Fast High-Resolution Simulation of the Gross Slip Wear of Axially Symmetric Contacts

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High Resolution Simulation of the Gross Slip Wear

A.V. Dimaki[1,2], A.I. Dmitriev[1,2], N. Menga[3], A. Papangelo[3], M. Ciavarella[3], V.L. Popov[2,4,5]

[1]Institute if Strength Physics and Materials Science, Russian Academy of Science, 634021 Tomsk (Russia)
[2]National Research Tomsk State University, 634050, Tomsk (Russia).
[3]Politecnico di BARI, V. le Gentile 182, 70125 Bari (Italy).
[4]Berlin University of Technology, 10623 Berlin (Germany).
[5]National Tomsk Polytechnic University, 634050, Tomsk (Russia)

In the present paper, we study development of wear profile under conditions of gross slip and assumption of the Reye-Archard wear criterion. Simulations are carried out using the Method of Dimensionality Reduction and a full FEM formulation. The calculation time of the proposed model is several orders lower than that of FEM-based models and allows for much higher spatial resolution. High resolution simulation revealed a feature which has not been previously reported: the pressure distribution in the worn region has a peak in the vicinity of the outer border of the contact region.

Keywords: Wear, gross-slip, method of dimensionality reduction

1. Introduction

Wear is inevitable and is one of the major causes of malfunctioning and disservice of engineering components in any mechanical part subjected to contact with friction. A large literature exists on the subject, both on experimental and modeling side, since wear regimes depend on many variables and conditions: for a given materials pair, the very first parameters are mechanical pressure and speed, which in turn influence temperature at interface and in the bulk. Lim and Ashby (Lim and Ashby, 1987) tried to locate regimes and mechanisms in wear maps, in order to explain the orders of magnitude variation in the wear coefficient. In the most practical cases, for theoretical description of wear, the simplest wear equation is used stating that the wear volume is proportional to the dissipated energy and inversely proportional to the hardness σ_0 of the worn material. This kind of wear criterion was first proposed by (Reye, 1860) and later justified in detail theoretically and experimentally for abrasive wear (Khrushchov and Babichev, 1960) and for adhesive wear (Archard and Hirst, 1956). The local formulation of this criterion means that the linear change $\Delta f(r)$ of the three-dimensional profile $f(r)$ (where r is the polar radius in the contact plane) is given by the equation

$$\Delta f(r) = \frac{k_{wear}}{\sigma_0} \tau(r) \ \Delta u_x^{(0)} - \Delta u_x^{(3D)}(r) \qquad (1)$$

where $u_x^{(0)}$ is the tangential displacement of the "indenter", $u_x^{(3D)}(r)$ is tangential displacement of the "substrate" due to elastic deformation, $\tau(r)$ is tangential stress and symbol Δ means the increment of corresponding parameter during one step of spatial displacement. In this wear law, all the influencing parameters are “absorbed” in the empirical dimensionless wear coefficient k_{wear}.

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Generally, the process of wear described by Eq. (1) is stable as regions of higher pressure are worn faster, and the general trend is therefore to achieve more uniform pressure. However, this is not always the case, since for example due to dynamical effects, the presence of corrugations can become unstable and grow in time (see e.g. the short pitch corrugation "enigma" in railways or metro lines (Ciavarella, Afferrante, 2009, Afferrante, Ciavarella, 2011). One greatly simplifying assumption is to consider a constant contact area and this is realistic in some geometries like in brakes systems (see e.g. the book of Goryacheva (Goryacheva, 1998)): this permits to use a large class of linear techniques, including eigenfunction expansion of the wear solution in various "modes" (Liu et al, 2014). In the general case, it is not possible to neglect the change of contact area during the wear process, and this requires a much more demanding solution since the contact area extension is possibly defined at the boundary between worn and unworn geometry, and an error in this edge region can grow in time and make unstable the entire procedure.

There are various FEM or BEM implementation of wearing contact problem using Archard law, see e.g. (Mattei et al, 2011, Põdra, Andersson, 1999, Sfantos, Aliabadi, 2007), and generally there is no alternative for parts of complex geometry. The most obvious implementation is an explicit formulation where the geometry of the contacting bodies is updated at each time step (an Euler formulation), and this is known to be only conditionally stable, requiring a sufficiently small time step. Implicit formulations have been also proposed with some success (Stupkiewicz, 2013), but the advantage in terms of time step - size comes at the cost that the shape transformation mapping constitutes an additional unknown (the problem is solved simultaneously for the nodal positions and displacements). Moreover, in the general case a re-mesh of the geometry is necessary, and this further adds to the computational cost. Clearly, even just for reference purposes, it would be useful to dispose of an analytical solution, if even in the simplest contact configurations, like the Hertz problem. However, the only approximate closed form solution known is that using the Winkler approximation for the elastic behavior, explained in the book of Goryacheva (Goryacheva, 1998) for the 2D case. In the best of the authors' knowledge, a full solution, necessarily numerical, for the Hertz problem between half-spaces, i.e. using the integral equation formulations, has not been attempted before. This will be the scope of the present paper. In particular, we shall provide a solution with the so-called Method of Dimensionality Reduction, that recently has been applied for a simulation of fretting wear (Dimaki et al, 2014, Li et al, 2014), which in the present form is exact as it corresponds to the classical Galin-Sneddon solution of contact problems (Galin, 1961, Sneddon, 1965), with an additional physical interpretation in transforming an axisymmetric geometry in a 1D Winkler problem. The results will then be compared with a full FEM simulation. Also we will demonstrate that MDR-based model is able to reproduce the effects that are difficult to obtain in FEM models (due to very high time cost), like fine details of pressure distribution.

2. General equations of the model and calculation algorithm

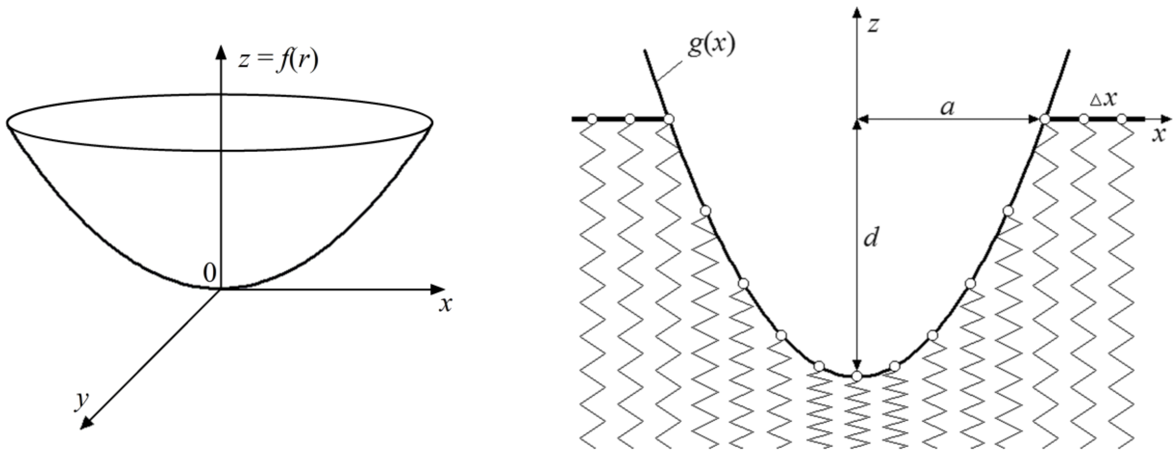
In this section we quickly recapitulate the main rules of the method of dimensionality reduction (Li et al., 2014, Popov and Hess, 2014). We consider a contact of a three-dimensional rotationally symmetric profile $z = f(r)$ and an elastic half-space. The profile is first transformed into a one-dimensional profile $g(x)$ according to the MDR-rule (Heß, 2012, Popov and Heß, 2013)

$$g(x) = |x| \int_0^{|x|} \frac{f'(r)dr}{\sqrt{x^2 - r^2}} \tag{2}$$

as illustrated in Fig. 1. The reverse transformation is given by the integral

$$f(r)=\frac{2}{\pi}\int_0^r \frac{g(x)}{\sqrt{r^2-x^2}}\,\mathrm{d}x \tag{3}$$

In case of a parabolic indenter $f(r)=r^2/2R$ the corresponding one-dimensional profile is $g(x)=x^2/R$.



a) b)

Fig. 1. The 3-dimensional body of revolution (a); and the corresponding one-dimensional MDR-transformed profile in a contact with the elastic foundation.

The profile (2) is pressed to a given indentation depth d into an elastic foundation consisting of independent springs with spacing Δx (Fig. 1b) whose normal and tangential stiffness are given by (Popov and Heß, 2013)

$$\begin{aligned} k_z &= E^*\Delta x \\ k_x &= G^*\Delta x \end{aligned} \tag{4}$$

where E^* is the effective elastic modulus

$$\frac{1}{E^*}=\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2} \tag{5}$$

and G^* the effective shear modulus

$$\frac{1}{G^*}=\frac{(2-\nu_1)}{4G_1}+\frac{(2-\nu_2)}{4G_2} \tag{6}$$

E_1 and E_2 are the Young's moduli, G_1 and G_2 the shear moduli of the indenter and the half - space, and ν_1 and ν_2 are their Poisson-ratios. Note that throughout this paper, we assume that the contacting materials satisfy the condition of "elastic similarity"

$$\frac{1-2\nu_1}{G_1}=\frac{1-2\nu_2}{G_2} \tag{7}$$

that guarantees the decoupling of the normal and tangential contact problems (Johnson, 1987). The vertical displacement of an individual spring is given by

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$$u_z(x) = d - g(x) \tag{8}$$

and the resulting normal force is equal to

$$F_z(x) = E^* \Delta x \;\; d - g(x) \;\; . \tag{9}$$

The linear force density is therefore

$$q_z(x) = \frac{F_z(x)}{\Delta x} = E^* u_z(x) = E^* \;\; d - g(x) \;\; . \tag{10}$$

The contact radius a is determined by the condition

$$g(a) = d\,. \tag{11}$$

The total normal force is obtained by integration over all springs in contact:

$$F_z^{total} = 2E^* \int\limits_0^a \; d - g(x) \; \mathrm{d}x\,. \tag{12}$$

According to the MDR rules, the distribution of normal pressure p in the initial three-dimensional problem can be calculated using the following integral transformation (Heß, 2012, Popov and Heß, 2013):

$$p(r) = -\frac{1}{\pi}\int\limits_r^\infty \frac{q_z{}'(x)}{\sqrt{x^2 - r^2}}\,\mathrm{d}x = \frac{E^*}{\pi}\int\limits_r^a \frac{g'(x)}{\sqrt{x^2 - r^2}}\,\mathrm{d}x \tag{13}$$

When simulating gross slip regime of wear, we may suggest (excluding the very beginning of the motion) that shear stresses are directly proportional to normal pressure:

$$\tau(r) = \mu p(r), \tag{14}$$

where μ is the coefficient of friction. In accordance with mentioned suggestion, the tangential displacement of indenter is always equal to

$$\Delta u_x(x) = \Delta u_x^{(0)} - \mu F_z(x) \,/\, k_x\,. \tag{15}$$

Distribution of relative tangential displacements in the initial three-dimensional problem is defined by equation similar to (3) (Popov and Heß, 2013):

$$\Delta u_x^{(0)} - \Delta u_x^{(3D)}(r) = \frac{2}{\pi}\int\limits_0^r \frac{\Delta u_x(x)dx}{\sqrt{r^2 - x^2}}\,, \tag{16}$$

Note that all above results obtained by the MDR represent *exact* solutions of the corresponding three-dimensional problem. As was shown by Galin (Galin, 1961), the transformation (2) maps the complete three-dimensional contact problem to a one-dimensional contact with an elastic foundation. All three-dimensional properties (as displacements, stresses and so on) can be obtained for the solution of the linear elastic foundation problem by appropriate integral transformations. This solution is *exact* and was used later in the well-known publication by Sneddon (Sneddon, 1965).

3. Verification of the model

In order to verify the proposed approach we developed an ANSYS axisymmetric model of a sphere and a rigid plane surface with about 250 nodes in the contact region. In order to drop down the computational effort, the sphere is “statically condensed” by using a “superelement” formulation, in which only the nodes potentially undergoing contact are retained in the systems of equations. Further, contact has been modeled using CONTA178 node-to-node contact

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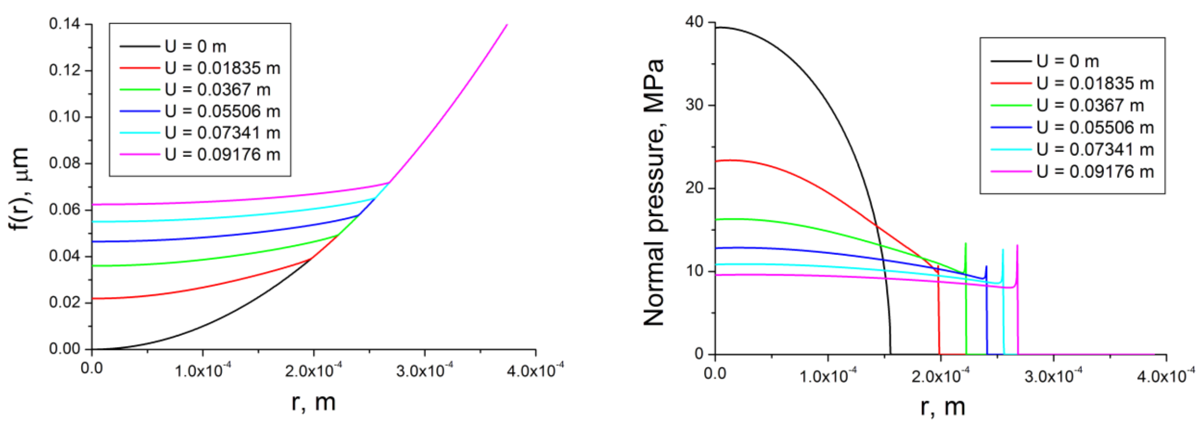
elements, and these elements allow introducing a virtual gap between bodies, which in our case is used to model wear. Hence, a subroutine computes the local wear depth proportional to the local pressure at the given step, and adds this length to the virtual gap of the corresponding contact element. This procedure is clearly possible because the geometry during sliding remains self-similar and only one of the two bodies is wearing. In general, it would not be possible to use these node-to-node elements, and in more complex node-to-surface elements, which allow large sliding features, it is not possible (at least in standard formulation) to simply add "wear" artificially as virtual gap. As expected in any Euler forward integration process, the solution process is stable only for sufficiently small time step (which depends on the spatial step size of the contact elements). This FEM procedure is very simple to implement, does not require re-meshing of the worn profiles, and used in conjunction with the superelement formulation, in which only the degrees pf freedom of the nodes in contact are active, results in a very fast process.

The verification has been carried out by means of the comparison between profiles of the worn indenter, obtained in MDR-based model and in FEM simulation (Fig. 2a). Also, we compared the distributions of normal pressure in the contact area (Fig. 2b). One can see good agreements between results of three-dimensional FEM calculation and one-dimensional model.



a) b)

Fig. 2 Profiles of worn indenter (a) and normal pressure distributions (b) obtained in the presented model and in three-dimensional FEM model and MDR-based model.[V1]

4. Pressure peaks at the border of the worn region

In Fig. 2b, one can see that the pressure distribution in the worn contact has a peak at the border of the worn region. To understand the nature of this peak, let us note that the worn profiles are very similar to "flattened spheres". The pressure distribution in a contact of a flattened sphere, however, has a singular peak at the border of contact. To show this, consider the profile of the blunted (flat) tip (Fig. 3a)wwhich cam be described by the equation

$$f(r) = \begin{cases} 0, 0 < r < b \\ \dfrac{r^2 - b^2}{2R}, r \geq b \end{cases}, \tag{17}$$

where R – is radius of curvature of paraboloid, b – is radius of blunted tip. The corresponding one-dimensional profile according to Eq. (2) is equal to

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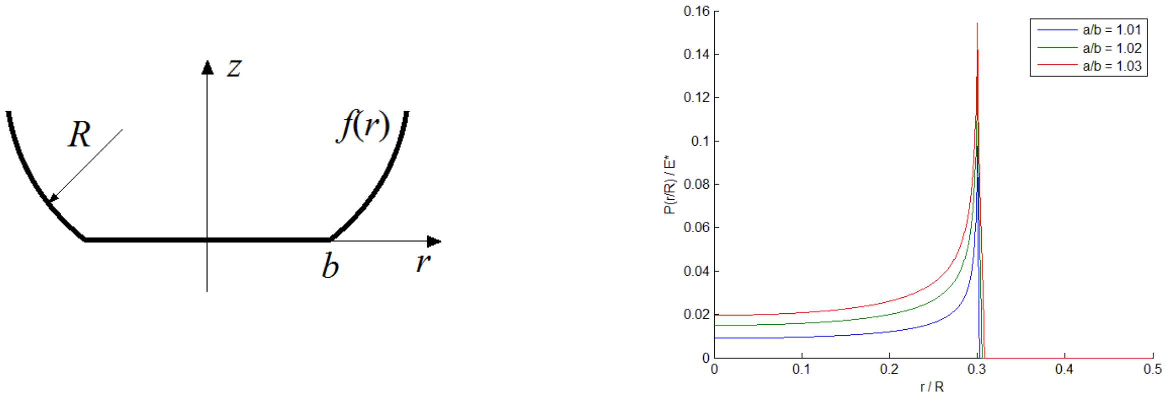
$$g(x) = \begin{cases} 0, 0 < |x| < b \\ \dfrac{|x|}{R}\sqrt{x^2 - b^2}, |x| \geq b \end{cases}. \tag{18}$$



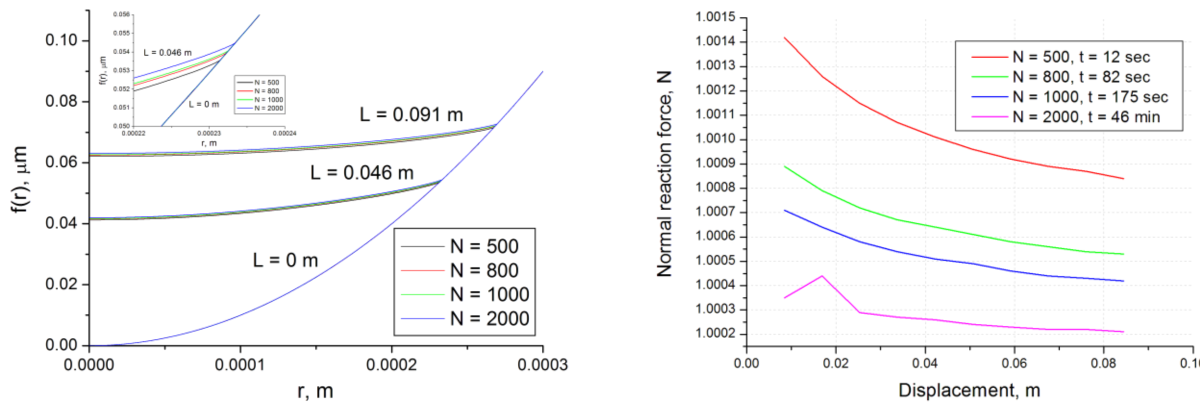
a) b)

Fig. 3 The profile of indenter with blunted tip (a), 3D pressure distribution in a contact with a flattened elastic sphere (b).

The distribution of the normal pressure in contact area is calculated using (13) with $q'(x)$

$$q'(x) = \frac{dq(x)}{dx} = \begin{cases} 0 & x < b \\ \dfrac{1}{R}\left(\sqrt{x^2 - b^2} + \dfrac{x^2}{\sqrt{x^2 - b^2}}\right) & b \leq x \leq a \\ 0 & x > a \end{cases} \tag{19}$$

Substituting (19) into (13) and integrating Eq. (13) numerically, we obtain the normal pressure distribution in the contact area (Fig. 3b). One can see that $p(r)$ has the singularity at $r = b$. Of course, during the wear process, the profile of the indenter cannot achieve the absolutely flat shape of the tip. However, the worn profiles seem to show relatively sharp edge, so that one should not wonder about the pressure peak at the contact border.



a) b)

Fig. 4 The profiles of worn parabolic indenter (a) and normal reaction force (b) versus tangential displacement, obtained for different numbers of spatial steps in the simulated region.

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The very important question that determines the applicability of a model is the rate of convergence of the results with decreasing of spatial step in simulated area, or, in other words, with increasing of a number of spatial steps for a given size of simulated area. Parameter studies have shown that the calculated profiles slightly change with the number of elements used, and the convergence is achieved at approximately $N = 800$ elements (see Fig. 4a). The same is true for the values of the normal reaction force (Fig. 4b). At that, the simulation up to "almost complete wear" takes about 1 minute for number of elements $N = 800$, while for $N = 500$ the results remain quite precise and the calculation takes about 10 second.

5. Conclusions

We have proposed the model of wear in gross slip regime, based on the method of dimensionality reduction. The model allows simulation of wear of indenter that represents a body of revolution. Comparison of the results of simulation with the results of direct FEM 3D modeling confirms within the precision of FEM model the correctness of proposed approach. However, the MDR based program is much faster and allows much higher spatial resolution compared with FEM model. In particular, the high resolution simulation with MDR revealed the appearance of a peak of pressure at the outer border of the contact area between an indenter and a substrate.

6. Acknowledgment

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johnson*Andrey V. Dimaki:*

is the researcher of the Laboratory of Computer-Aided Design of Materials at the Institute of Strength Physics and Materials Science since 2007. He studied computer science and received his PhD in 2006 at the Tomsk State University of Control Systems and Radioelectronics.

Andrey I. Dmitriev:

is the leading staff scientist of the Laboratory of Computer Aided Design of Materials at the Institute of Strength Physics and Materials Science of the Siberian Branch of the Russian Academy of Science (ISPMS SB RAS). Since 2006 he is the professor at the Tomsk State University. He habilitated in 2006 and received his PhD in 1997 at the ISPMS SB RAS. He studied physics at the Tomsk State University and graduated in 1997.

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Michelle Ciavarella:

is t

Valentin L. Popov:

is full professor at the Berlin University of Technology. He studied physics (1976-1982) and obtained his doctorate in 1985 from the Moscow State Lomonosov University. He worked at the Institute of Strength Physics of the Russian Academy of Sciences. Since 2002, he has headed the department of System Dynamics and the Physics of Friction at the Berlin University of Technology. His areas of interest include tribology, the influence of friction through ultrasound, numerical simulation of frictional processes, research regarding earthquakes, as well as mechanics of elastoplastic media, strength of metals and alloys, and shape memory alloys.