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PAPER

Optimal design of energy harvesting from vibration subject to stochastic colored Gaussian process

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Keywords: energy harvesting, random vibration, covariance analysis, piezoelectric materials

OPEN ACCESS

RECEIVED
8 December 2017

ACCEPTED FOR PUBLICATION
19 July 2018

PUBLISHED
11 February 2019

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Abstract

The problem treated in this paper is the optimal electromechanical setting of energy harvesters working with vibrations of random nature. We consider a simple system composed of two parts; one electrical circuit with a mechanical single-degree-of-freedom system, coupled to a piezoelectric element. The electric circuit is modeled as a simple one, composed of a resistance and a capacity only, while a linear viscoelastic model is used for the mechanical element. The only joint electro and mechanical element is the piezoelectric device, which is controlled by both the mechanical velocity and the electrical tension. This scheme is treated as dimensionless using the random vibration theory since the base excitation is considered as a stationary white noise Gaussian process. To consider the non-uniform frequency content, that characterizes many real vibration phenomena, the input is properly colored using simple linear filters. System response statistics is evaluated by covariance approach, producing both covariance matrix of system parameters and mean value of electric power under different filters and systems configurations. The optimal ratio between the periods of the mechanical and electric systems is thus defined maximizing the mean power value. This ratio is obtained numerically for some test cases in a graphic representation to evaluate the sensitivity response to selected parameters.

1. Introduction

Energy harvesting is the term used to describe processes of capturing and storing energy that derives from external environmental sources such as thermal energy, solar power, salinity gradients, wind energy, and kinetic energy. Despite that they provide weak power for low-energy electronics, they are still effective sources for some specific technologies such as small wireless autonomous devices like the ones used in wearable electronics and wireless sensor networks. This is because the energy source for energy harvesters is present as free ambient background. Environmental vibrations comprise a a very promising class among all the different possible ambient energy sources. Examples of environmental vibrations include earthquake ground shaking, pressure of wind or sea waves, aircraft or automotive vibrations; just to cite a few. Preliminary work on this type of energy harvesting has dealt with deterministic excitations, which have been the subject of extensive research over the last decade (Erturk and Inman 2011 is a complete and comprehensive state of the art in this field).

The design of energy harvesting devices (EHD) should be tailored to real environmental vibrations in order to maximize the energy efficiency. In some cases, the input should be assumed as a single frequency signal, so that most studies have designed resonant harvesting devices for pure sinusoidal excitations such as constant rotational regime machinery. In addition, these EHD have to be tuned to the main excitation frequency especially if they are strongly sensitive to input variations in such cases. On the other hand, there are many real cases where the ambient energy has a random and broadband nature. Thus, the design of the harvester must account for this excitation characteristic since the hypothesis of assuming a harmonic excitation input is

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sometime too restrictive. Such examples include the mechanical vibration of trains or cars, machinery with variable rotation regime, and traffic-induced vibrations. For instance, the conversion of vibrations into electrical energy in civil structures under normal loading conditions has been addressed in several studies. (Elvin *et al* 2006, Ali *et al* 2011, Galchev *et al* 2011, Peigney and Siegert 2013, McCullagh *et al* 2014, Zhang *et al* 2014, Maruccio *et al* 2016).

The inner nature of vibrations in all the mentioned cases is a random one. Hence, the use of the random vibration theory is necessary for an accurate study. Previous work on vibration energy harvesting due to random excitations considered the Gaussian broadband excitation of linear and nonlinear harvesters (Gammaitoni *et al* 2009, Daqaq 2010, Litak *et al* 2010, Khovanova and Khovanov 2011, Halvorsen 2013, Zhao and Erturk 2013, Green *et al* 2013). This approximation is widely used in many stochastic problems and focuses on using a stack configuration and on harvesting broadband vibration energy, which is a more practically available ambient source. It assumes that the ambient base excitation is stationary Gaussian white noise, which has a constant power-spectral density across the considered frequency range.

Subsequently, several publications proposed analytical, numerical, and optimization approaches for energy harvesters with random excitations (Adhikari *et al* 2009, Daqaq 2011, Daqaq 2012, Adhikari *et al* 2016). Most of these studies consider the ensemble average properties of the quantity of interests, such as the voltage or harvested power. In this field, the system response in two simple but relevant cases has recently been evaluated considering a white noise input (Adhikari *et al* 2009, Adhikari *et al* 2016). The two cases have involved the harvesting circuit with and without an inductor. The authors have also evaluated the mean power acquired from a piezoelectric vibration-based energy harvester subjected to random base excitation is derived using the theory of random vibrations. Considering the input as a white noise is not able to present an accurate description of the actual frequency content of the vibration input. This in turn reduces the efficiency of the design of piezoelectric energy harvesters that strongly depends on the realistic representation of the excitation. The importance of properly-described spectral content and evolutionary nature is related to the dependency of the power absorbed by a linear oscillator upon excitation by a pure white noise base acceleration on the mass of the oscillator and the spectral density of the base motion only. Under this observation, there is an upper bound on the energy that can be harvested from a linear oscillator under broadband excitation, regardless of the stiffness of the system or the damping factor. This work shows that the same result (upper limit to the energy harvested by a mechanical system) is valid to any multi-degree-of-freedom nonlinear system subjected to a white noise base acceleration. The only restriction to this result is that the internal forces are assumed to be a function of the instantaneous value of the state vector.

Moreover, this aspect is fundamental for designing such piezoelectric energy harvesters. This is because the total power absorbed by both a linear or nonlinear multi-degree-of-freedom electromechanical system subjected to a white noise base acceleration exclusively depends on the spectral density of the base acceleration and the total mass of the system. Hence, the improvement of performance of a broadband EHD is very limited in terms of design. Introducing nonlinear components or additional degrees of freedom will be effective only to the extent to which the mass of the system will change.

Nevertheless, all the reported studies are based on the hypotheses that the excitation is a white noise process, and the actual absorbed power is sensitive to the actual bandwidth of the excitation. The evaluation of the stochastic response of a piezoelectric mechanical system is of interest in the present study, for which a stochastic model of base motion must be employed. An ideal model should reasonably reflect the salient characteristics of the recorded motions pertaining to a real vibration phenomenon in different environmental situations. The most important characteristic that must be included is the variation in the frequency contents. In this paper, the base acceleration is modeled by means of a second-order linear filter, differently from what has been reported until now, in order to overcome the main limitation of simple white noise that presents the same energy for all frequency range.

This study upgrades the white noise model used for input vibrations considering the input as a filtered process that has specific frequency content instead of broadband white noise. This is done to preserve the real frequency content in real random vibrations.

The method of filtered white noise input is used to describe the response of a simple but relevant case, namely the piezoelectric harvesting circuit without an inductor coupled with a linear single mechanical system. The described system is a coupled electromechanical system with one-degree-of-freedom system where a mechanical system is coupled with an electric circuit. The coupling element is a piezoelectric device that transforms mechanical vibrations into tension and vice versa. The electromechanical system is modelled by two coupled differential equations and the random vibration analysis is performed using the covariance approach in time domain. The white noise hypothesis is thus relaxed adopting a double linear second order pre-filtered model of input. The Lyapunov covariance equation is formulated in a dimensionless way, obtaining system response statistics (covariance matrix) and mean value of harvested power.

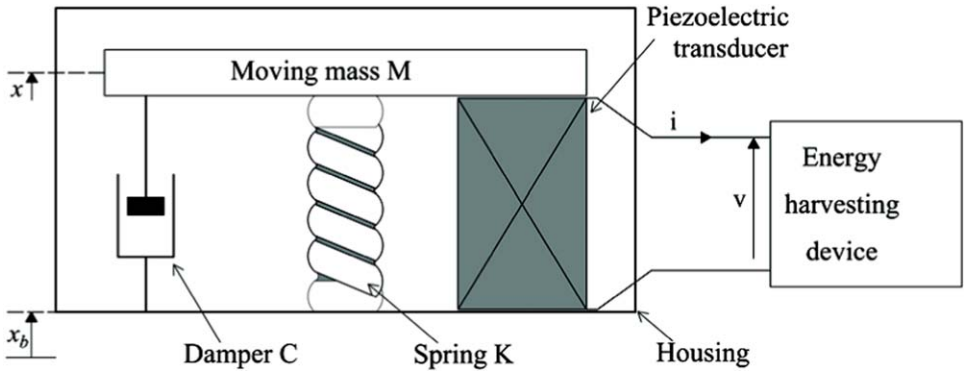


Figure 1. Scheme of SDOF linearly connected to a vibration base equipped with a piezoelectric device.

The optimal ratio between mechanical/electric time ratios is thus defined as the value that is able to maximize the dimensionless performance of the piezoelectric vibration-based energy harvesters. This ratio will provide the basic design guidelines for different conditions of vibration energy harvesters. Using this approach, the optimal ratio is thus obtained for some frequency content conditions by using a standard genetic algorithm. The results are presented for different dimensionless electromechanical coupling coefficients and structural damping. Results, posed in a graphical way, demonstrate the effectiveness of the proposed method to maximize the output power and efficiency of vibration energy harvesters in a wide number of real applications where detailed information of input frequency content are available. Furthermore, they demonstrate results that contribute strongly to environmental sustainability.

2. Piezoelectric electro-mechanical coupled model

In this paragraph, we consider a stack-type piezoelectric harvesting circuit. The mechanical and electric scheme of the circuit is depicted in figure 1. We consider the simplest type of electrical circuits (without inductor), and energy is harvested through base excitations, while piezoelectric device is operated in vertical direction. A simple single-degree-of-freedom model is used for the mechanical motion of the harvester, without losing the generality of analysis that should be extended to single-degree-of-freedom model that accounts for distributed mass effects.

This work only considers a linear model of the piezoelectric material along the vertical direction, which allows the application of the linear random vibration theory.

Various types of piezoelectric harvesting devices are available, integrating stack or patch transducers. The single-degree-of-freedom system chosen is this work can represent such mechanical behavior. The electric circuits coupled to the piezoelectric device is the simplest system considered in the literature, specifically consisting only of a resistor and a capacitor without inductors, as shown in figure 2.

The θ quantity is the electromechanical coupling coefficient. Assuming a piezoelectric system that is operating in the {33} mode, it is expressed by the following:

$$\theta = -\frac{e_{33}A_P}{t_P}(\mathrm{C\ m^{-1}}) \tag{1}$$

where e_{33} is the piezoelectric constant in the {33} direction, A_P and t_P are the piezoelectric cross-sectional area and thickness, respectively.

The considered system is a generic piezoelectric vibration energy harvester that can be modeled as a base-excited SDOF system coupled to a capacitive energy harvesting circuit, as reported in equations (2) and (3).

$$m\ddot{x} + c\dot{x} + kx - \vartheta v = -m\ddot{x}_b \tag{2}$$

$$\theta\dot{x} + C_P\dot{v} + \frac{1}{R}v = 0 \tag{3}$$

$\bar{x}$ and $\bar{x}_b$ are the mass and base displacement in equation (2), respectively. The dynamic force balance is formulated considering also a piezoelectric force f_{pz}. The standard nomenclature is used for mass m, damping c, and elastic stiffness k; while ϑ is the electromechanical coupling. The piezoelectric mechanical force is modeled as proportional to the voltage across the piezoelectric device $f_{pz} = \theta v(t)$, where $v(t)$ is the tension acting at the ends of piezoelectric device in the electrical circuit. It is modeled by equation (3), where the tension v_{pz} measured across the equivalent resistive load; R arises from the mechanical strain through the electromechanical coupling

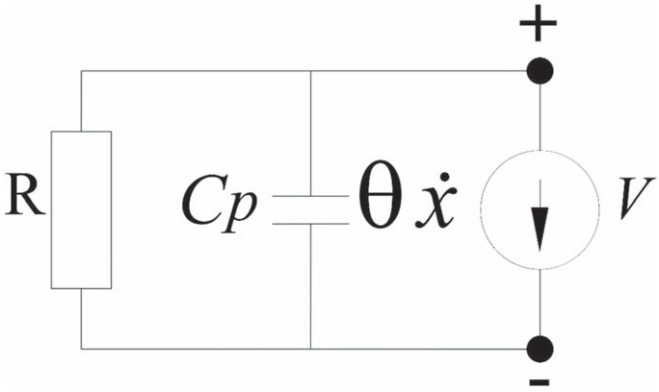


Figure 2. Electric energy harvesting circuit without an inductor.

ϑ as $v_{pz} = \theta\dot{x}$; and finally C_p is the capacitance of the piezoelectric element (Adhikari *et al* 2009, Erturk and Inman 2011, Adhikari *et al* 2016).

A common stochastic model used to describe the system base excitation $\ddot{x}_b$ is the broadband Gaussian white noise. However, in many real cases, the vibration phenomena of interest presents a well-defined frequency content, far from being considered as a constant one.

A consolidated and simple stochastic model for stationary signals with non-uniform frequency contents is achieved by filtering a white noise one, such as the well known Kanai–Tajimi model (Kanai 1957, Tajimi 1960). It uses a second-order linear single filter whose equation that describes the base EHD acceleration is expressed in equation (4):

$$\ddot{X}_b = \ddot{X}_g + w(t) = -2\xi_g\omega_g\dot{X}_g - \omega^2{}_gX_g \tag{4}$$

where the capitol letter is used for stochastic processes instead of deterministic time signals. The second order linear filter $\ddot{X}_g + 2\xi_g\omega_g\dot{X}_g + \omega^2{}_gX_g = w(t)$ is applied to an ideal stationary white noise process $w(t)$. $\ddot{X}_b$ is the filtered input process; ω_g and ξ_g are the two filters parameters. An upgrade to this model, developed to eliminate the quasi-static frequency content, has later been proposed by Clough and Penzien (Cloug considering a linear fourth-order filter forced by a modulated white noise. Therefore, the base acceleration $\overline{\ddot{X}}_b(t)$ is given by the following two differential equations:

$$\ddot{X}_b = -2\xi_p\omega_p\dot{X}_p - \omega^2{}_pX_p + 2\xi_g\omega_g\dot{X}_g + \omega^2{}_gX_g \tag{5}$$

$$\ddot{X}_p(t) + 2\xi_p\omega_p\dot{X}_p(t) + \omega^2{}_pX_p(t) = \ddot{X}_g(t) + w(t) \tag{6}$$

where a new second-order filter is applied with ω_p and ξ_p as parameters. The stochastic system that describes the filtered input is:

$$\begin{cases} \ddot{X}_b(t) = \ddot{X}_p(t) = -\omega^2{}_pX_p(t) - 2\xi_p\omega_p\dot{X}_p(t) + \omega^2{}_gX_g + 2\xi_g\omega_g\dot{X}_g(t) \\ \ddot{X}_p(t) + \omega^2{}_pX_p(t) + 2\xi_p\omega_p\dot{X}_p(t) = \ddot{X}_g(t) + w(t) = \omega^2{}_gX_g + 2\xi_g\omega_g\dot{X}_g(t) \\ \ddot{X}_g(t) + 2\xi_g\omega_g\dot{X}_g(t) + \omega^2{}_gX_g = -w(t) \end{cases} \tag{7}$$

Stationary gaussian white noise process is characterized by bilateral Power Spectral Density energy S_0. Several expressions that relate the Peak Acceleration (PA) to S_0 have been proposed in the literature, However, S_0 is considered a scattering variable of the problem in our case.

Thus, the spectral density for stationary condition is:

$$PSD(\omega) = S_0 \frac{1 + 4\xi^2{}_f\left(\frac{\omega}{\omega_f}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_f}\right)^2\right]^2 + 4\xi^2{}_f\left(\frac{\omega}{\omega_f}\right)^2} \frac{\left(\frac{\omega}{\omega_p}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_p}\right)^2\right]^2 + 4\xi^2{}_p\left(\frac{\omega}{\omega_p}\right)^2} \tag{8}$$

Eight different cases are reported in figure 3 as examples where only ψ_g varies obtaining different spectral configurations.

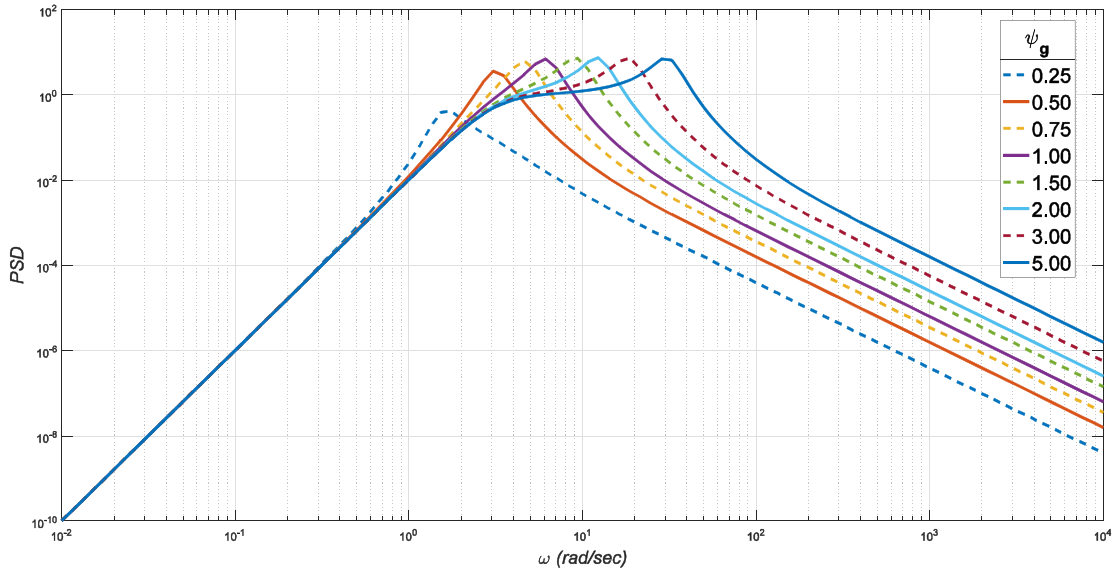


Figure 3. Clough Penzien spectra for different values of frequency ratio ψ_g. The main system period is assumed as 1 s ($\omega_0 = 2\pi$ (rad/ sec)) while $\psi_p = 0.5$ and $\xi_g = 0.2$ and $\xi_p = 0.7$.

Finally, using equation (7), the equations of the electromechanical system and input filter is:

$$\begin{cases} \ddot{X}(t) = -2\xi_s\omega_s\dot{X}(t) - {\omega_s}^2X(t) - \dfrac{\vartheta}{m}V - {\omega^2}_pX_p(t) + 2\xi_p\omega_p\dot{X}_p(t) + {\omega^2}_gX_g + 2\xi_g\omega_g\dot{X}_g(t) \\ \ddot{X}_p(t) = -{\omega^2}_pX_p(t) - 2\xi_p\omega_p\dot{X}_p(t) + {\omega^2}_gX_g + 2\xi_g\omega_g\dot{X}_g(t) \\ \ddot{X}_g(t) = -2\xi_g\omega_g\dot{X}_g(t) - {\omega^2}_gX_g - w(t) \\ \dot{V} = \dfrac{\theta}{C_P}\dot{X} - \dfrac{1}{C_PR}V \end{cases} \tag{9}$$

Assuming the state vector $\overline{Z}(t) = \left[X \quad X_p \quad X_g \quad V \quad \dot{X} \quad \dot{X}_p \quad \dot{X}_g\right]^T$, the dynamic equation describing the coupled system is

$$\dot{\overline{Z}}(t) = \mathbf{A}\overline{Z}(t) + \overline{F}(t) \tag{10}$$

where the 7×1 forcing vector is $\overline{F}(t) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad w(t)]^T$, and the system matrix **A** is:

$$\mathrm{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{R_cC} & \frac{\vartheta}{C} & 0 & 0 \\ -{\omega_0}^2 & -\omega_p^2 & +\omega_g^2 & \frac{\vartheta}{m} & -2\xi_0\omega_0 & -2\xi_p\omega_p & +2\xi_g\omega_g \\ 0 & -\omega_p^2 & +\omega_g^2 & 0 & 0 & -2\xi_p\omega_p & +2\xi_g\omega_g \\ 0 & 0 & -\omega_g^2 & 0 & 0 & 0 & -2\xi_g\omega_g \end{pmatrix} \tag{11}$$

The equation set obtained here can be posed in a non-dimensional form adopting the following transformations (see for instance, Adhikari *et al* 2009, Gammaitoni *et al* 2009, Daqaq 2010, Litak *et al* 2010, Daqaq 2011, Khovanova and Khovanov 2011, Daqaq 2012, Halvorsen 2013, Zhao and Erturk 2013, Green *et al* 2013).

$$\tilde{v} = \frac{C_P}{\theta l_C}v \tag{12}$$

$$\tilde{x} = \frac{x}{l_C} \tag{13}$$

$$t = \omega_0\tau = \sqrt{\frac{k}{m}}\tau \tag{14}$$

where l_C, which has the dimension of a length, is defined as the ratio between the area of the equivalent piezoelectric capacitor and the distance between its armors; parameter ω_n is the natural frequency of the mechanical system ($\omega_n = 2\pi/T_n$ where T_n is the natural period of the structure).

In case of dimensionless expression, the complete set of equations of (11) (dealing with processes, so that random vibration expressions) representing the filter plus electromechanical system equations are then written as:

$$\begin{cases} \ddot{\widetilde{X}} = -2\zeta\dot{\widetilde{X}} - \widetilde{X} - \kappa^2\widetilde{V} - \psi_p^2\widetilde{X}_p - 2\xi_p\psi_p\dot{\widetilde{X}}_p + 2\zeta_g\psi_g\dot{\widetilde{X}}_g + \psi_g^2\widetilde{X}_g \\ \ddot{\widetilde{X}}_p(t) = -\psi_p^2\widetilde{X}_p - 2\xi_p\psi_p\dot{\widetilde{X}}_p + 2\zeta_g\psi_g\dot{\widetilde{X}}_g + \psi_g^2\widetilde{X}_g \\ \ddot{\widetilde{X}}_g = -2\zeta_g\psi_g\dot{\widetilde{X}}_g - \psi_g^2\widetilde{X}_g - w(t) \\ \dot{\widetilde{V}} = \dot{\widetilde{X}} - \lambda\widetilde{V} \end{cases} \tag{15}$$

where $\zeta = \frac{c}{2m\omega_n}$ is the mechanical damping factor; $\kappa^2 = \frac{\theta^2}{C_p m\omega_n^2}$ is the dimensionless electromechanical coupling coefficient; and finally $\lambda = \frac{1}{\alpha} = \frac{1}{RC_p\omega_n}$ is the ratio between the mechanical and electrical time constants of the harvester. The dimensionless parameter α is the ratio between the time constants of the first-order electrical system and mechanical system frequency (Adhikari *et al* 2009).

Regarding the parameters of the filters, the indices are the ratios between the frequencies of the filters and that of the mechanical system as expressed by equations (16) and (17):

$$\frac{\omega_p}{\omega_n} = \psi_p \tag{16}$$

$$\frac{\omega_g}{\omega_n} = \psi_g \tag{17}$$

System (15) takes the following compact form:

$$\dot{\widetilde{\widetilde{Z}}}(t) = \widetilde{\mathbf{A}}\widetilde{\widetilde{Z}}(t) + \widetilde{\widetilde{F}}(t) \tag{18}$$

where the dimensionless state vector is:

$$\widetilde{\widetilde{\mathbf{Z}}}(t) = \left[\widetilde{X} \;\; \widetilde{X}_p \;\; \widetilde{X}_g \;\; \widetilde{V} \;\; \dot{\widetilde{X}} \;\; \dot{\widetilde{X}}_p \;\; \dot{\widetilde{X}}_g\right]^T \tag{19}$$

and the system matrix for the filtered RC circuit is:

$$\widetilde{\mathrm{A}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\lambda & 1 & 0 & 0 \\ -1 & -\psi_p^2 & +\psi_g^2 & -\kappa^2 & -2\xi & -2\xi_p\psi_p & +2\xi_g\psi_g \\ 0 & -\psi_p^2 & +\psi_g^2 & 0 & 0 & -2\xi_p\psi_p & +2\xi_g\psi_g \\ 0 & 0 & -\psi_g^2 & 0 & 0 & 0 & -2\xi_g\psi_g \end{pmatrix} \tag{20}$$

as the dimensionless state space system matrix.

Mechanical systems driven by random processes, as the case treated here, have been discussed by many authors in many well-known books within the scope of random vibration theory (see for example, Lin 1967, Nigam 1983, Bolotin 1984, Newland 1989, Roberts and Spanos 1990). In this case, starting from the state equation (18), the covariance analysis is then performed in dimensionless state space by solving the *Lyapunov* matrix differential equation in the following form:

$$\dot{\widetilde{\mathbf{R}}}(t) = \widetilde{\mathbf{A}}\widetilde{\mathbf{R}}(t) + \widetilde{\mathbf{R}}(t)\widetilde{\mathbf{A}}^{\mathbf{T}} + \widetilde{\mathbf{B}}(t) \tag{21}$$

matrix *differential algebraic* problem in cases respectively, usually known as *Lyuapunov* equations. In equation (21), $\widetilde{\mathbf{R}}(t) = \langle\widetilde{\widetilde{Z}}\widetilde{\widetilde{Z}}^T\rangle$ is the covariance matrix (the operator $\langle\circ\rangle$ is the mathematical average), while $\widetilde{\mathbf{B}}(t)$ contains all elements equal to zero except the last one that holds $2\pi S_0$, where S_0 is the intensity of the white noise.

Equation (21) is the general dimensionless solution for the electromechanical-coupled problem; it is written for non-stationary problems, and it should be simplified for stationary condition as an algebraic one:

$$\widetilde{\mathbf{A}}\widetilde{\mathbf{R}} + \widetilde{\mathbf{R}}\widetilde{\mathbf{A}}^{\mathbf{T}} + \widetilde{\mathbf{B}} = \mathbf{0} \tag{22}$$

The relative mechanical displacement variance is $\sigma^2_{\widetilde{X}} = \widetilde{\mathbf{R}}(1,\ 1)$, whereas the one for tension is $\sigma^2_{\widetilde{V}} = \widetilde{\mathbf{R}}(4,\ 4)$. The latter represents the variation of the tension generated by the piezoelectric device as represented in figure 1. Tension V and displacement X are Gaussian stochastic stationary processes, with zero mean and variance $\sigma^2_{\widetilde{V}}$. This information is extremely important for predicting electrical potential applications of these energy-harvesting systems.

Table 1. Different filter configurations.

Spectral characteristics	a	b	c	d
ξ_g	0.2	0.2	0.2	0.2
ξ_p	0.7	0.7	0.7	1.0
ψ_g	0.50	1.00	2.00	5.00
ψ_p	0.5	0.5	0.5	0.5

Following (Adhikari *et al* 2009, Adhikari *et al* 2016), the electrical energy is herein considered instead of the electrical power because the loading event (i.e., the seismic event) lasts for a finite time length. By making the time dependence explicit through the introduction of the time variable t, the energy harvested within a time window [0,T] is calculated as follows (Elvin *et al* 2006, Adhikari *et al* 2009):

$$\varepsilon = \frac{1}{\mathcal{R}} \int_0^T v^2(t)\,dt \tag{23}$$

The first-order moment of the harvested energy is:

$$\mu_\varepsilon(T) = \langle \varepsilon(T) \rangle = \frac{1}{R} \int_0^T \langle v^2(t) \rangle\, dt = \frac{1}{R} \int_0^T \sigma_V^2(t)\,dt \tag{24}$$

In case of stationary response, the integral (24) becomes

$$\mu_\varepsilon(T) = \frac{T\sigma_V^2}{R} \tag{25}$$

and the energy harvesting is the electrical net mean power:

$$\mu_P = \frac{\sigma_V^2}{R} \tag{26}$$

The dimensionless main quantity of interest in the energy harvesting is the electrical net mean power that has the following expression (Bobryk and Yurchenko 2015)

$$\mu_{\tilde{P}} = \lambda \kappa^2 \sigma_{\tilde{V}}^2 \tag{27}$$

3. Numerical analysis

In this paragraph, the method proposed is numerically implemented to evaluate the efficiency and sensitivity of results of average harvested energy under different input frequency contents. The main objective is to show the difference in the average harvested energy considering a filtered instead of a simple white noise Gaussian input. Different four spectral configurations (the four dimensionless parameters) are used. For each input spectral configuration, we have calculated the value of $\mu_{\tilde{P}}$ and compared it to $\mu_{\tilde{P}}$ at the same configuration in the case of pure white noise. The parameters of the spectral configurations are reported in table 1. The values are obtained for different values of structural damping ξ_0 and ratios of mechanical-electric time constants α. It must be stressed that the parameter α plays a key role representing the connection between the oscillation frequencies of the electrical system and the mechanical one. For each spectral configuration (set of filter parameters), we have considered three different values of structural damping (as representative of common mechanical situations) and have measured the maximum values of the obtained dimensionless power and the related α values for which this condition occurs.

In case of white noise input, the optimal value of α has been derived analytically from the equation (Adhikari *et al* 2009):

$$\alpha^2(1 - \kappa^2) = 1 \tag{28}$$

for which the optimal value of the ratio is α_{opt}^{WN} (where WN signifies white noise) is:

$$\alpha_{opt}^{WN} = \sqrt{\frac{1}{1+\kappa^2}} \tag{29}$$

It is worth noting that α_{opt}^{WN} only depends on the dimensionless electromechanical coupling coefficient κ. In contrast with the case of filtered input, the optimal value of the power (on average) not only varies considerably but also is obtained for different values than α_{opt}^{WN}. Figures 4(a)–(d) displays the plots of the dimensionless value of $\mu_{\tilde{P}}$ as a function of α, for four filter configurations. Continuous lines represent the filtered case, while dotted ones denote the simple white noise input. Different colors are used for the three different values of structural damping ξ_0. It is evident that the input spectral content causes differences in the energy efficiency $\mu_{\tilde{P}}$ and

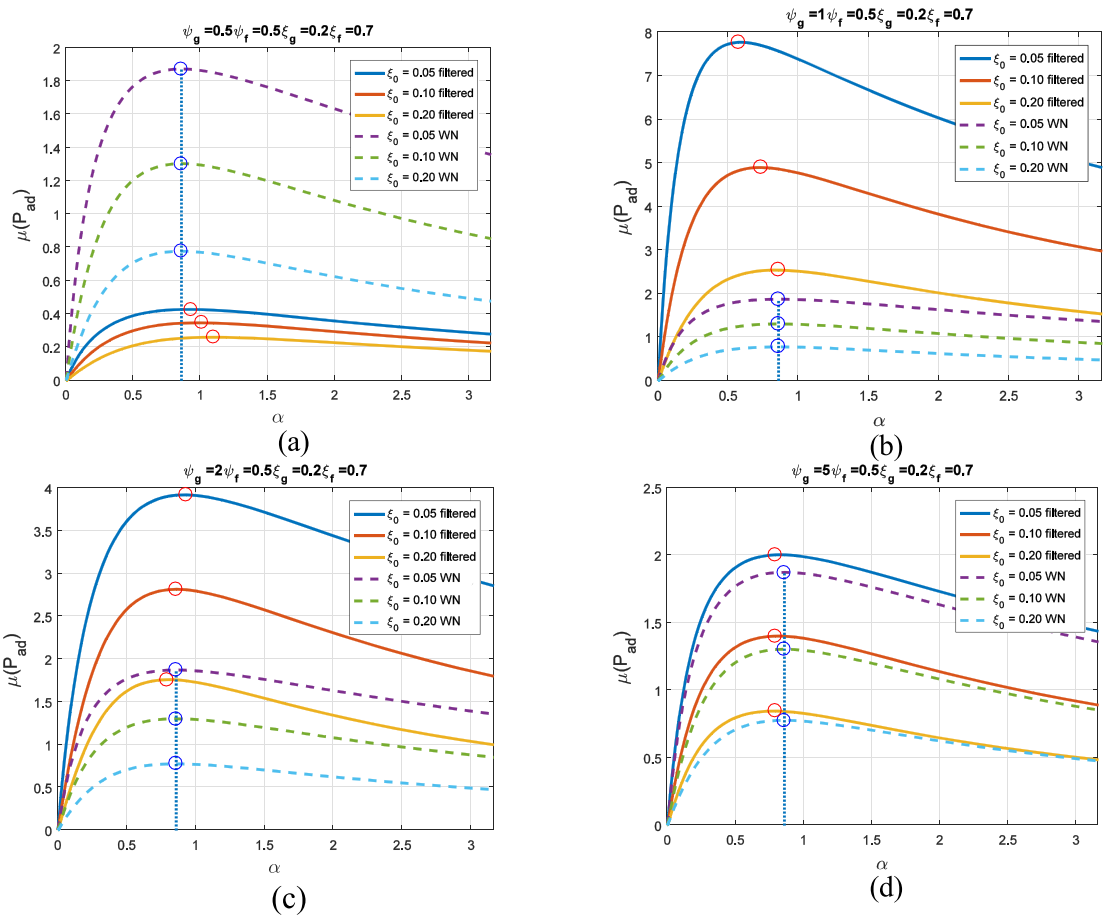


Figure 4. Energy efficiency assuming white noise (dotted lines) and filtered white noise (continuous lines) for different damping ratios and for different filter configurations. Maxima are reported with blue circles (white noise) and red circles (filtered input).

corresponding optimal value of α in both cases, (blue circles for white noise input, and red circles for filtered input).

Observing the four filter configurations, we can notice that the mean value of the non-dimensional power is greatly influenced by the characteristics of the filters, Furthermore, the optimum value of the alpha parameter (the ratio between the periods of the mechanical system and electrical circuit) can vary by departing from the one related to the subject system with the white noise only.

The optimal value for the ratio α_{opt}^{WN} in case of white noise input is independent of structural damping and dependent on parameter κ only. In addition, figures 4(a)–(d) clearly shows that there is a strong dependency on the optimal value and the structural damping.

For this reason, a comparison between α_{opt}^{WN} and the optimal value obtained in the case of colored white noise α_{opt}^{RC} is numerically obtained for different values of structural damping ξ_0. The range of the variation of this mechanical parameter is within the interval [0;0.5], covering a wide range of real cases using different materials and additional damping devices. The index RC is for the electric characteristic of the circuit.

Figures 5–8 report the optimal values of α_{opt}^{RC} for different structural damping and values of dimensionless electromechanical coupling coefficient κ, which represents the efficiency of piezoelectric material, subject to the input spectral conditions cases of table 1.

The optimal values of α_{opt}^{RC} are obtained using a genetic algorithm that must maximize the dimensionless mean value of energy power obtained by the energy harvesting device modeled as described in the previous chapter. Once the values of κ and ξ_0 are defined, the maximization problem deals with a single variable α.

The problem is relatively simple from computational point of view. The standard GA solver in MATLAB is sufficient to solve the problem without needing large number of populations or generations. This is definitely an advantage inferring that this strategy can be used in a wide number of cases as a design support for this type of devices.

Figures 4–7 illustrate the optimal values of α_{opt}^{RC} (continuous lines), which are sensitive to ξ_0, versus the optimal values of α_{opt}^{WN} obtained in the case of simplified white noise in equation (29), which are represented by dotted lines and are insensitive to damping.

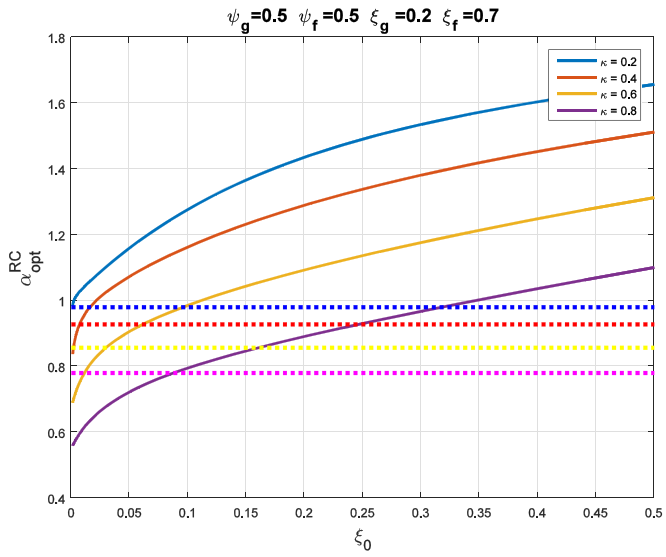


Figure 5. Optimal value of α assuming white noise (dotted lines) and filtered white noise (continuous lines) for different damping ratios; Spectral configuration (a).

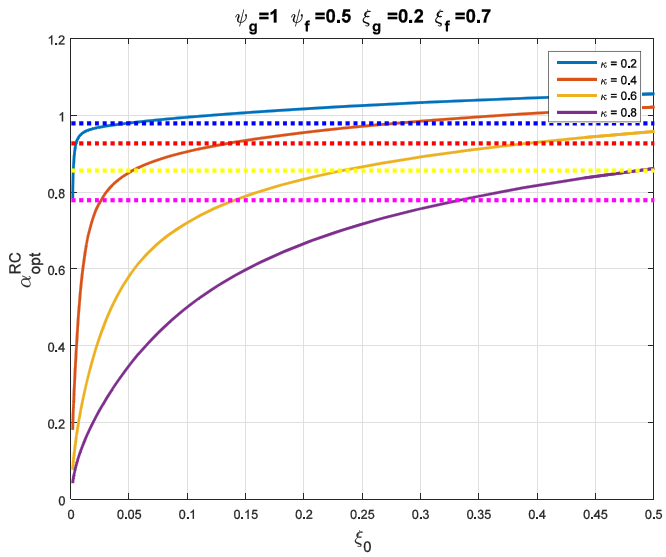


Figure 6. Optimal value of α assuming white noise (dotted lines) and filtered white noise (continuous lines) for different damping ratios; Spectral configuration (b).

The four colors are related to different values of κ parameter, which are assumed to be equal to 0.2, 0.4, 0.6, and 0.8, based on recent references from the literature.

It becomes more evident after analyzing the figures that the environmental conditions (frequency contents) play a fundamental role in defining the optimal α_{opt}^{RC} values. Indeed, there is no single trend as compared to the values of α_{opt}^{WN}.

4. Conclusion

An approach for piezoelectric energy harvester subject to random vibrations is presented. This approach is differently from the previously reported methods in considering the input frequency contents adopting a vibration source described as a filtered white noise.

Covariance approach has been implemented showing differences with varied degree of significane between the two cases (white noise and filtered) in terms of dimensionless mean value of produced power. It has also been

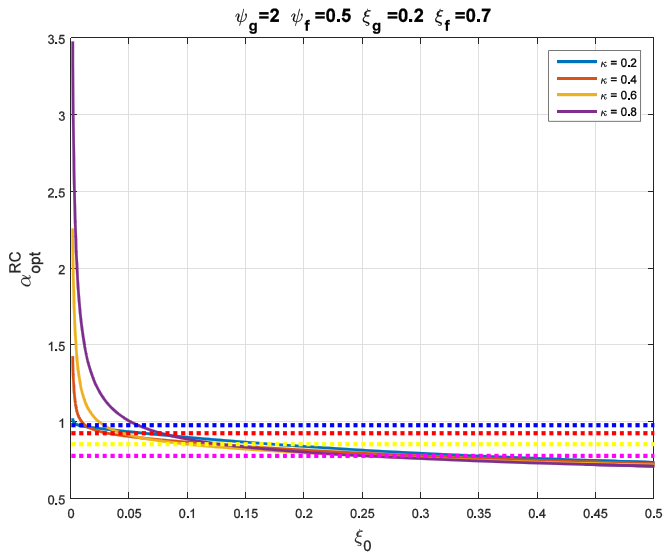


Figure 7. Optimal value of α assuming white noise (dotted lines) and filtered white noise (continuous lines) for different damping ratios; Spectral configuration (c).

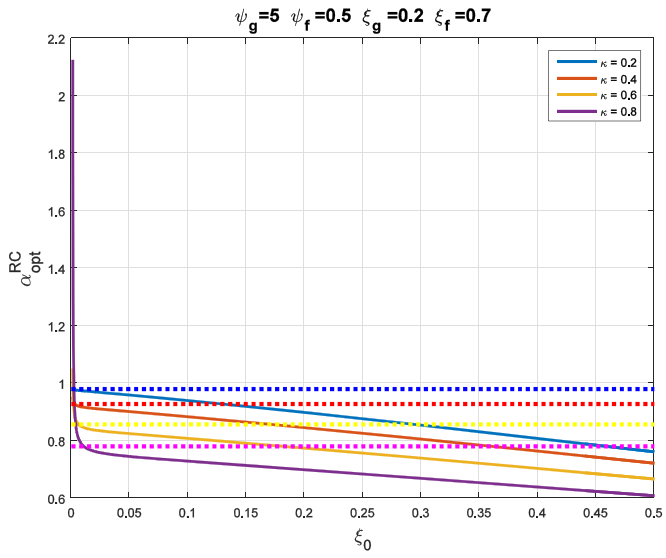


Figure 8. Optimal value of α assuming white noise (dotted lines) and filtered white noise (continuous lines) for different damping ratios; Spectral configuration (d).

observed that the optimal ratio between mechanical and electrical time constants (α) is dependent on structural damping. This dependency on α differs from the case of white noise where the analytical formulation deals only with the piezoelectric device efficiency. Numerical evaluation of optimal values of α in case of filtered input has been plotted for different frequency contents and environmental conditions, demonstrating this approach as an effective tool for designing piezoelectric devices to maximize energy efficiency under different input spectral configurations.

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