



12th International Conference on Computing and Control for the Water Industry, CCWI2013

## Supporting decision on energy vs. asset cost optimization in drinking water distribution networks

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### Abstract

One of the challenges for water utilities is the optimal asset design (i.e. maximum power of pump systems, tank volumes and pipe diameters) of water distribution networks (WDN) while optimizing operational efficiency (i.e. energy consumption and cost). Besides the classical minimization of capital cost while providing sufficient supply service, the operational sustainability is an emerging issue. As the reduction of each component of capital and energy costs are conflicting with each other, the optimization problem is multi-objective. This work presents the study of the robustness of solutions of the Pareto set as a further element to support the decision.

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Selection and peer-review under responsibility of the CCWI2013 Committee

Keywords: Water distribution network design; Asset management; Operational optimization; Pump scheduling; Multi-objective optimization.

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### 1. Introduction

In water distribution networks (WDNs), the interest in optimal asset design (i.e. maximum power of pump systems, tank volumes and pipe diameters) versus energy cost of pumping has recently increased. In fact, the efficiency of WDNs is of relevant interest for the water industry and operational optimization plays an important role together with the classical capital cost optimization related to optimal sizing of hydraulic capacity.

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Thus, the classical optimal pipe sizing challenge is currently expanded to the optimal design of upgrading of tank volumes and maximum pump powers in existing WDNs, together with energy consumption/cost minimization for pumping, accounting for the electricity tariff pattern over the typical operating cycle.

However, the reduction of each cost is conflicting with the reduction of some others, for example the reduction of the pipe diameters cost increases the energy consumption (and relevant operational cost).

For this reason, the construction of a Pareto set of optimal solutions in a multi-objective framework is a method to support the decision of water managers for the specific technical problem by providing solutions which are optimal trade-offs.

From a hydraulic standpoint the optimization problems involves, for each solution entailing the upgrade of the hydraulic system capacity, the optimal selection of the working pumps over time. This fact requires the prediction of the hydraulic system behavior over time varying the relevant boundary conditions (e.g. nodal demands, tank levels, etc.) and the decision on the pump scheduling strategy. In fact, pumping optimization means to schedule pumps over time in a relevant operating cycle, i.e. to select which pumps are running at any given time. This task is generally performed in two ways: (i) by programming the ON/OFF status over time or (ii) by setting the status of the pump (i.e., ON/OFF) based on the water level in a tank.

In the first strategy, the operating cycle is sampled using a time step for programming. The time step depends on the specific WDN and needs to track the relevant changes in boundary conditions such as nodal demands, state of control valves, tank levels, etc. In the second strategy, the working condition of pumps is controlled by tank levels, i.e. each pump switches ON or OFF depending on the level of its specific tank. In contrast to the first method, this kind of pump scheduling acts in a continuous way over the operating cycle and implies that the tank levels are significant indicators of WDN boundary condition changes over time, Giustolisi et al. (2013).

This work aims to study the robustness of the optimal solutions accounting for hydraulic capacity upgrading vs. energy costs with respect to the increasing or decreasing of the nodal demands and the two pump scheduling strategies. In this framework, the assessment of robustness is also a method to further support the decision of water managers.

The key idea is to test the sensitivity of each solution with respect to the demand pattern assumed during the optimization, which is designated here as “Deterministic”. To this purpose, the values of the nodal demands are randomly increased, (i.e. “Test plus”), and decreased, (i.e. “Test minus”), using the Latin Hypercube sampling method, McKay et al. (1979), and the uniform probability density function.

The exercise on TOWN-D network, Marchi et al. (2013), was used as case study in order to test and discuss the strategy.

## Nomenclature

$E_p$	electric power in KW
$H_k^{pump}$	pump static head installed along the $k$ th pipe
$H_s$	head level at the $s$ th tank node
$H_s^{ini}$	initial head level at the $s$ th tank node
$H_s^{max}$	maximum head level at the $s$ th tank node
$H_s^{min}$	minimum head level at the $s$ th tank node
$i$	subscript of internal nodes
$k$	subscript of pipes
$n_n$	number of nodes
$n_p$	number of pipes
$n_0$	number of tank nodes
$p$	subscript of pumps
$P_i$	model pressure at the $i$ th node
$P_i^{ser}$	pressure for correct or sufficient service of each $i$ th node
$Q_p$	pipe flow rates
$r_p$ and $c_p$	parameters of internal head loss of the pump installed along the $p$ th pipe

$s$	subscript of tank nodes
$t$	time variable representing the hydraulic system/model snapshot
$T$	number of EPS model snapshots of the operating cycle
$Z_i$	elevations of the $i$ th node
$\Delta H_p$	pump dynamic head
$\Delta T$	time interval of the real hydraulic system snapshot
$\eta$	pump efficiency
$\Omega_s$	cross-sectional area of tanks
EPS	Extended Period Simulation
WDN	Water Distribution Network

## 2. Materials and methods

### 2.1. Pumping of water optimization: problem formulation

Pumping optimization over an operating cycle  $T$  requires the prediction of the system behavior over time; therefore, the extended period simulation (EPS) of the WDN is required. EPS is the sequence of steady-state simulation runs to predict the hydraulic status of the WDN over time, Van Zyl et al. (2006), Todini (2011), Giustolisi et al. (2012), Giustolisi et al. (2013). The pump states over the operating cycle are the decision variables to be optimized considering the minimization of pumping cost with respect to the energy consumption and tariffs.

If the hydraulic capacity of the system is optimized contemporarily to pumping, the decision variables related to the asset components, i.e., the number of pumps in each installation, the tank volumes and pipes diameters, are comprised in the problem formulation

The optimization problem is also constrained by technical requirements and supply reliability (constraints on minimum pressures for sufficient service and on the minimum level of tanks, respectively), water overflows (constraint on maximum level of tanks) and global mass balance in each tank during an operating cycle (Brdys and Ulanicki 1994, Van Zyl et al., 2004). Thus, the general formulation of the optimization problem is given by,

$$\left\{ \begin{array}{ll}
 \text{EPS model} & \forall t = 0, \Delta T, 2 \times \Delta T, K, T \times \Delta T \\
 H_i - Z_i - P_i^{ser} \geq 0 & \forall i = 1, 2, K, n_n \\
 H_s^{mi}(0) \geq H_s^{mi}(T \times \Delta T) & \forall s = 1, 2, K, n_0 \\
 H_s(t) \geq H_s^{\min} & \forall s = 1, 2, K, n_0, \forall t = 0, \Delta T, 2 \times \Delta T, K, T \times \Delta T \\
 H_s(t) \leq H_s^{\max} & \forall s = 1, 2, K, n_0, \forall t = 0, \Delta T, 2 \times \Delta T, K, T \times \Delta T \\
 \min \{f_1(\text{Energy cost})\} \\
 \min \{f_2(\text{Pipe diameter cost})\} \\
 \min \{f_3(\text{Tank volume cost})\} \\
 \min \{f_3(\text{Pump installation cost})\}
 \end{array} \right. \quad (1)$$

where  $i$  = subscript of the  $i$ th node;  $s$  = subscript of the  $s$ th tank node;  $p$  = subscript of the  $p$ th pump;  $m$  = number of pumps;  $P_i$  = model pressure at the  $i$ th node varying over time;  $Z_i$  = elevations of the  $i$ th node;  $P_i^{ser}$  = pressure for sufficient service of each  $i$ th node;  $H_s^{\min}$  = minimum head level at the  $s$ th tank node;  $H_s^{\max}$  = maximum head level at the  $s$ th tank node;  $H_s^{mi}$  = initial head level at the  $s$ th tank node varying over time;  $H_s$  = head level at the  $s$ th tank node varying over time;  $T$  = number of EPS model time steps of the operating cycle;  $\Delta T$  = time interval of the real hydraulic system snapshot (in hours);  $E_p$  = electric power in KW.

The electric power for the energy cost is computed as:

$$E_p(t) = 9.806 \left( H_p^{pump} - r_p Q_p(t)^{c_p} \right) Q_p(t) / \eta = 9.806 \cdot \Delta H_p(t) Q_p(t) / \eta \quad (2)$$

where  $p$  = subscript of the  $p$ th pipe containing a pump;  $\Delta H$  = dynamic head varying over time  $t$ ;  $H_p^{pump}$  = static head;  $r_p$  and  $c_p$  = parameters of internal head loss;  $\eta$  is the variable pump efficiency.

## 2.2. The strategy to solve the constrained multi-objective optimization problem

The optimization problem in (1) is addressed here with a Multi-Objective (MO) strategy by using genetic algorithms and in particular OPTIMOGA, Laucelli and Giustolisi (2011). To accomplish the constraints, they are rearranged as an extra objective function as follows:

- the constraint on pressures for sufficient service is transformed in the number of times (number of snapshots) it is not satisfied ( $n_1$ );
- the constraint on minimum tank levels is transformed in the number of times (number of snapshots) it is not satisfied ( $n_2$ );
- the constraint on maximum tank levels is transformed in the number of times (number of snapshots) it is not satisfied ( $n_3$ );
- the constraint on initial level with respect to the final level in the operational cycle  $T$  for each tank is transformed in a global deficit of volume in the cycle as summation of the volume deficits in the tanks (see first Eq.(3)).

The objective function  $f_{\text{constraints}}$  to be minimized is the product of the previous four values, adding the unit value to each one in order to avoid the product with a null value, as follows:

$$\forall [\Omega_s H_s(T \times \Delta T) - \Omega_s H_s(0)] > 0 \quad V_{\text{deficit}} = \sum_s \Omega_s H_s(T \times \Delta T) - \Omega_s H_s(0) \quad (3)$$

$$f_{\text{constraints}} = (1 + V_{\text{deficit}})(1 + n_1)(1 + n_2)(1 + n_3)$$

It is worth noting that the use of the extra objective function for the constraints allows maintaining the solutions violating the constraints during the evolution, according to the amount of violation. This is useful to better explore the search space and to preserve diversity in the GA population. In addition, this method is more reliable when some constraints cannot be fully satisfied, although relevant solutions are technically feasible. Finally, during the optimization run the computation of EPS is stopped when the three constraints on minimum and maximum tank levels and minimum nodal pressures are not satisfied. This is a way to reduce the computational burden related to the need for computing an EPS for each individual (candidate solution) in the OPTIMOGA, Giustolisi et al. (2013).

## 3. Case study

The network used as case study is TOWN-D, Marchi et al. (2013), whose layout is reported in Fig. 1. The WDN is composed of 459 pipes and 407 nodes. A pumping system composed of three pumps is close to the reservoir; it pumps water in five districts. Four inline pumping systems composed of two pumps are installed upstream the four districts and seven tanks are filled and emptied during the operational cycle.

The original exercise (“battle”) deals with the optimal upgrading of the WDN hydraulic capacity (pipe diameters, number/type of pumps and tank volumes) and the optimal design of new area in order to contemporarily minimize energy cost (i.e. given the tariff variation over time), water age and carbon footprint. The operational cycle was set to one week ( $T=168\text{h}$ ).

To the purpose of the work, the same exercise is here slightly modified. The operational cycle was set  $T=24\text{h}$  while the minimization of water age and carbon footprint was not performed. The WDN optimization was executed assuming for the existing pipes the option of substituting or not with new pipes whose costs is reported in the

documentation of the “battle”, Marchi et al. (2013). Regarding the pumps it was assumed to add to the main installation (close to the reservoirs) a maximum number of six new pumps and for the other four installations a maximum number of two new pumps. This means to add a maximum of fourteen new parallel pipes with pumps.

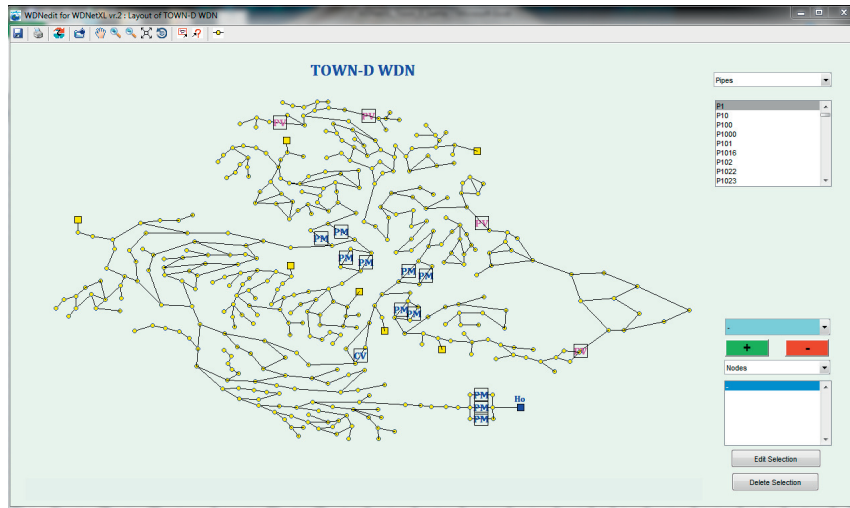


Fig. 1. Network Layout

Therefore, these candidate pumps were added to the problem as fourteen optional parallel pipes having the annualized cost of the pumps. Thus, the annualized costs of pipes and pumps were aggregated. Regarding the upgrading of the tank volumes the Table 1 was used.

The power for the annualized energy cost, see Eq. (2), was computed considering a variable efficiency and the curve of efficiency reported in Giustolisi et al. (2013) using the maximum efficiency equal to 0.85.

Table 1. Cross sectional area ( $\Omega$ ) of the new candidate tank and cost for each of the seven tanks.

Tank n.1		Tank n.2		Tank n.3		Tank n.4		Tank n.5		Tank n.6		Tank n.7	
$\Omega$	Cost	$\Omega$	Cost	$\Omega$	Cost	$\Omega$	Cost	$\Omega$	Cost	$\Omega$	Cost	$\Omega$	Cost
[m <sup>2</sup> ]	[\$]	[m <sup>2</sup> ]	[\$]	[m <sup>2</sup> ]	[\$]	[m <sup>2</sup> ]	[\$]	[m <sup>2</sup> ]	[\$]	[m <sup>2</sup> ]	[\$]	[m <sup>2</sup> ]	[\$]
148.06		769.45		40.04		54.50		111.03		339.14		106.41	
222.13	14020	846.37	14020	140.04	14020	145.41	14020	222.14	14020	423.89	14020	212.80	14020
296.21	30640	923.29	30640	240.04	30640	236.32	30640	333.26	30640	508.63	30640	319.18	30640
444.35	61210	1077.14	61210	440.04	61210	418.13	61210	555.48	61210	678.12	61210	531.95	61210
703.61	87460	1346.37	87460	790.04	87460	736.32	87460	944.37	87460	974.73	87460	904.29	87460
888.80	122420	1538.68	122420	1040.04	122420	963.59	122420	1222.14	122420	1186.60	122420	1170.24	122420
1629.54	174930	2307.91	174930	2040.04	174930	1872.68	174930	2333.26	174930	2034.06	174930	2234.07	174930

Therefore the multi-objective optimization aimed at the the minimization of the annualized pipe plus added pump cost vs. the annualized tank cost (upgrading of volume) vs. the annualized energy cost vs. the extra function on the violation of constraints, see Eq. (3).

About the pump scheduling optimization, it was preferred to schedule pump states by means of tank levels for two reasons: (i) the option is characterized by a reduced search space of the optimization problem with respect to temporal scheduling of the pump states; (ii) the pump scheduling by tanks can be transformed in the temporal

scheduling of the pump states for  $\Delta T=1h$ , although losing the continuity of the control in real system and in also EPS model depending on  $\Delta T$ . This last point is helpful in order to compare the robustness of the two pump scheduling strategies. In fact, the robustness, as sensitivity to the nodal demand variations, of each of the 114 solutions which were obtained using “deterministic” nodal demands, was tested using both the pump scheduling strategies. Each solution (hydraulic capacity and pump scheduling strategy) was tested:

1. increasing of 50% the “deterministic” nodal demands using the Latin Hypercube sampling method, McKay et al. (1979), i.e. “Test plus”;
2. decreasing of the “deterministic” nodal demands using the Latin Hypercube sampling method, McKay et al. (1979), i.e. “Test minus”.

#### 4. Results and Discussion

The optimization was performed using WNetXL ([www.hydroinformatics.it](http://www.hydroinformatics.it)) which is a collection of MS-Excel add-ins developed in the Matlab R2010b-32bit environment, Giustolisi et al. (2011). The notebook used was equipped with an Intel vPro i7 processor and Windows 7.0. OPTIMOGA provided 114 solutions running 2,000 generations in about 560 minutes. As for example, Fig. 2 reports the first 10 solutions (ordered by the annualized energy cost) as provided in MS-Excel format by WNetXL.

Solution	Pump Power [KW]	Energy Cost	Non Revenue Water Ccst/Volume	CO <sub>2</sub> Emission [Kg]	Average Pump Switches	Maximum Pump Switches	Mass balance [m <sup>3</sup> ]	Volume deficit [m <sup>3</sup> ]	% Violation Pserv	% Violation Min Level	% Violation Max Level	Pipes and Pump Cost	Tanks Cost	Capital Cost	CPU Time [min]
1	1972121	\$ 277,418	0	2,051,005	0.24	3	-1560.46	0.00	0.00%	0.00%	0.00%	\$ 74,000	\$ 58,680	\$ 132,680	560.11
2	1971972	\$ 277,394	0	2,050,851	0.24	3	-1560.17	0.00	0.00%	0.00%	0.00%	\$ 74,196	\$ 58,680	\$ 132,876	Model calls
3	1976940	\$ 272,173	0	2,056,018	0.12	1	-1339.42	0.00	0.00%	0.00%	0.00%	\$ 70,299	\$ 89,320	\$ 159,619	3734730
4	1976940	\$ 272,173	0	2,056,018	0.12	1	-1339.42	0.00	0.00%	0.00%	0.00%	\$ 70,366	\$ 89,320	\$ 159,686	Obj fun calls
5	1975336	\$ 271,911	0	2,054,349	0.12	1	-1512.93	0.00	0.00%	0.00%	0.00%	\$ 72,045	\$ 72,700	\$ 144,745	286946
6	1973266	\$ 271,637	0	2,052,197	0.12	1	-1333.07	0.00	0.00%	0.00%	0.00%	\$ 70,425	\$ 89,320	\$ 159,745	Avg calls
7	1972191	\$ 271,520	0	2,051,079	0.12	1	-1506.49	0.00	0.00%	0.00%	0.00%	\$ 72,103	\$ 72,700	\$ 144,803	13.02
8	1972191	\$ 271,520	0	2,051,079	0.12	1	-1506.49	0.00	0.00%	0.00%	0.00%	\$ 74,086	\$ 72,700	\$ 146,786	
9	1883607	\$ 258,430	0	1,958,951	0.16	1	-1004.07	0.00	0.00%	0.00%	0.00%	\$ 72,471	\$ 89,320	\$ 161,791	
10	1857651	\$ 258,120	0	1,931,957	0.28	3	-806.51	0.00	0.00%	0.00%	0.00%	\$ 74,217	\$ 58,680	\$ 132,897	

Fig. 2. Solutions of the Pareto set provided in MS-Excel by WNetXL (the example reports 10 out of 114 solutions)

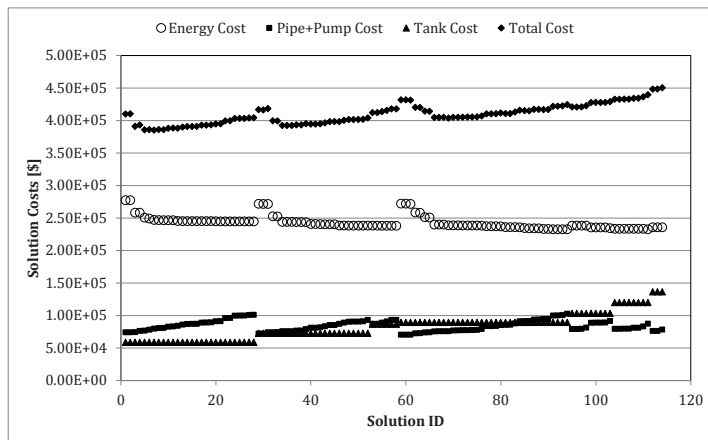


Fig. 3. Annualized cost of the solutions in the Pareto set ordered by the tank cost

During optimization the calls to the objective function were 286,946 while the WDN simulations were 3,734,730 which means the average value of 13.02 out of 24 snapshots of the EPS model. The Generalized WDN

model (Giustolisi et al., 2012) employed in WDN<sub>et</sub>XL never failed or had convergence troubles. The total annualized cost of the 114 solutions ranges from 385,294 \$ to 450,442 \$, see Fig. 3.

On the contrary, Fig. 5 demonstrates that temporal scheduling causes an insignificant reduction or increase of the energy consumption varying the demands. The marginal power variation is caused by the fact that the working point of each pump moves depending on demand variations. Figs 4 and 5 demonstrate that scheduling by means of tank levels adapts pumping and energy consumption to demand variations, while temporal scheduling is not adaptive and, for this reason, requires accurate demand forecasting and the real time pumping optimization.

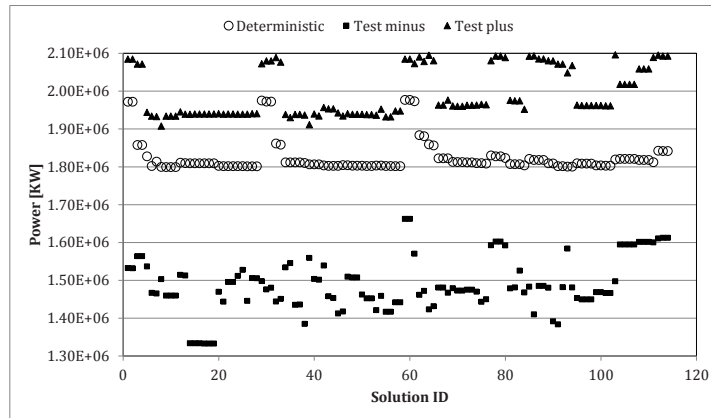


Fig. 4. Annualized pump power of the solutions ordered by the tank cost. Pumps controlled by tank levels

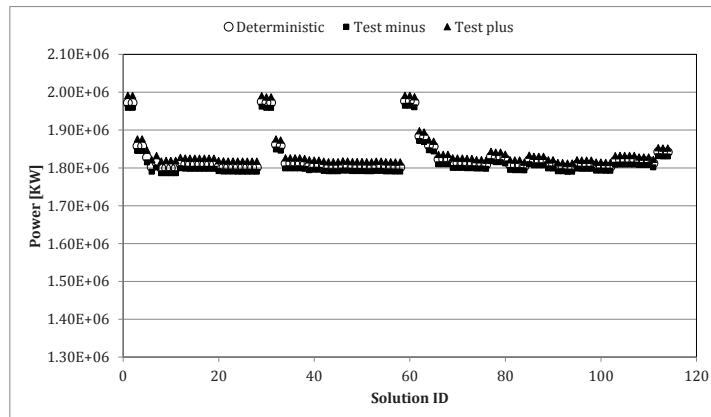


Fig. 5. Annualized pump power of the solutions ordered by the tank cost. Temporal scheduling of the pump states

Figs. 6 and 7 report the fraction of constraint violations over  $T$  in the “Test plus” scenario. Fig. 6 shows that the adaptability of scheduling by means of tank levels do not solve the problem of the insufficient pressure for a correct service, because it is related to the maximum hydraulic capacity of the system which was optimally designed with the “deterministic” nodal demands. The problem of the emptying of some tanks also occurs. It could be reduced by selecting the optimal solutions characterized by larger tanks.

Fig. 7 shows the same problems of pressure and minimum level violations of Fig. 6, but the inadaptability of temporal scheduling increases the occurrence of empty tanks even if the solutions with larger tanks are selected. In fact, a larger tank cannot completely solve the need of more pumping due to the increased demands.

Finally, the fact that the violation of the pressure for a sufficient service is not influenced by the pumping strategy confirms that it is related to the exceeding of the hydraulic capacity of the system, which was optimally designed with respect to the “deterministic” demand requests.

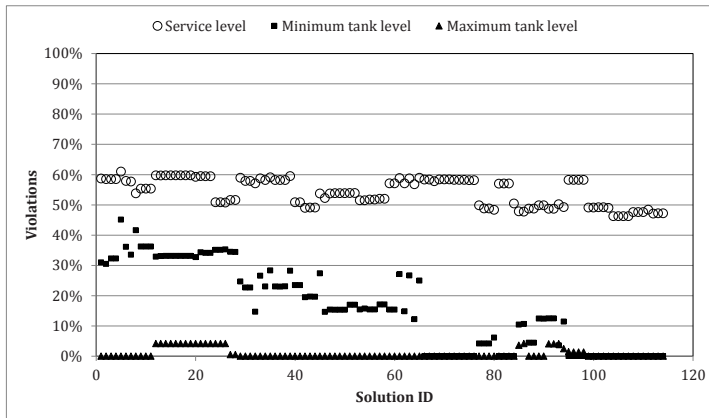


Fig. 6. Test plus: constraint violations of the solutions ordered by tank cost. Scheduling of the pump states by means of tank levels

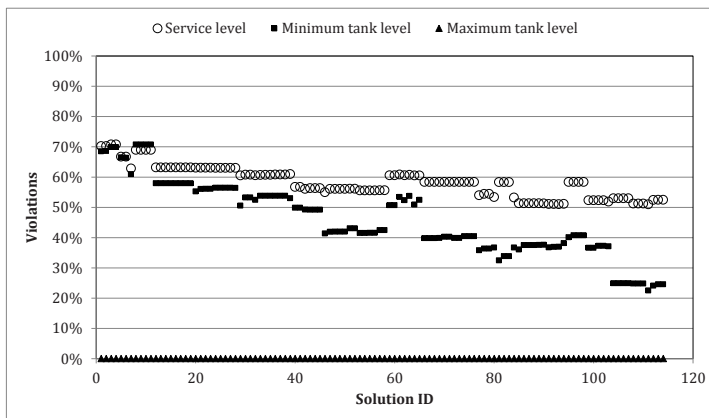


Fig. 7. Test plus: constraint violations of the solutions ordered by tank cost. Temporal scheduling of the pump states

Figs. 8 and 9 report the fraction of violations over  $T$  of the “Test minus” scenario. Fig. 8 shows that the adaptability of scheduling of the pump states by means of tank levels do not exclude the occurrence of an insufficient pressure for a correct service. The fact that the pressure violation does not occur for all the solutions is perhaps related to the sensitivity to the spatial variation of the demands. This is a good property to further support the selection of a specific solution. Fig. 8 also displays that the maximum level violations always occurs. This is related to the fact that, although adaptable, pumping is not optimized for the demand scenario. Fig. 9 shows the different behavior of the temporal scheduling when the demand request is lower. The problem of maximum level violation occur for all the solutions because pumping is not reduced. For the same reason, the other violations do not occur.



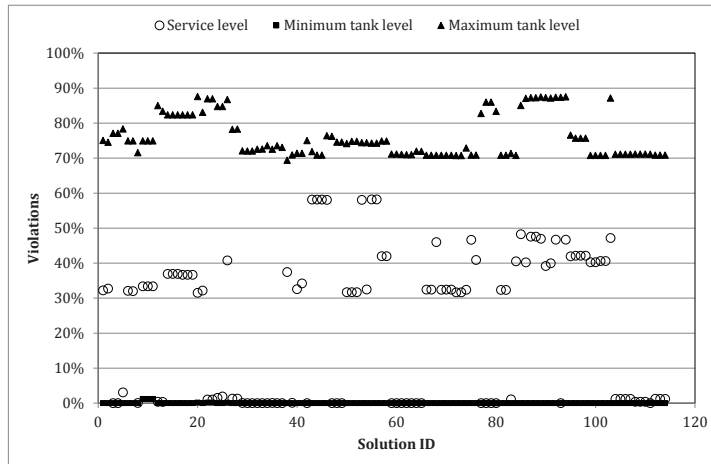


Fig. 8. Test minus: constraint violations of the solutions ordered by tank cost. Scheduling of the pump states by means of tank levels

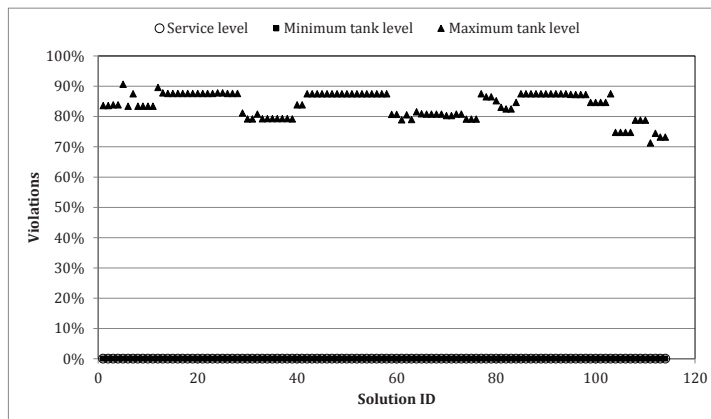


Fig. 9. Test minus: constraint violations of the solutions ordered by tank cost. Scheduling pump states by time

In general, it is possible to state that strategy of scheduling by means of tank levels is much more robust and adaptable to the demand variations than temporal scheduling. Scheduling by means of tank levels allows to reduce the energy consumption, then the carbon footprint. In addition, the sensitivity analysis allows the selection of a solution which is further robust with respect to service pressure violation, at least when the demand requests are lower than those used for optimization. On the contrary, the temporal scheduling is not adaptable to the increase of demands and much more energy consuming, requiring for this reason the real time pumping optimization by forecasting the water requests.

Finally, Fig. 10 reports the deficit of volume of the two demand scenarios, see Eq. (3), and the global mass balance in the tanks of the “Test minus” scenario. The deficit of volume for the “Test plus” scenario coincides with global mass balance. Negative values means that the water volumes increases, for each tank or globally, over  $T$ . Fig. 10 shows a variable deficit of volumes both for the “Test plus” and for the “Test minus” scenarios. The deficit of volume of the solutions could be then used as a further element to select a specific solution from the Pareto set. Clearly, the global mass balance of the “Test minus” scenario is always negative, although the values greatly varies through the solutions. Therefore, the worst and the global mass balance of the tanks are both indicators that could help to support decision. As for the same plot of Fig. 10 related to temporal scheduling, it is not reported here for

brevity. In addition, the fact that pump states remain fixed over time makes not particularly informative the tank mass balances varying the solutions and scenarios.

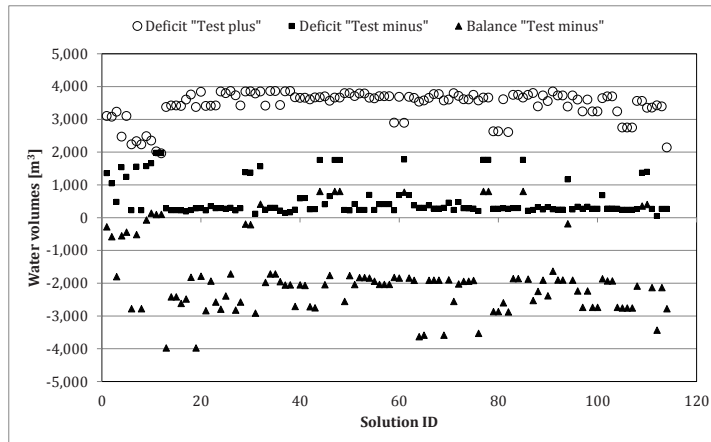


Fig. 10. Water balance of the solutions ordered by the tank cost. Scheduling of the pump states by means of tank levels

## 5. Conclusion

The work proposed a methodology to test the robustness of the solutions of the multi-objective optimization asset design vs. pump scheduling. The robustness was evaluated considering two scenarios of nodal demand variation: (i) increased or (ii) decreased with respect to the “deterministic” scenario used during optimization. The two scenarios were sampled using the Latin Hypercube technique and each solution of the Pareto set was tested using samples. The scheduling of the pump states by means of tank levels results into a strategy that is much more adaptable to the demand variation and less energy consuming than the temporal scheduling of the pump states. The sensitivity analysis of the optimal solutions with respect to the increase/decrease of demands is a method to further support the selection of a specific solution from the Pareto set and to achieve an additional insight on the characteristics of the optimal solutions.

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