

# Investigation of mid-infrared second harmonic generation in strained germanium waveguides

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**Abstract:** In this paper we present a detailed theoretical investigation of second harmonic generation in strained germanium waveguides operating at the mid infrared pump wavelength of 4  $\mu\text{m}$ . The effective second order susceptibility has been estimated through a multiphysics approach considering the residual stress of the  $\text{SiN}_x$  cladding film. Furthermore, general physical features have been investigated by means of a comparative analysis of SHG performance as a function of input pump power, linear and nonlinear phase mismatching, effective recombination carrier lifetime, and temperature, taking into account both continuous and pulsed regimes. Finally, periodically poled germanium devices have been explored with the aim to improve the SHG efficiency. In the same operative conditions, efficiencies of 0.6% and 0.0018% have been obtained in poled and not-poled waveguides, respectively.

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**OCIS codes:** (190.5890) Scattering, stimulated; (130.3060) Infrared; (130.4310) Nonlinear.

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## 1. Introduction

Second-order optical nonlinearity represents a foundational component of nonlinear photonics. Indeed, when a crystal possesses a strong second-order susceptibility ( $\chi^{(2)}$ ), it can produce a number of applications ranging from optical modulation (Pockels effect) to wavelength generation. In the latter case, the crystal generates shorter wavelengths by second-harmonic generation (SHG) or longer wavelengths by spontaneous parametric down-conversion of a single pump beam. Such a crystal can also nonlinearly mix two different beams, producing other wavelengths by a sum-frequency or difference-frequency generation process. Closely related to the wavelength generation is the Pockels effect. Both effects are caused by the same nonlinear coefficient, while the main difference is the frequency of the applied field. Lithium niobate is the most commonly used material for optical modulation due to its high second-order nonlinear susceptibility of 360 pm/V [1] and fast response time. However, lithium niobate modulators suffer from several considerable limitations, including high substrate cost, and difficulty of integration with the silicon-on-insulator (SOI) material platform through a CMOS-compatible fabrication process. Therefore, it should be desirable to create comparable  $\chi^{(2)}$  in silicon-based group IV photonic circuits. Although centrosymmetric crystals such as silicon and germanium lack second-order optical nonlinearity, the  $\chi^{(2)}$  could be induced by breaking the symmetry via induced mechanical stress [2–4]. In recent years, strain engineering is emerging as a new frontier in micro- and nano-technology in order to turn on physical and chemical properties absent in the unstrained material. For example, tensile strained germanium-on-silicon can be used as active material for short wavelength infrared light and it could represent an efficient solution for manufacturing monolithic lasers and optical amplifiers [5, 6]. On the other hand, the Pockels effect has been experimentally measured in strained silicon [7], making it a promising candidate for realizing optical modulators and switches. In this context, the measured value of 190 pm/V in a 300 nm-large rib waveguide has been achieved [8]. More recently, a theoretical and experimental validation for the existence of multiple nonzero components of the second-order nonlinear susceptibility tensor,  $\chi^{(2)}$ , generated via strain, has been reported [9]. In that work, the authors characterize the second order nonlinearity performing Pockels effect measurements on a Fabry–Perot waveguide resonator realized through a 250 nm-thick SOI wafer clad with 180 nm of compressively stressed (−1.275 GPa) silicon nitride ( $\text{Si}_3\text{N}_4$ ). At the same time, a number of works have been proposed in the literature with the aim to investigate the wavelength generation via  $\chi^{(2)}$  in silicon waveguide. For applications such as sum frequency generation (SFG) or second harmonic generation (SHG), a theoretical analysis of strained-silicon waveguides to provide perfect phase matching has been presented [10]. Moreover, second-harmonic-generation experiments and first-principle calculations have been carried out at optical wavelengths in a silicon waveguide by using a stressing silicon nitride overlayer. Consequently, a value of  $\chi^{(2)} = 40$  pm/V has been measured at 2300 nm [11]. Further, in-plane second and third harmonic generation with the frequency comb generation at visible wavelengths have been achieved experimentally in  $\text{Si}_3\text{N}_4$  ring resonators [12,13]. Recently, a new class of photonic devices based on periodic stress fields in silicon that enable second-order nonlinearity as well as quasi-phase matching, has been reported [14]. The proposed periodically poled silicon, where the second-order susceptibility periodically

changes from 6 to  $-12$  pm/V at  $1.3\ \mu\text{m}$ , has been used as an efficient device to generate different wavelengths.

It is generally recognized that group IV photonics leads to the integration of multiple optical functionalities with microelectronic devices. In this sense and on the basis of the third-order nonlinearities of silicon, functions such as amplification and lasing, wavelength conversion and optical processing have all been demonstrated in recent years. In parallel, germanium (Ge) has been seen for more than a decade as a very promising material for extending the operation of integrated photonic circuits from the conventional near-infrared (NIR) wavelength range to the vibrant mid-infrared (MIR) spectral window [15,16]. In this context, research efforts have been carried out in order to experimentally demonstrate fundamental photonic devices based on Ge technology (e.g., germanium-on-silicon (GOS) and germanium-on-SOI) and operating in the MIR [17,18]. Furthermore, nonlinear optical properties such as stimulated Brillouin scattering (SBS) [19], stimulated Raman scattering (SRS) [20], supercontinuum generation [21], and third harmonic generation [22] have been theoretically investigated in GOS waveguides, revealing competitive advantages with respect to the operation of Si- and Ge-based devices in the NIR wavelength range. However, third-order nonlinearities could require relatively high optical powers. In this sense, the second-order term of the nonlinear susceptibility tensor in the GOS platform could be exploited in order to investigate new potential for wavelength generation. Despite of a number of theoretical and experimental works based on silicon waveguides, investigations have not yet been proposed in literature in which the  $\chi^{(2)}$  susceptibility is induced in a GOS waveguide via the strain effect.

In this paper we present, for the first time to the best of our knowledge, a theoretical investigation of SHG in waveguides based on strained Ge material operating in the MIR. The paper is organized as follows. A detailed description of the mathematical modelling is reported in Section 2. The theoretical assumptions, the strain and  $\chi^{(2)}$  calculations in GOS waveguides are reported in Section 3.1, while Section 3.2 provides the theoretical performance of the SHG effect in the continuous or pulsed regime. In particular, theoretical investigations of the conversion efficiency are presented as a function of carrier recombination lifetime, input pump power and temperature. The possibility of inducing the poled effect in the GOS waveguide is also investigated in order to improve the SHG performances. Finally, Section 4 summarizes the conclusions.

## 2. Theoretical background

In our recent work, we have theoretically demonstrated the potential of Ge as a very efficient candidate to realize dielectric technology platforms for inducing important nonlinear effects such as Raman lasing [20], and super-continuum generation [21] in the MIR.

In this work, we investigate the possibility of inducing the SHG in strained GOS waveguides. Actually, the Ge centro-symmetry is broken by means of the stress effect produced by a deposited  $\text{SiN}_x$  cladding film, as sketched in Fig. 1. Thus, the induced  $\chi^{(2)}$  susceptibility can be used to realize the second order parametric process in which two pump photons generate a single photon at the second-harmonic (SH) frequency. In this context, the equations describing the power transfer among the pump ( $p$ ) and the SH waves can be written as Eqs. (1)-(2), where  $a_p$  and  $a_{SH}$  represent the slowly varying field amplitudes (time, and  $z$  functions) for the pump and SH waves. The coefficients  $\beta_{m,i}$  and  $\alpha_i$  ( $i = p, SH$ ) represent the  $m^{\text{th}}$ -order dispersion coefficient and the overall linear loss respectively,  $\beta_{i,j}^{\text{TPA}}$  are the TPA coefficients,  $\alpha_i^{\text{FCA}}$  and  $\Delta n_i$  represent the free-carrier absorption (FCA) and plasma dispersion effect as induced by the TPA process, respectively. The term  $\Delta k = k_{SH} - 2k_p$  is the waveguide

phase mismatch parameter between the pump and second harmonic waves, with  $k_{SH}$  and  $k_p$  the SH and pump wave-vectors, respectively. Moreover, the coefficients  $\gamma_i = n_2(\omega_i) \cdot \omega_i / c$  take into account the self-phase modulation (SPM), cross-phase modulation (XPM) effects as induced by Kerr nonlinearity, where  $\omega_i$  is the angular frequency of the  $i^{th}$  beam inside the waveguide structure and  $n_2$  is the nonlinear Kerr refractive index, evaluated as in [23].

$$\begin{aligned} \frac{\partial a_p}{\partial z} = & \sum_{m=1}^{\infty} \frac{j^{(m+1)}}{m!} \beta_{m,p} \frac{\partial^m a_p}{\partial t^m} - \frac{1}{2} (\alpha_p + \alpha_p^{FCA}) a_p - \frac{1}{2} \frac{\beta_{p,p}^{TPA}}{A_{p,p}^{TPA}} |a_p|^2 a_p - \frac{\beta_{p,SH}^{TPA}}{A_{p,SH}^{TPA}} |a_{SH}|^2 a_p + j \frac{2\pi}{\lambda_p} \Delta n_p a_p \\ & + j \left( \frac{\gamma_p}{A_{p,p}^{Kerr}} |a_p|^2 + 2 \frac{\gamma_p}{A_{p,SH}^{Kerr}} |a_{SH}|^2 \right) a_p + j \cdot f_{p,p,SH} \frac{\omega_p \epsilon_0 c_0^{3/2} \mu_0^{3/2}}{\sqrt{2} \sqrt{n_{SH} n_p^2}} \chi_{eff}^{(2)} a_{SH} a_p^* e^{+j\Delta k z} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial a_{SH}}{\partial z} = & \sum_{m=1}^{\infty} \frac{j^{(m+1)}}{m!} \beta_{m,SH} \frac{\partial^m a_{SH}}{\partial t^m} - \frac{1}{2} (\alpha_{SH} + \alpha_{SH}^{FCA}) a_{SH} - \frac{1}{2} \frac{\beta_{SH,SH}^{TPA}}{A_{SH,SH}^{TPA}} |a_{SH}|^2 a_{SH} - \frac{\beta_{SH,p}^{TPA}}{A_{SH,p}^{TPA}} |a_p|^2 a_{SH} + j \frac{2\pi}{\lambda_{SH}} \Delta n_{SH} a_{SH} \\ & + j \left( \frac{\gamma_{SH}}{A_{SH,SH}^{Kerr}} |a_{SH}|^2 + 2 \frac{\gamma_{SH}}{A_{SH,p}^{Kerr}} |a_p|^2 \right) a_{SH} + j \cdot f_{p,p,SH} \frac{\omega_{SH} \epsilon_0 c_0^{3/2} \mu_0^{3/2}}{2\sqrt{2} \sqrt{n_{SH} n_p^2}} \chi_{eff}^{(2)} a_p a_p e^{-j\Delta k z} \end{aligned} \quad (2)$$

In Eqs. (1)-(2), the effective nonlinear modal areas ( $A_{i,j}^{Kerr} = A_{i,j}^{TPA}$ ) for the third-order nonlinearities are calculated according with the full vectorial coupled mode theory (CMT) [24]. The terms  $n_p$  and  $n_{SH}$ , are the Ge refractive indices at the pump and SH wavelengths, respectively. Moreover, the coefficient  $f_{p,p,SH}$  represents the overlap integral between the pump and SH electric fields, defined as in [25, 26]. It plays a fundamental role since it strongly influences the efficiency with which the SHG process manifests itself inside the optical GOS waveguides. However, in order to include both FCA and plasma dispersion effect correctly, equations system (1)-(2) must be coupled with the rate equation, i.e., Eq. (3), which governs the time dynamics of the free carrier density ( $N_c$ ):

$$\frac{dN_c}{dt} = -\frac{N_c}{\tau_{eff}} + \frac{1}{2} \left[ \frac{\beta_{p,p}^{TPA} |a_p|^4}{(A_{p,p}^{TPA})^2 \hbar \omega_p} + \frac{\beta_{SH,SH}^{TPA} |a_{SH}|^4}{(A_{SH,SH}^{TPA})^2 \hbar \omega_{SH}} + \frac{2 \cdot \beta_{p,SH}^{TPA} |a_{SH}|^2 |a_p|^2}{(A_{p,SH}^{TPA} \cdot A_{p,p}^{TPA}) \hbar \omega_p} + \frac{2 \cdot \beta_{SH,p}^{TPA} |a_{SH}|^2 |a_p|^2}{(A_{SH,p}^{TPA} \cdot A_{SH,SH}^{TPA}) \hbar \omega_{SH}} \right] \quad (3)$$

where  $\tau_{eff}$  is the recombination effective lifetime.

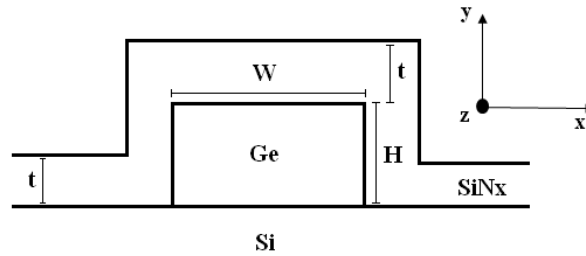


Fig. 1. Waveguide cross-section based on GOS technology platform with SiNx deposited film.

Finally,  $\chi_{eff}^{(2)}$  indicates the effective second order susceptibility responsible for the SHG process. The definition of this parameter will be better clarified in the following section, depending on the non-uniform strain distribution induced into the core waveguide. In this sense, we briefly describe the mathematical model used for waveguide strain calculations. Generally speaking, a thermal change ( $T - T_{ref}$ ) with respect to a reference temperature ( $T_{ref}$ )

or an initial stress distribution ( $\sigma_0$ ) stored in dielectric layers induce both strain and stress fields inside the structure, related as in Eq. (4):

$$\sigma = S(\epsilon - \alpha_{thermal}(T - T_{ref})) + \sigma_0 \quad (4)$$

where  $S$ ,  $\sigma$ , and  $\epsilon$  represent the stiffness, stress and strain tensors, respectively. The term  $\alpha_{thermal}$  is the linear thermal expansion coefficient. In addition, the stress induces changes in the material refractive index and, then, in the optical field distributions and effective refractive index. In turn, these variations can influence the values of waveguide phase mismatching parameter,  $f_{p,p,SH}$ , and nonlinear effective modal areas. Therefore, the stress-induced change in the refractive index is taken into account in our investigations by means of the relationship [27]:

$$\begin{cases} n_{x,i} - n_{0,i} = -\frac{n_{0,i}^3}{2E_{y,i}}(p_{11,i} - 2\nu_i p_{12,i})\sigma_{xx} - \frac{n_{0,i}^3}{2E_{y,i}}(-\nu_i p_{11,i} + (1 - \nu_i)p_{12,i})(\sigma_{yy} + \sigma_{zz}) \\ n_{y,i} - n_{0,i} = -\frac{n_{0,i}^3}{2E_{y,i}}(p_{11,i} - 2\nu_i p_{12,i})\sigma_{yy} - \frac{n_{0,i}^3}{2E_{y,i}}(-\nu_i p_{11,i} + (1 - \nu_i)p_{12,i})(\sigma_{xx} + \sigma_{zz}) \end{cases} \quad (5)$$

In Eq. (5),  $n_{0,i}$ ,  $E_{y,i}$ , and  $\nu_i$  are the refractive index of the material without stress, the Young's modulus and the Poisson's ratio relevant to the  $i^{th}$  layer (Si, Ge, SiN<sub>x</sub>), respectively. Moreover, the terms ( $p_{kl,i}$ ) are the elements of the photoelastic tensor, while  $\sigma_{jj}$  ( $j = x, y, z$ ) represent the stress components along the  $x, y$  and  $z$  reference axes.

### 3. Numerical results

#### 3.1 Stress-induced second-order susceptibility

The goal of this sub-section is to induce the second order susceptibility into the GOS waveguide. In this context, it is worth briefly describing the integrated procedure for the SHG calculations based on the model presented in the previous Section. With the aim of performing self-consistent simulations, we have implemented an integrated algorithmic procedure based on custom-made codes and commercial software using a full-vectorial finite element method (FEM) [28]. In particular, for a given operative temperature and initial stress ( $\sigma_0$ ), the stress field in the waveguide is numerically solved by FEM calculations. The static equilibrium equation is solved, satisfying at the same time the stress-strain relation, i.e., Eq. (4), the thermal effects and the strain-displacement relation for the displacement variables along  $x, y$ , and  $z$  directions and proper boundary conditions. Once the stress distribution in the vicinity of the ridge waveguide has been calculated, the local refractive index distribution can be found by Eq. (5). At this step, FEM approach is also used to solve Maxwell's equations and calculate the optical mode distributions and effective refractive index for both quasi-TE and quasi-TM polarizations at both pump and SH operative wavelengths (i.e.,  $\lambda_p$  and  $\lambda_{SH}$ ). It is worth outlining that the procedure is based on a multiphysics approach, i.e., the FEM electromagnetic module used in this step works together with the FEM stress module in order to take into account the stress effect on the material refractive index. Consequently, the electric-field distributions are used to calculate the nonlinear effective areas, the integral overlap, and the effective second-order susceptibility. Finally, the equations system (1)-(3) is solved by the custom-made codes implemented in order to evaluate the SH generation mechanism performance. Some comments about the evaluation of  $\chi^{(2)}$  are noteworthy. First, the breaking of the original crystal symmetry is needed to induce the second-order susceptibility in the Ge waveguide core. As a result, the non-uniform stress (i.e., strain)

distribution must be induced by the SiN<sub>x</sub> cladding. Indeed, a uniform strain distribution holds the crystal centrosymmetry. However, as outlined in [11], an explicit theory to relate the strain with  $\chi^{(2)}$  does not exist. In this context, *ab initio* models based on the time-dependent density functional theory have demonstrated that the displacement of atoms leads to induce second order nonlinear effects [11, 29, 30]. Despite the theoretical robustness of an *ab initio* model, it has the disadvantage of being computationally very time- and memory-consuming, without providing an explicit relationship between local strain and induced second order susceptibility. In this context, two heuristic approaches have been proposed in the literature. The first relates the second-order susceptibility with the strain gradient [9, 31], while the second approach expresses  $\chi^{(2)}$  as a function of the total strain.

Actually, a strain can be induced in the Ge core waveguide following a deposition process of SiN<sub>x</sub> on both the Ge ridge and silicon substrate, as shown in Fig. 1. In this context, the cladding deposition originates a change in the bow radii of the wafer before ( $R_1$ ) and after ( $R_2$ ) the film deposition, thus inducing a residual average stress in the deposited layer expressed by the Stoney's formula [32]:

$$\sigma_{film} = -\frac{1}{6R} \frac{E_{Y,sub} D^2}{(1-\nu_{sub})t} \quad (6)$$

In Eq. (6),  $E_{Y,sub}$ , and  $\nu_{sub}$  are the substrate Young modulus, and the Poisson ratio, respectively, while  $D$  and  $t$  indicate the substrate and film thickness, respectively. Finally,  $R$  represents the effective radius of curvature, i.e.,  $R_1 R_2 / (R_1 + R_2)$ , and is strongly influenced by the gaseous mixture, temperature and pressure of the deposition process. In this context, it is consistent to set  $\sigma_0 = \sigma_{film}$  in the uniform SiN<sub>x</sub> upper cladding film and far away from the waveguide ridge. This assumption leads to take into account correctly the effect of the residual stress of the cladding film on the germanium core waveguide.

**Table 1. Physical Parameters used in the Simulations.**

| Parameters                          | Values                                        |
|-------------------------------------|-----------------------------------------------|
| $E_{Y,Ge}$                          | 103 (GPa)                                     |
| $E_{Y,Si}$                          | 130 (GPa)                                     |
| $E_{Y,Si_3N_4}$                     | 160 (GPa)                                     |
| $\nu_{Ge}$                          | 0.26                                          |
| $\nu_{Si}$                          | 0.19                                          |
| $\nu_{Si_3N_4}$                     | 0.253                                         |
| $\alpha_{thermal,Ge}$ at 293 K      | $5.95 \times 10^{-6} \text{ (K}^{-1}\text{)}$ |
| $\alpha_{thermal,Si}$ at 293 K      | $2.5 \times 10^{-6} \text{ (K}^{-1}\text{)}$  |
| $\alpha_{thermal,Si_3N_4}$ at 293 K | $3.75 \times 10^{-6} \text{ (K}^{-1}\text{)}$ |

In Fig. 2, the strain component distribution inside the GOS waveguide is shown, assuming [110] waveguide orientation,  $T_{ref} = T = 300$  K,  $W = H = 2$   $\mu\text{m}$ , and  $t = 1$   $\mu\text{m}$ . As will be demonstrated in the following, the waveguide cross section considered meets very closely the phase matching condition. Moreover, the residual stress in the cladding film has been fixed to  $\sigma_{film} = 1$  GPa, according to the maximum values measured [32]. The parameters listed in Table 1 have been used to characterize the material used in our investigations. In particular,

$E_{Y,M}$  is the Young's modulus of the material M,  $\nu_M$  is the Poisson's ratio of the material M and  $\alpha_{thermal,M}$  is the thermal expansion coefficient of the material M as well.

It is worth outlining that the Young's modulus and Poisson's ratio are assumed independent of the waveguide orientation, while the values of the stiffness tensor elements used for stress-strain simulations are considered as influenced by the waveguide orientation [19].

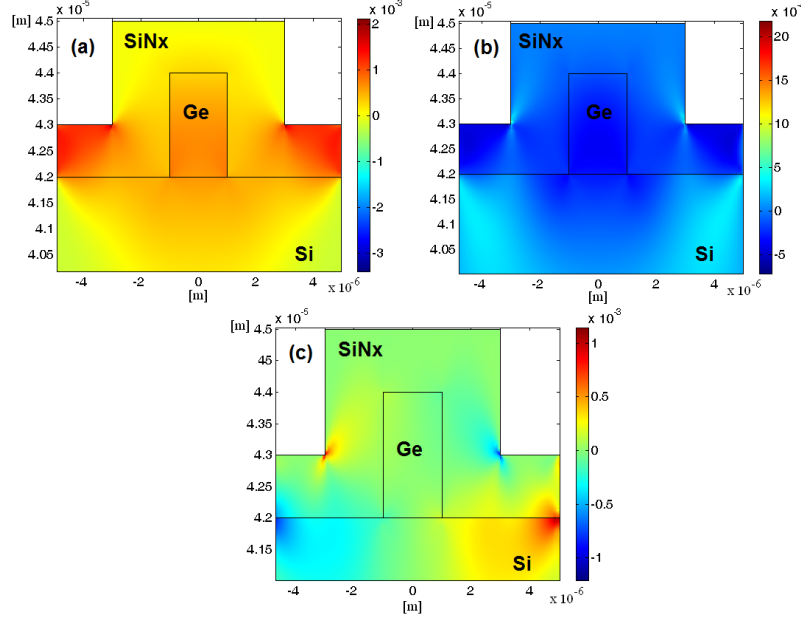


Fig. 2. In plane strain distribution: (a)  $\epsilon_{xx}$ ; (b)  $\epsilon_{yy}$ ; (c)  $\epsilon_{xy}$ .

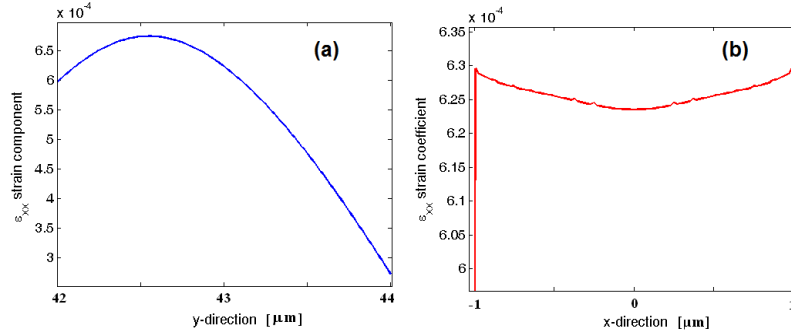


Fig. 3.  $\epsilon_{xx}$  strain component: (a)  $\epsilon_{xx}$  versus  $y$ -direction (at  $x = 0 \mu\text{m}$ , according to the plot of Fig. 2(a));  $\epsilon_{xx}$  versus  $x$ -direction (at  $y = 43 \mu\text{m}$ , according to the plot of Fig. 2(a)).

Two interesting features can be deduced by the colour map of Fig. 2 with the help of Figs. 3(a) and 3(b), where the  $\epsilon_{xx}$  component is plotted as a function of  $y$  (at  $x = 0 \mu\text{m}$ ) and  $x$  (at  $y = 43 \mu\text{m}$ ) coordinate directions, respectively. First, the strain distribution of the dominant component ( $\epsilon_{xx}$ ) inside the Ge core is not uniform, ensuring the possibility of inducing the second-order susceptibility. Moreover, the in plane distribution of  $\epsilon_{xx}$  assumes an even profile along the  $x$ -direction as in Fig. 3(b), inducing a zero-average value for the gradient component  $\partial\epsilon_{xx}/\partial x$ . In turn, this feature can be explained by considering that the SiN<sub>x</sub>

cladding is symmetrically deposited with respect to the waveguide vertical axes of symmetry. On the contrary, due to the presence of different materials on the bottom (Si-substrate) and on the top (SiN<sub>x</sub> film), the average gradient component  $\partial\epsilon_{xx}/\partial y$  is not null inside the core waveguide, see Fig. 3(a). After estimating the  $\chi^{(2)}$  tensor elements as,  $\chi_{xy}^{(2)} = C_{xy} \partial\epsilon_{xx}/\partial y$ ,  $\chi_{yy}^{(2)} = C_{yy} \partial\epsilon_{xx}/\partial y$ , and  $\chi_{xx}^{(2)} = C_{xx} \partial\epsilon_{xx}/\partial x$ , we can conclude that the GOS waveguide sketched in Fig. 1 could enable the SHG process as  $p_{TE} \rightarrow SH_{TM}$ , and  $p_{TM} \rightarrow SH_{TM}$ , while forbidding the nonlinear coupling  $p_{TE} \rightarrow SH_{TE}$ . Although the analysis based on the strain gradient leads us to qualitatively evaluate the non-null elements of the  $\chi^{(2)}$  tensor, it has the drawback of calculating  $\chi_{yy}^{(2)}$  and  $\chi_{xy}^{(2)}$  through unknown scaling coefficients ( $C_{xy}, C_{yy}$ ), determined on the basis of experimental data [9,31]. As a result, this approach is not suitable for the theoretical demonstration of the SHG process in new technology platforms based on centro-symmetric material such as GOS. As well outlined in [32] and [33], all the experimental measurements performed in silicon waveguides are in good agreement, while the theoretical results of  $\chi^{(2)}$  based on the strain gradient are consistent with each other but underestimate the measurements by three or four orders of magnitude. However, as interestingly evidenced in [32], a simple formula of the effective second-order susceptibility  $\chi_{eff}^{(2)}$ , i.e., Eq. (7), derived from a heuristic model based on the classical anharmonic oscillator in which the total strain in Eq. (8) is used instead of the gradient strain, produces values qualitatively in agreement with the measurements of [7]. Although this formula should not be interpreted rigorously, it represents a useful tool to avoid the disagreement between the experimental measurements and the theoretical results based on the strain gradient calculations. However, a full rigorous calculation of the nonlinearity induced in germanium by the applied stress still represents an open issue, but out of the scope for our work. In this context, due to the lack of experimental measurements of  $\chi^{(2)}$  in the GOS platform and with the aim of performing a self-consistent theoretical investigation, we estimate the second-order susceptibility using the heuristic approach as in [14, 32].

$$\chi_{eff}^{(2)} = \frac{4q^3}{m_0 \epsilon_0 \omega_p^4 a^4} S \quad (7)$$

$$S = \frac{\int \hat{\epsilon}(x, y) |\mathbf{E}(x, y)|^2 dx dy}{\int |\mathbf{E}(x, y)|^2 dx dy} \quad (8)$$

In Eq. (7),  $q$  is the electron charge,  $m_0$  is the electron mass, and  $a$  is the lattice constant. In Eq. (8),  $\mathbf{E}(x, y)$  is the electric field distribution of the GOS waveguide optical mode, and  $\hat{\epsilon}(x, y)$  is the spatial distribution of the strain tensor dominant component. It is worth outlining that the evaluation of Eq. (8) requires the multiphysics approach as mentioned before, in which the typical computation spatial domain needed for an accurate stress calculation is much larger (i.e., more than 70 times) than that required for the electromagnetic simulation. In this context, the effective second-order susceptibility  $\chi_{eff}^{(2)}$  can be evaluated by avoiding the use of any empirical coefficient.

Figure 4 shows  $\chi_{eff}^{(2)}$  as a function of the deposited film thickness for different values of  $\sigma_{film}$ , assuming the fundamental quasi-TM mode propagating at  $\lambda_p = 4 \mu\text{m}$ .

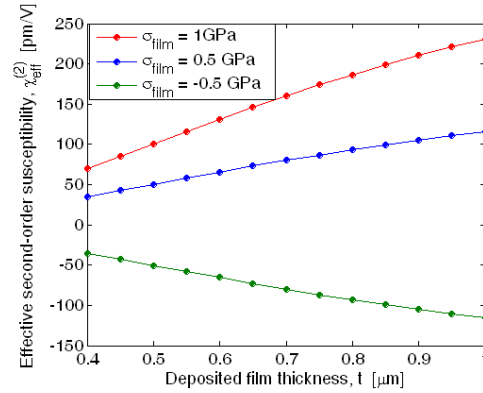


Fig. 4. Effective second-order susceptibility as a function of the deposited film thickness for different values of  $\sigma_{film}$ .

The plot indicates that the effective second-order susceptibility increases with the cladding thickness and residual stress stored in the  $\text{SiN}_x$  slab region after the deposition process. For a cladding thickness of 400 nm the simulated effective second order susceptibility assumes values less than 70 pm/V, depending on  $\sigma_{film}$ , and scales down with decreasing pump wavelength. Thus on the basis of the previous considerations and using the silicon results as indirect measurements [11,14], we can conclude that the values of  $\chi_{eff}^{(2)}$  obtained are consistent. Moreover, the curves in Fig. 4 suggest the possibility of tuning the sign of  $\chi_{eff}^{(2)}$  from positive to negative through setting a tensile or compressive value for  $\sigma_{film}$ . This is an essential property in designing periodically poled GOS devices for nonlinear optical purposes, such as those proposed in [14].

Table 2. Residual stress in  $\text{SiN}_x$  film deposited by LPCVD (40.5 Pa typically) [34].

| Deposition conditions:<br>[Temperature];[ $\text{NH}_3/\text{SiH}_4$ ratio] | Residual stress |
|-----------------------------------------------------------------------------|-----------------|
| [750 °C]; [0.4]                                                             | −200 (Mpa)      |
| [725 °C]; [0.3]                                                             | −500 (MPa)      |
| [775 °C]; [0.8]                                                             | 500 (MPa)       |
| [725 °C]; [0.8]                                                             | 700 (MPa)       |
| [725 °C]; [1.6]                                                             | 1000 (MPa)      |

In this context, we believe that the choice of  $\text{SiN}_x$  deposition over both the Ge core and Si substrate instead of on the exposed top surface of the Ge strip core could give more flexibility in the control of  $\chi_{eff}^{(2)}$  through  $\sigma_{film}$ . Indeed, several experimental measurements have been proposed in literature, in which the residual stress in slab  $\text{SiN}_x$  deposited on a silicon substrate has been evaluated as a function of the deposition process. In this sense and according to [34], we summarize in Table 2 the different deposition conditions suitable to induce positive and negative  $\sigma_{film}$  values.

### 3.2. Second harmonic generation

The goal of this Section is to elucidate the waveguide design rules required to induce the SH generation process in the GOS waveguides covered by  $\text{SiN}_x$  thin film.

Nowadays, increasing interest has been demonstrated in optical communications links around 2  $\mu\text{m}$  [35,36]. The reasons for this are mainly related to the possibility of using new spectral bands where fast and high-capacity photonic technologies can operate to satisfy the increasing and urgent demand of massive data volume processing in the globe. Similarly,

research efforts have been recently devoted to the development of new group-IV technologies for extending the operation of photonic circuits from the NIR to the MIR. In fact, Ge is seen as a promising material because of its compatibility with silicon and, in general, with standard CMOS technology processes, the very wide transparency (i.e., up to 16  $\mu\text{m}$ ), and the use of Ge-based devices as modulators, filters and routers in the MIR [37–40]. Further, the 2–4  $\mu\text{m}$  wavelength range can be exploited for the photonic sensing of chemical and biochemical species (e.g.,  $\text{CH}_4$ ,  $\text{CH}_2\text{O}$ ,  $\text{SO}_2$ ,  $\text{CO}_2$ ) characterized by strong absorption lines at the wavelengths said above. Consequently, we believe that the investigation of the SH generation process in GOS waveguides can be useful for future integration of multi-wavelength operating systems on chip that can allow signal processing around 2  $\mu\text{m}$  and sensing functionalities in the 2–4  $\mu\text{m}$  range to be performed simultaneously. Finally, it is worth specifying that tunable, commercial quantum cascade lasers emitting in the 4 to 8  $\mu\text{m}$  range are available and can be used for SH generation experiments.

In this Section we show the possibility of generating such signals by the SHG process with a pump at the wavelength of 4  $\mu\text{m}$ . Before proceeding to theoretical investigations, we would like to provide some working assumptions. In particular, we assume that the dominant element of the  $\chi^{(2)}$  tensor is  $\chi_{yyy}^{(2)}$  as typically takes place in SOI waveguide [9]. Thus, we consider Eq. (7) as representing the effective values of  $\chi_{yyy}^{(2)}$  ( $\chi_{eff,yyy}^{(2)}$ ). At the same time, the coefficient  $\chi_{eff,xyy}^{(2)}$  could be considered by introducing a parametric coefficient  $\delta = \chi_{eff,xyy}^{(2)} / \chi_{eff,yyy}^{(2)}$ .

Generally speaking, an efficient SHG process can be achieved in integrated waveguides if the phase-matching condition is satisfied. In particular, if  $\chi_{eff,yyy}^{(2)}$  is involved in the process, the second harmonic signal aligned as a quasi-TM mode is generated by a pump wave polarized as the fundamental quasi-TM mode. In this context, the phase matching condition requires that the effective refractive index of the propagating fundamental mode at the pump wavelength is equal to that of a high-order mode at the SH frequency. On the contrary, if  $\chi_{eff,xyy}^{(2)}$  is considered, the phase matching condition could be investigated considering only fundamental modes for the input quasi-TE pump wave and quasi-TM SH signal. Indeed, the nonlinear coupling between two different polarizations could induce a SHG improvement with respect to  $p_{TM} \rightarrow SH_{TM}$  excitation only if the fundamental modes are involved in the process. To this aim, we have performed a number of FEM simulations considering the GOS cross-section as sketched in Fig. 1, and assuming the waveguide height  $H = 2$   $\mu\text{m}$ , the cladding thickness  $t = 1$   $\mu\text{m}$ ,  $T_{ref} = T = 300$  K, and  $\sigma_{film} = 1$  GPa. In particular, we have varied the waveguide width,  $W$ , as a design parameter in order to achieve the phase matching condition between the pump fundamental quasi-TM mode ( $TM_{0,0}$ ) and the high-order SH quasi-TM modes ( $TM_{m,n}$ ). Furthermore, Sellmeier's equations for Ge, Si, and  $\text{SiN}_x$  bulk layers [41] have been included in our calculations in order to better estimate the phase matching condition as a function of chromatic dispersion. Thus, our investigations indicate that the phase matching condition can be obtained in the case of  $TM_{0,2}$  and  $TM_{2,0}$  SH modes, with waveguide widths,  $W = 1.881$   $\mu\text{m}$ , and 2.004  $\mu\text{m}$ , respectively. Similarly, the phase mismatching parameter as a function of the waveguide width has been investigated, assuming the pump and the SH waves polarized as the fundamental quasi-TE ( $TE_{0,0}$ ), and quasi-TM modes, respectively. The simulations reveal that the phase mismatching parameter  $\Delta k$  changes between  $8.5 \times 10^5 \text{ m}^{-1}$  and  $4.6 \times 10^5 \text{ m}^{-1}$  for  $W$  ranging from 1.6  $\mu\text{m}$  to 5  $\mu\text{m}$ , respectively. As a result, the potential advantage of high values of the integral overlap  $f_{p,p,SH}$  obtained by considering fundamental optical mode distributions is strongly compromised due

to high values of  $\Delta k$ . Thus, hereinafter only the quasi-TM polarized optical modes are taken into account in order to perform theoretical investigations for the SHG process.

With the aim of revealing the fundamental features of SHG process, the SH output power ( $P_{out}^{SH}$ ) has been calculated by solving Eqs. (1) and (2) in the continuous wave (CW) regime assuming as independent variable the phase mismatching parameter. Moreover, a reference straight waveguide with  $W = H = 2 \mu\text{m}$  has been selected in order to evaluate Eq. (7),  $f_{p,p,SH}$ , and  $A_{i,j}^{Kerr}$  by means of the full-vectorial FEM approach. The pump and SH waves are aligned as  $TM_{0,0}$ , and  $TM_{2,0}$  modes, respectively. Numerical results are plotted in Fig. 5 where the output SH power is plotted as a function of the propagation length and the waveguide phase mismatching parameter ( $\Delta k$ ), assuming the cladding thickness  $t = 1 \mu\text{m}$ ,  $T_{ref} = T = 300 \text{ K}$ , and  $\sigma_{film} = 1 \text{ GPa}$ . It is worth specifying that the coefficients  $\alpha_i$  ( $i = p, SH$ ), as detailed in [36], have been taken into account in simulations, also assuming the bulk material loss coefficient of  $1.5 \text{ dB/cm}$  [20].

Figure 5(a) shows the SHG effect in the ideal condition in which the third order nonlinearities are neglected, and considering an input pump power ( $P_{in}$ ) of  $500 \text{ mW}$ . The colour map of Fig. 5(a) indicates that the maximum SH output power of about  $25 \text{ mW}$  is achieved for propagation length larger than  $9.8 \text{ mm}$ , when the condition  $\Delta k = 0$  is guaranteed. Thus an efficiency conversion  $\eta = P_{out}^{SH} / P_{in}$  of  $5\%$  (i.e.,  $-13.01 \text{ dB}$ ) can be achieved. It is worth outlining that the assumption of neglecting all third order nonlinear effects leads to an evaluation of the theoretical efficiency limit for the SHG process in the proposed GOS platform. Similarly, Figs. 5(b)-5(d) show the output SH power as a function of the propagation length and  $\Delta k$ , when TPA, FCA, SPM, XPM, and plasma dispersion effects are all included and the input power is  $100 \text{ mW}$ ,  $500 \text{ mW}$  and  $1 \text{ W}$ , respectively. It is worth outlining that, although the pump wave does not suffer from any TPA effect (above the cut-off), the SH wave generated at  $2 \mu\text{m}$  is affected by a relative large value of the TPA coefficient. In particular, we have estimated the values  $\beta_{SH,SH}^{TPA} = 10^{-8} \text{ m/W}$  and  $\beta_{p,SH}^{TPA} = \beta_{SH,p}^{TPA} = 7.92 \times 10^{-9} \text{ m/W}$ , according to [23]. Moreover, generated TPA-induced free carrier density ( $N_c$ ) has been evaluated by Eq. (3) in the steady state regime assuming an effective e-h recombination lifetime  $\tau_{eff} = 20 \text{ ns}$ . Thus, according to [42], the FCA and plasma coefficients are given as:

$$\alpha_p^{FCA} = 1.66 \times 10^{-21} N_c^{1.012} + 1.25 \times 10^{-20} N_c^{0.989}$$

$$\alpha_{SH}^{FCA} = 6.3 \times 10^{-22} N_c^{1.007} + 1.03 \times 10^{-19} N_c^{0.935}$$

$$\Delta n_p = -3.43 \times 10^{-27} N_c^{1.024} - 4.64 \times 10^{-27} N_c^{1.007}$$

$$\Delta n_{SH} = -8.91 \times 10^{-28} N_c^{1.023} - 2.04 \times 10^{-21} N_c^{0.948}$$

Finally, the nonlinear modal areas calculated by the FEM approach are  $A_{SH,p}^{TPA} = 3.39 \mu\text{m}^2$  and  $A_{SH,SH}^{TPA} = 1.89 \mu\text{m}^2$ .

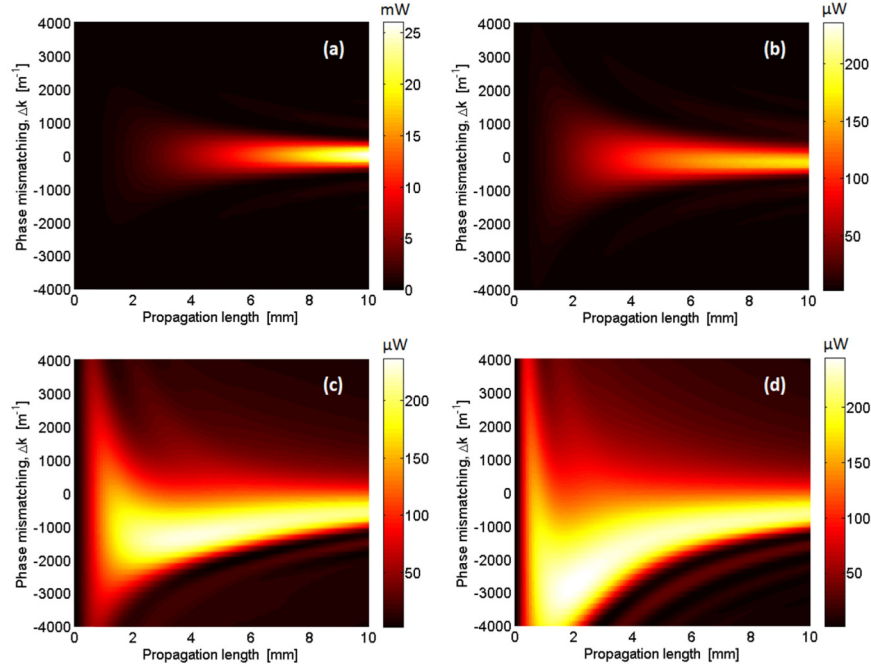


Fig. 5. Output SH power as a function of the propagation length and phase mismatching parameter  $\Delta k$ ; (a) without third-order nonlinearity,  $P_{in} = 500$  mW; (b) with third-order nonlinearity,  $P_{in} = 100$  mW; (c) with third-order nonlinearity,  $P_{in} = 500$  mW; (d) with third-order nonlinearity,  $P_{in} = 1$  W.

The plots of Fig. 5(b)-5(d) indicate clearly the detrimental effects induced mainly by TPA. Indeed, the maximum SH power generated changes weakly although the input pump power is strongly varied. Thus, a maximum conversion efficiency of about 0.18%, 0.044%, and 0.022% is obtained for  $P_{in}$  of 100 mW, 500 mW, and 1 W, respectively. This feature can be well explained by considering two opposite effects. An increase of the input pump power induces an initial increase of the SH power during the propagation. However, at the same time, the degenerate and non-degenerate TPA effects increase the overall SH propagation losses and the pump depletion. Furthermore, the free carrier density induced by TPA produces an increasing of FCA further limiting the parametric amplification induced by the input pump beam. Moreover, Figs. 5(b)-5(d) show that the SH peak power occurs for a negative value of  $\Delta k$  as a result of the additional nonlinear phase mismatching ( $\Delta k_{NL}$ ) induced by plasma, SPM and XPM effects. Our theoretical investigations demonstrate that the SHG reaches the highest efficiency when the net phase matching condition  $\kappa = 0$  is satisfied, where  $\kappa$  is given in Eq. (9):

$$\kappa = \Delta k + \Delta k_{NL} = \Delta k + \left[ 2P_{in} \left( \frac{\gamma_{SH}}{A_{SH,SH}^{Kerr}} - \frac{\gamma_p}{A_{p,p}^{Kerr}} \right) + \left( \frac{2\pi}{\lambda_{SH}} \Delta n_{SH} - \frac{4\pi}{\lambda_p} \Delta n_p \right) \right] \quad (9)$$

Numerical simulations indicate that the contribution of  $2P_{in}(\gamma_{SH}/A_{SH,SH}^{Kerr} - \gamma_p/A_{p,p}^{Kerr})$  is negligible with respect to the large positive term  $(2\pi\Delta n_{SH}/\lambda_{SH} - 4\pi\Delta n_p/\lambda_p)$ , confirming the results of Fig. 5(b)-5(d), where the maximum SH power is achieved for negative values of  $\Delta k$  for which the exact compensation of positive  $\Delta k_{NL}$  occurs.

According to Figs. 5(a)-5(d), the condition  $\kappa=0$  can be achieved by following two different approaches. The former consists of operating with a relatively low pump power level ( $< 100$  mW) in order to neglect the contribution of  $\Delta k_{NL}$  and then satisfying the relationship  $\Delta k=0$  (i.e., SH wave aligned as a  $TM_{2,0}$  mode,  $H=2$   $\mu\text{m}$ ,  $W=2.004$   $\mu\text{m}$ , and  $t=1$   $\mu\text{m}$ ). In the latter, we accept a negative value for  $\Delta k$ , which can be exactly compensated by adjusting the positive  $\Delta k_{NL}$  through the input pump power. Due to the fabrication tolerances, the rigorous control of the value  $W=2.004$   $\mu\text{m}$  could be a result too difficult to be achieved, then the second approach is adopted in the following analysis. Thus, a nominal waveguide cross section having  $H=2$   $\mu\text{m}$ , and  $W=2$   $\mu\text{m}$ , is considered, inducing a phase mismatching of about  $\Delta k=2.491 \times 10^3$   $\text{m}^{-1}$  as calculated by FEM simulations in the case of pump  $TM_{0,0}$  and SH  $TM_{2,0}$  modes. To better evidence the influence of the plasma dispersion effect on the SHG process, Fig. 6(a) shows the efficiency conversion as a function of the recombination effective lifetime  $\tau_{eff}$ , for different values of  $P_{in}$ , assuming a waveguide length  $L=1$  mm.

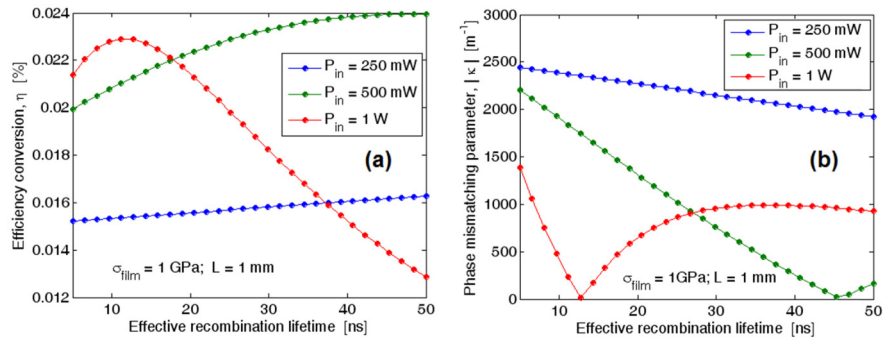


Fig. 6. (a) Efficiency conversion as a function of effective recombination lifetime for several value of input pump power; (b) Total phase mismatching parameter as a function of effective recombination lifetime for various values of input pump power.

The plot reveals that for a fixed value of  $P_{in}$ , a specific value of the effective recombination lifetime ( $\bar{\tau}_{eff}$ ) exists and is suitable for maximizing the efficiency conversion. In addition, the value of  $\bar{\tau}_{eff}$  decreases with increasing the input pump power. The presence of this peak can be explained by considering that the free carrier density decreases with  $\tau_{eff}$  in Eq. (3), inducing a FCA reduction and, then, a positive trend for the SHG process. On the other hand, lower values of  $\tau_{eff}$  can induce a plasma dispersion effect not strong enough to compensate the waveguide phase mismatching in Eq. (9). Thus, when  $\tau_{eff} < \bar{\tau}_{eff}$ , the SHG process acts with low nonlinear losses and large phase mismatching ( $\kappa \neq 0$ ). On the contrary, large nonlinear losses and phase mismatching occur when  $\tau_{eff} > \bar{\tau}_{eff}$ .

The maximum conversion efficiency occurs if the condition  $\tau_{eff} = \bar{\tau}_{eff}$  is satisfied, where  $\bar{\tau}_{eff}$  is the value of recombination lifetime for which the condition  $\kappa=0$  is achieved, as shown in Fig. 6(b). It is worth noting that, if  $\tau_{eff}$  is larger than 20 ns, the use of input pump power larger than 1 W will compromise the efficiency with respect to the case with  $P_{in} = 500$  mW. Thus, the combined TPA, FCA and plasma dispersion effects could inhibit the general trend of increasing the generated power as a function of the input pump power.

Hereafter, we assume that the effective recombination lifetime is around 20 ns. In this context, the good trade-off is obtained by assuming  $P_{in} = 800$  mW, and the waveguide length

$L = 2.5$  mm. Thus the previous results can be summarized as in the following. Although the pump and SH SPM effects can, in principle, induce nonlinear phase mismatch, their effects can be considered negligible for optical power up to 2 W. In this sense, the SHG process can be considered not influenced by third-order nonlinearities (achieving the maximum efficiency) only in the mid-IR wavelength range in which both degenerate and non-degenerate TPA effects are rigorously zero ( $\lambda_p \geq 7.2$   $\mu\text{m}$ ). Under this condition, the device design requires only satisfying the waveguide phase matching condition ( $\Delta k = 0$ ). Moreover, for a pump wavelength varying within 5.4  $\mu\text{m}$  (cut-off for indirect-gap TPA effect), the nonlinear pump depletion as induced by  $\beta_{p,p}^{TPA}$  can substantially inhibit the SHG into the germanium core. Moreover for pump wavelength changing in  $5.4 \leq \lambda_p < 7.2$   $\mu\text{m}$ , the conditions  $\beta_{p,p}^{TPA} = 0$ ,  $\beta_{p,SH}^{TPA} = 0$ , and  $\beta_{SH,SH}^{TPA} \neq 0$ , guarantee that the SHG process occurs with an efficiency growing with the pump wavelength as induced by the  $\beta_{SH,SH}^{TPA}$  reduction. Finally, for pump wavelength ranging in  $3.6 \leq \lambda_p < 5.4$   $\mu\text{m}$  (as the case under investigation), the non-degenerate TPA manifests its effect producing pump depletion and non-linear phase mismatching as induced by the plasma dispersion effects. In this context, the device design must proceed in order to satisfy Eq. (9). Thus, simulations and experiments would be useful in future to individuate a relationship between the GOS waveguide cross sections and recombination carrier lifetime in order to create a fitting function to relate the  $\kappa$  parameter to  $H$ ,  $W$ , and  $P_{in}$ .

Figures 7(a) and 7(b) show the efficiency conversion and the effective strain-induced second-order susceptibility ( $\chi_{eff,yyy}^{(2)}$ ) as a function of the temperature, respectively. Several values of  $\sigma_{film}$ , and  $t = 1$   $\mu\text{m}$  have been assumed in the simulations. The curves of Fig. 7(a) reveal that a minimum point exists for the temperature values corresponding to the transition from negative to positive values of  $\chi_{eff,yyy}^{(2)}$ . This feature can be explained by the fact that the thermal expansion induces tensile or compressive strain when  $T > T_{ref}$ , see Fig. 7(c), or  $T < T_{ref}$ , see Fig. 7(d), respectively. As a result, these contributions influence the tensile distribution induced by  $\sigma_{film}$ , producing the  $\chi_{eff,yyy}^{(2)}$  curves of Fig. 7(b). Then, Fig. 7(b) suggests the possibility of improving the SHG efficiency by tuning the second-order susceptibility by means of the temperature control. Indeed a slope  $d\chi_{eff,yyy}^{(2)}/dT$  of  $9.461 \times 10^{-4}\%$ /K has been achieved in the case of  $\sigma_{film} = 1$  GPa, and  $T > T_{ref}$ .

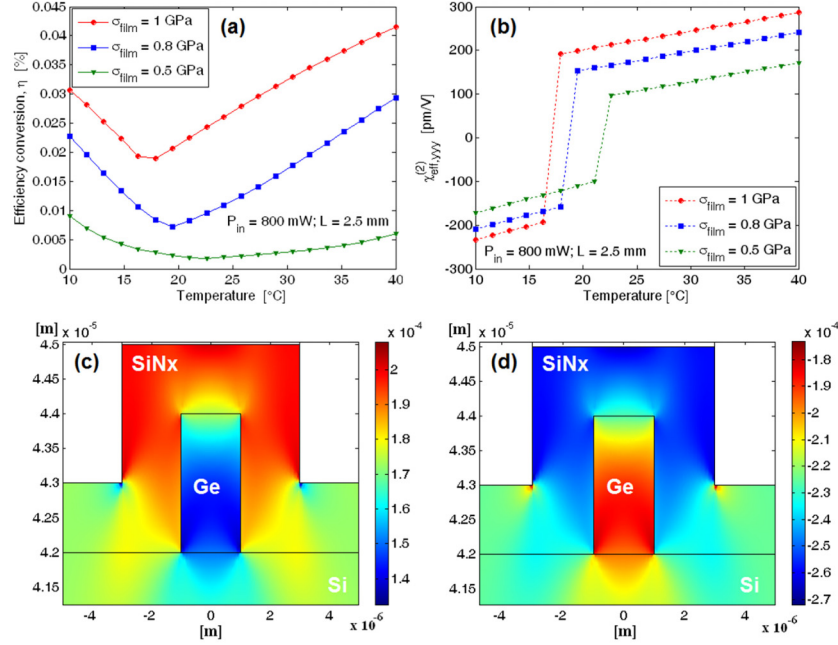


Fig. 7. (a) Efficiency conversion as a function of temperature; (b) Effective strain-induced second-order susceptibility as a function of temperature; (c)  $\epsilon_{xx}$  in-plane distribution for  $T = 313$  K,  $\sigma_{film} = 0$  GPa; (d)  $\epsilon_{xx}$  in-plane distribution for  $T = 383$  K,  $\sigma_{film} = 0$  GPa.

The previous results are rigorously valid for the CW regime. However, they can also be applied in the quasi-CW regime for pump pulse width  $t_0 > 0.1$  ns because the walk-off length  $L_w = t_0 / |\nu_{g,p}^{-1} - \nu_{g,SH}^{-1}|$  generally exceeds the waveguide length for such pulses. However, for ultrashort pulses of width below 100 ps,  $L_w$  becomes comparable with the typical waveguide length, compromising the SHG process. Indeed, the power transfer between the pump and SH waves is then limited by the group-velocity mismatch and can occur only over a distance of about  $L_w$  even if the waveguide length  $L > L_w$ . In this sense for very short pulses,  $L_w$  can assume a value too small if compared with the propagation length suitable to efficiently induce the SHG process.

The generalized model implemented allows us to simulate the SHG process in the pulsed regime. In this context, Fig. 8(a) shows the efficiency conversion as a function of the Gaussian pulse full width half maximum ( $t_{FWHM} = 1.665 t_0$ ) for two values of the peak pump pulse power, i.e.,  $P_0 = 600$  and 800 mW.

The simulation has been performed with  $t = 1 \mu\text{m}$ ,  $T_{ref} = T = 300$  K,  $\sigma_{film} = 1$  GPa, and  $L = 2.5$  mm. Moreover, according to FEM simulations, the group velocity, GVD and TOD coefficients are evaluated as  $\nu_{g,p} = 7.112 \times 10^7$  m/s,  $\nu_{g,SH} = 6.544 \times 10^7$  m/s,  $\beta_{2,p} = -0.339$  ps<sup>2</sup>/m,  $\beta_{2,SH} = 1.12$  ps<sup>2</sup>/m,  $\beta_{3,p} = 7.37 \times 10^{-3}$  ps<sup>3</sup>/m, and  $\beta_{3,SH} = 9.52 \times 10^{-3}$  ps<sup>3</sup>/m, respectively. The plot shows that it is possible to find a specific value of  $t_{FWHM}$  (named  $\bar{t}_{FWHM}$ ) for which the efficiency conversion is maximum. Moreover, the value of  $\bar{t}_{FWHM}$  shifts towards larger values with increasing the peak pump power  $P_0$ .

The colour map of Fig. 8(b) evidences the space-time evolution of the generated SH pulse, assuming a Gaussian pump pulse having  $P_0 = 800$  mW, and  $t_{FWHM} = \bar{t}_{FWHM} = 5$  ns. In particular, the plot reveals that the SH pulse reaches its maximum effect for a propagation length of about 2.766 mm, where the walk-off effect ( $L_w = 2.46$  mm) can be considered negligible and then the pump and SH pulses are overlapped. On the contrary, for propagation length larger than 3 mm the generated power decreases with the propagation length as a result of the temporal shift suffered by the SH pulse. It is worth outlining that both the pump and SH pulses do not suffer of the dispersive effects since the dispersive length  $L_D = t_0^2 / |\beta_{2,i}|$  is much larger than 10 mm considered in our simulations.

Finally, we investigate the possibility of realising the periodically poled GOS waveguides, in the same fashion proposed in [14]. According to the data summarized in Table 2, two different SiN<sub>x</sub> deposition steps could be performed in order to induce periodically tensile ( $\sigma_{film} = 500$  MPa), and compressive ( $\sigma_{film} = -500$  MPa) residual stress.

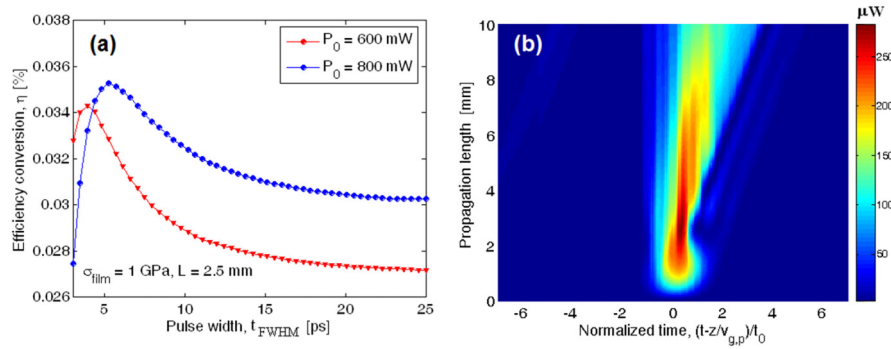


Fig. 8. (a) Efficiency conversion as a function of pulse FWHM. (b) Space-time pulse evolution,  $t_{FWHM} = 5$  ps.

Under this condition, the induced  $\chi_{eff,yyy}^{(2)}$  is estimated to have periodic values of  $\pm 115.48$  pm/V at  $\lambda_p$ . An intriguing property of the poled structure is the possibility of working far from the waveguide matching condition using the induced grating to achieve the quasi-phase-matching condition. As a result, the advantage of operating with low pump power and fundamental optical modes for both pump and SH waves could be exploited. In this sense, the processes  $p_{TM} \rightarrow SH_{TM}$ , and  $p_{TE} \rightarrow SH_{TM}$  have been analysed by assuming a pump power as low as 100 mW. Our simulations produce  $\Delta k = 6.75$  m<sup>-1</sup>, ( $6.99 \times 10^5$  m<sup>-1</sup>) and a poled period  $\Lambda = 9.307$   $\mu$ m (8.986  $\mu$ m) for the excitation  $p_{TM} \rightarrow SH_{TM}$  ( $p_{TE} \rightarrow SH_{TM}$ ) in order to satisfy the phase matching condition  $\Delta k' = \Delta k - 2\pi/\Lambda = 0$ . Under this condition, the solution of equations system (1)-(3) gives the curves of Fig. 9, where the efficiency conversion as a function of the propagation length is shown for both  $p_{TM} \rightarrow SH_{TM}$ , and  $p_{TE} \rightarrow SH_{TM}$  generations. Moreover, the parameter  $\delta = \chi_{eff,xyy}^{(2)} / \chi_{eff,yyy}^{(2)}$  has been introduced in order to compare the efficiency of the two processes considered. However, measurements performed in silicon waveguides [9] indicate that  $\chi_{eff,xyy}^{(2)} < \chi_{eff,yyy}^{(2)}$ . Therefore, supposing that the same trend holds for Ge, we can conclude that the process  $p_{TM} \rightarrow SH_{TM}$  is preferred for the periodically poled GOS waveguides as sketched in Fig. 1. The periodically poled GOS waveguide leads to obtaining an efficiency of about 0.6%, much larger than 0.0018% obtained in not-poled devices operating with the same input power and  $|\sigma_{film}|$ .

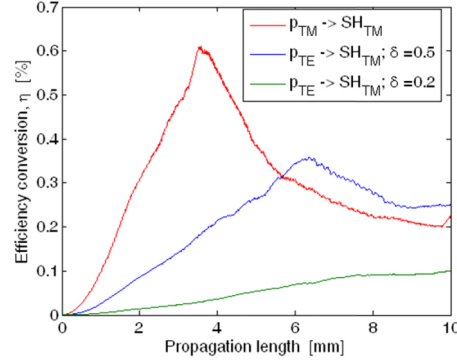


Fig. 9. Efficiency conversion versus the propagation length for both  $p_{TM} \rightarrow SH_{TM}$ , and  $p_{TE} \rightarrow SH_{TM}$ .

#### 4. Conclusions

In this paper, a detailed and rigorous engineering modeling based on a multiphysics approach has been implemented for investigating the feasibility and performance of the SHG process in the GOS technology platform. GOS waveguides with a  $\text{SiN}_x$  cladding thickness in the range of 0.4-1  $\mu\text{m}$  have been analyzed in order to induce non-uniform strain distribution and then the second-order susceptibility. The cladding film residual stress, as induced by the deposition process, has been considered as a design parameter. Thus, several waveguide cross sections with changing the waveguide widths  $W$  in the range of 1.6-5  $\mu\text{m}$  have been considered to attain the phase matching condition between pump and SH optical modes at wavelengths of 4  $\mu\text{m}$  and 2  $\mu\text{m}$ , respectively. The analysis of the efficiency conversion as a function of the input pump power and effective recombination lifetime has revealed that the TPA induced-plasma dispersion effect produces an additional nonlinear phase mismatching for input pump power larger than 100 mW. As a result, the design of a GOS waveguide with a low negative phase mismatching has been proposed in order to optimize SH generation. Therefore, the optimum selected waveguide structure is characterized by  $H=2$   $\mu\text{m}$ ,  $t=1$   $\mu\text{m}$  and  $W=2$   $\mu\text{m}$  inducing the SHG process for pump and SH waves polarized as  $TM_{0,0}$  and  $TM_{2,0}$ , respectively. The thermal effects on the SHG process have been taken into account for the estimation of the second-order susceptibility and efficiency conversion as a temperature function. Numerical results show the existence of a particular temperature value, depending on the  $\text{SiN}_x$  film residual stress, for which a minimum efficiency occurs. However, numerical simulations at room temperature reveal that a maximum efficiency of about 0.03% has been achieved in the case of a 2.5-mm long waveguide, assuming an input pump power of 800 mW, and linear loss coefficients of 1.5 dB/cm. Moreover, investigations in the pulsed regime have been performed showing the detrimental effects induced by the walk-off between the pump and SH waves. Finally, the possibility of realizing a periodically poled GOS waveguides has been explored, adjusting the tensile and compressive cladding residual stress by means of both deposition temperature and gaseous mixture. In this context, both  $p_{TM} \rightarrow SH_{TM}$ , and  $p_{TE} \rightarrow SH_{TM}$  excitations have been examined, considering fundamental optical modes for both waves involved in the process. Grating periods of 9.307  $\mu\text{m}$  and 8.986  $\mu\text{m}$  have been calculated in the case of  $p_{TM} \rightarrow SH_{TM}$  and  $p_{TE} \rightarrow SH_{TM}$  excitations, respectively. An efficiency of 0.6% has been achieved in the case of a 3.52-mm long waveguide, assuming an input pump power of 100 mW, and linear loss coefficients of 1.5 dB/cm. Moreover, we expect that the general second-plus-third-order modelling and physical analysis presented here can apply directly to SHG in GaAs and InP when the strain-induced  $\chi_{eff}^{(2)}$  is substituted by the material-induced

second-order susceptibility evaluated along with the proper crystal orientation for the waveguide core. In summary, we can conclude that the GOS technology platform, strained by means of the  $\text{SiN}_x$  cladding film, is suitable for the SHG process in the MIR wavelength range.