Experimental investigations of heat transport dynamics in a 1d porous medium column

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Abstract

The present study involves the experimental investigation of heat transport due to the forced convective flow through a thermally isolated porous medium column. The experiments regard the observation of thermal breakthrough curves obtained through a hot flow injection in correspondence of two thermocouples positioned along a thermally isolated column of porous medium. The experiment has been carried out for three flow rates in order to investigate the critical issues regarding heat transport phenomena such as the relationship between the thermal dispersion with the flow velocity and the validity of the local thermal equilibrium assumption between the fluid and solid phase.

Keywords: heat, thermal equilibrium, porous medium, Damköhler number

1. Introduction

In recent years, considerable interest has been given to the study of heat transfer through porous media because of its wide applications in many fields like crude oil extractions, petroleum reservoirs, agricultural engineering, coal
combustors, solar collectors, electronic cooling, energy storage units and nuclear waste repositories [1]. The study of heat transfer phenomena in the subsurface is also relevant for geothermal energy extraction.

<table>
<thead>
<tr>
<th>Nomenclature</th>
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Heat transfer dynamics in porous media are substantially different from solutes transport in that conduction is through both the matrix and the fluid and therefore conductive heat transport is more rapid than diffusive solute transport. On the contrary, the advective transport of heat (convection) is slower than advective solute transport since the heat capacity of the solid grains will retard the advance of the thermal front [2], [3].

Thermal dispersion is analogous to hydrodynamic dispersion and results from local velocity variations due to the mechanical interaction of the fluid with the porous medium structure [4]. But the hydrodynamic component of thermal dispersion is often neglected because thermal diffusion is more efficient than molecular diffusion by several orders of magnitude [4]. However, the literature contains conflicting descriptions of the thermal dispersivity coefficient [5]. An issue that has not been properly addressed experimentally is the quantification of thermal dispersivity as far as heat transport and its relationship with velocity.

Another issue to take into account is that the structure and porosity of the porous medium may affect the flow patterns and thermal transport phenomena in the porous channels [6].

Few authors have carried out laboratory experiments on heat transfer in porous media. Among those, the principal investigated issues have been the influence of non-linear flow regime, the relationship between the thermal dispersion with the flow velocity and the validity of the local thermal equilibrium assumption between the fluid and solid phase.
[6] investigated experimentally and theoretically the flow and heat transfer characteristics inside packed and fluidized beds. The purpose of their study was to investigate the heat transfer performance of the porous channels by using a modified version of the local thermal nonequilibrium model (LTNE) which neglected the effects of thermal dispersion in both fluid and solid. The results showed a highly non-Fourier behaviour which combined rapid thermal breakthrough with extremely long-tailing, that was attributed to disequilibrium between the fluid and the porous matrix. However, the adopted model was unable to fully capture the thermal breakthrough observed in some experimental runs.

[7] applied the continuous time random walk (CTRW) to three of the experiments carried out by Wu and Hwang (1998) over a range of different flow rates. CTRW is capable of quantifying both local equilibrium and nonequilibrium heat transfer in heterogeneous domains, and showed to successfully capture the observed nonequilibrium thermal breakthrough curves.

[5] carried out laboratory experiments on heat transfer in a specifically designed hydraulic tank containing well-sorted saturated sand. The experiments were aimed at analysing heat and solute transport behaviour separately, but under the same conditions, representative of naturally occurring groundwater flow systems.

They found that the thermal dispersion behavior for Darcy-related velocities in natural porous media did not exceed beyond a transition regime. The thermal dispersion can be approximated by a thermal dispersivity coefficient and a square dependency on the thermal front velocity. This result deviates from the linear description of thermal dispersion, which is assumed in analogy to solute transport. The difference can be explained with the different characteristics of heat and solute transport in porous media as expressed by the respective transport Peclet numbers. The results indicated that for relatively uniform coarse sand the thermal dispersivity term in the thermal dispersion equation can be neglected for Pe < 0.5.

This study is aimed at investigating the critical issues regarding heat transport phenomena in porous media by means of laboratory experiments. A physical model has been realised to analyse the forced convective flow and the related heat transport dynamics through a 1d porous medium column.

2. Theoretical background

2.1. Heat transport in one dimensional porous medium column

The behavior of heat transport in porous media is strongly dependent from the fluid velocity.

For high velocity flow, the interaction between the solid and fluid phase is rapid and then the solid and fluid phase cannot exchange sufficient amount of energy to establish local thermal equilibrium. At a given location the solid and fluid phases have different temperatures. In this situation each phase needs an energy equation for the description of heat transport. Assuming that porosity, densities and heat capacities are constant in time, energy equations can be written for the fluid and solid phase:

\[
\frac{\partial T_f}{\partial t} = -\nu \frac{\partial T_f}{\partial x} + \frac{\partial}{\partial x} \left( \frac{k_{eff}}{\rho_f c_f} \frac{\partial T_f}{\partial x} \right) + \frac{q_{fs}}{\rho_f c_f} \tag{1}
\]

\[
\frac{\rho_s c_s}{\rho_f c_f} \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial x} \left( \frac{k_s}{\rho_f c_f} \frac{\partial T_s}{\partial x} \right) - \frac{q_{fs}}{\rho_f c_f} \tag{2}
\]

The interaction between the two phases is represented by the sink/source terms \( q_{fs} \) given by following equation:

\[
q_{fs} = h^* s_f (T_s - T_f) \tag{3}
\]

The convective heat transfer coefficient can be expressed as:
\[ h^* = \left( \frac{d_p}{10k_x} + \frac{d_p}{\text{Nu} \left( \text{Pr}, \text{Re} \right) k_f} \right)^{-1} \]  

[8] redefined the Reynolds number to describe non-Darcy flow in porous media as:

\[ \text{Re} = \frac{\rho_f d_p v}{\mu} \frac{1}{1 - n} \]  

[9] suggest Re = 10 as a critical value for non-Darcy flow.

In low velocity flow regimes the solid and fluid phase are in contact for a sufficient period of time, and there exists the possibility for energy exchange locally and to establish a local thermal equilibrium. In such a case, only one energy equation is sufficient for the description of heat transport. The energy equation for the fluid and solid phase are combined into a single equation as:

\[ \left( 1 + \frac{1 - n}{n} \frac{\rho_s c_s}{\rho_f c_f} \right) \frac{\partial T_f}{\partial t} = \frac{\partial}{\partial x} \left( \frac{k_{\text{eff}}}{\rho_f c_f} \frac{\partial T_f}{\partial x} \right) - \nu \frac{\partial^2 T_f}{\partial x^2} \]  

Damköhler number \( Da \) can be used in order to evaluate the presence of local thermal equilibrium. \( Da \) relates the convection time scale to the exchange time scale between the two phases:

\[ Da = \frac{h^* s_f L}{\rho_f c_f v} \]  

When \( Da >> 1 \) the heat exchange between the two phases is rapid and there is instantaneous equilibrium between the two phases. On the contrary for \( Da << 1 \) the heat exchange velocity between the two phases is very low and it does not influence the heat propagation. When the convection time scale approaches the exchange time scale \( Da \approx 1 \), the impact of local thermal non equilibrium behavior of heat transport is stronger and the temperature distribution is characterized by a long tail.

3. Materials and methods

3.1. Experimental setup

The experiments have been performed on a laboratory physical model constituted by a thermal insulated plastic circular pipe with diameter of 0.11 m and height of 1.66 m filled with a porous medium with hydraulic and thermal parameters described in the Table 1.

Water inside the column flows from the bottom to the top according to the hydraulic head difference between the upstream tank connected to the inlet port positioned at the bottom and the outlet port positioned at the top. Water that enters into the column is heated by an electric water boiler with a volume of \( 10^{-3} \) m\(^3\). The instantaneous flow rate that flows across the block is measured by an ultrasonic velocimeter (DOP3000 by Signal Processing). Two thermocouples have been positioned at the center of the circular section of the pipe at the height of 0.25 m and 1.55 m respect to the inlet port. They have been connected to a TC – 08 Thermocouple Data Logger (pico Technology) and a sampling rate of 1 second has been used.
Table 1. Properties of porous medium.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
<tr>
<td>Porosity (-)</td>
<td>0.47</td>
</tr>
<tr>
<td>Average grain size (mm)</td>
<td>9.21</td>
</tr>
<tr>
<td>Average specific surface (m$^{-1}$)</td>
<td>337.90</td>
</tr>
<tr>
<td>Soil density (Kg⋅m$^{-3}$)</td>
<td>2210</td>
</tr>
<tr>
<td>Soil heat capacity (J⋅Kg$^{-1}$⋅K$^{-1}$)</td>
<td>840</td>
</tr>
<tr>
<td>Soil thermal conductivity (W⋅m$^{-1}$⋅K$^{-1}$)</td>
<td>2.15</td>
</tr>
</tbody>
</table>

3.2. Heat transport tests

Temperature tracer tests have been conducted through the following steps. First a hydraulic head difference between the upstream tank and the outlet port has been imposed. At time $t = 0$ s the cold water valve has been opened. At time $t = 60$ s the cold water valve has been closed and at the same time the hot water valve has been opened. In this manner the thermal breakthrough curves (BTCs) are measured by the thermocouples. The BTC measured by the first thermocouple located at the height of 0.25 from the inlet port is used as the injection temperature function $T_{inj}(t)$ whereas the BTC measured by the second thermocouple located at the height of 1.55 m from the inlet port is used as the observed temperature function $T_{obs}(t)$.

4. Results

4.1. Fitting thermal breakthrough curves and interpretation of estimated parameter models

Three tests have been conducted at different flow rates. As shown in the Table 2 for the investigated range of velocity the Damköhler number is much higher than the unit and then local thermal equilibrium model can be used to describe the behavior of heat transport.

Table 2. Specific discharge, fluid velocity, Reynolds number, heat transfer rate coefficient and Damköhler number at different flow rates.

<table>
<thead>
<tr>
<th>$Q$ (m$^3$s$^{-1}$)×10$^{-6}$</th>
<th>$q$ (m/s)×10$^{-3}$</th>
<th>$v$ (m/s)×10$^{-3}$</th>
<th>Re (-)</th>
<th>$\frac{h_{sf}}{\rho c_{f}}$ (s$^{-1}$)</th>
<th>Da (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.32</td>
<td>2.45</td>
<td>5.69</td>
<td>42.2</td>
<td>0.1847</td>
<td>23.56</td>
</tr>
<tr>
<td>1.60</td>
<td>1.68</td>
<td>3.61</td>
<td>29.0</td>
<td>0.1824</td>
<td>33.93</td>
</tr>
<tr>
<td>0.99</td>
<td>1.04</td>
<td>2.23</td>
<td>17.9</td>
<td>0.1774</td>
<td>53.30</td>
</tr>
</tbody>
</table>

Using the analogy with solute transport the Equation 6 can be rewritten as:

$$R \frac{\partial T_f}{\partial t} = \frac{\partial}{\partial x} \left( D_{eff} \frac{\partial T_f}{\partial x} \right) - v \frac{\partial T_f}{\partial x}$$  \hspace{1cm} (8)

Where:
\[
R = 1 + \frac{1-n}{n} \frac{\rho_s c_s}{\rho_f c_f}
\]  

(9)

\[
D_{\text{eff}} = D_0 + \alpha_L \nu
\]  

(10)

On the basis of the analytical solution for the instantaneous temperature injection of the Equation 8 the probability density function of the residence time (PDF) for the temperature in the one dimensional column of porous medium can be written as:

\[
PDF(x,t) = \frac{1}{\sqrt{\pi D_{\text{eff}} R^{-1} t}} \exp \left( \frac{x-v R^{-1} t}{4D_{\text{eff}} R^{-1} t} \right)
\]  

(11)

Using the convolution theorem the \(T_{\text{obs}}(t)\) can be related to \(T_{\text{inj}}(t)\) as:

\[
T_{\text{obs}}(t) = T_{\text{inj}}(t) * PDF(t)
\]  

(12)

The observed BTCs for different flow rates have been individually fitted using the Equation 10. Figure 1 shows the fitting results.

![Figure 1: Fitting BTCs at different specific discharge.](image)

Figure 2a shows the relationship between the transport parameter \(v/R\) and the convective velocity evaluated as \(v=q/n\). The estimated value of the retardation factor is \(R = 1.3772\) and it is close to the theoretical value that is equal to \(R = 1.5084\). Figure 2b shows the relationship between \(v\) and \(D_{\text{eff}}\). A linear relationship is evident between the thermal convective velocity and the effective thermal dispersion for the investigated range of velocity with a longitudinal dispersion coefficient equal to \(\alpha_L = 0.0121\) m. The estimated value of thermal diffusion is equal to \(D_0 = 9.74 \times 10^{-5}\) m\(^2\)/s and it is much higher than the theoretical value equal to \(D_0 = 2.82 \times 10^{-7}\) m\(^2\)/s.

5. Conclusions

Experimental investigations have been carried out to analyze the behavior of heat transport through a one dimensional porous medium column. For the investigated range of velocity the fluid and solid phases are in thermal equilibrium. \(Da\) is much higher than the unit, varying in the range between 23.5 – 53.3.

In a previous study that analyzed heat transfer dynamics in a fractured limestone block [10] the heat transfer velocity between the fluid in the fracture and the matrix was comparable with transport velocity and a non-local thermal equilibrium has been detected.
This puts into evidence that $s_f$ plays an important role on heat transport behavior. When $s_f$ reduces $h$ reduces consequently and then the heat transfer velocity between the fluid and solid phases could be comparable with the convective velocity giving rise to a strong local thermal non equilibrium effect.

Assuming valid the local thermal equilibrium model, the thermal BTCs have been fitted using the analytical solution of the 1d advection dispersion model. The estimated thermal convective velocity approaches the fluid convective velocity with an error in the range of 0.58% - 6.38% whereas regarding the effective thermal dispersion the results put into evidence a discrepancy between the estimated and theoretical values of the thermal diffusion coefficient.

The obtained results encourage further experimental work to increase the knowledge of the key parameters that govern heat propagation in porous media.

References