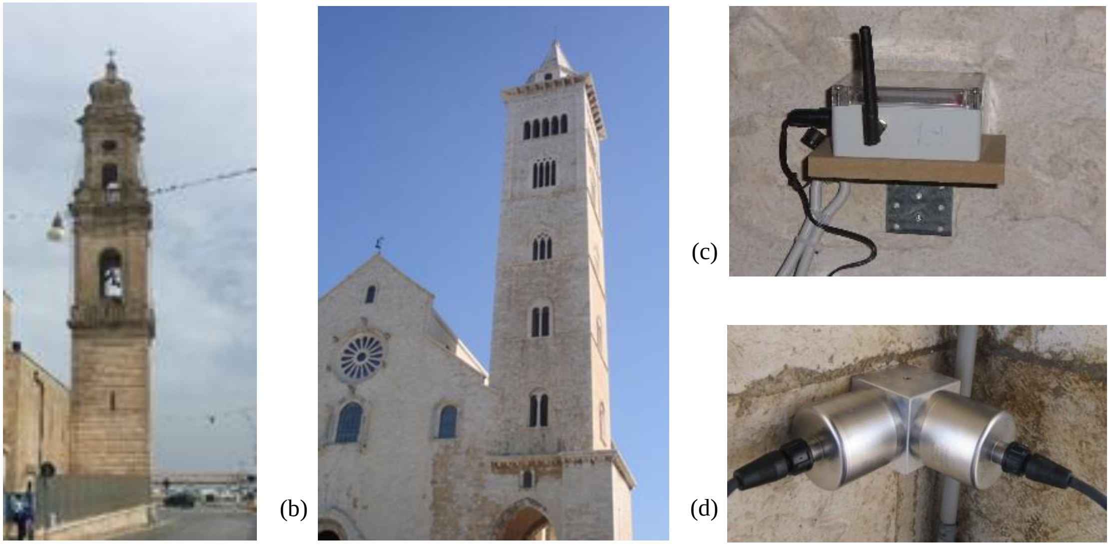


Figure 1. Two example of typical masonry towers: (a) "Santa Maria di Loreto tower, Mola (Italy); (b) Trani's Cathedral, Trani (Italy); two types of accelerometers: (c) wireless and (d) wired.

Table 1. Wide scientific overview of the historic masonry towers.

Case	Name	Location	Date	Reference
1	Sineo's Tower	Alba (Italy)	700 B.C.	Carpinteri et al [26]
2	Capua's Tower	Capua (Italy)	861 B.C.	Ferraioli et al [5]
3	Aversa's Tower	Aversa (Italy)	1053-1080 B.C.	Ferraioli et al [5]
4	Matildea Tower	Bondeno (Italy)	1114 B.C.	Milani et al [27]
5	San Luzi-bell Tower	Zouz (Switzerland)	1139 B.C.	Cantieni [28]
6	S. Vittore's Tower	Arcisate (Italy)	XII sec B.C.	Gentile et al [6]
7	Astesiano Tower	Alba (Italy)	XII-XIII sec B.C.	Carpinteri et al [26]
8	Trani's Cathedral	Trani (Italy)	1200 B.C.	Ivorra et al [7]
9	Soncino Tower	Cremona (Italy)	1200 B.C.	D'Ambrisi et al [8]
10	Gabbia Tower	Mantova (Italy)	1227 B.C.	Gentile et al [29]
11	Hagia Sofia Tower	Trabzon (Turkey)	1250-1260 B.C.	Bayraktar et al [30]
12	San Gimignano Tower	San Gimignano (Italy)	1300 B.C.	Bartoli et al [9]
13	Annunziata Tower	Corfù (Greece)	1394 B.C.	Diaferio et al [10]
14	San Domenico Tower	Mantova (Italy)	1466 B.C.	Gentile et al [31]
15	Roccaverano Tower	Asti (Italy)	1500 B.C.	Bonato et al [11]
16	Vistula Mounting Tower	Vistula Mounting	XV sec. B.C.	Tomaszewska et al. [32]
17	Clock Tower[a]	Bondeno (Italy)	1559 B.C.	Ramos et al [12]
18	S. Maria di Loreto Tower	Mola (Italy)	1587 B.C.	Ivorra et al. [20]
19	Monza's Cathedral	Monza (Italy)	1592-1605 B.C.	Gentile et al [13]
20	Maddalena's Cathedral	Mola di Bari (Italy)	1617 B.C.	Foti et al [14]
21	University of Coimbra	Lisbona (Portugal)	1728-1733 B.C.	Julio et al [15]
22	N. S.ra de la Misericordia	Valencia (Spain)	1740 B.C.	Ivorra et al [16]
23	Sant'Andrea's Tower	Venezia (Italy)	1850 B.C.	Russo et al [17], [18]
24	Clock Tower[b] Provincial Administration	Bari (Italy)	XX sen	Foti et al. [19]

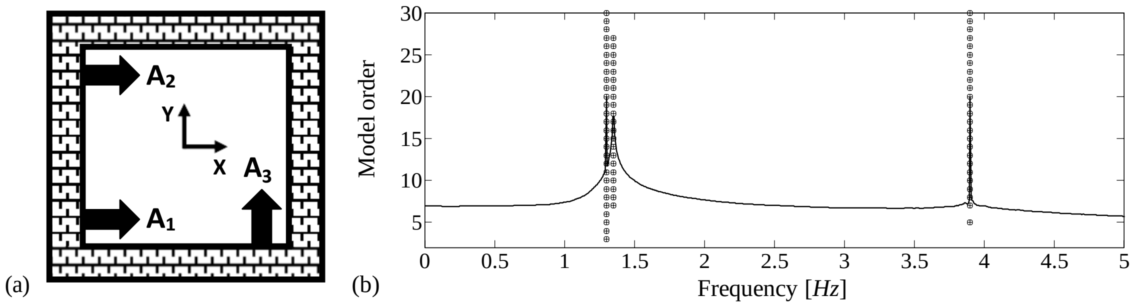


Figure 2. (a) Typical experimental setup needed to identify the main modes; (b) Stability diagram obtained applying the SSI on the acquired measurements (PSD on the background).

Other types of sensors are the Micro Electro-Mechanical Systems (MEMS) that can be of different nature (mechanical, electrical and electronic), whose principle is based on the variations of the electrical capacity produced by an imposed acceleration. They are suitable especially in the case of the development of a wireless sensor network. Moreover, in this last case it is possible also to install in the sensor board an open-source management software able to perform automatically some elementary algorithm (e.g. FFTs or comparisons between the measures in different sensors). Finally, among the more recent and innovative instrumentations for structural monitoring there are the optic fibers used especially to follow static or very slow movements [33]. Another important choice, regarding the experimental tests, is the design of the sensors' positions. For example, in the general case of frame structures or historic masonry towers one of the main target concerns the possibility to identify from the acquired data the main modes involved in the in-plane rigid motion. For this reason, in Figure 2a one of the most typical and useful experimental setup to reach this purpose is illustrated. It is constituted by three mono-axial accelerometers, two able to measure the accelerations in X-direction (A1 and A2) and the third (A3) in Y-direction. Subsequently, the acquired signals have to be processed through some traditional procedures to identify the modal characteristics of the main modes. For this reason, over the years, different processing techniques operating in the time, frequency and time-frequency domains have been developed and compared, [3]. Among the most and popular tools used for the analysis of AVT results, there are the Stochastic Subspace Identification (SSI) and the Power Spectral Densities (PSDs). In the first case, the procedure is able to develop approximate and statistical state-space models, which are not directly correlated to physical parameters like Young's modulus, Poisson coefficient etc, but however allow to identify the modal parameters through the identification of such "*grey model*" [34], [35].

The SSI results are visualized through a stability diagram (Figure 2b) in which the stable frequencies, i.e. the ones that appear in each model order, can be considered as frequencies associated to a structural mode. To confirm these results the PSDs can be calculated whose peaks correspond to the frequencies identified in the SSI procedure (see the background in Figure 2b). In the case of historic masonry towers the most common results found in the literature are shown in Figure 3.

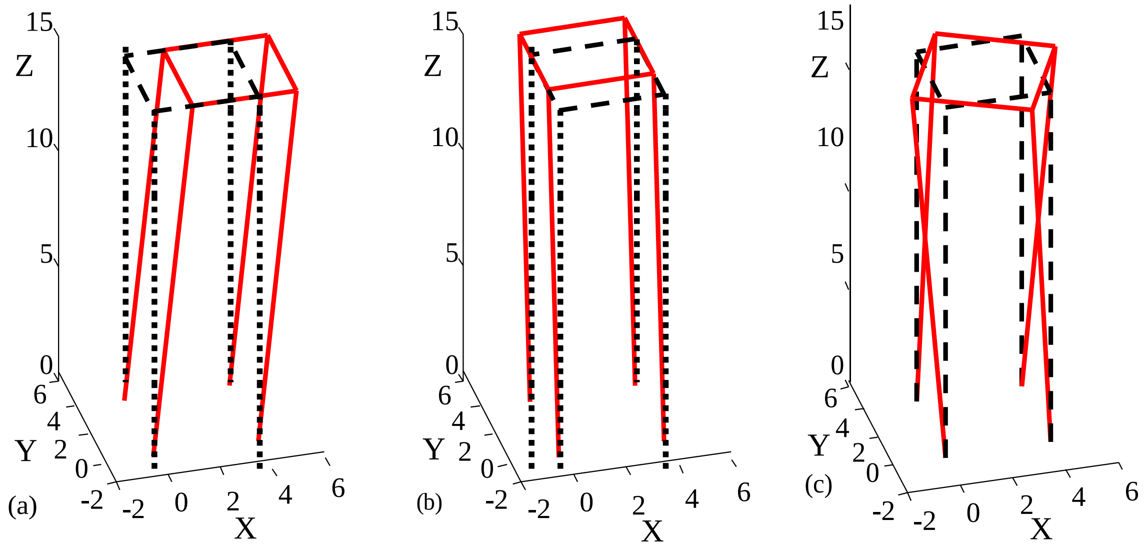


Figure 3. Main modal shapes identified for historic masonry towers: (a) flexural mode in X-direction, (b) flexural mode in Y-direction, (c) torsional mode.

The first two modal shapes are flexural and very close to each other because usually the stiffness along the two principal directions are almost the same, while the third frequency is quite far from the previous two and corresponds to a torsional mode. The dependence of the modal frequencies by the geometric characteristics of historic masonry towers will be in depth analyzed in the next section (sect. 3).

3. Evaluation of the modal characteristics

Historic masonry towers, even if built in different periods, show a very common and repetitive geometric configuration. For this reason, in the last few years different Authors tried to define empirical laws to calculate the main frequency that characterizes the tower's dynamic behavior. The knowledge of the dynamic proprieties can be very useful for different reasons: (1) to perform as accurately as possible a seismic analysis; (2) to carry out the model updating of the numerical and predictive finite element models; (3) to check the quality and robustness of the information coming from the experimental data processing; (4) to choose the optimum accelerometric sensor during the designing of a structural or seismic monitoring system. This section will show the ability and reliability of different laws that depend on one or two parameters to predict the main frequency.

3.1. Prediction of the fundamental period/frequency

Among the various laws used in the literature to predict the fundamental period/frequency in this section we have especially analyzed the ones that depend on one parameter only: the total height of the buildings (*H*). The expressions able to estimate the first natural frequency and that have been considered in this research are the following:

1. Italian Technical Standards for Buildings (NTC08) [21]:

$$f(H) = \frac{1}{0.05 \cdot H^{3/4}} \tag{1}$$

2. Empirical law proposed by Shakya et al. [4]

$$f(H) = \frac{1}{0.0151 \cdot H^{1.08}} \tag{2}$$

Empirical law proposed by Ranieri and Fabbrocino [23]

$$f(H) = \frac{1}{0.01137 \cdot H^{1.138}} \tag{3}$$

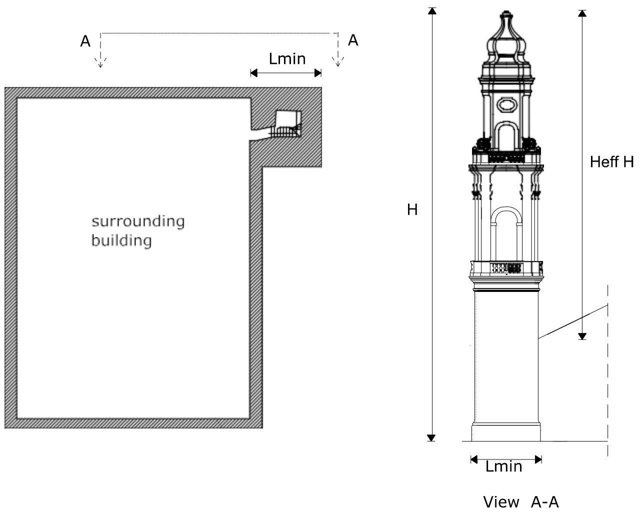
3. Empirical law proposed by Faccio et al [24]]

$$f(H) = \frac{1}{0.0187 \cdot H} \tag{4}$$

The laws look very simple to analyze and similar between each other. They all have an exponential behavior and are different only for the exponential factor and amplitude. The trend is quickly decreasing especially in the range between 10 *m* and 20 *m* of the total height, where the average fundamental frequency assumes a value of 5 *Hz* and 2.5 *Hz*, respectively. Instead, in the subsequent range and up to a total height of 80 m, the decreasing is much slower and the frequency variation is around 1 *Hz*. It is important to observe that the laws comply with a fundamental structural principle for which, considering equal geometry, mass distribution and material, the stiffness of a structure is inversely proportional to the height. In this research the issue regarding the prediction of the fundamental period for the towers has been faced classifying them on the base of their percentage of height constraint. For this reason, the towers introduced in Table 1 have been subdivided in bounded and isolated. The ones belonging to the first class have been reported in Table 2, while the second ones in Table 3. For each tower the first three experimental modal frequencies, the total height (*H*) and the minor side length (L_{min}) are reported (see Figure 4). Moreover, for the bounded case the percentage H_{eff} of the "*effective*" free height has been evaluated (see Figure 4). The values related to the total height for the bounded towers in average are very high (more or less 43 m), assuming a minimum value of 20 *m* up to a maximum of 74.10 *m*. This observation confirms that often to reach high positions for installing the experimental setup is not easy. Moreover, the access to the top is not always allowed through external or internal stairs (due to the period of towers' construction) and so basket platform or other self-propelled vehicles are needed. In the last column the minor side length of the tower's plant is reported, which has been used to calculate the fundamental frequency based on the following empirical law for masonry structures proposed by the Spanish Standards (NCSE 2002, [22])

$$f(H,L) = \frac{\sqrt{L}}{0.06H\sqrt{\frac{H}{2L+H}}} \quad (5)$$

where L is the base dimension in the direction of the oscillation and, consequently, the minor side length indicates the probable direction corresponding to the first modal shape. Also in the Spanish formula H represents the total height of the building. Moreover, the height and length parameters have to be introduced, in all laws, in meters.



(a) (b)

Figure 4. Main geometric features of the towers: (a) plan view; (b) frontal view.

Table 2. Bounded historic masonry towers: experimental frequencies, total height, and minor side length.

Case	Name	$f_{exp,1}$ [Hz]	$f_{exp,2}$ [Hz]	$f_{exp,3}$ [Hz]	H [m]	L_{min} [m]	H_{eff} [%]
1	Sineo’s Tower	-	-	-	39	5.90	62
3	Aversa’s Tower	1.05	1.37	4.81	45.50	14.00	80
4	Matildea Tower*	2.42	2.42	7.52	30.00	7.20	-
5	San Luzi-bell Tower	1.44	1.81	2.47	60.00	5.00	67
6	S. Vittore’s Tower	1.22	1.28	3.60	36.72	5.70	84
8	Trani’s Cathedral	2.04	2.26	7.03	57.00	7.50	75
9	Soncino Tower	1.08	1.11	-	41.80	6.00	74
10	Gabbia Tower	0.92	0.99	3.89	54.00	7.58	63
12	San Gimignano Tower	1.31	1.33	3.41	55.00	9.50	67
13	Annunziata Tower	2.62	2.83	5.51	20.00	2.76	60
15	Roccaverano Tower	1.66	2.26	4.67	23.00	4.25	52
18	S. Maria di Loreto Tower	1.70	1.75	5.30	38.3	4.57	75

19	Monza’s Cathedral	0.59	0.71	2.46	74.10	-	74
20	Maddalena’s Cathedral	4.57	9.15	13.70	34.70	4.11	57
21	University of Coimbra	2.13	2.47	6.56	34.00	5.00	56
22	N. S.ra de la Misericordia	1.29	1.49	4.32	41.00	5.60	72
24	Clock Tower[b] of Provincial Administration	2.30	2.43	4.17	60	9.00	57

*in this case only the numerical frequencies are available.

Table 3. Isolated historic masonry towers: experimental frequencies, total height, and minor side length.

Case	Name	$f_{exp,1}$ [Hz]	$f_{exp,2}$ [Hz]	$f_{exp,3}$ [Hz]	H [m]	L_{min} [m]
2	Capua’s Tower	1.26	1.29	3.10	41.00	11.30
7	Astesiano Tower	-	-	-	36.00	5.00
11	Hagia Sofia Tower	2.45	2.69	7.82	23.00	5.00
14	San Domenico Tower	1.14	1.31	7.26	30.00	5.30
16	Vistula Mounting tower	1.42	1.45	-	22.65	-
17	Clock Tower[a]	2.56	2.76	7.15	20.40	4.00
23	Sant’Andrea’s Tower	0.61	0.73	2.81	58.00	7.64

() The number in the apex, within the round brackets, is referred to the case study in the Table 1

Regarding the bounded towers in Table 2, it must be pointed out that both for the (1)Sineo tower and the (4)Matildea tower no experimental frequency values are available thus in the subsequent analysis these structures have not been taking into account. Moreover, the (8)Trani’s tower is made of assembled concrete and stiff masonry, thus also this case has been excluded in the following as its mechanical characteristics are highly different from the ones of the masonry towers. Also the Clock Tower of the (24)Provincial Administration of Bari has not been considered in the analysis as it is made by cyclopic concrete.

In Figure 5 the first four empirical functions that depend only on the height are plotted. In each graph the points related to the experimental cases have been reported. In particular, the first and second rows (i.e. 5 a-d and 5 e-h, respectively) illustrate the situation for the bounded and isolated towers, respectively. The points in each case are very close to the empirical functions but it is worth to notice an anomaly for the bounded towers’ group with respect to the (20)Maddelena’s Cathedral [14] (A-point in 5 a-d). In this case, the estimated frequency is highly different from the others obtained in similar cases, this circumstance may be due to various factors that are not easily identifiable and whose evaluation is beyond the scope of the present paper. It is however clear that this case represents an anomalous value and consequently it has been excluded in the subsequent analysis. This example was held in Table 2 to illustrate how a check of the robustness and reliability of the experimental results can be carried out also through a comparison with the ones coming from predictive formulations. On the other hand, possible discrepancies between such values can help to solve some uncertainties. Regarding the second group (isolated towers, second row in Figure 5), the formulations seem very good up to a height of 30 *m*, for higher values some differences appear. It must be pointed out that the (16)Vistula Mounting tower has not been taken into account due to its circular transversal section, which is very different from the other ones with a squared cross section, and also the (7)Astesiano tower has been excluded as no experimental frequencies are available.

In Figure 6. the iso-frequency curves based on the empirical law proposed by the Spanish Standards NCSE 2002 (Eq. (5)), are reported in the H and L_{min} parameters' space. The formula shows a rapid change of the predictive frequencies especially going towards high values of the minimum base length and low values of the total height; moreover, this path provides a trend gradually more rigid. It is very evident a wide band in 1-2 Hz frequency range where most of the experimental frequencies for both bounded and isolated cases are collocated. The experimental values seem to agree with the empirical law.

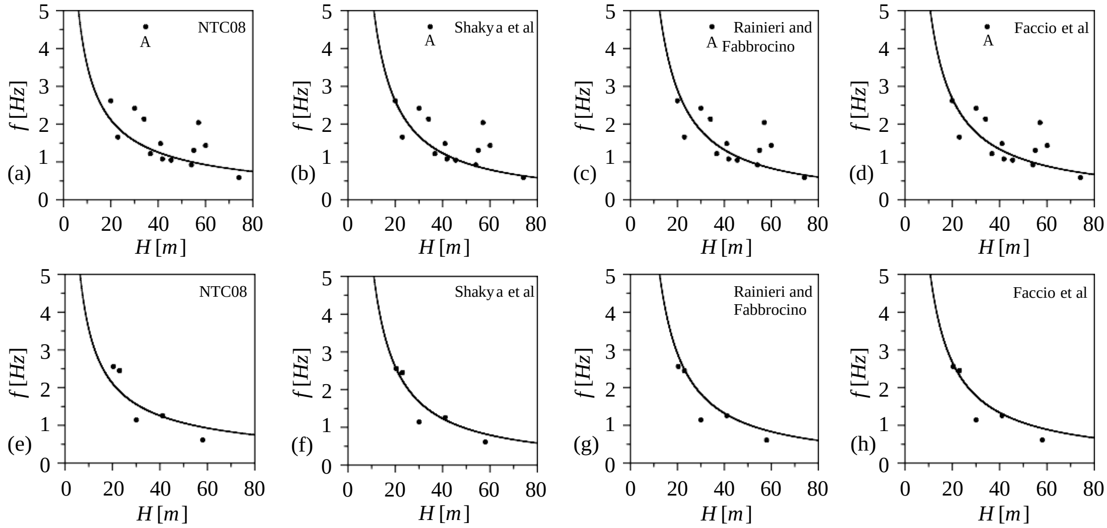


Figure 5. Prediction of the fundamental frequency: (a)-(d) bounded towers, (e)-(h) isolated towers. The "dots" are relative to the experimental frequencies while the function is the predictive function.

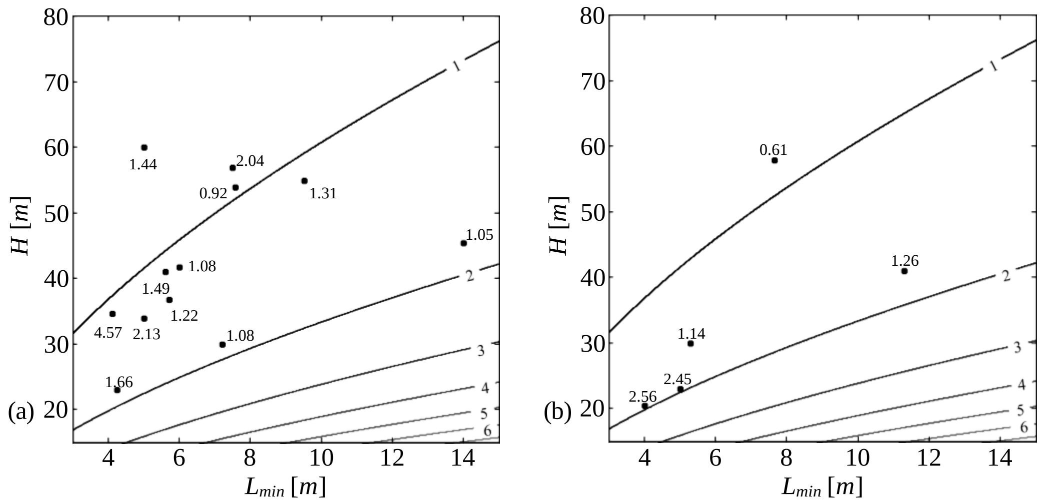


Figure 6. Development of iso-frequency curves based on the Spanish Standards NCSE 2002: (a) bounded tower, (b) isolated towers. The "dots" are relative to the experimental frequencies.

The robustness of all the formulations have been measured through the Mean Squared Error (MSE) whose expression is reported in Eq. (6). This coefficient expresses the mean of the squared difference between the experimental, x_i, and predictive frequencies, $\hat{x}_i$. In Table 4 the MSE values for all the laws previously described, and for both bounded and isolated towers, are illustrated. Looking at the results for the laws depending only on the total height (i.e. the first four columns), they seem very close to each other especially for the bounded towers. Some differences appear in the case of the isolated towers where the values are slightly smaller except for NTC08. In the Spanish formulation, the results are higher although the equation seemed more accurate, depending on two parameters. In any case, it must be noticed that such formulations are relative to generic masonry structures and not specifically to towers.

$$MSE = \frac{\sum_{i=1}^{n} (x_i - \hat{x}_i)^2}{n} \qquad (6)$$

Table 4. Mean Squared Error for some predictive fundamental frequency functions from the literature.

	NTC08	Shakya et al	Ran. and Fab.	Faccio et al	NCSE
Bounded	0.12	0.13	0.15	0.12	0.28
Isolated	0.16	0.08	0.13	0.11	0.27

4. Discussion and correlations

The available data have been analyzed in order to identify a new relation that may improve the accuracy of the first natural frequency's estimation. As aforementioned, the data have been treated by dividing the isolated and bounded towers. Figure 7. and Figure 8. show the plots where the experimental first natural frequency is compared with the ones obtained applying the laws (1)-(5) for both bounded and isolated towers and whose mean square error is listed in Table 4. In particular Figure 7. regards the laws that depend only on the total height, while Figure 8. is relative to the Spanish law that is a function of the minor side length and the total height. In each plot the bisectors that allow a visual evaluation about the goodness of the predictive laws are depicted in the background (dashed gray line). Indeed, more the points' cloud is close to the bisector, more the predicted frequency will be near to the experimental one. Moreover, the bisector separates the graph's area into two triangle zones, lower and upper; they allow to understand if the law tends to predict frequencies more or less rigid with respect to the experimental ones. In fact, if the points are in the upper sector, the predicted frequency will be less rigid (indicated with the sign "-" in Figure 7); vice-versa in the lower sector (indicated with the sign "+" in Figure 7.). It is worth to notice that, in the case of isolated towers (Figure 7. e-h), [16]Vistula Mounting hasn't been included for the reason explained above.

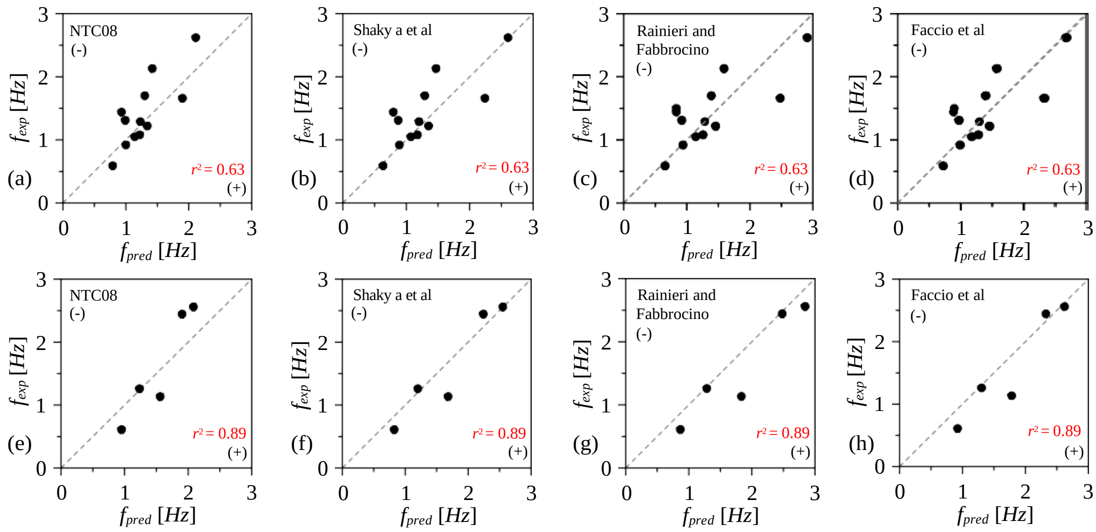


Figure 7. Comparison between experimental and estimated first natural frequencies for bounded (a)-(d) and isolated (e)-(h) towers.

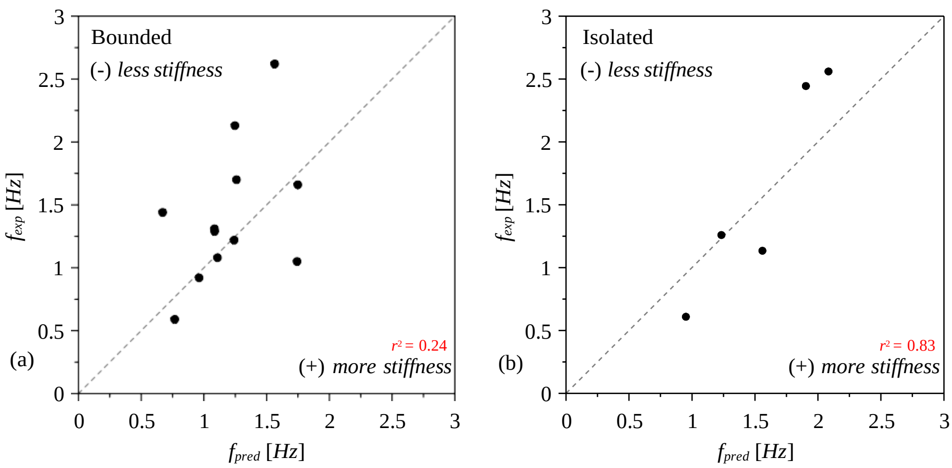


Figure 8. Comparison between experimental and estimated first natural frequency for the Spanish Standards NCSE 2002: (a) bounded, (b) isolated.

Here, the possibility of identifying an exponential law model, which expresses the first natural frequency as a function of the total height of the tower is focused as follow:

$$f = a H^{b} \tag{7}$$

where f_1 is the first natural frequency (expressed in *Hz*), *H* is the total height (expressed in meters), and *a* and *b* are the dimensionless correlation parameters to be evaluated by means of the regression analysis. Eq. (7) can be written as follows:

$$Ln(f_1) = a + b\,Ln(H) \tag{8}$$

where the parameters can be identified by minimizing the mean square error of the linear regression. Moreover, to compare the subsequent new laws, it has been used the determination coefficient "*r*" with the following expression [36]:

$$r^2 = 1 - \frac{\sum_{i=1}^{n} (f_{\exp,i} - f_{1,i})^2}{\sum_{j=1}^{n} \left(f_{\exp,j} - \frac{\sum_{i=1}^{n} f_{\exp,i}}{n} \right)^2} \tag{9}$$

where $f_{exp,i}$ is the *i*-th experimental first natural frequency, *n* the number of data that for the bounded tower is equal to 12, while for the isolated ones is equal to 5; $f_{1,i}$ is the *i*-th first natural frequency evaluated through the new correlation law. More this value is close to 1 and greater is the goodness of the correlation.

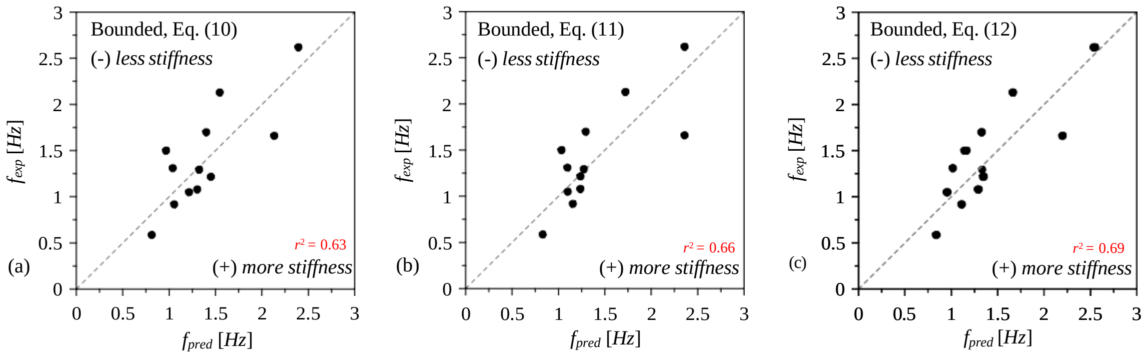


Figure 9. Performances of the two proposed laws (Eq. (10), Eq. (11) and Eq. (12)) for the main frequency's prediction for the bounded towers.

The first analysis has been performed on the bounded towers comparing the experimental first natural frequencies with the ones calculated in accordance with the laws (1)-(5). As aforementioned, the data have been analyzed adopting the exponential law model of Eq. (7). For the bounded towers the regression analysis gives:

$$f_1(H) = 28.35 H^{-0.83} \tag{10}$$

whose determination coefficient is equal to 0.63 and MSE is 0.10. Even if Eq. (10) improves the accuracy of the estimation with respect to the available laws, another analysis has been carried out in order to take into account the lateral constraints sometimes due to the presence of the church. In detail, starting from the observation that the existence of a connection with the surrounding buildings may increase the stiffness of the tower, and thus modifies the natural frequencies, the "*effective*" total height, H_{eff}, has been introduced, which corresponds to the total height above the lateral constraints (see Figure 4). Consequently, the multiple linear regression analysis has been performed assuming the first natural frequency as a function of the "*effective*" total height, the obtained correlation is:

$$f_1(H_{eff}) = 12.96\,H_{eff}^{-0.686} \qquad (11)$$

For Eq.(11), the determination coefficient becomes 0.656 while MSE is 0.103.
To improve further the accuracy, the correlation law has been considered as a function of H_{eff}, L_{min} and the total height H as follows:

$$f_1(L_{min}, H_{eff}, H) = 14.61\,L_{min}^{-0.254} H_{eff}^{-0.341} H^{-0.216} \qquad (12)$$

whose determination coefficient is 0.69 and MSE is 0.084. As in the previous Eqs. (10) and (11), all the coefficients of equation (12) have been determined minimizing the MSE of the linear regression. The performances of the Eqs. (10), (11) and (12) can be visualized in the Figure 9.
Next analysis regards the isolated towers' group for which the evaluated correlation is the following:

$$f_1(H) = 135.343\,H^{-1.32} \qquad (13)$$

In this case the determination coefficient is equal to 0.89 showing a good reliability of the equation. MSE is equal to 0.058. All predictive laws discussed before give a MSE clearly superior (on average more than twice) to the one associated to the Eq. (13).

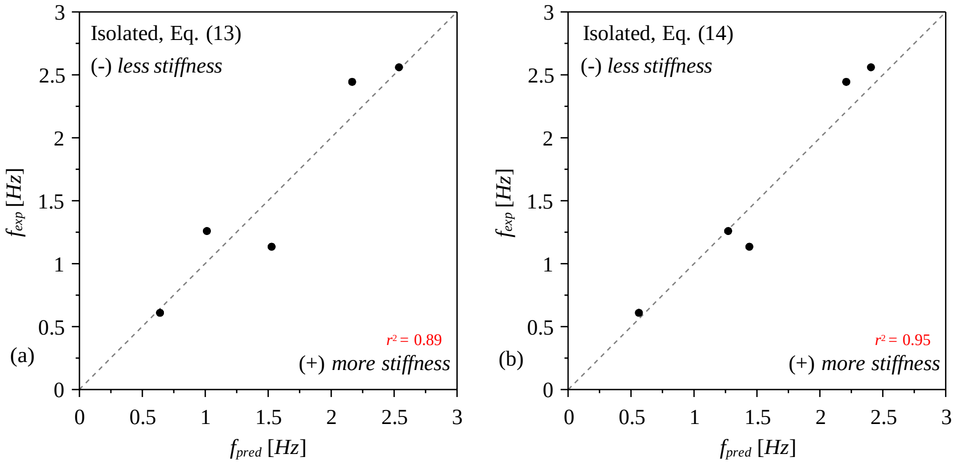


Figure 10. Performances of the two proposed laws (Eq. (13) and Eq. (14)) for the main frequency's prediction of the isolated towers.

To improve the accuracy of Eq. (13), another analysis has been performed taking into account the minimum side of the base transversal section, L_{min}. The application of the multiple linear regression analysis leads to the following law:

$$f_1(H, L_{min}) = 208.54 L_{min}^{0.55} H^{-1.73} \tag{14}$$

where L_{min} is expressed in meters. The determination coefficient is equal to 0.95, while MSE is equal to 0.035. In this case, Shakya et al. law [4] gives a MSE more than 2 times the one of Eq. (14). In order to evaluate the performance of the obtained formulation, the natural frequency evaluated by means of Eqs. (13) and (14) have been compared with the experimental first natural frequency of the examined towers (Figure 10.).

As it can be easily observed, the correlation law for the bounded towers is less reliable than the one evaluated for the isolated towers, this may be due to the different boundary conditions of the tower and/or irregularities along the height.

In the final part some correlations between the fundamental frequency and the subsequent two frequencies are presented. Indeed, such correlations are shown by means of the ratios of the second and third experimental frequencies with the first one for both groups of bounded and isolated towers. In the plots of Figure 11 the dashed-line indicates the mean ratio while the circles and triangles are the ratios for each single case. Regarding the second frequency, the mean is slightly greater than one (1.14 and 1.11 for the bounded and isolated towers, respectively) and it means that the first two frequencies are quite similar to each other. Moreover, the corresponding variance is very low (0.12 and 0.05), proving the robustness of the previous observation.

The examined ratio can be considered representative of the existence of different boundary conditions and/or irregularities along the two principal directions in plan. Otherwise, the situation is not as good, looking at the results related to the comparison between the third and first frequencies. In fact, first, the mean is quite different for the two cases (3.15 and 3.88 for the bounded and isolated towers, respectively, and second, the corresponding variances are very high especially for the isolated towers (see Table 5).

The relation between the second natural frequency and the first one has been evaluated for the bounded cases:

$$f_{2,bounded} = 1.1087\, f_1 + 0.045 \tag{15}$$

and for the isolated ones:

$$f_{2,isolated} = 1.0629\, f_1 + 0.051 \tag{16}$$

Figure 12. reports the behavior and goodness of Eqs. (15) and (16).

Finally, almost no experimental data are reported in the references for the structural damping. For this coefficient, the suggestion is to follow the indications of the different national standards.

The above described results show that the performance of the proposed equations can be considered good enough for a first and fast evaluation of the natural frequencies of a masonry tower. In fact, the main aim of the proposed procedure is not to obtain a complete and exhaustive description of the dynamic behavior of such towers, which is not possible with so simple laws due to the great amount of uncertainties that influence the response of such structures and also their identification procedure. However, these new laws constitute a friendly, easy and fast tool that may help during both the analysis of these structures and also in the assessing about the necessity of performing deeper tests. In this sense, the proposed laws have been evaluated only considering few geometrical parameters.

Table 5. Comparison between the first three experimental frequencies: mean and variance values.

	second frequency / first frequency		third frequency / first frequency	
	Mean	Variance	Mean	Variance
Bounded	1.14	0.013	3.15	0.659
Isolated	1.11	0.004	3.88	2.07

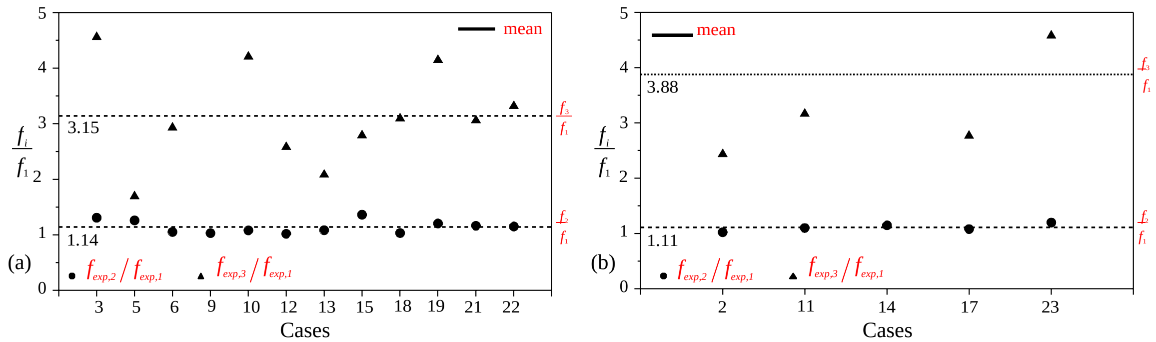


Figure 11. Comparison between the first three experimental frequencies: (a) bounded and (b) isolated towers.

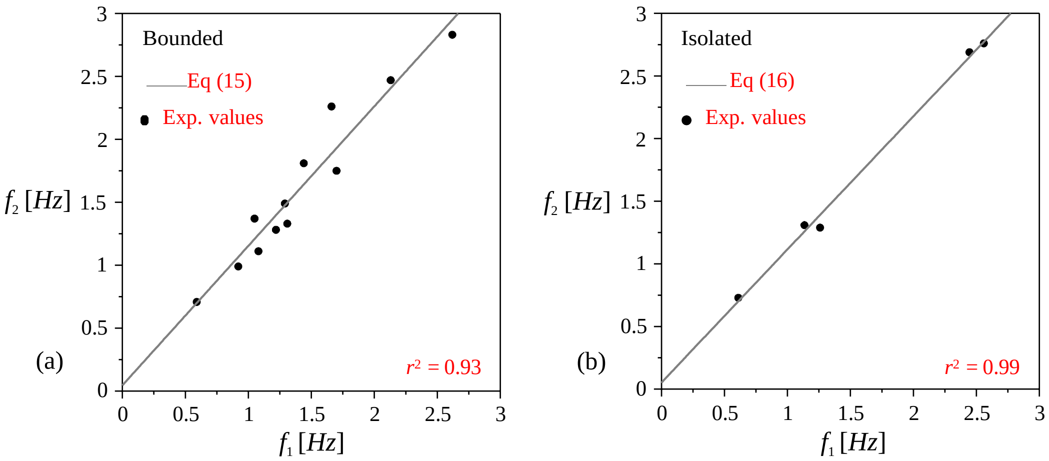


Figure 12. Ratio between second and first experimental natural frequencies: (a) Bounded and (b) Isolated towers. Comparison with the predictive laws of the ratios (continuous line).

5. Conclusions

In this paper a thorough investigation regarding a fast evaluation of the modal characteristics (especially natural frequencies and modal shapes) for historic masonry towers has been carried out. The data have been collected through the wide literature on single cases studied of historic towers. In many situations rapid dynamic tests or dynamic structural monitoring have been performed using more or less the same instrumentation (accelerometers) and technical processing

(SSI or PSD). For these reason, it has been verified if different empirical laws (some of them proposed in literature for generic masonry structures) could interpolate the experimental frequencies. In particular, these laws have been applied to two different groups of towers, bounded and isolated, and compared through the mean squared error. The results appeared quite good, nevertheless, some others new predictive functions have been proposed based on a model of exponential law for both bounded and isolated towers. These laws have been obtained minimizing the mean squared error of the linear regression and compared using the determination coefficient to evaluate the goodness of the correlation. The developed functions show a better approximation ability especially for the ones determined for the isolated towers. Finally, some new correlations between the first three frequencies have been proposed. Unfortunately, for lack of experimental data, no consideration can be given about the prediction of the structural damping and so further insights will be pursued in this direction.
Finally, the authors would derive from this scientific activity some general conclusions and helpful recommendations:

(1) a fast and accurate knowledge of the modal parameters constitute a good starting point for the dynamic investigations and, in this sense, the proposed laws have shown reliable results;
(2) the assessment of the fundamental frequency is very difficult for the bounded masonry tower where, above the height, the rigidity of the same constraint should be carefully considered;
(3) the second frequency seems to be of the same order of the first frequency while the third is larger of 3.5 times compared to the first two.

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